# Uncertain delivery and the market for lemons*

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**Abstract.** The notion of uncertain delivery is extended to study exchange economies in which agents have different abilities to distinguish between goods (for example a car in good condition versus a car in bad condition). Existence of equilibrium is established. Several examples are presented as an illustration.

**Keywords:** Uncertain delivery, General equilibrium, Asymmetric information.

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# 1 Introduction

Economic agents usually trade goods without having perfect knowledge of their characteristics. This applies to firms hiring workers with unknown productivity as well as to consumers buying used cars with unknown quality. Each trader enters the market with specific prior knowledge and observation abilities concerning the characteristics of the goods being traded.

For example, an agent may not be able to distinguish the quality of the melons that are being offered in a market, which may be good or bad. Suppose that, nevertheless, the agent decides to buy 10 good melons. The agent may get 10 good melons, or maybe 2 good and 8 bad, or, more likely, 10 bad melons.

Since some agents are able to distinguish the good from the bad melons, we assume that good melons and bad melons can be traded at different prices. But an agent that cannot observe the quality of the melons should be better off buying bad melons, because to buy good melons the agent has to pay a higher price and will end up receiving bad melons as well.

This is a particular kind of asymmetric information, famous since the seminal contribution of Akerlof (1970), which is designated as adverse selection. But observe that this information asymmetry differs from that considered in the models of trade with adverse selection of Prescott and Townsend (1984a, 1984b), Gale (1992, 1996), Bisin and Gottardi (1999, 2006) and Rustichini and Siconolfi (2008). In these models, agents enter the market having private information about their type (endowments and preferences in each of the possible states of nature).

In this paper, each agent's private information is described by a partition of the set of commodities, such that the agent can distinguish goods that belong to different sets of the partition. This formalization can also be found in Minelli and Polemarchakis (2001) and in the recent paper of Meier, Minelli and Polemarchakis (2006), which was the main motivation for this work.

In the model of Meier, Minelli and Polemarchakis (2006), there are only markets for classes of goods that everyone can distinguish. If there is an agent in the economy that does not distinguishes apples from oranges, then the other agents cannot trade apples (or oranges) among themselves. This seems very restrictive, and here an alternative framework is considered. An agent that does not distinguish good melons from bad melons may buy melons that are labeled as good. However, the agent should expect to receive bad melons instead.

Instrumental to the treatment of trade with adverse selection are the concepts of "pool" and "delivery rate", introduced by Dubey, Geanakoplos and Shubik (2005). We can think of a set of commodities, like melons (good and bad), as a pool. An agent that buys 10 units of good melons but is unable to distinguish the good from the bad melons, may receive 2 units of good melons and 8 units of bad melons (the delivery rates are 0,2 and 0,8). An agent that can distinguish the good melons would surely receive 10 units of good melons.

An agent that buys a good may be delivered one of a set of possibilities, and takes as given the probabilities of receiving each of the possible deliveries. This is closely related to what was termed as "uncertain delivery" by Correia-da-Silva and Hervés-Beloso (2007, 2008a and 2008b), in a series of papers that study ex-ante trade of contingent goods, with agents having different abilities to verify the occurrence of the exogenous states of nature.

The main result of this paper is the existence of equilibrium (Section 2).

To illustrate the main intuitions offered by the model, the examples presented by Meier, Minelli and Polemarchakis (2006) are explained and solved (Section 3).

## 2 The model

We consider an economy in which a finite number of agents, $i \in \{1, ..., n\}$, trade a finite number of goods, $l \in \{1, ..., L\}$.

Each agent has specific abilities to distinguish the different goods that are traded in the market. These observation abilities are described by a partition of the set of goods, $P_i$, such that $m \in P_i(l)$ if and only if agent $i$ cannot distinguish good $l$ from good $m$.

