Bertrand-Edgeworth competition under non-exclusivity

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Abstract

The work introduces a simple framework to study the relationship between competition and incentives under non-exclusivity. We characterize the equilibria of an insurance market where intermediaries compete over the contracts they offer to a single consumer in the presence of moral hazard. Non-exclusivity is responsible for under-insurance and positive profits in an otherwise competitive set-up. A relevant notion of optimality is also introduced and it is argued that market equilibria fail to be constrained-efficient.
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1 Introduction

The analysis of the aggregate implications of asymmetric information in competitive markets has become an important element of contemporary economic theory. The need for a theoretical understanding of the relationship between markets and contracts is at the center-stage of many recent developments in contract theory as well as in researches on general equilibrium. Furthermore, it constitutes a crucial step to extend traditional macroeconomic schemes to markets where agents are heterogeneous with respect to their access to information. This paper contributes to analyze the relationship between incentives and competition, when asymmetric information is explicitly taken into account.

We consider insurance and credit markets. We think of these markets as a setting where multiple intermediaries compete over the contract offers they are making to a representative agent. Our focus is on markets

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where contracts are non-exclusive: every single consumer can simultaneously accept the proposals of many financiers, generating contractual externalities among the competitors.

The choice to focus on non-exclusive competition on contract, rather than on an exclusive model of competition on contracts, stems from a series of observations that we report and discuss in detail.

Several financial markets seem not to operate (even implicitly) under exclusive assumptions. An example is the US credit cards market: consumers typically hold several cards and are often given incentives to open new accounts.

More generally, courts of law can enforce exclusive contractual agreements only if they have access to some form of centralized information over agents' characteristics. In addition, the relevance and the geographical extension of multi-banking relationship has been established by several researches. Ongena and Smith (2000b) emphasize that only less than 15 percent of European firms maintain a single banking relationship. Even though multi-banking is typically regarded as a relevant source of financing for big companies, Detragiache, Garella, and Guiso (2000) and Petersen and Rajan (1994) document its relevance for small firms for countries like the US, Italy and Portugal. The rise of multiple lending phenomena is frequently associated to the high cost of implementing exclusive, single-lending relationships. The argument is supported by a number of empirical studies emphasizing that only in a few cases debt covenants explicitly include exclusivity clauses (Asquith and Wizman (1990) and Smith and Warner (1979)).

As for what concerns insurance markets, both life insurance and annuity contracts are typically non-exclusive. In particular, the UK legislation explicitly requires every pension provider to allow the consumer to purchase annuities from other companies and this right is known as "Open Market Option" (OMO).

There is also evidence that competition in non-exclusive markets can hardly be reconciled with competitive outcomes. In a survey on the US credit cards market for the period $1981-1989$, Asubel (1991) found clear evidence of extra-profits for intermediaries: the rate of return to the credit card issuers is at least three times higher than the average of the banking sector. Looking at the insurance market, the 2006 British Survey of Annuity Pricing reports that there is strong evidence that both life insurance instruments and annuities are priced higher than what is suggested by actuarial considerations, despite the presence of a fairly high number of intermediaries. In addition, it is well-documented empirical fact that, in the US as well as in Europe, very few individuals voluntarily purchase life annuities, despite the fact that they look a valuable asset for risk-averse consumers in standard insurance set-ups.

This paper provides a simple characterization of equilibria in non-exclusive markets where many intermediaries strategically compete in the presence of moral hazard on the agent's side. We emphasize how under-insurance and positive extra-profits are a typical implication of this sort of competition. In addition, we identify a rationale for regulatory interventions in a non-exclusive context. More precisely, we emphasize that market equilibria fail to be constrained efficient.

If the relevant welfare criterion is that of a (benevolent) social planner who cannot control the agents' (hidden) actions, then the planner may indeed improve on market's allocations as long as he retains the power of controlling trades. If we consider the situation where the planner cannot modify the structure of markets, i.e. he *cannot* restrict financial intermediaries from offering further trade opportunities (i.e. side contracts), then the results change. The relevant welfare allocations belong to a tighter constrained-Pareto frontier. This seems a reasonable case to investigate in all those situations where a social planner is in the impossibility of enforcing exclusivity. Interestingly, we show that even in this very constrained scenario there emerge regulatory issues: public insurance provision turns out to be always welfare improving.

Many important researches have tried to provide a theoretical assessment of situations where competition on contracts was non-exclusive. In particular, the analysis of non-exclusive relationships under moral hazard initiated by the influential works of Pauly (1974), Arnott and Stiglitz (1991), Arnott and Stiglitz (1993) and Hellwig (1983) has been reformulated by Bizer and DeMarzo (1992), Kahn and Mookherjee (1998) and Bisin and Guaitoli (2004).

The works closely related to our are Parlour and Rajan (2001) and Bisin and Guaitoli (2004) who explicitly model the strategic interactions among intermediaries offering contracts to a single borrower-consumer simultaneously. Both researches find that non-exclusivity supports positive-profit equilibria and induces a distorted distribution of surplus. This paper studies the same economy, where the choice of hidden action (effort) is binary. We generalize their analysis to make the relationship between equilibrium allocations and equilibrium contracts fully explicit, in particular with those contracts which are issued but not bought at equilibrium. These are called *latent contracts*, and they play a crucial role in preventing standard Bertrand-like deviations, where an intermediary would offer a higher quantity at a lower price. More precisely, the presence of *latent* contracts transforms the agent into a coordinating device among the competing intermediaries. The threat of buying *latent* contracts with the induced change in effort following any deviation from a set of equilibrium offers, would make the deviation unprofitable. *Latent* contracts sustain equilibrium outcomes exhibiting under-insurance, positive profits for the active intermediaries and available insurance. Having fully explored the role of *latent* contracts, we show that those allocations can be as well supported in a simpler game where two competitors (principals) offer non-linear schedules to a single agent. The strategic role of the agent will then be to select her desired item in both menus. Since we consider a complete information case, there will be offers that are not accepted by any agent at equilibrium. The corresponding equilibria constitute an instance of the failure of the Revelation Principle in games with multiple principals.[1] Our characterization results provide simple intuitions about the welfare properties of the equilibria that these games exhibit. Differently from Parlour and Rajan (2001), none of our equilibrium allocations will satisfy a constrained-efficiency property.[2]

In addition, we provide a class of examples showing how pure equilibria may fail to be efficient even if the planner is subject to non-exclusivity clauses, which was neglected in Bisin and Guaitoli (2004).

The paper is organized as follows. Section 2 presents the general two-state, two-effort insurance economy we study. Section 3 analyzes the properties of the exclusive game, where the consumer is allowed to accept only one contract at a time. Section 4 presents general results about equilibrium characterization under non-exclusive competition. Section 5 and 6 examine the properties of equilibria with under-insurance in the situation where consumers' preferences are homogeneous, which allows us to provide foundation for our strategic approach and to relate our results to the Common Agency literature. Finally, section 7 discusses the welfare implications of our analysis.

2 The economy

We consider an insurance economy lasting two periods. It is populated by a single representative consumer and by a countable infinite set of intermediaries. To simplify the analysis, only two idiosyncratic states $\{1, 2\}$ are considered (we take 1 to be the "bad" state, and 2 the "good" one). The probability distribution over the set $\{1, 2\}$ depends on an unobservable effort $e = \{a, b\}$, with $a > b$. We take $\pi_a(\pi_b)$ to be the probability of occurrence of state 2 when $e = a(b)$, respectively, with $\pi_a > \pi_b$.
Effort is costly and we take the cost function to be linear, so that $K(e) = e$ with $e = \{a, b\}$.
The agent's utility from consuming c_s with $s = \{1, 2\}$, is state-independent and it is represented by the function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ that is continuous, increasing and concave. In addition, we denote agent's endowment array by $W = (w_1, w_2) \in \mathbb{R}^2_+$.

[1] See Martimort and Stole (2002) and Peters (2001). In particular, the relationship between the failure of the Revelation Principle and the existence of latent contract offers in games with complete information is stressed in Martimort and Stole (2003).

[2] Attar, Campioni, and Piaser (2006) show that all market equilibria studied by Parlour and Rajan (2001) indeed turn out to be constrained-efficient.

Trades are represented by insurance contracts offered by intermediaries to consumer.

We consider J intermediaries; the set of feasible offers for the $i-th$ intermediary is referred to as $\mathcal{D}^i$ and $\mathcal{D} = \times_{i\in J}\mathcal{D}^i$ is the collection of all the sets $\mathcal{D}^i$. Every intermediary is allowed to offer a continuum of insurance contracts. Offers are restricted to be linear: insurance companies propose a benefit-premium ratio and fix a maximum quantity to be sold; the consumer is then free to choose her desired amount of insurance from each contract. To represent this type of interaction, we assume that each intermediary i offers a contract consisting of a pair of state-contingent transfers $d^i = (d_1^i, d_2^i)$. The agent can buy a fraction $\lambda_i \in [0,1]$ of each contract.

Insurance relationships are non-exclusive: every single consumer chooses a subset of intermediaries to interact with and her trade decisions cannot be contracted upon. A pure strategy for the consumer is hence described by the map

$$s_c : \mathcal{D} \ \rightarrow \{a, b\} \times [0,1]^J,$$

that associates to every array of offered contracts a level of effort in $\{a, b\}$, and selects the desired amount of insurance from the (linear) menu offered by every single principal. We also take S_c to be collection of all the s_c maps.

The payoff to intermediary i is given by:

$$V^i = -((1-\pi_e)d_1^i + \pi_e d_2^i)\lambda_i,$$

when effort e is chosen and the fraction λ_i of the offer d^i is bought. In compact notation, we denote $\vec{\pi}_e = ((1-\pi_e), \pi_e)$, and then write $V^i = -\vec{\pi}_e \cdot d^i \lambda_i$.

The agent-consumer is risk-averse. Her preferences are represented by the VNM utility function:

$$\tilde{U}(c_1, c_2, e) = (1-\pi_e)u(c_1) + \pi_e u(c_2) - e,$$

where e denotes the cost of effort and $c_s = w_s + \sum_{i\in J}\lambda_i d^i$, for $s = \{1,2\}$ is the contingent consumption. Let C be the vector of contingent consumption, $C = (c_1, c_2)$. In what follows, we refer to $U(C) = \tilde{U}(C, e(C))$, with $e(C) \in \arg\max_e \tilde{U}(C, e)$. That is, we represent preferences through to the consumer's indirect utility $U(C)$, as it is standard in moral hazard economies. In addition, we take $\mathcal{A} = \{C \in \mathbb{R}_+^2 \ / \ e(C) = a\}$ to be the set of ex-post consumption profiles inducing the high effort choice

$e = a$, and $\mathcal{B} = \{C \in \mathbb{R}^2_+ \ / \ e(C) = b\}$ to be that inducing low effort $e = b$.

The frontier of the set $\mathcal{A}$ will be referred to as the Incentive frontier. We denote it by $\mathcal{G}$:

$$\mathcal{G} = \{(c_1, c_2) \in \mathbb{R}^2_+ : \tilde{U}(c_1, c_2, a) = \tilde{U}(c_1, c_2, b)\}$$

In the insurance market, multiple intermediaries compete over the contractual offers they are making to a risk-averse consumer. Strategic relationships are represented by the complete information game:

$$\Gamma = \{\tilde{U}(.), \left(V^i(.)\right)_{(i \in J)}, \mathcal{D}, S_c\}.$$

The game Γ can hence be thought as a Bertrand-Edgeworth game with a strategic (representative) consumer on the demand side. We will be focusing our attention on pure strategy Subgame Perfect Equilibria (SPE) of the game Γ. At any SPE of the game Γ:

i) The agent optimally chooses her effort level together with the desired composition of her portfolio. That is, for every offered array $d = (d^1, d^2, \ldots, d^J)$:

$$((\lambda_i(d))_{i \in J}, e(d)) \in \arg\max_{e,\lambda} \ (1 - \pi_e)u(c_1) + \pi_e u(c_2) - e \tag{1}$$

s.t.

$$c_s = w_s + \sum_{i \in J} \lambda_i d^i_s, \quad for \ s = \{1, 2\}$$

ii) For every array $d^{-i} = (d^1, \ldots, d^{i-1}, d^{i+1}, \ldots, d^J)$ of menu offers proposed by his competitors, the choice of d^i by intermediary i should maximize V^i subject to equation (1) that takes into account the agent's behavior and to the feasibility requirement $V^i \geq 0$.

