Residual-Based Tests for Cointegration and Multiple Deterministic Structural Breaks: A Monte Carlo Study*

Matteo Mogliani†

Paris School of Economics, France

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Abstract

The aim of this paper is to study the performance of residual-based tests for cointegration in the presence of multiple structural breaks via Monte Carlo simulations. As a support to most empirical applications, we focus on exogenous deterministic breaks. We consider the KPSS-type LM tests proposed in Carrion-i-Silvestre and Sansò (2006) and in Bartley, Lee and Strazicich (2001), as well as the Schmidt and Phillips-type LM tests proposed in Westerlund and Edgerton (2006). This exercise allow us to cover a wide set of single-equation cointegration estimators. From our Monte Carlo experiments, KPSS-type tests show large size distortions in finite sample simulations when the model accounts for multiple breaks. In addition, when regressors are endogenous, Schmidt and Phillips-type tests show large power distortions. The use of the DGLS estimator has nevertheless the best size-power performance overall.

Keywords: Cointegration; structural breaks; Monte Carlo simulations

JEL classification: C12, C13, C15, C22.

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† *Corresponding address*: Paris School of Economics, Paris-Jourdan Sciences Economiques, 48 Boulevard Jourdan, 75014 Paris (France). Tel: +33 (0)1 43 13 63 22. Fax: +33 (0)1 43 13 63 10. Email: mogliani@pse.ens.fr.

1 Introduction

Cointegration has been at the heart of a vast macroeconomic and econometric research since the seminal contribution of Engle and Granger (1987). This concept, *i.e.* the hypothesis that one stationary linear combination of individually non-stationary variables exists, has been widely used in empirical studies in many areas of economics. The development of cointegrating and error-correction models eventually allowed applied economists to analyze long-run and short-run economic relationships, such as money demand, balanced growth, purchase power parity and many others.

Many cointegration tests have been proposed in the econometric literature. Among them, the class of residual-based tests is the most popular, thanks to the simple computation and the straight interpretation in terms of economic theory. Following the unit-root testing approach, the literature has proposed tests for the null hypothesis of non-cointegration (Engle and Granger, 1987; Phillips and Ouliaris, 1990), as well as tests for the null hypothesis of cointegration (Hansen, 1992; Shin, 1994).

These tests show nevertheless serious size distortion when specific features of data are neglected. Indeed, one potential feature of long-run economic relationships is structural breaks, *i.e.* the significant change of one or more parameters affecting persistently the data generating process (DGP) of the economic model under consideration. This issue is addressed in Gregory and Hansen (1996), who extend the general framework of Engle and Granger (1987) and Phillips and Ouliaris (1990) to account for the presence of one structural break. However, as pointed out in Carrion-i-Silvestre and Sansò (2006), the statistical tests proposed in Gregory and Hansen (1996) are not able to discern between the situation of unstable cointegrating relationship and that of stability with regime-shifts, the null hypothesis of non-cointegration being tested against the alternative of cointegration with break. This is a crucial concern when assessing the instability of cointegration models, in particular when breaks are exogenously or endogenously known.

Residual-based tests recently proposed in the literature address this issue through the inclusion of structural breaks under both the null and the alternative hypothesis. The generalization of the break hypothesis makes the latter tests autonomous (stand alone tests for cointegration or non-cointegration), compared with the complementarity role of the former (auxiliary tests for spurious cointegration led by a neglected break). However, these recent contributions have only explored the "one structural break" hypothesis. This is mainly due to the econometric constraints related to the endogenous estimation of multiple breaks in cointegrating relationships. This issue has nonetheless attracted increasing attention in the econometric literature during the last decade (Bai and Perron, 1998, 2003; de Peretti and Urga, 2004; Kejriwal and Perron, 2008). In addition, as shown for instance in Mogliani *et al.* (2009), accounting for multiple breaks in economic relationships can be a crucial issue when dealing with emerging economies and/or long span datasets.

It is then worth studying the finite sample performance (size and power) of residual-based tests for cointegration in the case of multiple breaks. For this purpose, we run Monte Carlo simulations involving several single-equation cointegration estimators (OLS, DOLS, DGLS, FM and CCR) and breaks scenarios. To be consistent with most of empirical applications, we follow Perron (1989, 1990) and Hao (1996) and we only consider deterministic structural breaks (constant and trend). We also account for endogenous regressors and potential misspecification of model residuals.

We consider the residual-based tests for the null hypothesis of cointegration proposed in Bartley, Lee and Strazicich (2001) and Carrion-i-Silvestre and Sansò (2006). These works deal with the generalization of the univariate LM test of Kwiatkowski *et al.* (1992) - henceforth KPSS - as in Shin (1994), Hao (1996) and Lee (1999), to the case of cointegration with one structural break, while efficient estimates of the cointegrating relationship are carried out through the Canonical Cointegration Regression (Park, 1992), the dynamic OLS (Saikkonen, 1991; Stock and Watson, 1993) and the Fully-modified approach (Phillips and Hansen, 1990). We also consider testing procedure proposed in Westerlund and Edgerton (2006) and involving instead the null hypothesis of non-cointegration. This work extends the univariate LM test of Schmidt and Phillips (1992) - henceforth SP - to the cointegration with break framework. The proposed statistical tests are built upon the OLS estimate of the cointegrating relationship (Engle and Granger, 1987; Phillips and Ouliaris, 1990), and they are thus designed for strictly exogenous regressors.

Our main findings show that KPSS-based tests tend to be seriously over-sized when more deterministic breaks are included in the cointegration model. This is not true for the tests proposed in Westerlund and Edgerton (2006), which appear quite correctly sized across all our simulation exercises. These results are reverted in the power analysis: KPSS-based tests show high power against the alternative hypothesis in all simulations, while SP-based tests show very low power which tends to be close to the nominal size. It is worth noticing that these results are mainly driven by the presence of endogenous regressors, which tend to deteriorate the empirical size of KPSS-based tests and the power of SP-based tests. All in all, tests based on the DOLS and, in particular, on the DGLS estimators have the best size-power performance.

The remainder of the paper is as follows. In Section 2, we briefly describe the general cointegration with structural breaks model studied in this paper. in Section 3 we present, according with the estimator used in test regressions, the residual-based tests of cointegration under examination. In Section 4 we describe the design of our Monte Carlo experiments, with a particular attention to the DGP chosen for simulations and to the estimator of the long-run variance used by statistical tests. In Section 5 and 6 we discuss simulation results. The size and the power of statistical tests are compared under several alternatives, mainly involving the number of breaks and the sample size. Section 7 concludes.

2 The Cointegration with Structural Breaks Model

In this Section we introduce the general single-equation cointegration model with structural breaks used in simulations. Let assume that the data generating process (DGP) is of the form:

$$y_t = \alpha + g(t) + x_t'\beta + e_t \tag{1}$$

with

$$e_t = \rho e_{t-1} + \varepsilon_t$$

$$x_t = x_{t-1} + \mu_t$$

where $t = 1, \ldots, T$ is the time series index, x_t is the K-dimensional vector of I(1) regressors and ε_t and μ_t are i.i.d. processes with distribution $N(0, \Sigma)$. We define $g(t)$ as the function collecting the deterministic components of the model, except for the constant. Following Perron (1989, 1990), Hao (1996), Bartley *et al.* (2001) and Carrion-i-Silvestre and Sansò (2006), we choose to study an empirically relevant set of deterministic functions:

$$g(t) = \begin{cases} \theta_1 DU_t & \text{Model A} \\ \tau t + \theta_1 DU_t & \text{Model B} \\ \tau t + \theta_1 DU_t + \theta_2 DT_t & \text{Model C} \end{cases} \tag{2}$$

where $DU_t = (DU_{1,t}, \ldots, DU_{m,t})'$ and $DT_t = (DT_{1,t}, \ldots, DT_{m,t})'$ are the vectors of deterministic breaks and

$$DU_{j,t} = \begin{cases} 1, & \text{for } t > T_{jb} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad DT_{j,t} = \begin{cases} (t - T_{jb}), & \text{for } t > T_{jb} \\ 0, & \text{otherwise} \end{cases}$$

is the deterministic breaks structure at dates $T_{jb} = \lambda_j T$, with $\lambda_j \in (0, 1)$, for $j = 1, \ldots, m$, and m is the number of breaks. Model A allows for multiple level breaks without a linear trend. Model B allows for a linear trend and multiple level breaks. Finally, Model C allows for both multiple level and trend breaks, which are assumed for simplicity to pairwise occur at the same date.

3 Statistical Tests

3.1 A Test Based on the OLS Estimator

A test based on the standard OLS estimator of the cointegrating relationships (1) (Engle and Granger, 1987; Phillips and Ouliaris, 1990) is proposed in Westerlund and Edgerton (2006) - henceforth WE. Following Schmidt and Phillips (1992), WE (2006) propose an LM-type test for the null hypothesis of non-cointegration against the alternative of cointegration, with structural breaks under both the null and the alternative.

According to the LM (score) principle, cointegration test is obtained from the following regression:

$$\Delta \hat{S}_t = \text{constant} + \Phi \hat{S}_{t-1} + \text{error}, \tag{3}$$

where $\hat{S}_t = y_t - \hat{\alpha} - \hat{g}_i(t) - x_t'\hat{\beta}$ and $\hat{\alpha}$ is the restricted maximum likelihood estimate of $\tilde{\alpha} = \alpha + e_0$, given by $\hat{\alpha} = y_1 - \hat{g}_i(1) - x_1'\hat{\beta}$. Estimates of $\hat{\beta}$ and of parameters in $\hat{g}_i(t)$, for $i = \{\text{A, B, C}\}$, are obtained from the OLS regression of Δy_t over $\Delta g_i(t)$ and $\Delta x_t'$. It is worth noticing that expression $\Delta g_i(t)$ involves one-period jumps (ΔDU_t) and changes in drift (ΔDT_t), rather than constant (DU_t) and trend (DT_t) breaks. From Equation (3), the hypothesis of non-cointegration can be formulated as a test of $\Phi = 0$ against $\Phi < 0$, which can be verified through the OLS estimate of Φ or its LM t-statistic. WE (2006) then propose the following two statistical tests:

$$WE_{\Phi} = T \times \hat{\Phi} \qquad \text{and} \qquad WE_{t\text{-stat}} = \frac{\hat{\Phi}}{\hat{\sigma}} \times \sqrt{\sum_{t=2}^{T} (\hat{S}_{t-1})_p^2}, \tag{4}$$

where $\hat{\sigma}$ is the estimated standard error from regression (3) and $(\hat{S}_{t-1})_p$ is the error from projecting $\hat{S}_{t-1}$ onto its mean value. To account for autocorrelated and heteroskedastic errors, WE (2006) follow the parametric correction proposed in Ahn (1993) and include augmented terms in Equation (3) :

$$\Delta \hat{S}_t = \text{constant} + \Phi \hat{S}_{t-1} + \sum_{j=1}^{p} \psi_j \Delta \hat{S}_{t-j} + \text{error}, \tag{5}$$

where the optimal lag order p is chosen by following the "general to specific" procedure suggested by Perron (1989), Campbell and Perron (1991) and Ng and Perron (1995). In our Monte Carlo simulations we allow for a maximum number of 6 lags[1]. WE (2006) show that only the statistic WE_{Φ} is affected by the presence of autocorrelated errors. This requires the following correction:

$$WE_{\Phi} = T \times \hat{\Phi} \times \sqrt{\frac{\hat{\omega}}{\hat{\sigma}^2}}, \tag{6}$$

where $\hat{\sigma}^2$ is the residual variance from the augmented test regression (5) and $\hat{\omega}$ is the long-run variance of $\Delta \hat{S}_t$ evaluated at frequency zero:

$$\hat{\omega} = \frac{1}{T} \sum_{t=1}^{T} \Delta \hat{S}_t \Delta \hat{S}_t' + \frac{2}{T} \sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \Delta \hat{S}_t \Delta \hat{S}_{t-j}',$$

where $w(\cdot)$ and M are the kernel function and the bandwidth parameter, respectively. We follow WE (2006) and we use a Bartlett kernel with bandwidth parameter $M = p$.