The inability to distinguish between two goods, $l$ and $m$, implies that an agent that buys certain quantities of $l$ and $m$, say $y_i^l$ and $y_i^m$ may receive a different bundle, with $x_i^l$ and $x_i^m$ such that:

$$x_i^l + x_i^m = y_i^l + y_i^m.$$

More generally, when buying $y_i = (y_i^1, ..., y_i^L)$, agent $i$ will receive $x_i = (x_i^1, ..., x_i^L)$ such that:

$$\begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^L \end{bmatrix} = \begin{bmatrix} k_i^{11} & k_i^{12} & \cdots & k_i^{1L} \\ k_i^{21} & k_i^{22} & \cdots & k_i^{2L} \\ \vdots & \vdots & \ddots & \vdots \\ k_i^{L1} & k_i^{L2} & \cdots & k_i^{LL} \end{bmatrix} \cdot \begin{bmatrix} y_i^1 \\ y_i^2 \\ \vdots \\ y_i^L \end{bmatrix},$$

where $k_i^{lm}$ denotes the number of units of good $l$ that the agent $i$ receives for each unit of good $m$ that he buys.

The delivery matrix, $k_i$, is endogenous (equilibrating variable). The delivery matrices that are compatible with an agent's abilities to distinguish commodities are such that, for each $m$:

$$\begin{cases} \sum_{l \in P_i(m)} k_i^{lm} = 1 \\ k_i^{lm} = 0, \ \forall l \notin P_i(m) \end{cases}$$

The set of matrices that satisfy these conditions is denoted $K_i$, and $K = \prod_{i=1}^{n} K_i$.

It should be clear that if agent $i$ can distinguish all commodities ($P_i(l) = \{l\}, \ \forall l$), then $K_i$ has a single element, which is the identity matrix (agent $i$ receives exactly the bundle that he buys).

Agents wish to maximize their utility function, $U_i(x_i)$, which is continuous, concave and strictly increasing ($x_i \gg x_i' \Rightarrow U_i(x_i) > U_i(x_i')$).

Taking prices, $p$, and delivery rates, $k_i$, as given, agent $i$ buys a bundle, $y_i$, that maximizes utility, $U_i(k_i y_i)$, among those that satisfy its budget restriction, $y_i \in B_i(p)$.[1]

$$B_i(p) = \left\{ y_i \in \mathbb{R}^L_+ : \; p \cdot y_i \leq p \cdot e_i \right\}.$$

**Definition** 1

*A Nash-Walras equilibrium of the economy* $\mathcal{E} \equiv \{e_i, U_i, P_i\}_{i=1}^n$ *is composed by a price system,* $p^* \in \Delta^L_+$, *individual choices,* $y^* = (y_1^*, ..., y_n^*) \in \mathbb{R}^{nL}_+$, *delivery rates,* $k^* = (k_1^*, ..., k_n^*) \in K$, *and the resulting allocation,* $x^* = (x_1^*, ..., x_n^*) \in \mathbb{R}^{nL}_+$, *which are such that:*

$$y_i^* \in arg \max_{y_i \in B_i(p^*)} U_i(k_i^* y_i), \; \forall i;$$

$$x_i^* = k_i^* y_i^*, \; \forall i;$$

$$\sum_i x_i^* \leq \sum_i e_i.$$

**Assumption** 1

$$e_i \gg 0, \; \forall i.$$

**Theorem** 1

*Under Assumption 1, there exists a Nash-Walras equilibrium of the economy.*

**Proof:**

First, we consider bounded choice sets. Let $E = \sum_{i=1}^n \sum_{l=1}^L e_i^l + 1$ and define the following convex and bounded consumption sets:

$$\overline{Y}_i = \{y_i \in \mathbb{R}^L_+ : \; y_i^l \leq E, \forall l\}.$$

The budget set in this bounded economy are:

[1]All suppliers of a good $l$ receive the same price $p^l$ for a unit of this good, independently of their information.

$\overline{B}_i(p) = \Big\{ y_i \in \overline{Y}_i : \; p \cdot y_i \leq p \cdot e_i \Big\}.$

Let $\psi_i(y,p,k) = arg \max_{z_i \in \overline{B}_i(p)} \{U_i(k_i z_i)\}$.

