Insurance and Perceptions: How to Screen Optimists and Pessimists

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PRELIMINARY. COMMENTS VERY WELCOME.

Abstract

Individuals have differing beliefs about risks they face and their ability to mitigate these risks. Profit-maximizing firms screen identical agents with different beliefs by providing less insurance to optimists than to pessimists. Optimists perceive the risk to be less likely than pessimists given their respective choices of precautionary efforts. Depending on the nature of competition, the screening distortions in insurance coverage are determined by differences in beliefs about the likelihood or the marginal return to effort. I show that heterogeneity in beliefs strengthens the case for government intervention in insurance markets and can explain the negative correlation between risk occurrence and insurance coverage found in empirical studies.

Keywords: Insurance Markets, Moral Hazard, Risk perceptions, Adverse Selection

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1 Introduction

The perception of risk is inherently subjective.[1] Financial traders disagree about the risk of investments, mortgage bankers about the risk of defaulting homeowners, homeowners and renters about the risk of flooding, old and young drivers about the risk of a car accident. One person may perceive a risk as very likely, while another may perceive the same risk as unlikely. At the same time, the perception of the extent to which precautionary efforts mitigate the risk may differ as well. Both the perception of the likelihood of the risk and the perception of control are central to the design of insurance contracts. *Baseline-pessimistic* insurees, who underestimate the baseline likelihood of the risk, are willing to pay more for insurance. *Control-optimistic* insurees, who overestimate the marginal return to effort, exert more precautionary efforts and are therefore cheaper to insure.

This paper analyzes the role of heterogeneity in risk perceptions for the optimal design of screening contracts. In a model with moral hazard and adverse selection, I show how incentive compatibility imposes a very simple structure on the equilibrium contracts and I contrast the distortions in insurance coverage that arise with competing and monopolistic insurers. On the positive side, heterogeneity in risk perceptions offers an alternative explanation for the negative correlation between risk occurrence and insurance coverage found in empirical studies. On the normative side, the presence of agents with biased beliefs improves or worsens the welfare of agents with unbiased beliefs depending on the market structure and the differences in beliefs.

I consider a simple model with two states. Effort exerted by the insuree decreases the probability that a risk occurs, but insurees can have different perceptions about the probability of the risk as a function of effort. The insurer cannot observe the belief held by the insuree, but perceives her risk as independent of her belief. The insuree does not change her belief in response to the menu of insurance contracts being offered. That is, the insurer and the insurees 'agree to disagree' about the true underlying risk. The preferences satisfy a single-crossing property if the one insuree perceives the likelihood of the risk as lower than the other insuree for any given insurance contract. This is conditional on the effort levels chosen by the respective insurees. Optimism can therefore arise for two reasons; first of all, if an insuree is more optimistic about the baseline likelihood of the risk for the same level of effort and, second, if an insuree is more optimistic about the marginal return of effort and therefore exerts higher effort for the same insurance contract. If the single-crossing property is satisfied, the insurer can only separate the (more) optimistic insuree by offering her less insurance coverage than the (more) pessimistic insuree. This monotonicity property is independent of the nature of competition between insurers.

Optimistic agents receive less insurance, but still may be more risky ex-post if they

[1] Slovic (2000) surveys the research documenting the heterogeneity in the perception of risk and its determinants.

are pessimistic about their control and exert less precautionary effort. This contrasts with the property of positive correlation between insurance coverage and risk occurrence that arises in the standard adverse selection framework (Rothschild and Stiglitz 1976). However, many empirical papers find a correlation that is not significantly positive (Chiappori and Salanié 1997 and 2000, Cardon and Hendel 2001) or even negative (Cawley and Philipson 1999, De Meza and Webb 2001, Finkelstein and McGarry 2006). With two types of insurees who only differ in their beliefs, I show that it is sufficient that the one type is more baseline-optimistic and control-optimistic for the equilibrium to satisfy the positive correlation property. For the correlation to be negative, it is necessary that the control-pessimistic type is also more optimistic about the likelihood of the risk.

A prime issue for characterizing optimal contracts with private information is determining which incentive compatibility constraints are binding and thus which types' contracts are distorted compared to the case without private information. I show how this depends on the interaction between the nature of competition and the dimension in which beliefs are biased. Competing insurers distort the contract offered to the insuree who can be insured at lower cost, which depends on the exerted precautionary effort and thus the insuree's control beliefs. A monopolistic insurer distorts the contract offered to the insuree whose willingness to pay is lower, which depends on the insuree's baseline beliefs. Compared to someone who is unbiased, an optimist's willingness to pay is lower for an *insurance contract* providing more insurance than her outside option, but higher for an *incentive contract* providing less insurance than her outside option.

The distortions due to the screening of types imply that agents with heterogeneous perceptions impose information externalities on each other. An agent with biased beliefs imposes a negative externality on an agent with unbiased beliefs, when private insurers distort the unbiased agent's contract to discourage the biased agent from taking this contract. The externality is only positive when a monopolistic insurer pays a rent to the unbiased agent not to take the contract offered to the biased type. For agents with biased beliefs, the screening distortions may aggravate the distortion due to the biases in their beliefs (Spinnewijn 2009). Hence, heterogeneity in optimistic beliefs may strengthen the case for (paternalistic) government intervention through mandating insurance. This contrasts with the result in Sandroni and Squintani (2007) that heterogeneity in beliefs reduces the scope for government intervention. The heterogeneity in optimistic beliefs they consider implies that some agents with different risks perceive their risk to be the same and are pooled in equilibrium. The heterogeneity I consider implies that agents with the same underlying risk are separated.

Related Literature The paper studies the role of biased beliefs in the presence of both moral hazard and adverse selection. Spinnewijn (2009) considers only moral hazard, assuming that the bias in beliefs is known to the insurer. Jeleva and Villeneuve

(2004), Chassagnon and Villeneuve (2005) and Villeneuve (2005) consider only adverse selection. They introduce heterogeneity in risk types, but risk types may misperceive their risk. Sandroni and Squintani (2007) also introduce heterogeneity in risk types, but some agents of the high-risk type may be optimistic about being a low-risk type.

A small theoretical literature has suggested explanations for the advantageous selection with heterogeneous types that leads to negative correlation between risk occurrence and insurance coverage. Koufopoulos (2008) and Huang, Liu and Tzeng (2007) assume the presence of one type who exerts no precautionary effort, but is still more optimistic about the likelihood of the risk than the other type who exerts precautionary effort. This paper generalizes this intuition driven by heterogeneity in perceptions and characterizes how the correlation between risk occurrence and insurance coverage depends on the correlation between baseline and control beliefs. De Meza and Webb (2001) and Jullien, Salanié and Salanié (2006) explain the presence of advantageous selection by heterogeneity in risk preferences. Chiappori, Jullien, Salanié and Salanié (2006) show that such heterogeneity is not sufficient to explain the negative correlation if the competition in the insurance market is perfect. The correlation results in this paper are independent of the nature of competition.

This paper also relates to the literature that explores how firms exploit the bounded rationality of consumers, surveyed in Ellison (2006). In particular, Grubb (2009) and Eliaz and Spiegler (2008) analyze how firms exploit differences in overconfidence and optimism about future demand respectively with a menu of screening contracts. I also consider the externalities that biased agents and unbiased agents impose on each other. In a similar spirit, DellaVigna and Malmendier (2004) and Gabaix and Laibson (2006) analyze how sophisticated and non-sophisticated types affect each others' welfare.

The remainder of the paper is organized as follows. Section 2 introduces the model and defines the agent's beliefs. Section 3 analyzes properties of incentive compatible contracts with heterogeneity in beliefs. Section 4 characterizes the optimal screening contracts, contrasting the competitive equilibrium and the monopolistic optimum. Section 5 discusses welfare and policy implications. Section 6 presents a simple application with continuous output and linear contracts, as in Holmström and Milgrom (1987). Section 7 concludes the paper. All proofs are in the appendix.

2 Model

I consider a principal-agent model with two states. In the good state, the total endowment equals W. In the bad state, a loss L is incurred and the total endowment equals $W - L$. The agent's unobservable choice of effort determines the probability that the good or bad state occurs. When she exerts effort at additive cost $e \in E$, the good state occurs with probability $\pi(e)$ with $\pi' \geq 0, \pi'' < 0$. The bad state occurs with probability $1 - \pi(e)$. A risk-neutral principal offers a contract (w, Δ) to the risk-averse agent. With this contract, the agent can consume w in the good state and $w - \Delta$ in the bad state. Hence, the second argument Δ is the deductible, which determines the consumption risk left to the agent. The higher the deductible, the less insured the agent is. When the agent's outside opportunity is (w_0, Δ_0), the difference $w_0 - w$ denotes the insurance premium that the agent pays to reduce her consumption risk from Δ_0 to Δ.

I will allow the agent's outside option (w_0, Δ_0) to be different from the no insurance outcome (W, L). If the contract's deductible $\Delta < \Delta_0$, I call the contract an *insurance contract*. If the contract's deductible $\Delta > \Delta_0$, I call the contract an *incentive contract*. The principal's outside option equals $(W - w_0, L - \Delta_0)$ and the set of contracts that he can offer is restricted to

$$C \equiv \{(w, \Delta) \,|\Delta \in [0, L], w \in [\Delta, W]\}.$$

The agent cannot be overinsured, i.e. $\Delta \geq 0$, which follows immediately if the agent could make the bad state occur with certainty at zero cost.

2.1 The Agent's Beliefs

The agent's perception of the probability of success as a function of effort may differ from the true probability. I denote the agent's belief as $\hat{\pi}(e)$ with $\hat{\pi}' \geq 0, \hat{\pi}'' < 0$. I introduce these beliefs in the most general way, but the analysis shows that the differences in the levels and margins of the perceived probability functions are essential.

Definition 1 *Agent i is baseline-optimistic if $\hat{\pi}_i(e) \geq \pi(e)$ for all $e \in E$. Agent i is more baseline-optimistic than agent j if $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ for all $e \in E$.*

Definition 2 *Agent i is control-optimistic if $\hat{\pi}'_i(e) \geq \pi'(e)$ for all $e \in E$. Agent i is more control-optimistic than agent j if $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$ for all $e \in E$.*

For expositional purposes, I consider the sign of the differences to be the same for all effort levels. Baseline and control beliefs are related, but optimism in the one dimension does not exclude pessimism in the other dimension. Whether agents who are more optimistic about the baseline probability are also more optimistic about their control depends on the context, as in the following two examples.