For the case of Model B, it can be shown that WE_{Φ} and $WE_{t\text{-stat}}$ follow the asymptotic distributions derived in Schmidt and Phillips (1992). In addition, distributions are unaffected by the presence of multiple mean breaks, the number of regressors (K) and the breaks fraction (λ_j). For the case of Model A, our simulations show that the exclusion of the linear trend from the cointegrating equation affects the asymptotic distribution of both statistics. Nevertheless, distributions are unaltered by the presence of multiple mean breaks. Differently, for the case of Model C our simulations show that statistics under consideration follow asymptotic distributions which depend on the number of breaks (m)[2].

[1] We sequentially test at 5% level the significance of the last term in the augmented test regression (5), until either the optimal lag is found or $p = 0$.

[2] It must be verified whether the relative position of breaks in the sample (λ_j) also affects asymptotic distributions.

It is worth noticing that the testing procedure proposed in WE (2006) is valid until regressors x_t are strictly exogenous. Relaxing this assumption would imply a potential bias in the statistics under consideration which comes from the OLS estimate of $\hat{\beta}$ for the computation of $\hat{S}_t$. To correct for endogeneity bias, WE (2006) propose to estimate $\hat{\beta}$ by IV. In practice, finding out consistent instruments for endogenous regressors can be a hard task in the context of cointegrated macroeconomic time series. For this reason, in our simulations we prefer studying the performance of WE_{Φ} and $WE_{t\text{-stat}}$ statistics under endogeneity bias.

3.2 A Test Based on the Dynamic Leads-and-Lags Estimator

A test based on the leads-and-lags correction of the cointegrating regression (Saikkonen, 1991; Stock and Watson, 1993) is developed in Carrion-i-Silvestre and Sansò (2006) - henceforth CS. Following Shin (1994), CS (2006) propose a LM-type test for the null hypothesis of cointegration against the alternative of non-cointegration, with structural breaks under both the null and the alternative. Let define $v_t = \Delta x_t$ and $\eta_t = (e_t, v_t')$ and assume that η_t satisfies the multivariate invariance principle (Herrndorf, 1984; Phillips and Durlauf, 1986):

$$T^{-1/2}\Omega \sum_{t=1}^{[Tr]} \eta_t \Rightarrow W(r), \qquad 0 \leq r \leq 1,$$

where $\Rightarrow$ denotes weak convergence in probability and $W(r) = (W_1(r), W_{2K}(r)')'$ is a (K+1)-dimensional Wiener process. Ω is the long-run covariance matrix, which can be written (partitioned in conformity with η_t) as:

$$\Omega = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{T} E(\eta_j \eta_t') = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \Omega_{22} \end{bmatrix} = \Sigma + \Lambda + \Lambda',$$

where long-run variances ω_{11} and Ω_{22} of processes $W_1(r)$ and $W_{2K}(r)$ are positive definite to rule out multicointegration (Granger and Lee, 1989) and

$$\Sigma = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E(\eta_t \eta_t') = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \Sigma_{22} \end{bmatrix}$$

$$\Lambda = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{t} E(\eta_j \eta_t') = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12} & \Lambda_{22} \end{bmatrix}$$

Standard asymptotics cannot apply here because of the presence of correlation between disturbance terms. This means that regressors x_t are not strictly exogenous and the OLS estimator of the cointegration regression (1) is inefficient. To overcome this problem, CS (2006) propose to estimate the cointegration model (1) through the following Dynamic OLS regression:

$$y_t = \alpha_0 + g_i(t) + x_t'\beta + \sum_{j=-k}^{k} \Delta x_{t-j}'\xi_j + e_t^* \tag{7}$$

where k is the (finite truncated) number of leads and lags for first-differenced non-stationary regressors.

Provided that the errors e_t^* can be serially correlated and uncorrelated with regressors and all leads and lags, we follow Stock and Watson (1993) and we introduce the Dynamic GLS estimator. A feasible DGLS estimator is constructed by transforming regressors in (7) as $\tilde{x}_t = x_t'\hat{\varphi}(L)$, where $\hat{\varphi}(L)$ is an estimate of the lag polynomial of residuals $\varphi(L)$.

In our Monte Carlo experiments, we construct $\varphi(L)$ as an AR(1) model of residuals. We follow the Cochrane-Orcutt iterative procedure and we allow the AR(1) parameter to convergence across the sequential estimation. Finally, we allow the number of leads and lags to be selected by the SBC criterion, starting with a maximum number of 4.

The multivariate LM-type test proposed in CS (2006) is then given by:

$$CS_{\text{DOLS}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^*} \times \sum_{t=1}^{T} (S_t^*)^2 \qquad \text{and} \qquad CS_{\text{DGLS}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^*} \times \sum_{t=1}^{T} (S_t^*)^2 \tag{8}$$

where $S_t^* = \sum_{j=1}^{t} \hat{e}_j^*$, $\hat{e}_t^*$ are estimated residuals from DOLS/DGLS regression (7) and $\hat{\omega}_{11\cdot 2}^*$ is any consistent estimate of $\omega_{11\cdot 2} = \omega_{11} - \omega_{12}\Omega_{22}^{-1}\omega_{21}$, *i.e.* the endogeneity-corrected long-run variance of residuals e_t. In practice, a consistent estimate of $\omega_{11\cdot 2}$ can be obtained as follows:

$$\hat{\omega}_{11\cdot 2}^* = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t^*\hat{e}_t^{*\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{e}_t^*\hat{e}_{t-j}^{*\prime},$$

with $w(\cdot)$ and M being the kernel function and the bandwidth parameter, respectively. To avoid the inconsistency on the estimate of the long-run variance $\hat{\omega}_{11\cdot 2}^*$, we follow CS (2006) and we use the kernel and the bandwidth parameter proposed in Kurozumi (2002). This issue will be discussed in Paragraph 4.3.

For the case of one single break, CS (2006) show that the asymptotic distribution of the test statistic depends on the number of regressors (K), the break fraction (λ) and the deterministic model considered ($g_i(t)$). This result can be readily generalized to the case of multiple structural breaks. In this case, the number of breaks (m) and their relative position in the sample (λ_j) also affect the asymptotic distribution of the statistic.

3.3 A Test Based on the Fully-Modified Estimator

Carrion-i-Silvestre and Sansò (2006) also extend the test presented above to the Fully-modified estimator of cointegrating relationships (Phillips and Hansen, 1990), *i.e.* solving non-parametrically the issue of OLS inefficiency when regressors are non-strictly exogenous.

Consider the set of asymptotic assumptions illustrated in the first part of paragraph 3.2. We exploit here the long-run correlation properties of the innovations vector $\eta_t = (e_t, v_t')$ to rule out the bias due to endogeneity of the regressors x_t. Preliminary simulations suggest that cointegration tests based on the pre-whitened Fully-modified estimator gives better results in terms of size and power. We then follow Andrews and Monahan (1992) and Hansen (1992) and we build the Fully-modified correction by firstly fitting a VAR(1) to η_t and then consistently estimating the long-run covariance matrix from whitened residuals $\hat{\varepsilon}_t = \hat{\eta}_t - \hat{\eta}_{t-1}'\hat{\zeta}$:

$$\Omega_\varepsilon = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_t' + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j}'$$

with partitions

$$\Sigma_\varepsilon = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_t'$$

$$\Lambda_\varepsilon = \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j}'$$

where the kernel function $w(\cdot)$ used for our simulations is the Quadratic Spectral and its associated plug-in bandwidth estimator (Andrews, 1991)[3]. The long-run covariance matrix used for the Fully-modified estimation is then recolored: $\Omega = (I-\hat{\zeta})^{-1}\Omega_\varepsilon(I-\hat{\zeta}')^{-1}$ and $\Lambda = (I-\hat{\zeta})^{-1}\Lambda_\varepsilon(I-\hat{\zeta}')^{-1} - (I-\hat{\zeta})^{-1}\hat{\zeta}\Sigma$, where $\Sigma = 1/T\sum_t^T \hat{\eta}_t\hat{\eta}_t'$.

Fully-modified estimation is then computed by partitioning Ω and Λ, setting $\omega_{11\cdot 2} = \omega_{11} - \omega_{12}\Omega_{22}^{-1}\omega_{21}$ and $\lambda_{21}^{+} = \lambda_{21} - \Lambda_{22}\Omega_{22}^{-1}\omega_{21}$ and transforming the dependent variable $y_t^{+} = y_t - \omega_{12}\Omega_{22}^{-1}v_t'$. The Fully-modified estimator of cointegration parameters is obtained through the following OLS regression:

$$\beta_X^{+} = (X_t'X_t)^{-1}\left(X_t'y_t^{+} - \kappa\lambda_{21}^{+}\right)$$

where X_t is the vector of regressors (deterministic and stochastic) included in the cointegration regression (1) and $\kappa = [\mathbf{0}, I]$ is a matrix of dimension $(d+K)\times K$, with first $d\times K$ zero elements followed by a $K\times K$ identity matrix (d being the number of deterministic regressors in the model).

Fully-modified residuals $\hat{e}_t^{+} = y_t^{+} - X_t'\hat{\beta}_X^{+}$ are then used to compute the LM-type statistic:

$$CS_{\mathrm{FM}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^{+}}\times\sum_{t=1}^{T}\left(S_t^{+}\right)^2 \tag{9}$$

[3]The Quadratic Spectral kernel is defined as $w(x) = \frac{25}{12\pi^2x^2}\left(\frac{\sin(6\pi x/5)}{(6\pi x/5)} - \cos(6\pi x/5)\right)$ and its optimal bandwidth parameter is $M = 1.3221(\hat{\alpha}(2)T)^{1/5}$, where $\hat{\alpha}(2) = \sum_{a=1}^{p}\frac{4\rho_a^2\sigma_a^2}{(1-\rho_a)^8}\Big/\sum_{a=1}^{p}\frac{\sigma_a^2}{(1-\rho_a)^4}$ is obtained from an AR(1) model of each element $\varepsilon_{a,t}$, for $a = 1,\ldots,p$, of ε_t.

where $S_t^+ = \sum_{j=1}^t \hat{e}_j^+$ and the consistent estimate of the long-run variance of residuals e_t^+ is obtained as follows:

$$\hat{\omega}_{11\cdot 2}^+ = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t^+ \hat{e}_t^{+\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{e}_t^+ \hat{e}_{t-j}^{+\prime}$$

with $w(\cdot)$ and M being, respectively, the kernel function and the bandwidth parameter proposed in Kurozumi (2002) (see Paragraph 4.3).

For the case of one single break, CS (2006) show that the asymptotic distribution of the test statistic based on the Fully-modified correction is the same as that assuming x_t strictly exogenous. Again, the asymptotic distribution depends on the number of regressors (K), the break fraction (λ) and the deterministic model considered ($g_i(t)$). In the multiple breaks framework considered here, the asymptotic distribution also depends on the number of breaks (m) and their relative position in the sample (λ_j).

3.4 A Test Based on the Canonical Cointegration Estimator

A test based on the *feasible* Canonical Cointegration Regression estimator (Park, 1992) is developed in Bartley, Lee and Strazicich (2001) - henceforth BLS. The authors propose a LM-type test for the null hypothesis of cointegration against the alternative of non-cointegration, with structural breaks under both the null and the alternative.

