Reconciling the Consumption-Real Exchange Rate Anomaly by Love for Variety*

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Abstruct

This paper revisit the consumption real exchange rate anomaly known as the Baskus-Smith puzzle and provides an alternative view. The main claim of the paper is that the BS puzzle is not tested fully because we can not observe the welfare based real exchange rate which reflects the exact variation of the number of varieties. We first analyze in a simple two-country general equilibrium model which incorporates the endogenous entry of variety representing firms. An increase of the number of varieties following a positive productivity shock works as a demand shock under love for variety and tend to bring the terms of trade and observed real exchange rate in an appreciation. At the same time the consumption rises in the country which has received this positive shock. Hence the resolution of the puzzle arises very naturally. In the second part of the paper we introduce dynamic and more realistic extensions. The above mechanism is explored quantitatively using the standard calibration method in international real business cycle models. Another very important claim is that a realistic BS correlation is possibly observed when the elasticity of substitution between domestic and imported goods is high, consistent with the micro estimation in literature.

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1 Introduction

Perfect international risk sharing with complete asset markets predicts that the consumption in one country rises when the price of consumption in that country becomes relatively cheap. However, this relationship between relative consumptions and real exchange rate is not observed in the real world (See Backus and Smith (1993) and Kollmann (1995)). The correlation is close to 0 or even negative.

The recent attempts to solve this consumption real exchange rate anomaly known as the Backus-Smith (BS) puzzle focus on the absence of complete markets, a strong assumption on which the puzzle relies.[1] Breaking the tight link between relative consumptions and real exchange rate by introducing internationally held non-contingent bonds, Benigno and Thoenissen (2008) discuss the standard international real business cycle model which contains traded and non-traded sectors could successfully provide a realistic BS correlation with the well known Harrod-Balassa-Samuelson effect. Also, under incomplete asset markets Corsetti, Dedola and Leduc (2008) (henceforth CDL) emphasize the role played by the elasticity of substitution with differentiated imported goods. They argue that with a very low elasticity of substitution between local and imported goods, the terms of trade should appreciate after a positive productivity shock. This positive wealth effect that follows increases both consumption and domestic price index, hence providing a realistic BS correlation.[2]

This paper presents a complementary view of the BS puzzles. It is based on the idea that with entry and exist of varieties, the empirically constructed price indices and real exchange rates do not reflect such variations in terms of varieties. The puzzling BS correlation rests on the welfare based measure of price indices and real exchange rates. Our argument is that with entry and exit, the BS correlation can not be tested fully. Recent empirical studies in international trade emphasize the discrepancy between the empirical and "welfare" based price indices. For instance Broda and Weinstein (2004, 2006)point out that the large welfare gains stem from the increased number of imported varieties and note equally that the empirical based price indices have an inflation bias which historically does not consider such variety effect. They report a 1.2% upward bias per year for the U.S import price index. In Broda and Weinstein (2007) they report that the upward bias in CPI is around 0.8% per year from 1994 to 2003 in U.S.

Another important element in our paper is already present in Krugman (1989). Against the conventional role of the terms of trade which distributes the productivity gains through their depreciation, he points out this may not be the case. The love for variety and a resulted strong wage appreciation prevent the terms of trade from depreciating and even reverse their direction of appreciation.

[1] See for example Obstfeld and Rogoff (2000).

[2] Besides this very low elasticity they also point out such strong "wealth effect" arises when the persistance of the productivity shock rises. With a high persistance of shock, forward looking consumers tend to increase more easily the current demand for domestic goods above the supply, providing a realistic BS correlation as in the low elasticity case.

In such a case the real exchange rate also would appreciate reflecting directly these terms of trade or through the Harrod-Balassa-Samuelson effect amplified by this stronger wage appreciation due to the love for variety.[3]

The intuition of the paper is resumed as follows: in our model with love for variety, entry and exit, home bias in consumption and monopolistic competition, a productivity shock leads to entry of new varieties. Because of love for varieties, this entry results in higher demand for these new varieties. The increase in demand itself leads to an increase in the domestic relative wage and an appreciation of the terms of trade and the real exchange rate, at the same time consumption rises in this country. This means that when the real exchange rate is computed with a fixed number of varieties, it is possible to observe 0 or negative correlation between "statistically relevant" real exchange rate and consumption. However, this may not be the case in welfare basis where the real exchange rate may depreciate due to an increased number of varieties produced and consumed with home bias. In such a welfare basis it may well be possible to have a positive correlation between welfare based real exchange rate and consumption, hence providing an original puzzling correlation.

The theoretical model in this paper reflects the recent developments in international macroeconomics such as Corsetti, Martin and Pesenti (2007) and Ghironi and Melitz (2005) which provide a two-country general equilibrium model with endogenous entry of variety representing firms. In the first part of the paper we built a very simple static general equilibrium model in the spirt of Corsetti, Martin and Pesenti (2007) and show analytically the mechanism for providing a realistic BS correlation as stated above. In the second part of the paper the dynamics are introduced following Ghironi and Melitz (2005) with some realistic extensions. The purpose of this part is to reproduce such realistic BS correlation quantitatively using the standard calibration method in international real business cycle literature.

One of the most important emphasis of the paper is the role played by the elasticity of substitution between local and imported goods. Intuitively the higher the elasticity under some degree of love for variety is, the stronger the mechanism à la Krugman becomes, then the more probable we observe a realistic BS correlation. This point is shown analytically and quantitatively. With a moderated love for variety, the elasticity around 7, which is in the range of microestimation in trade literature, can provide quantitatively a realistic BS correlation which is close to 0 or negative. This is in sharp contrast to CDL who discuss the importance of a very low elasticity of substitution for the resolution of the puzzle.

We also analyze the role of market incompleteness to obtain our results. With complete markets, the introduction of extensive margin is able to reproduce the observed 0 or negative consumption-real exchange rate correlation. Due to risk sharing, it is not usually optimal when a positive productivity shock occurs to experience a terms of trade appreciation. Symmetrically, the model

[3] Empirical validity of the terms of trade appreciation following the expansion of trade in terms of extensive margin is recently documented for example in Hummels and Klenow (2005) and Galstyan and Lane (2008).

without extensive margin but with incomplete markets is unable to reproduce the observed BS correlation. We show that the combination of market incompleteness and extensive margin can reproduce this correlation, because, as explained above, the extensive margin means that a productivity shock generates a terms of trade appreciation. The principal discussion in this paper centers around supposing incomplete asset markets. However, for the purpose of the comparison both static and dynamic versions of the theoretical model and their implications with the BS puzzle under complete asset markets are presented in detail in the appendix.

The remaining of the paper is organized as follows. In the next section a general equilibrium model with endogenous entry but without investment dynamics is presented and the implication for the BS puzzle is discussed analytically. In the third section the investment dynamics are introduced and the mechanics of the analytical part is explored quantitatively. In particular we report the impulse response functions and the second moments of the theoretical model and perform the sensitivity analysis about the relating variables with the BS puzzles.

2 The static model with endogenous variety

A very simple two-country general equilibrium model which include endogenous variation of extensive margins is presented. There is no international lending or borrowing, hence trade is balanced. Investment takes place in the form of creation of new firms (varieties). However, at this point there is no investment dynamic in this first part of the paper. The variables are solved only for those steady state variations and especially for these between Home and Foreign. This is because it is enough for our purpose.

2.1 Households

Only Home representative agent and variables are discussed. Foreign variables are expressed with *. In each country there is 1 unit mass of population where the representative household supplies inelastically 1 unit of labor. The utility depends only on the consumption:

$$U = \frac{C^{1-\gamma}}{1-\gamma}, \tag{1}$$

where β is the discount factor. $\gamma \geq 1$ is the relative risk aversion. The consumption basket is defined as:

$$C = \left[\alpha^{\frac{1}{\omega}} C_H^{1-\frac{1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} C_F^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}}, \tag{2}$$

and

$$C_H = V_H \left[\int_0^N c_t(h)^{1-\frac{1}{\sigma}} dh \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_F = V_F^* \left[\int_0^{N^*} c(f)^{1-\frac{1}{\sigma}} df \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{3}$$

where $V_H \equiv N^{\psi - \frac{1}{\sigma-1}}$ and $V_F^* \equiv N^{*\psi - \frac{1}{\sigma-1}}$. ψ (≥ 0) represents the marginal increase of utility with one additional variety in each basket. This specification follows Benassy (1996) which claims the disconnection between firms' markup (the elasticity of substitution among varieties, σ) and the love for variety. The preference becomes Dixit-Stiglitz when $\psi = \frac{1}{\sigma-1}$. When $\psi = 0$ there is no love for variety. In such a case consumers are just satisfied by intensive margins (quantity of goods). α ($\geq 1/2$) is home bias in consumption. ω is the elasticity of substitution between Home and Foreign goods. σ is the elasticity of substitution among varieties. I suppose $\sigma \geq \omega$. $c(h)$ ($c(f)$) is the demand for individual Home (Foreign) variety. N (N^*) is the number of Home (Foreign) varieties. The corresponding price indices are

$$P = \left[\alpha P_H^{1-\omega} + (1-\alpha) P_F^{1-\omega} \right]^{\frac{1}{1-\omega}}, \tag{4}$$

and

$$P_H = \frac{1}{V_H} \left[\int_0^N p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F = \frac{1}{V_F} \left[\int_0^{N^*} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}. \tag{5}$$

Observe that the price index of each Home and Foreign goods decreases with an increase in the number of varieties. The deflation is larger the higher the love for variety, ψ. The similar expression holds for Foreign.

