Who Really Benefits from Pension Systems? When Life Expectancy Matters*

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Abstract

Using an overlapping generations model with a small open economy, we explain why the life expectancy differential can change the redistributive properties of unfunded pension systems. We use the concept of "net contribution" to measure this redistributivity of pension systems. We show that Beveridgian pension systems remain progressive. However, the poorest do not necessarily benefit the most from pension systems. For Bismarckian pension systems, net contributions are regressive. For mixed pension systems, it is possible that collected resources are redistributed in favour of the ends of the distribution of wages.

Keywords: Pension system, inequality, length of life, net contribution.

JEL Classification: H55.

Qui Bénéficie Réellement des Systèmes de Retraite ? Où l'Espérance de Vie Compte

Abstract

Une importante littérature empirique montre que l'espérance de vie dépend du niveau de salaire. En utilisant un modèle à générations imbriquées avec une petite économie ouverte, nous expliquons pourquoi ce résultat peut changer les propriétés redistributives des systèmes de retraite par répartition. Nous utilisons le concept de "contribution nette" pour mesurer cette redistributivité des systèmes de retraite. Nous montrons alors que les systèmes Beveridgiens restent progressifs mais que les plus pauvres ne bénéficient pas le plus de ces systèmes de retraite. Inversement, les systèmes Bismarckiens sont régressifs. Cela implique un transfert de resources des agents les plus pauvres vers les agents les plus riches. Quant aux systèmes mixtes, i.e. à la fois Beveridgiens et Bismarckiens, ils peuvent impliquer un transfert de resources des classes moyennes vers les plus pauvres et les plus riches.

Mots clés: Système de retraite, inégalité, espérance de vie, contribution nette.

1 Introduction

A wide literature studies the macroeconomic impact of the ageing of the population, notably on the sustainability of pension systems[1]. This ageing process implies that reforms have to be adopted by developed countries in order to limit the fiscal burden of pension systems. Another dimension of the length of life has been explored recently. Indeed, agents differ by their life expectancy. More particulary, these differences between socio-professional groups are wide. Mesrine (1999) studies the inequalities of length of life according to socio-professional groups in France[2]. The most striking feature of his paper is that a worker has a probability to die between 35 and 65 almost twice higher than that of an executive manager. Furthermore, their life expectancy at 35 is 38 and 44 respectively. The same qualitative results are observed in the United-States (Panis and Lillard 1995, Deaton and Paxson 2000). Finally, Robert-Bobbée and Cadot (2007) show that this inequality is also observed for elderly people. For agents who are 86, the ones with highest education level can expect to live 20% longer than the ones with lowest education level.

As socio-professional groups are linked to earnings, we can conclude from the previous results that earnings have an impact on the length of life. Some empirical studies deal with this link[3]. This life expectancy differential can have strong implications for the redistributive properties of pension systems. Indeed, as rich agents live and benefit from a pension for a longer period of time, pension systems are not as redistributive as they seem. Consequently, to study these properties, two dimensions have to be analyzed: (1) the distinction between Beveridgian and Bismarckian systems, and (2) the relationship between wages and life expectancy. Some authors have studied the empirical implications of this distinction. Coronado *et al.* (2000) and Liebman (2001) show that the pension system of the United-States is far less progressive than is usually mentioned. Legros (1994) and Bommier *et al.* (2003) find the same qualitative results on french data.

However, theoretical implications of these new results have not been clearly studied. Mitchell and Zeldes (1996) explain that:

"Despite its intent, the system [the pension system] is less progressive than it might

[1] See d'Autume (2003), Charpin (1999), Cremer and Pestieau (2000) or Galasso and Profeta (2004) among others. Here and in the rest of this chapter we only consider Pay-As-You-Go pension systems.

[2] These inequalities also depend on other factors such as sex or the geographical localization. For example, in France, the life expectancy of women is 84.1, whereas that of men is only 77.2 (INSEE, 2006). Moreover, Rican and Salem (1999) show that there are strong disparities according to the localization of people in France.

[3] See Attanasio and Emmerson (2001), Bommier *et al.* (2003) or Adams *et al.* (2003) for a survey.

seem, because there is a positive correlation between lifetime earnings and length of life."

They emphasize the main role played by the inequalities of length of life but this sentence raises a question: can a pension system be regressive?

Borck (2007) has exploited this idea. He shows that the size of a pension system can be determined by a coalition of elderly, very poor and very rich agents. Poor agents benefit from the Beveridgian part of the pension system, whereas rich agents benefit from the pension system for the longest period of time. This paper of Borck can be seen as a study on the consequences of the redistributive properties of pension systems when inequalities of length of life are taken into account. Nevertheless, it is not an analysis of the redistributivity itself. Consequently, this chapter is the next step to clarify this last point analytically.

A pension system is purely Beveridgian if every agent receives the same pension. Conversely, a pension system is purely Bismarckian if pensions completely depend on the wages of agents. A pension system is mixed if it has a Beveridgian and a Bismarckian component. The more Beveridgian a pension system is, the higher intra-generational transfers are. Countries highly differ by this intra-generational component. France, Germany and Italy have a Bismarckian structure. Canada, the Netherlands and New-Zeland are essentially Beveridgian. Finally, Japan, the United-Kingdom and the United States have mixed pension systems (Sommacal 2006, Casamatta *et al.* 2000).

In this chapter, we use an overlapping generations model in which agents differ by their wage and by their length of life. The pension system can be Beveridgian, Bismarckian or a mix of the two as in Casamatta *et al.* (2000). To study the redistributivity of pension systems we use the concept of "net contributions". The net contribution of an agent with a given wage, is the actualized difference (at the growth rate of the population) between the tax paid and the benefit from the pension system (Drouhin 2001). A positive net contribution implies that agents pay more than they receive from the pension system.