As for the case of test statistics based on the Fully-modified estimator, preliminary simulations suggest that tests based on the pre-whitened CCR estimator gives better results in terms of size and power. We then fit a VAR(1) to η_t and we compute consistent estimate of the long-run covariance matrix from whitened residuals $\hat{\varepsilon}_t = \hat{\eta}_t - \hat{\eta}_{t-1}^{\prime}\hat{\zeta}$:

$$\Omega_\varepsilon = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t^{\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_{t-j}^{\prime}$$

with partitions

$$\Sigma_\varepsilon = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t^{\prime}$$

$$\Lambda_\varepsilon = \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_{t-j}^{\prime}$$

$$\Gamma_\varepsilon = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t^{\prime} + \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_{t-j}^{\prime}$$

where the kernel function $w(\cdot)$ used for our simulations is the Quadratic Spectral and its associated plug-in bandwidth estimator (Andrews, 1991) (see footnote 3). It is worth noticing that $\Omega_\varepsilon = \Sigma_\varepsilon + \Lambda_\varepsilon + \Lambda_\varepsilon^{\prime} = \Gamma_\varepsilon + \Lambda_\varepsilon^{\prime}$. The long-run covariance matrix used for the

CCR estimation is then recolored: $\Omega = (I-\hat{\zeta})^{-1}\Omega_\varepsilon(I-\hat{\zeta}')^{-1}$ and $\Lambda = (I-\hat{\zeta})^{-1}\Lambda_\varepsilon(I-\hat{\zeta}')^{-1} - (I-\hat{\zeta})^{-1}\hat{\zeta}\Sigma$, where $\Sigma = 1/T\sum_t^T \hat{\eta}_t\hat{\eta}_t'$.

CCR estimation is computed by first transforming the regressand and the stochastic regressors and then estimating by OLS the corrected cointegration model (1):

$$y_t^\star = \alpha_0 + g_i(t) + x_t^{\star\prime}\beta^\star + e_t^\star \tag{10}$$

where $y_t^\star = y_t - (\Sigma^{-1}\Gamma_2\hat{\beta} + (0, \omega_{12}\Omega_{22}^{-1})')'\hat{\eta}_t$, $x_t^\star = x_t - (\Sigma^{-1}\Gamma_2)'\hat{\eta}_t$, $\Gamma_2 = (\gamma_{12}, \Gamma_{22})$ and $\hat{\beta}$ is the vector of estimated parameters obtained from the auxiliary regression of the uncorrected model (1).

CCR residuals $\hat{e}_t^\star = y_t^\star - \hat{\alpha}_0 - g_i(t) - x_t^{\star\prime}\hat{\beta}^\star$ are then used to compute the LM-type statistic:

$$BLS_{\text{CCR}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^\star} \times \sum_{t=1}^{T}\left(S_t^+\right)^2 \tag{11}$$

where $S_t^+ = \sum_{j=1}^t \hat{e}_j^\star$ and the consistent estimate of the long-run variance of residuals $e_t^\star$ is obtained as follows:

$$\hat{\omega}_{11\cdot 2}^\star = \frac{1}{T}\sum_{t=1}^{T}\hat{e}_t^\star\hat{e}_t^{\star\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{e}_t^\star\hat{e}_{t-j}^{\star\prime}$$

with $w(\cdot)$ and M being, respectively, the kernel function and the bandwidth parameter proposed in Kurozumi (2002) (see Paragraph 4.3).

For the case of one single break, BLS (2001) follow Choi and Ahn (1995) to derive the asymptotic distribution of the test statistic. It can be shown that the latter has the same distribution as that derived in CS (2006). Again, the asymptotic distribution will depend on the number of regressors (K), the break fraction (λ), the deterministic model considered ($g_i(t)$), the number of breaks (m) and their relative position in the sample (λ_j).

4 The Design of Monte Carlo Experiments

4.1 Data Generating Process

In this section we study the finite sample properties (size and power) of the statistical tests reviewed in Section 2. For this purpose, we simulate 20,000 series of dimension $T = \{100, 200\}$ using the following triangular system representation of the DGP (Gregory and Hansen, 1996; McCabe, Leybourne and Shin, 1997; Carrion-i-Silvestre and Sansò, 2006):

$$y_t = \alpha_0 + g_i(t) + \beta x_t + e_t \tag{12}$$

$$e_t = \rho e_{t-1} + \varepsilon_t \tag{13}$$

$$\varepsilon_t = \phi\varepsilon_{t-1} + u_t - \gamma u_{t-1} \tag{14}$$

$$\alpha_1 y_t - \alpha_2 x_t = w_t \tag{15}$$

$$w_t = w_{t-1} + \mu_t \tag{16}$$

where $g_i(t)$, for $i = \{\text{A, B, C}\}$, is the deterministic function as defined in (2). The error correction term (e_t) is assumed to be autocorrelated with coefficient $|\rho| \leq 1$, depending on the null hypothesis involved by the selected statistical test. We account for misspecification of residuals by allowing the error term ε_t to follow an ARMA(1,1) process, with AR parameter ϕ and MA parameter γ. Simple AR(1) and MA(1) processes can be simulated by setting either $\gamma = 0$ or $\phi = 0$, respectively. Finally, μ_t is the vector of x_t innovations. The system also accounts for the endogeneity of regressor x_t: when $\alpha_1 = 0$, x_t is strictly exogenous, while it isn't when $\alpha_1 = 1$.
In this general specification, u_t and μ_t are i.i.d. with distribution:

$$\begin{pmatrix} u_t \\ \mu_t \end{pmatrix} \sim \text{i.i.d.}N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta\sigma_\mu \\ \delta\sigma_\mu & \sigma_\mu^2 \end{pmatrix} \right]$$

where δ controls for the correlation between u_t and μ_t. To avoid data dependence on initial conditions, the actual Monte Carlo sample dimension is $T_{ac} = T + T_0$, where $T_0 = 100$ is the number of initial observations to be discarded.

4.2 Parameter Space

We consider two sets of parameter space, a first one common to all simulations and a second one dependent on each specific Monte Carlo exercise.

In the first set, we consider the parameter space $(\alpha_0, \tau, \beta, \alpha_1, \alpha_2, \rho, \sigma_\mu^2, \delta, \phi, \gamma)$, where $\alpha_0 = 1$, $\tau = \{0, 0.2\}$, $\beta = 1$, $\alpha_1 = \{0, 1\}$, $\alpha_2 = -1$, $\rho = \{0, 0.1, 0.9, 1\}$, $\sigma_\mu^2 = \{0.5, 1, 2\}$, $\delta = \{0, 0.5\}$, $\phi = \{0, 0.4\}$ and $\gamma = \{0, 0.4\}$.

In the second set, we consider the parameter space $(\theta_1, \theta_2, m, \lambda)$. For each Model $i = \{\text{A, B, C}\}$, the value of these parameters is defined as follows:

- $m = 1$, $\lambda = 50\%$, $\theta_1 = 0.5$, $\theta_2 = \{0, 0.2\}$.
- $m = 3$, $\lambda = (30\%, 50\%, 70\%)$, $\theta_1 = (0.5, -0.8, 0.5)$, $\theta_2 = \{(0, 0, 0), (0.2, -0.5, 0.2)\}$.
- $m = 5$, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $\theta_1 = (0.5, -0.8, 0.5, -0.2, 0.5)$,
$\theta_2 = \{(0, 0, 0, 0, 0), (0.2, -0.5, 0.2, -0.3, 0.4)\}$.

4.3 Long-run Variance Estimator

Some of the statistical tests reported in this paper require a consistent estimate of the long-run variance of cointegration residuals (ω_{11}). For this purpose, Andrews (1991) and Andrews and Monahan (1992) recommend the use of the HAC estimator involving a Pre-Whitened Quadratic-Spectral kernel and an automatic data-dependent rule for the selection of the bandwidth parameter. Nevertheless, recent literature points out the size distortions of statistical tests which may arise from the small sample bias of prewhitening coefficients (Kurozumi, 2002; Phillips and Sul, 2003; Sul *et al.*, 2005).

To avoid finite samples inconsistency problems, we report experimental results involving the modified bandwidth selection rules recently proposed in Kurozumi (2002). This is mainly the standard Bartlett kernel function

$$w(x) = \begin{cases} 1 - \frac{j}{M} & \text{if } \frac{j}{M} \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

with the bandwidth parameter M chosen following a modified automatic rule:

$$\tilde{M} = \min \left(1.1447 \left\{ \frac{4\hat{\rho}^2 T}{(1+\hat{\rho})^2(1-\hat{\rho})^2} \right\}^{1/3}, 1.1447 \left\{ \frac{4k^2 T}{(1+k)^2(1-k)^2} \right\}^{1/3} \right),$$

where $\hat{\rho}$ is the estimated AR(1) coefficient of $\hat{e}_t$, the estimated cointegration residual. The rule proposed in Kurozumi (2001) sets a boundary condition to the bandwidth parameter which depends on the predetermined value of k. Simulations show that $k = \{0.7, 0.8, 0.9\}$ is the best range of values for the power-size trade-off of the test. In this paper we follow CS (2006) and we fix $k = 0.8$.

5 Simulation Results - Empirical Size

We report in Tables 1 to 6 rejection frequencies at 5% nominal confidence level. The null hypothesis is cointegration for CS and BLS tests and non-cointegration for WE tests. Asymptotic critical values are computed by simulating 20,000 series of dimension $T_\infty = 2,000$ and picking up the 95-th percentile of the asymptotic distribution for CS and BLS tests and the 5-th percentile for WE tests.

Results from the one break case are reported in Tables 1 and 2. CS and BLS tests show the highest rates of rejection in all models when residuals are specified as an AR(1) process and when σ^2_μ is low. The MA(1) specification of residuals is instead associated to highest under-rejection rates. Under-rejection is also determined by the correlation of residuals (δ) for the CS_{FM} and the BLS_{CCR} tests. This result deteriorates with larger samples. These findings are exacerbated for the test based on the DGLS estimator in all models. The WE tests are instead well sized across all Monte Carlo experiments, except for some size distortions for the WE_Φ test when residuals are misspecified.

Results from the three breaks case are reported in Tables 3 and 4. As for the one break case, over rejection and under rejection rates are mainly led by the misspecification and the correlation of residuals. However, it is worth noticing that CS test based on the DGLS estimator seem to perform better than other estimators in model C, especially in the small sample simulation. The WE tests are in general well sized. Again, results from the WE_Φ test show some size distortions when residuals are misspecified.

Results from the five breaks case are reported in Tables 5 and 6. CS and BLS tests are in general highly oversized. In particular, the CS_{FM} and BLS_{CCR} show highest rejection rates in model C. CS test based on the DGLS estimator show best size performance in model B and C. The WE tests are extremely well sized when compared to other tests. All in all, WE tests do not seem to be affected by the inclusion of multiple breaks.

6 Simulation Results - Empirical Power

We report in Tables 7 to 12 size-adjusted rejection frequencies at 5% actual confidence level. The alternative hypothesis is non-cointegration for CS and BLS tests and cointegration for WE tests. Critical values are computed from the actual distribution of statistical tests by picking up the 95-th percentile for CS and BLS tests and the 5-th percentile for WE tests. For reasons of space, we only discuss power properties under the assumption of correct specification of residuals ($\gamma = \phi = 0$).

Results from the one break case are reported in Tables 7 and 8. CS and BLS tests have in general highest power in model A and C. In particular, the CS_{DGLS} have high power against the alternative of non-cointegration. However, it is worth noticing that rejection rates decrease slower than in other tests when we move away from this hypothesis. It is also interesting to note that WE tests show serious low power across models and simulations. Rejection rates are close to the nominal size, which implies that the tests are not actually able to discern between cointegration and non-cointegration. Preliminary simulations show that this result is mainly due to the uncorrected endogeneity of regressors.

