A normative justification of compulsory education

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Abstract

Using a household production model of educational choices, we characterise a free market situation in which some agents ("high-wagers") educate their children full-time and spend a sizable amount of resources on them, while others ("low-wagers") educate them only partially. The free-market equilibrium is inefficient and iniquitous. Public policy is thus called for: however, redistributive taxation alone is counter-productive, as it forces some agents to move away from full-time education for their kids, and educational price subsidies are only moderately effective, since they only work on the intensive margin. It is instead socially optimal to introduce a compulsory education package, using a redistributive tax system to finance it. Redistributive taxation and compulsory education are therefore best seen as complementary policies.

I Introduction

In what sense might parents constrain rather then favour the development of their children? Mostly by underinvesting in their education, a phenomenon which is by now accepted as a stylized fact in the literature. There are two competing explanations for this.

- First, there is the standard beckerian view (e.g. Becker et al. 1990) according to which parents see education as a consumption good whose enjoyment may be limited by liquidity constraints: that is, parents are altruistic towards their children, and would like to spend as much as possible in their education, but they might be unable to afford the level of outlay which would be optimal given the potential abilities of the children. The obvious remedy for this is a redistributive policy that transfers more resources towards the needy. A more market-oriented solution is difficult to find, as there is no credit market for the investment in education due to the lack of collateral (future income is normally unacceptable).

- An alternative view, that has gained popularity in recent years, sees education as an investment also from the point of view of the parents and not only of the children: this perspective is related to the "exchange model" of the family pioneered e.g. by Cigno (1993). Selfish family members engage in transfers regulated by self-enforcing rules specifying rewards for obedience and punishments for deviations. The resulting system may be inefficient for several reasons, the most relevant being that parents, when investing in their children's education, foresee that they will be able to reap only a fraction of the return, and tend therefore to underinvest. Redistribution is clearly ineffective, whereas the subsidization of educational expenditure, by lowering the cost of investment, might work (Anderberg and Balestrino, 2003).

The results from the empirical literature are hardly decisive. It is true that the testable implications of the altruistic model are usually not verified (e.g. Altonji et al. 1992, 1997), whereas those of the exchange model are more consistently found to be holding (e.g. Cigno et al. 1998, 2006). It has however been argued that the test usually employed for the altruistic model is unnecessarily restrictive, and that at this stage of our general knowledge there is no definitive case in favour of one or the other approach (McGarry 2000).

A point which we might want to stress is however that neither view recommends an education policy that includes, among other things, compulsory schooling. This is in stark contrast with what actually happens in virtually all the developed countries, and has been happening for the past 150-plus years. It is an historical fact that education policy was conceived in terms of free and mandatory public schooling (financed by public funds) when it was introduced during the XIX century in the West (Germany, France and later UK and US); and free and mandatory schooling is still at the basis of our educational systems today.[1] Compulsory schooling is, instead, still at stake in many less developed countries where universal primary education is far from having been achieved especially for girls, as it is shown by the importance given to such objectives by the Millennium Development Goals.

Economists are always suspicious towards policy interventions that seem to thwart individual freedom or consumer sovereignty. It has however been recognised in the literature, at least since the contributions of Neary and Roberts (1980) and Guesnerie and Roberts (1984), that in a second-best world quantitative restrictions may be welfare-improving inasmuch as they can

[1] For example, see Go (2013) for a paper presenting a political economy explanation for the American achievement of universal free public schooling according to a historical perspective.

enhance the efficiency and the redistributive impact of the tax system.[2] While these arguments certainly pave the way for our present line of research, they are too vague for our purposes. They refer to generic commodities, and not specifically to education, a service that can of course be bought, at least in principle, on the market as many others, but that has its own pecularities. Two aspects, normally recognised in the literature, are, in our opinion, worth emphasising:

1. unlike most commodities, education is purchased not by those who consume it, but by a third party (at least for primary and secondary education, the parents bear the costs of education, while the benefit will be reaped, in time, by the children);

2. the enjoyment of its fruits, no matter whether they are seen in terms of investment or consumption value, require out-of-pocket expenses *and* a large amount of time, i.e. ample opportunity costs (education is a long process: it goes on for years).

In order to account for these peculiarities, we employ a household economics approach. We recognise that there are two actors involved in the purchase and consumption of education, the parents and the children (point 1 above) and we model time allocation in a detailed way, trying to account for its key role in the educational process (point 2). From a normative standpoint, we develop an argument showing i) that education policy is socially desirable, and ii) that it must preferably include a period of compulsory schooling rather than following another intervention design.

The plan of the paper is the following. Section 2 presents the model of educational choice, and the *laissez-faire* outcome. Section 3 analyses different public policies, and finally, section 4 contains some concluding remarks.

II A model of educational choice

Consider a finite-horizon model. The economy is made of two-persons households: one parent and one child. We posit that, in order to earn an income, each parent supplies a certain amount of labour l to the market at a wage rate denoted by w; w varies across individuals. To be precise, we assume that w varies continuously on $[0, \overline{w}]$ according to a density function $f(w)$, and that the agents have unit mass.

[2] The surveys by Balestrino (1999, 2000) illustrate the state of the art in this stream of work at the end of the '90s. For a more recent outlook, see Blomquist et al. (2010).

Income can be spent on the parent's own consumption c^p, the child's consumption c^k, and the child's education e. The latter also requires time d (of the child): in fact, we attribute extreme importance to the fact that education is a very time-intensive activity. The time of the parent that is not employed on the market plus the time of the child that is not employed for educational purposes, denoted h^p and h^k respectively, are used to produce a non-marketable household public good y, nonrivalrous and nonexcludable within the family; for simplicity, no other input is required and there is no pure leisure. A perfect substitute for the households public good, z, is available on the market at the price p.

We assume that the parent is altruistically linked to the child. The degree of altruism towards the kid is represented by a parameter $\alpha \in (0,1)$, representing the weight that parents gives to their own utility (i.e. altruism is higher, the lower is α), and taken to be the same for all parents. This is the simplest setting in which the model can be developed: it could be replaced by one in which the link is strategic or purely selfish, at the cost of several complications (for example, we might suppose that the child makes a payment back to the parent in exchange for the money the parent herself spent on the child's education, as in Anderberg and Balestrino 2003, but then we would have to introduce an overlapping-generation structure[3]). In fact, all that is required for making sense of the analysis is that the child generates some benefit for the parent as well as costs; otherwise, the model would lose all its strength, because the parents would always trivially choose not to educate their children.