In order to understand the qualitative changes induced by the inequalities of length of life we first consider that every agent has the same length of life. In this specific case, it is possible to show that if pension systems are at least partially Beveridgian then it is also progressive. But if the pension system is Bismarckian then the net contribution for each agent is nil. Afterwards, we introduce inequalities of length of life and we consider the case of a Beveridgian, a Bismarckian and of a mixed pension system successively. If the pension system is Beveridgian then it is progressive, but the poorest do not necessarily benefit the most from the pension system. Furthermore, the share of the population who benefits from a negative net contribution changes.

If the pension system is Bismarckian, then we show that it is regressive because the poorest have the most positive net contribution and the richest the most negative one. If we have a mixed pension system our analytical results in terms of net contributions can be generalized only if pension systems tend towards either a Beveridgian or a Bismarckian structure. For intermediate cases, a numerical resolution calibrated on French data is used. It does not describe the exact structure of the French pension system but it emphasizes its important qualitative properties. We use different calibrations for the function which links the length of life to the wage level. We show that it is possible that the ends of the distribution of wages benefit from a negative net contribution.

This chapter is organized as follows. In section 2, we present our model. In section 3, the study of the redistributive properties of pension systems without inequalities of length of life is detailed. In the following sections we assume that the length of life is linked to the wage level. In section 4 and 5, we study a Beveridgian and a Bismarckian pension system respectively. In section 6, we emphasize the main properties of mixed pension systems. In section 7 we calibrate our model on french data. Section 8 provides some concluding remarks.

2 The Model

We consider a small open economy in which agents live two periods[4]. At each period t, the number of young agents is N_t. The population is assumed to grow at a constant rate n, such that $N_t = (1+n)N_{t-1}$. These agents are heterogenous since each of them has a wage w which belongs to the interval $\Omega_w = [w_-, w_+]$, with $w_- > 0$. Wages are distributed randomly among the population. $f(w)$ denotes the density function of the random variable w. Consequently, it is also the fraction of the population having a wage level w[5]. The average wage of this economy can be written:

$$\bar{w} = \int_{\Omega_w} w f(w) dw \tag{1}$$

Our framework is static because we assume that wages and the distribution of wages are constant over time.

Furthermore, we assume that agents live only a fraction T of their second period of life[6].

[4] The length of each period is normalized to 1.

[5] We assume that the size of each generation is sufficiently large to apply the law of large numbers.

[6] There is no uncertainty in our model. Consequently agents are sure to live until the end of the fraction T of their second period of life. However, our model can also be interpreted as a model in which $(1-T)$ would be the probability of dying at the end of the first period of life as in Drouhin (2001).

This length of life is supposed to be linked to the wage level of agents: $T = T(w)$, and more specifically we assume that $T'(w) > 0$. It represents the inequalities of length of life according to socio-professional groups. The average length of life is:

$$\bar{T} = \int_{\Omega_w} T(w) f(w) dw \tag{2}$$

The linkage between the wage level and the length of life is measured by the covariance:

$$COV_{T,w} = \int_{\Omega_w} T(w) w f(w) dw - \bar{T}\bar{w} \tag{3}$$

As we assume that $T'(w) > 0$, then we have $COV_{T,w} > 0$[7].

Moreover, we make the following assumption about the function $T(w)$:

Assumption 1: $T'(w) > 0$, $T''(w) < 0$, $E_{T/w} = T'(w)\frac{w}{T(w)} < 1$, and $-T''(w)\frac{w}{T'(w)} < 2$.

The first part of this assumption is standard and represents the decreasing marginal impact of wages on the length of life. The assumption on the elasticity implies that an increase in wages of $x\%$ implies an increase in the length of life of less than $x\%$. The last part of this assumption is only a technical hypothesis which will be used later in this chapter[8].

Each agent works when he is young and retires at the end of his first period of life[9]. It is the same assumption as the one used in Casamatta *et al.* (2000) or in Borck (2007). As long as an agent works, he pays a payroll tax τ. This tax is used to finance a PAYG pension system. When old, an agent receives a pension $p(w)$. Pensions are paid as long as agents are still alive, i.e. during a fraction $T(w)$ of their second period of life (d'Autume 2003). Furthermore, pensions per unit of time are partly indexed on the wage of the first period of the agent and on the average wage of the economy $(\lambda w + (1-\lambda)\bar{w})$. λ measures the size of the Bismarckian part of the pension system, whereas $(1-\lambda)$ measures the Beveridgian part of the pension system. When $\lambda = 1$ then

[7] See Appendix A.

[8] Some main functions respect this property. For example: $T(w) = aw + b$, or $T(w) = \gamma w^{\xi}$ with $a,b,\gamma > 0$ and $1 > \xi > 0$.

[9] In this chapter we do not consider the length of education and the retirement age even if they have an impact on the redistributive properties of pension systems. Indeed, the length of education and the retirement age are positively correlated with the wage level. Then, a strong link between education and wages reduces the progressivity of pension systems, whereas the positive correlation between the retirement age and the wage level increases the progressivity of pension systems. However, in this chapter we only analyse the redistributive properties of unfunded pension systems when the life expectancy differential is taken into account. In this way we emphasize the main role played by the mortality differential, neutralizing other channels.

the pension system is completely Bismarckian because pensions are only indexed on the wages of each agent. Conversely, when $\lambda = 0$ every agent receives the same pension. In that case, the pension system is Beveridgian.