2.1.1 First order conditions

Optimal consumption of Home and Foreign goods by Home household is shown respectively as below:

$$C_H = \alpha \left(\frac{P_H}{P} \right)^{-\omega} C, \qquad C_F = (1-\alpha) \left(\frac{P_F}{P} \right)^{-\omega} C, \tag{6}$$

and using symmetry among varieties optimal consumption for each individual firm is given by

$$c_h = V_H^{\sigma-1} \left(\frac{p_h}{P_H} \right)^{-\sigma} C_H, \qquad c_f = V_F^{\sigma-1} \left(\frac{p_f}{P_F} \right)^{-\sigma} C_F. \tag{7}$$

For notational convenience we define the relative price in terms of local consumption basket:

$$\rho_H = \frac{P_H}{P}, \qquad \rho_F = \frac{P_F}{P}, \qquad \rho_h = \frac{p_h}{P}, \qquad \rho_{f.} = \frac{p_f}{P},$$

$$\rho_H^* = \frac{P_H^*}{P^*}, \qquad \rho_F^* = \frac{P_F^*}{P^*}, \qquad \rho_h^* = \frac{p_h^*}{P^*}, \qquad \rho_f^* = \frac{p_f^*}{P^*}.$$

2.2 Firms

2.2.1 Entry cost

Prior to entry entrants must pay a sunk entry cost in terms of f_E units of effective labor:

$$f_E = z_E l_{EM}, \tag{8}$$

where f_E is considered as an exogenous variable which represents "deregulation" as discussed in Ghironi and Melitz (2005). z_E is the labor productivity required to set up the firm. As it will be clear we distinguish this productivity from that required for the goods production. In reality they are supposed to be correlated to some extent, l_{EM} is the labor demand for firm set up activity.

2.2.2 Production

For goods production only labor is used as input and its technology is linear:

$$y_h = zl, \tag{9}$$

where z is the labor productivity for intensive margins. The operational real profits (dividends) are expressed by

$$d_h = \left(\rho_h - \frac{w}{z}\right) y_h. \tag{10}$$

Goods market clearing implies

$$y_h = c_h + c_h^*, \tag{11}$$

where c_h (c_h^*) is the consumption demand from Home (Foreign) households. Thus using the optimal demands found in the previous section, y_h can be rewritten as

$$y_h = N^{\psi(\sigma-1)-1} \rho_h^{-\sigma} \rho_H^{\sigma-\omega} \left[\alpha C + (1-\alpha)\, Q^\omega C^*\right]. \tag{12}$$

Profits maximization gives standard pricing:

$$\rho_h = \frac{\sigma}{\sigma - 1} \frac{w}{z}. \tag{13}$$

Individual real price is the real marginal cost over markup. Because there is no transportation cost, the LOP holds for exported goods whose price is ρ_h^*, denominated in Foreign consumption basket:

$$\rho_h^* = Q^{-1}\rho_h, \tag{14}$$

where Q is the real exchange rate defined as the price of Foreign consumption in terms of Home:

$$Q = \frac{P^*}{P}. \tag{15}$$

Finally using the above optimal pricing and the fact that $\rho_H = N^{-\psi}\rho_h$ from the symmetry the real dividends are expressed by:

$$d_h = \frac{1}{\sigma}\rho_h^{1-\omega}N^{\psi(\omega-1)-1}\left[\alpha C + (1-\alpha)\, Q^{\omega}C^*\right]. \tag{16}$$

This form of presentation allows us to clearly see what causes the variation of real dividends. A lower price of individual variety, ρ_h, increases the dividends when the elasticity of substitution between local and imported goods is higher than unity ($\omega > 1$). In addition to this term, $N^{\psi(\omega-1)-1}$ represents an additional competing effect from a rise in the domestic number of varieties, N. When $\psi > \frac{1}{\omega-1}$ an increase of Home originated firm gives further real dividends while when $\psi < \frac{1}{\omega-1}$, the dividends decrease for the same rise in the number of local firms. High love for variety, ψ, or high elasticity of substitution, ω, tend to further increase the dividends. When it is low the additional competition effect among local varieties dominates from this second term. In particular when $\omega = \sigma$ and $\psi = \frac{1}{\sigma-1}$, the preference becomes Dixit-Stiglitz where the above two effects offset each other.

2.3 Free entry, labor market clearing and balanced trade condition

In this static version of the model there is no investment choice. We define the real investment for each firm creation with x_h and suppose all dividends (which are evenly held by Home households) are used to finance the investment (which are also evenly held by them). This implies $Nd_h = Nx_h$. At the equilibrium the investment (firm's value) is equal to the entry cost giving the following free entry condition:

$$x_h = \frac{f_E w}{z_E}. \tag{17}$$

On the other hand in general equilibrium the labor market clears. Exogenously supplied labor is used in the production (intensive margin) and in the firm creation. Home labor market clearing condition is given by

$$1 = Nl + Nl_{EM}, \tag{18}$$

and noting $y_h = (\sigma - 1) \frac{d_{ht}}{w} z$ and $l_{EM} = \frac{x_h}{w}$, the above condition can be written as

$$1 = (\sigma - 1) \frac{N d_h}{w} + \frac{N x_h}{w}. \tag{19}$$

The similar expression holds for Foreign.

Finally we suppose that trade is balanced. In terms of real this condition is

$$N \rho_h c_h^* = Q N^* \rho_f^* c_f. \tag{20}$$

Rewriting with optimal demands found in previous section, the balanced trade condition becomes

$$Q^{2\omega-1} N^{\psi(\omega-1)} \rho_h^{1-\omega} C^* = N^{*\psi(\omega-1)} \rho_f^{*1-\omega} C. \tag{21}$$

In what follows we solve the log linearized version of the model for the relative variables between Home and Foreign.

2.4 Solution for linearized relative variables

2.4.1 Real exchange rate, relative dividends and relative wage

We express the log-deviation from its steady state value with sans-serif font. Before examining the solution for relative variables, we discuss the behavior of real exchange rate and relative dividends. With endogenous variety, the real exchange rate variation is given by

$$\mathsf{Q} = (2\alpha - 1) \rho^R + \psi (2\alpha - 1) \mathsf{N}^R, \tag{22}$$

where ρ_t^R is the variation of the terms of trade and N^R is the variation of the relative number of varieties between Home and Foreign:

$$\rho^R = \left(\varepsilon + \mathsf{p}_f^*\right) - \mathsf{p}_h,$$

and

$$\mathsf{N}^R = \mathsf{N} - \mathsf{N}^*.$$

As usual, without home bias there is no variation of real exchange rate ($\alpha = \frac{1}{2}$). With home bias, the terms of trade appreciation (a decrease of ρ_t^R) also bring the real exchange rate into a state of appreciation. In addition, here the variation of the real exchange rate depends on that of the relative number of varieties, N^R. An increase of N^R makes the real exchange rate depreciated with the elasticity $\psi (2\alpha - 1)$, home bias weighted by the love for variety. Very naturally without love for variety ($\psi = 0$) this term disappears.

The log-linearized balanced trade condition yields:

$$\mathsf{C} - (\mathsf{Q} + \mathsf{C}^*) = (\omega - 1) \left[\rho^R + \psi \mathsf{N}^R + \mathsf{Q}\right], \tag{23}$$

and using the above relationship the relative dividends are expressed as (see appendix in detail):

$$\mathsf{d}^R = 2\alpha\left(\omega-1\right)\rho^R + \left[2\alpha\psi\left(\omega-1\right)-1\right]\mathsf{N}^R. \tag{24}$$

For 1% depreciation of terms of trade the relative dividends increase (decrease) $2\alpha\left(\omega-1\right)$ % under $\omega > 1$ ($\omega < 1$). The dividends rise or decrease with a rise of the number of Home originated firms depending on whether $2\alpha\psi\left(\omega-1\right)$ exceeds unity or not. At this point what is important is the love for variety (ψ) and the elasticity of substitution (ω). Combination of a low love for variety and low elasticity of substitution tend to give a negative impact on dividends with $\mathsf{N}^R > 0$. Intuitively this is because a strong competition effect takes place as the number of firms increases under such low love and elasticity. Without love for variety ($\psi = 0$) a rise in the number of varieties makes the dividends decrease and the competition effect dominates.

2.4.2 Relative wage and the relative number of varieties

Next we solve the model for the log-linearized relative variations from the two general equilibrium conditions discussed in the previous section, namely the labor market clearing and free entry conditions. The relative labor market clearing condition is

$$S_W \mathsf{w}^R = \left(\sigma-1\right) S_D \left(\mathsf{N}^R + \mathsf{d}^R\right) + S_I \left(\mathsf{N}^R + \mathsf{x}^R\right), \tag{25}$$

where S_W, S_D and S_I are respectively the steady state wage, dividends and investment relative to the steady state consumption. Because all dividends are just used for investment, the steady state total dividends and investment are equal and all wage income is used for consumption:

$$S_I \equiv \frac{N d_h}{C} = S_D \equiv \frac{N x_h}{C} = \frac{1}{\sigma} \text{ and } S_W \equiv \frac{w}{C} = 1. \tag{26}$$

Supposing $f_E^R = 0$ (no deregulation shock) and noting again $\mathsf{d}^R = x^R$, the free entry condition is expressed in log-deviation by

$$\mathsf{d}^R = \mathsf{w}^R - z_E^R. \tag{27}$$

Plugging d^R found in the previous section and noting that the terms of trade variation reflect the variation of wage and productivity such as $\rho^R = -\left(\mathsf{w}^R - z^R\right)$, the above two general equilibrium conditions determine w^R and N^R. These are:

$$\mathsf{w}^R = \frac{2\alpha\left(\omega-1\right)}{1+2\alpha\left(\omega-1\right)}\mathsf{z}^R + \psi\frac{2\alpha\left(\omega-1\right)}{1+2\alpha\left(\omega-1\right)}\mathsf{z}_E^R, \tag{28}$$

$$\mathsf{N}^R = \mathsf{z}_E^R. \tag{29}$$

[4]The solutions are very simple. The relative number of varieties N^R moves one by one to the labor productivity shock which improves the firm setting up efficiency z_E^R and no variation arises with that improving the productivity of intensive margins z^R. Contrary to this the relative wage varies with both shocks. When $0 < \omega < 1 - \frac{1}{2\alpha}$ the wage becomes appreciated, when $1 - \frac{1}{2\alpha} < \omega < 1$ the wage becomes depreciated and under $1 < \omega$ wage appreciates for both $\mathsf{z}^R > 0$ and $\mathsf{z}_E^R > 0$. Note especially that without love for variety ($\psi = 0$) the latter shock has no impact on relative wage. Put another way, a higher number of varieties due to the firm creation shock ($\mathsf{z}_E^R > 0$) adds a further wage appreciation.