Results from the three breaks case are reported in Tables 9 and 10. Very high rejection rates are achieved by the CS_{DGLS} test, mainly in model A. In model B and C, the CS_{DOLS} test also show great power performances. Similarly to the one break case, WE tests show serious low power.

Finally, results from the five breaks case are reported in Tables 11 and 12. The inclusion of many breaks do not seem to affect the power performance of CS and BLS tests. Rejection rates close to unity are even achieved by CS_{DGLS} and CS_{DOLS} tests in model A and C. Curiously, the CS_{FM} test show a very low power in model C, when $\delta = 0.5$ and $T = 100$, which disappears with larger samples.

7 Concluding Remarks

In this paper we compare the size-power performance of residual-based tests for cointegration with structural breaks. In particular, we focus on statistical tests recently proposed in Bartley *et al.* (2001), Carrion-i-SIlvestre and Sansò (2006) and Westerlund and Edgerton (2006). Through an extensive Monte Carlo study, we evaluate their performance in small samples when up to five (exogenous) deterministic breaks are included in the cointegration model. We consider several estimators of single-equation cointegrating relationships and three deterministic breaks scenarios (breaks in constant, with and without trend, and breaks in both constant and trend), as well as endogenous regressors and misspecifications in model residuals.

Simulation results on the empirical size suggest that the LM-type tests proposed in Bartley *et al.* (2001) and Carrion-i-SIlvestre and Sansò (2006) have highest over-rejection rates in the multiple breaks scenarios. In addition, the presence of residuals misspecification generally exacerbate size distortions. This is not the case for the tests proposed in Westerlund and Edgerton (2006), which have correct size across all our simulation exercises. Despite the presence of strong size distortions, simulation results on the empirical (size-adjusted) power show that the first two tests have quite high power against the alternative hypothesis across all simulations and models, while the latter have a very poor power close to the nominal size. The best size-power performance is shown by the LM-type tests associated with the DGLS and DOLS estimators of cointegrating relationships, although tests based on Fully-modified and Canonical cointegration corrections have mainly good power performance. Despite the strong power distortions, the LM-type tests proposed in Westerlund and Edgerton (2006) show impressive size performance. A correction for regressors endogenity, which is not accounted for in the OLS estimation of the test regression, would probably improve the overall size-power balance for these tests.

Our findings provide a twofold guideline for practitioners when dealing with cointegrating models and multiple deterministic structural breaks: the choice of the cointegration estimator and its size-power best statistical test.

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Table 1: Empirical Size (5% nominal size), 1 Break, $\lambda = 50\%$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0822	0.0682	0.1012	0.0620	0.0706	0.0219	0.0636	0.0207	0.0738	0.0735	0.0642	0.0663
		1	0.0743	0.0649	0.0566	0.0462	0.0543	0.0221	0.0562	0.0252	0.0722	0.0718	0.0656	0.0659
		2	0.0660	0.0505	0.0467	0.0383	0.0520	0.0257	0.0554	0.0277	0.0721	0.0714	0.0648	0.0627
0.4	0	0.5	0.1047	0.0756	0.4026	0.1798	0.1142	0.0335	0.1098	0.0411	0.0655	0.0681	0.0364	0.0481
		1	0.0962	0.0756	0.2196	0.0817	0.0972	0.0434	0.0984	0.0516	0.0678	0.0668	0.0295	0.0307
		2	0.0966	0.0734	0.1372	0.0583	0.0948	0.0600	0.0950	0.0648	0.0697	0.0697	0.0205	0.0164
0	0.4	0.5	0.0501	0.0431	0.0304	0.0286	0.0513	0.0261	0.0221	0.0145	0.0734	0.0746	0.0876	0.0767
		1	0.0314	0.0262	0.0189	0.0207	0.0297	0.0184	0.0190	0.0155	0.0755	0.0773	0.1063	0.1063
		2	0.0174	0.0158	0.0147	0.0151	0.0213	0.0154	0.0193	0.0150	0.0801	0.0866	0.1327	0.1648

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1089	0.0894	0.1122	0.0734	0.1025	0.0397	0.0853	0.0331	0.0826	0.0812	0.0719	0.0710
		1	0.0907	0.0761	0.0694	0.0541	0.0721	0.0329	0.0699	0.0350	0.0813	0.0816	0.0742	0.0737
		2	0.0735	0.0517	0.0545	0.0412	0.0576	0.0325	0.0612	0.0350	0.0816	0.0807	0.0717	0.0702
0.4	0	0.5	0.1640	0.1141	0.2842	0.1572	0.1472	0.0527	0.1414	0.0606	0.0732	0.0756	0.0388	0.0509
		1	0.1379	0.0976	0.2047	0.0912	0.1213	0.0540	0.1190	0.0621	0.0723	0.0703	0.0293	0.0304
		2	0.1227	0.0821	0.1492	0.0657	0.1015	0.0639	0.1062	0.0680	0.0747	0.0721	0.0196	0.0160
0	0.4	0.5	0.0610	0.0557	0.0376	0.0386	0.1120	0.0639	0.0380	0.0217	0.0846	0.0815	0.1026	0.0866
		1	0.0377	0.0319	0.0261	0.0273	0.0618	0.0363	0.0288	0.0228	0.0936	0.0920	0.1278	0.1254
		2	0.0198	0.0187	0.0168	0.0174	0.0389	0.0255	0.0283	0.0234	0.1000	0.1130	0.1598	0.1977

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1135	0.0873	0.1347	0.0827	0.1413	0.0560	0.1254	0.0506	0.0840	0.0818	0.0741	0.0746
		1	0.0952	0.0811	0.0750	0.0557	0.0900	0.0418	0.0921	0.0430	0.0833	0.0816	0.0767	0.0757
		2	0.0818	0.0546	0.0571	0.0389	0.0664	0.0347	0.0730	0.0383	0.0891	0.0858	0.0789	0.0757
0.4	0	0.5	0.1905	0.1171	0.4069	0.2254	0.2204	0.0719	0.2219	0.0990	0.0724	0.0750	0.0358	0.0524
		1	0.1605	0.1023	0.2647	0.1084	0.1696	0.0735	0.1733	0.0896	0.0759	0.0736	0.0282	0.0295
		2	0.1504	0.0972	0.1777	0.0794	0.1482	0.0846	0.1534	0.0945	0.0783	0.0784	0.0209	0.0159
0	0.4	0.5	0.0552	0.0502	0.0313	0.0318	0.2059	0.1731	0.0637	0.0703	0.0864	0.0839	0.1093	0.0911
		1	0.0311	0.0257	0.0186	0.0196	0.1130	0.1105	0.0430	0.0601	0.0944	0.0910	0.1378	0.1322
		2	0.0151	0.0137	0.0134	0.0118	0.0635	0.0658	0.0357	0.0517	0.1031	0.1180	0.1761	0.2169

Notes: The DGP is given in equations (12)-(16). x_t is endogenous ($\alpha_1 = 1$), $\alpha_2 = -1$, $\rho = 0$ under H_0 for the CiS and BLS tests, while $\rho = 1$ under H_0 for the WE tests. The LRV is computed as in Kurozumi (2001). Asymptotic critical values are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Estimated critical values for Model A are: 95% cv CiS = BLS = 0.1547; 5% cv t-stat = -2.871, 5% cv Φ = -14.206. Estimated critical values for Model B are: 95% cv CiS = BLS = 0.1063; 5% cv t-stat = -3.019, 5% cv Φ = -18.150. Estimated critical values for Model C are: 95% cv CiS = BLS = 0.0556; 5% cv t-stat = -3.333, 5% cv Φ = -22.084.

Table 2: Empirical Size (5% nominal size), 1 Break, $\lambda = 50\%$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0772	0.0732	0.0684	0.0549	0.0530	0.0180	0.0514	0.0184	0.0563	0.0565	0.0520	0.0534
		1	0.0753	0.0671	0.0553	0.0504	0.0503	0.0244	0.0514	0.0249	0.0588	0.0568	0.0571	0.0536
		2	0.0671	0.0566	0.0468	0.0435	0.0461	0.0256	0.0493	0.0259	0.0550	0.0599	0.0525	0.0558
0.4	0	0.5	0.0920	0.0794	0.2415	0.0946	0.0877	0.0248	0.0841	0.0276	0.0532	0.0540	0.0270	0.0374
		1	0.0872	0.0746	0.1107	0.0641	0.0846	0.0417	0.0822	0.0442	0.0532	0.0513	0.0199	0.0214
		2	0.0804	0.0733	0.0712	0.0532	0.0823	0.0600	0.0812	0.0610	0.0541	0.0568	0.0132	0.0133
0	0.4	0.5	0.0489	0.0490	0.0264	0.0305	0.0274	0.0107	0.0177	0.0087	0.0599	0.0558	0.0768	0.0649
		1	0.0365	0.0312	0.0220	0.0246	0.0214	0.0137	0.0185	0.0113	0.0644	0.0638	0.0999	0.0969
		2	0.0179	0.0168	0.0150	0.0156	0.0169	0.0115	0.0170	0.0111	0.0670	0.0752	0.1226	0.1506

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0898	0.0762	0.0825	0.0578	0.0618	0.0255	0.0591	0.0238	0.0632	0.0600	0.0582	0.0563
		1	0.0816	0.0723	0.0635	0.0549	0.0534	0.0303	0.0565	0.0294	0.0650	0.0643	0.0607	0.0604
		2	0.0697	0.0579	0.0489	0.0466	0.0491	0.0309	0.0534	0.0317	0.0678	0.0661	0.0632	0.0612
0.4	0	0.5	0.1170	0.0938	0.2400	0.1029	0.1014	0.0321	0.0988	0.0331	0.0527	0.0551	0.0268	0.0384
		1	0.1064	0.0857	0.1304	0.0711	0.0934	0.0477	0.0920	0.0496	0.0517	0.0545	0.0196	0.0218
		2	0.0943	0.0776	0.0881	0.0566	0.0868	0.0628	0.0866	0.0643	0.0581	0.0600	0.0126	0.0098
0	0.4	0.5	0.0538	0.0509	0.0301	0.0319	0.0512	0.0239	0.0238	0.0124	0.0683	0.0635	0.0882	0.0714
		1	0.0389	0.0334	0.0265	0.0281	0.0329	0.0215	0.0237	0.0165	0.0760	0.0742	0.1161	0.1144
		2	0.0198	0.0202	0.0169	0.0189	0.0250	0.0170	0.0220	0.0161	0.0872	0.0970	0.1547	0.1928

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0992	0.0843	0.0911	0.0664	0.0669	0.0221	0.0678	0.0215	0.0671	0.0662	0.0622	0.0625
		1	0.0914	0.0769	0.0638	0.0529	0.0559	0.0246	0.0632	0.0261	0.0659	0.0641	0.0623	0.0605
		2	0.0816	0.0664	0.0530	0.0491	0.0525	0.0269	0.0597	0.0297	0.0675	0.0682	0.0632	0.0627
0.4	0	0.5	0.1341	0.1023	0.3209	0.1374	0.1407	0.0369	0.1348	0.0402	0.0558	0.0585	0.0281	0.0403
		1	0.1178	0.0945	0.1619	0.0790	0.1211	0.0550	0.1170	0.0582	0.0557	0.0554	0.0187	0.0197
		2	0.1105	0.0923	0.1059	0.0649	0.1132	0.0794	0.1116	0.0807	0.0584	0.0588	0.0121	0.0089
0	0.4	0.5	0.0510	0.0497	0.0237	0.0275	0.0648	0.0344	0.0198	0.0134	0.0708	0.0672	0.0929	0.0770
		1	0.0325	0.0255	0.0166	0.0188	0.0357	0.0211	0.0179	0.0150	0.0757	0.0736	0.1237	0.1191
		2	0.0137	0.0133	0.0129	0.0124	0.0233	0.0154	0.0197	0.0150	0.0889	0.1024	0.1678	0.2139

Notes: See Table 1.

