

Kinship, Incentives and Evolution*

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Abstract

We analyze how family ties affect incentives, with focus on the strategic interaction between two mutually altruistic siblings. The siblings exert effort to produce output under uncertainty, and they may transfer output to each other. With equally altruistic siblings, their equilibrium effort is non-monotonic in the common degree of altruism, and it depends on the harshness of the environment. We define a notion of local evolutionary stability of degrees of sibling altruism, and show that this degree is lower than the kinship-relatedness factor. Numerical simulations show how family ties vary with the environment, and how this affects economic outcomes.

Keywords: altruism, family ties, free-riding, empathy, Hamilton's rule, evolutionary stability.

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[B]etween the frozen pole of egoism and the tropical expanse of utilitarianism [there is] (...) the position of one for whom in a calm moment his neighbour's utility compared with his own neither counts for nothing, nor 'counts for one', but counts for a fraction. (F.Y. Edgeworth, *Mathematical Psychics*, 1881, Appendix IV)

As much as economists cherish the assumption that individuals are selfish, altruistic behavior, such as gift giving, material assistance, and cooperation in social-dilemma-like situations, is common. While such behavior may arise as an equilibrium outcome in an indefinitely repeated interaction between selfish individuals, many economists, including Edgeworth (1881) and Becker (1974), have theorized that altruism exists. Most people would probably also find, by introspection, that they are sometimes willing to help others, even with no prospect of future rewards. An extensive theoretical and empirical literature has developed to investigate how altruism affects economic outcomes and how altruistic behaviors are sustained.¹ In this paper, we shed new light on both questions, with a focus on family ties.

Numerous empirical studies show that transfers within families are common,² and that such transfers function as a risk-sharing device.³ There is also evidence that family ties vary in strength across cultures, as suggested by Alesina and Giuliano's (2007) study of the World Values Survey. Analyses of rates of cohabitation between parents and their adult children show that such cohabitation is an inferior good in the U.S. (Rosenzweig and Wolpin, 1993) and a normal good in Italy (Manacorda and Moretti, 2006). Motivated by these observations, we analyze theoretically the effects of family ties on risk sharing and incentives, and we suggest a way to endogenize the strength of family ties.

If family members with higher earnings give transfers to those with lower incomes, and are willing and expected to do so, what is the effect of such family ties on incentives to exert productive effort? What is the welfare effect, if any? We analyze these classical issues by allowing for mutual altruism and an endogenous risk-reducing effort, where the earlier

¹For a recent collection of surveys, see Kolm and Ythier (2006).

²See Cox and Fafchamps (2008), and Fafchamps and Lund (2003).

³Cox, Galasso and Jimenez (2006) show that the average income of donor households exceeds that of recipient households. Fafchamps and Lund (2003) find that output shocks affect transfers between Filipino rural households. Using data from Thailand, Miller and Paulson (1999) show that remittances respond to shocks to regional rainfall. See also Udry (1990) and Townsend (1994).

literature has focused on either one-sided altruism, or on mutual altruism, but without risk. Furthermore, we analyze whether the incentive effect of family ties depends on the environment.⁴ If so, can this help explain the different strengths of family ties in different parts of the world?

Our model is simple, but, we believe, canonical: two risk-averse siblings each choose a costly action, “effort,” that determines the probability distribution over output levels. Once both siblings’ outputs have been realized, each sibling chooses whether to share some of his or her output with the other. The motive for intrafamily transfers is represented by a positive weight placed on other family members’ welfare: a form of true altruism.

In the case of equally altruistic siblings, an increase in the common level of altruism leads to larger transfers, and thus a stronger free-rider effect on effort, but also to a stronger empathy effect on effort, by which we mean the desire to be able to help one’s sibling if need be. Which effect dominates? As expected, both effects are absent when the common degree of altruism is sufficiently low — then no transfers are given. We show that, at the margin, the free-rider effect outweighs the empathy effect when altruism is of intermediate strength, and that the opposite holds when altruism is strong.⁵ Thus, mutual altruism has a negative net effect on work effort at intermediate levels of altruism; equally altruistic individuals exert less effort than egoists. Despite the non-monotonicity of effort with respect to the common degree of altruism, siblings’ welfare is highest when they are fully altruistic towards each other; then they internalize fully the strategic externalities in their interaction.⁶

⁴In a companion sequel paper, Alger and Weibull (2008), we analyze these questions in a setting in which family transfers are socially coerced rather than, as here, voluntary, and there we also compare the outcomes with those in perfectly competitive insurance markets.

⁵Using census data for Slavonia from 1698, Kohler and Hammel (2001) established that the number of different crops grown by a nuclear family tended to increase as the grain resources available within the extended family network (relative to the household’s own land resources, and controlling for physical distance) increased. The authors were expecting the opposite effect, namely that as a result of an increase in the amount and proximity of resources available for risk pooling within the extended family, a household would invest *less* in risk-reducing planting strategies. Our results provide an explanation for this pattern: when a family expects to help out another family, the expected benefit of the risk-reducing planting strategy is increased. However, there is also evidence of the free-rider effect: Azam and Gubert (2005) find that recipients of remittances in Mali decrease their work effort in response to an increase in remittances.

⁶Relatedly, Hwang and Bowles (2009) find that the equilibrium contribution to a public good in a repeated game between mutually altruistic individuals is non-monotonic in the common degree of altruism. In their model, altruism may have a negative effect on the equilibrium contribution because it lowers the willingness

Although full altruism would lead to the (*ex ante* expected) Pareto-efficient outcome, full altruism is not what we observe in reality.⁷ What level of intrafamily altruism should one expect, from first principles? Here we follow in the footsteps of Darwin, who was puzzled by the occurrence of altruism in nature: how can a behavior or trait, whereby the individual gives up resources for the benefit of others, survive? Since then, biologists have developed theories of kinship altruism (Haldane, 1955, and Hamilton, 1964a,b), reciprocal altruism (Trivers, 1971), and multilevel selection (Sober and Wilson, 1998). Our approach is closest to that of Hamilton (1964a,b), who developed a model “which is particularly adapted to deal with interactions between relatives of the same generation” (op. cit., p.2), leading to the conclusion that evolutionary forces will lead to a degree of altruism of approximately one-half between siblings: “This means that for a hereditary tendency to perform an action of this kind [which is detrimental to individual fitness] to evolve, the benefit to a sib must average at least twice the loss to the individual.” (op. cit., p.16) The general version of the so-called *Hamilton’s rule* can be summarized as the prediction that such an action will be taken if and only if $rb > c$, where c is the reduction of the actor’s fitness, b is the increase in the recipient’s fitness, and r is *Wright’s coefficient of relationship*, a coefficient that is one-half between siblings (Wright, 1922).⁸

When postulating his rule, Hamilton did not consider strategic aspects of the interaction between kin. Ted Bergstrom (1995, 2003) enriched Hamilton’s kinship theory by allowing for precisely such aspects. Inspired by Bergstrom’s (1995, 2003) approach, we develop here a notion of *local evolutionary stability of altruism* and apply this to the effort/transfer sibling interaction mentioned above.⁹ In the light of Hamilton’s rule, one might conjecture the locally evolutionarily stable degree of altruism to equal one-half: the coefficient of relationship between the siblings. This would indeed be true, had effort levels been exogenously fixed. However, in our model the endogeneity of efforts pushes the evolutionarily stable degree

to punish bad past behavior.

⁷Cox, Hansen, and Jimenez (2004), and Maitra and Ray (2003) find fairly strong evidence that, for low-income households, transfers are driven by altruistic motives, although there is no evidence that such altruism would be anywhere near full altruism.

⁸For recent accounts of Hamilton’s rule, see Grafen (2006) and Rowthorn (2006).

⁹Our formal analysis does not require altruism to be genetically determined. It only requires siblings’ degrees of altruism to be positively correlated. The correlation may be due to genetics, the common upbringing, or both.

of altruism down to a level below one-half. To see why, imagine a population consisting of individuals with a certain degree of altruism, say one-half. Suppose that a very small population share “mutates” to a lower degree of altruism. If efforts are endogenous, then such a mutant will exert a lower effort — since the mutant has a weaker incentive to help his or her sibling. Therefore, individuals with the “incumbent” degree of altruism are more vulnerable to exploitation by less altruistic mutants than when efforts are exogenous. Had siblings’ degrees of altruism been statistically independent, then only selfishness would prevail. However, since, in our model, siblings’ degrees of altruism are positively correlated, a mutant’s sibling is quite often another mutant. Such a positive correlation increases the probability that the benefits of an individual’s altruism are bestowed on another altruist.

Numerical simulations show that, in general, a stable degree of sibling altruism depends on the physical, economic and/or institutional environment. In particular, evolutionary forces tend to select for lower degrees of altruism in harsher environments — situations where output variability is higher and/or the returns to effort are lower. *Ceteris paribus*, individuals then work harder, and their marginal cost of effort is higher. As a result, a relatively selfish mutant enjoys a larger gain from slacking off, the harsher the environment, and he or she also stands to lose less from doing so, because a sibling with the incumbent degree of altruism works harder than in a less harsh environment.

The remainder of the paper is organized as follows. In the next section we discuss related literature. In Section 2 we set up the model, beginning with the case of a selfish atomistic individual and then introducing family ties between siblings. In Section 3 we characterize equilibria and conduct comparative-static analyses of the equilibrium outcome. In Section 4 we focus on the special case of equally altruistic siblings, and in Section 5 we develop a notion of local evolutionary stability of family ties and apply this to our model. Section 6 is devoted to generalized evolutionary processes. In Section 7 we discuss extensions and methodological issues, and Section 8 concludes. Mathematical proofs can be found in the appendix.

1 Related literature

Our work is linked to several lines of research. First, our baseline model is related to the literature on altruistic transfers, first formalized by Becker (1974). Most of this literature assumes one-sided altruism (see, e.g., Becker, 1974, Bruce and Waldman, 1990, Coate, 1995,

Chami, 1998, Gatti, 2005, Lindbeck and Nyberg, 2006, and Fernandes, 2008): transfers flow from the altruistic agent to the non-altruistic one, and, as a result, only a free-rider effect of altruism can arise — the empathy effect is absent. In existing models with two-sided altruism, either only one agent is free to choose an effort (see Laferrère and Wolff, 2006, for a recent survey), or there is no risk (Lindbeck and Weibull, 1988, and Chen and Woolley, 2001). We are not aware of any model that allows for both the empathy and the free-rider effect of altruism in the presence of risk.

In the literature about mutual insurance between selfish individuals, Arnott and Stiglitz (1991) ask whether, in the presence of insurance markets, supplemental informal insurance within the family improves welfare. They model “family insurance” as transfers within pairs of *ex ante* identical individuals, who, like here, choose a risk-reducing effort. Family transfers are the outcome of a joint agreement. If family members can observe each other’s effort, the agreement specifies equal sharing of total family income and the effort to be taken. Mathematically, this is equivalent to the special case of full altruism in our model. Other researchers have provided insights into how beneficial risk sharing may be sustained as an equilibrium in a repeated game between selfish individuals (see, e.g., Coate and Ravallion, 1993, and Genicot and Ray, 2003), between altruistic individuals (Foster and Rosenzweig, 2001), or as an equilibrium in networks (Bramoullé and Kranton, 2007). With a focus on the sustainability of risk sharing, this line of research has set aside the thorny issue of endogenous incomes, an issue we analyze here within a simple one-shot interaction that allows for informal risk sharing.¹⁰

We are not aware of any work leading to the main prediction of our evolutionary analysis, namely, that the evolutionarily stable degree of altruism is lower than the degree of relationship, and that it depends on the environment. The closest study appears to be Bergstrom (1995), who develops a methodology to determine evolutionarily stable strategies in games played by siblings. Bergstrom further derives sufficient conditions for a population consisting of individuals who discount the fitness benefit bestowed on their siblings by one-half to resist an invasion by mutants with a different discount factor (degree of altruism). Also related is the work by Eshel and Shaked (2001), who model partnerships in which individuals, related or unrelated, may protect each other against hazards in order to increase the likelihood of having someone around to help them back in the future. Hence, there is a strategic element in the interaction. However, Eshel and Shaked presume Hamilton’s rule,

¹⁰For a discussion of repeated interactions, see Section 7.

while here we derive a generalization of that rule. Sethi and Somanathan (2001) develop an evolutionary model of conditional altruism, that is, other-regarding preferences that depend on the other person’s type. For a class of aggregative games, including public-goods provision and common-pool extraction games, they show that “reciprocators,” individuals who are altruistic towards other reciprocators but spiteful against selfish individuals, can invade a population of selfish individuals. They also show that a monomorphic population of reciprocators is evolutionarily stable.

Tabellini (2008a) argues that in order to better understand the reason behind and the functioning of current institutions, we need to analyze the formation of values in society. Our evolutionary analysis also contributes to this growing literature, in which the closest articles to our work are those by Bisin and Verdier (2001), Bisin, Topa and Verdier (2004) and Tabellini (2008b). Those papers analyze cultural value transmission driven by parents’ incentives to foster their childrens’ taste for cooperation.¹¹

2 The model

2.1 Atomistic and selfish individuals

Consider a selfish individual living in autarky. The individual chooses an *effort* level $x \geq 0$ that determines the probability distribution over the possible returns, or *output* levels. The output is either high, y^H , or low, $y^L = \lambda y^H$, where $\lambda < 1$. The output is high with probability p and low with probability $1 - p$. The probability p for the high output level is increasing in the individual’s effort, $p = f(x)$, where $f : \mathbb{R}_+ \rightarrow [0, 1)$ is twice differentiable with $f(0) = 0$, $f' > 0$, $f'' < 0$ and $f(x) \rightarrow 1$ as $x \rightarrow +\infty$.

An effort level $x \geq 0$ results in the expected utility

$$f(x) u(y^H) + [1 - f(x)] u(y^L) - v(x), \tag{1}$$

where $u(y)$ is the utility from consuming an amount $y > 0$, and $v(x)$ is the disutility (or cost) of exerting effort $x \geq 0$. We assume that both u and v are twice differentiable with, $u', v' > 0$, $u'' < 0$, and $v'' \geq 0$.