3 Exclusivity

When exclusive dealings are considered, the agent can at most accept one contract at a time. We regard this as a benchmark case, since our concern is the analysis of situations where trade decisions cannot be restricted. For this reason, we devote the next paragraphs to investigate the properties of the insurance market when exclusivity can be enforced.

The equilibrium outcome of the exclusive game can be represented as the solution to the utility maximization problem of the agent under a non-negative profit constraint for the principal (see Chassagnon and Chiappori (1997)). We will therefore consider two possible consumption profiles:

- The allocation $E = (h_1, h_2)$ such that:

$$E = \big((1-\pi_b)\, w_1 + \pi_b\, w_1, (1-\pi_b)\, w_2 + \pi_b w_1\big),$$

  which lies at the intersection of the 45 degree line with the zero-profit line of slope $\dfrac{1-\pi_b}{\pi_b}$.

- The allocation $F = (f_1, f_2)$, which is defined by:

$$\begin{cases} u(f_2) - u(f_1) & = & \dfrac{a-b}{\pi_a - \pi_b} \\ (1-\pi_a)\, f_1 + \pi_a\, f_2 & = & (1-\pi_a) w_1 + \pi_a\, w_2 \end{cases}$$

That is, F is located at the intersection of the incentive frontier $\mathcal{G}$ with the zero-profit line of slope $\dfrac{1-\pi_a}{\pi_a}$.
One should notice that $E \in \mathcal{B}$ and corresponds to the consumption level appropriated by the single agent when $e = b$ is chosen, whereas $F \in \mathcal{A}$ corresponds to $e = a$.
It is straightforward to provide a characterization of (pure strategy) equilibrium allocations under exclusivity in terms of the relative cost of high effort, $\Delta = a - b$.

Lemma 1 *The nature of the exclusive equilibrium depends on the relative cost of implementing the high level of effort Δ. More precisely, there exists a cut-off level $\tilde{\Delta}$ such that whenever $\Delta > \tilde{\Delta}$ the effort $e = b$ will always be selected at equilibrium.*

Proof. Consider the contract $\tilde{d} = (\tilde{d}_1, \tilde{d}_2)$ defined by

$$\begin{cases} (1-\pi_b)\, u(\tilde{c}_1) + \pi_b\, u(\tilde{c}_2) & = & u((1-\pi_b)\, \tilde{c}_1 + \pi_b\, \tilde{c}_2) \\ -\vec{\pi}_a \cdot \tilde{d} & = 0, \end{cases}$$

where $\tilde{C} = W + \tilde{d}$ is the consumption allocation corresponding to the choice of the additional insurance $\tilde{d}$. Notice that $\tilde{d}$ is defined independently of Δ.
For every $\tilde{d}$, we let $\tilde{\Delta}$ be the cost differential guaranteeing that the corresponding $\tilde{C}$ lies on the incentive frontier $\mathcal{G}$:

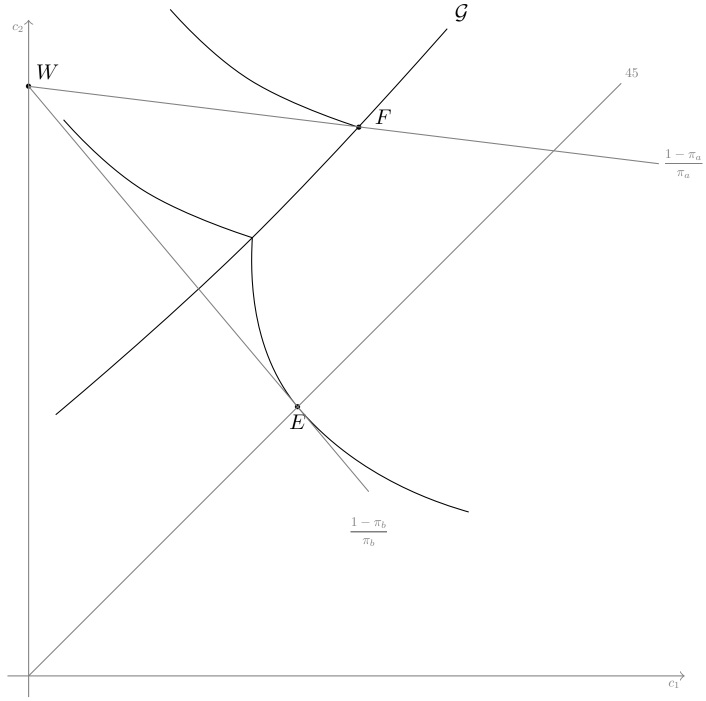


Figure 1: Equilibrium consumption under exclusivity

$$\tilde{\Delta} = (\pi_a - \pi_b)\big(u(\tilde{c}_2) - u(\tilde{c}_1)\big).$$

One should observe that if $\Delta = \tilde{\Delta}$, then $\tilde{C} = F$ and the single agent will be indifferent between the consumption allocations E and F.

For all $\Delta > \tilde{\Delta}$, the allocation $F(w)$ will lie between W and $\tilde{C}$. It follows that $U(\tilde{C}) = U\big(E\big) > U(F)$; that is, the effort $e = b$ will be selected, and the equilibrium allocation will be E.

With a similar reasoning one can establish that the consumer will strictly prefer F to E for all $\Delta < \tilde{\Delta}$, and will therefore take the high level of effort. ■

We now move to discuss the properties of non-exclusive environments.

4 Non-exclusivity

This section investigates the features of insurance relationships when exclusivity clauses are not enforceable. If contracts are exclusive, every single intermediary can offer allocations which are contingent on the consumer's trades with his competitors. In addition, a court can enforce exclusive agreements only if trades are observable and verifiable at no cost. Among other things, this requires an institutional setting where information about agents' trades is centralized and readily accessible.

When contracts are non-exclusive, a form of contractual externality arises. The decisions of the $j-th$ intermediary affect the $i-th$ intermediary's payoff through their effects on the agents' behavior. In such a strategic context, the single agent is therefore acting as a coordinating device among the competing principals. In particular, her portfolio choices crucially determine the structure of the market.

For every array of offered contracts $d = (d^1, d^2, \ldots, d^J)$, the single agent is allowed to choose her preferred consumption level by buying a fraction λ^j of principal j's proposal. The set of feasible allocations is given by:

$\mathcal{F} = \{(c_1, c_2) \in \mathbb{R}^2_+ : (c_1, c_2) = W + \sum_{i \in J} \lambda_i d^i\}$

and it is convex.

The agent's behavior crucially depends on the prices of all received proposals. We denote with p^i the price of the contract offered by the $i-th$ intermediary: $p^i = \left|\frac{d^i_2}{d^i_1}\right|$. Given the effort choice e, we also let τ^C be the marginal rate of substitution evaluated at $C = \{c_1, c_2\} \in \mathcal{F}$, i.e. $\tau^C = \frac{1-\pi_e}{\pi_e}\frac{u'(c_1)}{u'(c_2)}$.

For any given array of offers, we will henceforth order the prices of posted contracts in accordance with the index of intermediaries. That is, we denote:

$$p^1 \leq p^2 \leq \cdots \leq p^J.$$

Any optimal consumption choice will be located on the frontier of the $\mathcal{F}$ set, that is, the set of allocations which are not dominated by any other element in $\mathcal{F}$. We remark here that the frontier is piece-wise linear, and that the slope of the relevant segments is given by $p^1, p^2, \ldots, p^J$. Using the convexity of $\mathcal{F}$, one can get additional information on the structure of the frontier as well as on the properties of the optimal choice. Given the offers $d = (d^1, d^2, \ldots, d^J)$, we let $X = \sum_{i=1}^J \lambda_i d^i$ be any allocation on the frontier and we denote C the preferred consumption allocation.

In a first step we show that every allocation on the nonlinear part of the frontier of $\mathcal{F}$ can be identified in a unique way.

Lemma 2 ***Feasibility:*** *Take any $X = \Omega + \sum_{i=1}^K \lambda_i a^k$ on the frontier of $\mathcal{F}$,then, there is only one array $(\lambda_1, \lambda_2, \ldots, \lambda_K)$ supporting X. In addition, if all aggregate contracts are positive insurance and if $i < j$,* *then,*$\lambda_i = 0 \Rightarrow \lambda_j = 0$ *and* $\lambda_j > 0 \Rightarrow \lambda_i = 1$.

It is to be noticed that similar rules can be found with negative insurance contracts. The trick being that when some aggregate contracts are of negative insurance, then, it should be noticed that the set of feasible allocation is the same as the set of feasible allocation that would correspond to an initial endowment

$$\Omega' = \Omega + \sum_{d_i \text{ negative insurance}}$$

and to the same positive insurance contracts offered and at the place of any d_i negative insurance, the offer of $-d_i$. Then, the rules of the lemma can be applied to this "new" rationalization of the feasible set.

Proof.

- Suppose not. There should then be two arrays $\lambda_1, \lambda_2, \ldots \lambda_k$, $\lambda'_1, \lambda'_2, \ldots \lambda'_k$ with at least one i such that $\lambda_i \neq \lambda'_i$ and

  $$X = \Omega + \sum_{i=1}^{J} \lambda_i a^i = \Omega + \sum_{i=1}^{J} \lambda'_i a^i.$$

  Let $\mu_i = \min(\lambda_i, \lambda'_i)$. We let i_1 be the first index such that $\lambda_i \neq \lambda'_i$, and we assume that without loss of generality, we suppose that $\lambda_{i_1} > \mu_{i_1}$

  Then, we denote

  $$a = (\lambda_{i_1} - \mu_{i_1}) a_{i_1} \quad c = \sum_{i > i_1}^{J} (\lambda_i - \mu_i) a_i \quad b = \sum_{i=1}^{J} (\lambda'_i - \mu_i) a_i.$$

  One can hence write $X = \Omega' + a + c = \Omega' + b$, with $\Omega' = \Omega + \sum i = 1^J \ \mu_i a_i$. We then verify that $p_a < p_b < p_c$ (see Sub-figure 2-a).[3] Moreover, we remark that the contracts $\lambda_i a_i$, are bought, then, the contracts $(\lambda'_i - \mu_i) a_i$ are available, which implies that the allocations Ω', $\Omega' + a$, $\Omega' + a + b$, and $\Omega' + a + b + c$ are all feasible. But then, it is then straightforward to verify that any available allocation between $\Omega' + a$ and $\Omega' + a + b$ dominates X (see Sub-figure 2-b) which guarantees that X is not on the frontier.

[3] where p_a, p_b, p_c design the price of the particular contracts a, b and c.

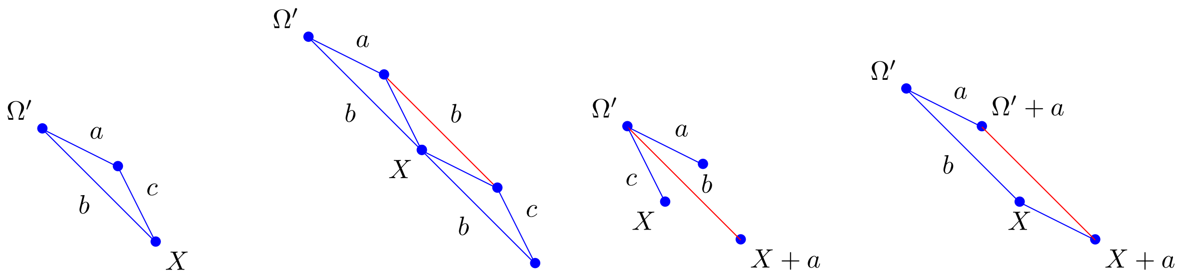


Figure 2:

- For the second part of the proof, let first suppose that $\lambda_{i_1} = 0$ and that $\sum_{j \geq i_1} \lambda_j > 0$, then, we denote

$$a = a_{i_1} \quad c = \sum_{j > i_1} \lambda'_j a_j \quad b = a + c.$$

Then, $c \neq 0$ and one can hence write $X = \Omega' + c$, with $\Omega' = \Omega + \sum_{i<i_1} \lambda_i a_i$, but then, as it should be that $p_a < p_b < p_c$ (because all agregate contracts are positive insurance contracts), we could apply the preceding line of reasoning and remark that the line connecting Ω' and $X + a$ dominates X (see Sub-figure 2-c) , which implies that X is not on the frontier, a contradiction.