Table 3: Empirical Size (5% nominal size), 3 Breaks, $\lambda = (30\%, 50\%, 70\%)$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1008	0.0774	0.1816	0.1056	0.1329	0.0660	0.1043	0.0505	0.0733	0.0749	0.0667	0.0670
		1	0.0867	0.0726	0.0842	0.0560	0.0848	0.0480	0.0825	0.0478	0.0723	0.0741	0.0652	0.0680
		2	0.0746	0.0526	0.0589	0.0408	0.0628	0.0444	0.0704	0.0452	0.0737	0.0747	0.0675	0.0666
0.4	0	0.5	0.1655	0.0965	0.6140	0.3490	0.2018	0.0633	0.1898	0.0814	0.0687	0.0705	0.0365	0.0485
		1	0.1417	0.0927	0.3741	0.1471	0.1550	0.0639	0.1543	0.0813	0.0684	0.0675	0.0299	0.0306
		2	0.1292	0.0883	0.2328	0.0914	0.1342	0.0789	0.1387	0.0916	0.0712	0.0720	0.0211	0.0181
0	0.4	0.5	0.0527	0.0448	0.0397	0.0339	0.1906	0.1810	0.0595	0.0774	0.0726	0.0735	0.0868	0.0768
		1	0.0303	0.0259	0.0200	0.0189	0.1062	0.1162	0.0460	0.0671	0.0762	0.0768	0.1074	0.1073
		2	0.0165	0.0162	0.0126	0.0119	0.0643	0.0807	0.0406	0.0617	0.0786	0.0881	0.1338	0.1627

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1541	0.1170	0.2111	0.1316	0.1909	0.1058	0.1500	0.0805	0.0792	0.0789	0.0694	0.0694
		1	0.1188	0.0945	0.1066	0.0736	0.1151	0.0736	0.1092	0.0669	0.0795	0.0816	0.0716	0.0717
		2	0.0995	0.0637	0.0774	0.0504	0.0813	0.0564	0.0878	0.0625	0.0791	0.0792	0.0687	0.0707
0.4	0	0.5	0.2599	0.1660	0.5852	0.3549	0.2654	0.0934	0.2578	0.1197	0.0698	0.0724	0.0375	0.0496
		1	0.2124	0.1399	0.3871	0.1706	0.2006	0.0872	0.2041	0.1100	0.0708	0.0691	0.0295	0.0294
		2	0.1850	0.1207	0.2595	0.1138	0.1679	0.0934	0.1729	0.1081	0.0730	0.0712	0.0192	0.0165
0	0.4	0.5	0.0746	0.0636	0.0542	0.0502	0.3034	0.2823	0.1006	0.1275	0.0828	0.0777	0.0984	0.0812
		1	0.0423	0.0344	0.0291	0.0267	0.1788	0.1904	0.0698	0.1089	0.0924	0.0891	0.1260	0.1238
		2	0.0221	0.0234	0.0169	0.0156	0.1097	0.1225	0.0638	0.0981	0.0961	0.1077	0.1538	0.1899

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.3010	0.2088	0.2737	0.1897	0.4860	0.5895	0.3990	0.3576	0.0951	0.0964	0.0932	0.0932
		1	0.1921	0.1268	0.1505	0.0891	0.3133	0.4307	0.2598	0.2871	0.0987	0.0960	0.0967	0.0956
		2	0.1394	0.0736	0.0977	0.0523	0.2008	0.2944	0.1843	0.2368	0.0968	0.0959	0.0926	0.0925
0.4	0	0.5	0.6039	0.4018	0.5960	0.4327	0.5617	0.2297	0.6081	0.3547	0.0819	0.0866	0.0463	0.0637
		1	0.4804	0.2641	0.4428	0.2341	0.4332	0.1676	0.4802	0.2572	0.0870	0.0832	0.0350	0.0384
		2	0.4029	0.2051	0.3239	0.1558	0.3480	0.1575	0.3864	0.2107	0.0849	0.0881	0.0249	0.0200
0	0.4	0.5	0.1125	0.0909	0.0719	0.0693	0.8025	0.9646	0.5012	0.7850	0.0988	0.0958	0.1336	0.1105
		1	0.0476	0.0415	0.0302	0.0267	0.7373	0.9475	0.3970	0.7461	0.1051	0.1072	0.1657	0.1656
		2	0.0266	0.0604	0.0152	0.0122	0.6736	0.9055	0.3820	0.7469	0.1105	0.1279	0.2071	0.2509

Notes: See Table 1. Estimated critical values for Model A are: 95% cv CiS = BLS = 0.0743; 5% cv t-stat = -2.873, 5% cv Φ = -14.154. Estimated critical values for Model B are: 95% cv CiS = BLS = 0.0599; 5% cv t-stat = -3.026, 5% cv Φ = -18.235. Estimated critical values for Model C are: 95% cv CiS = BLS = 0.0270; 5% cv t-stat = -3.849, 5% cv Φ = -29.467.

Table 4: Empirical Size (5% nominal size), 3 Breaks, $\lambda = (30\%, 50\%, 70\%)$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0906	0.0787	0.1087	0.0753	0.0710	0.0270	0.0641	0.0245	0.0590	0.0591	0.0549	0.0556
		1	0.0851	0.0751	0.0703	0.0566	0.0581	0.0278	0.0581	0.0292	0.0591	0.0567	0.0568	0.0544
		2	0.0748	0.0614	0.0532	0.0462	0.0521	0.0298	0.0577	0.0308	0.0567	0.0591	0.0542	0.0551
0.4	0	0.5	0.1182	0.0924	0.4121	0.1865	0.1279	0.0298	0.1203	0.0365	0.0536	0.0558	0.0281	0.0387
		1	0.1082	0.0883	0.1940	0.0943	0.1145	0.0472	0.1095	0.0533	0.0527	0.0532	0.0203	0.0224
		2	0.1019	0.0867	0.1186	0.0691	0.1054	0.0693	0.1055	0.0759	0.0539	0.0570	0.0148	0.0130
0	0.4	0.5	0.0511	0.0479	0.0286	0.0292	0.0613	0.0393	0.0239	0.0204	0.0608	0.0577	0.0776	0.0657
		1	0.0335	0.0268	0.0187	0.0210	0.0362	0.0284	0.0223	0.0211	0.0654	0.0631	0.1004	0.0988
		2	0.0148	0.0154	0.0118	0.0129	0.0253	0.0216	0.0225	0.0217	0.0662	0.0717	0.1231	0.1477

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1160	0.1023	0.1312	0.0905	0.0869	0.0370	0.0791	0.0343	0.0614	0.0615	0.0575	0.0582
		1	0.1031	0.0854	0.0837	0.0622	0.0660	0.0348	0.0692	0.0355	0.0636	0.0640	0.0606	0.0589
		2	0.0889	0.0714	0.0627	0.0558	0.0592	0.0376	0.0657	0.0391	0.0661	0.0665	0.0616	0.0601
0.4	0	0.5	0.1700	0.1319	0.4389	0.2142	0.1581	0.0419	0.1538	0.0500	0.0534	0.0558	0.0276	0.0387
		1	0.1452	0.1113	0.2257	0.1070	0.1311	0.0546	0.1277	0.0630	0.0518	0.0538	0.0202	0.0207
		2	0.1309	0.1059	0.1439	0.0838	0.1244	0.0843	0.1228	0.0880	0.0578	0.0575	0.0126	0.0104
0	0.4	0.5	0.0620	0.0582	0.0361	0.0384	0.1043	0.0706	0.0330	0.0305	0.0661	0.0629	0.0870	0.0700
		1	0.0387	0.0297	0.0227	0.0230	0.0548	0.0432	0.0298	0.0317	0.0740	0.0731	0.1154	0.1100
		2	0.0174	0.0195	0.0151	0.0153	0.0381	0.0332	0.0316	0.0320	0.0859	0.0963	0.1520	0.1856

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1916	0.1465	0.1846	0.1411	0.2130	0.1190	0.1580	0.0899	0.0703	0.0718	0.0694	0.0698
		1	0.1400	0.1054	0.0949	0.0735	0.1123	0.0751	0.1128	0.0767	0.0744	0.0693	0.0716	0.0679
		2	0.1067	0.0722	0.0663	0.0537	0.0787	0.0570	0.0871	0.0650	0.0734	0.0749	0.0694	0.0722
0.4	0	0.5	0.3628	0.2388	0.5252	0.3192	0.3376	0.0715	0.3430	0.1290	0.0589	0.0621	0.0294	0.0430
		1	0.2701	0.1681	0.3030	0.1475	0.2486	0.0748	0.2642	0.1194	0.0602	0.0590	0.0192	0.0186
		2	0.2244	0.1390	0.2006	0.0929	0.2049	0.1016	0.2166	0.1275	0.0615	0.0634	0.0115	0.0080
0	0.4	0.5	0.0805	0.0700	0.0383	0.0425	0.4427	0.5125	0.1260	0.2119	0.0755	0.0728	0.1084	0.0887
		1	0.0373	0.0277	0.0183	0.0207	0.2563	0.3419	0.0814	0.1847	0.0852	0.0824	0.1475	0.1408
		2	0.0128	0.0184	0.0102	0.0117	0.1425	0.2186	0.0711	0.1698	0.0915	0.1113	0.1947	0.2505

Notes: See Table 3.

Table 5: Empirical Size (5% nominal size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1887	0.1323	0.2891	0.1846	0.2729	0.2251	0.2037	0.1354	0.0728	0.0740	0.0640	0.0636
		1	0.1372	0.1023	0.1441	0.0893	0.1547	0.1461	0.1343	0.1080	0.0734	0.0726	0.0654	0.0674
		2	0.1102	0.0691	0.0943	0.0567	0.1064	0.1042	0.1035	0.0899	0.0745	0.0744	0.0661	0.0633
0.4	0	0.5	0.3695	0.2189	0.7246	0.4960	0.3774	0.1252	0.3712	0.1662	0.0690	0.0711	0.0381	0.0480
		1	0.2867	0.1666	0.5091	0.2497	0.2719	0.1069	0.2832	0.1398	0.0692	0.0692	0.0296	0.0318
		2	0.2442	0.1443	0.3590	0.1604	0.2207	0.1162	0.2340	0.1363	0.0719	0.0715	0.0222	0.0179
0	0.4	0.5	0.0830	0.0669	0.0716	0.0570	0.5055	0.6056	0.1905	0.3198	0.0741	0.0732	0.0883	0.0755
		1	0.0423	0.0329	0.0300	0.0241	0.3425	0.4729	0.1377	0.2700	0.0763	0.0761	0.1058	0.1052
		2	0.0232	0.0298	0.0166	0.0136	0.2407	0.3525	0.1213	0.2459	0.0781	0.0836	0.1272	0.1556