Given Assumption 1, the correspondence $\overline{B}_i(p) = \{y_i \in \overline{Y}_i : \; p \cdot y_i \leq p \cdot e_i\}$ is continuous with non-empty compact values.

The utility function, $U_i(k_i y_i)$, is continuous with respect to both $k_i$ and $y_i$.

Then, from Berge's Maximum Theorem,[2] we know that $\psi_i(y,k,p)$ is upper hemicontinuous with nonempty compact values. It is also convex-valued, because $U_i$ is concave.

Let $\psi_p(y,p,k) = arg \max_{q \in \Delta_+^L} \{q \cdot (k_i y_i - e_i)\}$.

And let $\psi_{k_i}(y,p,k) = arg \min_{z_i \in K_i} \{p \cdot z_i y_i\}$.

All the correspondences ($\psi_i$, $\psi_p$ and $\psi_{k_i}$) are upper hemicontinuous with nonempty compact and convex values. Therefore, the product correspondence, $\psi = \prod_{i=1}^n \psi_i \times \psi_p \times \prod_{i=1}^n \psi_{k_i}$, also is.

Applying the Theorem of Kakutani, we find that there exists a fixed point of $\psi$, that we denote by $(y^*, p^*, k^*)$.

To prove that it is an equilibrium, all we need is to show that the resulting allocation is feasible:

$\sum_i k_i^* y_i^* \leq \sum_i e_i.$

The individual budget restrictions imply that: $p^* \cdot \sum_i y_i^* \leq p^* \cdot \sum_i e_i$.

Since $k_i^* \in \psi_{k_i}(y^*, p^*, k^*)$, we are sure that $p^* \cdot k_i^* y_i^* \leq p^* \cdot y_i^*$ (because choosing $k_i$ to be the identity matrix implies equality).

Therefore: $p^* \cdot \sum_i k_i^* y_i^* \leq p^* \cdot \sum_i e_i$.

Then, if there were some good $m$ for which $k_i^{m*} y_i^{m*} > e_i^m$, there would be another, $j$, for which $k_i^{j*} y_i^{j*} < e_i^j$. But the goods in excess supply, as $j$, would have null prices, $p^{j*} = 0$, because $p^* \psi_p(y^*, p^*, k^*)$. In any case, we would have $p^* \cdot \sum_i k_i^* y_i^* >$

[2] See, for example, Aliprantis and Border (2006).

$p^* \cdot \sum_i e_i$, a contradiction. Existence of equilibrium in the bounded economy is established.

Observe that the bound on the choice sets is large enough for the individual choices to be interior. Since preferences are convex, we are sure that the bounds are not binding. The same $(p^*, y^*, k^*, x^*)$ are an equilibrium of the economy $\mathcal{E} \equiv \{e_i, U_i, P_i\}_{i=1}^n$.

**QED**

# 3 Examples

## 3.1 Cherry picking

Three individuals, $I = \{1, 2, 3\}$, trade three commodities, $L = \{0, r, g\}$, that we can think of as 'money', 'red cherries' and 'green cherries'.

Agent 1 is endowed with 'money', agent 2 with 'red cherries' and agent 3 with 'green cherries'.

$e_1 = \{12, 0, 0\}$;

$e_2 = \{0, 12, 0\}$;

$e_3 = \{0, 0, 12\}$.

All agents prefer 'green cherries' to 'red cherries'. Agent 2 does not like 'red cherries' at all. Preferences are described by the following utility functions:

$u_1(x_1) = ln(x_1^0) + ln(x_1^r) + 2ln(x_1^g)$;

$u_2(x_2) = ln(x_2^0) + 2ln(x_2^g)$;

$u_3(x_3) = ln(x_3^0) + ln(x_3^r) + 2ln(x_3^g)$.

There is asymmetric information because agent 1 cannot distinguish 'red cherries' from 'green cherries' while agents 2 and 3 are able to distinguish the three commodities.

