Trade Liberalization and Optimal Taxation with Pollution and Heterogeneous Workers

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Abstract

In this paper, we address two questions: (i) how should a government pursuing both environmental and redistributive objectives design domestic taxes when redistribution is costly, and (ii) how does trade liberalization affect this optimal tax system, and modify the economy's levels of pollution and inequalities ? Using a general equilibrium model under asymmetric information with two goods, two factors (skilled and unskilled labor) and pollution, we fully characterize the optimal mixed tax system (nonlinear income tax and linear commodity and production taxes/subsidies). We provide simulations highlighting the linkages between pollution, labor income redistribution and increasing globalization with our endogenous fiscal system. In the redistributive case (i.e. in favor of the unskilled workers) and when the dirty sector is intensive in unskilled labor, we show that (i) trade liberalization involves a clear trade-off between the reduction of inequalities and the control of pollution when the source of externality is mainly production ; this is not necessarily true with a consumption externality; (ii) under openness to trade, the source of the externality matters for redistribution, while it is not the case in autarky. Finally we discuss the impact of an increasing willingness to redistribute income and of a technological shock affecting emissions intensity.

JEL : H21, H23, F13, F18

Keywords:

Optimal tax theory, International trade, Pollution, Production efficiency

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February 5, 2014

1. Introduction

It is widely acknowledged that income distribution within a country is greatly determined by its trade regime and its fiscal system. Debates about governments' ability to redistribute the gains from trade, and compensate losers, have been revisited in the light of modern normative economic theory. It suggests that the limitations of actual redistributive tools only allow the attainment of second best objectives, and challenges the separation of efficiency and redistributive considerations - making tariffs possibly part of a country's optimal policy mix (Guesnerie, 2001).

Besides, increasing environmental concerns over the last decades translates into an evolution of the tax system in many European countries. Because pollution taxes bring up simultaneously new distributional concerns and new revenues for the government, the opportunity of a "Green Fiscal Reform" has given rise to many discussions among politics and economists. The idea is to recycle revenue from new environmental taxes to reduce pre-existing distortionary labor taxes, either to compensate -to some extent- low-income households for their added burden on the uses side (Fullerton & Monti, 2013), or to make regressive pollution taxes acceptable to any category of workers (Chiroleu-Assouline & Fodha, 2014).

Now, individual attitudes with regard to environmental policy are also influenced by openness to trade, as has been highlighted in a political economy framework (McAusland, 2003; Hotte & Winer, 2012). Clearly, international trade complicates the task of designing optimal environmental and redistributive policies ; moreover, the consequences of freer trade on pollution and inequalities also crucially depend on an economy's tax system. To our knowledge, no analysis has tried to tackle these issues together. The aim of this paper is twofold : (i) to characterize the optimal domestic tax system in a small economy with pollution and heterogeneous workers, when redistribution is costly (skills are not directly observable) and (ii) to examine the effects of trade liberalization on the tax system, on after-tax income inequalities, and on pollution.

The paper is organized as follows. Section 2 reviews the three strands of literature we bring together, and underlines our contributions. Section 3 presents the basic framework in a general formulation that will be used to characterize the optimal structure of taxation in a situation of trade (section 4) and in autarky (section 4.5). Section 5 is devoted to the simulation-based analysis of how increasing globalization impacts the optimal tax policy, the extent of inequality and pollution. Section 6 draws conclusions.

2. Related Literature and Contribution

Our analysis bridges the gap between three strands of the literature: the optimal taxation theory with externalities, the normative economics of international trade, and the trade-and-environment literature.

First, an important strand of the public economics literature has studied the optimal tax design in a closed economy, when the government has both redistribution and environmental or public goods objectives. Early contributions (Sandmo, 1975) assuming linear taxes, have shown that the optimal tax rate for a polluting commodity is a weighted sum of the inverse elasticity of demand for that good, and the marginal social damage. This additivity property holds true when agents differ in their wage earning abilities and the government is concerned with redistribution. Building on Mirrlees (1971) and on Atkinson & Stiglitz (1976)'s mixed taxation models, more recent contributions with weaker informational assumptions on the government side (unobservable skills and non-linear income taxes) have addressed the trade-off between environmental and redistributive goals in a closed economy: these papers highlight the role that pollution can play on the individual incentive compatibility constraints that weight on redistributive policy. Pirttila & Tuomala (1997) show that, while the externality's direct harmful impact raises marginal tax rates, its influence through the self-selection constraint tends to limit the rise. This model has been extended by Tenhunen (2007b) with general equilibrium feedbacks allowing wages to be endogenous: it shows that some of the harm from the externality may be compensated by gains in redistribution, and that the environmental efficiency and equity based parts in the commodity tax rates cannot necessarily be separated from each other. In a setting where agents differ along two dimensions (market ability and tastes), Cremer et al. (1998) compare the way externalities affect the optimal mixed taxation system when the government can observe individual consumption choices (allowing for non-linear commodity taxes as a tool for redistribution), and when only aggregate purchases can be observed (linear commodity taxes): they show that while externalities may affect the income tax structure in the second case, they never affect the formula of the taxes on (non-polluting) commodities. In a closed economy with fixed wages, Micheletto (2008) provides a quite general framework to design taxes optimally with asymmetric information and externalities, allowing in particular for pollution to affect agents in a differentiated way.

Second, another strand of the normative literature has focused on labor income distribution in open economies when government intervention is constrained by the unobservability of labor types.[1] Naito (1996, 2006) shows that in a small open economy, distorting the production sector through tariffs or taxes/subsidies may be optimal because it reduces the incentive problem of the non-linear income tax system, and lower costs of redistribution outweigh the inefficiencies created in production. In a similar framework, Spector (2001) has shown that trade, by making prices exogenous, reduces the government's set of redistributive policies, and as a result that opening borders may decrease welfare, even with optimal policy adjustment. Environmental issues are not considered in these contributions.

The present paper extends the frameworks developed by Naito and Micheletto (*op. cit.*) in several directions: we model explicitly the production sector so that wages are endogenous and hence depend on the public policy implemented; we consider the pollution and the trade issues simultaneously; we allow for both production and consumption externalities, and examine their respective impact on incentive compatibility constraints under different assumptions regarding the separability of the utility function in consumption, leisure and pollution; and we provide a full characterization of the optimal mixed tax system (nonlinear income tax and linear commodity and production taxes/subsidies) in two situations of interest, autarky and international trade. In particular we derive and discuss the marginal income and effective tax rate for skilled and unskilled workers.

Finally, our paper builds also on the theoretical literature on trade and the environment. The unified framework developed by Copeland & Taylor (2003) in a Heckscher-Ohlin like model with a dirty sector and endogenous environmental taxes helps clarifying many issues at the country level. The explicit decomposition of environmental outcomes into a scale, a composition and a technique effect shows that in a small open economy, the pattern of specialization does crucially determine whether trade liberalization results in higher or lower levels of emissions. The analytical framework used in this literature typically relies on a representative consumer economy, that is not suited to deal with redistributive concerns between heterogeneous agents within the country. We extend it in several directions: we assume heterogeneous workers, make labour supplies endogenous (factors are supplied inelastically in Copeland & Taylor), allow for consumption externalities in the dirty sector, and introduce redistributive policies, so that the consequences of trade liberalization on net incomes is not limited to Stolper-Samuelson mechanisms.

In our simulation-based analysis, we explore the impact of increasing globalization on the trade-redistribution-pollution nexus. We concentrate on the redistributive case, where the gov-

[1] See Verdier (2005) for a survey.

ernment wants to redistribute towards the unskilled workers, and on the situation where the polluting sector (e.g. industry) is intensive in unskilled labor. We show among other results that (i) trade liberalization involves a clear trade-off between the reduction of inequalities and the control of pollution when the source of externality is mainly production ; this is not necessarily true with a consumption externality; (ii) under openness to trade, the source of the externality matters for redistribution, while it is not the case in autarky ; (iii) an increasing willingness to redistribute income corresponds to a shrinking economy under autarky while only the polluting sector shrinks in a open economy ; (iv) the relative importance of production and consumption in the pollution generating process influences not only the direction of trade but also the social cost of income redistribution. Last, our analysis suggests that "green" technological progress, in the sense of a reduction of emissions intensity, may be desirable for more than just environmental objectives as it allows to reduce further income inequalities.