The *laissez-faire* outcome

Let us begin by considering what would happen in a free market, in which there is no government intervention. Using the notation given above, we assume that the household production function is linear,

$$y = (h^p + 1 - d)\,\gamma, \tag{1}$$

where $\gamma > 0$, we set c^k fixed at some conventional level $\widehat{c}$, and, finally, we assume additive separability for the utilities of the parent and the child. We can then write the parent's preferences as

$$\alpha\left[u\left(c^p\right) + f\left(\left(h^p + 1 - d\right)\gamma + z\right)\right] + (1-\alpha)\left[v\left(\widehat{c} + x\left(e, d; \theta\right)\right) + g\left(\left(h^p + 1 - d\right)\gamma + z\right)\right], \tag{2}$$

[3] Of course, also the present model with altruism could be recast in an OLG framework. All the results reached in the simpler case treated here would however carry over, and we would have to add many unnecessary details, with the consequent risk of making the model lose its focus.

where $u(\cdot)$ and $f(\cdot)$ as well as $v(\cdot)$ and $g(\cdot)$ are strictly concave functions. The only argument that we haven't introduced so far is $x(\cdot)$, that we take to be a linearly homogeneous and strictly concave function, representing the value of education for the child in consumption terms (the child's gross income). The parameter θ denotes the kid's ability, and, just like w, varies across individuals: we assume that $\partial x/\partial\theta > 0$. Technically, there would be no need to make any specific assumption on how w and θ happen to be correlated; however, the empirical literature (e.g. Mayer 1997 and Blau 1999) suggests that they might be positively, albeit not perfectly, correlated, and we follow this suggestion here, mainly for the purpose of interpretating the results. We take it that θ is known to the parent (this simplifies matters, as we are not interested in imperfect information issues here). Finally, we assume that e and d are technogical complements ($\partial x/\partial e\partial d = \partial x/\partial d\partial e > 0$); the more time you spend on education, the more effective is the money you spend on it and *viceversa*. Also, we assume that both time and money are essential to production:

$$x(0, d; \theta) = x(e, 0; \theta) = 0. \tag{3}$$

The time constraints for the parent and the kid, respectively, are:

$$h^p = 1 - l;\ h^k = 1 - d, \tag{4}$$

where the time endowment is normalised to unity for both types of agent. Further normalising the price of the consumption good to unity, the budget constraint for the workers is

$$c^p + c^k + pz + e = wl. \tag{5}$$

Using these elements, we might write the agent's problem as one of choosing c^p, h^p, z, e and d so as to

$$\begin{gathered}\text{Max } \alpha\left[u\left(c^p\right) + f\left(\left(h^p + 1 - d\right)\gamma + z\right)\right] + (1-\alpha)\left[v\left(\widehat{c} + x\left(e, d; \theta\right)\right) + g\left(\left(h^p + 1 - d\right)\gamma + z\right)\right] \\ \text{s.t. } c^p + \widehat{c} + pz + e + wh^p - w = 0; \\ h^p \geq 0;\ z \geq 0;\ e \geq 0;\ d \geq 0.\end{gathered}$$

The first order conditions (FOCs) are as follows:

$$\alpha u' = \lambda; \tag{6}$$

$$\left(\alpha f' + (1-\alpha)\, g'\right)\gamma \leq \lambda w, \text{ plus complementary slackness;} \tag{7}$$

$$\alpha f' + (1-\alpha)\, g' \leq \lambda p, \text{ plus complementary slackness;} \tag{8}$$

$$(1-\alpha)\, v' x_e \leq \lambda \text{ plus complementary slackness;} \tag{9}$$

$$(1-\alpha)\, v' x_d \leq \left(\alpha f' + (1-\alpha)\, g'\right)\gamma \text{ plus complementary slackness,} \tag{10}$$

where x_e and x_d are partial derivatives, and λ is the marginal utility of income.

To begin with, let us investigate the question whether the household public good is produced internally, or purchased on the market. Comparing (7) and (8), we can see that the choice between home production and market purchase depends on whether w/γ (a measure of the cost of home production) exceeds or is less than p (the cost of the market purchase). One way of seeing this is to say that all those household whose comparative advantage, as measured by the ratio between the marginal productivites on the market and at home w/γ, lies in household production will not purchase the good on the market, while the others will purchase it. The upshot is that we identify a threshold wage rate

$$w^* = p\gamma, \tag{11}$$

such that all agents with a higher rate purchase a positive quantity of z, while those with a lower wage rate do not buy it. Notice, for future use, that the threshold wage rate does *not* depend on either θ or α.

We can now describe the comparative statics separately for the high- and low-wagers.

Comparative statics

The fact that $z > 0$ drastically simplifies the analysis of the remaining choices for the high-wagers. Having no need to employ their time in home production, they set $h^p = h^k = 0$, that is $l = 1$ and $d = 1$: all the parent's time goes into working and all the kid's time goes into education. Income is then equal to w, and using FOCS (6), (8) and (9) above, we see that the consumption mix is determined by the equality

$$\alpha u' = \frac{\alpha f' + (1-\alpha)\, g'}{p} = (1-\alpha)\, v' x_e. \tag{12}$$

The value for e that emerges is then combined with $d = 1$ to give the equilibrium value for x.

For these households, income trivially increases with the wage rate. As for the optimal values of z and e, we expect them to be positively correlated with the wage rate. The comparative statics, whose details are reported in Appendix A, confirm this intuition:[4]

$$\partial z/\partial w > 0; \partial e/\partial w > 0. \tag{13}$$

We also find that

$$\partial z/\partial \theta < 0; \partial e/\partial \theta > 0.$$

In summary, all the kids from high-wage families are educated full-time; furthermore, the richer is the family, the more they spend on education. As a consequence, x increases in the wage rate for any given θ. On the other hand, for any given wage, the higher is the ability of the child, the more the family's expenditure is skewed towards the kid's education.