Let in a second time suppose that $\lambda_{j_1} > 0$ and that $\lambda_{i_1} < 0$. Then, we denote

$$a = a_{i_1} \quad b = \sum_{j > i_1} \lambda'_j a_j.$$

Then, $b \neq 0$ and one can hence write $X = \Omega' + b$, with $\Omega' = \Omega + \sum_{i<i_1} \lambda_i a_i$. But then, it is immediate that the segment $\Omega' X$ is below the segment $\Omega + a$ $X + a$ (see Sub-figure 2-d), which implies that X is not on the frontier, a contradiction. That ends to prove the second part of the lemma.

■

Lemma 3 ***Optimality:*** *Let $d = (d^1, d^2, \ldots, d^J)$ be an array of insurance contract offers such that the corresponding optimal consumption choice of the agent C does not lie on the Incentive Compatibility frontier $\mathcal{G}$, i.e. $C \notin \mathcal{G}$. Then:*

i)

ii) If intermediary i proposes a positive insurance contract, his offer will be entirely *bought (i.e. $\lambda_i = 1$) if $p^i < \tau^C$. Furthermore, whenever i's offer is bought ($\lambda_i \in (0, 1]$), one gets $p^i \leq \tau^C$.*

iii) If intermediary i proposes a negative insurance contract, then his offer will be entirely bought if $p^i > \tau^C$. Furthermore, whenever i's offer is bought ($\lambda_i \in (0, 1]$), one gets $p^i \geq \tau^C$.

iv) If $p^i = \tau^C$, then any insurance contract d^i proposed by any intermediary i will be partially bought ($0 < \lambda_i < 1$).

Proof.

We start by establishing the following statement. (A): consider the consumption allocation C; if d^i is a positive (negative) insurance contract and the consumer accepts it (i.e. $\lambda_i > 0$), then $\tau^C \geq p^i$ ($\tau^C \leq p^i$). To prove it, one has to observe that if d^i is a positive insurance contract, then, since $\lambda_i > 0$, the allocation $C - \lambda\ d_i$ is achievable by the consumer for every $\lambda < \lambda_i$. It follows that $U(C - \lambda\ d_i) \leq U(C)$, which indeed implies $\tau^C \geq p^i$. Following the same reasoning one can prove (A) for the case of d^i being a negative insurance contract.

In a next step, we prove statement (B): if d^i is a positive (negative) insurance contract which is not entirely accepted (i.e. $\lambda_i < 1$), then $\tau^C \leq p^i$ ($\tau^C \geq p^i$). Let us refer to the positive insurance case. Then, if d^i is not fully accepted, $C + \lambda\ d^i$ belongs to $\mathcal{F}$, for every $\lambda < 1 - \lambda_i$. Since d^i a positive insurance contract, one gets that $U(C + \lambda\ d^i) \leq U(C)$, which indeed implies $\tau^C \leq p^i$. Once again, one can use a similar argument to establish (B) for the case of d^i being a negative insurance contract.

Notice that if the d^i is a positive insurance contract, then (A) implies the second part of equivalence (i). To contradict the first part of (i), it should be $p^i < \tau^C$ and $\lambda_i = 0$. However, such a situation is impossible because it would directly violate (B). Point (ii) can be established along the same lines.

Finally, (iii) is an immediate implication of point (A) and point (B). ■

Notice that this statement applies to any local optimal allocation of $\mathcal{F}$.

The consumer will hence have a clear incentive to first buy those contracts which are offered at a lower price. If we take C to be the equilibrium consumption allocation, then any contract with a price (strictly) higher than τ^C will never be bought, while contracts offered at a price strictly lower than τ^C will be entirely accepted.

The remaining of this section is devoted to analyze the general properties of market equilibria under non-exclusivity. We re-establish, in a general context, several results for economies with moral hazard and non-exclusive contracting. As long as the incentive problem is binding, competition among an arbitrarily large number of intermediaries typically delivers non-competitive outcomes in the form of under-insurance and positive extra-profits. As a starting point, it is useful to remark that whenever the incentive problem is very mild, then the presence of non-exclusive contracts will be of no aggregate impact. In other words, we can prove in our context the following standard result:

Lemma 4 *If* $U(C) < U\big(\pi_b c_2 + (1-\pi_b)c_1\big)$ *for all* $C \in \mathcal{F}$*, then there exists a unique (pure strategy) equilibrium allocation characterized by:* $c_2 = c_1 = \pi_b w_2 + (1-\pi_b)w_1$.

Proof. See Arnott and Stiglitz (1993) and Bisin and Guaitoli (2004). ■

If the cost of high effort is sufficiently high, then this choice does not represent a credible threat and a standard Bertrand argument will apply.[4]

Our analysis will therefore be devoted to evaluate and discuss the structure of high effort equilibria. Even though our reference framework is quite abstract, the results presented in this section identify several properties that should be satisfied by any pure strategy equilibrium of the game Γ when the high effort is chosen. Throughout this section, let $C \in \mathcal{A}$ denote a pure strategy equilibrium outcome.

As a starting point, we show that negative insurance will never be issued at equilibrium. Therefore, this type of contracts will only be relevant to analyze the deviation stage.

Lemma 5 *Let* $C \in \mathcal{A}$ *be a pure strategy equilibrium outcome of the game* Γ*. Then, every accepted contract involves positive insurance.*

Proof. Suppose not. Then, let j be the intermediary offering the negative insurance contract d^j, with the agent buying a fraction λ_j of it. Let p^j be the price of this contract. It follows from Lemma 3 that:

$$p^j \geq \tau^j > \frac{1-\pi_a}{\pi_a}.$$

If the high effort is selected, the $j-th$ intermediary is earning a negative profit which implies that d^j cannot be part of an equilibrium. ■

The main results of this section are summarized in the next Proposition. The properties of high effort equilibria are identified depending on whether the corresponding allocations belong to the incentive frontier $\mathcal{G}$ or not. In the former case, equilibria will be supported by having all intermediaries fully active, while equilibrium allocations outside $\mathcal{G}$ feature at least one intermediary that is rationed. In other words, there will be some available insurance in the market that the consumer will not find convenient to buy.

Proposition 1 *The following statements are equivalent:*

[4] A similar argument is suggested in Proposition 1 and 2 in Parlour and Rajan (2001).

1. *The equilibrium outcome $C \in \mathcal{A}$ belongs to the Incentive Compatibility frontier $\mathcal{G}$.*

2. *C is supported by all intermediaries being active. In particular, the offer of each intermediary is fully accepted, i.e. $\lambda_i = 1 \; \forall i \in J$.*

3. *The consumer strictly prefers C to any other feasible outcome: $U(C) > U(C') \; \forall C' \in \mathcal{F} \cup \mathcal{B}$.*

Proof. The proof is developed in three steps.

The first step consists in showing that if the outcome C belongs to the frontier $\mathcal{G}$, then all contracts must be accepted, i.e. $\lambda_i > 0 \; \forall i$. By contradiction, suppose that at equilibrium there is one inactive intermediary, say the $i-th$ one. Then, he could profitably deviate by offering a positive insurance contract $d^{i\prime}$ with a price arbitrarily close to $\dfrac{1-\pi_b}{\pi_b}$ and strictly higher to make positive profits, i.e. $-\vec{\pi}_b \cdot d^{i\prime} > 0$. It is immediate to verify that the agent will always have the incentive to accept this offer and to select $e = b$.

Let us move to the second statement to show that the consumer will entirely buy all offered contracts and reach the allocation:

$$C = W + \sum_{j \in J} d^j.$$

If, indeed, there was (at least) one positive λ_i strictly lower than 1, then, for every $\lambda < \min(\lambda_i, 1-\lambda_i)$, both $C - \lambda\, d^i$ and $C + \lambda\, d^i$ should be feasible consumption choices, with $U(C - \lambda\, d^i) \leq U(C)$ and $U(C + \lambda\, d^i) \leq U(C)$. This contradicts the fact that the marginal rate of substitution under high effort evaluated at C is different from that calculated under low effort. Indeed, if $\tau = \left|\dfrac{d_2^i}{d_1^i}\right|$ is the price of d^i, then,

- inequality $U(C - \lambda\, d^i) \leq U(C)$ would imply that $\dfrac{1-\pi_a}{\pi_a}\dfrac{u'(c_1)}{u'(c_2)} \geq \tau$;

- inequality $U(C + \lambda\, d^i) \leq U(C)$ would imply that $\dfrac{1-\pi_b}{\pi_b}\dfrac{u'(c_1)}{u'(c_2)} \leq \tau$,

with the consequence that $\dfrac{1-\pi_a}{\pi_a} \geq \dfrac{1-\pi_b}{\pi_b}$, a contradiction.

The next step is to show that at any (pure strategy) equilibrium where all intermediaries are active, C will be strictly preferred to any other feasible outcome. To prove that, it is enough to remark that if at equilibrium $\lambda_i = 1 \;\; \forall i \in J$, then the set of feasible consumptions for the agent will entirely be contained in $\mathcal{A}$. This indeed follows from the fact that all contracts offered at equilibrium are positive insurance ones.

Using a standard convexity argument one hence gets $U(C) > U(C') \;\; \forall C' \in \mathcal{F}$.

Finally, we have to show that if there is no consumption allocation that guarantees the consumer the same utility as C, then it must be that C belongs to the frontier $\mathcal{G}$. By contradiction, suppose that $C \notin \mathcal{G}$. Two cases should be considered depending on whether the active intermediaries charge the same price or not. In the latter case, it must be $p^1 < \tau^C$, by Lemma 3. Then, take a $p \in (p^1, \tau^C)$ and denote $\mathcal{Z} \subset \mathcal{A}$ a neighborhood of C such that $\tau^C > p$ for all $C \in \mathcal{Z}$. Take now any contract d whose price is smaller than τ^C, and such that $-\vec{\pi}_a \cdot d > 0$. Then, consider the deviation $d^{1\prime} = d^1 + \epsilon d$ for intermediary 1, and let $C(\epsilon)$ be the optimal consumption choice of the agent, given the new array of offers. If ϵ is small enough, the price of $d^{1\prime}$ will be strictly below p and we will also have $C(\epsilon) \in \mathcal{Z}$. The contract $d^{1\prime}$ will hence be entirely bought ($\lambda_1 = 1$), which guarantees, by Lemma 3, that the deviation is profitable.

We now have to consider the case where at equilibrium all contracts are offered at the same price, i.e. $p^i = p \; \forall i$. Once again, let us develop an argument by contradiction. If $C \notin \mathcal{G}$, then any intermediary necessarily earns positive profits, i.e. $\tau > \dfrac{1-\pi_a}{\pi_a}$. If this was not the case, a single intermediary could always gain by offering a negative insurance contract at a price strictly smaller (but, arbitrarily close to) $\dfrac{1-\pi_b}{\pi_b}$, which the consumer will have an incentive to accept selecting the effort $e = b$.