Given the above solutions the variation of the terms of trade is given by:

$$\rho^R = \frac{1}{1 + 2\alpha(\omega - 1)} \mathsf{z}^R - \psi \frac{2\alpha(\omega - 1)}{1 + 2\alpha(\omega - 1)} \mathsf{z}_E^R. \tag{31}$$

The first term is the same term discussed in CDL where they argue for the possibility of the terms of trade appreciation ($\rho^R < 0$) following a positive productivity shock ($\mathsf{z}^R > 0$) with very low elasticity of substitution, $0 < \omega < 1 - \frac{1}{2\alpha}$. With high elasticity $1 - \frac{1}{2\alpha} < \omega$ terms of trade depreciates from this first term. However, supposing the two shocks are correlated in some extent, with love for variety ($\psi \neq 0$) there is the second term which counteracts and prevents the terms of trade from depreciating in the case of high elasticity. The higher the love for variety and the elasticity of substitution are, the higher this appreciation from the second term becomes. Intuitively when love for variety and the elasticity of substitution are high, the same rise in the number of varieties is further demanded and makes the wage further appreciated. This cost appreciation works to counteract the terms of depreciation due to more efficient production technology ($\mathsf{z}^R > 0$). This is in essence exactly the point discussed in Krugman (1989) . In Figure 1 we give a numerical example about the elasticity of terms of trade for different values of ω for both with and without love for variety ($\psi = 0.2$ and $\psi = 0$). In particular, the two shocks are supposed to be correlated perfectly, $\mathsf{z}^R = \mathsf{z}_E^R$. Home bias in consumption α is set to 0.72.[5]

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[4]There are the "hidden" restrictions on parameters so that the number of varieties in one country doesn't increase infinitely. This kind of thing happens if dividends increase more than the wage appreciation in that country following an increase in the number of firms. To avoid this it must be

$$1 > \psi \frac{4\alpha^2(\omega - 1)^2}{1 + 2\alpha(\omega - 1)}. \tag{30}$$

The above conditon may be broken down under very high love for variety and elasticity of substitution.

[5]Figure 1 (and Figure 2) satisfies the parameter restriction, (30).

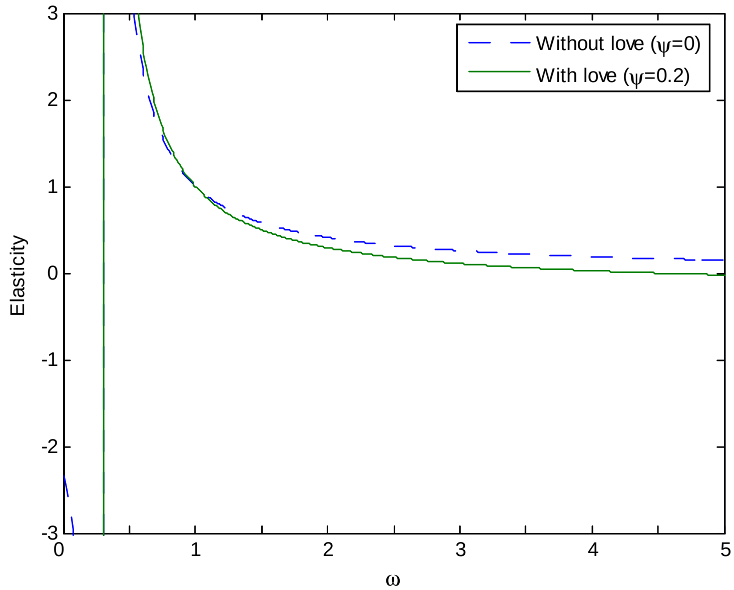

Figure 1: Terms of trade

Using the result found here we discuss next finally the implication with the BS puzzle.

2.4.3 Implication for the BS puzzle

Using the expression of real exchange rate the balanced trade condition can be rewritten as

$$\mathsf{Q} = \frac{2\alpha - 1}{2\alpha\omega - 1} \left(\mathsf{C} - \mathsf{C}^{*} \right). \tag{32}$$

This is exactly the same expression found in CDL. Home consume more when their basket becomes relatively cheaper under sufficiently high elasticity of substitution, $\omega > \frac{1}{2\alpha}$. With home bias in consumption ($\alpha > 1/2$), this is roughly $\omega > 1$. The only way to generate a negative BS relation is to suppose a very low elasticity as CDL.[6] However, the BS puzzle is the puzzle about the empirically observed real exchange rate and consumption. Contrast to the "welfare-based" such empirically observed real exchange rate doesn't contain

[6] It is worth noting that when $\alpha = 1/2$ and $\omega = 1$ it is possible to achieve the complete markets allocation without any financial assets. This is the case of Cole and Obstfeld (1991). The movement of the terms of trade transmits the productivity gains so that the relative consumption (empirical as well as welfare based) remains unchanged.

(or poorly measures) the variation of the number of varieties. For simplicity we suppose that the price indices don't reflect any variation of extensive margins. Noting such variation with $\widetilde{}$, the welfare based real exchange rate is broken down into two parts:

$$\mathsf{Q} = \widetilde{\mathsf{Q}} + \psi\,(2\alpha - 1)\,\mathsf{N}^R. \tag{33}$$

The consumptions are also badlly measured using such price indices. For example, total expenditure on consumption in Home is given by PC. Statistical agency divides this amount by $\widetilde{P}$ to measure the "consumption" $(\widetilde{C})$. So the variations of empirical based consumptions are

$$\widetilde{\mathsf{C}} \equiv \mathsf{P} + \mathsf{C} - \widetilde{\mathsf{P}} = \mathsf{C} - \psi\left[\alpha\mathsf{N} + (1-\alpha)\,\mathsf{N}^*\right]. \tag{34}$$

Thus the welfare based relative consumptions are also broken down into two parts those in terms of intensive $(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*)$ and extensive margins.$(\psi\,(2\alpha-1)\,\mathsf{N}^R)$:

$$\mathsf{C} - \mathsf{C}^* = \widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* + \psi\,(2\alpha - 1)\,\mathsf{N}^R. \tag{35}$$

Plugging (33) and (35) in (32) it yields

$$\widetilde{\mathsf{Q}} = \frac{2\alpha - 1}{2\alpha\omega - 1}\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*\right) - \psi\frac{2\alpha\,(2\alpha-1)\,(\omega-1)}{2\alpha\omega - 1}\mathsf{N}^R \tag{36}$$

The tight relationship which holds together the welfare based real exchange rate and consumptions is now broken. And it is possible to have a negative BS relation between $\widetilde{\mathsf{Q}}$ and $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*$ without relying on low elasticity.[7]

In particular, using the solution of terms of trade (31) and supposing $\mathsf{z}^R = \mathsf{z}_E^R$ the number of varieties is expressed in terms of $\widetilde{\mathsf{Q}}$.[8] Plugging this relationship in (36) the BS relation becomes:

$$\widetilde{\mathsf{Q}} = \frac{(2\alpha-1)\left[1 - 2\psi\alpha\,(\omega-1)\right]}{2\alpha\,(\omega-1)\left[1 + 2\psi\,(1-\alpha)\right] + 2\alpha - 1}\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*\right). \tag{38}$$

In addition to the very low range of elasticity of substitution ($0 < \omega < 1 - \frac{2\alpha-1}{2\alpha[1+2\psi(1-\alpha)]}$), for high enough elasticity ($\omega > 1 + \frac{1}{2\psi\alpha}$) the BS relation becomes negative. For very low elasticity the intuition is almost the same as discussed in CDL. When Home is hit by a positive productivity shock, Home provides more goods in terms of intensive as well as extensive margins. However, with a very low elasticity of substitution the condition of goods markets clearing requires the appreciation of terms of trade (expensive Home goods), therefore providing higher purchasing power for Home agents to absorb their own production.

For the same shock when the elasticity of substitution is high because of the demand addressed to these new varieties under love for variety, Home wage

[7] In the appendix we investigate the empirical validity of the above relationship in a crude way.

[8]
$$\mathsf{N}^R = \frac{1 + 2\alpha\,(\omega-1)}{2\alpha - 1}\,\frac{1}{1 - 2\psi\alpha\,(\omega-1)}\widetilde{\mathsf{Q}} \tag{37}$$

appreciates further. Counteracting the standard depreciation force due to higher productivity, the terms of trade tend to be appreciated bringing the observed real exchange rate also into appreciation. Of course at the same time Home consumption rises relative to Foreign, given by this high wealth from the terms of trade appreciation.[9] From this a even negative or close to 0 BS relation appears. Note that without love for variety ($\psi = 0$) the expression collapses to (32). In Figure 2 I give a numerical example with the same parameters in the previous subsection.

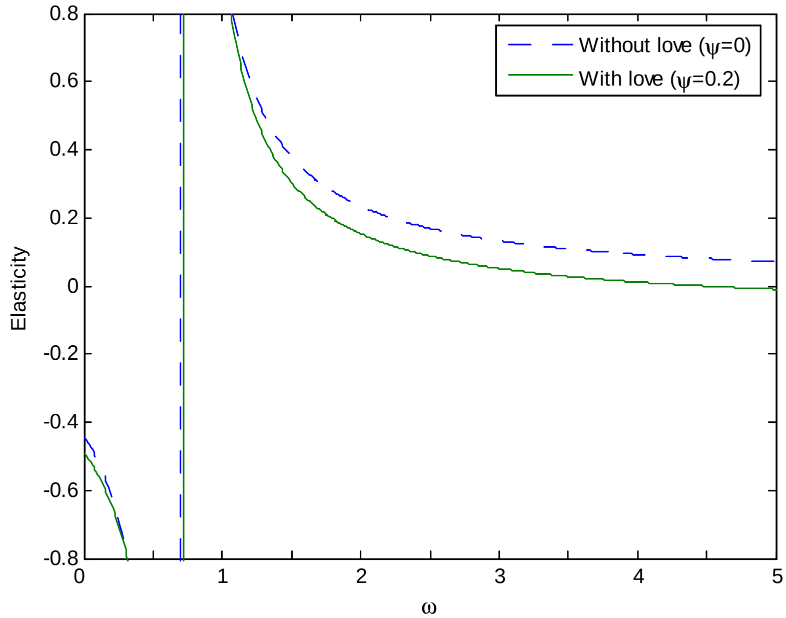

Figure 2: BS correlation

In the next section these intuitions are shown quantitatively introducing the dynamic.