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.2290	0.1587	0.2782	0.1776	0.3202	0.2576	0.2448	0.1664	0.0800	0.0809	0.0704	0.0715
		1	0.1585	0.1122	0.1432	0.0902	0.1878	0.1714	0.1618	0.1335	0.0837	0.0811	0.0757	0.0724
		2	0.1224	0.0713	0.0973	0.0576	0.1236	0.1168	0.1161	0.1056	0.0826	0.0808	0.0742	0.0726
0.4	0	0.5	0.4305	0.2705	0.6876	0.4613	0.4182	0.1538	0.4286	0.2020	0.0710	0.0743	0.0397	0.0512
		1	0.3400	0.2017	0.4858	0.2352	0.3062	0.1245	0.3230	0.1616	0.0731	0.0711	0.0311	0.0314
		2	0.2877	0.1636	0.3399	0.1538	0.2468	0.1255	0.2597	0.1429	0.0739	0.0721	0.0221	0.0181
0	0.4	0.5	0.0964	0.0733	0.0732	0.0571	0.5846	0.6770	0.2474	0.3818	0.0829	0.0789	0.0994	0.0822
		1	0.0474	0.0374	0.0340	0.0276	0.4197	0.5450	0.1715	0.3213	0.0960	0.0910	0.1277	0.1249
		2	0.0257	0.0351	0.0187	0.0149	0.2997	0.4171	0.1456	0.2863	0.0987	0.1067	0.1549	0.1857

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.6479	0.4954	0.4933	0.3683	0.8245	0.9456	0.7569	0.7719	0.0964	0.0981	0.1067	0.1080
		1	0.4531	0.2743	0.3235	0.1954	0.6985	0.9071	0.5845	0.7232	0.0967	0.0980	0.1061	0.1089
		2	0.2998	0.1902	0.2212	0.1054	0.5754	0.8271	0.4559	0.6898	0.0970	0.0943	0.1043	0.1025
0.4	0	0.5	0.8852	0.7674	0.7737	0.6456	0.8516	0.5885	0.8958	0.7166	0.0837	0.0871	0.0537	0.0714
		1	0.8190	0.6015	0.6610	0.4454	0.7593	0.4435	0.8233	0.5773	0.0820	0.0805	0.0385	0.0410
		2	0.7481	0.4726	0.5509	0.3335	0.6636	0.3739	0.7204	0.4665	0.0827	0.0852	0.0285	0.0226
0	0.4	0.5	0.2906	0.2270	0.1880	0.1669	0.9626	0.9996	0.8344	0.9827	0.0967	0.0985	0.1454	0.1252
		1	0.1340	0.1654	0.0823	0.0625	0.9562	0.9998	0.7809	0.9849	0.1024	0.1061	0.1777	0.1843
		2	0.1516	0.4297	0.0368	0.0310	0.9636	0.9992	0.8164	0.9883	0.1156	0.1283	0.2279	0.2754

Notes: See Table 1. Estimated critical values for Model A are: 95% cv CiS = BLS = 0.0488; 5% cv t-stat = -2.875, 5% cv Φ = -14.210. Estimated critical values for Model B are: 95% cv CiS = BLS = 0.0433; 5% cv t-stat = -3.015, 5% cv Φ = -18.085. Estimated critical values for Model C are: 95% cv CiS = BLS = 0.0178; 5% cv t-stat = -4.277, 5% cv Φ = -36.264.

Table 6: Empirical Size (5% nominal size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1372	0.1113	0.1767	0.1109	0.1153	0.0554	0.0888	0.0425	0.0583	0.0583	0.0545	0.0557
		1	0.1177	0.0941	0.0990	0.0717	0.0781	0.0457	0.0779	0.0428	0.0600	0.0583	0.0565	0.0539
		2	0.0951	0.0732	0.0713	0.0572	0.0659	0.0431	0.0675	0.0424	0.0568	0.0604	0.0530	0.0554
0.4	0	0.5	0.2154	0.1522	0.5641	0.3084	0.2120	0.0452	0.2003	0.0566	0.0534	0.0548	0.0268	0.0389
		1	0.1784	0.1291	0.3175	0.1493	0.1711	0.0599	0.1669	0.0727	0.0533	0.0533	0.0203	0.0220
		2	0.1560	0.1188	0.1967	0.1008	0.1486	0.0862	0.1515	0.0990	0.0549	0.0566	0.0151	0.0130
0	0.4	0.5	0.0685	0.0576	0.0372	0.0361	0.1693	0.1535	0.0455	0.0644	0.0606	0.0571	0.0774	0.0650
		1	0.0379	0.0265	0.0212	0.0207	0.0853	0.0935	0.0378	0.0592	0.0662	0.0633	0.1007	0.0971
		2	0.0160	0.0197	0.0121	0.0137	0.0550	0.0674	0.0369	0.0543	0.0654	0.0708	0.1209	0.1446

MODEL B

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1579	0.1222	0.1740	0.1103	0.1329	0.0675	0.1046	0.0523	0.0642	0.0649	0.0597	0.0608
		1	0.1284	0.0993	0.0985	0.0728	0.0838	0.0516	0.0852	0.0497	0.0650	0.0658	0.0614	0.0611
		2	0.1004	0.0723	0.0712	0.0567	0.0688	0.0485	0.0713	0.0501	0.0678	0.0670	0.0627	0.0613
0.4	0	0.5	0.2583	0.1789	0.5436	0.2923	0.2328	0.0573	0.2299	0.0709	0.0551	0.0576	0.0287	0.0412
		1	0.2105	0.1456	0.3027	0.1473	0.1825	0.0669	0.1875	0.0809	0.0532	0.0546	0.0216	0.0221
		2	0.1780	0.1294	0.1955	0.0998	0.1619	0.0937	0.1656	0.0995	0.0587	0.0584	0.0144	0.0108
0	0.4	0.5	0.0744	0.0633	0.0400	0.0379	0.2292	0.1917	0.0569	0.0722	0.0686	0.0646	0.0893	0.0725
		1	0.0399	0.0280	0.0233	0.0235	0.1162	0.1122	0.0439	0.0670	0.0764	0.0719	0.1153	0.1117
		2	0.0156	0.0202	0.0128	0.0138	0.0701	0.0781	0.0444	0.0658	0.0857	0.0960	0.1507	0.1875

MODEL C

			CS (2006)						BLS (2001)		WE (2006)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.4780	0.3412	0.3236	0.2456	0.5125	0.6028	0.3839	0.3369	0.0727	0.0740	0.0753	0.0780
		1	0.3126	0.1967	0.1786	0.1263	0.2928	0.3971	0.2429	0.2606	0.0764	0.0748	0.0784	0.0781
		2	0.1976	0.1059	0.1147	0.0799	0.1746	0.2474	0.1685	0.2126	0.0774	0.0791	0.0790	0.0813
0.4	0	0.5	0.8336	0.6562	0.7156	0.5189	0.6755	0.1927	0.7483	0.3289	0.0600	0.0641	0.0297	0.0462
		1	0.7013	0.4459	0.5038	0.2792	0.5410	0.1532	0.6210	0.2549	0.0633	0.0624	0.0196	0.0212
		2	0.5749	0.3231	0.3716	0.1763	0.4491	0.1721	0.5063	0.2330	0.0640	0.0663	0.0118	0.0087
0	0.4	0.5	0.1693	0.1252	0.0770	0.0765	0.9238	0.9923	0.5527	0.8531	0.0759	0.0735	0.1203	0.0967
		1	0.0609	0.0388	0.0315	0.0287	0.8247	0.9738	0.3936	0.8002	0.0845	0.0844	0.1633	0.1577
		2	0.0205	0.0538	0.0137	0.0124	0.6972	0.9304	0.3578	0.7870	0.1002	0.1229	0.2215	0.2896

Notes: See Table 5.

Table 7: Empirical Size-Corrected Power (5% actual size), 1 Break, $\lambda = 50\%$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.3248	0.3816	0.4174	0.5168	0.3981	0.5671	0.4177	0.5738	0	0.5	0.0492	0.0484	0.0648	0.0620
	1	0.3588	0.3845	0.5395	0.5707	0.4517	0.5756	0.4426	0.5624		1	0.0504	0.0560	0.0876	0.0940
	2	0.3857	0.4647	0.5649	0.6073	0.4559	0.5595	0.4419	0.5505		2	0.0604	0.0760	0.1273	0.1893
0.9	0.5	0.2115	0.2022	0.4335	0.5315	0.3056	0.4047	0.3340	0.4389	0.1	0.5	0.0509	0.0494	0.0687	0.0638
	1	0.1885	0.1478	0.5370	0.5322	0.3329	0.3963	0.3317	0.3978		1	0.0538	0.0601	0.0938	0.1038
	2	0.1724	0.1740	0.5333	0.4493	0.3166	0.3661	0.3134	0.3699		2	0.0658	0.0925	0.1398	0.2246
0.1	0.5	0.0507	0.0506	0.0763	0.0574	0.0562	0.0468	0.0570	0.0506	0.9	0.5	0.0628	0.0615	0.0733	0.0703
	1	0.0515	0.0520	0.0621	0.0533	0.0579	0.0496	0.0598	0.0507		1	0.0753	0.0953	0.0979	0.1306
	2	0.0518	0.0549	0.0574	0.0519	0.0586	0.0513	0.0592	0.0524		2	0.0980	0.1802	0.1348	0.2742

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.2460	0.2871	0.1479	0.2369	0.2309	0.3963	0.2564	0.4199	0	0.5	0.0551	0.0486	0.0749	0.0610
	1	0.2822	0.3206	0.2392	0.3051	0.2816	0.4314	0.2830	0.4181		1	0.0579	0.0599	0.0940	0.0993
	2	0.3294	0.4025	0.3043	0.3783	0.3216	0.4424	0.3176	0.4227		2	0.0683	0.0996	0.1459	0.2198
0.9	0.5	0.2407	0.2347	0.1273	0.2197	0.2502	0.3536	0.2794	0.3881	0.1	0.5	0.0568	0.0497	0.0782	0.0634
	1	0.2456	0.2188	0.2046	0.2479	0.2819	0.3561	0.2853	0.3512		1	0.0627	0.0666	0.1013	0.1106
	2	0.2497	0.2483	0.2467	0.2701	0.2941	0.3418	0.2918	0.3273		2	0.0788	0.1268	0.1600	0.2614
0.1	0.5	0.0537	0.0515	0.0658	0.0564	0.0559	0.0471	0.0580	0.0509	0.9	0.5	0.0690	0.0615	0.0735	0.0642
	1	0.0546	0.0526	0.0609	0.0546	0.0567	0.0483	0.0575	0.0497		1	0.0797	0.0985	0.0860	0.1057
	2	0.0567	0.0567	0.0610	0.0528	0.0578	0.0512	0.0594	0.0504		2	0.0997	0.1573	0.1142	0.1783

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.2800	0.3544	0.4063	0.5165	0.2751	0.5364	0.3094	0.5470	0	0.5	0.0528	0.0518	0.0743	0.0667
	1	0.3278	0.3573	0.5287	0.5896	0.4102	0.5925	0.4050	0.5801		1	0.0553	0.0614	0.0959	0.1057
	2	0.3527	0.5130	0.5857	0.6529	0.4911	0.6388	0.4601	0.6097		2	0.0684	0.0962	0.1492	0.2284
0.9	0.5	0.2552	0.2999	0.3660	0.4864	0.2504	0.4830	0.2857	0.4992	0.1	0.5	0.0565	0.0526	0.0796	0.0683
	1	0.2754	0.2602	0.4691	0.5012	0.3562	0.4989	0.3531	0.4904		1	0.0611	0.0684	0.1029	0.1177
	2	0.2853	0.3742	0.4920	0.5091	0.4180	0.5110	0.3878	0.4836		2	0.0818	0.1246	0.1645	0.2691
0.1	0.5	0.0536	0.0504	0.0785	0.0603	0.0554	0.0408	0.0583	0.0489	0.9	0.5	0.0641	0.0601	0.0658	0.0631
	1	0.0536	0.0525	0.0672	0.0550	0.0598	0.0440	0.0625	0.0477		1	0.0709	0.0823	0.0741	0.0903
	2	0.0564	0.0586	0.0627	0.0544	0.0638	0.0475	0.0640	0.0505		2	0.0842	0.1070	0.0921	0.1212

Notes: The DGP is given in equations (12)-(16). x_t is endogenous ($\alpha_1 = 1$), $\alpha_2 = -1$. The LRV is computed as in Kurozumi (2001).