Example I $\pi(e) = \theta e$ *and* $\hat{\pi}(e) = \hat{\theta} e$ *with* $e \in [0, \min\{1/\theta, 1/\hat{\theta}\}]$*:*

> When for a project the probability of success is complementary in the entrepreneur's ability θ and effort e, an entrepreneur who overestimates his ability (i.e. $\hat{\theta} > \theta$) is at the same time baseline-optimistic and control-optimistic.

Example II $1 - \pi(e) = \phi(1 - e)$ *and* $1 - \hat{\pi}(e) = \hat{\phi}(1 - e)$ *with* $e \in [0, 1]$:

> A driver who underestimates the probability to have an accident when exerting no effort (i.e. $\hat{\phi} < \phi$) is baseline-optimistic, but control-pessimistic.

The first two definitions are about the primitives of the probability functions. I introduce a third definition which involves the endogenous choice of effort by the respective agents and allows to describe a single-crossing property for the preferences of agents with different beliefs.

Definition 3 *Agent i is more optimistic than agent j if $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$ for all $c \in C$.*

An agent can be more optimistic either because she perceives the likelihood of the good state to be higher for the same level of effort or because she perceives the marginal return to effort to be higher and thus exerts more efforts.

Lemma 1 *An agent who is more baseline- and control-optimistic is more optimistic as well.*

2.2 The Agent's Preferences

The agent chooses the effort level that maximizes her perceived expected utility. Given the contract (w, Δ), the agent solves

$$U(w, \Delta) = \max_e \hat{\pi}(e) u(w) + (1 - \hat{\pi}(e)) u(w - \Delta) - e.$$

The agent's choice of effort $\hat{e}(w, \Delta)$ solves

$$\hat{\pi}'(\hat{e}(w, \Delta)) [u(w) - u(w - \Delta)] = 1.$$

The second order condition is satisfied since $\hat{\pi}'' < 0$. The effort choice is increasing in the agent's perceived return to search $\hat{\pi}'(\cdot)$ and the deductible Δ. When the outside option is chosen, the agent's perceived expected utility equals

$$U(w_0, \Delta_0) \equiv \hat{\pi}(e(w_0, \Delta_0)) u(w_0) + (1 - \hat{\pi}(e(w_0, \Delta_0))) u(w_0 - \Delta_0) - e(w_0, \Delta_0).$$

The utility in the outside option is increasing in the baseline belief about the probability $\hat{\pi}(\cdot)$ that the good state occurs. This increase is higher, the less insurance the outside option provides.

3 Incentive Compatibility with Heterogeneity in Beliefs

Pessimistic agents are willing to pay more for insurance coverage than optimistic agents because they perceive the risk as more likely. This single-crossing property of the preferences implies that only contracts providing more insurance to pessimistic agents than to optimistic agents can be incentive compatible. Whether pessimistic agents are also more risky ex-post depends on the agents' efforts and thus the agents' control beliefs.

The monotonicity in insurance coverage implies simple conditions for the correlation between risk occurrence and insurance coverage to be positive or negative.

I consider two types of agents who only differ in their beliefs. Type 1 and type 2 hold the beliefs $\hat{\pi}_1(\cdot)$ and $\hat{\pi}_2(\cdot)$ respectively, with $\hat{\pi}_1(\cdot) \neq \hat{\pi}_2(\cdot)$. These beliefs are unobservable to the insurers. The true probability of success $\pi(\cdot)$ is the same function of effort for both types. For the characterization of the equilibrium contracts, it does not matter whether these probability functions are actually the same or only perceived to be the same by the insurer. The outside option is the same for both types, but the perceived expected utility of the outside option may be different.

3.1 Single-Crossing Property

If the one type always perceives the probability of the good state to be greater than the other type for any possible contract, the two types' preferences satisfy a single-crossing property.

Assumption 1 *Type* 1 *is more optimistic than type* 2*.*

The higher the perceived probability that the bad state occurs, the higher the willingness to give up wealth w to decrease the deductible Δ. The perceived marginal rate of substitution between w and Δ for type i equals

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(w,\Delta))}{1-\hat{\pi}_i(\hat{e}_i(w,\Delta))}\frac{u'(w)}{u'(w-\Delta)}+1.$$

The effect through changes in effort on the perceived expected utility in response to dw and $d\Delta$ is of second order because of the envelope condition and does not impact the marginal rate of substitution. For different types, the marginal rates of substitution for a given contract (w,Δ) is ranked based on the respective perceived probability of success $\hat{\pi}_i(\hat{e}_i(w,\Delta))$. If type 1 is more optimistic than type 2, the marginal rates of substitution are ranked the same for any contract.

Lemma 2 *For any* $c \in C$,

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_1} \geq \left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_2}.$$

The profit-maximizing insurer cannot observe the type of insuree he is facing. By the revelation principle, we can restrict the analysis to contracts that are incentive compatible such that the different types will self-select into the contracts designed for them. A pair of contracts $\{(w_1,\Delta_1),(w_2,\Delta_2)\}$ is incentive compatible if and only if

$$U^i(w_i,\Delta_i) \geq U^i(w_j,\Delta_j) \text{ for } i,j=1,2,$$

with

$$U^i(w,\Delta) \equiv \max_e \hat{\pi}_i(e)\,u(w) + (1-\hat{\pi}_i(e))\,u(w-\Delta) - e.$$

Clearly, for any pair of incentive compatible contracts, one contract cannot offer more consumption in both states than the other contract. That is, if $w_1 > w_2$, then $w_1 - \Delta_1 < w_2 - \Delta_2$ and vice versa. I introduce the relation $x \rhd y$ to describe that the contract x provides less insurance than contract y in the sense that x provides lower coverage at a lower insurance premium than contract y.

Notation 1 $(w_i, \Delta_i) \rhd (w_j, \Delta_j) \Leftrightarrow w_i > w_j$ *and* $w_i - \Delta_i < w_j - \Delta_j$

Notation 2 $(w_i, \Delta_i) \trianglerighteq (w_j, \Delta_j) \Leftrightarrow w_i \geq w_j$ *and* $w_i - \Delta_i \leq w_j - \Delta_j$

I use the particular notation because $(w_i, \Delta_i) \rhd (w_j, \Delta_j)$ implies $(w_i, \Delta_i) > (w_j, \Delta_j)$. Notice that the opposite does not hold.

3.2 Monotonicity

In standard adverse selection problems the incentive compatibility constraints imply a monotonicity constraint on the separating contracts offered to different types, if the preferences satisfy a single-crossing property. The same is true here despite the presence of moral hazard.

The utility from one insurance contract can be expressed as the utility from any other insurance contract, plus the sum of the utility gains, positive or negative, from the incremental changes that lead from the latter to the former insurance contract. That is,

$$U^i(w_i, \Delta_i) = U^i(w_j, \Delta_j) + \int_{\Delta_j}^{\Delta_i} \left\{ U_w^i(\widetilde{w}(\Delta), \Delta)\, \widetilde{w}'(\Delta) + U_\Delta^i(\widetilde{w}(\Delta), \Delta) \right\} d\Delta,$$

for any continuous, differentiable function $\widetilde{w}(\Delta)$ with $\widetilde{w}(\Delta_j) = w_j$ and $\widetilde{w}(\Delta_i) = w_i$. I denote the gain in perceived expected utility for type i from switching from contract (w_2, Δ_2) to (w_1, Δ_1) by

$$\phi^i[(w_1, \Delta_1), (w_2, \Delta_2)] \equiv U^i(w_1, \Delta_1) - U^i(w_2, \Delta_2).$$

For contracts to be incentive compatible, the gain from switching to the other type's contract has to be negative for both types,

$$\phi^1[(w_1, \Delta_1), (w_2, \Delta_2)] \geq 0 \qquad (IC_1)$$
$$\phi^2[(w_2, \Delta_2), (w_1, \Delta_1)] \geq 0. \qquad (IC_2)$$

When choosing between two contracts, the more optimistic type puts relatively more weight on the change in consumption when successful and relatively less weight on the change in consumption when unsuccessful. This difference in weights is not sufficient to sign the difference for two types in utility gains from switching contracts, because the exerted effort levels differ as well. However, the single-crossing property can be used the evaluate the utility gains from all marginal changes in Δ and $\widetilde{w}(\Delta)$ for which changes in effort are of second order. When changing the contract from (w_j, Δ_j) to (w_i, Δ_i), the sign of the difference in utility gains for type i and type j from the marginal changes along the linear function $\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j)\frac{w_i - w_j}{\Delta_i - \Delta_j}$ exactly equals the sign of the difference in perceived likelihoods, $\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta), \Delta)) - \hat{\pi}_j(\hat{e}_j(\widetilde{w}(\Delta), \Delta))$. The more optimistic type suffers less from the marginal increase in Δ and gains more from the marginal increase in $\widetilde{w}(\Delta)$. This observation implies the following lemma.

Lemma 3 *If* $(w_1, \Delta_1) \triangleright (w_2, \Delta_2)$, *then*

$$\phi^1[(w_1, \Delta_1), (w_2, \Delta_2)] > \phi^2[(w_1, \Delta_1), (w_2, \Delta_2)].$$

The utility gain from switching to an insurance contract for which the insurance coverage and the insurance premium is lower, is greater for someone who is more optimistic about the risk not occurring. This implies that for two contracts to be incentive compatible, the insurance contract designed for the more optimistic type must provide less insurance, but at a lower insurance premium.

Proposition 1 *Type* 1 *receives less insurance than type* 2 *in any incentive compatible equilibrium, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2).$$

This monotonicity property follows immediately from the incentive compatibility constraints and Lemma 3. Assume, by contradiction, that (w_2, Δ_2) provides less insurance than (w_1, Δ_1). Since type 1 is more optimistic than type 2, the utility gain from switching to the contract providing less insurance is higher for type 1 than for type 2. However, for (w_2, Δ_2) to be incentive compatible for type 2, her gain from switching from (w_1, Δ_1) to (w_2, Δ_2) must be positive, which implies that the gain from switching from (w_1, Δ_1) to (w_2, Δ_2) is positive for type 1 as well. By consequence, (w_1, Δ_1) is not incentive compatible for type 1.