Take intermediary 1 to be the one earning the smallest (positive) profit at equilibrium and consider a small positive insurance contract d such that $\vec{\pi}_a \cdot d > 0$ and $U(C+d) > U(C)$. If intermediary 1 lowers his price deviating to $d^{1\prime} = d^1 + d$, the agent will accept this offer. If she accepts $d^{1\prime}$ together with other contracts, the deviating contract will be entirely bought (see Lemma 3), and the deviation will be profitable as long as $e = a$ is selected. If the agent chooses the high level of effort, the deviation will be profitable even in the extreme case when $d^{1\prime}$ is the only accepted contract at the deviation stage. To stress this point, it is enough to remark that $\vec{\pi}_a \cdot d^1 < \vec{\pi}_a \cdot \big(C - W\big)$, since at C all intermediaries earn a positive profit. Take any $\epsilon < \vec{\pi}_a \cdot \big(C - W\big) - \vec{\pi}_a \cdot d^1$. If ϵ is small enough to get $\pi_a \cdot \big(C'(\epsilon) - W\big) > \pi_a \cdot \big(C - W\big) - \epsilon$, where $C'(\epsilon)$ is the optimal consumption choice at the deviation stage, then one gets $\pi_a \cdot \big(C'(\epsilon) - W\big) > \pi_a \cdot d^1$, and the deviation is profitable.

Since the consumer could gain by accepting $d^{1\prime}$ and choosing $e = a$, for $C \in \mathcal{A}$ to be supported at equilibrium it must be that the effort $e = b$ is chosen at the deviation stage, so to make the deviation unprofitable. This remains true if d is arbitrarily close to $(0, 0)$. For $C \notin \mathcal{G}$ to be an equilibrium outcome, there must hence exist an outcome $L \in \mathcal{B}$ such that $U(C) = U(L)$, which generates a contradiction. ■

Equilibrium allocations outside the Incentive frontier $\mathcal{G}$ therefore involve insurance offers that are not accepted at equilibrium and block deviations from incumbent and from entrant intermediaries. This constitutes *latent* insurance. The structure of equilibria with "latent contracts" turns out to be significantly different from that of the equilibria on $\mathcal{G}$. The following pages explore this issue in greater detail.

4.1 High effort equilibria on $\mathcal{G}$: general properties

Every equilibrium outcome on $\mathcal{G}$ is supported by all contracts being fully accepted. That is, the single agent finds optimal not to ration any of the intermediaries. In addition, we show that every competitor must be charging the same price, so that the equilibrium will exhibit a fairly symmetric structure.

Proposition 2 *If the equilibrium outcome C belongs to the frontier G, then all intermediaries must set the same price:* $p^1 = p^2 = ... = p^J = \tau^C$

Proof. The proof is developed by contradiction. Let us consider that $p^1 < p^J$. Since all contracts have been (entirely) accepted, Lemma 3 guarantees that $p^J \leq \tau^C$. In addition, every contract will involve positive insurance (from Lemma 5). Let us denote $\overline{C}$ ($\underline{C}$) the optimal consumption choice of the agent when contract J (1) with price p^J (p^1) is removed. At equilibrium, it should be that $U(C) > U(\underline{C})$ and $U(C) > U(\overline{C})$. Take a neighborhood of C such that the corresponding agent's utility is strictly grater than both $U(\underline{C})$ and $U(\overline{C})$ for every element of the neighborhood.

Assume that intermediary 1 is deviating to $d^{1\prime} = d^1 + \epsilon d$, where d is a (small) negative insurance contract with price lower than $\dfrac{1-\pi_a}{\pi_a}$. If ϵ is small enough, a continuity argument guarantees that the offer $d^{1\prime}$ will always be at least partially bought. If $d^{1\prime}$ is entirely bought $(i.e.\ \lambda_1 = 1)$, the deviation is unambiguously profitable.

Given the array of offers $\{d^{1\prime}, d^2, ..., d^J\}$, let $C(\epsilon)$ be the corresponding optimal consumption choice. In addition, denote $\overline{C}(\epsilon)$ the outcome that would be selected at the deviation stage if all contracts with price p^J were removed. Observe that $U\big(\overline{C}(0)\big) = U(\overline{C}) < U\big(C(0)\big) = U(C)$. By continuity, $U\big(\overline{C}(\epsilon) < U\big(C(\epsilon)\big)$. This implies that, at the deviation stage, the agent will have an incentive to buy some of the contracts offered at price p^J. From Lemma 3 it follows that $\tau^{C(\epsilon)} \geq p^J$; and if ϵ is small enough, the price of $d^{1\prime}$ will be strictly smaller than p^J. Summarizing, we have that $\tau^{C(\epsilon)} > |\dfrac{d_2^{1\prime}}{d_1^{1\prime}}|$, so that $d^{1\prime}$ will be fully accepted, and the deviation is profitable.

■

If an equilibrium on $\mathcal{G}$ exists, it cannot be a zero-profit equilibrium. That is, the exclusive allocation F in Figure 1 cannot be supported at equilibrium under non-exclusivity. If this was the case, indeed, a single intermediary would always have an incentive to offer a small additional amount of insurance at a price $p \in \left(\frac{1-\pi_b}{\pi_b}, \tau^C\right)$. By doing so, he would increase his profit, induce the consumer to buy this additional amount of insurance and to select low effort.

Equilibria on $\mathcal{G}$ resemble (or can be related to) more traditional Cournot outcomes, with all intermediaries earning strictly positive profits.

With reference to moral hazard framework, Parlour and Rajan (2001), Attar, Campioni, and Piaser (2006), Martimort (2004) and others characterize pure strategy equilibria where all intermediaries are treated symmetrically. That is, they are equally sharing the surplus, leaving zero extra rent to the single agent.[5] These authors argue that a crucial condition to support these equilibrium outcomes is to make the single consumer indifferent between two alternative consumption profiles in the high effort subspace. More precisely, at equilibrium the consumer can get the same utility by buying all J contracts and by buying only $J-1$ contracts. This guarantees that deviations trying to reduce the agent's payoff will always be blocked. In other words, the equilibrium payoff will be available at the deviation stage.

The main difference between our set-up and those mentioned above consists in the fact that, for every given array of offers, the set of feasible consumption choices here is always convex. This greatly simplify our equilibrium analysis. In particular, we prove that there will not exist any pure strategy equilibrium where an allocation $C \in \mathcal{G}$ is implemented. Given the convexity of the set $\mathcal{F}$, it turns out that there will always be unilaterally profitable deviations implying an increase in prices. To develop this argument, we need to introduce some additional notation.

For $\alpha \in [0,1]$, let $C_\alpha = \left(c_1 + \pi_a\ \alpha, c_2 - (1-\pi_a)\alpha\right)$ be any allocation lying on the iso-profit line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C. Notice that if $\alpha = 0$, $C_\alpha = C$. We focus here on the case $\alpha < 0$, which implies that the corresponding C_α is at the north-west of C. For every given α, take $W_\alpha = \left(c_1 + \alpha\pi_a \frac{\tau^{C_\alpha} - \tau^a}{\tau^{C_\alpha} - \tau^C}, c_2 - \tau^C \alpha\pi_a \frac{\tau^{C_\alpha} - \tau^a}{\tau^{C_\alpha} - \tau^C}\right)$ to be the point on the equilibrium price line which is connected to C_α by a line of slope τ^{C_α}. That is, if the single consumer was offered the insurance contract $C_\alpha - W_\alpha$, then she would always have an incentive to entirely accept it (see Figure 3).[6]

[5] The related work of Bizer and DeMarzo (1992) also characterizes a situation where all contracts are take up at equilibrium. Given the different distribution of bargaining power, though, in Bizer and DeMarzo (1992) every single intermediary earns zero-profits at equilibrium.

[6] One has to observe that the sequence of points W_α converges to C when $\alpha \to 0$. This is implied by the two inequalities $0 \le W_{2\alpha} - C_2 \le -\alpha\pi_a\tau^C$ and $0 \le C_1 - W_{1\alpha} \le -\alpha\pi_a$.

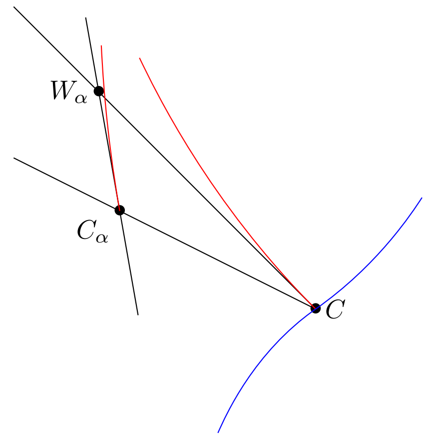


Figure 3: Relationship between W_α and C_α

The following proposition can now be established.

Proposition 3 *No allocation $C \in \mathcal{G}$ can be supported at a pure strategy equilibrium in the game* Γ.

Proof. We show that there always exists a profitable deviation for a single intermediary, say J, involving a price higher than the equilibrium price $p = \tau^C$. Consider any α small enough to guarantee that the corresponding W_α lies between $W^{-J} = W + \sum_{i=1}^{J-1} d^i$ (see Figure 4).

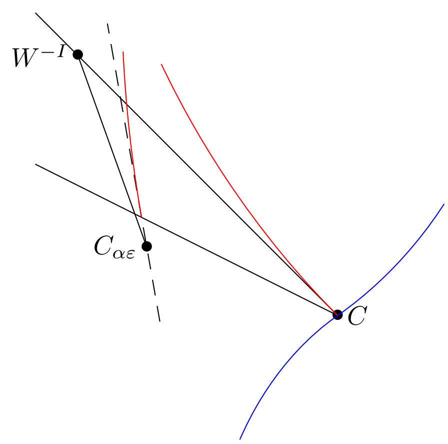


Figure 4: Existence of a profitable deviation for any equilibrium on $\mathcal{G}$

By construction, we have that:

$$\tau^{C_\alpha} > \frac{W_2^{-J} - C_{2\alpha}}{C_{1\alpha} - W_1^{-J}}.$$

Consider the following deviation of intermediary J: instead of offering d^J, he proposes $d_\varepsilon^{J\prime} = (1+\varepsilon)(C_\alpha - W^{-J})$.

Observe that if the agent decided to entirely accept all contracts at the deviation stage, she would achieve the allocation:

$$C_{\alpha\varepsilon} = C_\alpha + \varepsilon\big(C_\alpha - W^{-J}\big)$$

which is depicted in Figure 4. In particular, it is always possible to select an ε small enough to guarantee that the corresponding $C_{\alpha\varepsilon}$ belongs to $\mathcal{A}$ and to satisfy the following inequality:

$$\tau^{C_{\alpha\varepsilon}} > \frac{W_2^{-J} - C_{2\alpha\varepsilon}}{C_{1\alpha\varepsilon} - W_1^{-J}} \tag{2}$$

which exactly guarantees that the agent will have an incentive to accept all contracts.[7]. In addition, the deviating contract guarantees a profit higher than the original one:

$$-\vec{\pi}_a \cdot (C - W^{-J}) = -\vec{\pi}_a \cdot (C_\alpha - W^{-J})$$

where the original (positive) profit has been multiplied by $(1+\varepsilon)$. ■

4.2 High effort equilibria outside $\mathcal{G}$: general properties

Up to now, we know that allocations outside $\mathcal{G}$ may be supported as equilibria either through linear price, i.e. all intermediaries are offering the same price, or through non-linear prices. In both cases, some latent contracts will be issued at equilibrium. Given the array of contracts offered at equilibrium $d = (d^1, d^2, \ldots, d^J)$, we take $I = \{i \in J : \lambda_i(d) > 0\}$ to be the set of active principals and its complement I^c to be the set of inactive ones. As a consequence, we introduce the following definition:

Definition 1 *For every equilibrium outcome $C \in \mathcal{A}$, a latent outcome is a feasible point $L \in \mathcal{B}$ lying on the equilibrium indifference curve. The outcome L can in principle be supported by many alternative profiles of offered contracts. We say that an inactive principal $k \in I^c$ is "latent" if his equilibrium offer d^k can support a latent outcome L.*

[7] This is an immediate implication of our Lemma 3.

In what follows, we identify a set of necessary conditions guaranteeing that the candidate equilibrium $C \in \mathcal{A}$ is robust to deviations of active and latent intermediaries.

One has first to observe that from Proposition 1 point (iii), it follows that if $C \notin \mathcal{G}$ is a (pure strategy) equilibrium outcome, there must exist an allocation L such that $U(C) = U(L)$.

The next paragraphs provide a further characterization of the structure of the latent consumption L.