Table 8: Empirical Size-Corrected Power (5% actual size), 1 Break, $\lambda = 50\%$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.5573	0.5761	0.5511	0.5944	0.5872	0.7109	0.5857	0.6980	0	0.5	0.0543	0.0503	0.0735	0.0625
	1	0.5609	0.5767	0.5986	0.6163	0.5899	0.6808	0.5767	0.6741		1	0.0556	0.0599	0.0943	0.1047
	2	0.5921	0.6237	0.6257	0.6382	0.6096	0.6811	0.5894	0.6718		2	0.0669	0.0730	0.1424	0.1830
0.9	0.5	0.3219	0.2296	0.5644	0.6058	0.3877	0.4285	0.3925	0.4427	0.1	0.5	0.0559	0.0511	0.0774	0.0647
	1	0.2394	0.1663	0.6194	0.6191	0.3613	0.4101	0.3589	0.4265		1	0.0592	0.0627	0.1015	0.1154
	2	0.2033	0.1609	0.6372	0.4621	0.3568	0.4146	0.3453	0.4203		2	0.0755	0.0849	0.1555	0.2152
0.1	0.5	0.0517	0.0506	0.0591	0.0530	0.0581	0.0492	0.0579	0.0492	0.9	0.5	0.0778	0.0692	0.0957	0.0832
	1	0.0500	0.0504	0.0541	0.0518	0.0605	0.0508	0.0586	0.0504		1	0.1042	0.1518	0.1409	0.2137
	2	0.0525	0.0547	0.0519	0.0510	0.0615	0.0528	0.0606	0.0525		2	0.1595	0.3940	0.2245	0.5528

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.4002	0.4267	0.2265	0.3260	0.4604	0.6176	0.4614	0.6166	0	0.5	0.0547	0.0541	0.0778	0.0663
	1	0.4217	0.4408	0.3055	0.3472	0.4849	0.6031	0.4679	0.5850		1	0.0608	0.0609	0.1060	0.1105
	2	0.4431	0.4805	0.3759	0.4005	0.4963	0.5968	0.4807	0.5777		2	0.0683	0.0979	0.1507	0.2303
0.9	0.5	0.3224	0.2644	0.2045	0.3057	0.3575	0.4436	0.3719	0.4591	0.1	0.5	0.0569	0.0551	0.0817	0.0692
	1	0.2736	0.2008	0.2656	0.2866	0.3529	0.4181	0.3501	0.4112		1	0.0674	0.0692	0.1157	0.1248
	2	0.2467	0.1883	0.3226	0.2359	0.3347	0.4044	0.3246	0.3921		2	0.0787	0.1192	0.1698	0.2754
0.1	0.5	0.0523	0.0515	0.0597	0.0534	0.0570	0.0474	0.0591	0.0489	0.9	0.5	0.0860	0.0805	0.0938	0.0841
	1	0.0531	0.0512	0.0538	0.0498	0.0596	0.0502	0.0587	0.0508		1	0.1243	0.1788	0.1385	0.2027
	2	0.0535	0.0548	0.0547	0.0517	0.0612	0.0514	0.0595	0.0510		2	0.1763	0.4038	0.2017	0.4642

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.3848	0.4168	0.5666	0.6308	0.6205	0.7803	0.6098	0.7816	0	0.5	0.0507	0.0523	0.0749	0.0633
	1	0.4006	0.4312	0.6370	0.6746	0.6475	0.7780	0.6301	0.7702		1	0.0578	0.0613	0.1078	0.1134
	2	0.4188	0.4728	0.6702	0.6858	0.6563	0.7636	0.6369	0.7503		2	0.0719	0.1013	0.1644	0.2523
0.9	0.5	0.2930	0.2642	0.5143	0.5903	0.4923	0.6734	0.5015	0.6688	0.1	0.5	0.0536	0.0540	0.0806	0.0659
	1	0.2565	0.2080	0.5385	0.5240	0.4809	0.6349	0.4789	0.6190		1	0.0658	0.0697	0.1186	0.1309
	2	0.2361	0.2155	0.5225	0.4137	0.4631	0.5994	0.4501	0.5814		2	0.0840	0.1281	0.1862	0.2980
0.1	0.5	0.0512	0.0509	0.0624	0.0561	0.0608	0.0462	0.0617	0.0479	0.9	0.5	0.0781	0.0733	0.0855	0.0781
	1	0.0538	0.0520	0.0568	0.0528	0.0620	0.0482	0.0624	0.0495		1	0.1108	0.1482	0.1216	0.1675
	2	0.0549	0.0568	0.0529	0.0522	0.0666	0.0515	0.0642	0.0516		2	0.1518	0.2856	0.1739	0.3306

Notes: See Table 7.

Table 9: Empirical Size-Corrected Power (5% actual size), 3 Break, $\lambda = (30\%, 50\%, 70\%)$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.5081	0.5656	0.7796	0.8620	0.4186	0.6028	0.4742	0.6502	0	0.5	0.0498	0.0492	0.0646	0.0588
	1	0.5208	0.5582	0.8800	0.9098	0.5347	0.6520	0.5373	0.6549		1	0.0511	0.0530	0.0869	0.0927
	2	0.5442	0.6449	0.9085	0.9297	0.5966	0.6762	0.5866	0.6698		2	0.0568	0.0708	0.1203	0.1781
0.9	0.5	0.4063	0.4029	0.7739	0.8567	0.3136	0.4277	0.3713	0.4989	0.1	0.5	0.0514	0.0501	0.0674	0.0614
	1	0.3685	0.3210	0.8533	0.8535	0.3857	0.4417	0.3875	0.4614		1	0.0528	0.0571	0.0925	0.1009
	2	0.3401	0.3567	0.8443	0.7932	0.4052	0.4350	0.3991	0.4321		2	0.0617	0.0853	0.1336	0.2084
0.1	0.5	0.0540	0.0516	0.0919	0.0664	0.0533	0.0409	0.0586	0.0461	0.9	0.5	0.0600	0.0595	0.0704	0.0685
	1	0.0541	0.0513	0.0716	0.0573	0.0574	0.0430	0.0597	0.0450		1	0.0749	0.0899	0.0962	0.1258
	2	0.0550	0.0574	0.0628	0.0550	0.0590	0.0464	0.0628	0.0479		2	0.0951	0.1631	0.1301	0.2415

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.4207	0.4928	0.5016	0.6607	0.3323	0.5226	0.3968	0.5717	0	0.5	0.0553	0.0494	0.0729	0.0601
	1	0.4785	0.5273	0.6882	0.7591	0.4791	0.6053	0.4810	0.6097		1	0.0553	0.0631	0.0905	0.1010
	2	0.5133	0.6437	0.7574	0.8166	0.5633	0.6502	0.5431	0.6240		2	0.0649	0.0894	0.1424	0.2115
0.9	0.5	0.3707	0.4050	0.4729	0.6438	0.2790	0.4155	0.3439	0.4832	0.1	0.5	0.0577	0.0503	0.0767	0.0621
	1	0.3868	0.3707	0.6422	0.6840	0.3872	0.4490	0.3901	0.4639		1	0.0613	0.0687	0.0988	0.1107
	2	0.3843	0.4335	0.6726	0.6783	0.4373	0.4614	0.4161	0.4410		2	0.0754	0.1162	0.1583	0.2529
0.1	0.5	0.0556	0.0528	0.0831	0.0647	0.0517	0.0390	0.0589	0.0477	0.9	0.5	0.0703	0.0611	0.0717	0.0640
	1	0.0554	0.0524	0.0717	0.0571	0.0564	0.0417	0.0583	0.0458		1	0.0789	0.0973	0.0853	0.1059
	2	0.0563	0.0599	0.0662	0.0556	0.0609	0.0445	0.0612	0.0463		2	0.0957	0.1477	0.1117	0.1739

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.5866	0.6652	0.4279	0.5682	0.2084	0.2678	0.2722	0.3963	0	0.5	0.0510	0.0520	0.0752	0.0638
	1	0.6676	0.7516	0.6115	0.7228	0.3911	0.3788	0.4185	0.4798		1	0.0545	0.0591	0.0938	0.1003
	2	0.7279	0.8464	0.7100	0.8089	0.5404	0.5013	0.5385	0.5254		2	0.0677	0.0898	0.1446	0.2174
0.9	0.5	0.5697	0.6287	0.4002	0.5467	0.1925	0.2235	0.2498	0.3550	0.1	0.5	0.0528	0.0534	0.0783	0.0658
	1	0.6371	0.6974	0.5697	0.6701	0.3522	0.3041	0.3783	0.4023		1	0.0590	0.0645	0.1012	0.1114
	2	0.6913	0.7955	0.6541	0.7451	0.4849	0.3958	0.4911	0.4263		2	0.0772	0.1155	0.1588	0.2550
0.1	0.5	0.0678	0.0549	0.0782	0.0639	0.0483	0.0239	0.0559	0.0407	0.9	0.5	0.0573	0.0568	0.0619	0.0579
	1	0.0642	0.0566	0.0764	0.0597	0.0502	0.0227	0.0588	0.0348		1	0.0632	0.0658	0.0661	0.0686
	2	0.0661	0.0623	0.0711	0.0586	0.0512	0.0240	0.0610	0.0324		2	0.0684	0.0771	0.0701	0.0839

Notes: See Table 7.

Table 10: Empirical Size-Corrected Power (5% actual size), 3 Break, $\lambda = (30\%, 50\%, 70\%)$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.5553	0.5908	0.9082	0.9328	0.6868	0.8198	0.7048	0.8309	0	0.5	0.0519	0.0502	0.0702	0.0623
	1	0.5612	0.5915	0.9358	0.9487	0.7266	0.8186	0.7224	0.8127		1	0.0542	0.0610	0.0950	0.1037
	2	0.5916	0.6342	0.9517	0.9579	0.7369	0.8148	0.7259	0.8090		2	0.0642	0.0736	0.1386	0.1851
0.9	0.5	0.3637	0.2944	0.9102	0.9322	0.4620	0.5680	0.4906	0.6104	0.1	0.5	0.0540	0.0513	0.0733	0.0651
	1	0.2933	0.2216	0.9268	0.9074	0.4554	0.5377	0.4572	0.5519		1	0.0575	0.0655	0.1012	0.1137
	2	0.2630	0.2178	0.9115	0.7871	0.4337	0.5263	0.4173	0.5209		2	0.0706	0.0866	0.1522	0.2164
0.1	0.5	0.0516	0.0511	0.0664	0.0573	0.0576	0.0438	0.0598	0.0457	0.9	0.5	0.0745	0.0686	0.0897	0.0838
	1	0.0514	0.0509	0.0594	0.0543	0.0616	0.0481	0.0617	0.0488		1	0.1024	0.1469	0.1374	0.2092
	2	0.0536	0.0566	0.0548	0.0542	0.0643	0.0493	0.0619	0.0494		2	0.1528	0.3730	0.2193	0.5269