3.3 Positive vs. Negative Correlation

With heterogeneity in perceptions, either positive or negative correlation can arise between the ex-post probability that the risk occurs for a type and the insurance coverage provided to that type. An optimistic type necessarily receives more insurance than a pessimist, but whether the optimistic type is more risky depends on both her control beliefs and the insurance coverage.

Corollary 1 *If type 1 is more optimistic and control-optimistic than type 2, any separating equilibrium satisfies the 'positive correlation'-property, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2) \text{ and } \pi(\hat{e}_1(w_1, \Delta_1)) \geq \pi(\hat{e}_2(w_2, \Delta_2)).$$

If type 1 is more control-optimistic, she exerts more effort than type 2 for the same level of insurance. Since in addition type 1 receives less insurance, she exerts more effort in equilibrium and is less likely to suffer a loss. The observed correlation between risk occurrence and insurance coverage is positive.

Corollary 2 *Only if the optimistic type* 1 *is more control-pessimistic than type* 2*, a separating equilibrium may satisfy the 'negative correlation'-property, i.e.*

$$(w_1, \Delta_1) \rhd (w_2, \Delta_2) \ \text{ and } \pi\left(\hat{e}_1\left(w_1, \Delta_1\right)\right) < \pi\left(\hat{e}_2\left(w_2, \Delta_2\right)\right).$$

If type 1 is more control-pessimistic, she exerts less effort than type 2 for the same level of insurance. If she is sufficiently more control-pessimistic, she will still exert less effort despite receiving less insurance as well. The negative correlation between optimism and control-optimism across types is necessary for the negative correlation between risk occurrence and insurance coverage to occur.

Negative correlation arises naturally when one type believes his effort has no impact at all, but still perceives the probability that the good state occurs to be more likely than the other type. This extreme example is considered by Koufopoulos (2008) and Huang et al. (2007). Several recent papers show empirical evidence for negative correlation in insurance. Heterogeneity in risks and preferences cannot explain this negative correlation if insurance markets are competitive. Chiappori et al. (2006) show that the positive correlation between ex post risk and insurance is a robust property of competitive markets. However, with heterogeneity in risk aversion and imperfect competition, Jullien et al. (2007) show that the correlation can be negative as well. In contrast, Corollary 1 and 2 are independent of the nature of competition in the insurance market. The empirical question that arises is whether baseline beliefs and control beliefs are positively or negatively correlated. This will depend on the particular risk being considered. For instance, young drivers tend to overestimate the probability to avoid an accident, but underestimate the returns to driving safely (Finn and Bragg 1986, Tränkle et al. 1990). Similarly, women who overestimate the probability not to have breast cancer are less likely to take mammograms (Katapodi et al. 2004), plausibly because they underestimate the returns to preventive efforts, as argued by Polednak et al. (1991).

4 Optimal Insurance Contracts

I contrast the insurance contracts offered by competing insurers and a monopolistic insurer who cannot observe the beliefs of the insuree they are facing. Heterogeneity in beliefs drives a wedge between the insurer's cost of providing insurance and the insuree's willingness to pay for being insured. On the one hand, the insurer's cost of providing insurance depends on the insuree's effort choice, which is increasing in her control beliefs. When an insuree of type i accepts the contract (w, Δ), the insurer's expected profit equals

$$\Pi^i (w, \Delta) = W - w - (1 - \pi (\hat{e}_i (w, \Delta))) (L - \Delta) .$$

On the other hand, an insuree's willingness to pay for insurance is decreasing in her baseline beliefs. Similarly, her willingness to accept risk is increasing in her baseline beliefs.

The wedge between cost and valuation implies that whether a type's contract is distorted compared to the full-information contract crucially depends on the nature of competition between insurers. Competing insurers distort the contract offered to the 'low-cost' type to discourage the 'high-cost' type from pretending she has low cost. Control beliefs are thus central under competition. A monopolistic insurer distorts the contract to the 'low-valuation' type to discourage the 'high-valuation' type from pretending she has low valuation. Baseline beliefs are thus central under monopoly. Notice that when insurees only differ in risk, a riskier type values insurance more, but is also more costly to insure.

For the competitive equilibrium, I assume that insurers are competing as in Rothschild and Stiglitz (1976) with any contract offered in equilibrium making zero profit in expectation. For the monopolistic optimum, the insurees' participation constraints are central to the analysis. For a contract to be accepted, the insuree needs to expect higher utility from that contract than from her outside option. For the competitive case, I assume that the outside option provides no insurance and that the participation constraints are never binding in the competitive equilibrium. I relax both assumptions for the monopolistic case.

4.1 Full-Information Benchmark

I first characterize the profit-maximizing contract when the insurer knows the agent's perceived probability function $\hat{\pi}(e)$ and the agent's outside option equals $(w_0, \Delta_0) = (w, L)$. The insurer expects to pay insurance coverage $L - \Delta$ with probability $1 - \pi(\hat{e}(w, \Delta))$, whereas the agent expects to receive this coverage with probability $1 - \hat{\pi}(\hat{e}(w, \Delta))$. When acting as a monopolist, the profit-maximizing contract (w_m^*, Δ_m^*) solves

$$\max_{(w,\Delta)} W - w - (1 - \pi (\hat{e} (w, \Delta))) (L - \Delta)$$

such that

$$u(w) - (1 - \hat{\pi} (\hat{e} (w, \Delta))) [u (w) - u (w - \Delta)] - \hat{e} (w, \Delta) \geq U (w_0, \Delta_0) .$$

The competitive equilibrium (w_c^*, Δ_c^*) solves the dual problem with the equilibrium profits equal to zero,

$$\max_{(w,\Delta)} u(w) - (1 - \hat{\pi}(\hat{e}(w, \Delta)))[u(w) - u(w - \Delta)] - \hat{e}(w, \Delta)$$

such that

$$W - w - (1 - \pi(\hat{e}(w, \Delta)))(L - \Delta) \geq 0.$$

This implies the following proposition (Spinnewijn 2009).

Proposition 2 *The profit-maximizing contract* (w^*, Δ^*) *is characterized by*

$$\frac{\frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})}\frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})}u'(w^* - \Delta^*) - u'(w^*)}{u'(w^*)} = \varepsilon_{1-\pi(\hat{e}), w-\Delta}\frac{L - \Delta^*}{w^* - \Delta^*},$$

with $\hat{e} = \hat{e}(w^*, \Delta^*)$*. In monopoly, the perceived expected utility* $U(w^*, \Delta^*) = U(w_0, \Delta_0)$*. In competition, the expected profit* $\Pi(w^*, \Delta^*) = 0$*.*

The contracts optimally trade off the moral hazard cost of insurance and the consumption smoothing benefits of insurance, as perceived by the agent.[2] The moral hazard cost is determined by the elasticity of the probability that the bad state occurs with respect to the level of insurance coverage, $\varepsilon_{1-\pi(\hat{e}),w-\Delta}$. The perceived consumption smoothing benefits are determined by the wedge in marginal utilities in the good and the bad state, corrected for the baseline bias. When the agent is baseline-optimistic (i.e. $\frac{1-\hat{\pi}(\hat{e})}{1-\pi(e)}\frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})} < 1$), the perceived consumption smoothing benefit of actuarially fair insurance is lower than the true consumption smoothing benefit. Since a baseline optimist perceives insurance as less valuable than an unbiased agent, the insurance coverage offered to a baseline-optimistic agent is unambiguously lower. The optimal response to control optimism is ambiguous though. If an insuree becomes more control-optimistic, less risk is required to induce her to exert the same level of effort. Hence, the insurers substitute towards inducing more effort, but given the control optimism, could do so by giving at the same time more insurance. The insurance coverage can therefore be higher for either the more control-optimistic or the more control-pessimistic insuree in the full-information equilibrium. This is in stark contrast with Proposition 1. If beliefs are private and one insuree is more optimistic than the other (e.g. because she is more control-optimistic), she receives less insurance coverage in any incentive compatible equilibrium.

4.2 Binding Incentive Compatibility

A prime issue for characterizing optimal contracts with private information is determining which incentive compatibility (IC) constraints are binding. The difference in

[2] I assume that the first order condition is sufficient for the characterization of the optimum (see Spinnewijn 2009).

control beliefs determines which IC constraint is binding in the competitive equilibrium. The difference in baseline beliefs determines which IC constraint is binding at the monopolistic optimum.

Control Beliefs and Zero Profit Contracts A control-optimistic type chooses a higher effort level than a type with unbiased beliefs, when given the same contract. The insurer's profit for a contract is increasing in the effort choice of the agent. Hence, conditional on a contract being accepted by both types, an insurer generates more profit from the control-optimistic type than from the unbiased type. In a competitive equilibrium, the expected profit from any contract equals zero. A control-optimistic type can be offered better terms than an unbiased type. By revealed preference, the control-optimistic type always prefers her full information contract to the full information contract offered to the unbiased types. The latter contract would make non-negative profits on the control-optimistic type, but since it is not offered in equilibrium, it must be that the control-optimistic type prefers the former contract. However, the unbiased type may prefer the full information contract offered to the control-optimistic type. This implies the following lemma.

Lemma 4 *If type i is more control-optimistic than type j, the IC constraint for type i is never binding in a separating competitive equilibrium.*

Since the true risk is the same function of effort for both types, the control beliefs need to differ and effort needs to have a non-negligible impact on the outcome for the zero-profit conditions not to coincide. If the zero-profit conditions coincide, types with different beliefs prefer different contracts that satisfy this zero-profit condition. Hence, if only beliefs differ and there is no moral hazard, the full-information contracts are separating. The presence of the one type does not distort the contract offered to another type.