Finally, agents receive only a fraction (ν) of this weighted average ($\lambda w + (1-\lambda)\bar{w}$) per unit of time. ν denotes the average replacement rate of the pension system. Consequently, the pension $p(w)$ which an agent receives during his second period of life is[10]:

$$p(w) = \nu(\lambda w + (1-\lambda)\bar{w})T(w) \tag{4}$$

We assume that there is no debt in this economy. It implies that all pensions have to be financed by a tax on wages. The budget constraint of the government can be written:

$$N_t \int_{\Omega_w} \tau w f(w) dw = N_{t-1} \int_{\Omega_w} \nu(\lambda w + (1-\lambda)\bar{w})T(w) f(w) dw$$

Some straightforward calculations imply that:

$$\tau = \nu \frac{\lambda COV_{T,w} + \bar{w}\bar{T}}{(1+n)\bar{w}} \tag{5}$$

Let us first study the case in which every agent has the same length of life, that is: $T(w) = \bar{T}$, $\forall w$. In that case we have $COV_{T,w} = 0$, and the tax rate becomes (d'Autume 2003):

$$\tau = \nu \frac{\bar{T}}{(1+n)}$$

The higher the replacement rate is, the higher the tax rate used to finance the pension system is. Indeed, an increase in the generosity has to be financed by a higher tax on young agents. Furthermore, the higher the old age dependency ratio $\left(\frac{\bar{T}}{(1+n)}\right)$ is, the higher the tax rate has to be. If for each worker there are more old agents, and for a given generosity of the pension system, then the tax rate has to increase to finance these additional pensions.

Let us now consider the case where $COV_{T,w} > 0$, i.e. there are inequalities of length of life. *Ceteris paribus*, the introduction of inequalities of length of life has a positive impact on the tax rate of the pension system. Indeed, the richer agents are, the longer they live and then the longer the period during which they receive a pension is. Conversely, the poorer agents are, the shorter the period during which they receive a pension is. Even if pensions are partially indexed on the wage of agents (by a coefficient λ) then the highest pensions are paid to people who live for the longest period. The tax rate has to increase to finance this additional spending. Now,

[10]The aim of this chapter is not to capture the entire complexity of pension systems but just to give some intuitions about their redistributive properties through a theoretical model. That is why we use the simplifying formula (4).

if the pension system has a Beveridgian structure ($\lambda = 0$), then even if the covariance is large, the decrease in spending for agents with a length of life smaller than $\bar{T}$ exactly compensates the increase in spending for the others[11].

At each period t, a group of agents with a wage w pays: $\tau w f(w) N_t$, whereas at the same time agents with the same productivity receive $p(w)f(w)N_{t-1}$. Our main objective is to know if, at each period, a group of agents with a wage w receive more from the pension system than they pay for it, i.e. if $p(w)f(w)N_{t-1}$ is larger than $\tau w f(w) N_t$. But the model also makes it possible to determine the wage of the group of people who benefit the most from the pension system, i.e. who receive the most given the amount they pay.

At each period t, the net contribution of a group with a wage w is:

$$CN_{w,t} = \tau w f(w) N_t - p(w) f(w) N_{t-1}$$

or,

$$CN_{w,t} = N_t f(w) \left[\tau w - \frac{p(w)}{1+n} \right]$$

The member between brackets represents the net contribution of an agent with a wage w if he uses the growth rate of the population as actualisation rate (Drouhin 2001). A positive net contribution means that a group pays more for the pension system than he receives from it. Using equations (4) and (5) we obtain:

$$CN_{w,t} = \nu f(w) N_{t-1} \left[\frac{\lambda COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} w - (\lambda w + (1-\lambda)\bar{w}) T(w) \right] \quad (6)$$

Integrating this function over the interval Ω_w, it is straightforward to show that $\int_{\Omega_w} CN_{w,t} dw = 0$.

But this amount is biased by the size of each group. Indeed, net contributions are correlated with the size of the groups of agents. So it is better to use the individual net contribution for each group. It can be written:

$$CN^i_{w,t} = \nu \left[\frac{\lambda COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} w - (\lambda w + (1-\lambda)\bar{w}) T(w) \right] \equiv \nu A(w) \quad (7)$$

Note that the size of the pension system (ν) has only a quantitative effect because it does not influence the sign of $A(w)$. ν amplifies the net contribution of each agent. If every agent has the same length of life ($\bar{T}$), then an increase in the wage level has a positive impact on the net contribution. Indeed, the increase in the tax paid is higher than the increase in the amount of

[11] Because of the linearity of pensions in (4).

pensions received. However, once inequalities of length of life are introduced ($T'(w) > 0$), then the increase in the wage level can have a stronger impact on the amount of pensions received. Indeed, the wage level has a positive impact on the length of life, which implies that agents endowed with a high wage level, benefit from the pension system for a longer period of time. Thus, an increase in the wage level can have a negative impact on the net contribution of agents.

Consequently, the form and the sign of the function $A(w)$ are not trivial because of the function $T(w)$. To understand the main implications of such a function we first study the case where there are no inequalities of length of life. Then, we introduce inequalities of length of life and we study the case where $\lambda = 0$ and the case where $\lambda = 1$ successively. Finally, we give the main properties of a mixed pension system, i.e. a system with a Beveridgian and a Bismarckian part.

3 The Benchmark Case

This section details the results for the case in which each agent in our economy has the same length of life: $T(w) = \bar{T}$, $\forall w$. It is an usual assumption. Agents only differ by their wages. This uni-dimensionality of the heterogeneity often simplifies the analysis but masks a very different reality. Let us first study the conclusions that would be obtained if we had only considered wage inequalities.

If $T(w) = \bar{T}$, $\forall w$, then $COV_{T,w} = 0$ and finally:

$$A_1(w) = (1 - \lambda)\bar{T}(w - \bar{w}) \tag{8}$$

$A(w)$ is a strictly increasing function of w as long as $\lambda \in [0, 1)$ (see figure 1). Furthermore, the net contribution is negative for agents having wages below the average. It implies that poor agents receive more from the pension system than they pay for it. Conversely, the net contribution is positive for agents having wages above the average wage.