3. The model

We consider a two agent types, two factors and two goods model of a small, perfectly competitive economy, facing fixed world prices. Consumer-workers choose their consumption of private goods and labour supply, while the government chooses tax instruments, namely a nonlinear income tax and linear commodity taxes and production subsidies. One of the two goods (say good 1) is chosen as the numeraire and without loss of generality goes untaxed so that the design of the indirect tax system is reduced to the choice of the commodity tax and production subsidy on good 2. We denote p_k^* the international price of good k, $k = 1, 2$ whereas p_k and q_k denote respectively the producer and consumer price in the country. As good 1 is the numeraire, we also normalize for convenience the international price of good 1 to 1. This implies that $p_1^* = p_1 = q_1 = 1$. We assume that the good 2 produced at home and in the rest of the world are perfect substitutes and hence it follows that the government does not need to impose a tariff on good 2 as this instrument would be redundant given the production subsidy and the commodity tax on good 2. To save on notations, we denote hereafter p^*, p and q the international, production and consumer price for good 2. Hence, the production subsidy on good 2 is given by $p - p^*$ and the commodity tax on good 2 is given by $q - p^*$.

There are two types of consumer-workers, skilled or unskilled (denoted respectively by superscript s and u), in respective proportions π^s and π^u (with $\pi^s + \pi^u = 1$). Emissions e stem from both consumption and production of good 2. Hence, we model a situation with both production and consumption externalities. We assume that agents of different types face the same pollution (assumption of an atmospheric externality) but need not be equally vulnerable to the externality. Denoting the consumption of good 2 by agent of type i as c_2^i, the corresponding aggregate consumption by c_2 and the aggregate production level of good 2 as y_2, we have

$$e = E(c_2, y_2) \tag{1}$$

where $c_2 = \pi^s c_2^s + \pi^u c_2^u$ and we assume that $E(.,.)$ is increasing in each argument.

3.1. The consumers

Workers have preferences over two consumption goods, labor supply l and the public bad (pollution) e, given by a strictly quasi-concave utility function $U^i(c_1^i, c_2^i, l^i, e)$, $i = s, u$. We assume that $U^i(.)$ is increasing in consumption of the two goods and decreasing in labor supply and public bad. We assume that both private goods and leisure are normal goods.[2]

At this stage, we do not make any assumption regarding the separability of pollution, leisure and consumption. This allows to take into account both the impact of pollution on consumption and on labor supply. Indeed, a change in the pollution level may well influence the pattern of consumption: for instance an improvement in environmental quality may increase the marginal utility of consuming outdoor activities. Furthermore, pollution and leisure may be complements or substitutes as well: improving the quality of the environment may increase the disutility of labor (because one would want to consume more of leisure activities) or may decrease it because of reduced pollution-related health problems.

We break the individual's optimization problem into two parts: (i) given disposable income, choose the optimal commodity basket and obtain the conditional indirect utility function; (2) choose hours of work.

Let I^i denoting the disposable (or after tax) income for consumer $i = s, u$. We define the conditional indirect utility function as follows:

$$V^i(q, I^i, l^i, e) = \max_{c_1^i, c_2^i} U^i(c_1^i, c_2^i, l^i, e) \text{ s.t. } c_1^i + qc_2^i \leq I^i$$

and the optimal c_1^i and c_2^i of this maximization program are the conditional demand functions, denoted respectively $c_1(q, I^i, l^i, e)$ and $c_2(q, I^i, l^i, e)$. Note that the influence of pollution on the consumption pattern depends essentially on how the marginal rate of substitution between the two goods $((\partial U^i/\partial c_2)/(\partial U^i/\partial c_1))$ evolves with pollution.

[2]Normality of goods and leisure is a sufficient condition for the "single-crossing" condition to hold.

At the second stage of utility maximization, the number of hours worked l^i is chosen to maximize indirect utility subject to the relationship between primary (or before tax) income and disposable income as implied by the income tax schedule:

$$l^i(q, I^i, e) = \arg\max V^i(q, I^i, l^i, e) \text{ s.t. } I^i = w^i l^i - T(w^i l^i)$$

where w^i denotes the wage rate for workers of type i and $T(w^i l^i)$ the income tax.

3.2. The producers

Concerning production, we assume the standard Heckscher-Ohlin framework. There are two industries that each exhibits constant returns to scale and a concave production function, $y_k = F_k(L_k^u, L_k^s)$, $k = 1, 2$. Hence, production y_k is made from both skilled labor L_k^s and unskilled labor L_k^u. Each industry maximizes its profit taking as given the price of goods and wages. Let w^u and w^s denote wages for unskilled and skilled workers respectively. We assume that at equilibrium the production is diversified and two goods are always produced. We assume also that one of the two industries is always skilled-labour intensive for each pair of wages.

Let $C_k(w^s, w^u)$ denote the cost function for one unit of good k, then perfect competition and constant returns to scale imply (when production is diversified):

$$\begin{aligned} C_1(w^s, w^u) &= 1 \\ C_2(w^s, w^u) &= p. \end{aligned}$$

Hence, given p, these two equations determine the wages uniquely and so the wage ratio $\frac{w^u}{w^s}$ as a function of p. It is useful to recall the Stolper-Samuelson theorem here, which tells us that if the producer price p increases, then the wage of labour force intensively used in sector 2 will increase while the other wage will decrease. In particular, if the polluting sector is intensive in unskilled labor then we have $\partial(w^u/w^s)/\partial p > 0$.

Finally, from the Shephard's lemma, factor demands are:

$$L_k^i = y_k \frac{\partial C_k(w^i, w^j)}{\partial w^i} \tag{2}$$

for $i, j = s, u$, $i \neq j$ and $k = 1, 2$.

3.3. Production and equilibrium

Concerning the labor market, we assume that labor is perfectly mobile across sectors within the small economy, but not internationally. Hence, labor market equilibrium conditions are:

$$\pi^i l^i = L_1^i + L_2^i$$

for $i = s, u$. Using the labor demands given by (2) and labor supplies, it follows that we can write production y_k as a function of p, $\pi^s l^s$ and $\pi^u l^u$: $y_k(p, \pi^s l^s, \pi^u l^u)$.

Goods market equilibrium conditions are:

$$\pi^s c_1^s + \pi^u c_1^u = y_1 + m_1 \quad (3)$$

$$\pi^s c_2^s + \pi^u c_2^u = y_2 + m_2 \quad (4)$$

where m_k represents the amount of good k that is imported/exported when the economy is open to trade, while balanced trade implies that:

$$m_1 + p^* m_2 = 0$$

Hence, replacing the expressions of m_1 and m_2 using (3) and (4), we have

$$\pi^s c_1^s + \pi^u c_1^u - y_1 + p^* \left(\pi^s c_2^s + \pi^u c_2^u - y_2\right) = 0. \quad (5)$$

3.4. The government

The objective of the government is to design a tax system to maximize a weighted sum of unskilled and skilled class of workers. Let λ^s and λ^u represent the weights of the skilled and unskilled workers in the government objective ($\lambda^s + \lambda^u = 1$), which may differ from the actual proportions π^s and π^u. Thus $\lambda^u > \pi^u$ (or equivalently $\lambda^s < \pi^s$) represents the "redistributive" case, when the government's concern for unskilled workers' welfare is more than proportional. Since the government cannot differentiate taxes by skills, because it observes only primary income and not the agent's type, the maximization problem is constrained by the incentive compatibility constraints: each type should weakly prefer and select the bundle of disposable income-primary income (I, wl) intended for it instead of mimicking the one intended for the other. Formally, the incentive compatibility constraint for type i writes as follows:

$$V^i(q, I^i, l^i, e) \geq V^i(q, I^j, \frac{w^j l^j}{w^i}, e)$$

for any $i \neq j, i, j = s, u$. Indeed, a type-i worker whose wage is w^i is obliged to work $\frac{w^j l^j}{w^i}$ to mimic the primary income $w^j l^j$ get by worker j. To save on notations, we will denote in the following $V^{ij} = V^i(q, I^j, \frac{w^j l^j}{w^i}, e)$ for $i, j = s, u$.

Finally, the government is constrained by its budget constraint that writes as:

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) - (p - p^*) y_2 + \sum_{i=s,u} (q - p^*) \pi^i c_2^i \geq 0$$

recalling that sector 1 goes untaxed and that sector 2 is subject to a production subsidy $p - p^*$ per unit and a consumption tax $q - p^*$ per unit.