Lemma 6 shows that if all latent contracts taken together support L, the same latent outcome L remains available if any of the latent contracts is removed.

Lemma 6 *Let $C \in \mathcal{A}$ be a pure strategy equilibrium outcome of the game Γ. Then, the latent consumption L remains available to the consumer if any of the latent contracts d^k is removed.*

Proof. Suppose not. In this case, whenever (the latent) principal k removes his offer, the highest utility that the agent could earn by choosing $e = b$ would be strictly lower than $U(C)$. Then, as in point (iii) of Proposition 1, intermediary k could propose (instead of d^k) a small positive insurance contract that the agent would accept while keep choosing $e = a$. This would guarantee positive profits to the deviating latent intermediary k, a contradiction. ■

This allows us to prove our main Proposition:

Proposition 4 *If the equilibrium outcome C does not belong to the frontier G, then:*

i) All latent contracts will be offered at the same price $p^k = p^L$ and it will always be $\tau^L = p^L$.

ii) The two points $C \in \mathcal{A}$ and $L \in \mathcal{B}$ are connected by a line of slope p^L.

iii) The two vectors $\big(\nabla U(C) - \nabla U(L)\big)$ and $\vec{\pi}_a = \big(\pi_a, (1 - \pi_a)\big)$ are collinear.

Proof. To prove (i), consider the latent offer d^k and let $\lambda_k \in (0, 1]$ be the fraction of that offer which the consumer is willing to buy when choosing low effort. That is, $L = W + \sum_{i \neq k} \big(\lambda_i d^i\big) + \lambda_k d^k$. For every $\epsilon \in (0, \lambda_k)$ the outcome $L - \epsilon d^k$ is hence a feasible choice for the consumer. It follows that $U(L) \geq U(L - \epsilon d^k)$. One should also observe that, by Lemma 6, the outcome $L + \epsilon d^k$ is feasible for any $\epsilon \in [0, 1]$. Therefore, we get:

$$U(L) \geq max\{U(L - \epsilon d^k), U(L + \epsilon d^k)\}$$

for every k. This directly implies $p^k = p^L = \tau^L \; \forall k$.

For convenience, let us choose arbitrarily some positive contract d^L of price p^L.

To prove (ii), let us first show that all active contracts with price lower than p^L will be fully bought at equilibrium. If such contracts exist, denote with I the last active intermediary whose contract is offered at price lower than p^L. Then, for $k > I$, let α^k be the (unique) real number (positive or negative) such that $d^k = \alpha^k \, d^L$. Since L is the consumer's optimal choice inside $\mathcal{B}$, we know from Lemma 3 that all contracts offered at price lower than τ^L should be bought in order to reach L. That is:

$$L = W + \sum_{i=1,\ldots,I} d^i + \big(\sum_{k>I} \lambda_k \alpha^k\big) d^L \tag{3}$$

Suppose by contradiction that for some $i^0 \le I$, $\lambda_{i^0} < 1$. The consumer can still achieve the outcome C with the new array $\{d^1, \ldots, \lambda_{i^0} d^{i^0}, \ldots, d^I, d^{I+1}, ..., d^J\}$. Observe also that given these offers, the allocation L will still be available, otherwise the intermediary i^0 could profitably decrease his price.

In particular, L can be decomposed as follows:

$$L = W + \big(\sum_{i=1,\ldots,I} d^i\big) - (1 - \lambda_{i^0}) d^{i^0} + \big(\sum_{k>I} \lambda'_k \alpha^k\big) d^L, \tag{4}$$

which contradicts the previous statement unless $\lambda_{i^0} = 1$.

Since all active contracts must be entirely bought, it follows that $C = W + \big(\sum_{i=1,\ldots,I} d^i\big)$, and that equation (3) can be rewritten as:

$$L - C = \big(\sum_{k>I} \lambda_k \alpha^k\big) d^L, \tag{5}$$

that guarantees that the line connecting L and C has slope τ^L.

Finally, notice that

$\dfrac{l_2 - c_2}{c_1 - l_1} = \dfrac{1 - \pi_b}{\pi_b} \dfrac{u'(l_1)}{u'(l_2)}$,

follows from (i) and (ii), and it corresponds to:

$$\tau^L l_1 + l_2 = \tau^L c_1 + c_2. \tag{6}$$

To prove part (iii) of the Proposition, one has to show that there exists a $\theta \in \mathbb{R}$ such that:

$$\nabla U_L - \nabla U_C = \theta \vec{\pi}_a, \tag{7}$$

with $\nabla U_X = \big((1-\pi_e)u'(x_1), \pi_e u'(x_2)\big)$ for $X = \{C, L\}$.

Let $D = (d^1, d^2, \ldots, d^J)$ be the array of contracts offered at equilibrium and consider a unilateral deviation of one of the active intermediaries, say the $i-th$, offering $d^{i\prime} = d^i + d$ instead of d^i, with d small enough. In particular, $d^i + d$ is taken to be a positive insurance contract with price lower than $p^L = \tau^L$

Consider first the agent's behavior. At the deviation stage, she can achieve the consumption outcome $C + d \in \mathcal{A}$, as well as all the allocations on a line of slope τ^L passing through $C + d$. In particular, let $L'(d) = L + \delta$ be the corresponding optimal consumption choice of the agent in the set $\mathcal{B}$. We will have:

$$\tau^L(l_1 + \delta_1) + (l_2 + \delta_2) = \tau^L(c_1 + d_1) + (c_2 + d_2) \tag{8}$$

Using (6), equation (8) can be reduced to:

$$\tau^L \delta_1 + \delta_2 = \tau^L d_1 + d_2,$$

which corresponds to:

$$\nabla U_L \cdot \delta = \nabla U_L \cdot d.$$

For the deviation $d^{i\prime} = d^i + d$ to be profitable, it should be $-\vec{\pi}_a \cdot d > 0$. Given the new array of offers, the maximum utility that the agent could achieve in the low effort region is given by $U(L) + \nabla U_L \cdot \delta$. Since $C \in \mathcal{A}$ is an equilibrium outcome, deviations like $d^{i\prime}$ should be blocked. For this to be guaranteed, $U(C + d)$ must necessarily be (strictly) lower than the maximum utility that the agent could reach, at the deviation stage, selecting a consumption profile in $\mathcal{B}$. Since we are restricting to deviations that take place in a neighborhood of C, the agent can always achieve (at least) the payoff $U(C + d) = U(C) + \nabla U_C \cdot d$, by selecting high effort. Therefore,

$$-\vec{\pi}_a \cdot d > 0 \implies U(L) + \nabla U_L \cdot d > U(C) + \nabla U_C \cdot d.$$

Since $U(C) = U(L)$, one gets:

$$-\vec{\pi}_a \cdot d > 0 \implies (\nabla U_L - \nabla U_C) \cdot d > 0. \tag{9}$$

These inequalities show that the two operators $-\vec{\pi}_a$ and $\nabla U_L - \nabla U_C$ are collinear, so that there exists a real number θ satisfying:

$$\nabla U_L - \nabla U_C = \theta \, \vec{\pi}_a.$$

■

5 Existence and characterization: the homogeneous case

From now on, we will develop the analysis on existence and the characterization of equilibria with under-insurance and high effort in common agency games with moral hazard, concentrating on the case of CRRA utility functions. In particular, for reasons of tractability we will specify $u(c) = c^\gamma$, with $\gamma \in (0, 1)$.[8]

Taking the agent's utility of the form $u(c) = c^\gamma$ has the advantage to simplify the equilibrium characterization leading to sharper welfare predictions, as will be clarified in the following pages.

The first result we show is that in our context it will never be possible for an allocation $C \notin \mathcal{G}$ to be supported as a pure strategy equilibrium of the game Γ by having all intermediaries charging the same price. This is indeed a consequence of the following Lemma.

Lemma 7 *Consider two points $C \in \mathcal{A}$ and $L \in \mathcal{B}$ with the same MRS $\tau^C = \tau^L = p$ and such that:*

$$pc_1 + c_2 = pl_1 + l_2 \quad (10)$$

$$c_1 \leq l_1. \quad (11)$$

Then, it must be that:

$$\pi_a u'(c_2) \geq \pi_b u'(l_2)$$

Proof. Since $\tau^C = \tau^L = p$, one gets:

$$\pi_a = \frac{u'(c_1)}{u'(c_1) + pu'(c_2)} \qquad \pi_b = \frac{u'(l_1)}{u'(l_1) + pu'(l_2)}$$

[8] The results we will provide hold for the general class of CRRA utility functions.

We have to prove that:

$$\frac{1}{\frac{p}{u'(c_1)}+\frac{1}{u'(c_2)}} > \frac{1}{\frac{p}{u'(l_1)}+\frac{1}{u'(l_2)}}.$$

A sufficient condition for the result is that the function $(c_1, c_2) \mapsto \frac{p}{u'(c_1)}+\frac{1}{u'(c_2)}$ be increasing along the line connecting C and L. Consider $f(x) = \frac{p}{u'(c_1+x)}+\frac{1}{u'(c_2-px)}$, whose derivative is such that:

$$f'(x)/p = \frac{-u''(c_1+x)}{(u'(c_1+x))^2} - \frac{-u''(c_2+x)}{(u'(c_2+x))^2} = (1-\gamma)(\frac{1}{c_1+x} - \frac{1}{c_2+x} = \frac{(1-\gamma)(c_2-c_1)}{(c_1+x)(c_2+x)} > 0$$

that completes the proof. ■

Lemma 8 *Consider the game* Γ *where consumer preferences are represented by* $u(c) = c^\gamma$ *with* $\gamma \in (0,1)$. *If the allocation* $C \notin \mathcal{G}$ *is a (pure strategy) equilibrium outcome in which high level of effort is chosen, then at least two contracts must be offered at different prices.*

Proof. By contradiction, assume $C \notin \mathcal{G}$. Recall that at equilibrium every intermediary necessarily earns positive profits, i.e. $p = \tau^C > \frac{1-\pi_a}{\pi_a}$, where p is the equilibrium price (see Proposition 1).
Now, take intermediary 1 to be the intermediary that earns the smallest (positive) profit at equilibrium and consider a small positive insurance contract d such that $\vec{\pi}_a \cdot d > 0$ and $U(C+d) > U(C)$. If intermediary 1 deviates to $d^{1\prime} = d^1 + d$, the agent will accept it. To generate the contradiction it would be enough to show that the deviation $d^{1\prime}$ induces the effort choice $e = a$. Since we are considering a small d, the agent's optimal consumption choices in the subspaces $\mathcal{A}$ and $\mathcal{B}$ will be on a line of slope $p = \tau^C$. Denote the corresponding allocations $C' = C + (c_1, c_2)$ and $L' = L + (y_1, y_2)$, respectively. The relevant utilities can be approximated by:

$$U(C') = U(C) + (1-\pi_a)u'(c_1)c_1 + \pi_a u'(c_2)c_2 = \pi_a u'(c_2)(\tau^C c_1 + c_2)$$

and

$$U(L') = U(L) + (1-\pi_b)u'(l_1)y_1 + \pi_b u'(l_2)y_2 = \pi_b u'(l_2)(\tau^C y_1 + y_2).$$

At equilibrium $U(C) = U(L)$ and since C' and L' are connected by a line of slope τ, we also know that:

$\tau^C c_1 + c_2 = \tau^C y_1 + y_2$. Now, given that $\pi_a u'(c_2) > \pi_b u'(l_2)$, $U\big(C'\big)$ will be strictly greater than $U\big(L'\big)$, and the agent's optimal effort will be $e = a$. ■