Proposition 3.1 *(i) Every agent with a wage below (above) the average wage has a negative (positive) net contribution. (ii) The poorest benefit the most from the redistributive properties of the pension system. This net benefit is a decreasing function of w.*

Proof: (i) $A(w)$ is negative for $w < \bar{w}$ and positive for $w > \bar{w}$. (ii) $A(w)$ is a strictly increasing function of w.□

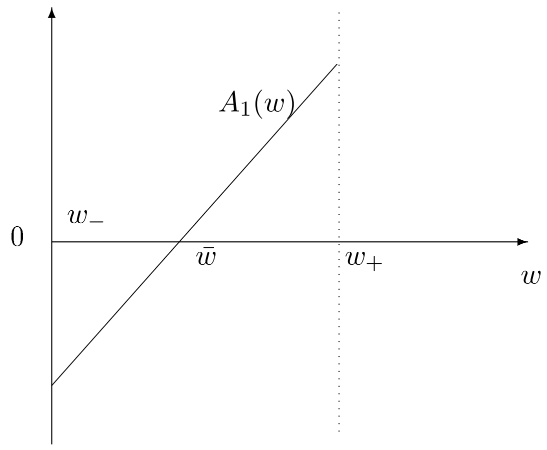

Figure 1: $A_1(w)$ for $T(w) = \bar{T}$ and $\lambda \in [0, 1)$

The second part of this proposition implies that the poorest have the highest negative net contribution. This result depends on the assumption that the pension system is at least partly Beveridgian ($\lambda \in [0, 1)$). If $\lambda = 1$ then the pension system is completely Bismarckian because the pension is only indexed on the wage of agents.

Proposition 3.2 *If $\lambda = 1$ then the net contribution of each group is nil. The pension system is not redistributive.*

Proof: See equation (8).□

This result ensures that a Bismarckian pension system, in an economy in which agents only differ by their wages, is neutral in terms of net contribution. Pensions exactly compensate contributions to the pension system.

These two results are usual in the economic literature (Casamatta *et al.* 2000). The following sections show that these results depend on the assumption that every agent has the same length of life. If it is not the case, then the higher the wage is, the longer agents live, and the more they benefit from the pension system. Consequently, the intuition is that the redistributive properties mentioned above change. Firstly because the poorest do not necessarily benefit the most from a Beveridgian pension system. Secondly because the Bismarckian pension system is not neutral.

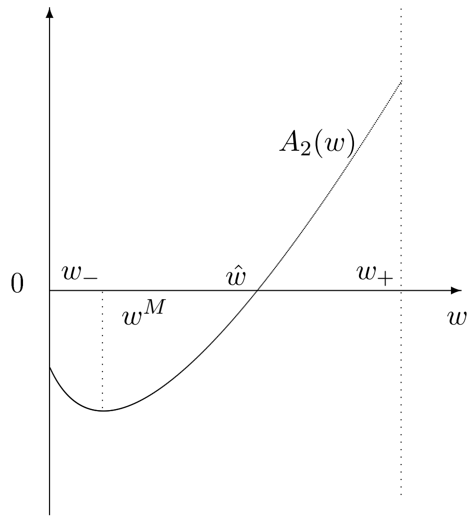

Figure 2: $A_2(w)$ with $w^M > w_-$

4 Pure Beveridgian Pension Systems

For pure Beveridgian pension systems we have $\lambda = 0$, i.e. every agent receives the same pension. This pension is indexed on the average wage of the economy. From this section we assume that agents also differ by their length of life: $T(w)$, with $T'(w) > 0$. The expression for $A(w)$ becomes:

$$A_2(w) = w\bar{T} - \bar{w}T(w) \tag{9}$$

The properties of this function are such that: $A_2'(w) = \bar{T} - \bar{w}T'(w)$ and $A_2''(w) = -\bar{w}T''(w)$. The sign of $A_2'(w)$ is indeterminate, but $A_2''(w)$ is clearly positive under assumption 1 (see figure 2).

Proposition 4.1 *There exists a threshold $\hat{w}$ such that the net contribution is negative (positive) for $w < \hat{w}$ ($w > \hat{w}$). Furthermore $\hat{w} > \bar{w}$ iff $\bar{T} < T(\bar{w})$.*

Proof: Under assumption 1, we know that $T(w)/w$ is a decreasing function of w. Furthermore, $A_2(w) > 0$ iff $T(w)/w < \bar{T}/\bar{w}$. $A_2(w)$ cannot be positive for every w since the sum of net contributions is equal to 0. Finally, as $T(w)/w$ is a decreasing function of w, we conclude that there exists a threshold value $\hat{w}$ such that $A_2(w) < 0$ for $w < \hat{w}$, and $A_2(w) > 0$ for $w > \hat{w}$. $\hat{w} > \bar{w}$ iff $A_2(\bar{w}) < 0$.□

This result ensures that poor agents (with a wage below $\hat{w}$) have a negative net contribution. Moreover, if the average length of life is smaller than the length of life of the average wage, then a larger group benefits from the redistributive effect of the pension system. The more $\bar{T}$ is different from $T(\bar{w})$, the more $\hat{w}$ removes away from $\bar{w}$. This result is intuitive. Indeed, a high average length of life implies that the taxe rate of the pension system is higher. Then the share of the population who benefits from a negative net contribution decreases ($\hat{w} < \bar{w}$).

But we also have to compare the net contribution $A_2(w)$ with $A_1(w)$. It is easy to show that $A_2(w) > A_1(w)$ ($<$) as long as $T(w) < \bar{T}$ ($>$). It implies that poor workers benefit less from the pension system and that rich workers pay less for the pension system.