In principle, the low-wagers could choose not to educate their children. This possibility would raise interesting questions but would take us too far afield, and therefore we focus on interior solutions. In this context, we begin by investigating the role of the wage rate, that represents the opportunity cost of time spent in education, but is of course also the only source of income. Using the FOCs (6), (7), (9) and (10) for an interior solution, we have that the consumption mix is determined by

$$\alpha u' = \frac{\alpha f' + (1-\alpha)\, g'}{w} = \frac{(1-\alpha)\, v' x_d}{w} = (1-\alpha)\, v' x_e. \tag{14}$$

Assuming that

$$-\frac{u''}{u'} < \frac{1}{w\,(1-h^p)}, \tag{15}$$

which is a restriction on the size of the coefficient of absolute risk aversion, we find that the comparative statics signs w.r.t. the wage rate (for the details, see again Appendix A) are as follows:

$$\partial h^p/\partial w < 0;\ \partial e/\partial w > 0;\ \partial d/\partial w < 0. \tag{16}$$

This means that the labour supply curve is always increasing in the (l, w)-space: the higher the wage rate, the more the parent works outside home, and the higher is her income. Also, the higher the wage rate, the more the parent spends on her kid's education, but the lower the time

[4] In our setting with separable utility, strict concavity is enough to guarantee the expected signs. With a general utility function, the same result would have been obtained with a few assumptions and restrictions on the sign and the magnitude of the cross-derivatives of the utility function. The same remark applies to all comparative statics results below.

that she allots to the kid's education. The facts that the more a parent earns on the market, the more she will be prone *not* to employ her child's time for educational purposes is apparently counterintuitive, but is based on the fact that a higher wage rate makes working on the market more actractive: if the child's time can be used for home-production purposes, the parent has more available time for working outside the home, therefore she will substitute money for time in the educational input mix.

As for the impact of θ, none of the derivatives can be signed: unlike the high-wagers, who have a fixed time allocation, the low-wagers choose the allocation of their time as well as of their budget, and changes in θ induce a chain of adjustments that starts from the educational inputs mix and reverberates over the whole decision process.

The properties of the equilibrium

The picture that we obtain is then one in which all the agents whose wage rate is relatively large (exceeds w^*) give their children a full-time education. Expenditure grows with income, and therefore, the higher is w, the higher will be the return to education x for the kids (*ceteris paribus*); also, expenditure is increasing in θ, so that, still *ceteris paribus*, the higher is the child's ability, the more educated she will be. Low-wagers instead perform somewhat unexpectedly: for those below w^*, the time spent in education varies *inversely* with the wage rate, while the money goes in the same direction as the wage. The effects of θ are not discernible at this level of generality. We can say that the children of these families will definitely spend less time in education than the others (some might in fact receive no education at all, although we do not study that case here), but there is no clear pattern emerging within the group.

We conclude the analysis of family choices in *laissez-faire* by asking whether they are efficient. Normally, we would ascertain this by comparing the actual market equilibrium with the one prevailing under $\alpha = 1/2$, for in that case the parent would actually maximise a utilitarian social welfare function, i.e. would achieve an efficient outcome. However, in our case, the presence of a skill level for children complicates matters: efficiency clearly requires that the education package is tailored on the θ's – so that for example kids with the same skill should be educated in the same way – but this is not guaranteed in this model. Suppose, for the sake of the argument, that θ is fixed across individuals. What happens as α varies? Recall that w^* does not depend on α, i.e. the partition between high-wagers who educate their child in full and low-wagers who educate them less or not at all is the same for all values of α. This

implies that the kids, despite having all the same skill level, receive different education packages: altruism is irrelevant here, as the decision is taken only on the basis of comparative advantage considerations. The picture worsens if we re-introduce differences in θ: with less than perfect correlation between w and θ, it may happen that a kid from a low-wage household with a higher θ than one from a high-wage household ends up being less educated (or, in an extreme case, not educated at all).

III Policy analysis

It is then clear that, for the usual equity and efficiency reasons, there will be room for some policy intervention. Let us now consider the policy instruments in two stages. First, we will assess the performance of the policy package that economists usually recommend: a linear income tax, with tax rate $\tau > 0$ and lump-sum subsidy $T > 0$, plus an educational subsidy, $\sigma > 0$.[5] We will remark that this policy combination is hardly effective. Therefore, we replace the subsidy with a compulsory education package E, D where E is per-child expenditure and D is the years of mandatory schooling, and perform a full policy analysis of this case.

Redistributive taxation and educational subsidy

Within this standard policy framework, the budget constraint becomes:

$$c^p(w) + \widehat{c} + pz(w) + (1-\sigma)\, e(w) + w(1-\tau)\, h^p(w) = w(1-\tau) + T. \tag{17}$$

We assume that also the future income of the children is taxed at the same rate as that of the parents *so that children* earn $(1-\tau)\, x(e,d)$ after tax. The utility function of the parent therefore is:

$$\begin{aligned} &\alpha\left[u(c^p) + f\left((h^p + 1 - d)\gamma + z\right)\right] + \\ &+ (1-\alpha)\left[v\left(\widehat{c} + (1-\tau)\, x(e,d;\theta)\right) + g\left((h^p + 1 - d)\gamma + z\right)\right]. \end{aligned} \tag{18}$$

[5] For simplicity, we restrict ourselves to linear policy instruments throughout the paper.

Let us check the effects on the model. The FOCs become

$$\alpha u' = \lambda; \tag{19}$$

$$\left(\alpha f' + (1-\alpha) g'\right) \gamma \leq \lambda (1-\tau) w, \text{ plus complementary slackness;} \tag{20}$$

$$\alpha f' + (1-\alpha) g' \leq \lambda p, \text{ plus complementary slackness;} \tag{21}$$

$$(1-\alpha) v' (1-\tau) x_e \leq (1-\sigma) \lambda \text{ plus complementary slackness;} \tag{22}$$

$$(1-\alpha) v' (1-\tau) x_d \leq \left(\alpha f' + (1-\alpha) g'\right) \gamma \text{ plus complementary slackness,} \tag{23}$$

which imply that a new cut-off wage rate

$$w^* (p, \gamma; \tau) = \frac{p\gamma}{1-\tau}. \tag{24}$$

The distinction between high- and low-wagers works in the same way as in the free-market, in the sense that the former purchase z and have a fixed time allocation, while the latter rely on the domestically produced good, and therefore choose both their time and their budget allocation.

It is interesting to comment on the impact of the policy tools on the cut-off wage. We see that both σ and T are totally ineffective, while the impact of the marginal tax rate τ is somewhat perverse, as it implies a rise in w^*:

$$\partial w^* / \partial \tau = \frac{p\gamma}{(1-\tau)^2} > 0. \tag{25}$$

Relative to the free-market situation, then, in an equilibrium with policy there will be less agents who educate their kids full time! This is because the marginal tax rate τ alters the comparative advantage situation: working outside home become less advantageous, and more agents choose to produce the household public good domestically. Of course, we expect that the expenditure on education for the parents who still send their kids to school will increase, because they receive a lump-sum transfer T and can rely on an educational subsidy σ: but there will be less households in this position, all concentrated on the upper tail of the income distribution, so that the redistributive nature of the policy is dubious at best.

In our setup, then, the standard beckerian prescription of using redistributive policy to make even the less well-off prone to educate their children is ineffective, despite the assumption of altruism within the family. Rather, redistribution has a strong distortionary effect, since the presence of a marginal tax rate forces some individuals not to educate their children. This cannot be remedied neither by the lump-sum transfer nor by the education subsidy, as these only work on the intensive margin, leaving the extensive one unaffected.