We know that every equilibrium outcome $C \notin \mathcal{G}$ will be supported by non-linear prices, let us now identify some restrictions on the consumer's initial wealth so that such an equilibrium exists. More precisely, consider the allocations C and L that satisfy the following set of conditions:

$$U(C) = U(L) \tag{12}$$

$$\tau^L c_1 + c_2 = \tau^L l_1 + l_2 \tag{13}$$

$$\pi_a(1-\pi_a)\big(u'(c_1) - u'(c_2)\big) = \pi_a\ (1-\pi_b)\ u'(l_1) - \pi_b\ (1-\pi_a)\ u'(l_2) \tag{14}$$

$$\frac{(1-\pi_b)}{\pi_b} \geq \tau^L > \tau^C > \frac{w_2 - c_2}{c_1 - w_1} > \frac{(1-\pi_a)}{\pi_a}, \tag{15}$$

$$w_1 + 2(l_1 - c_1) = w_2 + 2(l_2 - c_2), \tag{16}$$

$$\frac{w_2 + 3(l_2 - c_2)}{w_1 + 3(l_1 - c_1)} < \frac{l_2}{l_1}, \qquad -\vec{\pi}_a \cdot \tilde{d} < -\vec{\pi}_a \cdot \big(C - W\big). \tag{17}$$

Let us provide an interpretation to the system of equations (12)-(17). (12) requires the single agent to be indifferent between the outcomes C and L, and (13) states that C and L are connected by a line of slope τ^L in the space (c_1, c_2). That is, if there are contracts offered at price τ^L, the consumer is indifferent between buying only the insurance that takes her up to C (and choosing $e = a$) and buying it together with the additional contracts that make her reach L (and choose $e = b$). (14) is simply a restatement of the gradient condition discussed in Proposition 4, stating that the gradient of the agent's utility evaluated at C is parallel to that evaluated at L. The second and the third inequality in (15) guarantee that accepting the offer $C - W$ is an optimal choice for the consumer when she selects $e = a$. The last inequality implies that the active intermediaries earn a positive profit offering $C - W$, at a price $|\frac{w_2 - c_2}{w_1 - c_1}|$. Given (16), the consumer can achieve full insurance buying only the latent offers $L - C$. Finally, (17) provides a characterization of the amount of latent insurance needed to support the equilibrium consumption. More precisely, the first inequality in (17) guarantees that there is enough latent insurance to make the low-effort threat credible, while the second inequality puts a lower bound on that amount, that is sufficient to block deviations to negative insurance offers.[9]

[9] The amount $\tilde{d}$ defines the positive insurance contract offered at a price τ^L that is necessary to exactly achieve full insurance

If $u(c) = c^\gamma$, it is possible to show that there always exists a pair of consumption allocations $\big(C, L\big)$ that satisfies the system $(12) - (17)$.[10] Any allocation $\big(C, L\big)$ identified by $(12) - (17)$ can be supported at equilibrium in the game Γ.

Proposition 5 *If $u(c) = c^\gamma$, there exists a pure strategy equilibrium of the game Γ with the following players' behaviors:*

i) $d^1 = C - W, \quad \Lambda^1 = \{0, 1\}$,

ii) $d^2 = d^3 = L - C, \quad \Lambda^2 = \Lambda^3 = [0, 1]$,

iii) $d^4 = d^5 = \cdots = d^N = (0, 0)$,

iv) $\lambda_1 = 1, \lambda_2 = \lambda_2 = \cdots = \lambda_N = 0$*, and* $e = a$,

where C and L are determined by $(12) - (17)$.

Proof. See Attar and Chassagnon (2008). ■

The geometric structure of the equilibrium is depicted in Figure 5.

6 Take-it or leave-it offers and menus

This section aims at examining the relationship between our discussion and the game theoretic approach to common agency games, to clarify the foundations in terms of strategic behavior for the analysis of non-exclusive markets. Throughout the paper, we have been referring to situations in which intermediaries compete à la Bertrand-Edgeworth. That is, each intermediary is choosing a price and fixing the maximal amount of insurance that the consumer can buy. A natural question to raise would therefore be: could a single intermediary gain from a unilateral deviation to a simple take-it or leave-it offer? Since there does not seem to be any reason to assume that singleton contracts are not feasible in insurance and financial markets, the robustness of our equilibrium characterization to such an enlargement of the intermediaries' strategy spaces would be an important property to establish.

When an intermediary deviates to a take-it or leave-it contract, the strategic role of the agent is reduced, since she is not allowed to select her favorite amount of insurance at the given price, anymore. Therefore,

starting from the consumption point $\big(\frac{c_1 + w_1}{2}, \frac{c_2 + w_2}{2}\big)$. The role of $\tilde{d}$ will be carefully examined in the next section.

[10] See Lemma 1 in Attar and Chassagnon (2008).

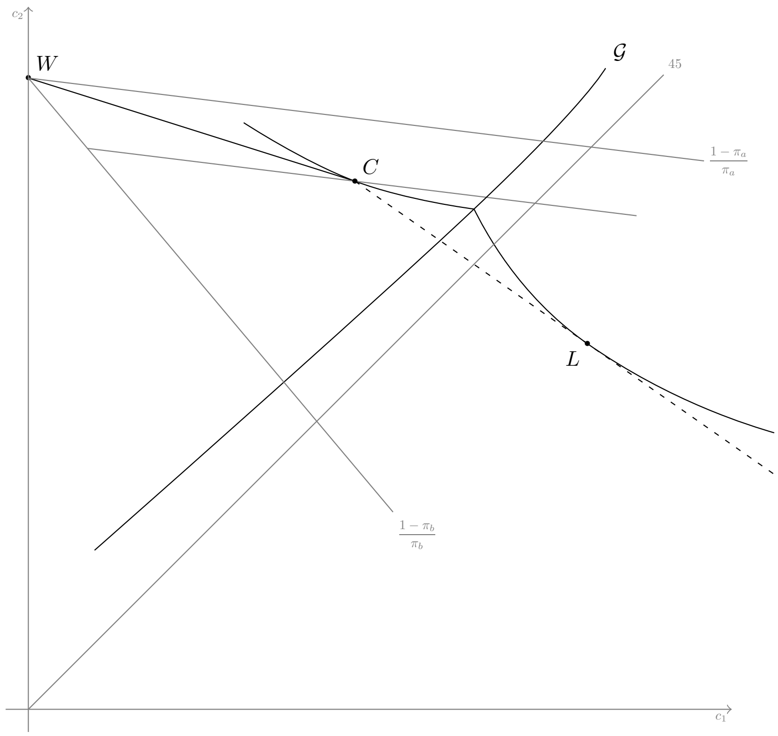


Figure 5: A pure strategy equilibrium with latent contracts

deviations to take-it or leave-it contracts may constitute a severe form of punishment for the consumer, and break the conjectured equilibrium.

In the following paragraphs we show that each of our (pure strategy) equilibria indeed satisfies a robustness property. More precisely, the allocation characterized in Proposition 5 can be supported as an equilibrium of a simpler game in which two competitors (principals) (are allowed to) offer menus of alternatives to the single agent. As will become clearer, this result enables us to relate our findings with those of the results of the literature on common agency games under moral hazard.

Consider the new game Γ', in which two principals compete on the menu offers they are simultaneously making to a single agent. If $\mathcal{D}^i$ is the set of offers available to principal i, let $\mathcal{P}\left(\mathcal{D}^i\right)$ be the set of available menus.[11] A strategy for principal i will hence be a map from $\mathcal{P}\left(\mathcal{D}^i\right)$ to itself. For every array of offered menus, the agent will choose one item from the menu of each competitor and take her (binary) effort decision.

In the next paragraphs, we show how the equilibrium outcome $C \notin \mathcal{G}$ of the game Γ can be supported as a symmetric equilibrium in the menu game Γ'.

[11] That is, $\mathcal{P}\left(\mathcal{D}^i\right)$ is the power set of $\mathcal{D}^i$.

Take $C \notin \mathcal{G}$ as defined in Proposition 5, and consider a situation where each of the two competitors offers the following menu of insurance contracts to the single agent:

$$\mathcal{M}^i = \left\{\mu^i \frac{d^1}{2} + \lambda^i \tilde{d}\right\}, \qquad \text{for } \; i \in \{1, 2\}$$

$\mu^i \in [0,1]$ and $\lambda^i \in [0,1]$ define the consumer's selected item, and the insurance contracts d^1 and $\tilde{d} = L - C$ have been defined in Proposition 5. Notice that every menu $\mathcal{M}^i$ is convex. As a special case, by selecting $\mu = 1$ and $\lambda = 0$ in the menu of both principals the agent can achieve the outcome C. Similarly, choosing $\mu = 0$ and $\lambda = 1$ in both menus, the consumer can get to the consumption point L. This follows from the fact that the maximum amount of latent insurance that the agent can buy when each competitor offers the menu $\mathcal{M}$ is $2\tilde{d}$, that is always greater than $2d$.[12]

The reader should observe that to construct the equilibrium in menus we let every principal offer the amount of latent insurance, say $\tilde{d}$, that corresponds to that available off equilibrium in the game Γ. More precisely, when a single principal removes his offer, the frontier of the set of feasible consumption pairs for the agent is that represented in Figure 6. In Figure 6 we also represented the iso-profit line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C.

[12]This result can be checked with simple manipulations of the system $(12) - (17)$.

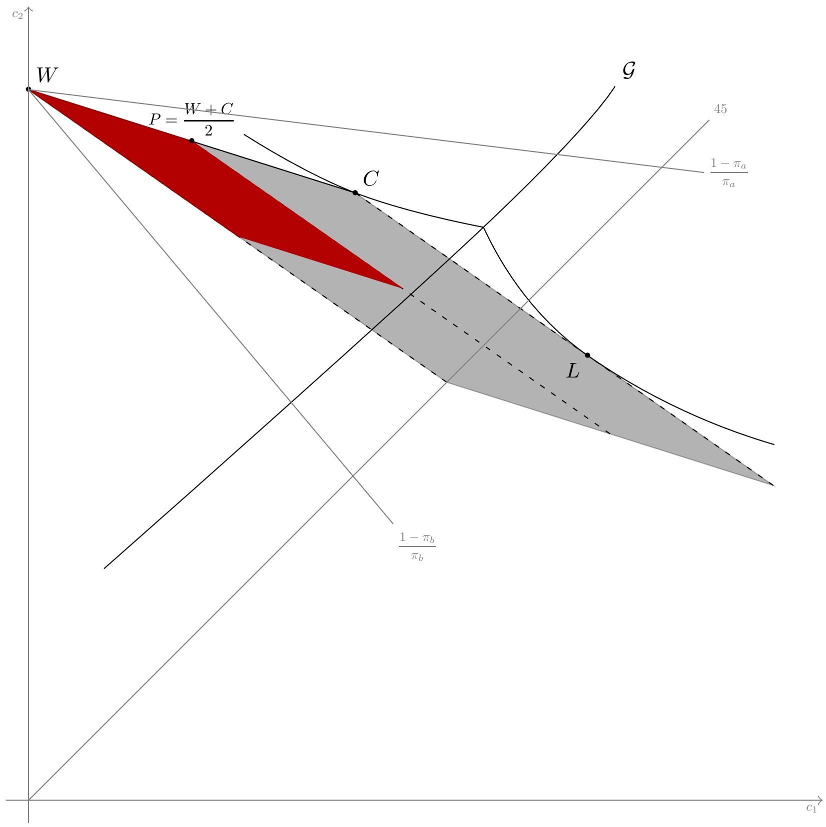


Figure 6: Available consumption choices at the deviation stage

That is, the agent can achieve $P = (\dfrac{c_1 + w_1}{2}, \dfrac{c_2 + w_2}{2}) \in \mathcal{A}$ as well as the corresponding allocation Q on the $45^{\cdot}$, which is connected to P by a line of slope τ^L. In particular, one gets:

$$Q = (\frac{\tau^L(c_1 + w_1) + (w_2 + c_2)}{\tau^L + 1}, \frac{\tau^L(c_1 + w_1) + (w_2 + c_2)}{\tau^L + 1}) \in \mathcal{B}.$$

To show that the outcome C can be supported at a symmetric equilibrium through the menu offers $\mathcal{M}$, we need to fully describe the consumer's behavior. Given the specific utility function we chose, $u(c) = c^\gamma$, we know that the agent's optimal consumption choices will be on linear expansion paths starting from the origin. That is, these choices would belong to the lines $\dfrac{c_2}{c_1}$ and $\dfrac{l_2}{l_1}$ starting from the origin. To complete the characterization of consumer's behavior, we find it convenient to introduce the outcome C_α.