Baseline Beliefs and Outside Options An insuree can always choose the outside option (w_0, Δ_0). The perceived utility increase from taking the contract (w_i, Δ_i) rather than the outside option (w_0, Δ_0) has to be non-negative,

$$\phi^i \left[(w_i, \Delta_i), (w_0, \Delta_0) \right] \geq 0 \text{ for } i = 1, 2.$$

With heterogeneity in beliefs, the perceived expected utility in the outside option $U^i(w_0, \Delta_0)$ is type-dependent.[3] As for any other contract, this perceived expected utility is increasing in the agent's baseline optimism. The wedge between the expected utility levels as perceived by a baseline optimist and by a type with unbiased beliefs is greater the less insurance the outside option provides. The wedge disappears when the

[3] Jullien (2000) analyzes screening contracts when the utility of outside options is type-dependent.

outside option provides full insurance. Baseline optimists require less compensation for an increase in the deductible, but value a decrease in the deductible less. If contracts provide more insurance than the outside option, the pessimistic type is tempted to take the favorable insurance contract offered to the optimistic type. If contracts provides less insurance than the outside option, the optimistic type is tempted to take the favorable incentive contract offered to the pessimistic type. Hence, it is the combination of the insurance provided in the outside option together with the difference in baseline beliefs that determines which incentive compatibility constraint will be binding for the monopolist.

Lemma 5 *In a separating monopolistic optimum with type i more optimistic than type j, the IC constraint is binding for type i and the IR constraint is binding for type j if $\Delta_0 = 0$. The reverse is true if $\Delta_0 = L$.*

4.3 Competitive Equilibrium

I now further characterize the competitive equilibrium. I restrict the analysis to equilibrium contracts that provide more insurance than the outside option. The control beliefs are central. In this subsection, I assume that type 1 is more control-optimistic than type 2 and characterize the competitive equilibrium depending on whether type 1 is more optimistic or more pessimistic.

Assumption 1' *Type* 1 *is more control-optimistic than type* 2.

The contract offered under full information in the competitive equilibrium to type 1 would make negative profit if chosen by type 2. There are two exceptions. Two contracts always make zero profits, regardless of the beliefs of the agent: the *full insurance contract* with $(w, \Delta) = (W - (1 - \pi(0)) L, 0)$ and the *no insurance contract* with $(w, \Delta) = (W, L)$. I show this graphically in Figure 1. The respective zero-profit curves are denoted by Π_1 and Π_2. Both curves connect the full insurance contract (on the 45°-line) and the no insurance contract (on the x-axis). However, the zero-profit curve for type 1 connects contracts that provide more consumption in the good and bad state than the zero-profit curve for type 2. The indifference curves are represented by U_1 and U_2. U_1 crosses U_2 once by the single-crossing property: from above if type 1 is more optimistic, from below if type 1 is more pessimistic. The full-information equilibrium contract for a type is determined by the tangency point between the zero-profit curve and the indifference curve for that type.

The single-crossing property allows to fully characterize the separating equilibrium, if it exists. I first introduce the two contracts (w^h, Δ^h) and (w^l, Δ^l).

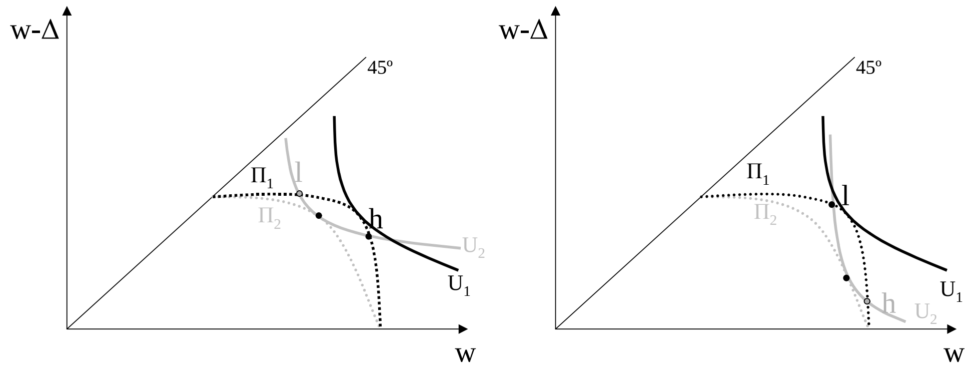

Figure 1: Competitive Equilibrium: Positive vs. Negative Correlation

Definition 4 *Contracts* $\left(w^h, \Delta^h\right)$ *and* $\left(w^l, \Delta^l\right)$ *such that for* $i = h, l$,

$$\begin{cases} \left(w^i, \Delta^i\right) \sim_2 \left(w^*_{c,2}, \Delta^*_{c,2}\right) \\ W - w^i = \left(1 - \pi\left(\hat{e}_i\left(w^i, \Delta^i\right)\right)\right)\left(L - \Delta^i\right), \end{cases}$$

and

$$\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right).$$

Both contracts satisfy the zero-profit condition of type 1 and leave type 2 indifferent with his full-information contract $\left(w^*_{c,2}, \Delta^*_{c,2}\right)$. The contract $\left(w^h, \Delta^h\right)$ provides less insurance coverage at a lower insurance premium than $\left(w^l, \Delta^l\right)$. The contracts are indicated by h and l in Figure 1.

Proposition 3 characterizes the separating equilibrium when type 1 is both more optimistic and more control-optimistic than type 2. I assume that the perceived expected utility is concave in consumption in the good the and the bad state.

Proposition 3 *If a separating equilibrium exists and type* 1 *is both more optimistic and more control-optimistic, the equilibrium contracts equal*

$$\begin{aligned} \left(w^{**}_{c,1}, \Delta^{**}_{c,1}\right) &= \left(w^h, \Delta^h\right) \\ \left(w^{**}_{c,2}, \Delta^{**}_{c,2}\right) &= \left(w^*_{c,2}, \Delta^*_{c,2}\right), \end{aligned}$$

unless $\left(w^*_{c,2}, \Delta^*_{c,2}\right) \succeq_2 \left(w^*_{c,1}, \Delta^*_{c,1}\right)$, *in which case the full-information contracts are separating.*

The presence of type 1 who is more control-optimistic has no impact on the contract offered to type 2, by Lemma 4. The presence of type 2 has no impact on the equilibrium contract offered to type 1 either if the full information equilibrium is separating. For instance, if type 1 is very optimistic, she will not be offered any insurance, regardless of the presence of a pessimistic type. However, if the full information equilibrium is not separating, it is because the more control-pessimistic type 2 prefers type 1's full information contract. Contracts h and l are natural alternatives, since type 2 is exactly indifferent between her full information contract $\left(w_{c,2}^*, \Delta_{c,2}^*\right)$ and these contracts. Contract h will be offered in equilibrium though, since the optimistic type 1 prefers the high deductible contract $\left(w^h, \Delta^h\right) \rhd \left(w_{c,1}^*, \Delta_{c,1}^*\right)$ to the low deductible contract $\left(w^l, \Delta^l\right) \lhd \left(w_{c,1}^*, \Delta_{c,1}^*\right)$ by the single-crossing property. The two types are thus separated by decreasing the insurance coverage for the optimistic type 1. I show this graphically in the left panel of Figure 1. The correlation between the ex-post risk and insurance coverage will be positive by Corollary 1.

Type 1's contract is distorted in the opposite direction if she is more control-optimistic, but at the same time more pessimistic than type 2.

Proposition 4 *If a separating equilibrium exists, utility is concave in consumption and type* 1 *is more control-optimistic, but more pessimistic than type* 2*, the equilibrium contracts are*

$$\left(w_{c,1}^{**}, \Delta_{c,1}^{**}\right) = \left(w^l, \Delta^l\right)$$
$$\left(w_{c,2}^{**}, \Delta_{c,2}^{**}\right) = \left(w_{c,2}^*, \Delta_{c,2}^*\right),$$

unless $\left(w_{c,2}^*, \Delta_{c,2}^*\right) \succeq_2 \left(w_{c,1}^*, \Delta_{c,1}^*\right)$*, in which case the full-information equilibrium is separating.*

The separating equilibrium contract for the control-pessimistic type 2 is still $\left(w_{c,2}^*, \Delta_{c,2}^*\right)$ as a consequence of Lemma 4. However, the pessimistic type 1 now prefers $\left(w^l, \Delta^l\right)$ to $\left(w^h, \Delta^h\right)$, because of the reversed single-crossing property. Since $\left(w^l, \Delta^l\right) \lhd \left(w_{c,2}^*, \Delta_{c,2}^*\right)$, the two types are separated by increasing the insurance coverage for type 1. I show this graphically in the right panel of Figure 1. Type 1 now receives less insurance than type 2. If type 2 is sufficiently control-pessimistic compared to type 1, the correlation between ex post risk and insurance coverage is negative, in line with Corollary 2.

Propositions 3 and 4 together imply that the insurance coverage of the control-optimistic type 1 may be non-monotonic in the baseline beliefs of type 2. When type 2 is sufficiently baseline-pessimistic, type 1's equilibrium contract $\left(w_{c,1}^*, \Delta_{c,1}^*\right)$ is separating. When type 2 becomes more baseline-optimistic, type 1's separating contract becomes $\left(w^h, \Delta^h\right)$, providing less insurance than $\left(w_{c,1}^*, \Delta_{c,1}^*\right)$ by Proposition 3. The more baseline-optimistic type 2 becomes, the more type 1's contract needs to be distorted, providing even less insurance, to keep the contracts separated, i.e. Δ^h increases. If type 2 becomes so baseline-optimistic that despite her relative control-pessimism she becomes more optimistic than type 1, the equilibrium contract for type 1 jumps to $\left(w^l, \Delta^l\right)$ with $\Delta^l < \Delta^h$ by Proposition 4. Hence, the insurance coverage jumps up. This contract may provide more insurance than $\left(w_{c,1}^*, \Delta_{c,1}^*\right)$.[4] Moreover, if type 2's baseline optimism further increases, type 1's insurance coverage decreases again, i.e. Δ^l increases, until eventually the full information contract $\left(w_{c,1}^*, \Delta_{c,1}^*\right)$ becomes separating again.

[4]This holds with certainty if type 1's zero-profit curve is decreasing in $(w, w\text{-}\Delta)$ - space.

I have ignored pooling equilibria in this analysis, which may survive if the single-crossing property does not hold. If the single-crossing property does hold, a pooling contract can never be offered in an equilibrium because of cream skimming (Rothschild and Stiglitz 1976). However, they may inhibit the existence of a separating equilibrium if the ratio of type 2 agents is sufficiently low.