3 Introducing the dynamic and more realistic extensions

The large part of the model is almost identical to the static version. Now all variables have the time index t. Investment dynamic(in terms of new firm

[9] To be precise Home consumption rises relative to Foreign in welfare based (including extensive margins) as well as in empirical based (only intensive margins) when terms of trade appreciate. See also the result of calibration of the second part of the paper.

creations, thus $N_{E.t}x_{h.t}$ where $N_{E.t}$ is the number of new entrants) is introduced following Ghironi and Melitz (2005). The number of existing firms in current period N_t behaves exactly as "state variable" as capital accumulation process in the standard RBC model. The investment is financed by the purchase of the "mutual found" of all existing firms including new entrants. Other than dynamic we add three more realistic extensions compared to the very simplified static model: endogenous labor supply, internationally held non-contingent bonds and the entry cost paid in terms of labor as well as capital goods. Below only these modified points are discussed.

3.1 Households

The Home representative household maximizes $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t$ at period t. The instantaneous utility now depends on consumption and labor supply as following:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \tag{39}$$

In the above expression γ (≥ 1) is the elasticity of marginal utility of consumption (relative risk aversion), χ (> 0) represents the degree of non satisfaction from supplying labor, L_t, φ is the Frisch elasticity of labor supply[10]. With this specification the marginal disutility in providing one additional labor is increasing. The structure of the consumption basket C_t is identical.

3.2 Budget constraint with non-contingent bonds

The Home representative households's budget constraint including non-contingent bonds is identical to that discussed in Ghironi and Melitz (2005). This is given by

$$\begin{aligned} &B_{t+1} + Q_t B_{*.t+1} + \frac{\vartheta}{2} B_{t+1}^2 + \frac{\vartheta}{2} Q_t B_{*.t+1}^2 + s_{h.t+1} \left(N_t + N_{E.t} \right) x_{h.t} + C_t \\ &\quad = \left(1 + r_t \right) B_t + Q_t \left(1 + r_t^* \right) B_{*.t} + s_{h.t} N_t \left(d_{h.t} + x_{h.t} \right) + T_t^f + w_t L_t, \end{aligned} \tag{40}$$

where B_{t+1} ($B_{*.t+1}$) is the non-contingent real bond holding into t+1 denominated in Home (Foreign) consumption basket. Q_t is the real exchange rate defined as the price of Foreign consumption in terms of Home. To ensure the 0 bond holding at the steady state, the quadratic cost for bond holdings captured by the parameter, ϑ, is introduced. T_t^f is the free rebate of that cost which is exogenous for households. r_t (r_t^*) is the real interest rate for holding Home (Foreign) bonds. Investment takes place only domestically, $s_{h.t+1}$ is the share holding into t+1. $x_{h.t}$ is the real share price of Home mutual fund. As it will be discussed later, because the exogenous "death shock" hits at the very end

[10] With $\varphi = \infty$ the marginal disutility of supplying labor becomes constant, χ. When $\varphi = 0$ the marginal disutility becomes infinite and the labor supply becomes inelastic.

of the period and the household does not know which firms are going to die, she finances all existing firms, N_t, including new entrants, $N_{E.t}$. $d_{h.t}$ is the real dividends. w_t is the real wage and L_t is the labor supply.

Optimal consumptions are similar to the previous case. Beside the representative household decides also its optimal share and bond holdings and labor supply. Using the equation which will discussed later about the motion of firms the Euler equation about share holding becomes:

$$x_{h.t} = \beta (1-\delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (x_{h.t+1} + d_{h.t+1}), \tag{41}$$

and the Eular equations for Home and Foreign bonds are respectively given by:

$$C_t^{-\gamma} (1 + \vartheta B_{t+1}) = \beta (1 + r_{t+1}) E_t C_{t+1}^{-\gamma}, \tag{42}$$

$$C_t^{-\gamma} (1 + \vartheta B_{*.t+1}) = \beta \left(1 + r_{t+1}^*\right) E_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma}. \tag{43}$$

Finally, optimal labor supply decision becomes:

$$\chi (L_t)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}. \tag{44}$$

Similar conditions hold for Foreign.

3.3 Firms

3.3.1 Entry cost

Different from the previous specification, new entrants are supposed to need labor as well as capital goods to set up. This type of specification is proposed in Billbie, Ghironi and Melitz (2007a, 2007b). One firm/variety creation needs an amount of firms setting up goods, f_E. The production of such goods is done by the following Cobb-Douglas technology using labor $l_{EM.t}$ and capital goods K_t as inputs:

$$f_E = \left(\frac{z_t l_{EM.t}}{\theta} \right)^{\theta} \left(\frac{K_t}{1-\theta} \right)^{1-\theta}, \tag{45}$$

where θ $(1-\theta)$ is the share of labor (capital) cost in total cost for firm setup. For simplicity we suppose that capital goods K_t has the same composition as the consumption goods. The cost minimization problem yields the following factor demand,

$$l_{EM.t} = \frac{\theta}{w_t} \mu_t f_E, \qquad K_t = (1-\theta) \mu_t f_E, \tag{46}$$

where $\mu_t = \left(\frac{w_t}{z_t} \right)^{\theta}$ is the real cost of firm set up. In particular when $\theta = 1$ only labor is used for entry as in the previous specification. The similar conditions hold for Foreign.

3.3.2 Motion of firms

The number of firms, N_t behaves as a state variable in this model as follows:

$$N_t = (1 - \delta)(N_{t-1} + N_{Et-1}), \tag{47}$$

where δ is the depreciation of the number of firms embodied in the economy. This "death shock" takes place at the very end of the period after investment has finished (financed by Home mutual fund). The production takes place only one period after entry. New entrants need this one period of "time-to build". The similar conditions hold for Foreign.

3.3.3 Production

The production of intensive margins is almost identical to the previous static model. However, with the demand for capital goods required for entry the demand addressed for each firm now includes these terms:

$$y_{h.t} = c_{h.t} + c^*_{h.t} + N_{E.t}k_{h.t} + N^*_{E.t}k^*_{h.t}, \tag{48}$$

where $k_{h.t}$ ($k^*_{h.t}$) is the capital demand from Home (Foreign) new entrants. The dividends are written as

$$d_{h.t} = \frac{1}{\sigma}\rho_{h.t}^{1-\omega}N_t^{\psi(\omega-1)-1}\left[\alpha M_t + (1-\alpha)Q_t^{\omega}M_t^*\right], \tag{49}$$

where M_t (M_t^*) is the consumption and investment goods demand in each country:

$$M_t = C_t + N_{E.t}K_t \text{ and } M_t^* = C_t^* + N^*_{E.t}K_t^*. \tag{50}$$

Note especially, using factor demand (46), when $\theta = 1$ (only labor is used as input) this is similar to the previous specification. The same expressions hold for Foreign.

3.4 Free entry and labor market clearing conditions

At the equilibrium the real share price (discounted some of dividends) is equal to the cost of entry. The free entry condition holds in the following form,

$$x_{h.t} = f_E\left(\frac{w_t}{z_t}\right)^{\theta}. \tag{51}$$

Endogenously supplied labor is used in the production (intensive margin) and in the firm creation. Home labor market clearing condition gives:

$$L_t = N_t l_t + N_{E:t}l_{EM.t}, \tag{52}$$

noting $y_{h.t} = (\sigma-1)\frac{d_{h.t}}{w_t}z_t$ and $l_{EM.t} = \theta\frac{x_{h.t}}{wt}$, the above condition can be written as,

$$L_t = (\sigma - 1)\frac{N_t d_{h.t}}{w_t} + \theta \frac{N_{E.t} x_{h.t}}{w_t}. \tag{53}$$

Note again with $\theta = 1$ only labor is used for the creation of extensive margins. The parameter θ governs the "redistribution" between labor and financial income. The similar expression holds for Foreign.

3.5 Bond market clearings and net foreign asset

Bond market clearings conditions for both Home and Foreign bonds are:

$$B_{t+1} + B^*_{t+1} = 0, \qquad B_{*.t+1} + B^*_{*.t+1} = 0. \tag{54}$$

Finally, because the world as a whole is a closed economy the following net foreign asset evolutions holds:

$$\begin{aligned} B_{t+1} + Q_t B_{*.t+1} &= (1 + r_t) B_t + Q_t (1 + r^*_t) B_{*.t} \\ &\quad + \frac{1}{2}\left[L_t w_t + N_t d_{h.t} - Q_t \left(L^*_t w^*_t + N^*_t d^*_{f.t}\right)\right] \\ &\quad - \frac{1}{2}\left[N_{E.t} x_t + C_t - Q_t \left(N^*_{E.t} x^*_t + C^*_t\right)\right]. \end{aligned} \tag{55}$$

The above expression says that Home net foreign asset accumulation is expressed as returns on bonds plus the difference of net export (or another way to put it, that of income and spending) between Home and Foreign.[11]

Finally under incomplete markets the model contains 31 equations and 31 variables in which 8 are endogenous state variables ($N_t, N^*_t, B_t, B^*_t, B_{*.t}, B^*_{*.t}, r_t$ and r^*_t) and 2 are exogenous shocks (z_t and z^*_t). For the full set of equations and the detail about the steady state see the appendix. In what follows we calibrate the linearized version of the model and explore quantitatively the mechanism in generating a realistic BS correlation.

4 Calibration

The model is calibrated by the parameters in Table 1.