For every $\alpha \in \mathbb{R}$, C_α is the point at the intersection between the (iso-profit) line of slope $\frac{1-\pi_a}{\pi_a}$ passing through C and the line connecting αC and αL (with slope τ_L). Hence, $C_\alpha = (c_1 + \pi_a f(\alpha), c_2 - (1 - \pi_a)f(\alpha))$, with $f'(\alpha) > 0$ and $f(1) = 0$.

Let us also consider the outcome $E = (e_1, e_2)$ such that $\frac{1-\pi_a}{\pi_a}\frac{u'(e_1)}{u'(e_2)} = \tau^L$, i.e. E is the optimal choice of

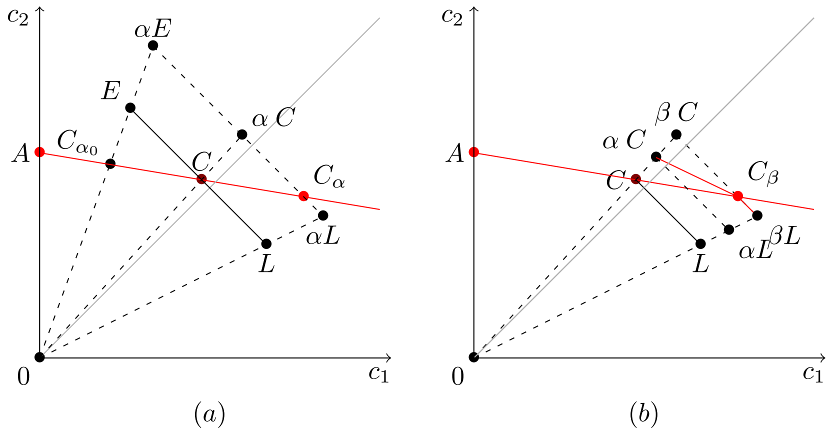


Figure 7: Properties of the optimal consumer's choices

the agent in the set $\mathcal{A}$ if she was offered divisible contracts at price τ^L.

For every $\alpha \in \mathbb{R}$, αE, αC, C_α and αL will all lie on the same line. We will prove that in this set-up, any unilateral deviation to positive insurance contracts will induce the agent to switch to low effort $e = b$. Conversely, a single intermediary who tries to profitably deviate proposing a negative insurance contract would determine a switch to low effort $e = b$. These results follow from the following Lemma.

Lemma 9 *If $u(c) = c^\gamma$ and the outcomes C and L satisfy $(12) - (17)$, then:*

1. *$U(C_\alpha) < U(\alpha L) \quad \forall \alpha \neq 1 \quad$ such that $\quad C_\alpha \in \mathcal{A}$,*

2. *$U(K) < U(\alpha L) \quad \forall K \in \mathcal{A} \quad$ lying between $\quad C_\alpha$ and αL,*

3. *$U(\alpha C) < U(\beta L) \quad \forall (\alpha, \beta)$ with $\quad \beta > \alpha > 1$ and such that the line connecting αC and C_β has slope τ^C.*

Points 1 and 2 refer to Figure 2 (a) and point 3 refers to Figure 2 (b).

Proof. See Attar and Chassagnon (2008). ■

With this characterization, it is now possible to show that a single principal, say principal 1 (P1), cannot profitably deviate to any menu different from $\mathcal{M}$. To show this, it will be enough to consider simple deviations $d^{1\prime}$ in take-it or leave-it offers.[13]

First, consider the deviations $d^{1\prime}$ to a negative insurance contract, which are easier to handle.

[13] This is true since the consumer is allowed to select at most one allocation from every menu.

If $d^{1\prime}$ is a negative insurance contract, to guarantee positive profits when the agent keeps taking high effort $e = a$, its price should be below $\frac{1-\pi_a}{\pi_a}$. Then, whenever $d^{1\prime}$ is bought, the corresponding choice of the agent K will lie to the left of the line connecting P and Q. But there will always be an element of this line which the consumer will strictly prefer to any point inside the triangle WPQ. If P1 deviates to a negative insurance contract and the agent reacts choosing $e = b$, the relevant price should be higher than τ^L, for the agent to have an incentive to buy it, and lower than $\frac{(1-\pi_b)}{\pi_b}$, to be profitable for the deviator.[14] However, by buying $\tilde{d}$ from the non-deviating principal, the agent could achieve an allocation like H on the 45 degree line, and there is no contract, with price is between τ^L and $\frac{(1-\pi_b)}{\pi_b}$, that could increase the agent's utility.

Consider now deviations to a positive insurance contract $d^{1\prime}$ which induce the agent to select high effort $e = a$ at the deviation stage. In particular, consider deviations having a price $p' = |\frac{d_2^{1\prime}}{d_1^{1\prime}}| \in (p^C, p^L]$, where $p^C = \dfrac{w_2 - c_2}{c_1 - w_1}$ is the price of the contract d^1. If such a deviation is chosen, by Lemma 3 the agent will still select $\beta = 1$ in the menu of the non-deviating principal to get the outcome P. However, for the deviation to be profitable, we should have: $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a \cdot d^1 = -\vec{\pi}_a \cdot (C - W)$. (17) implies that $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a(C - W) = -\vec{\pi}_a \cdot \tilde{d}$. Therefore, $d^{1\prime}$ can be decomposed into the sum of $\tilde{d}$ and $d^{1\prime\prime}$ a positive insurance contract at price lower than τ^L, i.e. $d^{1\prime} = \tilde{d} + d^{1\prime\prime}$. After the deviation the new array of contracts $(d^{1\prime}, d^2, ..., d^N)$ will determine a new terminal point on the frontier of feasible consumption choices, $W + d^{1\prime\prime} + 2\tilde{d}$, that will fall below the line passing through the origin and L, as it is the case for the point $W + 3d^2$.[15] Therefore, at the deviation stage, the agent can achieve her optimal choice in the subset $\mathcal{B}$, which belongs to the consumption expansion path passing through L(i.e. the ray starting from the origin and passing through L).

Let us now look at deviations $d^{1\prime}$ which induce the agent to select $\beta < 1$ from the non-deviating principal menu and to take the effort $e = a$. This sort of deviations involve a price $p' \leq p^C$. In addition, since P is available, it must be that $U(K) > U(P)$ where K is the agent's choice at the deviation stage. The corresponding feasible consumption choices are depicted in Figure 8. By definition, the agent will not have any incentive to select the allocation $M = K + d^1$ and to choose $e = a$. For every $K \in \mathcal{A}$ she can always get an outcome like γL that is on the ray connecting the origin to L.[16] We want to prove that there will not be any profitable $d^{2\prime}$ inducing $e = a$. Take P_β to be the point at the intersection between the line of slope $\frac{1-\pi_a}{\pi_a}$ passing through P and the line connecting K and M. Denote βL (with $\beta < \gamma$) the projection of P_β

[14] If the high level of effort was indeed selected, any negative insurance contract involving a price higher than τ^L would earn negative profits.

[15] See the equilibrium condition (17).

[16] The availability of γL is guaranteed by construction.

through a line of slope τ^L onto the ray connecting 0 and L. Then, let $\alpha P \in \mathcal{A}$ be the optimal choice of the agent along the line of slope τ^P passing through P_β. For every $K \in \mathcal{A}$, we can establish that:

$$U(K) \leq U(\alpha P) \tag{18}$$

$$U(\beta L) \leq U(\gamma L) \tag{19}$$

The first inequality is satisfied by construction, since $|\frac{d_1^1}{d_2^1}| \leq \tau^C$. With respect to the second one, remark that $\vec{\tau}^L \cdot K \leq \vec{\tau}^L \cdot M$, and, since P_β is between K and M, we have $\vec{\tau}^L \cdot P_\beta \leq \vec{\tau}^L \cdot M$, with $\vec{\tau}^L = \left(\tau^L, 1\right)$. That is, βL will fall to the left of γL. This indeed implies $U(\beta L) \leq U(\gamma L)$.

Then,using Lemma 2 we get $U(\beta L) > U(\alpha C)$, which, given the inequalities above, implies that $U(\gamma L) > U(K)$. This proves that there is no profitable deviation inducing $e = a$.

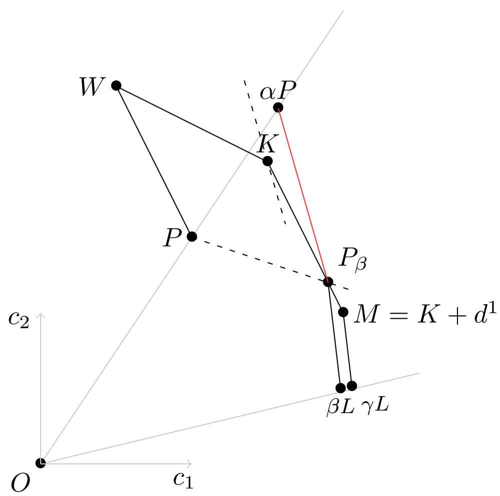


Figure 8: A deviation of intermediary 2

To complete the proof, we have to show that there cannot be any profitable deviation $d^{1\prime}$ inducing low effort. Such a deviation could only be profitable if its price was higher than $\frac{1-\pi_b}{\pi_b}$. The agent, though, will never have any incentive to buy such a contract since she can already achieve the allocation Q on the 45 degree line at a price $\tau^L \leq \frac{1-\pi_b}{\pi_b}$.[17]

Recent developments of oligopoly theory stressed the relevance of situations where many principals strate-

[17] The availability of this outcome is guaranteed by (17).

gically compete over the contract offers they make to a single agent. In particular, those contexts where the agent is allowed to contract with any subset of principals are referred to as of delegated common agency. When common agency games of complete information are considered (i.e. when the agent's type space is a singleton), it is now well established that the set of equilibrium outcomes supportable when principals are allowed to compete by offering menus is in general larger than the set of equilibria associated to "simple" competition in take-it or leave-it offers. More precisely, let us consider a game where two principals interact in the presence of a single agent who is taking a non-contractible effort choice. Then, the particular equilibrium allocation we supported could not have been implemented by letting the two principals compete over take-it or leave-it (singleton) contracts.[18]. It should also be remarked that, when common agency games of moral hazard are considered, there is very weak (little) support for the restriction to take-it or leave-it offers (Attar, Piaser, and Porteiro (2007)).

7 Welfare analysis

In the standard Second Best analysis the planner retains control on the consumption/insurance allocations, but is subject to the same informational problems about agent's decisions of the players. In other words, there is a correspondence between second-best efficiency and exclusivity on contracts. Hence, the planner directly controls (or can contract upon) the mechanisms that the principals of the game will use. This direct control implies that the social planner can always implement the desired allocation by means of point contracts (take-it-or-leave-it offers), hence the agent's choice on λ_is is *de facto* limited to $\{0, 1\}$.

Our equilibrium analysis has already emphasized that the externalities induced by competition under moral hazard are sufficient to guarantee that every equilibrium allocation of the game Γ will fail to be Second Best efficient.

In a non-exclusive interaction, the planner looses control on the mechanisms of the principals, hence indirectly on the total insurance the agent has available for purchases. We will hence consider a situation where both the agent's effort and her trade decisions are not-contractible.

As a consequence, the social planner does not control the hidden action (effort) of the agent, as well as the agent's choice on the number/quantity of insurance contracts she will accept (λ_is). Hence, the optimal allocation must be robust to side-contracting. In the definition of the relevant efficiency concept, we must

[18]This is an instance of the so-called failure of the Revelation Principle in games of multiple principals, see Martimort and Stole (2002) and Peters (2001)

take into account the strategic behavior of the principals as well as the agent's one.

The planner can use/design transfers to all the players, as long as these transfers satisfy aggregate feasibility. Note that a transfer to the agent effectively changes the endowment point w, and hence may equivalently be thought of as public insurance. Let $d^0 = (d_1^0, d_2^0)$ be the state-contingent transfer offered by the planner. That is, we let the planner be denoted as firm 0.