4.4 Monopolistic Optimum

I now further characterize the monopolistic optimum. I allow the outside option to be different than the contract that provides no insurance. The beliefs about the likelihood of the risk are central to the analysis. I again assume that type 1 is more optimistic than type 2 and I characterize the monopolistic optimum depending on the extent to which the agent is insured the outside option. I generalize the methodology in Jullien et al. (2007) who analyze monopolistic screening in the presence of moral hazard and adverse selection due to unobserved heterogeneity in CARA preferences.

Assumption 1 *Type* 1 *is more optimistic than type* 2*.*

If the outside opportunity provides no insurance, the monopolist needs to pay a rent to the pessimist to induce her not to choose the contract offered to the optimist. Since the optimist needs to be compensated less than the pessimist for an increase in risk, the monopolist reduces the rent paid to the pessimist by imposing more risk on the optimist. The separating contract offered to the pessimist, however, is constrained efficient.

If the outside opportunity provides full insurance, it is the optimist who needs to be paid a rent in order to be separated from the pessimist. The monopolist now imposes less risk on the pessimist to reduce the rent paid to the optimist. The separating contract offered to the optimist is constrained efficient. Proposition 5 summarizes these results.

Proposition 5 *If the monopolist separates types, the optimal contract satisfies that*

$$\begin{aligned}\left(w_{m,1}^{**},\Delta_{m,1}^{**}\right) \rhd \left(w_{m,1}^{*},\Delta_{m,1}^{*}\right) \text{ when } \Delta_0 &= L,\\ \left(w_{m,2}^{**},\Delta_{m,2}^{**}\right) \lhd \left(w_{m,2}^{*},\Delta_{m,2}^{*}\right) \text{ when } \Delta_0 &= 0.\end{aligned}$$

In both cases, the contract offered to the other type is constrained efficient.

The monopolist may exclude a type if the profit from this type does not compensate for the rents paid to the other type to induce her to select the contract proposed to her. In that case, the latter type is proposed the full-information contract. If $\Delta_0 = L$ and the agent is optimistic with sufficiently low probability κ, the optimistic type is excluded. If $\Delta_0 = 0$ and the agent is optimistic with sufficiently high probability κ, the pessimistic type is excluded. Finally, a pooling contract may dominate any separating contract. This happens when the solution to the profit-maximizing problem constrained to a binding participation constraint for one type and a binding incentive compatibility constraint for the other type gives more insurance to the optimistic type than to the pessimistic type.[5]

A sufficient condition under which these results generalize for outside opportunities that provide some but not full insurance is that the full-information problem is convex. One special case may arise when the outside opportunity provides partial insurance; if the full information contracts specify deductibles $\Delta_2^* < \Delta_0 < \Delta_1^*$, then these contracts are incentive compatible.[6]

Proposition 6 *If type* 1 *is more optimistic than type* 2 *and the full-information problem is convex, the optimal contract with types separated satisfies*

$$\begin{aligned} \left(w_{m,1}^{**}, \Delta_{m,1}^{**}\right) &\rhd \left(w_{m,1}^{*}, \Delta_{m,1}^{*}\right) \ \textit{ if } \Delta_0 > \max\left\{\Delta_1^*, \Delta_2^*\right\}, \\ \left(w_{m,2}^{**}, \Delta_{m,2}^{**}\right) &\lhd \left(w_{m,2}^{*}, \Delta_{m,2}^{*}\right) \ \textit{ if } \Delta_0 < \min\left\{\Delta_1^*, \Delta_2^*\right\}. \end{aligned}$$

The contract offered to the other type is constrained efficient in both cases. Also, if $\Delta_2^* < \Delta_0 < \Delta_1^*$, $\left(w_{m,i}^{**}, \Delta_{m,i}^{**}\right) = \left(w_{m,i}^{*}, \Delta_{m,i}^{*}\right)$ *for* $i = 1, 2$.

If both contracts are incentive contracts (i.e. $\Delta_i > \Delta_0$), the optimistic type receives a rent and the contract for the pessimistic type is distorted towards less incentives. If both contracts are insurance contracts (i.e. $\Delta_i < \Delta_0$), the pessimistic type receives a rent and the contract of the optimistic type is distorted towards less insurance.

5 Welfare Analysis

The normative implications of heterogeneity in risk perceptions are twofold. First, the presence of one type imposes an externality on the other type if the insurer changes the terms of the contracts to separate types. Second, this externality may aggravate or reduce the distortion due to an agent's bias in beliefs.

I first focus on the informational externality biased agents impose on unbiased agents. In equilibrium, competing insurers offer contracts that maximize the insuree's perceived expected utility and make zero profits in expectation. Insurees with different perceptions have different preferences. Hence, if the set of contracts that make zero profits is the same for two types of insurees, the presence of the one does not affect

[5] Notice that with CARA preferences and monetary costs of efforts, as considered in Jullien et al. (2007) and in Section 6, the monopolist pays a rent to the potentially imitating agent i by increasing w_i, but keeping Δ_i unchanged at Δ_i^*. Hence, if for the full-information contracts $\Delta_1^* \geq \Delta_2^*$, then a pooling contract can never be optimal.

[6] If for a given outside opportunity (w_0, Δ_0) the full information contracts are such that $\Delta_1^* < \Delta_0 < \Delta_2^*$, then the two types are likely to be pooled. This 'irregular case' is also treated by Jullien et al. (2007).

the contract offered to the other. Moral hazard and disagreement about the returns to effort are therefore necessary ingredients for informational externalities to occur in the competitive equilibrium. If these ingredients are present, the insurance contract for the more control-optimistic type may be distorted compared to the full-information contract by Lemma 4. The contract offered to the more control-pessimistic type is unchanged. This has immediate welfare implications for an agent with unbiased beliefs, in line with Proposition 3 and 4.

Corollary 3 *In a separating competitive equilibrium, an agent with unbiased beliefs never gains from the presence of an agent with biased beliefs and may strictly lose only if that agent is control-pessimistic.*

In a separating monopolistic optimum, the contracts offered to both types may change when the insurees' perceptions are not observable. If one IC constraint binds at the optimum, one type receives a rent not to switch to the other type's contract. The first type ends up strictly better off than with the full-information contract. The second type will still be indifferent about switching to the outside option, but her contract is distorted compared to the full-information contract to reduce the rent paid to the first type. The cases in which an agent with unbiased beliefs is paid a rent follow immediately from Proposition 6.

Corollary 4 *In a separating monopolistic optimum, an agent with unbiased beliefs gains from the presence of a pessimistic agent when offered incentive contracts and from the presence of an optimistic agent when offered insurance contracts.*

These results generalize for an agent with biased beliefs in terms of her perceived expected utility, but not necessarily in terms of her true expected utility. Evaluated in terms of true expected utility, competing insurers offer too little insurance to baseline-optimistic or control-optimistic agents in an equilibrium without private information. Profit-maximizing insurers respond to the low perceived value of insurance for a baseline-optimistic insuree and do not correct the high incentives due to control-optimistic beliefs. Increasing the insurance coverage would increase the agent's true expected utility, when

$$\{\hat{\pi}(e) - \pi(e)\} \geq \left\{\pi'(e) - \hat{\pi}'(e)\right\} \frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)}. \tag{1}$$

This is Corollary 3 in Spinnewijn (2009). Hence, heterogeneity in risk perceptions may aggravate or mitigate the distortion due to biases in beliefs. In a competitive equilibrium with optimistic agents for whom (1) holds, the insurance coverage and therefore the true expected utility decreases further due to heterogeneity in risk perception if types who are more control-optimistic are also more optimistic. This increases the scope for government intervention through insurance mandates (Rothschild and Stiglitz 1976).

In general, the heterogeneity in risk perceptions creates screening opportunities for the insurers. This is in contrast with the conclusions in Sandroni and Squintani (2007). Central to their analysis is that some types of agents perceive their risk to be the same, although their true risk is different. These agents are necessarily pooled in any equilibrium. This is exactly opposite to the analysis here where agents with the same risk are separated based on their heterogeneity in beliefs. In particular, Sandroni and Squintani (2007) analyze the case where some of the high-risk type agents are optimistic about being a low-risk type agent and always choose the contract designed for the latter. The low-risk type therefore necessarily subsidizes the optimistic high-risk type in equilibrium. Given the higher insurance premium, it may be that the low-risk type agent does not prefer to have more insurance than what is offered in a separating equilibrium. Since the incentive compatibility constraint is not binding, mandating insurance is detrimental to low-risk agents in their analysis.

6 Example: Continuous Output and Linear Contracts

In this section, I consider an example with a continuum of states. I characterize the optimal linear screening contracts in the framework considered by Holmström and Milgrom (1987).

Output $q = \theta e + \varepsilon$ is additive in effort and a normal noise term ε with variance σ^2. The return to effort depends on the agent's ability θ. However, the agent may perceive her ability differently. I consider two types. Type 1 perceives the return to effort to be $\hat{\theta}_1$. Type 2 perceives this return to be $\hat{\theta}_2$, with $\hat{\theta}_1 > \hat{\theta}_2$. Hence, type 1 is both more baseline-optimistic and control-optimistic, and thus more optimistic than type 2 by Lemma 1. The agent is of type 1 with probability κ. Both types have the same constant absolute risk aversion η and face the same monetary costs of effort $\psi\frac{e^2}{2}$. The outside option is the same, but the perceived expected utility of the outside option depends on the ability perception. Denote the respective expected utility levels by $\hat{u}_1$ and $\hat{u}_2$.

Given a linear contract $t + sq$, the certainty equivalent of type i's perceived utility equals

$$CE_i\left(s,t\right) = t + s\hat{\theta}_i e - \psi\frac{e^2}{2} - \frac{\eta}{2}s^2\sigma^2.$$

Type i's effort choice is

$$\hat{e}_i\left(s\right) = \frac{\hat{\theta}_i}{\psi}s.$$

The expected change in output in response to a change in the sharing rule, $\frac{dEq_i}{ds_i}$, equals $\theta\frac{\hat{\theta}_i}{\psi}$ and thus depends both on the true and perceived ability. The expected profit for the insurer equals

$$\Pi_i(s,t) = (1-s)\,\theta\hat{e}_i(s) - t.$$

I first characterize the optimal linear sharing rule with full information as a benchmark.