¹¹Hauk and Saez-Martí (2002) use a similar framework to analyze parents’ incentives to foster honesty, and Lindbeck and Nyberg (2006) analyze parents’ incentive to instill a work norm in their children.

Alternatively, if the individual directly chooses his or her success probability p , at a cost or disutility $\psi(p)$, the expected utility can be written as

$$pu(y^H) + (1 - p)u(y^L) - \psi(p), \quad (2)$$

where u is defined as above, and ψ can be derived from v and f as follows: $\psi(p) = v(f^{-1}(p))$. The previous assumptions on v and f imply that the disutility associated with a success probability p is increasing and strictly convex in p : $\psi', \psi'' > 0$ and $\psi'(p) \rightarrow +\infty$ as $p \rightarrow 1$. A positive success probability p is thus uniquely determined by the first-order condition

$$\psi'(p) = u(y^H) - u(y^L), \quad (3)$$

which simply requires that the marginal disutility of increasing the success probability should equal the marginal benefit thereof. We note that the success probability determined by (3) is higher, the higher the variability λ of the environment is, given y^H . If the marginal disutility when the success probability is zero exceeds the utility difference between the high and low outputs, then the individual will optimally choose to exert no effort, so then $p = 0$. In the sequel, we will let x^0, p^0, y^0 and V^0 denote the effort, success probability, expected income, and expected utility of an atomistic and selfish individual.

Example 1 *We will subsequently illustrate some of the results for the special case when the success probability function is exponential, $f(x) = 1 - e^{-\theta x}$, and the consumption-utility function is of the CRRA form, $u(y) = y^{1-\rho}/(1-\rho)$. Here $\theta > 0$ represents one aspect of the harshness of the environment, how much effort is needed in order to obtain a given success probability, and $\rho \in (0, 1)$ is the individual's degree of relative risk aversion. We abuse notation slightly and include $\rho = 1$ as the case $u(y) = \ln y$. We will let the disutility from effort be of the form $v(x) = \gamma x$ for some $\gamma > 0$, the individual's dislike of effort. With these functional forms, the expected material utility, written as a function of the success probability p , becomes, for $\rho \in (0, 1)$,*

$$p \frac{(y^H)^{1-\rho}}{1-\rho} + (1-p) \frac{(y^L)^{1-\rho}}{1-\rho} + \frac{\gamma}{\theta} \ln(1-p), \quad (4)$$

and for $\rho = 1$,

$$p \ln y^H + (1-p) \ln y^L + \frac{\gamma}{\theta} \ln(1-p). \quad (5)$$

The success probability that maximizes the expected material utility is, for $\rho \in (0, 1)$,

$$p^0 = \max \left\{ 0, 1 - \frac{\gamma(1-\rho)}{\theta [(y^H)^{1-\rho} - (y^L)^{1-\rho}]} \right\},$$

and for $\rho = 1$,

$$p^0 = \max \left\{ 0, 1 + \frac{\gamma}{\theta \ln \lambda} \right\}.$$

2.2 Individuals with family ties

Now assume that individuals work individually as described above, but belong to families in which members have altruistic feelings towards each other. In the case of unequal individual output levels, those who obtained a high output may want to share some of their output with less fortunate members. More precisely, consider two siblings, A and B , who interact over two periods, along the lines of the model in the preceding subsection. Thus, in the first period, both siblings simultaneously choose their individual success probabilities. Let $\mathbf{p} = (p_A, p_B)$ be the success-probability vector. The output y_i of each individual $i = A, B$ is realized at the end of the first period. The probability for the output pair (y^H, y^H) is, by independence, $p_A p_B$, that for (y^H, y^L) is $p_A (1 - p_B)$, that for (y^L, y^H) is $(1 - p_A) p_B$, and that for (y^L, y^L) is the residual probability.

At the beginning of the second period, the siblings observe each other's outputs.¹² The *state* at the outset of period two is the vector $\omega = (y_A, y_B) \in \Omega = \{y^L, y^H\}^2$. Having observed the state ω , both siblings simultaneously choose whether to make a transfer to the other, and if so, how much to transfer. After these transfers have been made, the *disposable income*, or *consumption*, of each sibling therefore equals his or her output plus any transfer received from the other sibling minus any transfer given.

In this two-stage game, a pure strategy for player $i \in \{A, B\}$ is a pair $s_i = (p_i, \tau_i)$, where $p_i \in [0, 1)$ is i 's success probability, and $\tau_i : \Omega \rightarrow [0, y_H]$ is a function that satisfies $0 \leq \tau_i(y_A, y_B) < y_i$ and that specifies what transfer, if any, to give in each state ω . Each strategy profile $\mathbf{s} = (s_A, s_B)$ determines the *total utility* to each sibling $i = A, B$ in each state ω :

$$U_i(\mathbf{s}, \omega) = V_i(\mathbf{s}, \omega) + \alpha_i V_j(\mathbf{s}, \omega), \quad (6)$$

where $j \neq i$. Here, V_i is sibling i 's *material utility*,

$$V_i(\mathbf{s}, \omega) = u(y_i - \tau_i(\omega) + \tau_j(\omega)) - \psi(p_i)$$

and $\alpha_i \in [0, 1]$ represents i 's degree of *altruism* towards his or her sibling.¹³ An individual

¹²As will be seen later, our results are unchanged if the siblings also observe each other's efforts.

¹³For $\alpha_i \alpha_j < 1$, Equation (6) can be shown to be equivalent with a model where U_i is proportional to

i with $\alpha_i = 0$ will be called *selfish*, and an individual with $\alpha_i = 1$ *fully altruistic*. As the solution concept for this two-stage game G , we use sequential equilibrium (Kreps and Wilson, 1982). Since all four states ω are reached with positive probability when both individuals make positive efforts, all Nash equilibria with $p_A, p_B > 0$ are also sequential equilibria.

3 Equilibrium

In each state $\omega \in \Omega$ at the beginning of the second stage, each sibling i wants to make a transfer to the other if and only if his own marginal material utility from consumption is lower than his sibling's, when the latter is weighted by i 's degree of altruism. In order to make his transfer decision, individual i also has to figure out whether the sibling is simultaneously planning to give a transfer to him. All that matters to each sibling, however, is the *net* transfer to the other. It is straightforward to prove that, except for the case when both individuals are fully altruistic, in equilibrium at most one sibling makes a transfer, and this transfer is unique. Should both siblings be fully altruistic ($\alpha_A = \alpha_B = 1$), the transfers are not uniquely determined, but the resulting allocation is uniquely determined. For each state $\omega \in \Omega$, let $G(\omega)$ be the continuation game in stage two, a two-player simultaneous-move game in which each player's strategy is his or her transfer to the other player.

Proposition 1 *For each $\omega \in \Omega$, there exists at least one Nash equilibrium of $G(\omega)$. If $\alpha_A \alpha_B < 1$, then this equilibrium is unique, and at most one sibling makes a transfer. A transfer is never made from a poorer to a richer sibling, and the size of the transfer does not depend on the poorer sibling's degree of altruism. If $\alpha_A = \alpha_B = 1$, then there is a continuum of Nash equilibria, all resulting in equal sharing of the total output.*

Let us describe this in some detail. A positive equilibrium transfer is made by a “rich” sibling — a sibling with the high output y^H — to a “poor” sibling — a sibling with the low output y^L . Let $t(\alpha)$ denote the transfer that a rich sibling with altruism α gives, in equilibrium, to his or her poor sibling (whose degree of altruism does not matter). This transfer is positive if and only if the rich sibling is sufficiently altruistic, in the sense that $\alpha u'(y^L) > u'(y^H)$, or, equivalently, if and only if $\alpha > \hat{\alpha}$, where

$$\hat{\alpha} = u'(y^H) / u'(y^L) \in (0, 1). \tag{7}$$

$V_i(\mathbf{s}, \omega) + \alpha_i U_j$ for $i = A, B$, and $j \neq i$. Hence, for such parameter combinations, the current formulation is consistent with “pure” (or “non-paternalistic”) altruism; see Lindbeck and Weibull (1988).

For each $\alpha > \hat{\alpha}$, the transfer $t(\alpha) \in (0, y^H)$ is uniquely determined by the first-order condition

$$u'(y^H - t(\alpha)) = \alpha u'(y^L + t(\alpha)). \quad (8)$$

In sum: the transfer $T(\alpha)$ that a rich sibling with altruism $\alpha \in [0, 1]$ makes to his or her poor sibling is

$$T(\alpha) = \max\{t(\alpha), 0\}, \quad (9)$$

where $t(\alpha)$ is defined by (8).

Example 2 *With the parametric specification of Example 1, we obtain*

$$T(\alpha) = \max\left\{0, \frac{\alpha^{1/\rho} - \lambda}{1 + \alpha^{1/\rho}}\right\} \cdot y^H, \quad (10)$$

(where $\lambda = y^L/y^H$). This transfer is increasing in the donor's altruism α , from zero for all $\alpha < \hat{\alpha} = \lambda^\rho$, towards $(y^H - y^L)/2$ as $\alpha \rightarrow 1$. Furthermore, given α , it is increasing in the degree of risk aversion ρ .

The equilibrium transfer function $T : [0, 1] \rightarrow [0, y^H]$ is continuous, positive if $\alpha > \hat{\alpha}$, and zero otherwise. Moreover, T is differentiable for all $\alpha \neq \hat{\alpha}$, with

$$T'(\alpha) = -\frac{u'(y^L + t(\alpha))}{u''(y^H - t(\alpha)) + \alpha u''(y^L + t(\alpha))} > 0 \quad (11)$$

for all $\alpha > \hat{\alpha}$. Hence, as one would expect, a rich sibling gives more the more altruistic he or she is, for all degrees of altruism above the critical lower bound, $\hat{\alpha}$. Moreover, a rich sibling with altruism $\alpha \in (\hat{\alpha}, 1)$ remains richer than his or her poor sibling after the transfer:

$$c^H = y^H - T(\alpha) > y^L + T(\alpha) = c^L.$$

When $\alpha = 1$, total output is shared equally: $y^H - T(\alpha) = y^L + T(\alpha)$. As expected, for a given level of altruism $\alpha > \hat{\alpha}$, a rich sibling makes a smaller transfer, the less poor is the poor sibling: the higher y^L is, the smaller is $T(\alpha)$, *ceteris paribus*. However, an increase in y^L is not fully offset by the decrease in the transfer: it leads to higher consumption levels for both siblings, when one is rich and the other poor. Formally:¹⁴

¹⁴In a model with an altruistic parent and a selfish child, Altonji, Hayashi, and Kotlikoff (1997) showed that an increase in the child's income by \$1 would lead to a decrease of \$1 in the parent's transfer to the child. This result was derived in a model where the parent makes transfers to the child in two subsequent periods, and it hinges on the assumption that the child is liquidity constrained in the first period. Hence, Proposition 2 is not in contradiction with their result.

Proposition 2 : Both $c^H = y^H - T(\alpha)$ and $c^L = y^L + T(\alpha)$ are increasing in y^L , *ceteris paribus*.

Remark 1 *The equilibrium transfers would have been the same, had the siblings observed each others' effort. This follows from the assumed additive separability of material utility, see equation (2).*

We now turn to the first period, in which the siblings simultaneously choose their individual success probabilities (or, equivalently, efforts). In equilibrium, they both anticipate the subsequent transfers in each of the four possible states in the second period. The *ex ante expected total utility* for each sibling i is thus a function of their choices of success probabilities:

$$\begin{aligned}
U_i(p_i, p_j) = & p_A p_B (1 + \alpha_i) u(y^H) \\
& + (1 - p_A)(1 - p_B)(1 + \alpha_i) u(y^L) \\
& + p_i(1 - p_j)[u(y^H - T(\alpha_i)) + \alpha_i u(y^L + T(\alpha_i))] \\
& + p_j(1 - p_i)[u(y^L + T(\alpha_j)) + \alpha_i u(y^H - T(\alpha_j))] \\
& - \psi(p_i) - \alpha_i \psi(p_j),
\end{aligned} \tag{12}$$

for $i = A, B$ and $j \neq i$. The four first terms represent the distinct second-period states: both being rich, both being poor, i rich and j poor, and i poor and j rich (for $i = A, B$ and $j \neq i$). The last two terms represent the two siblings' disutility from effort.

The pair (U_A, U_B) defines the payoff functions in a two-player normal-form game G^* in which a pure strategy for each player i is his or her success probability $p_i \in [0, 1)$. A necessary and sufficient condition for a strategy pair $(p_A, p_B) \in (0, 1)^2$ to constitute a Nash equilibrium of G^* is that it satisfies the following generalization of the first-order condition for the autarky case:

$$\begin{cases} \psi'(p_A) = u(y^H) - u(y^L) + g(p_B, \alpha_A, \alpha_B) \\ \psi'(p_B) = u(y^H) - u(y^L) + g(p_A, \alpha_B, \alpha_A) \end{cases} \tag{13}$$

where, for any $p, \alpha, \beta \in [0, 1]$:

$$\begin{aligned}
g(p, \alpha, \beta) = & (1 - p) \cdot (u[y^H - T(\alpha)] + \alpha u[y^L + T(\alpha)] - [u(y^H) + \alpha u(y^L)]) \\
& - p \cdot (u[y^L + T(\beta)] + \alpha u[y^H - T(\beta)] - [u(y^L) + \alpha u(y^H)]).
\end{aligned} \tag{14}$$

Just as in the autarky case (equation (3)), the equation system (13) requires that, when both siblings make positive efforts, the marginal cost of increasing one's success probability (or effort) should equal the expected marginal benefit thereof.

Compared to the autarky case, here the marginal benefit has an additional term, $g(p, \alpha, \beta)$, defined in (14). First, increasing one's success probability *increases* the probability of being able to help one's sibling, should the sibling become poor. This is the first term in the expression for $g(p, \alpha, \beta)$. Second, increasing one's success probability *decreases* the probability of being helped out by one's sibling, should the sibling become rich. This is the second term.