Note first, that by Walras' law, the balance trade equation and the budget constraint are necessarily equivalent. Also, as noted earlier, a tariff would be redundant with our mix of production subsidy and consumption tax. Nevertheless, it is possible to reinterpret the model by specifying an indirect tax policy based on a tariff equal to $q - p^*$ and a tax on production equal to $q - p$. Indeed, using (4) and rearranging, the budget constraint becomes now

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + (q-p)y_2 + (q-p^*)m_2 \geq 0.$$

Finally, note that, in autarky ($m_2 = 0$ or equivalently $y_2 = \sum_{i=s,u} \pi^i c_2^i$), the budget constraint simply becomes

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + (q-p)y_2 \geq 0.$$

4. Optimal tax policy in a small open economy

The program of the government can be written as follows:

$$\max_{I^i, l^i, q, p, e} \sum_{i=s,u} \lambda^i V^i(q, I^i, l^i, e)$$

$$\text{s.t.}$$

$$V^i(q, I^i, l^i, e) \geq V^i(q, I^j, \frac{w^j l^j}{w^i}, e) \quad \forall i, j = s, u, \ i \neq j$$

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + \sum_{i=s,u} (q-p^*)\pi^i c_2^i(q, I^i, l^i, e) - (p-p^*)y_2(p, \pi^s l^s, \pi^u l^u) \geq 0$$

$$e = E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) \qquad \forall i, j = s, u$$

Let μ^i, ν and ρ be the Lagrange multipliers of the type-i workers' incentive-compatibility constraint, the budget constraint (balanced trade condition) and the pollution definition, respectively. The corresponding Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = &\sum_{i=s,u} \lambda^i V^i(q, I^i, l^i, e) + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left[V^i(q, I^i, l^i, e) - V^i(q, I^j, \frac{w^j l^j}{w^i}, e) \right] \\ &+ \nu \left[\sum_{i=s,u} \pi^i \left[(w^i l^i - I^i) + (q-p^*)c_2^i(q, I^i, l^i, e) \right] - (p-p^*)y_2(p, \pi^s l^s, \pi^u l^u) \right] \\ &+ \rho \left[E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) - e \right] \end{aligned} \tag{6}$$

The set of equations corresponding to the first-order conditions characterizes the Pareto-efficient allocations constrained by self-selection in this small open economy with pollution from both production and consumption. In the following subsections, we will derive and discuss the optimal commodity tax ($q-p^*$), the harmfulness of pollution (ρ), the optimal production subsidy ($p-p^*$) and finally we will characterize the marginal effective (τ'^i) and income tax (T'^i).

4.1. Optimal commodity tax

The first issue is to see whether there is an interest for the planner to introduce distortions on consumption (i.e. by having consumption price q being different from the world price p^* in the polluting sector 2). This established in the following proposition.

Proposition 1. *The optimal consumption tax structure satisfies:*

$$q - p^* = -\frac{\rho}{\nu}\frac{\partial E}{\partial c_2} + t_{NS} \tag{7}$$

with $t_{NS} = \frac{\sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left(c_2^j - c_2^{ij}\right) \frac{\partial V^{ij}}{\partial I}}{\nu \sum_{i=s,u} \pi^i \frac{\partial \tilde{c}_2^i}{\partial q}}$.

Proof. See Appendix B. ■

The optimal distortion of the consumption price with regard to the world price consists of two terms: the first part of the right hand side of equation (7) represents the pigovian term ($-\frac{\rho}{\nu}\frac{\partial E}{\partial c_2}$) while the second one t_{NS} originates from the incentive compatibility constraints where NS stands for Non Separability (as it will be clear below). First, the pigovian term reflects the marginal social cost of consumption externalities ($\rho\frac{\partial E}{\partial c_2}$) weighted by the marginal cost of public funds.

The component t_{NS} conveys the differential consumption of a mimicker and a truly unskilled type. Why would a mimicker with the same income than an unskilled worker select a different consumption basket? This is because their hours of work/leisure differ: let H represent individuals total endowment in hours, and $h^i_{leisure}$ the number of hours worked for type i. As a skilled mimicker works $\frac{w^u l^u}{w^s}$ hours to get the gross income of an unskilled worker $w^u l^u$, his time available for leisure is $h^s_{leisure} = H - \frac{w^u l^u}{w^s} > H - l^u = h^u_{leisure}$. In other words, unless the preferences are weakly separable over consumption and leisure, a mimicker would not pick the same consumption basket. This opens the way for the planner to distort consumption as this helps to relax incentive constraints. Indeed, if the skilled mimicker consumes more (less) than the low skilled worker (i.e. $c_2^{su} - c_2^u > (<) 0$ or equivalently, good 2 and leisure are complementary (substitute) goods), then it is efficient to tax (subsidize) consumption of good 2 in order to deter the skilled worker from mimicking. When leisure and consumption are weakly separable in the utility function, this distortion naturally vanishes and only the pigovian term remains.

The sign of the pigovian term in Proposition 1 is related to the sign of ρ because both $\frac{\partial E}{\partial c_2}$ and ν are positive. We will show that despite consumers are willing to pay for a reduction of pollution, it is not guaranteed that ρ is always negative due to some general equilibrium effects. We may as well have a commodity subsidy for the polluting good as we explain in the next section.

We sum up our discussion in the following corollary.

Corollary 2. *Assume that pollution is globally harmful ($\rho < 0$). Concerning the tax policy on consumption, there is accordance (conflict) between environmental and redistributive concerns when the polluting good and leisure are complements (substitute).*

4.2. The harmfulness of pollution

Derivating (6) with respect to e gives the following first-order condition with respect to pollution:

$$\frac{\partial \mathcal{L}}{\partial e} = \sum_{i=s,u} \lambda^i \frac{\partial V^{ii}}{\partial e} + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left(\frac{\partial V^{ii}}{\partial e} - \frac{\partial V^{ij}}{\partial e}\right) + \nu(q-p^*) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} + \rho \sum_{i=s,u} \pi^i \frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial e} - \rho = 0. \quad (8)$$

Replacing in (8) the expression of $q - p^*$ expressed in (7) and rearranging yields the following expression for the marginal social cost of pollution:

$$\rho = \sum_{i=s,u} \lambda^i \frac{\partial V^{ii}}{\partial e} + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left(\frac{\partial V^{ii}}{\partial e} - \frac{\partial V^{ij}}{\partial e}\right) + \nu t_{NS} \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e} \quad (9)$$

There are three components in the RHS of (9). The first one is:

$$\sum_{i=s,u} \lambda^i \frac{\partial V^{ii}}{\partial e}$$

and reflects the direct globally negative impact of pollution on the workers' utility. Indeed, it represents the social marginal willingness to pay to avoid pollution.

The second one is:

$$\sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left(\frac{\partial V^{ii}}{\partial e} - \frac{\partial V^{ij}}{\partial e}\right)$$

and represents the impact of pollution on the incentive compatibility constraints. The planner takes into account that by manipulating the tax system, pollution will change and will impact the cost of redistributing labor incomes. Indeed, this exploits the difference between the marginal valuation of pollution by a mimicker and that one by a truthful revealing agent, in order to weaken the self-selection constraint. For instance, in the *redistributive case*, that is when the social weight for unskilled workers exceeds their proportion in the population ($\lambda^u > \pi^u$), then only the incentive compatibility constraint of the skilled workers is binding, $\mu^s > 0$ and consequently $\mu^u = 0$. Assume that the marginal disutility of labor increases with pollution (because of health related effects). This is equivalent to say that leisure and pollution are complements. Since the mimicker is a skilled worker, he needs to work less to mimic the primary income of an unskilled worker and hence he has more leisure time than a truthful revealing skilled agent. When pollution and leisure are complements, then the marginal willingness to pay to avoid pollution is lower for the mimicker than for the truth telling agent. Consequently, the term $\mu^s \left(\frac{\partial U^{ss}}{\partial e} - \frac{\partial U^{su}}{\partial e}\right)$ is negative and it adds to the (negative) direct impact of pollution. In other words, decreasing pollution diminishes the desirability of mimicking.

Hence, complementarity between pollution and leisure provides an additional motivation to reduce pollution as this helps relaxing the incentive constraint of skilled workers. The environmental objective is in accordance with the redistributive objective in that case. Alternatively, substitution between pollution and leisure provides some motive to increase pollution and hence in this case there is a conflict between protecting the environment and redistributing labor incomes.

Now, the impact of pollution on consumption decisions also plays a role in the second term. The direction of the distortion brought to pollution through consumption changes in order to relax incentive constraints remains difficult to ascertain without specifying more the utility function.

Finally, the third term in the RHS of equation (9) is $\nu t_{NS} \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial e}$. When considering a marginal change in pollution, one has to anticipate that this will modify the consumption of good 2 and in turn the money collected from consumers through the redistributive part of the consumer tax (or transferred to them in case of a subsidy), in order to favor labor by skilled workers. Assume that t_{NS} is positive at the optimum, which means that one has to tax good 2 in order to alleviate incentive compatibility constraints. It remains to ascertain whether pollution encourages or discourages the consumption of goods. If the former holds, then a increase in pollution will increase the tax collected on goods which in turn is socially beneficial in terms of budget. This impact is to be valued using ν the shadow cost of money and taken into account in the shadow price of pollution ρ.