$P_1 = \{\{0\}, \{r, g\}\}$;

$P_2 = \{\{0\}, \{r\}, \{g\}\}$;

$P_3 = \{\{0\}, \{r\}, \{g\}\}$.

In the lines of the model of an economy with uncertain delivery, agents 2 and 3 can trade all goods, while agent 1 can only trade the two groups of commodities that he can distinguish: 'money' and 'cherries'. This restriction reflects the difficulty of enforcing trade contracts under asymmetric information. Formally, agent 1 can actually buy 'green cherries', but the seller can deliver 'red cherries' anyway (agent 1 would not notice it).

The group of commodities designated as 'cherries' can be seen as a list containing two alternatives: 'red cherries' and 'green cherries'. In the original context of an economy with uncertain delivery, which was of ex-ante contracting for future contingent delivery (agents could not distinguish states of nature, but could distinguish commodities), delivery had to be either of 'red cherries' or 'green cherries'.[3] Here, additional complexity arises from the fact that there is an infinite number of possibilities for the delivery of 10 'cherries' (10 'red' and 0 'green', 5 'red' and 5 'green', 7 'red' and 3 'green', etc.).

Since agent 2 will not buy 'red cherries', his budget restriction is (notice that the price of 'money' is normalized to $p^0 = 1$):

$$p^0 x_2^0 + p^r x_2^r + p^g x_2^g = p^r e_2^r \Rightarrow x_2^0 + p^g x_2^g = 12p^r.$$

The optimality condition implies the equality between the ratios of the marginal utility over price, for each good demanded:

$$x_2^0 = 0.5 p^g x_2^g.$$

[3]See Correia-da-Silva and Hervés-Beloso (2007, 2008a and 2008b) for a detailed description.

From the budget restriction and the optimality condition, we find the demand of agent 2:

$$x_2^0 + p^g x_2^g = 12p^r \Rightarrow x_2^0 + 2x_2^0 = 12p^r \Rightarrow x_2^0 = 4p^r;$$

$$x_2^r = 0;$$

$$x_2^0 + p^g x_2^g = 12p^r \Rightarrow 0.5p^g x_2^g + p^g x_2^g = 12p^r \Rightarrow x_2^g = 8\frac{p^r}{p^g}.$$

Similarly, we can obtain the demand function of agent 3:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 12p^g \\ x_3^0 = p^r x_3^r = 0.5p^g x_3^g \end{cases} \Rightarrow$$

$$\Rightarrow x_3^0 + x_3^0 + 2x_3^0 = 12p^g \Rightarrow x_3^0 = 3p^g;$$

$$\Rightarrow p^r x_3^r + p^r x_3^r + 2p^r x_3^r = 12p^g \Rightarrow x_3^r = 3\frac{p^g}{p^r};$$

$$\Rightarrow 0.5p^g x_3^g + 0.5p^g x_3^g + p^g x_3^g = 12p^g \Rightarrow x_3^g = 6.$$

Looking at the demand of agents 2 and 3 for 'green cherries', we find that $p^r < p^g$, otherwise there would be an excess demand. More precisely:

$$8\frac{p^r}{p^g} + 6 \leq 12 \Rightarrow p^r \leq 0.75p^g.$$

Suppose that agent 1 buys a quantity $x_1^{rg}$ of 'cherries' (a list that guarantees delivery of 'red cherries', $x_1^r$, and 'green cherries', $x_1^g$, such that $x_1^r + x_1^g = x_1^{rg}$). According to the model of an economy with uncertain delivery, if 'red cherries' are cheaper than 'green cherries', then the agent should receive only 'red cherries', and no 'green cherries'.

If the agent received some 'green cherries', then one could wonder why someone is delivering these 'green cherries', instead of trading them in the market for 'red cherries' plus 'money' and delivering the 'red cherries' while keeping the 'money' (there would be an arbitrage opportunity).