Result 1 *With θ and $\hat{\theta}_i$ the true and perceived ability, the linear sharing contract offered by a monopolist or competing insurers under full information determines a linear sharing rule*

$$s_i^* = \left[1 + \frac{1}{\hat{\theta}_i\theta}\psi\eta\sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta}\right]^{-1} \equiv \left[\Xi_i - \frac{\hat{\theta}_i - \theta}{\theta}\right]^{-1}.$$

The contract is more performance-dependent the more optimistic the agent is about her ability. An increase in $\hat{\theta}_i$ increases the agent's perceived ability to increase output and thus the responsiveness to incentives $\frac{dEq_i}{ds_i}$. An increase in $\hat{\theta}_i$ also increases the probabilistic weight the agent puts on states with high output. Private insurers respond to this baseline optimism by decreasing $[s_i^*]^{-1}$ by $\frac{\hat{\theta}_i-\theta}{\theta}$. The linear sharing rule that maximizes the true expected utility is not affected by baseline optimism, but corrects for control optimism. The socially optimal sharing rule equals $\left[\Xi_i + \frac{\hat{\theta}_i-\theta}{\theta}\right]^{-1}$, providing less incentives (and more insurance) than the competitive sharing rule when $\hat{\theta}_i > \theta$.

When types cannot be distinguished, the full information contracts may not be incentive compatible. In any equilibrium, the optimist's contract is at least as performance-dependent as the pessimist's contract. This result is in line with Proposition 1.

Result 2 *In any incentive compatible contract, type 1's contract is more performance-dependent than type 2's contract,*

$$s_1^{**} \geq s_2^{**} \text{ and } t_1^{**} \leq t_2^{**}.$$

In competition, the pessimist prefers the full information contract offered to the optimist. The profit for a given contract is higher when it is accepted by an agent who is optimistic about her ability and therefore exerts more effort. For the insurer to make zero profit, the fixed transfer t given to the optimist will thus be relatively high. Hence, with asymmetric information about the agent's beliefs, the contract of the optimist is distorted more towards performance to discourage the pessimist from switching to the optimist's contract. This result is in line with Proposition 3.

Result 3 *In any competitive equilibrium, the performance-dependence is distorted upward for type* 1*,*

$$s_{c,1}^{**} \geq s_1^* \text{ and } s_{c,2}^{**} = s_2^*.$$

With a monopolist, the outside opportunities are relevant. I denote by $\hat{u}_i$ the perceived expected utility of the outside option for type i. If the perceived expected utility from the outside option is the same for the optimist and the pessimist (i.e. $\hat{u}_1 = \hat{u}_2$), the monopolist can pay a relatively low transfer t to the optimist to convince her to take the contract. The optimist prefers the monopolistic full information contract offered to the pessimist. In the separating optimum, the optimist receives a rent and the linear sharing rule for the pessimist is distorted downwards, providing less incentives. If the optimist's perceived expected utility of the outside option is higher than some upper bound h, the monopolist needs to pay a relatively high transfer t to convince the optimist to take the contract. The pessimist then prefers the full information contract offered to the optimist. In the separating optimum, the linear sharing rule of the optimist's contract is distorted upward, providing more incentives, unless the optimists are excluded. If the optimist's perceived expected utility of the outside option is below this upper bound h, but above some lower bound l ($> \hat{u}_2$), the full-information contracts are incentive compatible.[7] This is summarized in the following result.

Result 4 *If the monopolist separates type* 1 *and type* 2 *with* $\hat{\theta}_1 > \hat{\theta}_2$*, the linear sharing rules of the separating contracts equal*

$$\left\{s^{**}_{m,1}, s^{**}_{m,2}\right\} = \begin{cases} \{[\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta}]^{-1}, [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta} + \frac{\kappa}{1-\kappa}\frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta\hat{\theta}_2}]^{-1}\} \text{ for } \hat{u}_1 < l \\ \{[\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta} - \frac{1-\kappa}{\kappa}\frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta\hat{\theta}_1}]^{-1}, [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta}]^{-1}) \text{ for } \hat{u}_1 > h. \end{cases}$$

The sharing rules are the same as in the full-information contracts $\left(s^{*}_{m,1}, s^{*}_{m,2}\right)$ *for* $\hat{u}_1 \in [l, h]$.

If $\hat{\theta}_1 > \hat{\theta}_2 > \theta$, private insurers provide too little insurance to the optimistic workers. The screening distortion aggravates this distortion in the competitive equilibrium. In the monopolistic case, this is only true if the optimistic type perceives the expected utility of the outside option to be sufficiently higher than the pessimistic type.

7 Conclusion

People often face the same risk, but may have very different perceptions about the likelihood and the controllability of the risk. I analyze how insurance companies separate people based on the heterogeneity in their perceptions. People with heterogeneous abilities or risks, but identical perceptions cannot be separated. People with different perceptions can be separated with a menu of screening contracts, even when the true abilities or risks are the same.

The differences in perceptions are at the heart of adverse selection and the distinction between baseline and control beliefs is essential. First, I show that contracts offer less insurance and thus provide more incentives to optimists than to pessimists. This very general result does not depend on the nature of competition. Second, I analyze

[7] The bounds are $l \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi}(s^*_2)^2$ and $h \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi}(s^*_1)^2$, as derived the appendix.

which type's contract is distorted due to the heterogeneity in beliefs. This analysis crucially depends on the nature of competition. A monopolistic insurer distorts the contract offered to the type with low willingness to pay. Competing insurers distort the contract offered to the type to whom insurance can be provided at low cost. Heterogeneity in beliefs, in contrast with heterogeneity in risks, drives a wedge between the insuree's willingness to pay for insurance and the insurer's cost of providing insurance.

Heterogeneity in beliefs can explain why contracts offer too little insurance in some markets (e.g., health insurance, car insurance) and in other markets provide no incentives at all, although a small risk suffices to induce effort (e.g., no-limit contracts on rented cars mileage, cell phone usage). However, unobserved differences in risk or ability could have the same positive implications. Heterogeneity in beliefs can also explain why in many insurance markets the correlation between risk occurrence and insurance coverage is negative. However, unobserved differences in preferences could explain this as well. Identifying to what extent results are driven by true differences or misperceived differences, or by heterogeneity in beliefs or heterogeneity in preferences is clearly a challenge (Manski 2004). The distinction is however essential for policy and welfare analysis. It seems that the natural approach to quantify the importance of heterogeneity in beliefs is to identify beliefs directly, either by eliciting expectations through surveys (Spinnewijn 2009) or by relating anomalies in behavior to biases in perceptions (Köszegi and Rabin 2007). These challenging approaches have been followed recently in the literature on CEO compensation (Landier and Thesmar 2009, Malmendier, Tate and Yan 2007). Extending these empirical approaches to the analysis of private insurance markets and social insurance schemes seems a promising avenue for future research.

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Appendix: Proofs

Proof of Lemma 1

If $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ for any e and $\hat{e}_i(c) = \hat{e}_j(c)$, than $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$ for any c. If also $\hat{\pi}_i'(e) \geq \hat{\pi}_j'(e)$ for any e, than $\hat{e}_i(c) \geq \hat{e}_j(c)$ for any c. The Lemma follows, since $\hat{\pi}'(e) > 0$.□

Proof of Lemma 2

The marginal rate of substitution (MRS) between Δ and w equals

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(c))\,u'(w) + (1-\hat{\pi}_i(\hat{e}_i(c)))\,u'(w-\Delta)}{(1-\hat{\pi}_i(\hat{e}_i(c)))\,u'(w-\Delta)} = \frac{\hat{\pi}_i(\hat{e}_i(c))}{(1-\hat{\pi}_i(\hat{e}_i(c)))}\frac{u'(w)}{u'(w-\Delta)} + 1.$$

Since $\frac{u'(w)}{u'(w-\Delta)} > 0$, the MRS is increasing in $\hat{\pi}_i(\hat{e}_i(c))$. The lemma follows, since $\hat{\pi}_1(\hat{e}_1(c)) \geq \hat{\pi}_2(\hat{e}_2(c))$ for any c.□

Proof of Lemma 3

Given $\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j)\frac{w_i - w_j}{\Delta_i - \Delta_j}$,

$$U_w^i(\widetilde{w}(\Delta),\Delta)\,\widetilde{w}'(\Delta) + U_\Delta^i(\widetilde{w}(\Delta),\Delta) =$$
$$\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta),\Delta))\,u'(\widetilde{w}(\Delta))\frac{w_1-w_2}{\Delta_1-\Delta_2} - (1-\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta),\Delta)))\,u'(\widetilde{w}(\Delta)-\Delta)\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2}.$$

Hence,

$$\phi^1[(w_1,\Delta_1),(w_2,\Delta_2)] - \phi^2[(w_1,\Delta_1),(w_2,\Delta_2)] =$$
$$\int_{\Delta_2}^{\Delta_1}\{[\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) - \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))]\times$$
$$[u'(\widetilde{w}(\Delta))\frac{w_1-w_2}{\Delta_1-\Delta_2} + u'(\widetilde{w}(\Delta)-\Delta)\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2}]\}d\Delta.$$

Since $\frac{w_1-w_2}{\Delta_1-\Delta_2} > 0$ and $\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2} > 0$, given $(w_1,\Delta_1) \rhd (w_2,\Delta_2)$, the term in the integral between squared brackets is greater than zero. Since the integration is from Δ_2 to Δ_1 with $\Delta_1 > \Delta_2$,

$$\phi^1[(w_1,\Delta_1),(w_2,\Delta_2)] - \phi^2[(w_1,\Delta_1),(w_2,\Delta_2)] > 0,$$

if $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) \geq \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for all $\Delta \in [\Delta_2,\Delta_1]$ and $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) > \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for some $\Delta \in [\Delta_2,\Delta_1]$.□

Proof of Lemma 4

If type 1 is (strictly) more control-optimistic than type 2, $\Pi_1(w,\Delta) > \Pi_2(w,\Delta)$. In any separating equilibrium,

$$\Pi_1(w_1,\Delta_1)=\Pi_2(w_2,\Delta_2)=0.$$

Assume by contradiction that $(w_2,\Delta_2)\sim_1(w_1,\Delta_1)$ in a separating equilibrium, but $\Pi_2(w_2,\Delta_2)=0$. Hence, if preferences are continuous and the single-crossing property is satisfied, an insurer can change the contract (w_2,Δ_2) to (w_2',Δ_2') such that $\Pi_2(w_2',\Delta_2')=0$, but $(w_2',\Delta_2')\succ_1(w_1,\Delta_1)$ and $\Pi_1(w_2',\Delta_2')>0$. This is a profitable deviation.□

Proof of Lemma 5

If $\Delta_0=L$, then for any interior solution $\Delta_1<\Delta_0$ by the risk aversion of the agent and the risk neutrality of the principal. This implies $\phi^2[(w_1,\Delta_1),(w_0,\Delta_0)]>\phi^1[(w_1,\Delta_1),(w_0,\Delta_0)]$. Given the binding IC/IR constraints, this implies

$$\phi^2[(w_2,\Delta_2),(w_0,\Delta_0)]=\phi^2[(w_1,\Delta_1),(w_0,\Delta_0)]>0.$$

Moreover, since $(w_1,\Delta_1)>(w_2,\Delta_2)$ if the contracts are separating, $\phi^1[(w_1,\Delta_1),(w_0,\Delta_0)]>\phi^1[(w_2,\Delta_2),(w_0,\Delta_0)]$ by Lemma 3. This in turn implies that $\phi^1[(w_1,\Delta_1),(w_0,\Delta_0)]=0$.