The right-hand sides in the equation system (13) are decreasing affine functions of the *other* sibling's success probability. Hence, the higher one's sibling's success probability, the weaker is the incentive to increase one's own success probability. This disincentive effect can be decomposed into two components: when i 's sibling's success probability (effort) increases, then (a) the probability that i will be put in a position to help, if successful, decreases, and (b) the probability of being helped out, if unsuccessful, increases.

We saw previously that the transfer from a rich to a poor sibling is increasing in the donor's level of altruism. Will higher levels of altruism therefore lead to lower levels of effort? To answer this question, we first ask how changes in the individual degrees of altruism affect the equilibrium efforts. Thus, consider an increase in sibling i 's altruism: this has only one effect on the transfers, namely, that sibling i would make a larger transfer to his sibling j should i be rich and j poor. Clearly, this will reduce j 's incentive to provide effort. But how about sibling i ? Sibling i gets to keep less if he is rich and j is poor—intuitively this should have a negative impact on i 's effort. However, now, sibling i also cares more about j , and this should have a positive impact. It happens that the latter, positive effect outweighs the former, negative effect. This claim can be made precise when the equilibrium is unique and the Jacobian of the equation system (13) is non-null at that point (a condition that holds generically).

Proposition 3 *Suppose that $(p_A^*, p_B^*) \in (0, 1)^2$ is the unique Nash equilibrium of G^* . If $\alpha_A, \alpha_B < \hat{\alpha}$, then a marginal change of α_A or α_B has no effect on (p_A^*, p_B^*) . If (15) holds and $\alpha_i > \hat{\alpha}$, then a marginal increase in α_i causes an increase in p_i^* and a decrease in p_j^* (for $i \in \{A, B\}$ and $j \neq i$).*

$$\psi''(p_A^*) \cdot \psi''(p_B^*) \neq \frac{\partial g(p_A^*, \alpha_B, \alpha_A)}{\partial p_A} \cdot \frac{\partial g(p_B^*, \alpha_A, \alpha_B)}{\partial p_B} \quad (15)$$

In other words: if individual A plans to transfer some amount to B, in case A is rich and B poor, then A's equilibrium effort is increasing in her own altruism, *ceteris paribus*. The motive is twofold: first, to increase the chance to have something to give, and, secondly, to decrease the risk that B will be put in a situation in which B will feel the need to help A.¹⁵ Likewise, if individual A plans to transfer some amount to B, in case A is rich and B poor, then B's equilibrium effort is decreasing in A's altruism, *ceteris paribus*, since the chance to receive help from A has increased, and thus the expected material utility from the low output has increased.

In sum, a more altruistic individual not only gives a larger transfer, but also makes a greater effort to obtain the high output level. We call this positive effect of altruism the *empathy effect* (from own altruism). In contrast, an individual may choose a lower success probability if the sibling's altruism increases, *ceteris paribus*. This is the well-known *free-rider effect* of others' altruism.

If both siblings become more altruistic, will the empathy or free-rider effect dominate? We answer this question for the case of equally altruistic siblings.

4 Equally altruistic siblings

Consider a pair of siblings with the same degree of altruism, $\alpha_A = \alpha_B = \alpha$. The game G^* then has a unique symmetric equilibrium, and this can be characterized in terms of equation (13). Formally:

Proposition 4 *If $\alpha_A = \alpha_B = \alpha$, then G^* has a unique symmetric equilibrium, (p^*, p^*) . If $p^* > 0$, then it solves the equation*

$$\begin{aligned} \psi'(p) &= u(y^H) - u(\lambda y^H) \\ &+ (1-p) \cdot (u[y^H - T(\alpha)] + \alpha u[y^L + T(\alpha)] - [u(y^H) + \alpha u(y^L)]) \\ &- p \cdot (u[y^L + T(\alpha)] + \alpha u[y^H - T(\alpha)] - [u(y^L) + \alpha u(y^H)]). \end{aligned} \tag{16}$$

¹⁵Transfers are voluntary, but it is better for a sibling to be in a state in which both siblings receive the high output.

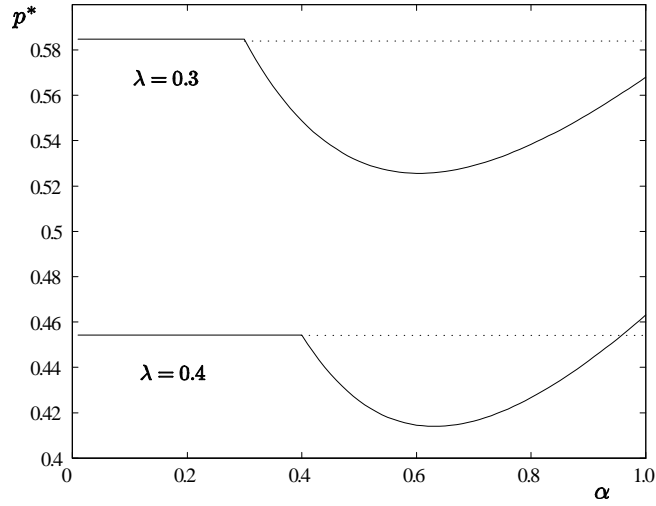


Figure 1: The equilibrium success probability as a function of the common degree of altruism.

Example 3 *In the parametric specification of Example 1, for relative risk aversion $\rho = 1$ and $\alpha > \hat{\alpha}$, equation (16) boils down to*

$$(1 - p) \left[(1 - p - \alpha p) \ln \left(\frac{1 + \lambda}{1 + \alpha} \right) + (\alpha - p - \alpha p) \ln \left(\frac{\alpha(1 + \lambda)}{\lambda(1 + \alpha)} \right) - \ln \lambda \right] = \frac{\gamma}{\theta}. \quad (17)$$

For given parameter values, the left-hand side of this equation is a polynomial of degree two in p . Figure 1 plots its solution, the equilibrium success probability p^ , as a function of α , for $\gamma/\theta = 0.5$, $\lambda = 0.3$ (the upper curve) and $\lambda = 0.4$ (the lower curve). When altruism is weak ($\alpha \leq \hat{\alpha} = \lambda$), the siblings expect no transfers from each other and therefore choose their autarky effort. As α increases beyond λ , each sibling expects to give (receive) a transfer, should he become rich (poor) and the other sibling poor (rich). The equilibrium effort is non-monotonic in the common degree of altruism. Moreover, in the more risky environment, $\lambda = 0.3$, the equilibrium effort is higher for relatively selfish individuals ($\alpha \leq \hat{\alpha}$) than for relatively altruistic individuals ($\alpha > \hat{\alpha}$), while in the less risky environment, $\lambda = 0.4$, high degrees of altruism (α close to 1) have a positive net incentive effect on effort, pushing it above the autarky level. In less risky environments, the autarky effort is lower, and, hence, so is the marginal disutility of effort. The free-rider effect is therefore weaker in less risky environments. The corresponding diagrams for lower degrees of relative risk aversion, $\rho < 1$, are qualitatively similar.*

Does the non-monotonicity of the success probability p^* , with respect to the common degree of altruism in the above example, hold generally? More altruistic individuals are, by

definition, more concerned about the “external effects” that their behavior has on others, and hence the empathy effect is stronger and the free-rider effect weaker on such an individual’s behavior than on a less altruistic individual. This suggests that the free-rider effect should dominate at low common levels of altruism and the empathy effect at high such levels. However, as both siblings’ degrees of altruism increase, so does the return from free-riding on one’s sibling’s increased altruism, therefore the net effect is *a priori* ambiguous. The following proposition shows that this counter-effect is never strong enough: the free-rider effect always dominates at low degrees of altruism, while the empathy effect dominates at high degrees of altruism. Let $p^*(\alpha)$ denote the common success probability when the common degree of altruism is α .

Proposition 5 *Consider the unique symmetric Nash equilibrium (p^*, p^*) of G^* . If $p^*(\hat{\alpha}) > 0$ and $p^*(1) > 0$, then there is an $\bar{\varepsilon} > 0$ such that $p^*(\hat{\alpha} + \varepsilon) < p^*(\hat{\alpha})$ and $p^*(1 - \varepsilon) < p^*(1)$ for all $\varepsilon \in (0, \bar{\varepsilon})$.*

Next, we briefly consider the effects of changes in the low output level, y^L , one aspect of the harshness of the environment of the pair of siblings, on their equilibrium efforts. Not surprisingly, a decrease in y^L leads to an increase in the equilibrium success probability, a generalization of the example in Figure 1.

Proposition 6 *Consider the unique symmetric Nash equilibrium (p^*, p^*) of G^* , for $p^* > 0$. A decrease in y^L , *ceteris paribus*, leads to an increase in p^* .*

Turning from behavior to welfare, we note that a sibling’s expected material utility in the unique symmetric equilibrium of G^* may be written as

$$\begin{aligned} V^*(\alpha) &= [p^*(\alpha)]^2 \cdot u(y^H) + [1 - p^*(\alpha)]^2 \cdot u(y^L) \\ &\quad + p^*(\alpha)[1 - p^*(\alpha)] \cdot [u(y^H - T(\alpha)) + u(y^L + T(\alpha))] \\ &\quad - \psi[p^*(\alpha)]. \end{aligned}$$

Using this expression, it is straightforward to show that the common degree of altruism that leads to the highest *expected material utility*, in equilibrium, is *full* altruism:

Proposition 7 $V^*(1) \geq V^*(\alpha)$ for all $\alpha \in [0, 1]$.

The intuition is simple: fully altruistic individuals completely internalize the external effect of their own behavior on their sibling's material utility.¹⁶ Hence, siblings' incentives are perfectly aligned, with each sibling acting like a utilitarian social planner. For lower degrees of altruism, however, their incentives are imperfectly aligned and there is room for some free-riding. From this, it is not difficult to show that the expected equilibrium outcome of the interaction between two equally altruistic siblings is, *ex ante*, Pareto efficient, in terms of their (imperfectly or perfectly) altruistic preferences, if and only if both siblings are fully altruistic.

Corollary 1 *The symmetric Nash equilibrium (p^*, p^*) of G^* is Pareto efficient if and only if $\alpha = 1$.*

At first sight, it may come as a surprise that the outcome is inefficient even when the siblings are purely selfish ($\alpha = 0$). Should not the independent strife of selfish individuals lead to a Pareto-efficient outcome? The explanation is that both individuals' utility can be increased by keeping their common success probability at its equilibrium level, but by having the rich sibling transfer a small amount to the poor sibling, whenever they end up with distinct outputs. Such consumption smoothing across states is beneficial, *ex ante*, because of the assumed risk aversion (concavity in the utility from consumption). Hence, two selfish siblings would prefer to write such an (incomplete and mutual) insurance contract, also involving their efforts, had this been possible.

While very high levels of altruism are beneficial, it is a non-trivial matter whether moderate levels of altruism are beneficial in terms of the expected material utility. As was shown above, the success probability, and therefore also the expected output, declines as altruism increases from an initially moderate level. It turns out, however, that the expected material utility increases:

Proposition 8 *If $p^*(\hat{\alpha}) > 0$ there is an $\bar{\varepsilon} > 0$ such that $V^*(\hat{\alpha} + \varepsilon) > V^*(\hat{\alpha})$ for all $\varepsilon \in (0, \bar{\varepsilon})$.*

¹⁶ Assuming that the siblings are fully altruistic is mathematically equivalent to assuming that they are selfish, but that together they can commit to an effort level and a transfer from a rich individual to a poor individual that maximize their joint expected material utility, a situation modelled by Arnott and Stiglitz (1991).

In the proof of the proposition, we use $V(\alpha, \beta)$ to denote the expected material utility obtained by an individual with altruism α in equilibrium play with a sibling with altruism β :

$$\begin{aligned}
V(\alpha, \beta) = & p(\alpha, \beta)p(\beta, \alpha)u(y^H) \\
& + [1 - p(\alpha, \beta)][1 - p(\beta, \alpha)]u(y^L) \\
& + p(\alpha, \beta)[1 - p(\beta, \alpha)]u[y^H - T(\alpha)] \\
& + [1 - p(\alpha, \beta)]p(\beta, \alpha)u[y^L + T(\beta)] - \psi[p(\alpha, \beta)].
\end{aligned} \tag{18}$$

Here $p : [0, 1]^2 \rightarrow (0, 1)$ is the function that to each pair of sibling altruism levels, (α, β) , specifies the equilibrium success probability of the α -altruist.¹⁷ Thus, if an individual has altruism α and his or her sibling has altruism β , then $p(\alpha, \beta)$ is the individual's own success probability, and $p(\beta, \alpha)$ is that of the sibling. Such a pair of success probabilities, when positive, necessarily satisfy the system of equations (13). Let \mathcal{A} be the degrees of altruism $\alpha \in [0, 1]$ such that $V : [0, 1]^2 \rightarrow \mathbb{R}$ is differentiable at the point (α, α) . It follows from (18) that for $\alpha \in \mathcal{A}$ the function p is differentiable at (α, α) as well. Let its partial derivatives at such a point, with respect to its first and second arguments, be denoted $p_1(\alpha, \alpha)$ and $p_2(\alpha, \alpha)$, respectively.

5 Evolutionarily stable family ties

A pair of siblings would fare best, in terms of their expected material utility, if they were both fully altruistic. But if sibling altruism is a trait that is inherited from parent to child (where inheritance could be cultural or genetic), is such a high degree of altruism stable against “mutations” towards lower degrees of altruism? Here we propose a method to determine stable degrees of altruism. In this exploration, we follow and extend somewhat Bergstrom's (1995, 2003) approach. More specifically, suppose that a child adopts its father's or mother's degree of sibling altruism (“family values”), with equal probability for both events, and with statistical independence.¹⁸ Thus, if the father's degree of altruism is α_f and the mother's is

¹⁷We restrict attention to cases in which there is a unique equilibrium. Uniqueness holds, for instance, in the parametric example in Section 2.1 (for details, see Alger and Weibull, 2007). The uniqueness assumption will, in fact, be used only when α and β are (infinitesimally) close to each other.