Following these lines, since $p^r < p^g$, then $x_1^g = 0$. Assuming that agent 1 is aware of this (delivery rates are anticipated and taken as given), we can find his demand function. The quantity $x_1^r$ and the price $p^r$ can also be interpreted as the quantity and price of (undistinguished, or pooled) 'cherries', $x_1^{rg}$ and $p^{rg}$.[4]

$$\begin{cases} x_1^0 + p^r x_1^r = 12 \\ x_1^0 = p^r x_1^r \end{cases} \Rightarrow \begin{cases} x_1^0 + x_1^0 = 12 \Rightarrow x_1^0 = 6 \\ p^r x_1^r + p^r x_1^r = 12 \Rightarrow x_1^r = \frac{6}{p^r} \end{cases}$$

For demand to equal supply:

$$\begin{cases} x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 4p^r + 3p^g = 6 \\ x_1^r + x_2^r + x_3^r = 12 \Rightarrow \frac{6}{p^r} + 3\frac{p^g}{p^r} = 12 \\ x_1^g + x_2^g + x_3^g = 12 \Rightarrow 8\frac{p^r}{p^g} = 6 \Rightarrow p^r = 0.75p^g \end{cases}$$

These equations allow the determination of equilibrium prices, $p^* = (1; 0.75; 1)$:

$$4p^r + 3p^g = 6 \Rightarrow 4 \cdot 0.75p^g + 3p^g = 6 \Rightarrow p^g = 1 \Rightarrow p^r = 0.75;$$

$$x_1^r + x_2^r + x_3^r = 12 \Rightarrow \frac{6}{p^r} + 3\frac{p^g}{p^r} = 12;$$

$$x_1^g + x_2^g + x_3^g = 12 \Rightarrow 8\frac{p^r}{p^g} = 6 \Rightarrow p^r = 0.75p^g.$$

The allocation is, therefore (deliveries are truthful):

$$x_1^* = (6, \frac{6}{p^r}, 0) = (6, 8, 0);$$

$$x_2^* = (4p^r, 0, 8\frac{p^r}{p^g}) = (3, 0, 6);$$

$$x_3^* = (3p^g, 3\frac{p^g}{p^r}, 6) = (3, 4, 6).$$

## 3.2 Modified cherry picking

Is it always the case that agent 1 does not consume 'green cherries'? Agent 1 could consume 'green cherries' if the incentives for agents 2 and 3 to give him only 'red cherries' disappear.

[4]Agent 1 prefers any interior bundle ($x_1 \gg 0$) to a bundle that is in the frontier of the consumption set. But he cannot get any 'green cherries' and, therefore, his utility is infinitely negative. Here we are assuming that, among bundles (in the frontier of the consumption set) with $x_1^g = 0$, the preferences of agent 1 are described by $u_1(x_1) = ln(x_1^0) + ln(x_1^r)$.

This may be the case if 'green cherries' become much more abundant than 'red cherries'. Let's double the endowments of agent 3 (green cherries):

$$\begin{cases} e_1 = \{12, 0, 0\} \\ e_2 = \{0, 12, 0\} \\ e_3 = \{0, 0, 24\}. \end{cases}$$

Demand of agent 2 remains unaltered:

$$\begin{cases} x_2^0 + p^g x_2^g = 12p^r \\ x_2^0 = 0.5p^g x_2^g \end{cases} \Rightarrow$$

$$\Rightarrow x_2^0 + 2x_2^0 = 12p^r \Rightarrow x_2^0 = 4p^r;$$

$$\Rightarrow x_2^r = 0;$$

$$\Rightarrow 0.5p^g x_2^g + p^g x_2^g = 12p^r \Rightarrow x_2^g = 8\frac{p^r}{p^g}.$$

While demand of agent 3 doubles:

$$\begin{cases} x_3^0 + p^r x_3^r + p^g x_3^g = 24p^g \\ x_3^0 = x_3^r p^r = 0.5x_3^g p^g \end{cases} \Rightarrow$$

$$\Rightarrow x_3^0 + x_3^0 + 2x_3^0 = 24p^g \Rightarrow x_3^0 = 6p^g;$$

$$\Rightarrow p^r x_3^r + p^r x_3^r + 2p^r x_3^r = 24p^g \Rightarrow x_3^r = 6\frac{p^g}{p^r};$$

$$\Rightarrow 0.5p^g x_3^g + 0.5p^g x_3^g + p^g x_3^g = 24p^g \Rightarrow x_3^g = 12.$$