If $\Delta_0=0$, then for any interior solution $\Delta_2>\Delta_0$, since the agent cannot overinsure. This implies $\phi^1[(w_2,\Delta_2),(w_0,\Delta_0)]>\phi^2[(w_2,\Delta_2),(w_0,\Delta_0)]$. Given the binding IC/IR constraints, this implies

$$\phi^1[(w_1,\Delta_1),(w_0,\Delta_0)]=\phi^1[(w_2,\Delta_2),(w_0,\Delta_0)]>0.$$

Moreover, since $(w_1,\Delta_1)>(w_2,\Delta_2)$ if the contracts are separating, $\phi^2[(w_2,\Delta_2),(w_0,\Delta_0)]>\phi^2[(w_1,\Delta_1),(w_0,\Delta_0)]$ by Lemma 3. This in turn implies that $\phi^2[(w_2,\Delta_2),(w_0,\Delta_0)]=0$.□

Proof of Proposition 1

Assume, by contradiction, that $(w_2,\Delta_2)\rhd(w_1,\Delta_1)$, although type 1 is more optimistic than type 2. In that case,

$$\phi^1[(w_2,\Delta_2),(w_1,\Delta_1)]\geq\phi^2[(w_2,\Delta_2),(w_1,\Delta_1)]\geq0,$$

by Lemma 3. But this contradicts $\phi^1[(w_1,\Delta_1),(w_2,\Delta_2)]\geq0$, since $\phi^1[x,y]=-\phi^1[y,x]$.□

Proof of Proposition 2

The competing insurers and the monopolist solve

$$\max\hat{\pi}(e)[u(w)-u(w-\Delta)]+u(w-\Delta)-e$$

such that

$$\begin{aligned}\hat{\pi}'(e)\left[u(w)-u(w-\Delta)\right] &= 1\\ W-w-(1-\pi(e))\left[L-\Delta\right] &\geq \bar{\Pi},\end{aligned}$$

where $\bar{\Pi}$ equals 0 in the competitive equilibrium and $\bar{\Pi}$ equals the expected profits such that $U(w,\Delta)=U(w_0,\Delta_0)$ in the monopolistic optimum.

I define the insurance coverage $b=w-\Delta$ and the tax on the good state $\tau=W-w$, as in Spinnewijn (2009). Denote by $\hat{e}_i(b)$ the level of effort and $\hat{\tau}_i(b)$ the tax that solve the IC constraint and the respective profit constraints for a given level of insurance coverage b with $i=c,m$. The change in the required tax τ when the insurance coverage increases equals

$$\tau_i'(b)=\frac{1-\pi(\hat{e}_i(b))}{\pi(\hat{e}_i(b))}\left[1+\frac{\tau_i(b)+b-W+L}{b}\varepsilon_{1-\pi(\hat{e}_i(b)),b}\right].$$

Notice that $\bar{\Pi}$ does not enter the expression for this derivative directly.

The first order condition with respect to b gives

$$(1-\hat{\pi}(\hat{e}_i(b)))\,u'(b)-\hat{\pi}(\hat{e}_i(b))\,u'(w-\tau_i(b))\,\tau_i'(b)=0.$$

The effect through effort is of second order by the envelope condition. Plugging in for $\tau_i'(b)$ and rewriting the expression, one finds

$$\frac{\frac{1-\hat{\pi}(\hat{e}_i(b))}{1-\pi(\hat{e}_i(b))}u'(b)-\frac{\hat{\pi}(\hat{e}(b))}{\pi(\hat{e}(b))}u'(w-\tau_i(b))}{\frac{\hat{\pi}(\hat{e}_i(b))}{\pi(\hat{e}_i(b))}u'(w-\tau_i(b))}=\frac{\tau_i(b)+b-W+L}{b}\varepsilon_{1-\pi(\hat{e}_i(b)),b}.$$

With $b=w-\Delta$ and $\tau=W-w$, the proposition immediately follows.□

Proof of Proposition 3

If the full-information contracts are incentive compatible, the competitive equilibrium coincides with the full-information equilibrium. If the full-information contracts are not incentive compatible, then the IC constraint of type 1 is still not binding in a separating equilibrium, by Lemma 4. Hence, if a separating equilibrium exists, type 2's equilibrium contract equals the full-information contract. The actuarial contract that maximizes the perceived expected utility of type 1, but is not strictly preferred by type 2 to its full-information contract is (w^h,Δ^h). Since $(w^h,\Delta^h)\sim_2(w^l,\Delta^l)$ and $(w^h,\Delta^h)\rhd(w^l,\Delta^l)$, $(w^h,\Delta^h)\succ_1(w^l,\Delta^l)$ by Lemma 3. Then, since utility is concave in consumption, $(w^h,\Delta^h)\succ_1(w,\Delta)$ for any actuarial contract for which $(w,\Delta)\rhd(w^h,\Delta^h)$ or $(w,\Delta)\lhd(w^l,\Delta^l)$. Hence, type 1's equilibrium contract equals (w^h,Δ^h).□

Proof of Proposition 4

The proof is analogue to the proof of Proposition 3. Since the single-crossing property is reversed now, $\left(w^h, \Delta^h\right) \sim_2 \left(w^l, \Delta^l\right)$ and $\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right)$ imply that $\left(w^l, \Delta^l\right) \succ_1 \left(w^h, \Delta^h\right)$. Type 1's equilibrium contract equals $\left(w^l, \Delta^l\right)$.□

Proof of Proposition 5

The monopolist solves

$$\max \kappa \left\{W - w_1 - \left(1 - \pi\left(\hat{e}_1(1)\right)\right)\left[L - \Delta_1\right]\right\} + (1-\kappa)\left\{W - w_2 - \left(1 - \pi\left(\hat{e}_2(2)\right)\right)\left[L - \Delta_2\right]\right\}$$

such that

$$\begin{aligned}
\phi_1\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right) &= \max\left\{0, \phi_1\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right)\right\} \\
\phi_2\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right) &= \max\left\{0, \phi_2\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right)\right\}.
\end{aligned}$$

If $\Delta_0 = L$, then by Lemma 5, the IC/IR constraints for a separating optimum simplify to[8]

$$\begin{aligned}
\phi_1\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right) &= 0 \\
\phi_2\left(\left(w_2, \Delta_2\right), \left(w_1, \Delta_1\right)\right) &= 0.
\end{aligned}$$

Assume $(w_1, \Delta_1) < (w_1^*, \Delta_1^*)$, then $\phi_2\left(\left(w_1, \Delta_1\right), \left(w_1^*, \Delta_1^*\right)\right) > 0$ by Lemma 3, so the utility rent paid to type 2 implied by the binding IC constraint is higher if type 1 receives (w_1, Δ_1) rather than (w_1^*, Δ_1^*). Since the profit made on type 1 is higher in (w_1^*, Δ_1^*) as well, (w_1, Δ_1) can never be optimal. Hence, $(w_1, \Delta_1) \geq (w_1^*, \Delta_1^*)$.

Assume $(w_1, \Delta_1) = (w_1^*, \Delta_1^*)$, one can find a contract (w_1', Δ_1') such that $(w_1', \Delta_1') \sim_1 (w_1^*, \Delta_1^*)$ and $(w_1', \Delta_1') > (w_1^*, \Delta_1^*)$ (and thus, $(w_1', \Delta_1') \prec_2 (w_1^*, \Delta_1^*)$), but sufficiently close to (w_1^*, Δ_1^*) such that the loss in profit on type 1 is of second order, but the reduction in the rent paid to type 2 is of first order. Hence, $(w_1, \Delta_1) > (w_1^*, \Delta_1^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 1 is slack. Hence, the problem becomes separable for type 2. The contract (w_2, Δ_2) solves the full information problem with type 2's outside opportunity equal to (w_1, Δ_1). Clearly, this contract is efficient.

If $\Delta_0 = 0$, then by Lemma 5, the IC/IR constraints for a separating optimum simplify to[9]

$$\begin{aligned}
\phi_1\left(\left(w_1, \Delta_1\right), \left(w_2, \Delta_2\right)\right) &= 0 \\
\phi_2\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right) &= 0.
\end{aligned}$$

[8] Notice that the optimum is only separating if the solution of this simplified constrained problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \geq (w_2, \Delta_2)$. If not, the two types are pooled.

[9] Again, the optimum is only separating if the solution of this problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \geq (w_2, \Delta_2)$.

Assume $(w_2, \Delta_2) > (w_2^*, \Delta_2^*)$, then $\phi_1((w_2, \Delta_2), (w_2^*, \Delta_2^*)) > 0$ by Lemma 3, so the utility rent paid to type 1 implied by the binding IC constraint is higher if type 2 receives (w_2, Δ_2) rather than (w_2^*, Δ_2^*). Since the profit made on type 2 is higher in (w_2^*, Δ_2^*) as well, (w_2, Δ_2) can never be optimal. Hence, $(w_2, \Delta_2) \leq (w_2^*, \Delta_2^*)$.