Overall, it is clear that the sign of ρ is generally ambiguous. In particular, it is possible to have a *positive* sign if the last two terms are sufficiently positive, that is if the impact of pollution on the self-selection constraints is such that one would want to increase pollution to save on redistribution cost and if pollution increases sufficiently the demands for good 2.

4.3. Optimal production subsidy

We now turn to the issue of fixing the production price. The next Proposition describes the optimal distortion brought to the production sector.

Proposition 3. *The optimal production subsidy structure satisfies:*

$$p - p^* = \frac{\rho}{\nu}\frac{\partial E}{\partial y_2} + t_{EW} \tag{10}$$

with $t_{EW} = -\frac{1}{\nu \frac{\partial y_2}{\partial p}} \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i l^j \frac{\partial V^{ij}}{\partial l} \frac{\partial\left(\frac{w^j}{w^i}\right)}{\partial p}$ *where EW stands for endogenous wages.*

Proof. See Appendix C ■

Let us assume that sector 2 is intensive in unskilled labor. Assume also that the income tax is redistributive (towards the unskilled) so that $\mu^s > 0$ and $\mu^u = 0$. The second term t_{EW} of the right hand side of (10) is positive because of the Stolper-Samuelson effect (and because $\frac{\partial y_2}{\partial p} > 0$) while the first (pigovian) term is negative if $\rho < 0$. This means that on the one hand, one would want to subsidy production because this allows to relax the incentive constraints (due to the impact on wages) and on the other hand one would want to tax production because of harmful pollution. Whether there is a subsidy or a tax on production (i.e. whether $p > p^*$ or not) depends on the trade-off between these two forces. In any case, the production system is not efficient.

Another way to interpret the result is as follows. Introducing a pure pigovian tax on an otherwise untaxed good may decrease welfare. It is only if the environmental tax takes into account the redistributive concern (i.e. the impact on the distribution of wages) that there is a Pareto-improvement. In other words, the optimal production tax is non pigovian.

Finally, suppose on the contrary that the polluting sector is intensive in skilled labor, then the redistributive term is negative (because the Stolper-Samuelson effect goes in the other way, i.e. $\frac{\partial\left(\frac{w^u}{w^s}\right)}{\partial p} < 0$). It is then optimal to tax production for two reasons, redistributive and environmental concerns.

We sum up our results as follows.

Corollary 4. *Assume that pollution is globally harmful ($\rho < 0$). Concerning the tax/subsidy policy on production, there is conflict (accordance) between environmental and redistributive concerns when the polluting sector is intensive in the type of labor (not) favored by redistribution.*

4.4. Income and effective marginal tax rates

Let us turn to the optimal non-linear income tax in this small open economy. Recall that the primary income is $w^i l^i$ while the disposable income is $w^i l^i - T(w^i l^i)$ where $T(.)$ denotes the income tax. Since we have only two types, it is sufficient to establish two points on the income tax schedule.

The second step of consumer optimization, i.e. when the worker maximizes his utility w.r.t. his labor supply subject to a given tax schedule, enables the marginal income tax rate to be expressed in terms of the utility function:

$$T'^{i} \equiv T'(w^i l^i) = 1 + \frac{\frac{\partial V^i}{\partial l}}{w^i \frac{\partial V^i}{\partial I}}. \tag{11}$$

Similarly, the effective marginal tax can be defined by taking into account the indirect tax system. Indeed, the total tax paid by a consumer-worker of type i is defined as follows:

$$\tau(w^i l^i) = T(w^i l^i) + (q - p^*) c_2^i(q, I^i, l^i, e)$$

and the marginal effective tax rate is:

$$\tau'^i \equiv \tau'(w^i l^i) = T'(w^i l^i) + (q - p^*) \left[\left(1 - T'(w^i l^i)\right) \frac{\partial c_2^i}{\partial I} + \frac{1}{w^i} \frac{\partial c_2^i}{\partial l} \right] \tag{12}$$

The next Proposition characterizes the income and effective marginal tax rates for this small open economy.

Proposition 5. *Under openness, the marginal effective tax rate is given by:*

$$\tau'^i = \frac{t_{EW}}{w^i} \frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu \pi^i w^i} \left(\frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} - \frac{\partial V^{ji}}{\partial I} \frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}} \right) - \frac{\rho}{\nu w^i} \frac{\partial E}{\partial c_2} \left. \frac{dc_2^i}{dl} \right|_{dV^i = 0} \tag{13}$$

and the marginal rate of income taxation is:

$$T'^i = \frac{t_{EW}}{w^i} \frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu \pi^i w^i} \left[\frac{\partial V^{ji}}{\partial l} \frac{w^i}{w^j} - \frac{\partial V^{ji}}{\partial I} \frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}} \right] - \frac{t_{NS}}{w^i} \left. \frac{dc_2^i}{dl} \right|_{dV^i = 0}. \tag{14}$$

Proof. See Appendix D. ■

From (14), note that the presence of externalities does not influence directly the marginal rate of income taxation, but only indirectly through the levels of activities. This is consistent with the Principle of Targeting: it does not pay to distort labor supply in order to reduce pollution in this model. Taxing the consumption and the production of the polluting good is sufficient to internalize the externalities, even if wages are endogenous.

Assume once again the redistributive case ($\mu^s > 0$ and $\mu^u = 0$). Concerning the skilled workers, the marginal rate of income tax reduces to (the second term in (14) disappears):

$$T'^s = \frac{t_{EW}}{w^s}\frac{\partial y_2}{\partial L^s} - \frac{t_{NS}}{w^s}\left.\frac{dc_2^s}{dl}\right|_{dV^s=0}$$

The first term comes from the endogeneity of wages and is always negative whatever the sector intensive in skilled labor. Indeed, if sector 2 is intensive in unskilled labor, then $t_{EW} > 0$ and $\frac{\partial y_2}{\partial L^s} < 0$ (because of the Rybczynski theorem) and vice versa if sector 2 is intensive in skilled labor. Intuitively, it always pays for a government to distort upwards the labor supply from skilled workers in order to diminish their equilibrium wage and thereby to increase the wage of unskilled workers. This is why the first term tends to make the marginal income tax negative. The second term is due to the non separability between consumption of the polluting good and labor.

Concerning the unskilled workers, the marginal income tax becomes:

$$T'^u = \frac{t_{EW}}{w^u}\frac{\partial y_2}{\partial L^u} + \frac{\mu^s}{\nu\pi^i w^i}\left[\frac{\partial V^{su}}{\partial l}\frac{w^u}{w^s} - \frac{\partial V^{su}}{\partial I}\frac{\frac{\partial V^u}{\partial l}}{\frac{\partial V^u}{\partial I}}\right] - \frac{t_{NS}}{w^u}\left.\frac{dc_2^u}{dl}\right|_{dV^u=0}$$

Here the first term is positive: the government has interest to tax the labor supply of unskilled workers in order to increase their equilibrium wage which in turn allows to reduce the wages inequality. The second term corresponds to the usual Mirrleesian distortion and is positive: by taxing the labor supply of unskilled workers, one increases the cost of mimicking for skilled workers which then allows to reduce the overall cost of redistribution under asymmetric information. The third term represents the incentives to distort labor supply because of the non separability between consumption and leisure.

4.5. Optimal fiscal policy in autarky

For completeness, we now describe what would happen in an otherwise identical but closed economy.[3] First, under autarky, the optimal consumption tax naturally aggregates the four terms identified in the optimal indirect tax system for an open economy. Indeed, the optimal consumption is composed of the two pigovian taxes (one for the consumption externality and the other one for the production externality), plus the tax due to the non separability between consumption and labor at the utility level and the subsidy (when the polluting sector is intensive

[3]All the results are straightforward and are established in an appendix available upon request (see Appendix F).

in unskilled labor and for the redistributive case) due to the endogeneity of wages:

$$q - p = -\frac{\rho}{\nu}\left(\frac{\partial E}{\partial c_2} + \frac{\partial E}{\partial y_2}\right) + t_{NS} - t_{EW}.$$

Moreover, the marginal income tax rate remains unchanged,so that the comparison of the optimal mixed tax system under autarky and trade is straightforward:

Proposition 6. *The expressions of the optimal income tax rate and the total indirect taxes are similar for a small economy under an autarky and a trade situation. Only their levels differ.*

Only the expression of the marginal effective tax rate differs under autarky and is now given by:

$$\tau'^{i} = \frac{t_{EW}}{w^i}\left(\frac{\partial y_2}{\partial L^i} + 1\right) + \frac{\mu^j}{\nu \pi^i w^i}\left(\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j} - \frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right) - \frac{\rho}{\nu w^i}\left(\frac{\partial E}{\partial c_2} + \frac{\partial E}{\partial y_2}\right)\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}.$$

This difference in formulation results from the incorporation of terms (the cost of production externality and the implicit subsidy due to the endogeneity of wages) in the consumption tax, due to the endogeneity of producer prices in a closed economy.