Because of the convexity of the function $A_2(w)$, if there exists a value w^M such that $A_2'(w^M) = 0$, then it is a minimum. It has two implications for our analysis. (i) The richest contribute the most to the pension system because of the convexity of $A_2(w)$. They have the highest positive net contribution. (ii) The poorest do not necessarily benefit the most from the redistributivity of the pension system.

Lemma 4.1 *$\hat{w} > w^M > w_-$ iff $A_2'(w_-) < 0$. $w^M = w_-$ iff $A_2'(w_-) \geq 0$.*

Considering $A_2'(w_-)$, this expression is negative if $T'(w_-)$ is sufficiently large. It means that the marginal impact of wages on the length of life is large for the low values of wages. Agents with a wage slightly higher than w_- can expect to live much longer than agents with a wage w_-. They receive a pension during this additional time. Consequently, the agents from the group w^M benefit the most from the pension system and w^M can be different from w_-.

The two main implications are: (i) The share of the population who benefits from the pension system can differ from the interval $[w_-, \bar{w}]$. (ii) The agents who benefit the most from the pension system are not necessarily the poorest. It depends on the properties of the function $T(w)$.

5 Pure Bismarckian Pension Systems

For pure Bismarckian pension systems, pensions are only indexed on wages ($\lambda = 1$). In section 3, we obtained that the net contributions are nil for every group if there are no inequalities of length of life. But the introduction of these inequalities changes the qualitative results considerably.

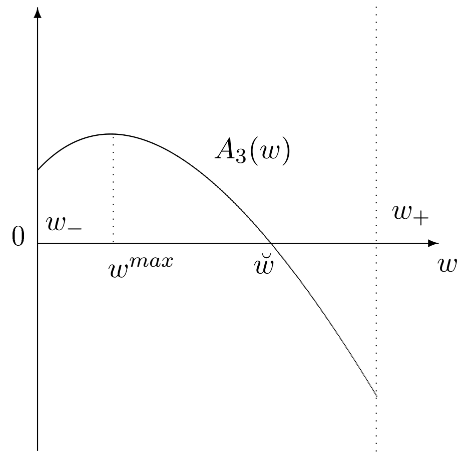

Figure 3: $A_3(w)$ for $w^{max} > w_-$

For $\lambda = 1$, $A(w)$ can be written:

$$A_3(w) = w\frac{COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - wT(w) \tag{10}$$

The sign of $A_3'(w) = \frac{COV_{T,w}+\bar{w}\bar{T}}{\bar{w}} - T(w) - wT'(w)$ is indeterminate, but $A_3''(w) = -2T'(w) - T''(w)w$ is clearly negative under assumption 1. Then $A_3(w)$ is a concave function of w (see figure (3)).

Proposition 5.1 *There exists a threshold $\breve{w}$ such that the net contribution is positive (negative) for $w < \breve{w}$ ($w > \breve{w}$). $\breve{w}$ is determined by the following equation:*

$$T(\breve{w}) = \frac{COV_{T,w}}{\bar{w}} + \bar{T}$$

Proof: $A_3(w) > 0$ iff $T(w) < \frac{COV_{T,w}+\bar{w}\bar{T}}{\bar{w}}$. As $T(w)$ is a strictly increasing function of w and as the sum of the net contributions is equal to 0, then as long as $w < \breve{w}$ we have $T(w) < T(\breve{w})$ and the net contribution is positive. But for $w > \breve{w}$ the net contribution is negative.□

This result is completely different from that of proposition 2. Rich agents benefit from a negative net contribution whereas poor agents have a positive net contribution. But the result of this section is even more surprising. Indeed, the highest negative value of $A_3(w)$ is obtained for $w = w_+$. Furthermore, as $A_3(w)$ is a concave function of w then if there exists a value w^{max} such that $A_3'(w) = 0$, it is a maximum. Given the result of proposition 4 we can write that

$w^{max} \in [w_-, \breve{w})$, and more precisely:

Lemma 5.1 *$w^{max} = w_-$ as long as $A'_3(w_-) = \frac{COV_{T,w}+\bar{w}\bar{T}}{\bar{w}} - T(w_-) - w_- T'(w_-) \leq 0$. $w^{max} \in (w_-, \breve{w})$ iff $A'_3(w_-) > 0$.*

$A'_3(w_-)$ is positive if $T'(w_-)$ is not too large, i.e. if agents with a wage slightly higher than w_- can expect to have a life expectancy only just higher than that of agents with a wage w_-.

Two main conclusions can be drawn from this analysis. (i) Pure Bismarckian pension systems are regressive because poor agents have a positive net contribution and rich agents have a negative one. (ii) If $T'(w_-)$ is sufficiently large, then the poorest have the highest positive net contribution. The concavity of $A_3(w)$ also implies that the richest have the highest negative net contribution, which reinforces our previous conclusion.

6 Mixed Pension Systems

Let us assume from now on that $\lambda \in (0,1)$. If λ tends towards 1 then the pension system becomes more Bismarckian. Conversely, if λ tends toward 0 the pension system becomes more Beveridgian as the pension depends less on the wage of agents. The function $A(w)$ can be written as:

$$A_4(w,\lambda) = w\frac{\lambda COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - (\lambda w + (1-\lambda)\bar{w})T(w) \quad (11)$$

But this function has indeterminate properties. Indeed $\partial A_4(w,\lambda)/\partial w$ can be positive, negative or null. And $\partial^2 A_4(w,\lambda)/\partial w^2$ can also be positive, negative or null.