[11] Aggregation implies the following net foreign assets accumulation for each country:

$$B_{t+1} + Q_t B_{*.t+1} = (1 + r_t) B_t + Q_t (1 + r^*_t) B_{*.t} + L_t w_t + N_t d_{h.t} - N_{E.t} x_t - C_t. \tag{56}$$

$$\frac{B^*_{t+1}}{Q_t} + B^*_{*.t+1} = \frac{(1 + r_t) B^*_t}{Q_t} + (1 + r^*_t) B^*_{*.t} + L^*_t w^*_t + N^*_t d^*_{f.t} - N^*_{E.t} x^*_t - C^*_t. \tag{57}$$

The above two equations (eliminating bonds position by Foreign using bond market clearings (54)) yields (55).

Baseline parametrization

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	7
ω	between Home and Foreign goods	6
α	home bias in consumption	0.72
δ	death shock	0.025
ϑ	bonds holding cost	0.0025
θ	share of labor in entry cost	0.64
ψ	love for variety	0.2

Table 1

Constant risk aversion (γ), discount factor (β), Frisch elasticity of labor supply (φ) come from Bilbiie, Ghironi and Melitz (2007) who choose them based on the standard RBC literature. The death shock (δ) comes from Ghironi and Melitz (2005) where it is chosen to match the U.S. empirical level of 10 percent job destruction per year. The bond holding cost (ϑ) is set also following them.[12] The elasticity of substitution between Home and Foreign goods (ω) is set to 6. Given this value, the elasticity of substitution among varieties (σ) is set to 7. This choice of elasticities may be considered too high compared to the low value used in the literature. However, this is well in the rage of microestimate in trade literature. Recently Imbs and Méjan (2009) argue the conventional estimation about the elasticity of substitution has downward bias without considering the heterogeneity of these values among sectors. They propose the elasticity ω around 7. In generating the second moments of the theoretical model for the purposes of comparison we also consider the low elasticity $\omega = 2$ which is used as benchmark elasticity in the Benigno and Thoenissen (2008). With such a low elasticity following Ghironi and Melitz (2005) we set $\sigma = 3.8$. A great ambiguity surrounds the parameter ψ, the love for variety. We set it at 0.2 arbitrary. This is higher than for the implied love under Dixit-Stiglitz preference of benchmark elasticity (1/(7-1)=0.17) and lower for $\sigma = 3.8$ (1/(3.8-1)=0.36). Home bias in consumption (α) is taken from Corsetti, Dedola and Leduc (2008), the value ranged in that used in recent literature. We choose the share of labor (θ) in entry cost as 0.64 from Heathcote and Perri (2002). Again this is the standard value in the model including capital and labor in its production function.

The productivity process is taken from Backus, Kehoe and Kydland (1992) such as $\mathsf{Z}_{t+1}= \Omega \mathsf{Z}_t + \xi_t$ where $\mathsf{Z}_t = \begin{bmatrix} z_t, & z_t^* \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi_t^* \end{bmatrix}'$, and the correlation of the shock and error terms are given by

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}, \text{ and } V(\xi) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix}. \tag{58}$$

[12] % deviation of bond positions are defined relative to the steady state consumption C in the linearized version of the model.

4.1 Intuition by impulse response functions

To see the intuition of the quantitative model the impulse response functions for the real exchange rate, terms of trade and relative consumptions are reported in Figure 3. For real exchange rate and consumptions both welfare and empirical based are shown. The shock is 1% increase in Home productivity. For simplicity we omit the spillover term in the correlation matrix (58) in this exercise. On impact of the shock new entry takes place in Home while firms exit in Foreign. The relative number of varieties increases steadily and over time the terms of trade appreciates for Home in spite of the persistent efficiency gains due to the positive productivity shock. The reason is identical to the one explained in the first part of the paper. High elasticity of substitution and a mild love for variety of baseline parameters are sufficient to change the terms of trade from a depreciation to an appreciation (dotted lines on upper and lower panels). Reflecting these terms of trade movement, the empirical based real exchange rate appreciates also (solid line on upper panel). However, the welfare based real exchange rate depreciates reflecting a higher number of Home originated varieties (solid line on lower panel).

So what happens to the consumptions? It is well known that with non-contingent bonds the BS relation holds only in first difference. Home and Foreign agents try to stabilize their consumption using such bonds in the aftermath of a shock. As a result Home and Foreign consumption tend to diverge more on impact compared to the complete markets case (crossed line on lower panel. See Figure A-1 in the appendix for impulse response functions in complete markets). However, in spite of the divergence in relative consumptions due to non-contingent bonds it is easy to see that the relative consumptions and real exchange rate remain positively correlated in welfare based(crossed and solid line on lower panel). This suggest that market incompleteness itself is not possible to create an observed BS correlation in our model and parametrization. Contrary to this, with empirical based they are negatively correlated or their correlation is close to 0 (crossed and solid lines on upper panel). Consumption in Home increases relative to Foreign in terms of intensive as well as extensive margins given by higher wealth due to the terms of trade appreciation. As has been already explained the empirical based real exchange rate only reflects this terms of trade appreciation the observed BS correlation is negative or close to 0.

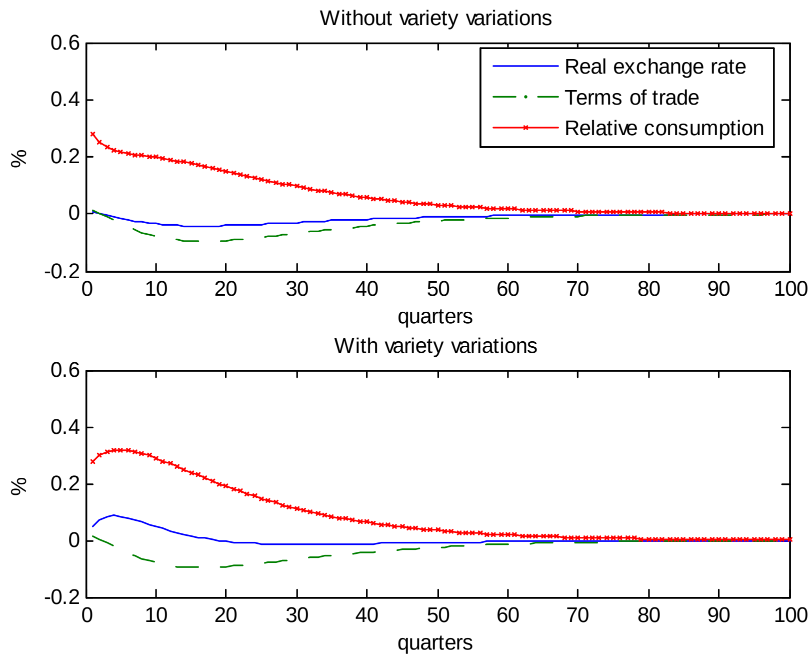

Figure 3: IRF under Incomplete markets.

4.2 Characteristics of the theoretical model

The characteristics of the theoretical model with baseline parameters are documented here in Table 2-1 and Table 2-2. The US data comes from either Backus, Kehoe and Kydland (1992) or Heathcote and Perri (2002) except for the BS correlation (-0.27) drawn from CDL, the medium of OECD countries relative to the ROW. The moments of the theoretical model are calculated using the frequency domain technique presented in Uhlig (1999) for HP filtered series. The smoothing parameter is set to 1600. All the variables are "empirical" based. They do not contain any variation in the number of varieties for simplicity. GDP in this model is $\widetilde{Y}_t = w_t L_t + N_t \widetilde{d}_{h.t}$ (labor +financial income) The investment is $N_E \widetilde{x}_h$, the share price multiplied by the number of new entrants. The net export (trade balance) is defined as $\widetilde{TB}_t = \left(\widetilde{X}_t - \widetilde{IM}_t\right) / \widetilde{Y}_t$ where $\widetilde{X}_t$ and $\widetilde{IM}_t$ are the export and import. For the purpose of comparison we report the moments with low elasticity ($\sigma = 3.8$ and $\omega = 2$) and under complete markets as well.

At a glance it is noticed that the model shares the principle (bad) characteristics of the standard two-country real business cycle model, such discussed in Heathcote and Perri (2002): lower volatility in terms of trade and real exchange rate, higher cross country correlation of consumption than output and negative cross country correlation of investment and employment. Under incomplete markets with benchmark parameters, compared to the data, high investment volatility (15.25) and strong negative international cross correlation of investment (-0.91) appear. This high endogenous entry (investment) is a natural consequence of high elasticity of substitution between Home and Foreign goods under love for variety. Intuitively when the elasticity is high with some love the increase of dividends (hence the share price of firms) attracts more entry in spite of the wage (cost) appreciation in that country (which happens also due to this endogenous entry)[13]. Because firm creation requires labor force as a natural result employment is correlated negatively (-0.91) as well as output (-0.52). However, on the other hand this high elasticity is what provides a realistic BS correlation (0.17) along the mechanism discussed in the previous sections. With alternative elasticity ($\sigma = 3.8$ and $\omega = 2$) the investment volatility declines (8.38) and Home and Foreign investment becomes less correlated (-0.77) while the BS correlation remains in the puzzle (0.95).

The moments under complete markets are quite similar to those under incomplete markets. This is considered as reminiscent of Heathcote and Perri (2002) who discuss that only balanced trade case is quite different. However, under complete markets the investment volatility increases (18.93) and it becomes correlated more negatively (-0.94). This can be seen as the result discussed in Corsetti, Martin and Pesenti (2007). They show that under complete markets there are more entry compared to incomplete markets in more efficient country. Intuitively because the consumptions are perfectly insured under complete markets the equilibrium allocation of firms follow just the principle of efficiency so that the world has the maximum number of varieties[14]. The BS correlation becomes negative (-0.19) but the reason is quite different from that discussed for incomplete markets. Under complete markets when this happens it means that there is very strong positive transmission via the terms of trade depreciation (see Figure A-1 for the impulse response). As a result Foreign consumption in terms intensive margins rises more than Home consumption while the real exchange rate depreciated. However, generally speaking and very intuitively as it is shown in Corsetti, Dedola and Leduc (2006) in their VAR model, following a positive shock empirical based consumption rises in that country relative to the rest of the world. Hence such a mechanism under complete markets to generate a realistic BS correlation would not be realistic. We place a detailed discussion about complete markets in the appendix.