In the simulations presented in the next section, the autarky situation will be replicated by an open economy for a specific value of the world price p^* that entails no trade with the rest of the world.

5. Trade liberalization, pollution and inequalities

In section 4, we have characterized the tax system that maximizes welfare for a given vector of social weights (λ^i) and given the incentive compatibility constraints, and shown that the expressions of optimal taxes remain similar under autarky and trade. In order to investigate the impact of freer trade on the *level* of taxes and outcomes like emissions and inequalities, we need to put more structure on the model by specifying preferences and the production/pollution functions.

5.1. Specification and parameterization

5.1.1. Preferences

We assume the following additive separable form of the utility function:

$$U^i(c_1^i, c_2^i, l^i, e) = u(c_1^i, c_2^i) - \psi(l^i) - \varphi(e) \tag{15}$$

where $u(c_1^i, c_2^i) = \ln\left(c_1^i\right)^{\alpha}\left(c_2^i\right)^{1-\alpha}$ is the (homothetic) subutility function of consumption, $\psi(l^i) = \gamma\frac{\left(l^i\right)^{1+1/\xi}}{1+1/\xi}$ is the (isoelastic) disutility of labor and $\varphi(e) = \omega\frac{(e)^{1+1/\phi}}{1+1/\phi}$ is the (isoelastic) disutility of pollution. Parameters γ and ω are positive scale parameters of the labor and pollution disutilities respectively.[4] Parameter ξ is the Frisch elasticity of labor supply and is usually estimated between 0.1 and 1 in econometric estimations. Parameter $\phi > 0$ plays a similar role in the pollution disutility function.

5.1.2. Production and pollution

Regarding production of good $k = \{1, 2\}$, we assume a Cobb-Douglas technology with constant returns to scale:

$$F_k(L_k^u, L_k^s) = A_k\,[L_k^s]^{\sigma_k}[L_k^u]^{1-\sigma_k} \tag{16}$$

and the assumption that the polluting sector (good 2) is intensive in unskilled labour is captured in our parameterization with $\sigma_2 < \sigma_1$.

We also assume that pollution in the economy is described by the following linear specification:

$$e = \beta\,[\delta c_2 + (1-\delta)y_2] \tag{17}$$

where parameter β captures the *intensity of emissions* (pollution per unit of dirty good) and parameter δ (resp. $(1-\delta)$) the share of consumption (resp. production) in emissions.

5.1.3. Consequences for the optimal policy

Our specification above allows several simplifications with regard to our previous results. First, by assuming homothetic preferences for consumption, we can study purely redistributive expenditures, i.e., those expenditures that do not alter the composition or the size of demand (Alesina and Perotti, 1997).[5]

Second, separability between labor and consumption implies that there is no need to distort consumption in order to alleviate the cost of incentive compatibility constraints. Therefore, the term t_{NS} (arising from non-separability between consumption and labour decision) vanishes in the expression of the optimal consumption tax (equation (7)) and in the expressions of the marginal rates (14) and (13). It follows that the optimal consumption tax consists only in the following pigovian term:

$$q - p^* = -\frac{\rho}{\nu}\beta\delta.$$

[4]For the clarity of exposition, we assume that pollution hurts the skilled and the unskilled workers in the same manner. Actually, introducing a differentiated impact of pollution on consumers (through some type-dependant scale parameters $\omega^s \neq \omega^u$) would not change the optimal policy in this setting as long as the "global" scale parameter $\omega = \omega^s + \omega^u$ is kept constant, because welfare is the sum of individual indirect utilities.

[5]Due to homothetic preferences, the consumption functions are $c_1^i(I^i, q) = \alpha I^i$ and $c_2^i(I^i, q) = (1-\alpha)\frac{I^i}{q}$ and the indirect utility function V^i is increasing in real income defined as the ratio of net income I^i to a consumer price index $F(q)$: $V^i(I^i, l^i, q, e) = \ln\frac{I^i}{F(q)} - \psi(l^i) - \varphi(e)$ where $F(q) = \frac{q^{1-\alpha}}{\alpha^\alpha.(1-\alpha)^{1-\alpha}}$.

Regarding marginal income tax rates, since $t_{NS} = 0$, skilled workers will always face a negative marginal rate:[6]

$$T'^s = \frac{t_{EW}}{w^s}\frac{\partial y_2}{\partial L^s} < 0 \tag{18}$$

while unskilled workers will face a positive marginal rate consisting in a (positive) endogenous wages term and a (positive) mirrleesian term:

$$T'^u = \frac{t_{EW}}{w^u}\frac{\partial y_2}{\partial L^u} + \frac{\mu^s}{\nu \pi^u w^u}\left[\frac{\partial V^{su}}{\partial l}\frac{w^u}{w^s} - \frac{\partial V^{su}}{\partial I}\frac{\frac{\partial V^u}{\partial l}}{\frac{\partial V^u}{\partial I}}\right] > 0 \tag{19}$$

This means that a government concerned with costly redistribution in favor of the unskilled does optimally distort upward the labour supply from the skilled, and does distorts downward the labour supply from the unskilled, in order to increase their equilibrium wages w^u. Although pollution does not explicitly appear in the expressions of the marginal rates, we will discuss its indirect effects through the level of the other variables.

Second, the separability between pollution and consumption/labor decisions leads to a simple expression of the social cost of pollution ρ defined in (9): because pollution has no influence on the incentive compatibility constraint, ρ only reflects the weighted sum of each type's marginal disutility of emissions:

$$\rho = \sum_{i=s,u} \lambda^i \frac{\partial V^{ii}}{\partial e} = -\omega e^{1/\phi}$$

that is, for $\phi < 1$, an increasing and convex function of aggregate emissions.

Third, solving the first order conditions with regard to I^s, I^u and q yields a simple expression for the optimal levels of net incomes:

$$I^s = \frac{\lambda^s + \mu^s}{\nu \pi^s} \text{ and } I^u = \frac{\lambda^u - \mu^s}{\nu \pi^u}.$$

Combining these two equations, we obtain a simple expression for the cost of public funds:

$$\nu = \frac{1}{\sum \pi^i I^i} \tag{20}$$

and a measure of after-tax income inequalities:

$$\frac{I^u}{I^s} = \frac{(\lambda^u - \mu^s)/\pi^u}{(\lambda^s + \mu^s)/\pi^s} \tag{21}$$

[6] As argued in section 3.4. In this section, we assume more specifically that sector 2 in intensive in unskilled labour, so that $t_{EW} > 0$ and from the Rybczynski theorem $\frac{\partial y_2}{\partial L^s} < 0$.

Clearly, asymmetric information (the effect of which is captured by the multiplier μ^s) decreases the regulator's ability to leave unskilled workers with a higher after-tax income with respect to skilled workers. Since the value of μ^s is determined endogenously for given parameters, our simulations will allow us to discuss the consequences of trade liberalization in terms of redistribution costs and after-tax income inequalities.

5.1.4. Parameters values for numerical simulations

The complexity of the whole specified system does not allow to obtain closed-form expressions for our (eleven) variables, so we turn to simulations.[7] Table 1 indicates the parameter values we hold constant in all sets of simulations.

Fraction of individuals in each group	$\pi^s = \pi^u = 0.5$
Parameters of the utility function	$\alpha = 0.6;\ \xi = 0.3;\ \ \phi = 0.5; \gamma = 1; \omega = 0.75$
Parameters of the production functions	$A_1 = A_2 = 1;\ \sigma_1 = 0.8;\ \sigma_2 = 0.6$

Table 1: Fixed parameters

Results are organized as follows: we first investigate the consequences of trade liberalization (section 5.2) on pollution, the level of optimal taxes, and inequalities. In line with the literature, trade liberalization is modelled as a decrease in trade costs for sector 2, assuming there are no trade barriers for the numeraire: this allows to interpret a move toward freer trade as a variation in the relative world price p^*.[8]

We start by solving numerically the system in the case of a pure production externality ($\delta = 0$), for the whole range of world prices such that the output of both goods is positive (partial specialization of the economy). This first set of simulations constitutes our "benchmark" scenario of trade liberalization, and the assumptions regarding the values of the parameters used in this scenario are given in Table 2.