¹⁸If the transmission is genetic, this corresponds to the *sexual haploid reproduction* case, where each parent carries one copy of the gene, and the child inherits either the father's or the mother's gene. The

$\alpha_m \neq \alpha_f$, then, with probability 1/4 the two siblings will both have altruism α_f , with the same probability they will both have altruism α_m , and with probability 1/2 one sibling will have altruism α_f and the other α_m . As in Bergstrom’s (1995) model, mating is monogamous and mate selection is random.¹⁹

5.1 Local evolutionary stability

Consider a sequence of successive, non-overlapping generations, living for one time period each. In each time period, those individuals who survived to the age of reproduction mate in randomly matched pairs. Each pair has exactly two children, and each sibling pair plays a symmetric game once.²⁰ This game may be the game in Subsection 2.2. In the first generation, all individuals have the same degree of sibling altruism $\alpha \in [0, 1]$. Suppose that a “mutation” occurs in the second generation: a small population share of those who are about to reproduce switch to another degree of altruism, $\alpha' \neq \alpha$. Such a switch could be caused by genetic drift, a cultural shift in family values, or it could be due to the immigration of individuals with other family values. Random matching of couples takes place as before and reproduction occurs. We call the “incumbent” degree of altruism α *evolutionarily stable against* α' if a child carrying the incumbent degree of altruism obtains, on average, a higher material utility than a child carrying the mutant degree, for all sufficiently small population shares of the “mutant” degree of altruism, α' . The “incumbent” degree α is (globally) *evolutionarily stable* if this holds for every $\alpha' \neq \alpha$.

As we will presently see, the condition for the above-mentioned incumbent degree of altruism α to be (globally) evolutionarily stable against a mutant degree $\alpha' \neq \alpha$ boils down

human species uses *sexual diploid reproduction*: then, each individual has two sets of chromosomes; one set is inherited from the father, and the other from the mother. Whether a gene is expressed or not depends on whether it is recessive (two copies are needed for the gene to be expressed), or dominant (one copy is sufficient for the gene to be expressed). Bergstrom’s (2003) analysis of games between relatives shows that the condition for a population carrying the same gene to resist the invasion by a mutant gene in the haploid case is the same as the condition for a population carrying the same recessive gene to resist the invasion by a dominant mutant gene in the diploid case.

¹⁹In Section 6 we generalize the evolutionary process to allow for assortative mating as well as societal influences.

²⁰Somewhat more generally, each pair may have an even number of children and they interact in pairs.

to the following inequality:

$$V(\alpha, \alpha) > \frac{1}{2} [V(\alpha', \alpha) + V(\alpha', \alpha')], \quad (19)$$

where $V(\alpha, \beta)$ denotes the expected material utility to an individual with altruism α whose sibling has altruism β (see (18)). Formally, we define a degree of sibling altruism $\alpha \in [0, 1]$ to be *evolutionarily stable* if it meets (19) for all $\alpha' \neq \alpha$.²¹

To see that (19) is sufficient for evolutionary stability as informally defined above, note that the left-hand side, $V(\alpha, \alpha)$, approximates the expected material utility to a child with the incumbent degree of altruism, α . For if the population share of mutants in the parent generation, $\varepsilon > 0$, is close to zero, then, with near certainty, both parents of this child are α -altruists, implying that the child's sibling is also almost surely an α -altruist. Likewise, the expression on the right-hand side approximates the expected material utility to a child with the mutant degree of altruism, α' . Because, for ε close to zero, such a child almost certainly has exactly one parent with the mutant degree of altruism (the probability that both parents are mutants is an order of magnitude smaller, ε^2 , and the probability that none is, is zero). Therefore, with probability close to 1/2, this child's sibling has the incumbent degree of altruism, α , and, with the complementary probability, the sibling has the mutant degree of altruism, α' .²²

The process by which mutations appear in a population may affect the extent to which the mutant degree of altruism differs from the incumbent degree. In particular, "cultural drift" in a society's values may arguably lead to smaller differences between incumbents and mutants, while immigration from another community or society may sometimes give rise to

²¹Bergstrom (1995, 2003) derives a condition similar to (19) in a slightly different model, in which each individual is programmed to play a strategy in a symmetric two-player game. Bergstrom shows that for a sexual haploid species, a sufficient condition for a population consisting of x -strategists to be stable against a small invasion of y -strategists is

$$\Pi(x, x) > \frac{1}{2}\Pi(y, x) + \frac{1}{2}\Pi(y, y),$$

where $\Pi(s, s')$ denotes the payoff to strategy s against strategy s' .

²²The exact condition for evolutionary stability, as informally defined in the text above, is that for every α' there exists an $\bar{\varepsilon} \in (0, 1)$ such that

$$(2 - \varepsilon)V(\alpha, \alpha) + \varepsilon V(\alpha, \alpha') > (1 - \varepsilon)V(\alpha', \alpha) + (1 + \varepsilon)V(\alpha', \alpha')$$

for all $\varepsilon \in (0, \bar{\varepsilon})$. By continuity, (19) is sufficient for this, and its weak-inequality version is necessary.

larger such differences. Thus, the relevant evolutionary stability criterion against “cultural drift” is a local version of the above definition. Formally:

Definition 1 *A degree of altruism $\alpha \in [0, 1]$ is **locally evolutionarily stable** if (19) holds for all $\alpha' \neq \alpha$ in some neighborhood of α .*

Let us elaborate the notions of evolutionary stability and local evolutionary stability a bit. First, note that a degree of altruism α is evolutionarily stable if and only if the right-hand side of (19), viewed as a function of $\alpha' \in [0, 1]$, has its unique global maximum at $\alpha' = \alpha$. Second, a degree of altruism α is *locally* evolutionarily stable if and only if the right-hand side of (19), again viewed as a function of $\alpha' \in [0, 1]$, has a strict local maximum at $\alpha' = \alpha$. Recalling that \mathcal{A} is the set of degrees of altruism $\alpha \in [0, 1]$ such that $V : [0, 1]^2 \rightarrow \mathbb{R}$, defined in (18), is differentiable at the point (α, α) , define $D : \mathcal{A} \rightarrow \mathbb{R}$ by

$$D(\alpha) = V_1(\alpha, \alpha) + \frac{1}{2}V_2(\alpha, \alpha), \quad (20)$$

where V_k is the partial derivative of V with respect to its k 'th argument, for $k = 1, 2$. For any degree of altruism $\alpha \in \mathcal{A}$, $D(\alpha)$ is the derivative of the right-hand side of (19), viewed as a function of α' , evaluated at $\alpha' = \alpha$. We will refer to the function D as the *evolutionary drift function*.

If the incumbent degree of altruism in a society is $\alpha \in \mathcal{A}$, then $D(\alpha) d\alpha$ is the marginal effect of a slight increase in a mutant's degree of altruism, from α to $\alpha + d\alpha$, on its child's expected material utility (achieved in the child's equilibrium play with its sibling) if the child inherits its mutant parent's degree of altruism. The first term is the effect of an increase in the child's own altruism on his or her expected material utility, whereas the second term is the effect of an increase in the child's sibling's altruism, multiplied by one half — the conditional probability that also the sibling is a mutant.

If $D(\alpha) > 0$, then the mutant child, if slightly *more* altruistic than the incumbent population, will outperform the incumbents' children in terms of expected material utility. Likewise, if $D(\alpha) < 0$, then it is instead a mutant child who is slightly *less* altruistic than the incumbents that will outperform the incumbents' children. Hence, in order for an incumbent degree of altruism $\alpha \in \mathcal{A}$ to be locally evolutionarily stable, it is necessary that $D(\alpha) = 0$. Let $\text{int}(\mathcal{A}) \subset \mathcal{A}$ be the set of *interior* points in \mathcal{A} , that is, degrees of altruism α such that V is continuously differentiable at all points (α', α') near (α, α) . For such degrees of altruism

more can be said:²³

Proposition 9 *A necessary condition for a degree of altruism $\alpha \in \mathcal{A}$ to be locally evolutionarily stable is $D(\alpha) = 0$. A necessary and sufficient condition for a degree of altruism $\alpha \in \text{int}(\mathcal{A})$ to be locally evolutionarily stable is (i)-(iii), where:*

- (i) $D(\alpha) = 0$
- (ii) $D(\alpha') > 0$ for all nearby $\alpha' < \alpha$
- (iii) $D(\alpha') < 0$ for all nearby $\alpha' > \alpha$.

In other words: wherever the evolutionary drift function is well-defined, a necessary condition for local evolutionary stability is that there be no drift, and that there be upward (downward) drift at slightly lower (higher) altruism levels. Clearly, if D is differentiable at α and satisfies (i), then $D'(\alpha) < 0$ implies (ii) and (iii).

5.2 Application to the present sibling interaction

Here, we derive the drift function for the case where siblings interact according to the model analyzed in Sections 2-4. Recall from Proposition 3 that if $p(\alpha, \beta) > 0$ and $p(\alpha, \beta) > 0$, then $p_1(\alpha, \beta) > 0$ and $p_2(\alpha, \beta) < 0$ whenever $\alpha, \beta > \hat{\alpha}$. Straightforward calculations based on (18) and the envelope theorem lead to:

Lemma 1 *For any $\alpha \in \text{int}(\mathcal{A})$ with $p(\alpha, \alpha) > 0$:*

$$D(\alpha) = (1/2 - \alpha) \cdot F(\alpha) + [(1/2 - \alpha)p_1(\alpha, \alpha) + (1 - \alpha/2)p_2(\alpha, \alpha)] \cdot G(\alpha) \quad (21)$$

where

$$F(\alpha) = p(\alpha, \alpha)(1 - p(\alpha, \alpha)) \cdot u'[y^L + T(\alpha)]T'(\alpha)$$

and

$$G(\alpha) = p(\alpha, \alpha) \cdot (u(y^H) - u[y^H - T(\alpha)]) + [1 - p(\alpha, \alpha)] \cdot (u[y^L + T(\alpha)] - u(y^L)).$$

²³This follows from the fact that local evolutionary stability is equivalent with local strict maximization of the right-hand side of (19).

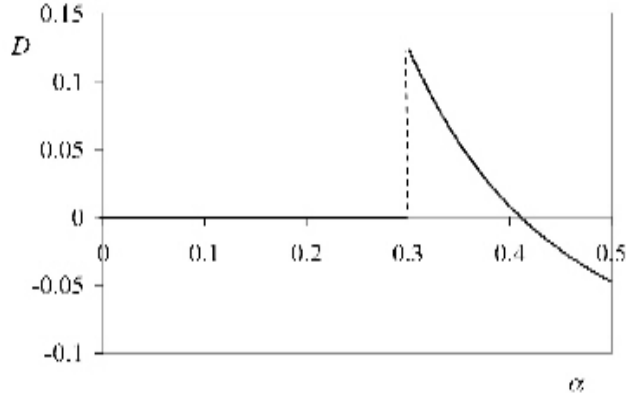


Figure 2: The evolutionary drift $D(\alpha)$.

Figure 2 shows the graph of a drift function D , with the common degree of altruism, α , on the horizontal axis.²⁴ The evolutionary drift, $D(\alpha)$, is zero for all $\alpha < \hat{\alpha}$ (recall that $\hat{\alpha} = \lambda$). At $\alpha = \hat{\alpha} < 1/2$, $D(\alpha)$ jumps to a positive value, from which it declines continuously from positive to negative, as α increases towards unity. According to Proposition 9, the intersection of the down-sloping curve and the horizontal axis defines a locally evolutionarily stable degree of sibling altruism. At lower (higher) degrees of sibling altruism there is upward (downward) evolutionary drift. The diagram shows that the locally evolutionarily stable degree of altruism, in this example, is unique. Since local evolutionary stability is necessary for evolutionary stability, the unique locally stable degree of altruism is also the only potentially globally stable degree. Numerical simulations suggest global evolutionary stability (see also further simulations below).

We are now in a position to derive a number of general and qualitative results. First, the evolutionarily stable degree of altruism must be large enough for rich individuals to help out their poor siblings:

Proposition 10 *For any λ , if $\alpha \in \text{int}(\mathcal{A})$ is locally evolutionarily stable, then $\alpha > \hat{\alpha} = u'(y^H)/u'(y^L)$.*

²⁴The diagram is based on the parametric representation in Example 1, for $\rho = 1$, $\gamma = 1$, $\theta = 2$, and $\lambda = 0.3$.

Incumbents with $\alpha \leq \hat{\alpha}$ do not give any transfers. Since a mutant sibling with altruism α' near α does not give any transfer either, and hence it obtains the same expected material utility as a sibling with the incumbent degree of altruism, α , it follows that no degree of altruism $\alpha \leq \hat{\alpha}$ is locally evolutionarily stable.

In the light of Hamilton's rule (Hamilton, 1964a), mentioned in the introduction, one might expect $\alpha = 1/2$ to be the stable degree of kinship altruism. This is never true here, however. In fact, an evolutionarily stable degree of altruism must be smaller than $1/2$:

Proposition 11 *If $\alpha \in \text{int}(\mathcal{A})$ is locally evolutionarily stable, then $\alpha < 1/2$.*

Consider a population where the degree of altruism is one half (or higher) and where the equilibrium transfer from a rich to a poor sibling is positive. Such a population would be vulnerable to the “invasion” by slightly less altruistic mutants: for any $\alpha \geq 1/2$ (and $\alpha > \hat{\alpha}$), $D(\alpha) < 0$ (see equation (21)).

Propositions 10 and 11 together imply that there exists an evolutionarily stable degree of altruism only if $\hat{\alpha} < 1/2$, or, equivalently, only if

$$\lambda < \hat{\lambda} = \frac{1}{y^H} (u')^{-1} [2u'(y^H)]. \quad (22)$$

This condition says that the environment is such that the critical degree of altruism for a transfer to occur is lower than Wright's coefficient of relationship between the siblings. In *gentle* environments (where $\lambda \geq \hat{\lambda}$), the marginal utility at the low output is so close to the marginal utility at the high output level that siblings with altruism $\alpha = 1/2$ do not give any transfers to each other, and from Proposition 10 this cannot be stable.