In summary we can say that a high elasticity of substitution brings the BS correlation into a realistic range. On the other hand other puzzles in international business cycle continue to be observed without significant quantitative improvement. In the next subsection the sensitivity of the BS correlation with

[13] This aspect is not captured in a very simplified model in the first part of this paper where the entry (number of firms) moves one by one in response of the productivity shock only.

[14] Again this point is not captured in our very simplified analytical model in the first part of the paper. Under complete markets also the number of varieties moves one by one follwing the productivity shock exactly as under balanced trade.

regards to the elasticity of substitution and the love for variety are examined.

Table 2-1

% std.dev.

(Relative to $\widetilde{Y}$)

	$\widetilde{Y}$	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\widetilde{TB}/\widetilde{Y}$	ρ^R	$\widetilde{Q}$
US data	1.71	0.84	5.38	0.66*	0.45	2.99*	3.73*
	(1.00)	(0.49)	(3.15)	(0.34*)		(1.79*)	(2.23*)
Incomplete	1,46	0,48	15,25	0,83	0,46	0,09	0,04
	(1.00)	(0, 33)	(10, 42)	(0, 56)		(0, 06)	(0, 03)
$\sigma = 3.8$ $\omega = 2$	1,40	0,45	8,38	0,82	0,19	0,33	0,15
	(1.00)	(0, 32)	(5, 98)	(0, 58)		(0, 24)	(0, 10)
Complete	1,76	0,47	18,93	1,19	0,74	0,09	0,04
	(1.00)	(0, 27)	(10, 76)	(0, 68)		(0, 05)	(0, 02)
$\sigma = 3.8$ $\omega = 2$	1,47	0,44	8,90	0,92	0,17	0,42	0,19
	(1.00)	(0, 30)	(6, 07)	(0, 63)		(0, 29)	(0, 13)

Contemporaneous correlation with output

	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\widetilde{TB}/\widetilde{Y}$	ρ^R	$\widetilde{Q}$
US data	0.76	0.90	0.87*	−0.28	−0.24*	0.13*
Incomplete	0.64	0.89	0.78	-0.18	0.55	0.55
$\sigma = 3.8$ $\omega = 2$	0.68	0.96	0.81	-0.67	0.67	0.67
Complete	0.39	0.87	0.86	0.06	0.91	0.91
$\sigma = 3.8$ $\omega = 2$	0.61	0.96	0.83	-0.65	0.65	0.65

Table 2-2

Cross country correlation

	$\widetilde{Y}$	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\frac{\widetilde{C}}{\widetilde{C}^*}, \widetilde{Q}$	$\frac{C}{C^*}, Q$
US data	0.58*	0.36*	0.30*	0.42*	-0.27^{CDL}	
Incomplete	-0.52	0.90	-0.91	-0.91	0.17	0.85
$\sigma = 3.8$ $\omega = 2$	-0.33	0.93	-0.77	-0.95	0.95	0.97
Complete	-0.67	0.98	-0.94	-0.96	-0.19	1.00
$\sigma = 3.8$ $\omega = 2$	-0.39	0.99	-0.80	-0.96	0.84	1.00

Data with * comes from Heathcote and Perri (2002). Others from BKK.

4.3 Sensitivity of the BS correlation

Figure 4 and 5 examines the BS correlation with different values of the elasticity of substitution (ω) and the love for variety (ψ) given the baseline parameters. It becomes steadily weaker and finally becomes negative as the elasticity increases.

For $\omega = 7$, a proposed value in Imbs and Méjan (2009), the BS correlation is -0.23.

For the change of the love for variety almost exactly the same pattern is observed. The BS correlation becomes weaker and arrives in the negative range as the love for variety increases. Again the intuition is shed light by the analytical part of this paper where the interaction between the elasticity of substitution and the love for variety has been emphasized.

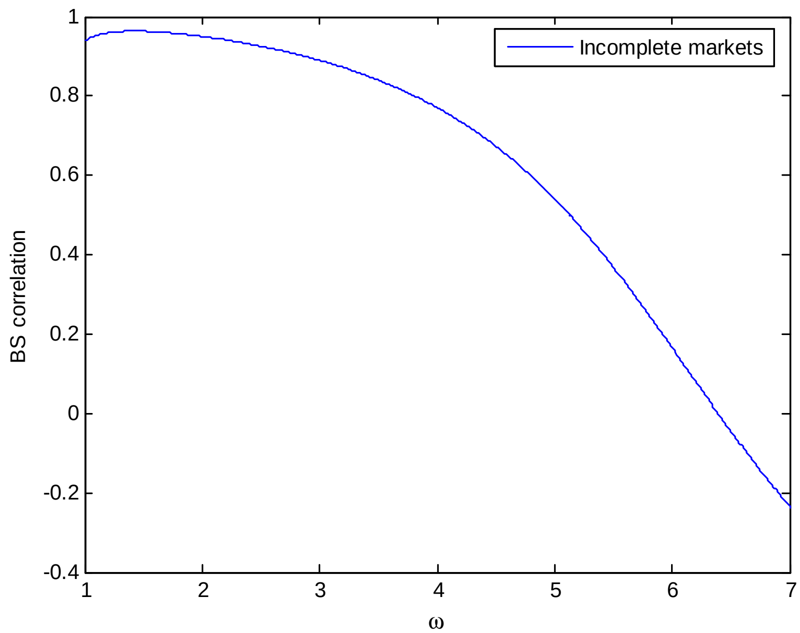

Figure 4 : BS correlation and the elasticity of substitution.

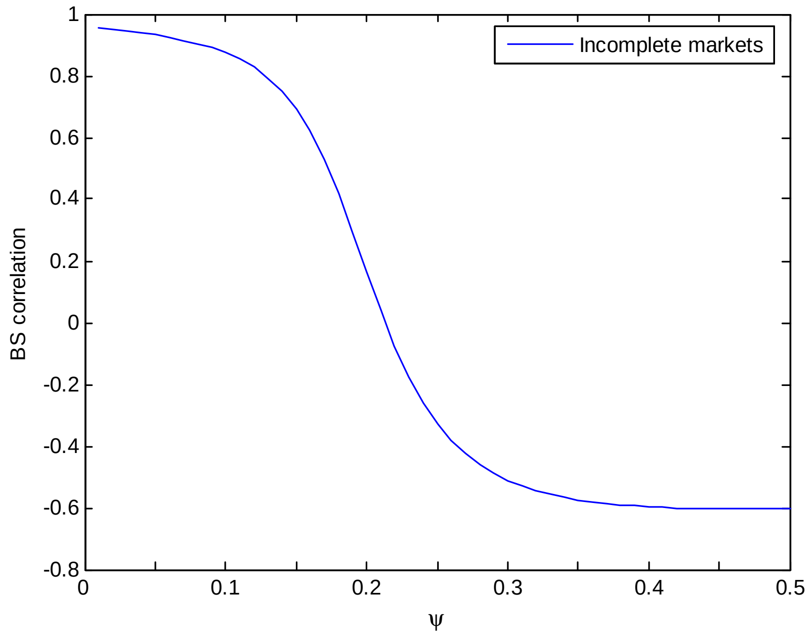

Figure 5 : BS correlation and the love for variety.

5 Conclusion

This paper revisits the consumption real exchange rate anomaly known as the Baskus-Smith puzzle and provides the mechanism of reconciliation by the love for variety. An increase in the number of exported varieties following a positive productivity shock, works as a demand shock under love for variety and tends to bring the terms of trade in an appreciation counteracting their depreciation due to the decrease in marginal cost. Reflecting such terms of trade movement, the empirically observed real exchange rate tends to appreciate while consumption rises for high income country. In contrast when we count correctly the arising extensive margins in the price indices, the real exchange rate tends to be depreciated reflecting these higher number of varieties consumed with home bias. Thus the positive correlation between them remains. Another very important claim of the paper is that a realistic BS correlation is possibly observed when the elasticity of substitution between Home and Foreign goods is high.

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A Complete markets

In this appendix we give the complete markets case for the static and dynamic version of the model. The purpose is to show that whilst it is very possible even under complete markets to have a realistic BS correlation, however, this requires a not realistic empirical based consumption movement. Following a positive productivity shock under complete markets it is usually not optimal to experience terms of trade appreciation. Thus when we observe a realistic BS correlation (negative or close to 0) under complete markets, Home observed consumption decreases relatively than Foreign consumption reflecting a strong positive transmission from Home to Foreign due to the terms of trade depreciation. However, as shown in Corsetti, Dedola and Leduc (2006) with their VAR model, consumption increases after a positive shock for major industrialized countries. Given this unrealistic characteristic under complete markets, we conclude that the combination of extensive margins and market incompleteness is important to generate a realistic real exchange rate and relative consumption variations, hence for the resolution of the BS puzzle.

A.1 Static model

.Under complete markets, the marginal utility which stems from an additional nominal wealth is the same across countries:

$$Q = \left(\frac{C^*}{C}\right)^{-\gamma}. \tag{59}$$

An increase of Home consumption relative to Foreign should be associated with a real depreciation. The relative dividends are expressed as

$$\mathsf{d}^R = (\omega - 1)\,\rho^R + [\psi\,(\omega - 1) - 1]\,\mathsf{N}^R + (2\alpha - 1)\,[\mathsf{C} - (C^* + \mathsf{Q})] - (2\alpha - 1)\,(\omega - 1)\,\mathsf{Q}, \tag{60}$$

Replacing the balanced trade condition (23) by the above (log-linearised) perfect risk sharing condition and using the variation of the real exchange rate (22) the variation of the relative dividends is given by

$$\mathsf{d}^R = (\lambda - 1)\,\rho^R + [\psi\,(\lambda - 1) - 1]\,\mathsf{N}^R, \tag{61}$$

where

$$\lambda \equiv \omega\left[1 - (2\alpha - 1)^2\right] + (2\alpha - 1)^2\,\frac{1}{\gamma}.$$

When lamda exceeds the unity, $\lambda > 1$ (this is roughly the elasticity of substitution between local and imported goods is higher than the unity, $\omega > 1$) as we expected relative profits decrease with the terms of trade appreciation. When $\lambda < 1$ it increases. Under $\psi > \frac{1}{\lambda - 1}$ an increase of relative number in Home originated varieties increases the relative profits. When $\psi < \frac{1}{\lambda - 1}$ it decreases. The intuition is the same as in the incomplete markets case. Given a certain elasticity of substitution between Home and Foreign ω (roughly this is λ) a high love for variety gives a further increase of relative profits, contrary to a low love for variety which induces an additional competition effect on relative profits.