This represents a familiar scenario in the trade and environment general equilibrium literature (*à la* Copeland and Taylor), with emissions stemming only from the production process of good 2, with some additional features: labor is heterogeneous and elastically supplied, the regulator has a moderate preference for redistribution towards the unskilled workers ($\lambda^u > \pi^u$), and due to

[7]The Mathematica notebooks with the detailed calculations and resolution algorithms are available upon request.

[8]If the economy has a comparative advantage for the good 2 (exports the polluting good), then trade liberalization is equivalent to an increase in the relative world price p^* . Conversely, if it has a comparative advantage for the clean good (imports the polluting good), then the impact of trade liberalization is captured in a decrease in p^* .

Weight of the unskilled workers in regulator's preferences	$\lambda^u = 0.6 \quad (\lambda^s = 0.4)$
"Emission intensity"	$\beta = 2$
Share of consumption in total emissions	$\delta = 0$

Table 2: Values of the variable parameters for the benchmark scenario

asymmetric information, optimal redistributive policies involve a distortion of both the domestic producer price and the labor supplies (through the type-specific marginal income tax rate). With a pure production externality, the parameter β captures the intensity of emissions e/y_2 in the polluting sector.[9] The value for β in this scenario is arbitrary, but we will discuss in section 5.4 the consequences of an exogenous shock (technological progress) in the emission intensity.

Then, in section 5.3, we compare the results in terms of pollution and inequalities with a pure consumption externality ($\delta = 1$), which is also a familiar assumption in the optimal taxation literature, and also discuss the effects of freer trade on the pollution-inequality nexus in situations where emissions may stem both from production and consumption of the polluting good.

Finally, we let some of the parameters in Table 2 vary and study the consequences, for this small open economy, of a political shock on government's redistributive preferences (section 5.5), and the consequences of a technological shock on the intensity β of emissions (section 5.4).

5.2. Trade liberalization with a pure production externality
Does trade liberalization with optimal policies increase pollution?

In the case of a pure production externality, emissions are proportional to output in sector 2, i.e. $e = \beta y_2(p, l^s, l^u)$, where the producer price p and the labour supplies l^i are changing with both the economic conditions (change in the world price p^* after trade liberalization) and the domestic environmental and redistributive policies.

Total emissions from production of good 2 are drawn in Fig 1 below as a function of the world price. Starting from the *Autarky* situation (when the relative world price level is such that no trade at all occurs), total emissions will increase (decrease) with p^* as the economy becomes increasingly specialized in good 2 (resp. good 1) of the polluting good.

Thus, Copeland and Taylor's result (for a 2 good, 2 factors H-O-S model with pollution proportional to output in one sector) holds true when we allow for endogenous factor supplies

[9] In this paper, for simplicity, we do not take into consideration the possibility for polluting firms to abate emissions. For this, one would need to consider an augmented production function with three inputs, pollution and the two types of labor.

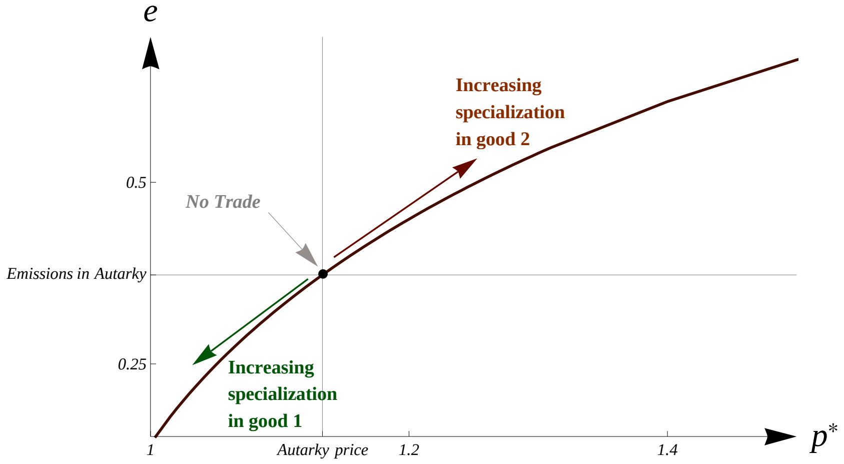

Figure 1: *Impact of trade liberalization on emissions: the pure production externality case*

and redistributive policies, distorting both labour supplies and producer prices: the direction in which trade liberalization drives the specialization of the economy (the composition effect), is the crucial determinant of the increase/decrease of pollution. With a pure production externality, trade liberalization will increase pollution if and only if the economy is (or becomes) an exporter of the polluting good.

The intuition for this result can be described as follows. From the expression of y_2 in Appendix E, we know it is an increasing function of p and l^u, and a decreasing function of l^s. Our simulations show that p increases with p^* (recall that $p = p^* + t_{EW} - t_{pigou}$, where $t_{pigou} = \rho\beta/\nu$ and where all terms are increasing in p^* - even though $|t_{pigou}| >> t_{EW}$, see Figure 2). The optimal labour supplies are such that l^s decreases with p^* while l^u displays an inverted U shape. Thus an increase in p^* increases the output of good 2 because (i) of an unambiguous increase in producer price, (ii) of a decrease in l^s and (iii) the initial increase in l^u also tends to increase y_2, while its decrease for high values of p^* does not offset the increasing trend of y_2 due to the other variables.

How does freer trade affect the level of taxes?

In the case of a pure production externality, the government's environmental policy is implemented by the way of a pigovian tax on producers. Her redistributive objectives are implemented using two instruments: (i) a subsidy t_{EW} to the sector that makes intensive use of the unskilled labour in order to reduce the wage gap between skilled and unskilled workers and thus reduces μ^s, a measure of the cost of redistribution; and (ii) a non-linear income tax system designed in order to distort upward the labour supply from the skilled, and downward the labour supply from the unskilled.

The expression of the optimal "endogenous-wages" subsidy is:[10]

$$t_{EW} = \frac{\mu^s}{\nu} \frac{\left[\frac{w^u}{w^s} l^u\right]^{1+1/\xi}}{\frac{\partial y_2}{\partial p}} \frac{\partial \frac{w^u}{w^s}}{\partial p}$$

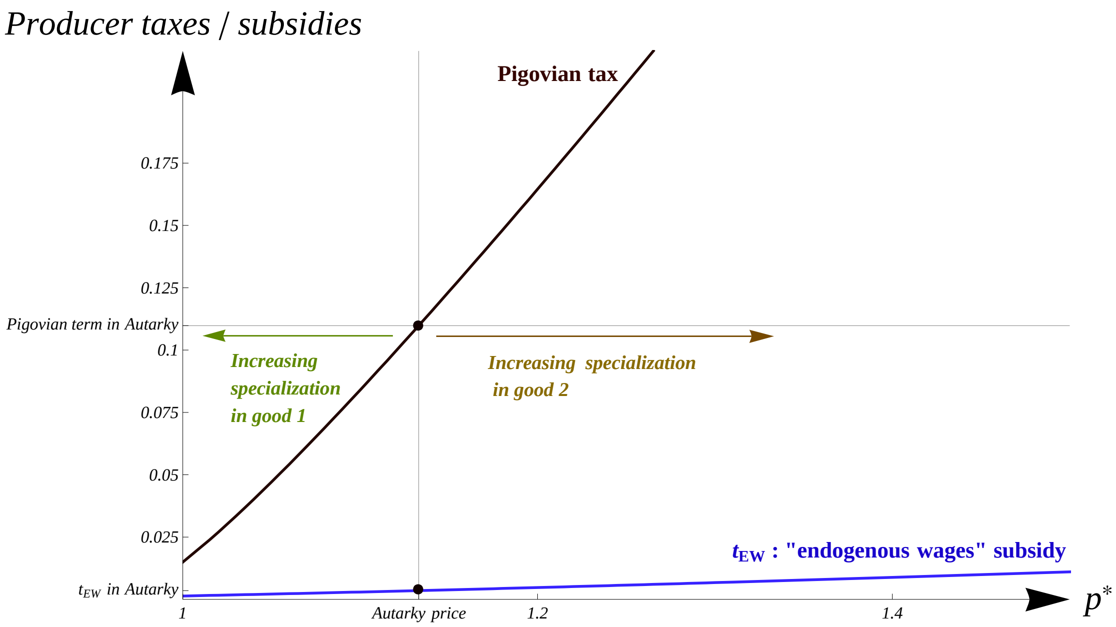

Figure 2: *Impact of trade liberalization on the optimal distortion of the producer price*

From figure 2, we note that both the pigovian tax (in absolute value) and the endogenous wages subsidy are increasing with p^*, i.e., when trade liberalization results in an increased specialization in the polluting, unskilled-labour intensive, sector 2.