Assume $(w_2, \Delta_2) = (w_2^*, \Delta_2^*)$, one can find a contract (w_2', Δ_2') such that $(w_2', \Delta_2') \sim_2 (w_2^*, \Delta_2^*)$ and $(w_2', \Delta_2') < (w_2^*, \Delta_2^*)$ (and thus, $(w_2', \Delta_2') \prec_1 (w_2^*, \Delta_2^*)$), but sufficiently close to (w_2^*, Δ_2^*) such that the loss in profit on type 2 is of second order, but the reduction in the rent paid to type 1 is of first order. Hence, $(w_2, \Delta_2) < (w_2^*, \Delta_2^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 2 is slack. Hence, the contract (w_1, Δ_1) solves the full information problem with type 1's outside opportunity equal to (w_2, Δ_2). Clearly, this contract is efficient.□

Proof of Proposition 6

If $\Delta_0 < \min\{\Delta_1^*, \Delta_2^*\}$, then any contract $(w_2, \Delta_2) < (w_0, \Delta_0)$ such that $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$, is dominated by offering (w_0, Δ_0) to type 2 and (w_1^*, Δ_1^*) to type 1. These contracts are incentive compatible by Lemma 3. Given that the full-information problem is convex and $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$, the insurer makes lower profit when offering $(w_2, \Delta_2) < (w_0, \Delta_0)$. Moreover, the insurer cannot make higher profits than the full-information profits on type 1. Any contract $(w_1, \Delta_1) < (w_0, \Delta_0)$ such that $(w_1, \Delta_1) \succeq_1 (w_1^*, \Delta_1^*)$ is also dominated by offering (w_0, Δ_0) to type 2 and (w_1^*, Δ_1^*) to type 1. Again, the profit on type 1 cannot be higher than in (w_1^*, Δ_1^*). Moreover, by Lemma 3, $(w_1, \Delta_1) \succeq_2 (w_0, \Delta_0)$ and thus incentive compatibility requires $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$. Again, given that the full-information problem is convex and $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$, the insurer makes lower profit when offering a contract $(w_2, \Delta_2) < (w_0, \Delta_0)$ rather than (w_0, Δ_0). Hence, both (w_1, Δ_1) and (w_2, Δ_2) need to be greater than the outside option (w_0, Δ_0) in order to be optimal. If $\Delta_0 > \max\{\Delta_1^*, \Delta_2^*\}$, the argument is exactly the same, mutatis mutandum. In this case, both (w_1, Δ_1) and (w_2, Δ_2) need to be smaller than the outside option (w_0, Δ_0) in order to be optimal.

Now, if either

$$(w_1, \Delta_1) > (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) > (w_0, \Delta_0)$$

or

$$(w_1, \Delta_1) < (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) < (w_0, \Delta_0),$$

Lemma 5 and exactly the same argument as in Proposition 5 applies. This proves the first part of the proposition.

If $\Delta_2^* < \Delta_0 < \Delta_1^*$, $\phi_1[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] = 0$ implies that $\phi_2[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] < 0$ and $\phi_2[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] = 0$ implies that $\phi_1[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] < 0$, by Lemma 3. Hence, both $\phi_1[(w_1^*, \Delta_1^*), (w_2^*, \Delta_2^*)] \geq 0$ and $\phi_2[(w_2^*, \Delta_2^*), (w_1^*, \Delta_1^*)] \geq 0$. The incentive compatibility constraints are satisfied for the full-information contracts.□

Proof of Result 1

The optimal linear sharing rule maximizes the total surplus, that is the sum of the perceived certainty equivalent for the agent and the expected profit for the firm. With $\hat{e}_i(s) = \frac{\hat{\theta}_i}{\psi} s$, this simplifies to

$$\max_s (1-s)\,\theta\hat{\theta}_i \frac{s}{\psi} + s\hat{\theta}_i^2 \frac{s}{\psi} - \frac{\hat{\theta}_i^2}{\psi}\frac{s^2}{2} - \frac{\eta}{2}s^2\sigma^2.$$

The derivation of this problem yields

$$s_i^* = \frac{1}{1 + \frac{1}{\hat{\theta}_i\theta}\psi\eta\sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta}},$$

for both the competitive and the monopolistic contract. The performance-independent transfer t is such that the profit of the firms is zero in competition and such that the perceived expected utility of the agent equals $\hat{u}_i$ with a monopolist.□

Proof of Result 2

For an agent with belief $\hat{\theta}$, the marginal rate of substitution between s and t equals

$$\left.\frac{ds}{dt}\right|_{\hat{\theta}} = \hat{\theta}\hat{e}(s) - \eta\sigma^2.$$

Hence, $\left.\frac{ds}{dt}\right|_{\hat{\theta}_1} \geq \left.\frac{ds}{dt}\right|_{\hat{\theta}_2}$, since $\hat{\theta}_1 \geq \hat{\theta}_2$. Given this single-crossing property, any competitive equilibrium must satisfy $s_1 > s_2$ and $t_1 < t_2$.□

Proof of Result 3

Since type 1 exerts more effort for any given contract, any contract making zero profit from type 2 makes non-negative profits from type 1. Hence, by revealed preference, type 1 prefers his full information contract. If type 2 prefers type 1's full information contract as well, type 1 will be given the contract that makes zero profit from type 1 and makes type 2 indifferent between the two contracts. Hence,

$$t^h = \left(1 - s^h\right)\theta\hat{\theta}_1 \frac{s^h}{\psi}$$

and

$$(1 - s_2^*)\,s_2^* \frac{\theta\hat{\theta}_2}{\psi} + (s_2^*)^2 \left[\frac{\left(\hat{\theta}_2\right)^2}{2\psi} - \eta\sigma^2\right] = \left(1 - s^h\right)s^h \frac{\theta\hat{\theta}_1}{\psi} + \left(s^h\right)^2 \left[\frac{\left(\hat{\theta}_2\right)^2}{2\psi} - \eta\sigma^2\right].$$

The result follows.□

Proof of Result 4

The monopolist solves

$$\max \kappa \left[\left(1-s_1\right)\theta e_1 - t_1 \right] + \left(1-\kappa\right)\left[\left(1-s_2\right)\theta e_2 - t_2\right]$$

such that

$$\begin{aligned} t_i + s_i\hat{\theta}_i e_i - \psi\frac{e_i^2}{2} - \frac{\eta}{2}s_i^2\sigma^2 &\geq t_j + s_j\hat{\theta}_i e_i^j - \psi\frac{\left(e_i^j\right)^2}{2} - \frac{\eta}{2}s_j^2\sigma^2 && (IC_i) \\ t_i + s_i\hat{\theta}e_i - \psi\frac{e_i^2}{2} - \frac{\eta}{2}s_i^2\sigma^2 &\geq \hat{u}_i && (IR_i) \end{aligned}$$

and

$$e_i = \frac{\hat{\theta}_i}{\psi}s_i \text{ and } e_i^j = \frac{\hat{\theta}_i}{\psi}s_j \text{ for } i,j = 1,2.$$

First, consider the case that IC_2 and IR_1 are binding, then type 2 is given a rent

$$R = s_1\left(\hat{\theta}_2 e_2^1 - \hat{\theta}_1 e_1\right) - \psi\frac{\left(\left(e_2^1\right)^2 - \left(e_1\right)^2\right)}{2}$$

such that

$$t_2 + s_2\hat{\theta}_2 e_2 - \psi\frac{e_2^2}{2} - \frac{\eta}{2}s_2^2\sigma^2 = \hat{u}_1 + R.$$

The monopolist's problem simplifies to

$$\begin{aligned} \max \kappa & \left[\theta e_1 + s_1\left(\hat{\theta}_1 - \theta\right)e_1 - \psi\frac{\left(e_1\right)^2}{2} - \frac{\eta}{2}s_1^2\sigma^2\right] \\ & + \left(1-\kappa\right)\left[\theta e_2 + s_2\left(\hat{\theta}_2 - \theta\right)e_2 - \psi\frac{\left(e_2\right)^2}{2} - \frac{\eta}{2}s_2^2\sigma^2 - R\right] \end{aligned}$$

with

$$e_i = \frac{\hat{\theta}_i}{\psi}s_i \text{ for } i = 1,2.$$

Except for the rent R paid to type 2, this problem is the same as in the full-information problem. Since R only depends on s_1, the optimal sharing rule for type 2 is the same as in the full-information problem. The monopolist reduces the rent by the distorting the sharing rule for type 1. The result in the proposition follows immediately from differentiating with respect to s_1.

Second, consider the case that IC_1 and IR_2 are binding. The problem is the same, mutatis mutandum. A rent is paid to type 1 now, so only the sharing rule for type 2 will be different from the full-information problem. The sharing rule for type 2 is distorted compared to the full-information problem.

Finally, from the IC constraints, we find that the full-information sharing rules for type 1 and type 2 are optimal if respectively

$$\hat{u}_1 \geq \hat{u}_2 + \frac{\left(\hat{\theta}_1\right)^2 - \left(\hat{\theta}_2\right)^2}{2\psi} \left(s_2^*\right)^2$$

and

$$\hat{u}_2 \geq \hat{u}_1 + \frac{\left(\hat{\theta}_2\right)^2 - \left(\hat{\theta}_1\right)^2}{2\psi} \left(s_1^*\right)^2 .$$

Since $\hat{\theta}_1 > \hat{\theta}_2$ and $s_1^* > s_2^*$, it immediately follows that for $\hat{u}_1 < l \equiv \hat{u}_2 + \frac{\left(\hat{\theta}_1\right)^2 - \left(\hat{\theta}_2\right)^2}{2\psi} \left(s_2^*\right)^2$, IC_1 and IR_2 are binding. For $\hat{u}_1 \in [l, h]$ with $h \equiv \hat{u}_2 + \frac{\left(\hat{\theta}_1\right)^2 - \left(\hat{\theta}_2\right)^2}{2\psi} \left(s_1^*\right)^2$, the full-information contracts are optimal. For $\hat{u}_1 > h$, IC_2 and IR_1 are binding.□