The result in Proposition 11 is due to the “strategic externality” that one sibling's altruism exerts on the other's choice of effort: each sibling adjusts his or her productive effort not only to the exogenous environment, but, also, to the anticipated transfer from the other sibling. To see that this “strategic externality” influences what degree of altruism is evolutionarily stable, suppose that both siblings' success probabilities instead were fixed, at some exogenously given level. What levels of sibling altruism α would then be evolutionarily stable? An application of Proposition 9 provides the answer:

Corollary 2 *Suppose that $\lambda < \hat{\lambda}$ and that efforts are exogenously fixed and equal. Then the unique evolutionarily stable degree of sibling altruism is $\alpha = 1/2$.*

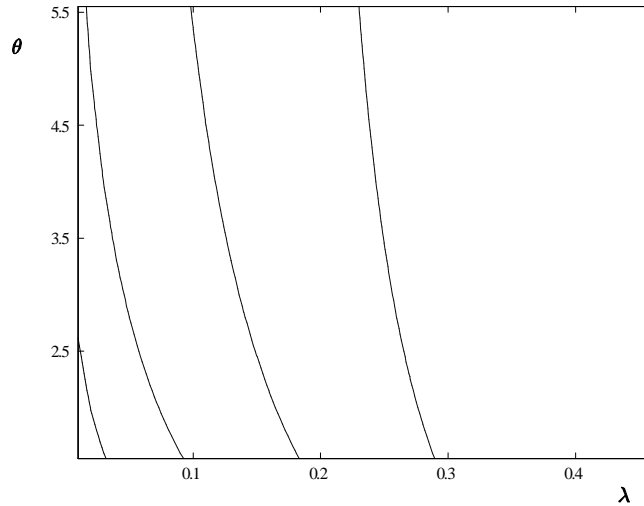


Figure 3: The evolutionarily stable degree of altruism as a function of output variability λ and return to effort θ .

Our result for evolutionary stability thus confirms Hamilton’s rule in settings in which the only interaction between siblings is “pie division,” that is, how a resource of given size will be split. In the present model, the siblings not only divide the “pie,” but also influence its size, by way of their (costly) efforts. It is this additional and strategic element that drives down the evolutionary stable degree of sibling altruism, below $1/2$. This raises two questions: “By how much?” and “How does this depend on other aspects of the environment in which they live?” Given the analytical complexity, we resort to numerical simulations.

5.3 Evolutionary comparative statics

Here, we use the parametric specification in Example 1 to explore how different aspects of the environment may affect the evolutionary stability of different degrees of altruism, and thereby also indirectly effort, income, and material welfare. In order to limit the number of parameters, we henceforth set $\gamma = \rho = 1$. In this case $\hat{\lambda} = 1/2$, see equation (22).

Figure 3 shows the stable degree of altruism as a function of the parameters λ and θ , where a high λ means a less risky environment (the low output level being a larger share of the high output level) and a high θ means a higher (absolute and marginal) return to effort (in terms of the resulting probability for the high output level). We see that higher degrees of altruism are selected for in environments that are milder in the sense of having higher

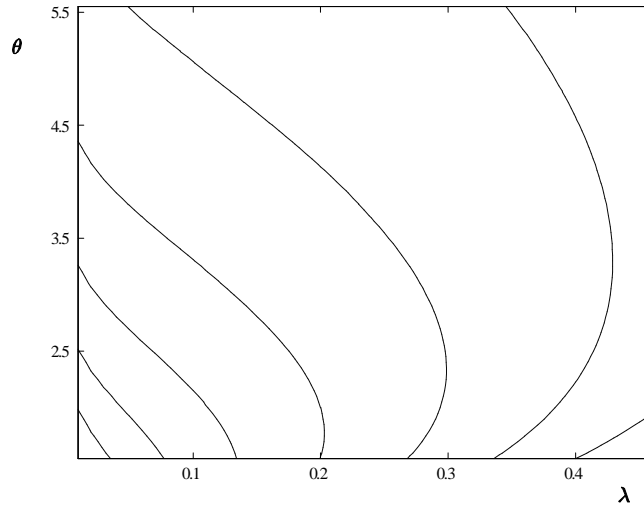


Figure 4: Equilibrium effort x^* as a function of output variability λ and return to effort θ , and for stable altruism levels.

λ and/or θ . This might, at first sight, appear counterintuitive, since risk sharing between siblings would seem to have a lower survival value in milder environments. However, this intuition neglects the incentive effect of the environment on effort: that the “size of the pie” is not exogenous. Indeed, the milder the environment, the less vulnerable is a population with high kinship altruism to an “invasion” by less altruistic mutants. For example, consider an individual, A, whose sibling, B, is less altruistic. A suffers doubly from the relative selfishness of B: the latter *both* makes a lesser effort (Proposition 3) *and* gives a lower transfer, when B is rich and A poor. The altruistic individual is thus more likely to have to help out his sibling, is less likely to be helped out, and receives a lower transfer, than if his sibling had been like him. Since, in milder environments, both siblings make lower efforts (Proposition 6), a slightly less altruistic sibling B may have less to gain from his free-riding on the altruistic individual A, in milder environments.

In the subsequent diagrams, we have calculated the equilibrium effort and income as indirect functions of the environment (λ, θ) , by first letting the degree of sibling altruism adapt to its unique evolutionarily stable value in each environment, and then letting the siblings choose their corresponding equilibrium efforts. Figure 4 shows the resulting effort, x^* , as such an indirect function of the environment (λ, θ) . For a given value of θ , siblings (with the corresponding evolutionarily stable degree of altruism) exert less work effort in environments with lower output variability (higher λ). Hence, in milder environments their

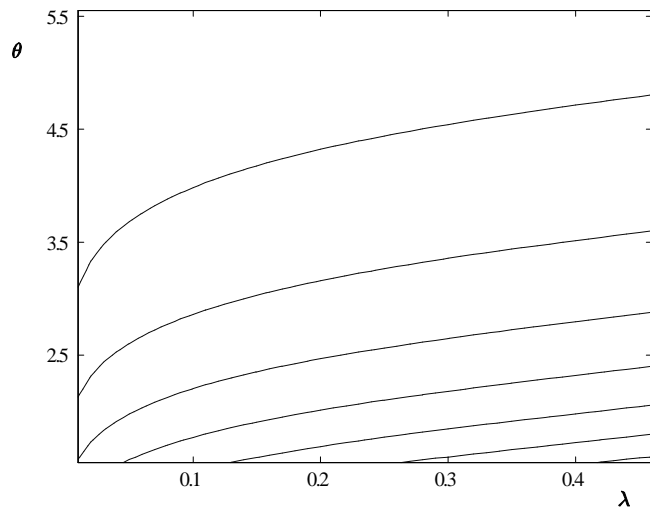


Figure 5: Equilibrium income Y^* as a function of output variability λ and return to effort θ , and for stable altruism levels.

family ties are stronger and they work less hard. The effect of θ , the return to effort, is not as clear-cut: for some values of λ , the equilibrium effort, as an indirect function of the environment, is non-monotonic in θ . This is due to two opposing effects: a sort of substitution effect and a sort of income effect. *Ceteris paribus*, an increase in θ has a positive incentive (substitution) effect, but, in the new and slightly milder environment, the stable level of altruism is a bit higher, and this has a disincentive (income) effect on effort. Our numerical simulations show that the equilibrium effort level, given the associated stable degree of altruism, is lower than the corresponding autarky effort level, for all values of θ .

The lower effort exerted in milder environments does not always yield lower incomes. Indeed, when family ties adapt to the environment, the expected income may increase as the environment becomes milder, see Figure 5. However, even if the expected income is lower in a milder environment (with higher λ , say), and people thus are poorer, they may be “happier”; their expected material utility may be higher. Figure 6 shows the expected *material utility* when family ties have adapted to the unique stable degree of sibling altruism in each environment.

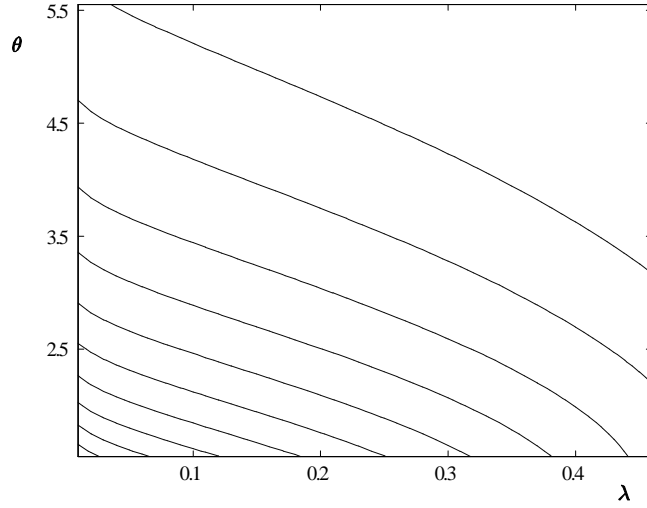


Figure 6: Equilibrium material utility V^* as a function of output variability λ and return to effort θ , and for stable altruism levels.

6 Generalized evolutionary processes

Our evolutionary stability criterion builds upon an evolutionary process in which mating is random and family values are “inherited” only from parents. Here, we generalize the analysis to allow for other evolutionary processes.

Suppose that the conditional probability that the sibling to a child with the mutant degree α' of altruism also has the mutant degree of altruism, α' , is $(1 + \kappa)/2$, for some $\kappa \in [-1, 1]$. The evolutionary stability condition (19) then generalizes to

$$V(\alpha, \alpha) > \frac{1}{2} [(1 - \kappa) V(\alpha', \alpha) + (1 + \kappa) V(\alpha', \alpha')], \quad (23)$$

and the drift function D becomes

$$D(\alpha) = V_1(\alpha, \alpha) + \frac{1}{2} (1 + \kappa) V_2(\alpha, \alpha). \quad (24)$$

For any $\kappa \in [-1, 1]$ and $\alpha \in \text{int}(\mathcal{A})$ with $p(\alpha, \alpha) > 0$, let

$$D(\alpha) = \left(\frac{1 + \kappa}{2} - \alpha \right) \cdot F(\alpha) + \left[\left(\frac{1 + \kappa}{2} - \alpha \right) p_1(\alpha, \alpha) + \left(1 - \frac{(1 + \kappa)\alpha}{2} \right) p_2(\alpha, \alpha) \right] \cdot G(\alpha) \quad (25)$$

where F and G are defined in Lemma 1. Proposition 11 generalizes to:

Proposition 12 *Suppose that $\lambda < \hat{\lambda}$. If $\alpha \in \text{int}(\mathcal{A})$ is locally evolutionarily stable, then $\hat{\alpha} < \alpha \leq 1/2 + \kappa/2$, and α is increasing in κ . In particular, α may exceed $1/2$ if and only if $\kappa > 0$. Moreover, $\alpha = 1$ is a locally stable degree of altruism if $\kappa = 1$.*

The more likely a mutant is to have a mutant sibling, the higher is the stable degree of altruism. A relatively selfish mutant is worse off the higher the probability is that its sibling is also relatively selfish, for such a mutant is more likely to receive a low transfer and is also less likely to receive a transfer (since the empathy effect is weaker on a relatively selfish sibling).

6.1 Assortative mating

Suppose that grown-ups have a tendency towards assortative mating. With probability $\mu \in [0, 1]$ a given mutant grown-up is selective and settles only for a match with another mutant, while with the complementary probability, $1 - \mu$, the mutant is non-selective and has a random match.²⁵ For a small population share $\varepsilon > 0$ of mutants, the conditional probability that the sibling to a child with the mutant degree α' of altruism also has altruism α' is then approximately equal to $(1 - \mu)/2 + \mu$ (instead of $1/2$). Equation (25) and Proposition 12 may be applied, by setting $\kappa = \mu$. When $\mu > 0$, that is, when individuals are more likely to marry someone with the same “family values,” then the marginal fitness effect on a child from a mutation towards a slightly higher level of sibling altruism is higher, and, as a result, so is the evolutionarily stable degree of such altruism.

6.2 Social evolution

Suppose that mating is random but children may be influenced by the values in the surrounding society. With probability $1 - \zeta$ a child “inherits” one of its parents’ degree of family altruism (“parental influence”), with equal probability for both parents. With probability $\zeta \in [0, 1]$, the child imitates a randomly drawn grown-up from the population at large. For a small population share $\varepsilon > 0$ of grown-up mutants, the conditional probability that the sibling to a child with the mutant degree α' of altruism also has altruism α' is then close to $(1 - \zeta)/2$. Equation (25) and Proposition 12 apply, with $\kappa = -\zeta$. A higher likelihood ζ

²⁵This formalization is in line with Cavalli-Sforza and Feldman (1981), see also Wright (1921) and Bergstrom (2003).

that children’s family values are influenced by society at large, rather than by their parents, implies that the marginal value to a child’s fitness, from a mutation to a slightly higher level of kinship altruism, is lower, since this decreases the likelihood that the child’s sibling also is a mutant. Consequently, the evolutionarily stable degree of altruism is lower.

7 Discussion

Our basic model is very simple. Here we briefly discuss some variations of our assumptions.

7.1 Commitment

In our base-line model, there is no possibility to precommit to efforts or transfer levels. Suppose that both siblings are committed to effort level, $\bar{x} > 0$. Such commitment may be the result of strong social norms concerning work effort (“work ethic”), social norms that may be internalized or socially sanctioned. The success probabilities would then be fixed, so an increase in an individual’s degree of altruism could only lead to an increase in his or her (voluntary) transfer to his or her sibling. Corollary 2 implies that, with random mating and pure parental influence, the unique evolutionarily stable level of altruism would then be $\alpha = 1/2$, irrespective of the committed effort level $\bar{x} > 0$: Hamilton’s rule would apply.

Next, suppose that a rich sibling is committed to transfer $\bar{T} \in (0, y^H)$ to a poor sibling. Such commitment may again be the result of social norms.²⁶ The preceding analysis may be applied directly, by setting $T(\alpha) \equiv \bar{T}$ (and thus $T'(\alpha) = 0$). Let $\bar{p}(\alpha, \alpha)$ denote the effort in the unique symmetric equilibrium of the corresponding game, when both individuals have sibling altruism α . Proposition 3 still holds; an increase in own altruism leads to an increase in effort. Interestingly, however, if instead the common degree of altruism is increased, *the empathy effect always outweighs the free-rider effect*, at the margin. The non-monotonicity result in Proposition 5 does not carry over to this setting; with an exogenous transfer \bar{T} , the unique symmetric equilibrium effort is increasing in the common degree of altruism.²⁷

²⁶We refer to Alger and Weibull (2008) for an analysis of what level of such commitment would be stable as a social norm, and how the resulting intrafamily risk sharing compares to risk sharing by way of formal insurance systems.