Redistributive taxation and compulsory education

Given the weak performance of the standard set of policy tools, and taking for granted that redistribution is one of the tasks that policy is required to accomplish, it makes sense to investigate whether compulsory education can be more effective in actually achieving it.

Consider then a compulsory education package E, D where E is per-child expenditure and D is the years of mandatory schooling. The utility function becomes

$$\alpha \left[u\left(c^p\right) + f\left(\gamma h^p + z\right)\right] (1-\alpha) \left[v\left(\widehat{c} + (1-\tau)\, x\left(E+e, D+d; \theta\right)\right) + g\left(\gamma h^p + z\right)\right]; \tag{26}$$

while the budget constraint is

$$c^p + \widehat{c} + pz + e + (1-\tau)\, wh^p = (1-\tau)\, w + \widetilde{T}. \tag{27}$$

In this case, e is the amount of expenditure that the parent can employ for topping up the compulsory E. Notice that it is possible to write the budget constraint also as

$$c^p + \widehat{c} + pz + (e+E) + (1-\tau)\, wh^p = (1-\tau)\, w + T, \tag{28}$$

that is, as if the agent were paying the educational expenditure herself. In fact, as it is formally shown in Appendix B, T now also cover E.

The problem of the agent is then to maximise (26) by choice of c^p, h^p, z, d and e s.t. (28) and the non-negativity constraints. The FOCs are as follows:

$$\alpha u' = \lambda; \tag{29}$$

$$\left(\alpha f' + (1-\alpha)\, g'\right) \gamma \leq \lambda\, (1-\tau)\, w, \text{ plus complementary slackness;} \tag{30}$$

$$\left(\alpha f' + (1-\alpha)\, g'\right) \leq \lambda p, \text{ plus complementary slackness;} \tag{31}$$

$$(1-\alpha)\, v'\, (1-\tau)\, x_e \leq \lambda \text{ plus complementary slackness;} \tag{32}$$

$$(1-\alpha)\, v'\, (1-\tau)\, x_d \leq \left(\alpha f' + (1-\alpha)\, g'\right) \gamma \text{ plus complementary slackness.} \tag{33}$$

The choice between home production and market purchase depends on the measure of comparative advantage, just as before. The threshold wage rate is as in (24), and the distinction between high- and low-wagers works in the same way, with the added twist that, for the high-wagers, the presence of D is of no consequence: at most, it is $D = 1$, which is what the parents would have chosen anyway.

The indirect utility for both types can be written as a function of the policy instruments, $W = W(\tau, T, E, D)$, and the derivatives w.r.t. the policy tools are

$$\frac{\partial W}{\partial T} = \lambda > 0; \ \frac{\partial W}{\partial \tau} = -\lambda w (1 - h^p) - (1 - \alpha) v' x < 0; \tag{34}$$

$$\frac{\partial W}{\partial E} = (1 - \alpha) v' (1 - \tau) x_e - \lambda < 0 \ \text{ if } \ e = 0 \tag{35}$$

$$\frac{\partial W}{\partial D} = (1 - \alpha) v' (1 - \tau) x_d - \left(\alpha f' + (1 - \alpha) g'\right) \gamma < 0 \ \text{ if } \ d = 0. \tag{36}$$

Notice that $h^p > 0$ for low-wagers and $h^p = 0$ for high-wagers in the expression for $\partial W/\partial \tau$. The sign of $\partial W/\partial E$ depends on whether E exceeds the quantity that the agent would have chosen in the free market or not: $\partial W/\partial E$ is negative if it does, equal to 0 otherwise. Similarly, the sign of $\partial W/\partial D$ depends on whether D exceeds the time that the agent would have chosen in the free market or not: $\partial W/\partial D$ is negative if it does, equal to 0 otherwise.

The next step requires us to check the comparative statics in this new setting with the policy instruments. This task is made extremely cumbersome by the fact that there are many possibilities concerning the extent to which the compulsory educational package actually constrains the family choices. We noticed that D is always inframarginal for the high-wagers; but E may or may not bite. And for the low-wagers, it might be the case the neither D nor E bite, or that they both do, or that only one does. Here we focus on the case that seems more interesting,[6] i.e. the one in which E constrains the choices of all households, while D constrains all the low-wagers. Therefore, $e = 0$ for all agents, while $d = 0$ for the low-wagers. This is the scenario in which the quantity constraints interfere the most with the free choices of the agents: can in this extreme case those constraints be welfare-improving?

Let us start from the comparative statics for both the high- and the low-wagers (calculations are found in Appendix C). Notice that, besides setting d so as to reach full time education for their children,[7] the high-wagers only choose z. We confirm that $\partial z/\partial w > 0$ as in *laissez faire*, and we find that

$$\partial z/\partial \tau < 0; \ \partial z/\partial T > 0; \ \partial z/\partial E < 0. \tag{37}$$

As for the low-wagers, they only choose h^p. We confirm that $\partial h^p/\partial w < 0$, and, using

$$-\frac{u''}{u'} < \frac{1}{(1 - \tau) w (1 - h^p)}, \tag{38}$$

[6] As far as the effects of the education policies are concerned, the conclusions in the other cases are analogous to those discussed here. The results for the other cases are available from the authors.

[7] This implies $d > 0$ if $D < 1$, and $d = 0$ if $D = 1$.

which is analogous to (15), we find that

$$\partial h^p/\partial\tau > 0;\ \partial h^p/\partial T > 0;\ \partial h^p/\partial E < 0;\ \partial h^p/\partial D > 0. \tag{39}$$

Optimal redistributive and educational policy

As a benchmark we consider the first-best case with purely redistributive taxation. Social welfare is

$$\int_0^{\overline{w}} \beta(w)\, W(T(w); w)\, f(w)\, \mathrm{d}w, \tag{40}$$

where β is a welfare weight, with $\beta > 0$, and $\beta' < 0$. Maximising (40) under the constraint that $\int_0^{\overline{w}} T(w) = 0$, which implies purely redistributive taxation, gives us

$$\beta(w)\, \alpha u'(w) = \mu, \tag{41}$$

where μ is the Lagrange multiplier and where we we use $\lambda = \alpha u'$. Then, all socially weighted marginal utilities of income are equalised. If $\beta = 1$ for all agents (utilitarian SWF) and recalling that α is constant across agents, the outcome is that all agents must have the same after-tax income. This fact does not alter, however, the distinction between high- and low-wagers, because the FOCs of the consumer are the same as in *laissez-faire*, and therefore, it is still true that some agents educate the children full-time, while others don't. Of course, the low-wagers would purchase more e than without redistributive taxation, but d would not reach unity. Given the complementarity of time and expenditure, it would still be true that the high-wagers have more educated kids – irrespective of the value of θ.