Using the above expression of dividends, the labor markets clearings and free entry conditions give the solution for w^R and N^R:

$$\mathsf{w}^R = \frac{\lambda - 1}{\lambda}\mathsf{z}^R + \frac{\psi\,(\lambda - 1)}{\lambda}\mathsf{z}_E^R, \tag{62}$$

$$\mathsf{N}^R = \mathsf{z}_E^R. \tag{63}$$

The relative number of varieties reacts one by one to the investment shock as in the balanced trade case.. Relative wage increases (decreases) with the labor productivity in intensive margin when $\lambda > 1$ ($\lambda < 1$). The second term comes from under the love for variety only. With $\mathsf{z}_E^R > 0$ a higher number of varieties appears in Home relative to Foreign. The interaction between the love for variety and sufficiently high elasticity of substitution ($\lambda > 1$) makes

Home wage appreciated. When the elasticity is low ($\lambda < 1$) it depreciates. The corresponding restriction on parameters to guarantee the steady state is $1 > \psi\frac{(\lambda-1)^2}{\lambda}$ under complete markets.

The variation of the terms of trade is given by

$$\rho^R = \frac{1}{\lambda}\mathsf{z}^R - \frac{\psi(\lambda-1)}{\lambda}\mathsf{z}_E^R. \tag{64}$$

In contrast to the terms of trade under balanced trade analyzed in the paper, with complete markets for the productivity shock in the creation of intensive margins, z^R, they never appreciate no matter what the elasticity of substitution (λ, roughly this is ω). As in the balanced trade case a positive investment shock, z_E^R, and induced variation in the number of varieties tend to give an appreciation in the terms of trade under love for variety. However, for whatever correlation of the shocks, the chance of appreciation is less compared to the case of balanced trade.

A.1.1 Implication for the BS puzzle

Again the BS relation under complete market is rewritten with the variety term as follows:

$$\widetilde{Q} = \gamma\left(\widetilde{C} - \widetilde{C}^*\right) + (\gamma-1)\psi(2\alpha-1)\mathsf{N}^R, \tag{65}$$

and the real exchange rate is given by using the terms of trade,

$$\widetilde{Q} = \frac{2\alpha-1}{\lambda}\mathsf{z}^R - \frac{2\alpha-1}{\lambda}\psi(\lambda-1)\mathsf{N}^R, \tag{66}$$

so by plugging this in (65) and $\mathsf{N}^R = \mathsf{z}_E^R$, the consumption variation of intensive margins becomes

$$\gamma\left(\widetilde{C} - \widetilde{C}^*\right) = \frac{2\alpha-1}{\lambda}\mathsf{z}^R - \frac{2\alpha-1}{\lambda}\psi(\lambda\gamma-1)\mathsf{z}_E^R. \tag{67}$$

The above two expressions are not equal because of the parameter, γ (risk aversion). Without the love for variety ($\psi = 0$) the two expressions are identical and we are in the original puzzle. However, the existence of the love for variety creates a discrepancy between them and the BS correlation is possibly negative depending on the parameters and the relative strength of the two shocks. Inspection of the complete market condition (65) depicts that following the positive shocks the BS correlation becomes negative only when the observed real exchange rate depreciates ($\widetilde{Q} > 0$) and when the consumption in intensive margins decreases ($\widetilde{C} - \widetilde{C}^* < 0$) and thus achieving relatively high welfare gains from the extensive margins ($\mathsf{N}^R > 0$). In other words, this is possible when there is a very strong positive transmission via the terms of trade depreciation (in spite of the higher number of varieties which brings them into an appreciation) to the extent that the relative consumption in terms of intensive margins decreases.

In particular, with perfect correlation between the two shocks, $\mathsf{z}^R = \mathsf{z}_E^R$, the BS relation is given by:

$$\widetilde{Q} = \frac{1-\psi(\lambda-1)}{1-\psi(\lambda\gamma-1)}\gamma\left(\widetilde{C} - \widetilde{C}^*\right). \quad (68)$$

Without love for variety ($\psi = 0$) or log utility case ($\gamma = 1$), we are in the original puzzle. But depending on the parameters the BS relation would become negative or positive. However, we consider this case under complete markets is not realistic because the negative BS relation requires the less consumption in terms of intensive margins in Home at the same time as the terms of trade depreciation following a positive shock.

A.2 Dynamic model

Here the dynamic version of the model under complete markets is presented. With complete markets the model becomes simpler. We discuss only these modified points. The real budget constraint for the Home representative household now contains the state-contingents securities in the place of non-contingent bonds. This is:

$$\begin{aligned} C_t &+ s_{h.t+1} x_{h.t} (N_t + N_{E.t}) \\ &+ \sum_{s_{t+1}} b_{t+1}(S_{t+1}) q_t (S_{t+1} \mid S_t) + Q_t \sum_{s_{t+1}} b_{t+1}^*(S_{t+1}) q_t^* (S_{t+1} \mid S_t) \\ &= w_t L_t + s_{h.t} N_t (x_{h.t} + d_{h.t}) + b_t(S_t) + Q_t b_t^*(S_t), \quad (69) \end{aligned}$$

where $s_{h.t+1}$ is the share holding into t+1, $x_{h.t}$ is the real share price of Home mutual fund. N_t is the number of existing firms, and $N_{E.t}$ is the number of new entrants. The household has access to the full set of Arrow-Debreu securities denominated Home and Foreign goods which give one unit in the next period. $b_{t+1}(S_{t+1})$ ($b_{t+1}^*(S_{t+1})$) is the holding into t+1 indexed by the state of nature S_{t+1}. $q_t(S_{t+1} \mid S_t)$ ($q_t^*(S_{t+1} \mid S_t)$) is the date t real price of such security which is conditional on the current state of nature, S_t.

A.2.1 First order conditions

First order conditions for the Arrow-Debreu securities are given by:

$$q_t (S_{t+1} \mid S_t) = \beta \Pr(S_{t+1} \mid S_t) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \quad (70)$$

$$q_t^* (S_{t+1} \mid S_t) = \beta \Pr(S_{t+1} \mid S_t) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{Q_{t+1}}{Q_t}. \quad (71)$$

where $\Pr(S_{t+1} \mid S_t)$ is the probability of the realization of state of nature S_{t+1} conditional on S_t.[15] Other first order conditions are the same.

[15] Those of Foreign counterparts are

A.2.2 Perfect risk sharing condition

Finally using the optimal conditions for Arrow-Debreu securities in both countries,

$$\frac{Q_{t+1}}{Q_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}=\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}, \tag{74}$$

and imposing the symmetry at the initial steady state, the following complete market condition holds

$$Q_t=\left(\frac{C^*_t}{C_t}\right)^{-\gamma}. \tag{75}$$

The Euler equations about non-contingent bonds, bonds markets clearings and the evolution of the net foreign asset are no more needed. Finally the model contains 25 equations and 25 variables of which 2 are endogenous state variables (N_t and N^*_t) and 2 are exogenous shocks ($z_t$and z^*_t).

A.3 (Unrealistic) characteristics under complete markets

Here we show quantitatively how it is unrealistic for there to be a realistic BS correlation under complete markets. For this purpose we perform the same exercise as with under incomplete markets with the same baseline parameters and shock (Figure A-1). Following an increase in Home productivity the terms of trade always depreciate for Home in spite of a relatively high number of Home originated firms, which potentially brings them into an appreciation. Welfare based real exchange rate strongly depreciates reflecting this higher Home extensive margins consumed with home bias. Elimination of such effect from the consumption basket makes the variation of relative consumption negative. In other words, Home consumes less in terms of intensive margins whilst consuming more in terms of extensive margins in order to achieve a perfect risk sharing. At the same time the real exchange rate remains depreciated, creating a negative BS correlation.

Next we perform the sensitivity analysis for the BS correlation with different values for the elasticity of substitution and the love for variety (Figure A-2 and Figure A-3). For complete markets as the elasticity of substitution increases the BS correlation declines and afterwards starts to increase. The analytical solution again sheds light on this non-lineality under complete markets. As can be seen from (64), when the elasticity of substitution or love for variety becomes high the terms of trade start to appreciate. The variation of relative consumption in

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$$q^*_t\left(S_{t+1}\mid S_t\right)=\beta\Pr\left(S_{t+1}\mid S_t\right)E_t\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma} \tag{72}$$

$$q_t\left(S_{t+1}\mid S_t\right)=\beta\Pr\left(S_{t+1}\mid S_t\right)E_t\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}\left(\frac{Q_{t+1}}{Q_t}\right)^{-1} \tag{73}$$

terms of intensive margins remains negative, hence starting to provide a positive BS correlation. As it has been previously discussed in the analytical section when the BS correlation becomes realistic it requires the negative variation of the empirical based consumption following a positive productivity shock. And this case would be considered unrealistic.