While the shape of the pigovian term is not surprising, and reflects the increasing social cost of pollution in the economy when production of good 2 increases, the shape of the production subsidy t_{EW} might seem counter-intuitive at first sight: indeed, when the relative price of the polluting good increases, the wage gap between the unskilled and the skilled decreases ($\frac{w^u}{w^s}$ increases), so why should the regulator increase a subsidy that aims precisely at reducing the before-tax wage gap? Two mechanisms are at stake: first, even though it appears from our simulations that μ^s decreases with p^* , so does the cost of public funds ν. The variation of

[10] See details in *Appendix E*.

the ratio $\frac{\mu^s}{\nu}$ with p^* is not monotonic and U-shaped, meaning that above a threshold level in prices, the decrease in the cost of public funds dominates the one in redistribution cost, i.e. the social cost of distorting production becomes relatively lower. Second, it appears that the marginal increase in the wage ratio $\frac{\partial \frac{w^u}{w^s}}{\partial p}$ is also an increasing function of p^*. In other words, even though an increase in the relative price of the polluting good makes redistribution less necessary, the optimal subsidy to the sector 2 increases because it becomes both less distorting and more efficient.

Let's turn to the changes in marginal tax rates when trade barriers decrease. Simulations confirm that the optimal marginal rate for the skilled workers is negative, while it is positive for the unskilled. Figure 3 illustrates how trade liberalization changes the marginal income tax rate for the skilled (left panel) and the unskilled workers.

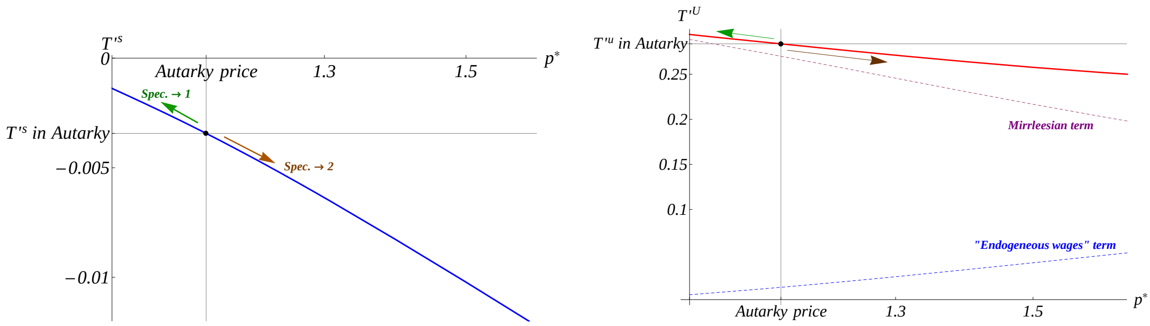

Figure 3: *Impact of trade liberalization on marginal income tax rates*

As it appears from the expression of T'^s in (18), for the skilled workers, the increase (in absolute value) of the marginal income tax rate with p^* is due to the increase in t_{EW} described above, and to the decrease in w^s (Stolper-Samuelson effect). In other words, if trade liberalization results in an increase in the relative price of the dirty good, the economy becomes an exporter of the good using intensively unskilled labour: the optimal marginal income tax goes down (becomes more negative) with respect to its autarky level, in order to compensate the decrease in skilled workers' labour demand. Conversely, if the economy specializes in the clean good, trade liberalization stimulates demand of skilled labour and the optimal marginal income tax rate goes down (in absolute value) with regard to the autarky.

For the unskilled, the expression of T'^u is made of two terms, a mirleesian distortion and an "endogenous wages" term (see 19). As can be seen from the right panel in Figure 4, the "endogenous wages" component of T'^u is positive and increasing in p^*, while the "mirleesian" distortion is decreasing, resulting in a decreasing marginal income tax rate. The shape of the endogenous wages term can be explained by the increase in t_{EW} as already discussed above.[11] The "mirleesian" distortion aims at increasing the cost of mimicking for the skilled workers. With our separable specification of utility, the incentive compatibility constraint simplifies to:

$$V^{ss}(I^s, l^s, q, e) > V^{su}(I^u, l^u \frac{w^u}{w^s}, q, e) \Leftrightarrow ln\frac{I^s}{I^u} > \psi(l^s) - \psi(l^u \frac{w^u}{w^s}) \quad (22)$$

where $\psi(.)$ is the increasing labour disutility function. When trade liberalization results in an increase in the relative price of good 2, p^*, the wage ratio $\frac{w^u}{w^s}$ increases, so that the RHS of (22) decreases: mimicking becomes less attractive, and the mirrleesian distortion necessary to prevent it decreases unambiguously.

Note that in this benchmark scenario, unskilled workers always obtain a net income transfer from the government. As it can be seen in Figure 4, $T(w^u l^u)$ has an inverted-U shape, but always remains negative. For skilled workers, the burden of income taxes decreases as the economy specializes towards the dirty good. This reflects the fact that the gross income of skilled workers unambiguously decreases with trade liberalization when p^* increases: both w^s (because of the Stolper-Samuelson effect) and l^s (because of lower wages) are decreasing functions of p^*. Eventually, when trade liberalization drives the economy to a high degree of specialization in the dirty good, the optimal income tax schedule is a subsidy for both the skilled and the unskilled. Note that the subsidy t_{EW} is also high in this case: all these transfers are permitted by the very high level of environmental taxes, which become the only source of government revenue.

How does trade with optimal policies affect the pollution-inequality nexus?

As has been highlighted in the theoretical model, when the polluting sector is also the sector making intensive use of unskilled labour, there are *conflicting environmental and redistributive objectives in terms of indirect taxes*: in our benchmark, the optimal distortion of the producer price is made of two conflicting elements, a production subsidy and a pigovian tax, and it appears from above that both are increasing with p^*. Figure 2 illustrates how these components tend to become more (less) conflicting for an exporter (importer) of a polluting good.

Does this polarization between social and environmental objectives in optimal policies appear in the pollution/inequalities outcomes of trade liberalization? Figure 5 represents all the reachable level of pollutions (on the y-axis) for a given level of inequality (x-axis), for all possible

[11] Note that, for the unskilled, the increase in w^u in the denominator tends to limit the increase in the "endogenous wages" term, while for the skilled, the decrease in w^s tends to amplify it.

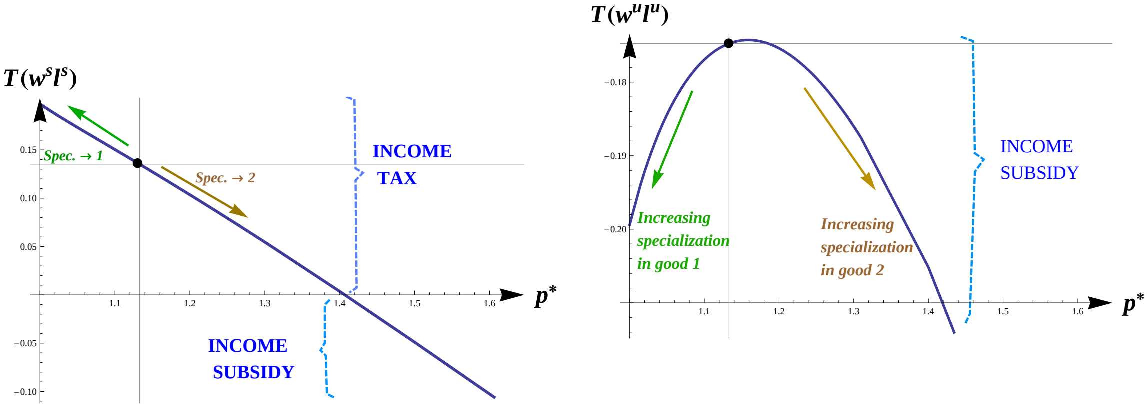

Figure 4: *Impact of trade on net income taxes: the pure production externality case*

relative price-changes induced by trade.