Consider now a second-best setting with compulsory education. We have the following policy problem

$$\max_{\tau,T,E,D} \int_0^{\overline{w}} \beta(w)\, W(\tau, T, E, D; w)\, f(w)\, \mathrm{d}w$$
$$\text{s.t. } \tau \int_0^{\overline{w}} x(E,D)\, f(w)\, \mathrm{d}w + \tau \int_0^{\overline{w}} w f(w)\, \mathrm{d}w - \tau \int_0^{w^*(\tau)} \left[w h^p(w)\, f(w)\right] \mathrm{d}w - T = R,$$

where R is a fixed revenue requirement. The FOCs are

$$-\int_0^{\overline{w}} \beta(w) \frac{\partial W}{\partial \tau} f(w)\, \mathrm{d}w + \mu \left\{ \int_0^{\overline{w}} x(w)\, f(w)\, \mathrm{d}w + \int_0^{\overline{w}} w f(w)\, \mathrm{d}w - \int_0^{w^*(\tau)} w h^p(w)\, f(w)\, \mathrm{d}w - \right.$$
$$\left. -\tau \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial \tau} f(w)\, \mathrm{d}w - \tau w^* h^p(w^*)\, f(w^*) \frac{\partial w^*}{\partial \tau} \right\} = 0; \tag{42}$$

$$\int_0^{\overline{w}} \beta(w) \frac{\partial W}{\partial T} f(w)\, \mathrm{d}w - \mu \left\{ \tau \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial T} f(w)\, \mathrm{d}w + 1 \right\} = 0; \tag{43}$$

$$\int_0^{\overline{w}} \beta(w) \frac{\partial W}{\partial E} f(w)\,\mathrm{d}w + \mu\tau \left\{ \int_0^{\overline{w}} x_e f(w)\,\mathrm{d}w - \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial E} f(w)\,\mathrm{d}w \right\} = 0; \tag{44}$$

$$\int_0^{\overline{w}} \beta(w) \frac{\partial W}{\partial D} f(w)\,\mathrm{d}w + \mu\tau \left\{ \int_0^{\overline{w}} x_d f(w)\,\mathrm{d}w - \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial D} f(w)\,\mathrm{d}w \right\} = 0, \tag{45}$$

where the derivatives with respect to the indirect utility functions are given by (34), (35) and (36). We can then state the main results concerning the policy rules.

First, consider the second FOC:

$$\frac{\int_0^{\overline{w}} \beta(w) (\partial W/\partial T) f(w)\,\mathrm{d}w}{\mu} - \tau \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial T} f(w)\,\mathrm{d}w = 1; \tag{46}$$

the net social marginal utility of income, inclusive of its effect on revenue and weighted by μ, equals unity. This is a standard result that characterises the optimal T.

The third FOC can be rearranged as follows:

$$\tau \left\{ \int_0^{\overline{w}} x_e f(w)\,\mathrm{d}w - \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial E} \right\} f(w)\,\mathrm{d}w = -\frac{\int_0^{\overline{w}} \beta(w) (\partial W/\partial E) f(w)\,\mathrm{d}w}{\mu}, \tag{47}$$

that is the marginal benefit in terms of increased revenue must equal the marginal cost in terms of forcing the agents out of the chosen consumption bundle – recall that $\partial W/\partial E < 0$ when the ration bites. Increased revenue depends on the fact that an increase in E implies an increase in the future income of the children x, as well as less home-production time, or equivalently more time devoted to market work, ($\partial h^p/\partial E < 0$) and therefore more taxable income from the parents.

Consider now the fourth FOC:

$$\tau \int_0^{\overline{w}} x_d f(w)\,\mathrm{d}w = \tau \int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial D} f(w)\,\mathrm{d}w - \frac{\int_0^{\overline{w}} \beta(w) (\partial W/\partial D) f(w)\,\mathrm{d}w}{\mu}. \tag{48}$$

At the optimum, the advantage of creating more future revenue by pushing x up (the l.h.s) must equal the cost in terms of reduced current revenue (we know from the comparative statics that $\partial h^p/\partial D > 0$ for the low-wagers) and of forcing the agents out of the chosen consumption bundle (again, $\partial W/\partial D < 0$ when the ration bites).

This analysis of the third and fourth FOC presupposes that $\tau > 0$. In order to check whether this is the case, we can rearrange the first FOC. To this end, define

$$\Psi \equiv \int_0^{\overline{w}} x(w) f(w)\,\mathrm{d}w + \int_0^{\overline{w}} w f(w)\,\mathrm{d}w - \int_0^{w^*(\tau)} w h^p(w) f(w)\,\mathrm{d}w. \tag{49}$$

We can then write

$$\tau = \frac{\Psi - \int_0^{\overline{w}} \beta(w) (\partial W/\partial \tau) f(w)\,\mathrm{d}w/\mu}{\int_0^{w^*(\tau)} w \frac{\partial h^p(w)}{\partial \tau} f(w)\,\mathrm{d}w + w^* h^p(w^*) f(w^*) \frac{p\gamma}{(1-\tau)^2}}, \tag{50}$$

where we used (25).

From the fact that $\partial h^p/\partial\tau > 0$ we deduce that the denominator in (50) is positive. This term represents the total revenue loss associated with a marginal increase in τ: the reduction in labour supply implies a reduction of tax base, and the fact that w^* varies inversely with τ implies a further reduction because more agents start employing home-production to get the household public good, and therefore work less. The larger is this term, the smaller will be τ.

Instead, the first term at the numerator, Ψ, is, as we just said, the marginal revenue gain from the tax. It is positive because

$$\int_0^{\overline{w}} x(w) f(w)\,\mathrm{d}w + \int_0^{\overline{w}} w f(w)\,\mathrm{d}w > \int_0^{w^*(\tau)} w h^p(w) f(w)\,\mathrm{d}w. \tag{51}$$

Further, the second term at the numerator of (50), also positive, is the marginal welfare loss. Therefore, if the revenue gain exceeds the welfare loss, the numerator is positive as well. In that case, we have $\tau > 0$; and the larger is the difference between the two terms above the line, the larger is the tax rate.

The fact that $\tau > 0$ implies that, at the optimum, we have $D > 0$ and $E > 0$: if some form of redistributive taxation is in place, it is optimal to force some agents i) to spend on education more than they would have done in a free-market, and ii) to devote more time to it than they would have done in a free-market. The desirability of the quantity controls is justified by the fact that they imply a gain in revenue terms. If this gain is large enough to compensate the costs in terms of displaced consumption, then some form of quantitative restriction is welfare-improving.