B The system

Price indices (or variety effect)

$$\alpha\rho_{H.t}^{1-\omega} + (1-\alpha)\,\rho_{F.t}^{1-\omega} = 1 \tag{76}$$

$$\rho_{H.t} = N_t^{-\psi}\rho_{h.t}, \qquad \rho_{F.t} = N_t^{*-\psi}\rho_{f.t} \tag{77}$$

$$\alpha\rho_{F.t}^{*1-\omega} + (1-\alpha)\,\rho_{H.t}^{*1-\omega} = 1 \tag{78}$$

$$\rho_{F.t}^{*} = N_t^{*-\psi}\rho_{f.t}^{*}, \qquad \rho_{H.t}^{*} = N_t^{-\psi}\rho_{h.t}^{*} \tag{79}$$

Pricing

$$\rho_{h.t} = \frac{\sigma}{\sigma-1}\frac{w_t}{z_t} \tag{80}$$

$$\rho_{f.t}^{*} = \frac{\sigma}{\sigma-1}\frac{w_t^{*}}{z_t^{*}} \tag{81}$$

Profits

$$d_{h.t} = \frac{1}{\sigma}N_t^{\psi(\sigma-1)-1}\rho_{h.t}^{1-\sigma}\rho_{H.t}^{\sigma-\omega}\left[\alpha D_t + (1-\alpha)\,Q_t^{\omega}D_t^{*}\right] \tag{82}$$

$$d_{f.t}^{*} = \frac{1}{\sigma}N_t^{*\psi(\sigma-1)-1}\rho_{f.t}^{*1-\sigma}\rho_{F.t}^{*\sigma-\omega}\left[\alpha D_t^{*} + (1-\alpha)\,Q_t^{-\omega}D_t\right] \tag{83}$$

Consumption and investment goods demand

$$D_t = C_t + (1-\theta)\,N_{E.t}x_{h.t} \tag{84}$$

$$D_t^{*} = C_t^{*} + (1-\theta)\,N_{E.t}^{*}x_{f.t}^{*} \tag{85}$$

Free entry

$$x_{h.t} = f_E\left(\frac{w_t}{z_t}\right)^{\theta} \tag{86}$$

$$x_{f.t}^{*} = f_E^{*}\left(\frac{w_t^{*}}{z_t^{*}}\right)^{\theta} \tag{87}$$

Number of firms

$$N_t = (1-\delta)\left(N_{t-1} + N_{E.t-1}\right) \tag{88}$$

$$N_t^* = (1-\delta)\left(N_{t-1}^* + N_{E.t-1}^*\right) \tag{89}$$

Euler shares

$$x_{h.t} = \beta(1-\delta) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(x_{h.t+1} + d_{h.t+1}\right) \tag{90}$$

$$x_{f.t}^* = \beta(1-\delta) E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} \left(x_{f.t+1}^* + d_{f.t+1}^*\right) \tag{91}$$

Euler equation (bond) (under incomplete market)
(Home)

$$C_t^{-\gamma}\left(1 + \vartheta B_{t+1}\right) = \beta\left(1 + r_{t+1}\right) E_t C_{t+1}^{-\gamma} \tag{92}$$

$$C_t^{-\gamma}\left(1 + \vartheta B_{*.t+1}\right) = \beta\left(1 + r_{t+1}^*\right) E_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma} \tag{93}$$

(Foreign)

$$C_t^{*-\gamma}\left(1 + \vartheta B_{*.t+1}^*\right) = \beta\left(1 + r_{t+1}^*\right) E_t C_{t+1}^{*-\gamma} \tag{94}$$

$$C_t^{*-\gamma}\left(1 + \vartheta B_{t+1}^*\right) = \beta\left(1 + r_{t+1}\right) E_t \frac{Q_t}{Q_{t+1}} C_{t+1}^{*-\gamma} \tag{95}$$

Optimal labor supply

$$\chi\left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma} \tag{96}$$

$$\chi\left(L_t^*\right)^{\frac{1}{\psi}} = w_t^* C_t^{*-\gamma} \tag{97}$$

Labor Market clear

$$L_t = (\sigma-1)\frac{N_t d_{h.t}}{w_t} + \theta\frac{N_{E.t} x_{h.t}}{w_t^*} \tag{98}$$

$$L_t^* = (\sigma-1)\frac{N_t^* d_{f.t}^*}{w_t^*} + \theta\frac{N_{E.t}^* x_{f.t}^*}{w_t^*} \tag{99}$$

Perfect risk sharing condition (under complete markets)

$$Q_t = \left(\frac{C_t^*}{C_t}\right)^{-\gamma} \tag{100}$$

Net foreign Asset (under incomplete markets)

$$B_{t+1} - (1+r_t)\, B_t + Q_t \left[B_{*.t+1} - (1+r_t^*)\, B_{*.t} \right] =$$
$$+\frac{1}{2} \left[L_t w_t + N_t d_{h.t} - Q_t \left(L_t^* w_t^* + N_t^* d_{f.t}^* \right) \right]$$
$$-\frac{1}{2} \left[N_{E.t} x_t + C_t - Q_t \left(N_{E.t}^* x_t^* + C_t^* \right) \right] \quad (101)$$

Bond market clearing (under incomplete markets)

$$B_{t+1} + B_{t+1}^* = 0 \quad (102)$$

$$B_{*.t+1} + B_{*.t+1}^* = 0 \quad (103)$$

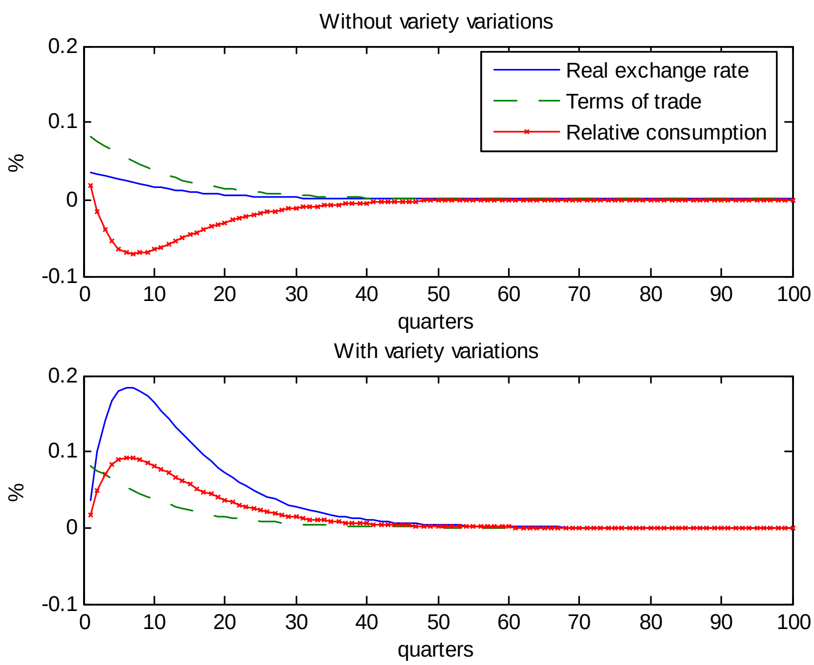

Figure A-1: IRF under complete markets.

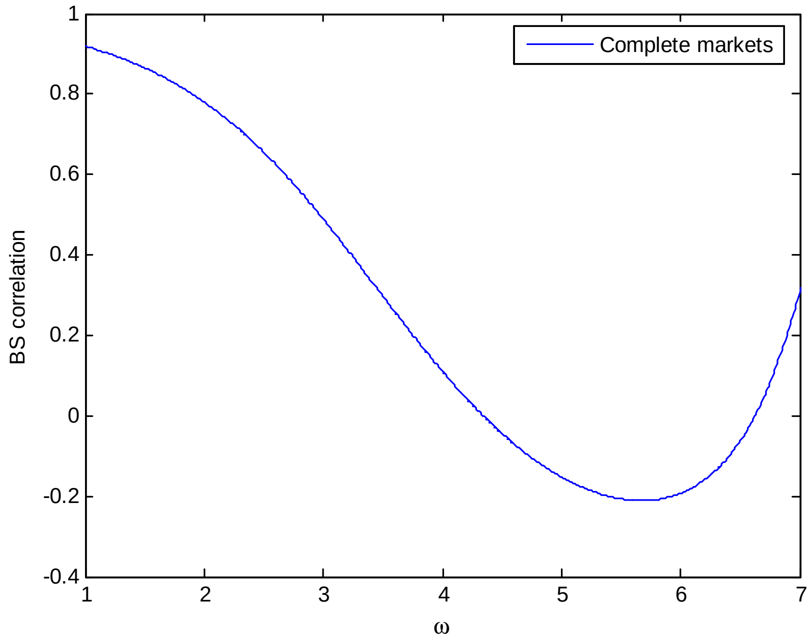

Figure A-2: BS correlation and the elasticity of substitution.

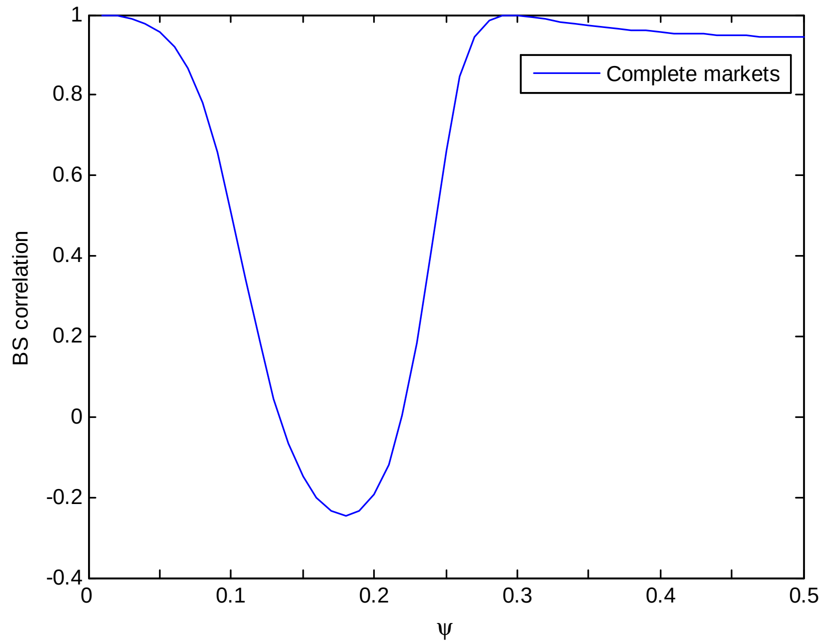

Figure A-3: BS correlation and the love for variety.