In this case, demand of agents 2 and 3 for 'green cherries' does not exceed supply as long as $p^r \leq 1.5p^g$:

$$x_2^g + x_3^g = 8\frac{p^r}{p^g} + 12 \leq 24 \Rightarrow p^r \leq 1.5p^g.$$

There are three possibilities: (a) the price of 'green cherries' is higher than the price of 'red cherries' and thus agent 1 only consumes 'red cherries' (as in the previous example); (b) the price of 'red cherries' is higher than the price of 'green cherries' and thus agent 1 only consumes 'green cherries'; (c) the prices of 'green cherries' and 'red cherries' coincide.

In case (a), there would be excess supply of 'green cherries', as aggregate consumption is lower than 20.

$$x_1^g + x_2^g + x_3^g = 0 + 8\frac{p^r}{p^g} + 12 < 20.$$

In case (b), there would be excess supply of ‘red cherries’, as aggregate consumption is lower than 6.

$$x_1^r + x_2^r + x_3^r = 0 + 0 + 6\frac{p^g}{p^r} < 6.$$

Thus, in equilibrium we must have case (c): $p^r = p^g = p^{rg}$.

$$\Rightarrow \begin{cases} x_2 = (4p^{rg}, 0, 8) \\ x_3 = (6p^{rg}, 6, 12) \end{cases} \Rightarrow x_2 + x_3 = (10p^{rg}, 6, 20).$$

The only candidate equilibrium allocation gives agent 1 the following consumption bundle: $x_1 = (12 - 10p^{rg}, 6, 4)$.

To check whether this is an equilibrium, we need to find the demand of agent 1. A problem that we face is that agent 1, through his demand, may influence the quality of the cherries (the proportion between red and green cherries).

It is assumed that he takes the proportions of delivered red and green cherries as given. In equilibrium, this proportion must be fulfilled (otherwise it would not be an equilibrium). Since we already have a single candidate for an equilibrium ($x_1 = (12 - 10p^{rg}, 6, 4)$), we must assume that he expects to receive 60% red cherries and 40% green cherries.

The utility and the maximization condition of agent 1 are (with $x_1^{rg} = x_1^r + x_1^g$):

$$u_1(x_1) = lnx_1^0 + ln(0.6x_1^{rg}) + 2ln(0.4x_1^{rg}) \Rightarrow x_1^0 = \frac{1}{3}p^{rg}x_1^{rg}.$$

Putting this together with the budget restriction, demand is obtained:

$$x_1^0 + p^{rg}x_1^{rg} = 12 \Rightarrow 4x_1^0 = 12 \Rightarrow x_1^0 = 3;$$

$$x_1^0 + p^{rg}x_1^{rg} = 12 \Rightarrow x_1^{rg} = \frac{9}{p^{rg}}.$$

For demand of ‘money’ to equal supply:

$$x_1^0 + x_2^0 + x_3^0 = 12 \Rightarrow 3 + 4p^{rg} + 6p^{rg} = 12 \Rightarrow p^{rg} = 0.9.$$

$$\Rightarrow x_1^{rg} = \frac{9}{p^{rg}} = 10.$$

Equilibrium prices are, therefore, $p^* = (1; 0.9; 0.9)$, and the allocation is:

$x_1^* = (3; 6; 4)$;

$x_2^* = (3.6; 0; 8)$;

$x_3^* = (5.4; 6; 12)$.

In this case, agent 1 consumes both 'red cherries' and 'green cherries', which are traded at the same price. Agents 2 and 3 optimize by delivering to 3.6 units of 'red cherries' and 5.4 units of 'green cherries' for agent 1 to consume.