The set of allocations which are feasible for a planner who is subject to non-exclusivity is indeed given by:

- The incentive compatibility on the agent's side:

$$(e, \{\lambda_i\}_{i=1}^N) = \arg \max_{e', \{\lambda_i'\}_{i=1}^N} \ (1-\pi_{e'})u(w_1 + \sum_{i=1}^N \lambda_i' d_1^i) + \pi_{e'} \ u(w_2 + \sum_{i=1}^N \lambda_i' d_2^i) - e' \tag{20}$$

- Robustness of the optimal allocation: $\left(d^i, d^{-i}\right)$ has to be such that:

$$V^i\big(d^i, e(d^i, d^{-i}, W), \lambda_i(d^i, d^{-i}, W)\big) \geq -\tilde{\lambda}_i[(1-\pi_{\tilde{e}})\tilde{d}_1^i + \pi_{\tilde{e}}\tilde{d}_2^i] \ \ \forall i \ \text{ and } \ \forall \tilde{d}^i \in \mathbb{R}^2. \tag{21}$$

One should notice that offering a public insurance contract to the single agent indeed corresponds to control the endowment allocation $W = (w_1, w_2)$. The planner must hence take into account the optimal behavior of all players given alternative public insurance offers.

To show that market equilibria may fail to be constrained efficient, we present our discussion in the context of the Example of Proposition 5.

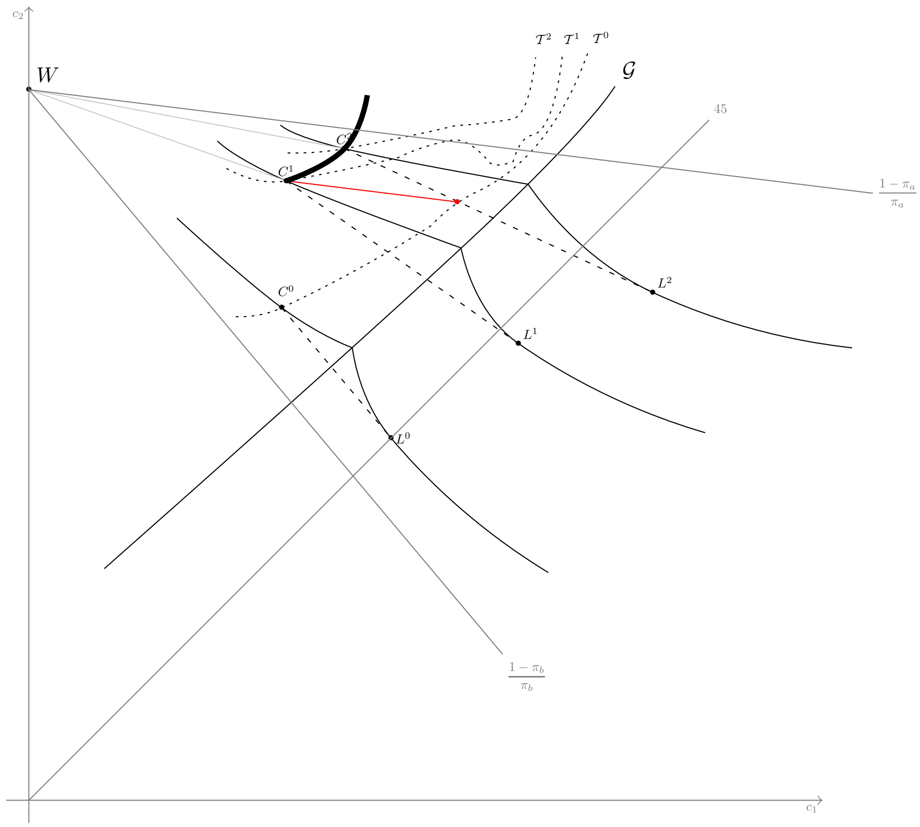
Let us take any $\tau < \dfrac{1-\pi_b}{\pi_b}$ which is supported at equilibrium. Given τ, at every equilibrium allocation $C = (c_1, c_2)$ it must be the that:

$$U(C) = U(L) \tag{22}$$

$$\tau^L c_1 + c_2 = \tau^L l_1 + l_2 \tag{23}$$

$$\tau^L = \frac{(1-\pi_b)u'(l_1)}{\pi_b u'(l_2)} \tag{24}$$

We let $T(\tau)$ be the relationship between c_2 and c_1 which is induced by equations (22)-(24). The equilibrium allocation corresponding to the latent price τ will belong to $T(\tau)$ but, clearly, only one point of $T(\tau)$ constitutes an equilibrium allocation. The geometry of the $T(\tau)$ curves is described in the following Figure.



Importantly, one can show that T curves which correspond to different τ will not intersect. In addition, the curve $T^2 = T(\tau^2)$ will always lie to the left of a curve like $T^2 = T(\tau^2)$ for $\tau^2 < \tau^1$.

In other words, we can think of T^0 as the frontier of the set of feasible allocations in our problem.

Let us first show that in the particular case $u(c) = c^\gamma$, the curve $\mathcal{T}^0$ is increasing, and hence it cuts the relevant iso-profit line passing through C. In this case, the candidate high-effort equilibrium at point C is not robust to individually profitable deviations of some active insurer. Hence, in the power utility case there cannot be an equilibrium with $e = a$ and $\tau^L = \frac{1-\pi_b}{\pi_b}$.

Lemma 10 *In the c^γ-case conditions* (22) *and* (23) *together with the additional requirement* $\tau^L = \frac{1-\pi_b}{\pi_b}$ *identify an increasing curve* $\mathcal{T}^0$.

Proof. Given the requirement that $\tau^L = \frac{1-\pi_b}{\pi_b}$, L belongs to the 45 degree line and equations (22) and (23) can be written in terms of the only two variables (c_1, c_2). In fact, by equation (23), $L = (l, l)$ with

$l = (1 - \pi_b)c_1 + \pi_b c_2$. The equation we have to consider is then:

$$
\begin{aligned}
&(1-\pi_a)u(c_1) + \pi_a u(c_2) = u(l) \quad \Longleftrightarrow \\
&\mathcal{T}^0 = (1-\pi_a)u(c_1) + \pi_a u(c_2) - u\left((1-\pi_b)c_1 + \pi_b c_2\right) = 0 \qquad (25)
\end{aligned}
$$

It should be first noted that it cannot be that both conditions $\dfrac{\partial \mathcal{T}^0}{\partial c_1} < 0$ and $\dfrac{\partial \mathcal{T}^0}{\partial c_2} < 0$ hold together. Assume they hold together, the corresponding first-order inequalities would be:

$$
\begin{cases}
(1-\pi_a)u'(c_1) & < & (1-\pi_b)u'(l) \\
\pi_a \, u'(c_2) & < & \pi_b u'(l)
\end{cases}
$$

Since the marginal utility in convex, using $u''' \geq 0$, we can majorate $u'(l)$ with $(1-\pi_b)u'(c_1) + \pi_b u'(c_2)$ and then

$$
\begin{aligned}
\frac{\partial \mathcal{T}^0}{\partial c_2} &\geq \pi_a \, u'(c_2) - \pi_b\big((1-\pi_b)u'(c_1) + \pi_b u'(c_2)\big) \qquad (26) \\
&= (\pi_a - \pi_b)u'(c_1) + \pi_b^2\big(u'(c_1) - u'(c_2)\big) > 0 \qquad (27)
\end{aligned}
$$

Hence, $\dfrac{\partial \mathcal{T}^0}{\partial c_2} > 0$. ■

If the locus $\mathcal{T}^0$ is increasing in the consumption space, there is room for deviations. In particular, those deviations that reduce the utility of the agent without altering her effort choice become profitable.

Hence, any high-effort equilibrium for which $\tau^L = \frac{1-\pi_b}{\pi_b}$ can be destroyed by profitable deviations of an active principal.

In the c^γ-case, every equilibrium is therefore inefficient with respect to the third-best outcome.

Is that possible that the action of the planner be such that none of the private insurers wants to enter the market for insurance? The answer is yes.

We shall argue, in particular, that it is possible to implement any allocation located on the T^0, which equation is:

$$(1-\pi_a)u(c_1) + \pi_a u(c_2) - u((1-\pi_b)c_1 + \pi_b c_2) - (a-b) = 0 \qquad (28)$$

Let's indeed consider a consumption bundle $C^0 = (c_1^0, c_2^0)$ verifying (28); we also let $L^0 = (l^0, l^0)$ be the corresponding latent allocation. One should notice that $l^0 = (1-\pi_b)c_1^0 + \pi_b c_2^0$.

We then consider the following behavior for the public planner (which we can also call intermediary 0) and the private intermediaries $1, 2, \ldots, J$:

- The planner proposes the following non-linear menu

$$\mathcal{M}^0 = (d^0 + \lambda \tilde{d}^0)$$

  where $d^0 = C - W$ and it is offered at a price $p^0 > \frac{1-\pi_a}{\pi_a}$, $\tilde{d}^0 = L - C$ and it is offered at a price $p_L = \frac{1-\pi_b}{\pi_b}$, and $\lambda \in [0, 1]$;

- the private insurance companies propose the null contract, $d^1 = d^2 = \cdots = d^J = (0, 0)$.

These behaviors support the equilibrium allocation C^0 which belongs to the T^0 curve. The following proposition emphasizes that this allocation can indeed be implemented by the planner:

Proposition 6 *When the planner provides public insurance by means of the piece-wise linear menu $\mathcal{M}^0$, then it is a best reply for the single agent to accept the element $d^0 \in \mathcal{M}'$ (i.e. to set $\lambda = 0$) and for every private insurer to offer the null insurance contract.*

Proof. We should verify that intermediary $i \in \{1, 2, \ldots, J\}$ has no interest in deviating. Take intermediary 1 as a candidate deviator. If he proposes positive insurance, it should be at a price lower than $\dfrac{1-\pi_a}{\pi_a}$ to be accepted by the agent. Then, he could only make profit in the case the agent remains in the high effort area. Let us take $K = (k_1, k_2) \in \mathcal{A}$ as the optimal consumption allocation selected by the agent at the deviation stage. It hence follows from Lemma 9 that:

$$(1-\pi_a)u(k_1) + \pi_a u(k_2) - u((1-\pi_b)k_1 + \pi_b k_2) - (a-b) < 0,$$

which implies in turn that the agent will prefer the feasible allocation $(1-\pi_b)k_1 + \pi_b k_2$ to K, that is a contradiction.

If he proposes negative insurance, the only way for the agent to accept the offer is to remain in $\mathcal{A}$, making the deviation non profitable. A contradiction. ■

In particular, we can state the following:

Proposition 7 *Given any positive-profit high-effort equilibrium $C \in \mathcal{A}$ satisfying (22), (23), (24), there exists a system of transfers and of public insurance that Pareto dominates it.*

Proof. Let us now construct a system of transfers, that satisfies aggregate feasibility and allow to redistribute the planner's profits to the private insurers.

The system of public insurance is a non-linear menu of the type $\mathcal{M}^0 = (\alpha d^0 + \beta \tilde{d^0})$ as defined above, where the single agent has to choose α and β, appropriately. The system of transfers to the players in the economy is the following: each private insurer receives $\frac{1}{J}$-th of the total profit from the purchase of the insurance (V).

Given the offered menu $\mathcal{M}^0$, it is a best reply for the agent to choose $\alpha = 1$ and $\beta = 0$. In fact, since $U(C) = U(L)$ the agent is indifferent between C and L. Choosing $\alpha < 1$ would allow to reach lower indifference curves, and even supplementing with $\beta \geq 1$ would still keep the agent below point L.

For the private insurers, we have previously shown that offering the null contract is a best reply when m^0 is available. Receiving the transfer $\frac{1}{J}\,V$, does not change the main argument of the proof. Each individual insurer takes into account that in the aggregate C and L are available, and that there is no profitable deviation for him. ■

At the equilibrium with transfers, the social planner proposes a piece-wise linear menu of insurance. The private insurers offer the null contract, the agent buys the insurance proposed by the planner and exerts high effort. Finally, the planner redistributes by means of un-contingent transfers the profits to the private insurers according to his objective function.

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