A remarkable feature of this result is that redistributive taxation and education policy in the form of compulsory education appear to be strongly intertwined. Neither works without the other, in the sense that the redistributive taxation *per se* may damage the future earnings of (some of) the children, and thus requires a specific education policy, while compulsory education cannot be optimal without redistributive taxation.

IV Concluding remarks

We began by asking whether there is a reason why education policy should involve a mandatory and (virtually) free-of-charge schooling period, as it commonly does in the Western countries. Economists should be particularly interested in obtaining an answer, as quantity controls are traditionally considered outperformed by price controls in standard economic theory. Taking for granted that education policy must, for some reason, be implemented, many would argue

that it should take the form of a price subsidy (making education less costly should make agents more prone to purchase it for their children) or simply be embedded in tax policy (redistributing resources in favour of the poor should automatically help them to send their children to school).

Now, it is well-known, at least since Guesnerie and Roberts (1984), that the superiority of price controls is only valid in first-best, and that quantity controls can be welfare-improving in a variety of second-best contexts: the last 20 years have seen a vast research effort on this that traces its roots to the contributions of Blomquist and Christiansen (1995) and Boadway and Marchand (1995) and continues to this day (e.g. Blomquist et al. 2010). Our work follows this stream of the literature, and aims to fill a blank space, because none of those works has dealt specifically with education as we believe it should be characterised, namely as i) an extremely expensive and time-consuming process that ii) involves a decision-maker (the parent) who is not the direct beneficiary (the child).

Using a model that accounts for both these features, we have first depicted a free market situation in which some agents ("high-wagers") educate their children full-time and spend a sizable amount of resources on them, while others ("low-wagers") educate them only partially (and in principle might even not educate them at all). This outcome is generated by the presence of an alternative usage of the children's time: rather than be sent to school, they can be employed in producing a household public good. The high-wagers can in fact afford to replace this home-produced good with a marketed substitute; the low-wagers' comparative advantage, instead, lies in home-production. This free-market equilibrium is inefficient even when the parents are fully altruistic, because kids with the same ability receive different educations depending on whether they are born in a high-wage or a low-wage family; and is iniquitous where parents are concerned (due to their having different exogenous skills), and also where children are concerned, because the differences in the education they receive today imply that there will be a disparity in earning abilities tomorrow, which, from their point of view, is "exogenous" (in the sense that it depends on choices made by the parents, not by themselves).

Public policy is thus called for, due to the usual equity and efficiency reasons. In this framework, we argued that it is indeed socially optimal to introduce a compulsory education package, using a standard redistributive tax system to finance it. Mandatory schooling forces the parents away from their equilibrium choices, and that is the cost of the policy; but it entails also an advantage in terms of increased revenue that can be used to finance the poll-subsidy, which, it will be remembered from the formal analysis, covers also the costs of the kids' educational

expenditure for the mandatory period.

Also, a compulsory education policy is shown to be vastly superior to the use of price subsidies, that only work on the intensive margin, i.e. boost education expenditure for those who would have educated their children full-time anyway (in a free market), but are unable to induce those who didn't educate their kids full-time to start doing so. And we also argued that redistributive taxation alone is in fact counter-productive, as it forces more agents than in *laissez-faire* to avoid educating their children full-time, because it tips the comparative advantage balance in favour of making child household work more desirable. This suggests the conclusion we reached above, namely that redistributive taxation and compulsory education are best seen as complementary policies.

Appendix A - Comparative statics in *laissez-faire.*

High wagers

In the case of high wagers, we can write the maximisation problem as

$$\max_{z,e} \alpha \left(u \left(w - \widehat{c} - pz - e\right) + f\left(z\right)\right) + \left(1-\alpha\right)\left(v\left(\widehat{c} + x\left(e,d\right)\right) + g\left(z\right)\right). \tag{A1}$$

The FOCs are

$$-\alpha u' p + \alpha f' + \left(1-\alpha\right) g' = 0 \;\; \left(z\right); \tag{A2}$$

$$-\alpha u' + \left(1-\alpha\right) v' x_e = 0 \;\; \left(e\right). \tag{A3}$$

By totally differentiating, we have:

$$-\alpha u'' p \mathrm{d}w + \left[\alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g''\right] \mathrm{d}z + \alpha u'' p \mathrm{d}e = 0; \tag{A4}$$

$$-\alpha u'' \mathrm{d}w + \alpha u'' p \mathrm{d}z + \left[\alpha u'' + \left(1-\alpha\right)\left(v''\left(x_e\right)^2 + v' x_{ee}\right)\right] \mathrm{d}e = 0. \tag{A5}$$

Therefore:

$$\begin{bmatrix} \alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g'' & \alpha u'' p \\ \alpha u'' p & \alpha u'' + \left(1-\alpha\right)\left(v''\left(x_e\right)^2 + v' x_{ee}\right) \end{bmatrix} \begin{bmatrix} \mathrm{d}z/\mathrm{d}w \\ \mathrm{d}e/\mathrm{d}w \end{bmatrix} = \\ = \begin{bmatrix} \alpha u'' p \\ \alpha u'' \end{bmatrix}. \tag{A6}$$

The signs then are as follows

$$sgn\ \mathrm{d}z/\mathrm{d}w = sgn\ \left[\alpha u'' p\left(\alpha u'' + (1-\alpha)\left(v''\left(x_e\right)^2 + v' x_{ee}\right)\right)\right] - \left(\alpha u''\right)^2 p > 0; \tag{A7}$$

$$sgn\ \mathrm{d}e/\mathrm{d}w = sgn\ \left[\alpha u'' p^2 + \alpha f'' + (1-\alpha) g''\right] \alpha u'' - \left(\alpha u'' p\right)^2 > 0. \tag{A8}$$

Low wagers

In the case of low wagers, we can write the maximisation problem as

$$\max_{h^p,e,d}\ \alpha\left(u\left(w - \widehat{c} - pz - e - wh^p\right) + f\left(\left(h^p + 1 - d\right)\gamma\right)\right) + \\ + (1-\alpha)\left(v\left(\widehat{c} + x\left(e,d\right)\right) + g\left(\left(h^p + 1 - d\right)\gamma\right)\right). \tag{A9}$$

The FOCs for an interior solution are:

$$-\alpha u' w + \alpha f'\gamma + (1-\alpha) g'\gamma = 0\ \ (h^p); \tag{A10}$$

$$-\alpha u' + (1-\alpha) v' x_e = 0\ \ (e); \tag{A11}$$

$$-\alpha f'\gamma - (1-\alpha) g'\gamma + (1-\alpha) v' x_d = 0\ \ (d). \tag{A12}$$