²⁷To see this, set $T'(\alpha) = 0$ in equation (33), in the proof of Proposition 5.

Moreover, the drift function now becomes

$$D(\alpha) = [(1/2 - \alpha)\bar{p}_1(\alpha, \alpha) + (1 - \alpha/2)\bar{p}_2(\alpha, \alpha)] \cdot G(\alpha) \quad (26)$$

where $\bar{p}_1(\alpha, \alpha)$ and $\bar{p}_2(\alpha, \alpha)$ are the two partial derivatives of the equilibrium success probability $\bar{p}(\alpha, \beta)$, evaluated at $\beta = \alpha$. We obtain

$$G(\alpha) = \bar{p}(\alpha, \alpha) \cdot [u(y^H) - u(y^H - \bar{T})] + [1 - \bar{p}(\alpha, \alpha)] \cdot [u(y^L + \bar{T}) - u(y^L)].$$

Since $\bar{p}_1(\alpha, \alpha) > 0$ and $\bar{p}_2(\alpha, \alpha) < 0$, equation (26) implies that the conclusion of Proposition 11 holds up when the transfer is exogenously fixed: any evolutionarily stable degree of altruism α is necessarily lower than $1/2$ (under random mating and purely parental influence).

7.2 Repetition

Above, we have analyzed a two-stage interaction that takes place only once. An interesting extension would be to let this interaction occur repeatedly over time. Suppose that the interaction takes the form of an infinitely repeated game between two (altruistic or selfish) siblings, with equal discounting. Under perfect monitoring, any feasible and individually rational outcome (defined in terms of their potentially altruistic preferences) would then be achievable in subgame perfect equilibrium, if the siblings were sufficiently patient. Repeated play of the equilibrium of our baseline model would constitute one of these subgame perfect equilibria, for all discount factors. However, the standard repeated-games model might not be fully satisfactory. First, if output is storable, then stored output (wealth) would constitute a state-variable of potential strategic relevance, and hence the long-run game would not be a repeated game, but a stochastic game. Second, finite life spans and age-dependent conditional survival probabilities would tend to make siblings less patient as they grow older, leading to a reduction of the set of subgame-perfect equilibria, though presumably still containing repeated play of the baseline equilibrium analyzed here. Despite these complications, repetition is certainly an important aspect well worth analyzing: a subject that we leave for future studies.

7.3 Preference evolution vs. strategy evolution

We have analyzed the evolutionary stability of preferences, rather than of strategies, and we focused on a setup where the players of the game are not randomly matched. Each

of these two deviations from the standard approach has been analyzed separately before. The so-called indirect evolutionary approach (see, e.g., Güth and Peleg, 2001, and Heifetz, Shannon, and Spiegel, 2007) concerns the evolutionary stability of preferences under random matching, whereas Bergstrom (1995) analyzes the evolutionary stability of strategies when the players are siblings, so that their “strategy types” are correlated in a specific way.

To clarify how these approaches differ from ours, reconsider the two-stage interaction analyzed in Sections 2-4. Had we studied the evolutionary stability of preferences with randomly matched players, then pure selfishness would have been selected for, since mutants would be almost sure to be matched with incumbents. What if we had studied strategy evolution with non-randomly matched players? A pure strategy in this symmetric two-player game is an effort level and a transfer function that assigns transfers to each of the four second-period states. Assume that strategies, rather than degrees of altruism, are inherited from parent to child. Evolutionary stability of strategies would lead to a condition formally identical to (19), with the incumbent degree of altruism, α , replaced by an incumbent strategy, s , and with the mutant degree of altruism, α' , replaced by a mutant strategy, s' . This approach would differ from ours in two important respects. First, while, in our model, incumbents and mutants adapt their own behavior to their sibling’s “preference type,” the behavior of an incumbent or mutant in the strategy approach is independent of the sibling’s “strategy type.” Secondly, in our model, a mutant’s degree of altruism (given the sibling’s “preference type”) determines both the mutant’s transfer and effort. This restricts the set of possible mutant behaviors. By contrast, in a model where individuals inherit strategies, the set of possible mutant behaviors would be richer; any mutant effort could be combined with any transfer function. Due to these differences, it is a non-trivial matter whether the strategy approach would lead to the same evolutionarily stable behaviors as our approach. On the one hand, the richer set of mutant behaviors would make it “harder” for any incumbent behavior to be stable in the strategy approach. On the other hand, the inability of mutants to adjust their behavior would make it “easier” for an incumbent behavior to be stable in the strategy approach. We feel that the preference approach is more relevant than the strategy approach in applications to human siblings, who have the cognitive capacity and “social intelligence” to understand each other and adjust to each others’ personality.

7.4 Public insurance

In our analysis, risk was pooled within the family only. This was arguably the case for many societies throughout human history, prior to the advent of insurance arrangements beyond the family sphere. As such, the analysis indicates how intrafamily altruism may have evolved in the past. Here, we indicate how our analysis may be extended to allow for compulsory redistribution/insurance systems that pool the risks of a large group of individuals.

Suppose that after outputs are realized, but before transfers are made, each rich individual pays an income tax or insurance premium, $\Delta \cdot y^H$, and every poor individual receives a public transfer or indemnity (net of the premium), $\delta \cdot y^L > 0$. For exogenously given values of δ and Δ , our analysis can be directly applied, by way of replacing y^H with $(1 - \Delta) y^H$, and y^L with $(1 + \delta) y^L < (1 - \Delta) y^H$ in equations (7), (8), and (16). From the individual's viewpoint it is as if $\lambda = y^L/y^H$ were replaced by

$$\lambda_{\delta, \Delta} = \frac{1 + \delta}{1 - \Delta} \cdot \lambda > \lambda.$$

This has a disincentive effect: an increase in λ induces lower efforts. Moreover, the introduction of social insurance will *crowd out* intrafamily transfers, perhaps even eliminate them. Our evolutionary analysis suggests that an increase in λ , due to the introduction of public redistribution/insurance, may lead to a higher degree of sibling altruism, unless intrafamily transfers are completely crowded out. We also note that lower institutional quality may lead to stronger family ties. To see this, compare two identical societies with different insurance systems, say (δ, Δ) and (δ', Δ') with $\delta' < \delta$ and $\Delta' > \Delta$. The second society has lower institutional quality, in the sense that it taxes more and delivers less. According to our evolutionary comparative-statics analysis, family ties will tend to be stronger in the latter.

While each individual takes δ and Δ as exogenous and fixed when choosing effort, these parameters may, in a policy-formation process, depend on each other through a governmental budget-balance condition. From each dollar collected as a tax or insurance premium from the rich, let $\eta \in [0, 1]$ be the share that reaches the poor. The rest goes to administrators and government officials. If all individuals choose the same success probability p , the expected *per capita* tax revenue is $\eta p \Delta y^H$, while the expected *per capita* transfer or indemnity to those with low output is $(1 - p) \delta y^L$. Aggregate (expected) budget balance, given the “leakage” factor η , requires that these two amounts are equal, or, equivalently, that

$$p = \frac{\delta \lambda}{\eta \Delta + \delta \lambda}, \tag{27}$$

where $\lambda = y^L/y^H$. Replacing p by the equilibrium success probability in this equation determines the set of budget-balanced policies (δ, Δ) , given α and η . Let $M(\alpha, \eta)$ be the maximum expected material utility that may be achieved across the set of all feasible policies, given α and η . Clearly $M(\alpha, \eta)$ is increasing in the institutional quality parameter η , given α . As before, let $V^*(\alpha)$ be the expected material utility in our base-line model without public insurance/redistribution. For each α , let

$$\eta(\alpha) = \min \{ \eta \in [0, 1] : M(\alpha, \eta) \geq V^*(\alpha) \}.$$

This set defines the lowest institutional quality η for which the optimal public social insurance system is superior to *laissez-faire*. If $\eta(\alpha) > \eta(\alpha')$ for some α and α' , then a society consisting of α' -altruists will more likely adopt a public social insurance system, *ceteris paribus*, than a society with α -altruists, since the optimal public policy is superior to *laissez-faire* for a wider range of institutional qualities in the former than in the latter society. Preliminary numerical simulations suggest that harsher environments may induce lower degrees of sibling altruism and a wider range of parameters under which public social insurance is superior to *laissez-faire* — a topic for future research.

8 Conclusion

Family ties are arguably stronger in some parts of the world than in others, and there is empirical evidence that the strength of family ties may endure for several generations after changes in the environment.²⁸ This suggests that current differences in family ties may be due to differences in the distant past, perhaps even prior to the industrial revolution. These observations prompted us to ask first, how family ties affect economic outcomes, and, second, whether evolutionary forces may have shaped family ties differently in different environments. We focused on the family's role as sole insurance provider for its members. We modelled a family as a pair of mutually altruistic siblings, who may provide partial and informal insurance to each other by way of voluntary transfers. The siblings choose their risk-reducing efforts, anticipating possible future transfers between them. We found that,

²⁸U.S. data collected by Keefe et al, (1979) indicates that second and third generation Mexican-American (people of Mexican descent but born in the U.S.) families have stronger kinship ties than white Anglo families, even after controlling for many variables. Using another data set, Gonzales (1998) shows that Mexican-Americans tend to live closer to, and have more contact with, kin than Anglos, even after several generations in the U.S.

in our model, altruism is a double-edged sword: it enhances the extent to which individuals internalize the external effects of their actions, but it also enhances free-riding. We found that these two opposing effects of altruism on effort, the free-rider effect and the empathy effect, result in a risk-reducing effort that is non-monotonic in the extent of family risk sharing. The presence of the free-rider and empathy effects are, arguably, quite a general consequence of mutual altruism in situations that include some productive activity. How far one can generalize our result, that the empathy effect outweighs the free-rider effect at high levels of altruism while the opposite holds at intermediate levels of altruism, remains an open question.

For a given level of intrafamily altruism, we studied how environmental factors, such as output variability and the marginal return to effort, affect individual family members' productive efforts. We then studied how evolutionary forces, be they genetic or sociocultural, may affect the level of sibling altruism in a society. We showed that neither very weak nor very strong family ties are stable against evolutionary drift. We also showed that the degree of altruism dictated by Wright's degree of relationship ($1/2$ for siblings) is unstable. If a few individuals in such a society were to become slightly less altruistic towards their kin, these individuals would do better in terms of material utility. We showed how this generalizes Hamilton's rule (Propositions 10-11) from exogenous to endogenous production, and showed that intermediate degrees of family altruism are locally evolutionarily stable in many environments. Numerical simulations suggested that the harsher the environment, the weaker are the family ties.²⁹

Max Weber (1951) argued that "the great achievement of [...] the ethical and ascetic sects of Protestantism was to shatter the fetters of the sib [the extended family]. These religions established [...] a common ethical way of life in opposition to the community of blood, even to a large extent in opposition to the family." (p.237)³⁰ Likewise, Banfield (1958) thought that the "amoral familism" that he observed in certain parts of Italy was an impediment to economic development. Macfarlane (1978, 1992) argues that the individualism that developed in northwestern Europe (in particular in England) was key in favoring the advent of

²⁹The idea that individuals in a society may exhibit traits that depend on exogenous factors, such as climate, goes back to Montesquieu (1748) at least.

³⁰Some historians argue that religions may develop largely in response to underlying social factors, see, e.g., Ozment (1975, 1992). Social factors may in turn evolve in response to exogenous factors, perhaps along the lines suggested here.

markets and the industrial revolution.³¹ Our analysis suggests a reason why weaker family ties may have developed in the arguably often harsher climates in the Protestant countries, and how this may have induced stronger economic growth. This opens an avenue for future theoretical and empirical research.

The results reported here are indeed derived under heroic simplifications. The sibling interaction that we model is very simple and stylized. Its precise mathematical form, is, of course, only one of many possibilities. However, we believe it is canonical for the interplay between human effort and nature. Future work in this area may take many directions, including those discussed in the preceding section. Moreover, we have focused on the evolution of altruism in a society in an exogenous environment. Yet, many societies or groups compete with each other for resources, sometimes by way of armed conflicts. A group or society with a higher degree of internal altruism may then become materially richer and perhaps also more successful in such conflicts. There could thus be selection of altruism at two levels, along the lines proposed by Wilson and Wilson (2007). Using prehistoric evidence on violence, Bowles (2009) finds that conflicts between groups indeed were common enough to favor the evolution of altruism in such a fashion. Finally, it would be of interest to develop models that allow for more than two siblings, other kin-relations, and a richer menu of actions and outcomes. We hope that the notion of local evolutionary stability of preferences, along with the analytical machinery developed here, can be useful for further studies of social and economic interactions.

³¹Macfarlane (1992) cites many studies showing that several centuries ago a large fraction of youngsters in England were servants or laborers outside of the parental home, some leaving the parents at the age of 10. For instance, Kussmaul (1981) reports that as far back as 1380, more than half of men in East Anglian villages were employees (servants or labourers), and Hajnal (1982) provides similar evidence for the 17th-19th centuries. Thus, in 17th century England, “the unit of production was the husband and the wife and hired labor, not children” (Macfarlane, 1978). By contrast, in southern and eastern Europe, during the same period, hired labor was scarce and children would typically work on the parents’ farm.