In order to obtain clear analytical results we specify the function $T(w)$. We assume it has the following form:

$$T(w) = \varrho w^{\xi} \quad (12)$$

with $\varrho > 0$ and $1 > \xi > 0$. This function respects each property of assumption 1. Let us first study the concavity and the convexity of the function $A_4(w)$. Note that:

$$\frac{\partial A_4(w,\lambda)}{\partial w} = \frac{\lambda COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - T'(w)(\lambda w + (1-\lambda)\bar{w}) - \lambda T(w) \quad (13)$$

and that:

$$\frac{\partial^2 A_4(w,\lambda)}{\partial w^2} = -T''(w)(\lambda w + (1-\lambda)\bar{w}) - 2\lambda T'(w) \quad (14)$$

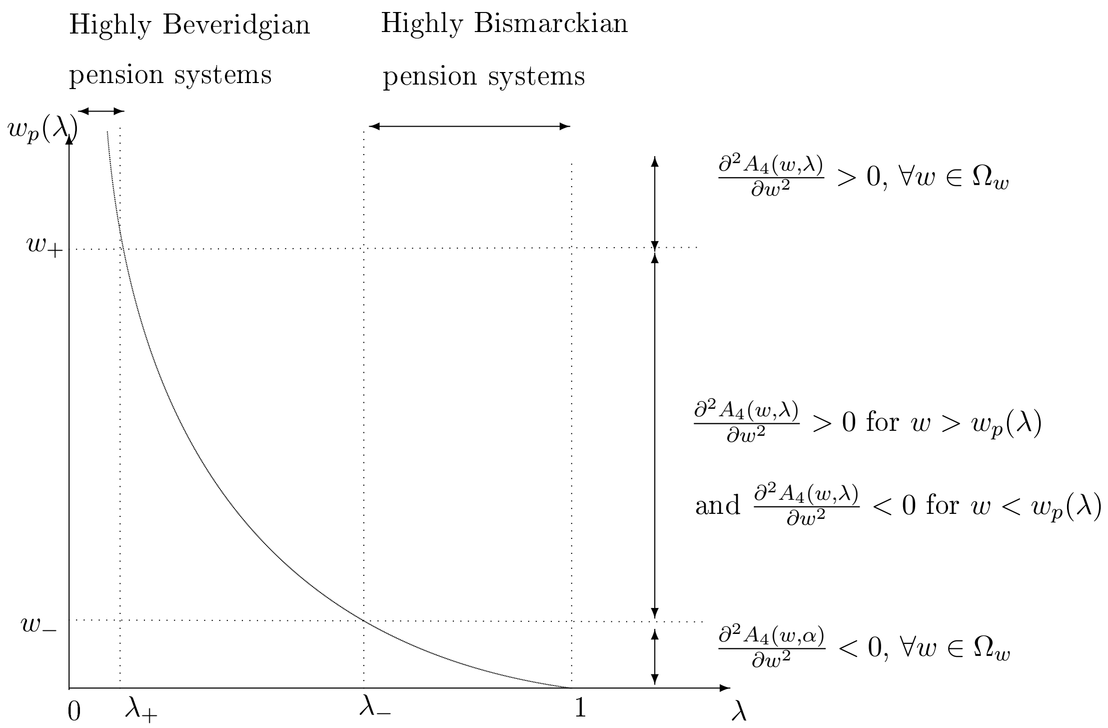

Figure 4: Illustration of lemma 3

Lemma 6.1 *There exists a threshold value $w_p(\lambda)$ such that if $w < w_p(\lambda)$ ($> w_p(\lambda)$) then $\frac{\partial^2 A_4(w,\lambda)}{\partial w^2} > 0$ (< 0). It is such that $\frac{\partial w_p(\lambda)}{\partial \lambda} < 0$. Furthermore, there exists an interval $(0, \lambda_+)$ such that for $\lambda \in (0, \lambda_+)$ we have $w_p(\lambda) > w_+$; and a second interval $(\lambda_-, 1)$ such that for $\lambda \in (\lambda_-, 1)$ we have $w_p(\lambda) < w_-$, with $\lambda_+ < \lambda_-$.*

Proof: Using equation (12) we obtain that $T''(w)/T'(w) = (\xi - 1)/w$. Then using equation (14) it is straightforward to show that $\frac{\partial^2 A_4(w,\lambda)}{\partial w^2} > 0$ if and only if $w < \bar{w}\frac{(1-\xi)(1-\lambda)}{\lambda(1+\xi)} \equiv w_p(\lambda)$, with $w_p'(\lambda) < 0$. $w_p(0) = +\infty$ and $w_p(1) = 0$. Figure 4 illustrates this lemma.□

This result gives the properties in terms of convexity and of concavity of the function $A_4(w)$. It implies that for a value of λ sufficiently small (inferior to λ_+) $A_4(w)$ is convex. Conversely, with α sufficiently high (superior to λ_-) $A_4(w)$ is concave.

Proposition 6.1 *If $A(w_-, \lambda_-) > 0$ and if $A(w_-, \lambda_+) < 0$ then for $\lambda \in (0, \lambda_+)$ net contributions behave qualitatively as in the pure Beveridgian case (figure 2). And for $\lambda \in (\lambda_-, 1)$ net contributions behave qualitatively as in the pure Bismarckian case (figure 3).*

Proof: The first two conditions ensure that the poorest have a positive (negative) net contribution in the interval $(\lambda_-, 1)$ $((0, \lambda_+))$.□

This result extends the qualitative properties of Bismarckian and Beveridgian pension systems to intervals for the parameter λ. Each pension system with a λ in the interval $(\lambda_-, 1)$ is regressive. Conversely, each pension system with a λ inferior to λ_+ is redistributive, whereas the poorest do not necessarily benefit the most from this pension system.

For $\lambda \in (\lambda_+, \lambda_-)$, we have to calibrate our model to know the form of net contributions.