The quantities of red and green cherries that agent 1 buys ($y_1^r$ and $y_1^g$) are irrelevant, given that they add to 10. In any case, the delivery rates adjust to be such that delivery is surely as calculated above: $x_1^r = 6$ and $x_1^g = 4$.

## 3.3 A job market

Consider an economy with two firms, $A$ and $B$, that have an initial endowment of 'money' and hire labor. There are two types of labor, 1 and 2 (type 2 is more productive than type 1). Money is designated as good 0. Both firms have the same preferences and endowments (8 units of 'money'):

$e_A = \{8, 0, 0\}$;

$e_B = \{8, 0, 0\}$;

$u_A(x_A) = x_{A0} + x_{A1} + 4x_{A2}$;

$u_B(x_B) = x_{B0} + x_{B1} + 4x_{B2}$.

Price of money is normalized to 1, and the wages of each type of labor are denoted $w_1$ and $w_2$.

Two workers supply labor, at the expense of their time of leisure. Their endowments are 4 units of time.

$e_1 = \{0, 4, 0\}$;

$$e_2 = \{0, 0, 4\};$$

$$u_1(x_1) = x_{10} - \frac{1}{x_{11}};$$

$$u_2(x_2) = x_{20} - \frac{1}{x_{22}}.$$

Asymmetric information is present because firm $A$ can distinguish between the two types of labor, but firm $B$ cannot.

$$P_A = \{\{0\}, \{1\}, \{2\}\};$$

$$P_B = \{\{0\}, \{1, 2\}\}.$$

For demand of labor by firm $A$ to be finite, it is necessary that $w_1 \geq 1$ and $w_2 \geq 4$. In fact, it is clear that in equilibrium we will have $w_2 = 4$, otherwise there would be no demand for labor of type 2. Notice that, at this wage, firm $A$ is willing to hire any quantity of labor of type 2.

Optimization by the workers yields demand for leisure. Agent 2 dedicates half of the time to work and the other half to leisure.

$$w_2 x_{22}^2 = 1 \Rightarrow x_{22} = w_2^{-1/2} = 2.$$

If $w_1 = 4$, then the worker of type 1 would also wish to sell 2 units of labor. Only firm $B$ could use this labor (firm $A$ would not find it profitable to hire labor of type 1 at $w_1 = 4$). But, clearly, firm $B$ would not be willing to pay $w_1 = 4$, knowing that the labor would not be 100% of type 2. Thus: $w_1 < 4$ and all the labor hired by firm $B$ is of type 1 (workers of type 2 are not be willing to work for a wage lower than 4, which is what they receive from firm $A$).

Firm $B$ is aware of this reasoning, and knows, therefore, that the labor hired is 100% of type 1. For firm $B$ to demand a finite amount of labor, it is necessary that $w_1 = 1$. In sum, these considerations imply that equilibrium prices are $w_1^* = 1$ and $w_2^* = 4$.

Again, optimization by the worker yields agent 1's demand for leisure.

$$w_1 x_{11}^2 = 1 \Rightarrow x_{11} = w_1^{-1/2} = 1.$$

From the budget restrictions, we obtain the money income of each agent:

$x_{10} + w_1 x_{11} = w_1 e_{11} \Rightarrow x_{10} = w_1(4 - x_{11}) \Rightarrow x_{10} = 3;$

$x_{20} + w_2 x_{22} = w_2 e_{22} \Rightarrow x_{20} = w_2(4 - 2) \Rightarrow x_{20} = 8.$

The corresponding money income and leisure time of each agent are:

$x_1^* = (1, 3, 0);$

$x_2^* = (8, 0, 2);$

Ignoring the indeterminacy on the allocation of labor of type 1 (firm $B$ is assumed to hire all the supply), we find:

$x_A^* = (x_{A0}^*, x_{A1}^*, x_{A2}^*) = (4, 0, 2);$

$x_B^* = (x_{B0}^*, x_{B1}^*, x_{B2}^*) = (7, 1, 0).$

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