9 Appendix

9.1 Proposition 1

Let $\hat{\tau}_i : \Omega \rightarrow [0, y^H]$ be the function that defines, for every state $\omega \in \Omega$, the transfer that individual i would like to make to his or her sibling *if the latter makes no transfer to i* . Then $\hat{\tau}_i(\omega) = 0$ if $u'(y_i) \geq \alpha_i u'(y_j)$ for $j \neq i$, otherwise the optimal transfer $\hat{\tau}_i(\omega)$ is positive and equates i 's marginal material utility to that of his sibling's when weighted by his own (i 's) degree of altruism:

$$u'(y_i - \hat{\tau}) = \alpha_i u'(y_j + \hat{\tau}). \quad (28)$$

Thus:

$$\hat{\tau}_i(\omega) = \max\{0, \hat{\tau}\}, \quad (29)$$

where $\hat{\tau}$ is uniquely defined by (28). Since the material utility function is separable in consumption and effort, efforts play no role when determining the transfers, only outputs matter. The claims in Proposition 1 follow from the following lemma:

Lemma 2 *For each $\omega \in \Omega$, the transfer pair $(\hat{\tau}_A(\omega), \hat{\tau}_B(\omega))$ constitutes a Nash equilibrium of $G(\omega)$. If $\alpha_A \alpha_B < 1$, then this equilibrium is unique. If $\alpha_A = \alpha_B = 1$, then there is a continuum of Nash equilibria, all resulting in equal sharing of the total output.*

Proof: We first prove that if $\alpha_A \alpha_B < 1$, in any equilibrium at most one transfer is strictly positive. Suppose that $\alpha_A \alpha_B < 1$, and that (t_A, t_B) is a Nash equilibrium with $t_A, t_B > 0$. Then the following two first-order conditions must both hold:

$$u'(y_A - t_A + t_B) = \alpha_A u'(y_B + t_A - t_B)$$

$$u'(y_B + t_A - t_B) = \alpha_B u'(y_A - t_A + t_B)$$

and hence

$$u'(y_A - t_A + t_B) = \alpha_A \alpha_B u'(y_A - t_A + t_B)$$

implying $\alpha_A \alpha_B = 1$, contradicting our hypothesis that $\alpha_A \alpha_B < 1$.

Suppose that $\alpha_A \alpha_B < 1$. Then the previous observation, together with (29), implies that $(t_A, t_B) = (\hat{\tau}_A(\omega), \hat{\tau}_B(\omega)) = (0, 0)$ is the unique equilibrium in the two states ω in which $y_A = y_B$. The previous observation further implies that, in the states ω where $y_A \neq y_B$, the individual with the low output y^L gives no transfer, since $u'(y^L - \hat{\tau}) > \alpha_i u'(y^H + \hat{\tau})$

for all $\alpha_i \in [0, 1]$ and $\hat{\tau} \geq 0$. Hence, if $y_A > y_B$, the unique Nash equilibrium of $G(\omega)$ is $(t_A, t_B) = (\hat{\tau}_A(\omega), 0)$, and, if $y_B > y_A$, it is $(t_A, t_B) = (0, \hat{\tau}_B(\omega))$.

Suppose now that $\alpha_A \alpha_B = 1$; then it is straightforward to verify the following claims:

- if $y_A > y_B$ any $(t_A, t_B) = (\hat{\tau}_A(\omega) + \varepsilon, \varepsilon)$ is a Nash equilibrium of $G(\omega)$ for all $\varepsilon \in (0, y_A - \hat{\tau}_A(\omega))$
- if $y_B > y_A$ any $(t_A, t_B) = (\varepsilon, \hat{\tau}_B(\omega) + \varepsilon)$ is a Nash equilibrium of $G(\omega)$ for all $\varepsilon \in (0, y_B - \hat{\tau}_B(\omega))$
- if $y_A = y_B$ any $(t_A, t_B) = (\varepsilon, \varepsilon)$ is a Nash equilibrium of $G(\omega)$ for all $\varepsilon \in [0, y_A]$.

9.2 Proposition 2

The first-order condition (8) implicitly defines the transfer t as a differentiable function of λ . An application of the implicit function theorem gives

$$\frac{dt}{d\lambda} = -\frac{\alpha u''(\lambda y^H + t)}{\alpha u''(\lambda y^H + t) + u''(y^H - t)} \cdot y^H,$$

where, by strict concavity of u , the ratio on the right-hand side is a number in the open unit interval. Hence

$$\frac{d(\lambda y^H + t)}{d\lambda} = \left[1 - \frac{\alpha u''(\lambda y^H + t)}{\alpha u''(\lambda y^H + t) + u''(y^H - t)} \right] \cdot y^H > 0,$$

and

$$\frac{d(y^H - t)}{d\lambda} = \frac{\alpha u''(\lambda y^H + t)}{\alpha u''(\lambda y^H + t) + u''(y^H - t)} \cdot y^H > 0.$$

9.3 Proposition 3

First, assume that $\alpha_A, \alpha_B < \hat{\alpha}$. Then $T(\alpha_A) = T(\alpha_B) = 0$, and inspection of (14) shows that the equation system (13) is independent of α_A and α_B . Hence, its solution set is unaffected by a marginal increase in any one, or both, of these parameters.

Second, assume that condition (15) is met. Then the Jacobian of the equation system (13) is non-null, a condition, which, by the Inversion Theorem (see, e.g., Theorem 41.8 in Bartle, 1976), guarantees local uniqueness of the solution to (3). Suppose that $\alpha_i > \hat{\alpha}$, and consider an increase in α_i .

Step 1: First, we prove that, for each success probability of the other individual, p_j , individual i 's best response is strictly increasing in α_i . From (13) and noting that $(\psi')^{-1}$ is an increasing function, this claim holds if

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_i} > 0.$$

Using the first-order condition (8) for the transfer $T(\alpha_i)$, we obtain:

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_i} = (1 - p_j) \cdot [u(y^L + T(\alpha_i)) - u(y^L)] + p_j \cdot [u(y^H) - u(y^H - T(\alpha_j))]. \quad (30)$$

The expression on the right-hand side is positive, since $\alpha_i > \hat{\alpha}$ implies $T(\alpha_i) > 0$.

Step 2: Secondly, we prove that, for each success probability p_i , individual j 's best response is strictly decreasing in α_i . For this claim, it is sufficient to show that:

$$\frac{\partial g(p_i, \alpha_j, \alpha_i)}{\partial \alpha_i} < 0.$$

Using the first-order condition (8) for the transfer $T(\alpha_j)$, we obtain:

$$\frac{\partial g(p_i, \alpha_j, \alpha_i)}{\partial \alpha_i} = -p_i \cdot (1 - \alpha_i \alpha_j) \cdot u'(y^L + T(\alpha_i)) T'(\alpha_i). \quad (31)$$

The expression on the right-hand side is strictly negative, since $\alpha_i > \hat{\alpha}$.

In sum: for $\alpha_i > \hat{\alpha}$ an increase in α_i causes an upward shift in i 's success-probability reaction function, and a downward shift in j 's success-probability reaction function. In the case of a unique equilibrium, the best-response curves intersect in such a way that either shift is sufficient, *per se*, to yield that p_i^* increases and p_j^* decreases.

9.4 Proposition 4

To establish the uniqueness claim, note that, by hypothesis, the left-hand side of (16) is continuous and increasing from zero to plus infinity, while the right-hand side is a decreasing affine function with positive intercept. The latter property becomes transparent after some algebraic manipulation: equation (16) can be written in the form

$$\psi'(p) = a - (1 + \alpha)bp \quad (32)$$

for

$$a = u(y^H - T(\alpha)) - u(y^L) + \alpha [u(y^L + T(\alpha)) - u(y^L)]$$

and

$$b = u(y^L + T(\alpha)) - u(y^L) - [u(y^H) - u(y^H - T(\alpha))],$$

where $a, b > 0$. That a is positive follows from our earlier observation that a donor remains richer than the recipient, $u(y^H - T(\alpha)) > u(y^L)$. That b is positive follows from the concavity of u , implying that the recipient's material utility increases more from the transfer than the donor's material utility decreases.

9.5 Proposition 5

Using equation (16), we obtain

$$\begin{aligned} \frac{dp^*}{d\alpha} &= \frac{(1-p^*)}{K} \cdot [u(y^L + T(\alpha)) - u(y^L)] + \frac{p^*}{K} \cdot [u(y^H) - u(y^H - T(\alpha))] \\ &\quad - \frac{p^*(1-\alpha^2)}{K} \cdot u'(y^L + T(\alpha)) T'(\alpha) \end{aligned} \quad (33)$$

where

$$K = \psi''(p^*) + (1+\alpha) ([u(y^L + T(\alpha)) - u(y^L)] - [u(y^H) - u(y^H - T(\alpha))]). \quad (34)$$

As $\alpha \downarrow \hat{\alpha}$, at which point p^* is not differentiable, the first two terms in (33) both tend to zero, while the third term is positive. Since it is to be subtracted, we conclude that $dp^*/d\alpha < 0$ for all $\alpha > \hat{\alpha}$ close to $\hat{\alpha}$. Likewise, as $\alpha \uparrow 1$, the third term tends to zero while the first two are positive. Hence, $dp^*/d\alpha > 0$ for all $\alpha < 1$ close to 1.

9.6 Proposition 6

Using equation (16), we obtain

$$\begin{aligned} \frac{dp^*}{d\lambda} &= -\frac{(1-p^*)\alpha y^H}{K} \cdot [u'(\lambda y^H) - u'(\lambda y^H + t)] - \frac{(1-p^*)y^H}{K} \cdot u'(\lambda y^H) \\ &\quad - \frac{p^*\alpha^2 y^H}{K} \cdot u'(\lambda y^H + t) - \frac{p^*(1-\alpha^2)}{K} \left[y^H + \frac{dt}{d\lambda} \right] \cdot u'(\lambda y^H + t), \end{aligned}$$

where t is defined in (8) and $K > 0$ in (34). Since u is strictly increasing and concave, and $|dt/d\lambda| < y^H$ (see proof of Proposition 2), $dp^*/d\lambda < 0$.

9.7 Proposition 7

We proceed in two steps. First, we characterize the socially optimal probability p and transfer t , to be given by the rich to the poor, under a Benthamite social welfare function. Secondly, we verify that these coincide with the equilibrium probabilities p_A^* and p_B^* , and transfers $T(\alpha_A)$ and $T(\alpha_B)$ if and only if $\alpha_A = \alpha_B = 1$.

Step 1: Consider a hypothetical social planner who chooses a probability p and transfer t so as to maximize the expected material utility of one individual (since they are *ex ante* identical, this also maximizes the sum of their material utilities):

$$\begin{aligned} W(p, t) = & [p^2 u(y^H) + (1-p)^2 u(y^L)] \\ & + p(1-p) [u(y^H - t) + u(y^L + t)] - \psi(p). \end{aligned} \quad (35)$$

The necessary first-order condition for an interior solution for p is

$$2pu(y^H) - 2(1-p)u(y^L) + (1-2p)[u(y^H - t) + u(y^L + t)] = \psi'(p). \quad (36)$$

Moreover, for any value of p , the value of t that maximizes $W(p, t)$ is such that both individuals end up with the same consumption in all states: $y^H - t = y^L + t$.

Step 2: When positive, the equilibrium transfer satisfies (8). Strict concavity of u implies that $y^H - T(\alpha) = y^L + T(\alpha)$ if and only if $\alpha = 1$. Hence, $\alpha = 1$ is a necessary condition for the equilibrium outcome to coincide with the Benthamite optimum. It is also a sufficient condition, since the first-order condition that defines the equilibrium success probability p^* , equation (16), for $T(\alpha) = (1-\lambda)y^H/2$, coincides with (36), the necessary first-order condition for an interior social optimum, if and only if $\alpha = 1$.

9.8 Corollary 1

Given the symmetry of the unique equilibrium outcome, this is Pareto efficient if and only if it maximizes the sum of both individuals' expected welfare, as defined in equation (6). If each individual chooses a success probability p and gives a transfer t when rich and the other is poor, the mentioned sum is $S(p, t) = (1+\alpha)W(p, t)$, where $W(p, t)$ is defined in (35). For any $\alpha \in [0, 1]$, $S(p, t)$ is clearly strictly increasing in $W(p, t)$. But, by Proposition 7, the equilibrium expected material utility V^* coincides with the maximum value of $W(p, t)$ if and only if $\alpha = 1$.

9.9 Proposition 8

The claim in the proposition holds if

$$\lim_{\alpha \downarrow \hat{\alpha}} \left[\frac{\partial V(\alpha, \beta)}{\partial \alpha} + \frac{\partial V(\alpha, \beta)}{\partial \beta} \right]_{|\beta=\alpha} > 0,$$

where $V(\alpha, \beta)$, defined in (18), is the expected equilibrium material utility to an individual whose degree of altruism is α and whose sibling's degree of altruism is β . The corresponding success probabilities, $p(\alpha, \beta)$ and $p(\beta, \alpha)$, necessarily satisfy the following system of first-order equations, a generalization of (13):

$$\begin{cases} \psi' [p(\alpha, \beta)] = [1 - p(\beta, \alpha)] \cdot (u[y^H - T(\alpha)] + \alpha u[y^L + T(\alpha)] - [u(y^H) + \alpha u(y^L)]) \\ -p(\beta, \alpha) \cdot (u[y^L + T(\beta)] + \alpha u[y^H - T(\beta)] - [u(y^L) + \alpha u(y^H)]) + [u(y^H) - u(y^L)] \\ \psi' [p(\beta, \alpha)] = [1 - p(\alpha, \beta)] \cdot (u[y^H - T(\beta)] + \beta u[y^L + T(\beta)] - [u(y^H) + \beta u(y^L)]) \\ -p(\alpha, \beta) \cdot (u[y^L + T(\alpha)] + \beta u[y^H - T(\alpha)] - [u(y^L) + \beta u(y^H)]) + [u(y^H) - u(y^L)]. \end{cases} \quad (37)$$