7 Calibration on French Data

In order to calibrate our model we first have to specify wage inequalities. We assume that wages belong to the interval: $\Omega_w = [0.2, 9]$. This interval implies that the wage w_+ is 45 times higher than w_-. Piketty (2002), studying the distribution of wages in France, finds a ratio of 5 between the wages of the first and of the last decile. The gap between this empirical fact and our calibration can be explained by the fact that we use the two extreme values of a *continuum* and as a consequence wage inequalities are greater. We choose this interval for Ω_w because once it is combined with the density function of w, our model matches the Gini coefficient of the wage distribution calculated by Hairault and Langot (2008) on French data.

The density function of the distribution of wages among the population has to respect the essential property: mode<median<mean (Lambert 2001, p.23). This property is a common feature of most industrialized countries. It implies that the wage distribution among the population is asymetric. The most common income level is less than the median wage. And because of strong wage inequalities the median wage is less than the average wage of the economy. Furthermore, the Gini index of wages has to tend towards 0.32 (Hairault and Langot 2008). Lambert (2001) shows that the Gini index can be calculated as:

$$G = -1 + 2 \int_{w_-}^{w_+} \frac{wF(w)f(w)}{\bar{w}} dw$$

A useful density function is the density function of Weibull. It is asymmetric and it has the following form:

$$f(w) = \frac{c}{b} \left(\frac{w-a}{b} \right)^{c-1} e^{-\left(\frac{w-a}{b}\right)^c} \tag{15}$$

if $w > a$, and $f(w) = 0$ for $w \leq a$, with b and $c > 0$. The only problem with the use of this function is that the $\int_{\Omega_w} f(w)dw$ is not exactly 1. But the following calibration is such that this

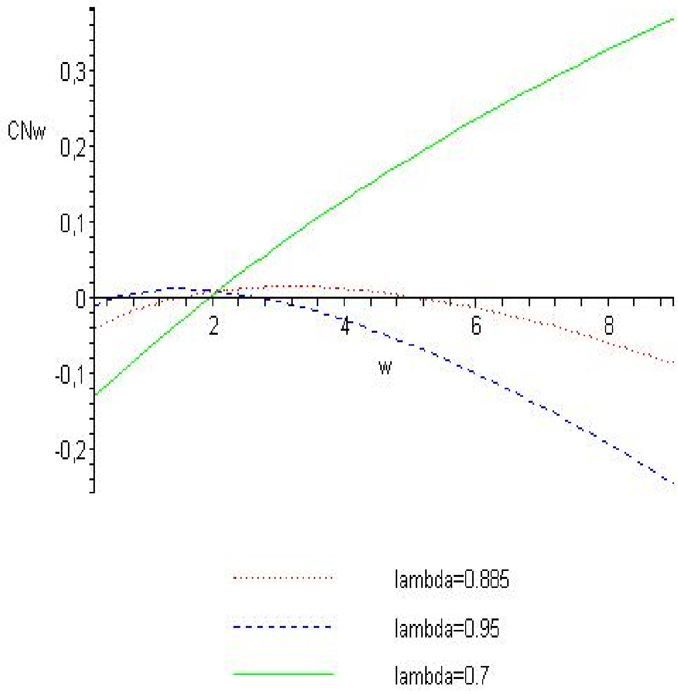

Figure 5: Net Contributions for different values of λ and for $\xi = 0.09$

integral is approximately 1 over our interval Ω_w: $a = 0.2$, $b = 2$ and $c = 1.55$. Furthermore, it implies that $F(\bar{w}) > 0.5$, with $F()$ the cumulative distribution function, $\bar{w} = 2$, $w_{median} = 1.78$ and $G = 0.3245$.

We now have to specify the relationship between wages and the length of life. We assume that it has the following form:

$$T(w) = \varrho w^{\xi} \tag{16}$$

with ξ the elasticity of the length of life with respect to wages. ϱ is a scale parameter. We assume that $\varrho = 0.2$. In the following analysis we study the impact of a change in the parameter ξ, knowing that this value is clearly less than 1 (Bommier *et al.* 2006).

Finally, we assume that $\lambda = 0.885$ (Hairault and Langot 2008), but we study what happens if λ varies around this benchmark value.

For the moment, let us consider that $\xi = 0.09$, i.e. an increase in wages of 1% implies an increase in the length of life of 0.09%. Figure 5 illustrates this case.

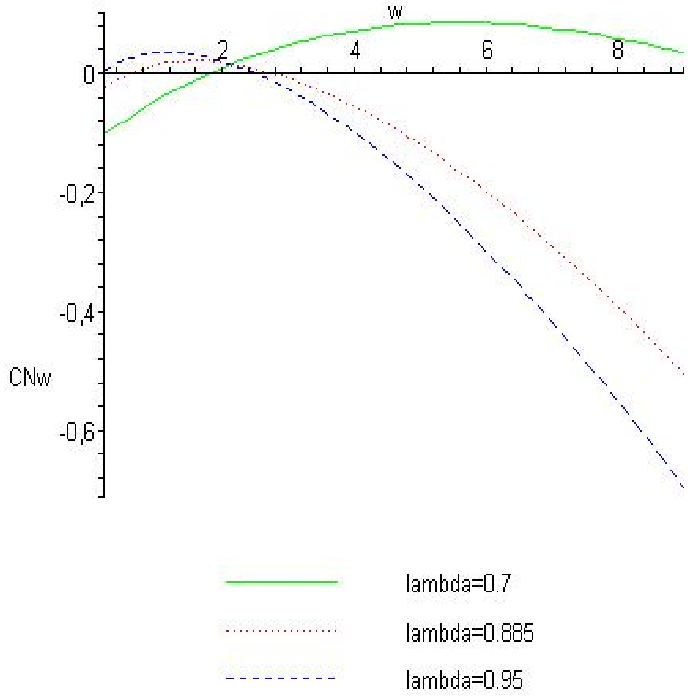

Figure 6: Net Contributions for different values of λ and for $\xi = 0.18$

For the value of λ calculated by Hairault and Langot (2008), then it is clear that agents with a wage below 1.6 and above 5.2 benefit from the pension system. The richest have a net contribution more negative than the net contribution of the poorest. Agents with a wage in the interval [1.6,5.2] have a positive net contribution and finance the negative net contributions of the two extremes. If λ increases, i.e. the pension system becomes more Bismarckian, then less poor agents benefit from a negative net contribution whereas the net contributions becomes negative for wages above 2.8. It is almost a regressive pension system. Finally, if $\lambda = 0.7$, then the redistributive properties of the pension system endure. The poorest benefit the most from the pension system and the richest have the highest positive net contribution.