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.4918	0.5233	0.7195	0.7933	0.6438	0.7894	0.6558	0.7954	0	0.5	0.0558	0.0505	0.0777	0.0628
	1	0.5301	0.5649	0.8075	0.8473	0.6954	0.8004	0.6881	0.7923		1	0.0601	0.0606	0.1032	0.1089
	2	0.5448	0.6101	0.8497	0.8591	0.7153	0.7852	0.6910	0.7810		2	0.0684	0.0929	0.1510	0.2248
0.9	0.5	0.3760	0.3275	0.6980	0.7762	0.4707	0.5989	0.4943	0.6259	0.1	0.5	0.0589	0.0516	0.0817	0.0657
	1	0.3346	0.2739	0.7610	0.7598	0.4740	0.5736	0.4771	0.5658		1	0.0661	0.0695	0.1137	0.1224
	2	0.2965	0.2643	0.7634	0.6349	0.4611	0.5310	0.4364	0.5234		2	0.0790	0.1158	0.1697	0.2702
0.1	0.5	0.0532	0.0508	0.0644	0.0592	0.0582	0.0435	0.0598	0.0462	0.9	0.5	0.0878	0.0770	0.0940	0.0831
	1	0.0541	0.0514	0.0604	0.0541	0.0603	0.0457	0.0614	0.0475		1	0.1230	0.1727	0.1361	0.1966
	2	0.0535	0.0564	0.0552	0.0532	0.0647	0.0472	0.0617	0.0482		2	0.1734	0.3845	0.1994	0.4431

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.7453	0.8021	0.7065	0.7590	0.4935	0.7245	0.5703	0.7710	0	0.5	0.0498	0.0507	0.0794	0.0670
	1	0.8070	0.8619	0.8203	0.8551	0.6914	0.8065	0.6900	0.8053		1	0.0600	0.0640	0.1165	0.1271
	2	0.8598	0.9167	0.8692	0.8893	0.7823	0.8426	0.7474	0.8225		2	0.0713	0.0989	0.1817	0.2742
0.9	0.5	0.6959	0.6984	0.6505	0.7221	0.4314	0.6145	0.5119	0.6842	0.1	0.5	0.0537	0.0529	0.0838	0.0707
	1	0.7276	0.7125	0.7483	0.7607	0.5951	0.6478	0.5913	0.6646		1	0.0676	0.0741	0.1279	0.1461
	2	0.7662	0.7664	0.7777	0.7446	0.6779	0.6737	0.6367	0.6570		2	0.0874	0.1326	0.2047	0.3288
0.1	0.5	0.0564	0.0540	0.0693	0.0631	0.0577	0.0338	0.0627	0.0428	0.9	0.5	0.0699	0.0665	0.0725	0.0701
	1	0.0564	0.0536	0.0624	0.0557	0.0606	0.0354	0.0635	0.0402		1	0.0888	0.1143	0.0985	0.1286
	2	0.0566	0.0595	0.0594	0.0551	0.0656	0.0403	0.0649	0.0416		2	0.1064	0.1663	0.1232	0.1925

Notes: See Table 7.

Table 11: Empirical Size-Corrected Power (5% actual size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.6812	0.7526	0.8817	0.9397	0.4566	0.5728	0.5565	0.6910	0	0.5	0.0500	0.0482	0.0686	0.0606
	1	0.7379	0.7733	0.9528	0.9687	0.6296	0.6669	0.6642	0.7316		1	0.0516	0.0546	0.0882	0.0894
	2	0.7473	0.8338	0.9658	0.9777	0.7190	0.7351	0.7190	0.7582		2	0.0553	0.0676	0.1199	0.1683
0.9	0.5	0.5888	0.6230	0.8593	0.9238	0.3315	0.3960	0.4386	0.5406	0.1	0.5	0.0516	0.0495	0.0708	0.0627
	1	0.6033	0.5713	0.9189	0.9235	0.4637	0.4285	0.5041	0.5226		1	0.0547	0.0576	0.0955	0.0982
	2	0.5682	0.6209	0.9143	0.8938	0.5265	0.4767	0.5292	0.5146		2	0.0613	0.0837	0.1306	0.1954
0.1	0.5	0.0567	0.0524	0.0943	0.0708	0.0509	0.0316	0.0581	0.0411	0.9	0.5	0.0627	0.0579	0.0742	0.0673
	1	0.0575	0.0545	0.0774	0.0616	0.0528	0.0333	0.0622	0.0406		1	0.0762	0.0895	0.0971	0.1246
	2	0.0569	0.0619	0.0721	0.0587	0.0577	0.0367	0.0616	0.0427		2	0.0924	0.1538	0.1273	0.2325

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.6113	0.6897	0.7076	0.8329	0.3710	0.5389	0.4755	0.6338	0	0.5	0.0524	0.0500	0.0753	0.0596
	1	0.6689	0.7255	0.8505	0.8992	0.5635	0.6284	0.5828	0.6700		1	0.0556	0.0620	0.0869	0.1008
	2	0.7097	0.8127	0.8953	0.9319	0.6715	0.7019	0.6725	0.7095		2	0.0676	0.0888	0.1374	0.2024
0.9	0.5	0.5498	0.6017	0.6700	0.8160	0.2974	0.4235	0.4051	0.5395	0.1	0.5	0.0551	0.0511	0.0782	0.0623
	1	0.5853	0.5849	0.8068	0.8413	0.4518	0.4543	0.4802	0.5155		1	0.0600	0.0683	0.0936	0.1103
	2	0.5892	0.6596	0.8256	0.8366	0.5370	0.5075	0.5435	0.5238		2	0.0766	0.1128	0.1528	0.2381
0.1	0.5	0.0607	0.0546	0.0891	0.0689	0.0505	0.0322	0.0588	0.0431	0.9	0.5	0.0680	0.0626	0.0732	0.0642
	1	0.0592	0.0548	0.0751	0.0597	0.0529	0.0341	0.0620	0.0401		1	0.0780	0.0953	0.0825	0.1024
	2	0.0616	0.0609	0.0700	0.0575	0.0559	0.0375	0.0612	0.0410		2	0.0962	0.1411	0.1084	0.1642

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.6099	0.7339	0.4023	0.5673	0.2582	0.0559	0.3377	0.2754	0	0.5	0.0513	0.0507	0.0724	0.0594
	1	0.7414	0.8434	0.6174	0.7518	0.3687	0.0777	0.4937	0.3035		1	0.0515	0.0520	0.0929	0.1001
	2	0.8146	0.8943	0.7327	0.8498	0.4917	0.1454	0.6208	0.3031		2	0.0664	0.0941	0.1467	0.2166
0.9	0.5	0.5960	0.7107	0.3766	0.5505	0.2385	0.0390	0.3178	0.2339	0.1	0.5	0.0545	0.0519	0.0739	0.0611
	1	0.7236	0.8116	0.5833	0.7155	0.3341	0.0546	0.4591	0.2509		1	0.0564	0.0584	0.0980	0.1080
	2	0.7987	0.8685	0.6936	0.8073	0.4492	0.0991	0.5862	0.2390		2	0.0765	0.1193	0.1568	0.2454
0.1	0.5	0.0788	0.0671	0.0718	0.0673	0.0370	0.0154	0.0538	0.0279	0.9	0.5	0.0534	0.0535	0.0545	0.0531
	1	0.0782	0.0648	0.0777	0.0641	0.0324	0.0129	0.0563	0.0235		1	0.0574	0.0599	0.0602	0.0654
	2	0.0760	0.0522	0.0778	0.0651	0.0280	0.0126	0.0514	0.0194		2	0.0593	0.0686	0.0627	0.0714

Notes: See Table 7.

Table 12: Empirical Size-Corrected Power (5% actual size), 5 Break, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.7141	0.7674	0.9772	0.9870	0.7543	0.8606	0.7942	0.8812	0	0.5	0.0542	0.0513	0.0703	0.0609
	1	0.7374	0.7792	0.9865	0.9907	0.8203	0.8821	0.8146	0.8858		1	0.0531	0.0579	0.0921	0.1026
	2	0.7612	0.8166	0.9922	0.9934	0.8409	0.8833	0.8269	0.8841		2	0.0621	0.0700	0.1349	0.1790
0.9	0.5	0.5344	0.4945	0.9716	0.9846	0.5427	0.6331	0.6080	0.6906	0.1	0.5	0.0558	0.0520	0.0737	0.0633
	1	0.4820	0.4080	0.9779	0.9677	0.5779	0.6130	0.5761	0.6440		1	0.0570	0.0626	0.0978	0.1140
	2	0.4425	0.4022	0.9679	0.9123	0.5635	0.6015	0.5373	0.6074		2	0.0695	0.0808	0.1497	0.2082
0.1	0.5	0.0529	0.0518	0.0727	0.0637	0.0593	0.0377	0.0625	0.0423	0.9	0.5	0.0741	0.0682	0.0899	0.0809
	1	0.0540	0.0509	0.0641	0.0564	0.0636	0.0422	0.0643	0.0444		1	0.0993	0.1407	0.1332	0.2020
	2	0.0543	0.0576	0.0578	0.0538	0.0656	0.0450	0.0636	0.0474		2	0.1461	0.3509	0.2119	0.4993

MODEL B

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.6784	0.7398	0.9053	0.9421	0.6856	0.8187	0.7341	0.8426	0	0.5	0.0534	0.0505	0.0757	0.0636
	1	0.7195	0.7715	0.9436	0.9589	0.7779	0.8475	0.7757	0.8478		1	0.0573	0.0596	0.1005	0.1071
	2	0.7669	0.8228	0.9617	0.9710	0.8042	0.8509	0.7981	0.8439		2	0.0713	0.0916	0.1500	0.2210
0.9	0.5	0.5569	0.5476	0.8861	0.9310	0.5271	0.6328	0.5935	0.6866	0.1	0.5	0.0571	0.0514	0.0796	0.0656
	1	0.5342	0.4862	0.9100	0.9056	0.5818	0.6146	0.5905	0.6322		1	0.0639	0.0668	0.1094	0.1198
	2	0.5456	0.5113	0.9089	0.8388	0.5818	0.5962	0.5694	0.5889		2	0.0819	0.1134	0.1672	0.2630
0.1	0.5	0.0529	0.0534	0.0726	0.0618	0.0568	0.0392	0.0640	0.0449	0.9	0.5	0.0865	0.0748	0.0916	0.0807
	1	0.0551	0.0527	0.0614	0.0545	0.0623	0.0404	0.0659	0.0443		1	0.1181	0.1695	0.1356	0.1933
	2	0.0566	0.0582	0.0586	0.0549	0.0635	0.0442	0.0669	0.0447		2	0.1768	0.3674	0.2004	0.4274

MODEL C

		CS (2006)						BLS (2001)				WE (2006)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.9125	0.9474	0.7671	0.8159	0.5379	0.6822	0.6492	0.8018	0	0.5	0.0555	0.0491	0.0794	0.0652
	1	0.9523	0.9738	0.8735	0.9164	0.7874	0.8027	0.8128	0.8550		1	0.0551	0.0610	0.1131	0.1215
	2	0.9738	0.9912	0.9182	0.9432	0.8901	0.8807	0.8848	0.8869		2	0.0729	0.1068	0.1870	0.2925
0.9	0.5	0.8892	0.9126	0.7161	0.7851	0.4794	0.5344	0.5960	0.7008	0.1	0.5	0.0593	0.0508	0.0847	0.0680
	1	0.9324	0.9426	0.8255	0.8596	0.7348	0.6436	0.7653	0.7373		1	0.0634	0.0708	0.1240	0.1388
	2	0.9539	0.9719	0.8631	0.8749	0.8336	0.7374	0.8329	0.7593		2	0.0915	0.1418	0.2097	0.3525
0.1	0.5	0.0651	0.0618	0.0767	0.0652	0.0556	0.0240	0.0618	0.0346	0.9	0.5	0.0669	0.0624	0.0670	0.0647
	1	0.0647	0.0595	0.0671	0.0600	0.0559	0.0220	0.0697	0.0324		1	0.0735	0.0928	0.0786	0.0975
	2	0.0640	0.0680	0.0637	0.0604	0.0577	0.0268	0.0709	0.0321		2	0.0931	0.1297	0.1003	0.1452

Notes: See Table 7.