Letting V_1 and V_2 denote the partial derivatives of V , with respect to the first and second argument, respectively, and likewise, using p_1 and p_2 to denote the partial derivatives of p , with respect to the first and second argument, respectively, we get:

$$\begin{aligned} V_1(\alpha, \beta) &= [p(\beta, \alpha) p_1(\alpha, \beta) + p(\alpha, \beta) p_2(\beta, \alpha)] u(y^H) \\ &\quad - ([1 - p(\beta, \alpha)] p_1(\alpha, \beta) + [1 - p(\alpha, \beta)] p_2(\beta, \alpha)) u(y^L) \\ &\quad + ([1 - p(\beta, \alpha)] p_1(\alpha, \beta) - p(\alpha, \beta) p_2(\beta, \alpha)) u[y^H - T(\alpha)] \\ &\quad - (p(\beta, \alpha) p_1(\alpha, \beta) - [1 - p(\alpha, \beta)] p_2(\beta, \alpha)) u[y^L + T(\beta)] \\ &\quad - p(\alpha, \beta) [1 - p(\beta, \alpha)] u' [y^H - T(\alpha)] T'(\alpha) \\ &\quad - \psi' [p(\alpha, \beta)] p_1(\alpha, \beta) \end{aligned} \quad (38)$$

and

$$\begin{aligned} V_2(\alpha, \beta) &= [p(\beta, \alpha) p_2(\alpha, \beta) + p(\alpha, \beta) p_1(\beta, \alpha)] u(y^H) \\ &\quad - ([1 - p(\beta, \alpha)] p_2(\alpha, \beta) + [1 - p(\alpha, \beta)] p_1(\beta, \alpha)) u(y^L) \\ &\quad + ([1 - p(\beta, \alpha)] p_2(\alpha, \beta) - p(\alpha, \beta) p_1(\beta, \alpha)) u[y^H - T(\alpha)] \\ &\quad - (p(\beta, \alpha) p_2(\alpha, \beta) - [1 - p(\alpha, \beta)] p_1(\beta, \alpha)) u[y^L + T(\beta)] \\ &\quad + p(\beta, \alpha) [1 - p(\alpha, \beta)] u' [y^L + T(\beta)] T'(\beta) \\ &\quad - \psi' [p(\alpha, \beta)] p_2(\alpha, \beta). \end{aligned} \quad (39)$$

From the equation system (37) we have:

$$\begin{aligned}
\psi' [p(\alpha, \beta)] &= p(\beta, \alpha) (1 + \alpha) u(y^H) \\
&\quad - [1 - p(\beta, \alpha)] (1 + \alpha) u(y^L) \\
&\quad + [1 - p(\beta, \alpha)] [u(y^H - T(\alpha)) + \alpha u(y^L + T(\alpha))] \\
&\quad - p(\beta, \alpha) [u(y^L + T(\beta)) + \alpha u(y^H - T(\beta))].
\end{aligned}$$

Using this to replace $\psi' [p(\alpha, \beta)]$ in (38) and (39), and simplifying, yields

$$\begin{aligned}
V_1(\alpha, \beta) &= [p(\alpha, \beta) p_2(\beta, \alpha) - \alpha p(\beta, \alpha) p_1(\alpha, \beta)] u(y^H) \\
&\quad - ([1 - p(\alpha, \beta)] p_2(\beta, \alpha) - \alpha [1 - p(\beta, \alpha)] p_1(\alpha, \beta)) u(y^L) \\
&\quad - p(\alpha, \beta) p_2(\beta, \alpha) u[y^H - T(\alpha)] \\
&\quad + p(\beta, \alpha) p_1(\alpha, \beta) \alpha u[y^H - T(\beta)] \\
&\quad + [1 - p(\alpha, \beta)] p_2(\beta, \alpha) u[y^L + T(\beta)] \\
&\quad - [1 - p(\beta, \alpha)] p_1(\alpha, \beta) \alpha u[y^L + T(\alpha)] \\
&\quad - p(\alpha, \beta) [1 - p(\beta, \alpha)] u' [y^H - T(\alpha)] T'(\alpha)
\end{aligned}$$

and

$$\begin{aligned}
V_2(\alpha, \beta) &= [p(\alpha, \beta) p_1(\beta, \alpha) - \alpha p(\beta, \alpha) p_2(\alpha, \beta)] u(y^H) \\
&\quad - ([1 - p(\alpha, \beta)] p_1(\beta, \alpha) - \alpha [1 - p(\beta, \alpha)] p_2(\alpha, \beta)) u(y^L) \\
&\quad - p(\alpha, \beta) p_1(\beta, \alpha) u[y^H - T(\alpha)] \\
&\quad + p(\beta, \alpha) p_2(\alpha, \beta) \alpha u[y^H - T(\beta)] \\
&\quad + [1 - p(\alpha, \beta)] p_1(\beta, \alpha) u[y^L + T(\beta)] \\
&\quad - [1 - p(\beta, \alpha)] p_2(\alpha, \beta) \alpha u[y^L + T(\alpha)] \\
&\quad + p(\beta, \alpha) [1 - p(\alpha, \beta)] u' [y^L + T(\beta)] T'(\beta).
\end{aligned}$$

Evaluating these two expressions at $(\alpha, \beta) = (\alpha, \alpha)$, and rearranging terms, we obtain

$$\begin{aligned}
V_1(\alpha, \alpha) &= p(\alpha, \alpha) [p_2(\alpha, \alpha) - \alpha p_1(\alpha, \alpha)] (u(y^H) - u[y^H - T(\alpha)]) \\
&\quad + [1 - p(\alpha, \alpha)] [p_2(\alpha, \alpha) - \alpha p_1(\alpha, \alpha)] (u[y^L + T(\alpha)] - u(y^L)) \\
&\quad - p(\alpha, \alpha) [1 - p(\alpha, \alpha)] u' [y^H - T(\alpha)] T'(\alpha)
\end{aligned} \tag{40}$$

and

$$\begin{aligned}
V_2(\alpha, \alpha) &= p(\alpha, \alpha) [p_1(\alpha, \alpha) - \alpha p_2(\alpha, \alpha)] (u(y^H) - u[y^H - T(\alpha)]) \\
&\quad + [1 - p(\alpha, \alpha)] [p_1(\alpha, \alpha) - \alpha p_2(\alpha, \alpha)] (u[y^L + T(\alpha)] - u(y^L)) \\
&\quad + p(\alpha, \alpha) [1 - p(\alpha, \alpha)] u' [y^L + T(\alpha)] T'(\alpha).
\end{aligned} \tag{41}$$

Finally, using the first-order equation (8) that defines $T(\alpha)$ for $\alpha > \hat{\alpha}$, and rearranging terms, we get:

$$\begin{aligned} V_1(\alpha, \alpha) + V_2(\alpha, \alpha) &= (1 - \alpha) [p_1(\alpha, \alpha) + p_2(\alpha, \alpha)] \cdot p(\alpha, \alpha) [u(y^H) - u(y^H - T(\alpha))] \\ &\quad (1 - \alpha) [p_1(\alpha, \alpha) + p_2(\alpha, \alpha)] \cdot [1 - p(\alpha, \alpha)] [u(y^L + T(\alpha)) - u(y^L)] \\ &\quad + p(\alpha, \alpha) [1 - p(\alpha, \alpha)] (1 - \alpha) u'(y^L + T(\alpha)) T'(\alpha). \end{aligned}$$

This tends to a positive number as α tends to $\hat{\alpha}$ from above, since the first two terms then tend to zero while the last term tends to a positive number.

9.10 Lemma 1

Using the expressions in (40) and (41) for the partial derivatives V_1 and V_2 , we obtain from (20):

$$\begin{aligned} D(\alpha) &= \left[V_1(\alpha, \alpha) + \frac{1}{2} V_2(\alpha, \alpha) \right] \\ &= p(\alpha, \alpha) [1 - p(\alpha, \alpha)] \left[\frac{1}{2} u'(y^L + T(\alpha)) - u'(y^H - T(\alpha)) \right] T'(\alpha) \\ &\quad + \left[\left(\frac{1}{2} - \alpha \right) p_1(\alpha, \alpha) + \left(1 - \frac{\alpha}{2} \right) p_2(\alpha, \alpha) \right] \cdot \\ &\quad \cdot [p(\alpha, \alpha) (u(y^H) - u(y^H - T(\alpha))) + [1 - p(\alpha, \alpha)] (u(y^L + T(\alpha)) - u(y^L))] \\ &= \left(\frac{1}{2} - \alpha \right) \cdot p(\alpha, \alpha) [1 - p(\alpha, \alpha)] u'(y^L + T(\alpha)) T'(\alpha) \\ &\quad + \left[\left(\frac{1}{2} - \alpha \right) p_1(\alpha, \alpha) + \left(1 - \frac{\alpha}{2} \right) p_2(\alpha, \alpha) \right] \cdot \\ &\quad \cdot [p(\alpha, \alpha) (u(y^H) - u(y^H - T(\alpha))) + [1 - p(\alpha, \alpha)] (u(y^L + T(\alpha)) - u(y^L))]. \end{aligned}$$

9.11 Proposition 10

The result follows from Proposition 9, together with the observation that, for any $\alpha < \hat{\alpha}$, $D(\alpha) = 0$ (simply set $T(\alpha) = T'(\alpha) = 0$ in (21)).

9.12 Proposition 11

Recall that $T(\alpha) = 0$ for all $\alpha \leq \hat{\alpha}$, and that $T(\alpha)$ tends to zero as $\alpha \downarrow \hat{\alpha}$. Recall also that $p_1 > 0$ and $p_2 < 0$ for all $\alpha > \hat{\alpha}$, see Proposition 3. The drift function D being continuous

at each $\alpha \in \mathcal{A}$, we obtain, by applying the definition of local evolutionary stability in Proposition 9:

1. Suppose that $\lambda \geq \frac{1}{y^H} (u')^{-1} [2u'(y^H)]$, so that $\hat{\alpha} \geq 1/2$: then there exists no locally evolutionarily robust degree of altruism, since $D(\alpha) = 0$ for any $\alpha < \hat{\alpha}$, and $\alpha > \hat{\alpha} \geq 1/2 \Rightarrow D(\alpha) < 0$.
2. Suppose that $\lambda < \frac{1}{y^H} (u')^{-1} [2u'(y^H)]$, so that $\hat{\alpha} < 1/2$: then if there exists a locally evolutionarily robust degree of altruism $\alpha \in \mathcal{A}$, it must be that $\alpha \in (\hat{\alpha}, 1/2)$, since $\lim_{\alpha \downarrow \hat{\alpha}} D(\alpha) > 0$ (because $G(\alpha)$, *i.e.*, the very last term in the expression for D above, tends to 0 as $\alpha \downarrow \hat{\alpha}$), and $D(1/2) < 0$.

9.13 Corollary 2

Assume that $\lambda < \hat{\lambda}$ (so that $\hat{\alpha} < 1/2$) and that the success probabilities are exogenously fixed and equal: $p_A = p_B = p \in (0, 1)$. For every $\alpha \in \text{int}(\mathcal{A})$, we then have

$$D(\alpha) = p(1-p) \cdot \left[\frac{1}{2} u' [y^L + T(\alpha)] - u' [y^H - T(\alpha)] \right] T'(\alpha). \quad (42)$$

Since $T'(\alpha) = 0$ for all $\alpha < \hat{\alpha}$, by Proposition 9 no $\alpha \leq \hat{\alpha}$ is locally evolutionarily stable. Turning now to $\alpha > \hat{\alpha}$, $T'(\alpha) > 0$ for all $\alpha > \hat{\alpha}$ and $T(\alpha)$ satisfies the first-order condition (8). Hence, for every $\alpha \in \text{int}(\mathcal{A})$ exceeding $\hat{\alpha}$:

$$D(\alpha) = p(1-p) \cdot \left(\frac{1}{2} - \alpha \right) u' [y^L + T(\alpha)] T'(\alpha).$$

By Proposition 9, this implies that α is locally evolutionarily stable if and only if $\alpha = 1/2$. From (18) we obtain that $\mathcal{A} = \{\alpha \in [0, 1] : \alpha \neq \hat{\alpha}\}$. In particular, $(\hat{\alpha}, 1) \subset \text{int}(\mathcal{A})$. Hence, $\alpha = 1/2$ is the only locally stable degree of altruism.

9.14 Proposition 12

Recall the following properties:

- for all $\alpha \leq \hat{\alpha}$: $T(\alpha) = 0$, and hence $F(\alpha) = G(\alpha) = 0$
- for all $\alpha > \hat{\alpha}$: $T(\alpha) > 0$ and $T'(\alpha) > 0$, and hence $F(\alpha) > 0$ and $G(\alpha) > 0$

- for all $\alpha > \hat{\alpha}$: $p_1 > 0$ and $p_2 < 0$ (see Proposition 3)
- $p_1 + p_2 > 0$ for α slightly below 1 (see Proposition 5).

The drift function D being continuous at each $\alpha \in A$, we obtain, by applying the definition of local evolutionary stability in Proposition 9:

1. There exists no locally evolutionarily robust degree of altruism $\alpha < \hat{\alpha}$, since $D(\alpha) = 0$ for all $\alpha < \hat{\alpha}$.
2. Assume now that $\kappa < 1$, and consider the two cases:
 - (a) $\hat{\alpha} > \frac{1+\kappa}{2}$: then there exists no locally evolutionarily robust degree of altruism, since $\alpha > \hat{\alpha} \geq \frac{1+\kappa}{2} \Rightarrow D(\alpha) < 0$.
 - (b) $\hat{\alpha} < \frac{1+\kappa}{2}$: then if there exists a locally evolutionarily robust degree of altruism $\alpha \in \mathcal{A}$, then it must be that $\alpha \in (\hat{\alpha}, \frac{1+\kappa}{2})$, since $\lim_{\alpha \downarrow \hat{\alpha}} D(\alpha) > 0$ (because $G(\alpha)$ tends to 0 as $\alpha \downarrow \hat{\alpha}$), and $D(\frac{1+\kappa}{2}) < 0$.
3. Assume now that $\kappa = 1$. Then $D(\alpha) = (1 - \alpha) [F(\alpha) + (p_1(\alpha, \alpha) + p_2(\alpha, \alpha)) \cdot G(\alpha)]$. The properties recalled above imply that $D(\alpha) = 0$ if $\alpha = 1$, and that $D(\alpha) > 0$ for α close to 1.
4. An increase in κ leads to an upward shift in $D(\alpha)$ for all $\alpha \in (\hat{\alpha}, 1)$. Hence, if there exists a locally evolutionarily robust degree of altruism $\alpha \in (\hat{\alpha}, 1)$, then it is strictly increasing in κ .

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