Let us now assume that $\xi = 0.18$. The elasticity of the length of life is twice as high as before. Then the previous qualitative results endure (see figure 6), but the pension system is clearly regressive because only very small wages benefit from a negative net contribution. Agents with a wage above 2.4 have a negative net contribution and the richest benefit the most from the pension system. This result is also truer for $\lambda = 0.95$. Finally, even for $\lambda = 0.7$ the redistributive properties of the pension system are less clear.

8 Conclusion

Our contribution clarifies the debate on the redistributive properties of pension systems theoretically, when there are inequalities of length of life. We explain more precisely the sentence from Mitchell and Zeldes (1996) cited above. It is shown that Beveridgian pension systems are less progressive than they seem and that Bismarckian pension systems are regressive. Moreover, it makes it possible to show that for mixed pension systems, there can be a redistribution of resources from the middle to the ends of the distribution of wages. This last point would have to be taken into account for empirical analyses which study the progressivity of pension systems.

Our theoretical analysis is a first attempt to clarify the debate on the progressivity of pension systems once the life expectancy differential is taken into account. The next step on our research agenda would be to use a micro-simulation model, as in Liebman (2001), on French data. It would permit to quantify the impact of the life expectancy differential on the redistributive properties of pension systems.

9 APPENDIX

APPENDIX A

Indeed, the covariance can also be written as: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw$. But as $\int_{\Omega_w}(w-\bar{w})f(w)dw = 0$, we can write that: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw = \int_{\Omega_w}(w-\bar{w})(T(w)-X)f(w)da$, with X a constant, whatever the value of X. So it is particulary true for $X = T(\bar{w})$. Then we can write that: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw = \int_{\Omega_w}(w-\bar{w})(T(w)-T(\bar{w}))f(w)dw$. The right-hand-side is positive as it is an integral on a product of terms which have the same sign because $T'(w) > 0$.□

REFERENCES

Adams, P., Hurd, M.D., McFadden, D., Merrill, A., Ribeiro, T., 2003. "Healthy, Wealthy and Wise ? Tests for Direct Causal Paths between Health and Socio-Economic Status". *Journal of Econometrics*, 112, 3-56.

Attanasio, O., Emmerson, C., 2001. "Differential Mortality in the UK". *IFS Working Paper* 01/16.

d'Autume, A., 2003. "Vieillissement et choix de l'âge de la retraite. Que peut nous dire le modèle à générations ?". *Revue Economique*, 54 (3), 561-571.

Bommier, A., Magnac, T., Rapoport, B., Roger, M., 2006. "Droit à la Retraite et Mortalité Différentielle". *Economie et Prévision*, 168, pp.1-16.

Borck, R., 2007. "On the Choice of Public Pensions when Income and Life Expectancy are Correlated". *Journal of Public Economic Theory*, 9, 711-725.

Casamatta, G., Cremer, H., Pestieau, P., 2000. "The political economy of social security". *Scandinavian Journal of Economics*, 102, pp.503-522.

Charpin, J.M., 1999. "L'avenir de nos retraites". *La documentation française.*

Coronado, J.L., Fullerton, D., Glass, T., 2000. "The Progressivity of Social Security". *NBER working papers* 7520.

Cremer, H., Pestieau, P., 2000. "Reforming our pension system: Is it a demographic, financial or political problem?". *European Economic Review*, 44(4-6), pp.974-983.

Drouhin, N., 2001. "Inégalités face à la mort et systèmes de retraite". *Revue d'Economie Politique*, Hors-Serie: Epargne et Retraite.

Galasso, V., Profeta, P., 2004. "Lessons for an ageing society: the political sustainability of social security systems". *Economic Policy*, 19(38), pp.63-115.

Hairault, J.O. and Langot, F., 2008. "Inequality and Social Security Reforms". *Journal of Economic Dynamics and Control*, 32 (2), 386-410.

Lambert, P.J., 2001. "The Distribution and Redistribution of Income". *Manchester University Press.*

Legros, F., 1994. "Caractère Redistributif des Systèmes de Retraite". *Revue Economique*, 4, 805-817.

Liebman, J.B., 2001. "Redistribution in the Current U.S. Social Security System". *NBER working paper 8625.*

Mesrine, A., 1999. "Les différences de mortalité par milieu social restent fortes". *Données Sociales*, INSEE, pp.228-235.

Mitchell, O.S., Zeldes, S.P., 1996. "Social Security Privatization: A Structure for Analysis". *American Economic Review*, 88 (2), pp.363-367.

Monteil, C., Robert-Bobée, I., 2005. "Les différences sociales de mortalité : en augmentation chez les hommes, stables chez les femmes". *INSEE première*, No.1025.

Piketty, T., 2002. "L'économie des Inégalités". *Repères: La découverte*.

Robert-Bobée, I., Cadot, O., 2007. "Mortalité aux Grands Ages : encore des écarts selon le diplôme et la catégorie sociale". *INSEE première*, No.1122.