Totally differentiating, we have:

$$-\alpha\left[u''\left(1-h^p\right)w + u'\right]\mathrm{d}w + \left[\alpha\left(u''w^2 + f''\gamma^2\right) + (1-\alpha) g''\gamma^2\right]\mathrm{d}h^p + \left[\alpha u'' w\right]\mathrm{d}e - \\ - \left[\alpha f''\gamma^2 + (1-\alpha) g''\gamma^2\right]\mathrm{d}d = 0; \tag{A13}$$

$$-\alpha\left[u''\left(1-h^p\right)\right]\mathrm{d}w + \left[\alpha u'' w\right]\mathrm{d}h^p + \left[\alpha u'' + (1-\alpha) v''\left(x_e\right)^2\right]\mathrm{d}e + \\ + \left[(1-\alpha) v'' x_e x_d\right]\mathrm{d}d = 0; \tag{A14}$$

$$0\mathrm{d}w - \left[\alpha f''\gamma^2 + (1-\alpha) g''\gamma^2\right]\mathrm{d}h^p + \left[(1-\alpha) v'' x_d x_e\right]\mathrm{d}e + \\ + \left[\alpha f''\gamma^2 + (1-\alpha) g''\gamma^2 + (1-\alpha) v''\left(x_d\right)^2\right]\mathrm{d}d = 0 \tag{A15}$$

Now let

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} \alpha\left(u''w^2 + f''\gamma^2\right) + (1-\alpha)\, g''\gamma^2 \\ \alpha u'' w \\ -\left[\alpha f''\gamma^2 + (1-\alpha)\, g''\gamma^2\right] \end{bmatrix} \tag{A16}$$

$$\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} \alpha u'' w \\ \alpha u'' + (1-\alpha)\, v''\,(x_e)^2 \\ (1-\alpha)\, v'' x_e x_d \end{bmatrix} \tag{A17}$$

$$\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \begin{bmatrix} -\left[\alpha f''\gamma^2 + (1-\alpha)\, g''\gamma^2\right] \\ (1-\alpha)\, v'' x_e x_d \\ \alpha f''\gamma^2 + (1-\alpha)\left[g''\gamma^2 + v''\,(x_d)^2\right] \end{bmatrix} \tag{A18}$$

Then

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \mathrm{d}h^p/\mathrm{d}w \\ \mathrm{d}e/\mathrm{d}w \\ \mathrm{d}d/\mathrm{d}w \end{bmatrix} = \begin{bmatrix} \alpha\left[u''\,(1-h^p)\, w + u'\right] \\ \alpha u''\,(1-h^p) \\ 0 \end{bmatrix} \tag{A19}$$

Given that

$$-\frac{u''}{u'} < \frac{1}{(1-h^p)\, w}, \tag{A20}$$

the comparative statics signs are as follows. Take $\mathrm{d}h^p/\mathrm{d}w$ first.

$$sgn\ \mathrm{d}h^p/\mathrm{d}w =$$

$$= -sgn\left\{\alpha\left[u''\,(1-h^p)\, w + u'\right]\left(\underbrace{\underset{-}{a_{22}}\underset{-}{a_{33}} - \underset{-}{a_{32}}\,\underset{-}{a_{23}}}_{?}\right) + \underbrace{\alpha u''\,(1-h^p)}_{-}\underbrace{\left(\underset{-}{a_{23}}\underset{+}{a_{31}} - \underset{-}{a_{21}}\underset{-}{a_{33}}\right)}_{-}\right\}. \tag{A21}$$

The second term is positive. Doing the actual computations we can verify that also the first term is positive. In fact

$$\underbrace{\alpha^2 u'' f''\gamma^2}_{+} + \underbrace{\alpha u''\,(1-\alpha)\, g''\gamma^2}_{+} + \underbrace{\alpha u''\,(1-\alpha)\, v''\,(x_d)^2}_{+} + \underbrace{(1-\alpha)\, v''\,(x_e)^2\, \alpha f''\gamma^2}_{+} +$$

$$+\underbrace{(1-\alpha)^2\, v''\,(x_e)^2\, g''\gamma^2}_{+} + +\underbrace{(1-\alpha)^2\left(v''\right)^2 (x_e)^2\,(x_d)^2}_{+} > \underbrace{(1-\alpha)^2\left(v''\right)^2 (x_e)^2\,(x_d)^2}_{+}, \tag{A22}$$

Hence we can conclude that

$$dh^p/dw < 0. \tag{A23}$$

Consider then $\mathrm{d}e/\mathrm{d}w$.

$$sgn\ \mathrm{d}e/\mathrm{d}w =$$

$$-sgn\left\{\underbrace{\alpha u''\left(1-h^p\right)}_{-}\underbrace{\left(\underset{-}{a_{11}}\underset{-}{a_{33}}-\underset{+}{a_{31}}\underset{+}{a_{13}}\right)}_{?}+\underbrace{\alpha\left[u''\left(1-h^p\right)+u'\right]}_{+}\underbrace{\left(\underset{-}{a_{32}}\underset{+}{a_{13}}-\underset{-}{a_{12}}\underset{-}{a_{33}}\right)}_{-}\right\} \tag{A24}$$

which is positive because

$$\underset{-}{a_{11}}\underset{-}{a_{33}}-\underset{+}{a_{31}}\underset{+}{a_{13}}>0. \tag{A25}$$

From the calculation in fact we obtain

$$\begin{aligned}&\alpha u''w^2\left[\alpha f''\gamma^2+(1-\alpha)\,g''\gamma^2+(1-\alpha)\,v''\left(x_d\right)^2\right]+\\&+\left[\alpha f''\gamma^2+(1-\alpha)\,g''\gamma^2\right]\left[\alpha f''\gamma^2+(1-\alpha)\,g''\gamma^2\right]+\\&+\left(\alpha f''\gamma^2+(1-\alpha)\,g''\gamma^2\right)\left((1-\alpha)\,v''\left(x_d\right)^2\right)>\left[-\left[\alpha f''\gamma^2+(1-\alpha)\,g''\gamma^2\right]\right]^2.\end{aligned} \tag{A26}$$

Finally, let us consider $\mathrm{d}d/\mathrm{d}w$.

$$sgn\ \mathrm{d}d/\mathrm{d}w =$$

$$=-sgn\left\{\underbrace{\alpha\left[u''\left(1-h^p\right)+u'\right]}_{+}\underbrace{\left(\underset{-}{a_{12}}\underset{-}{a_{23}}-\underset{+}{a_{13}}\underset{-}{a_{22}}\right)}_{+}+\underbrace{\alpha u''\left(1-h^p\right)}_{-}\underbrace{\left(\underset{-}{a_{21}}\underset{+}{a_{13}}-\underset{-}{a_{11}}\underset{-}{a_{23}}\right)}_{-}\right\} \tag{A27}$$

The term in curly brackets is always positive, then

$$\mathrm{d}d/\mathrm{d}w<0. \tag{A28}$$

Appendix B - Equivalence of budget constraints

To see that budget constraint (27) is equivalent to budget constraint (28), let us first write the government budget if E is paid for by the government itself and then if E is paid by the parent:

$$\tau\int_0^{\overline{w}} w\left(1-h^p\left(w\right)\right)f\left(w\right)\mathrm{d}w+\tau\int_0^{\overline{w}} x\left(E+e,D+d\right)f\left(w\right)\mathrm{d}w-E=\widetilde{T} \tag{B1}$$

$$\tau\int_0^{\overline{w}} w\left(1-h^p\left(w\right)\right)f\left(w\right)\mathrm{d}w+\tau\int_0^{\overline{w}} x\left(E+e,D+d\right)f\left(w\right)\mathrm{d}w=T. \tag{B2}$$

To check the equivalence, integrate (27) to yield

$$\int_0^{\overline{w}}\left(c^p+\widehat{c}+pz+e\right)f\left(w\right)\mathrm{d}w-(1-\tau)\int_0^{\overline{w}} w\left(1-h^p\right)f\left(w\right)\mathrm{d}w=\widetilde{T}, \tag{B3}$$

and substitute the revenue constraint (B1); then integrate (28) to yield

$$\int_0^{\overline{w}} \left(c^p + \widehat{c} + pz + e\right) f\left(w\right) \mathrm{d}w + E - \left(1-\tau\right) \int_0^{\overline{w}} w \left(1 - h^p\right) f\left(w\right) \mathrm{d}w = T, \tag{B4}$$

and substitute (B2) (recall that the agents have unit mass). It is immediate to see that the resource constraints computed using the two procedures coincide:

$$\begin{aligned} \int \left(c^p + \widehat{c} + pz + e\right) f\left(w\right) \mathrm{d}w + E + \tau \int_0^{\overline{w}} x\left(E+e, D+d\right) f\left(w\right) \mathrm{d}w = \\ = \int_0^{\overline{w}} w \left(1 - h^p\right) f\left(w\right) \mathrm{d}w. \end{aligned} \tag{B5}$$

Appendix C - Comparative statics under policy

High wagers

Having no need to employ their time in home production, the high wagers set $h^p = h^k = 0$, that is $l = 1$ and $d = 1 - D$: all the parent's time goes into working and all the kid's time goes into education. The presence of D is of no consequence: at most, it equals $D = 1$, which is what the parents would have chosen anyway. Since, on the contrary, E is a constraint ($e = 0$), high-wagers choose z to maximise

$$\alpha \left(u\left(\left(1-\tau\right) w + T - \widehat{c} - pz - E\right) + f\left(z\right)\right) + \left(1-\alpha\right) \left(v\left(\widehat{c} + \left(1-\tau\right) x\left(E, 1\right)\right) + g\left(z\right)\right). \tag{C1}$$

The FOC is:

$$-\alpha u' p + \alpha f' + \left(1-\alpha\right) g' = 0, \tag{C2}$$

and it follows that

$$\frac{\partial z}{\partial w} = -\frac{-\alpha u'' p \left(1-\tau\right)}{\alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g''} > 0; \tag{C3}$$

$$\frac{\partial z}{\partial \tau} = -\frac{\alpha u'' w p}{\alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g''} < 0; \tag{C4}$$

$$\frac{\partial z}{\partial T} = -\frac{-\alpha u'' p}{\alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g''} > 0; \tag{C5}$$

$$\frac{\partial z}{\partial E} = -\frac{\alpha u'' p}{\alpha u'' p^2 + \alpha f'' + \left(1-\alpha\right) g''} < 0. \tag{C6}$$

Low wagers

Low-wagers are constrained by both E and D ($d = 0$, $e = 0$). Consequently, they only choose h^p to maximise

$$\alpha\left(u\left(\left(1-\tau\right)w+T-\widehat{c}-E-\left(1-\tau\right)wh^p\right)+f\left(\left(h^p+1-D\right)\gamma\right)\right)+$$
$$+\left(1-\alpha\right)\left(v\left(\widehat{c}+\left(1-\tau\right)x\left(E,D\right)\right)+g\left(\left(h^p+1-D\right)\gamma\right)\right). \tag{C7}$$

The FOC is

$$-\alpha\left(1-\tau\right)u'w+\left[\alpha f'+\left(1-\alpha\right)g'\right]\gamma=0. \tag{C8}$$

Hence, assuming

$$-\frac{u''}{u'}<\frac{1}{\left(1-\tau\right)w(1-h^p)}, \tag{C9}$$

we have that

$$\frac{\partial h^p}{\partial w}=-\frac{-\alpha\left(1-\tau\right)u'-\alpha\left(1-\tau\right)^2w\left(1-h^p\right)u''}{\alpha u''\left(1-\tau\right)^2w^2+\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}=$$
$$-\frac{\overbrace{-\alpha\left(1-\tau\right)}^{-}\overbrace{\left(u'+\left(1-\tau\right)w\left(1-h^p\right)u''\right)}^{+}}{\alpha u''\left(1-\tau\right)^2w^2+\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}<0; \tag{C10}$$

$$\frac{\partial h^p}{\partial \tau}=-\frac{\alpha w\overbrace{\left[u'+\left(1-\tau\right)w\left(1-h^p\right)u''\right]}^{+}}{\alpha u''\left(1-\tau\right)^2w^2+\alpha\left[f''+\left(1-\alpha\right)g''\right]\gamma^2}>0; \tag{C11}$$

$$\frac{\partial h^p}{\partial T}=-\frac{-\alpha\left(1-\tau\right)wu''}{\alpha u''\left(1-\tau\right)^2w^2+\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}>0; \tag{C12}$$

$$\frac{\partial h^p}{\partial E}=-\frac{\alpha\left(1-\tau\right)wu''}{\alpha u''\left(1-\tau\right)^2w^2+\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}<0; \tag{C13}$$

$$\frac{\partial h^p}{\partial D}=-\frac{-\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}{\alpha u''\left(1-\tau\right)^2w^2+\left[\alpha f''+\left(1-\alpha\right)g''\right]\gamma^2}>0. \tag{C14}$$

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