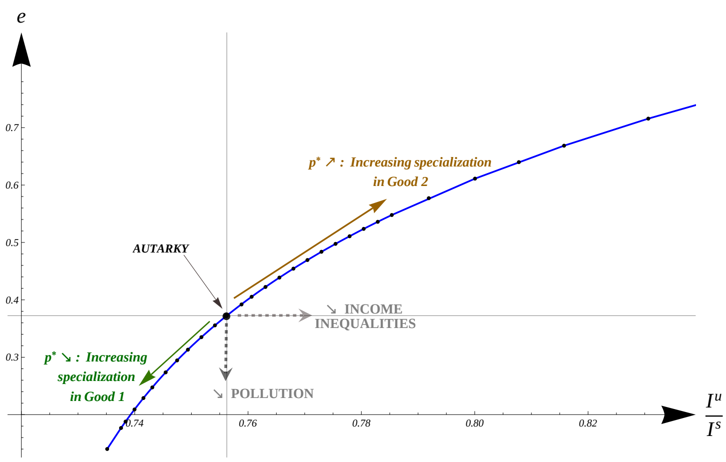

Figure 5: *The Pollution-Inequality Nexus of trade with a pure production externality*

It shows that in our benchmark scenario, there is no "free lunch": for an exporter of the dirty good, trade liberalization will increase pollution but decrease inequalities, while for an importer, the decrease in pollution comes with an increase in inequalities.

5.3. Trade liberalization with consumption externalities

We now revisit the main conclusions drawn with the benchmark scenario (pure production externality) and allow for an increasing share of emissions from consumption of the dirty good.

Impact of trade on emissions

Fig 6 illustrates how trade liberalization, i.e. a change in the relative price p^*, affects the level of emissions for alternative assumptions regarding the share δ of emissions from consumption in total emissions. Our benchmark scenario (pure production externality, $\delta = 0$) is reminded in blue, while the situation of a pure consumption externality ($\delta = 1$) is drawn in red and detailed in the right panel.

Let us first have a look at the situation under Autarky. Since producer and consumer prices of good 2 depend on the level of pigovian taxes, reflecting the respective share of production and consumption in the social cost of aggregate emissions, the autarky price (or more precisely, the world price at which no trade occurs) varies with δ, as shown in Figure 6. However, the level of emissions under Autarky remains constant whatever the respective share of production and consumption in total emissions, due to the constraint of $c_2 = y_2$.

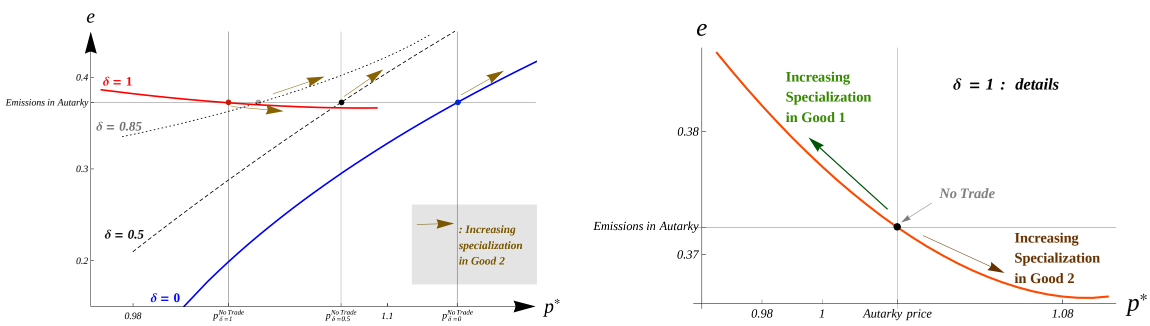

Figure 6: *Impact of trade on pollution: increasing the share δ of emissions from consumption*

In the case of a pure consumption externality, the impact of trade on pollution appears to be the opposite of our benchmark scenario: emissions increase when the economy imports more of the polluting good. Indeed, if the economy has a comparative advantage in sector 1 (so that a decrease in trade barriers results in an increased specialization in sector 1), the relative price of the polluting good will go down, so that imports, consumption and associated emissions will increase. Conversely, a specialization in the dirty good comes with an initial reduction in emissions. However, the shape of emissions w.r.t. p^* is no longer monotonic and appears to be U-shaped, reflecting some income effects of trade liberalization. Indeed, total emissions are given by the aggregate level of consumption of good 2:

$$e = \beta[\pi^s c_2^s(I^s, q) + \pi^u c_2^u(I^u, q)] = \beta \frac{(1-\alpha)}{q} \sum_{i=s,u} \pi^i I^i$$

From (20), we know that variations in the (nominal) aggregate after-tax income $\sum_{i=s,u} \pi^i I^i$ are measured by the change in the inverse of the opportunity cost of public funds ν . Thus the impact of trade on emissions can be decomposed in a (negative) price-effect of trade on consumption (q is unambiguously increasing in p^*), and a positive income effect which is ambiguous (the variation of ν w.r.t. p^* is not monotonic -inverted U-shape- but always decreasing for an exporter of good 2). Hence, it is difficult to draw general conclusions on the consequences of trade liberalization on pollution with at least a fraction of emissions stemming from consumption.

How does freer trade affect the level of taxes?

Regarding the indirect taxes/subsidies (see fig 7) and comparing with the production-externality, it is easy to see that the producer subsidy t_{EW} remains an increasing function of p^*, whatever the value of δ (left panel of the figure). However, even though the level of subsidy is the same under autarky, it appears that for a given value of world price, t_{EW} is higher when emissions from production get lower (higher δ). Regarding the environmental taxes, in the case of a pure consumption externality, the pigovian term on consumer price is U-shaped (right panel of the figure).

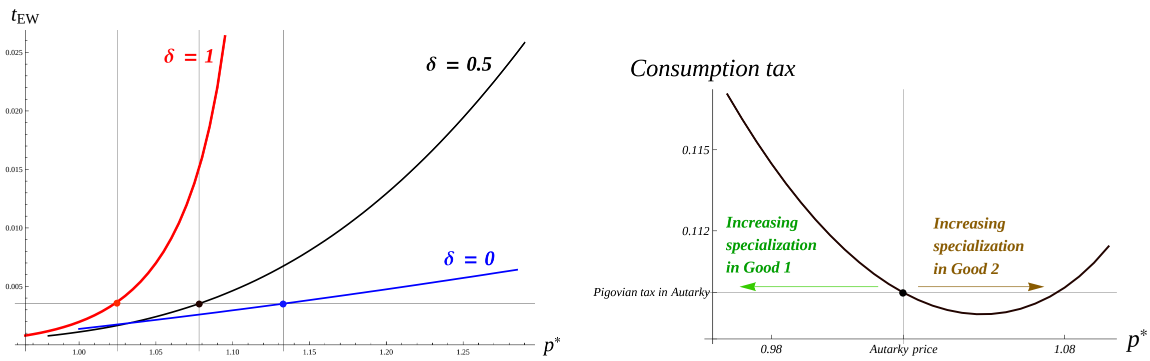

Figure 7: *Production subsidy / Consumption tax with consumption externalities*

This simply reflects the fact that the pigovian term in consumption taxes is proportional to the social cost of pollution, which in turn -with our separable specification of preferences- is an increasing and convex function of aggregate emissions described above.

The marginal tax rates for the skilled and the unskilled are drawn in Figure 8 . The effect of trade liberalization on marginal rates remains comparable to our benchmark, so we won't detail it again; but interestingly, it appears that in an open economy, a greater share of consumption externalities involves frequently a higher distortion in the labour from the skilled, and a lower distortion of the labour from the unskilled.[12]

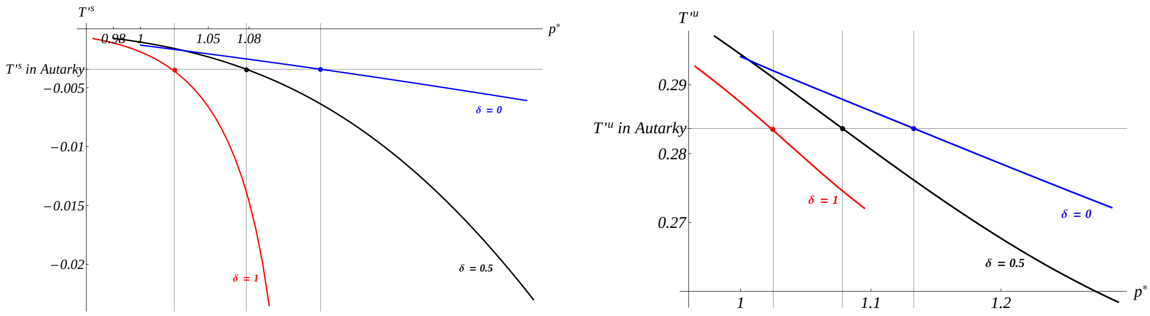

Figure 8: *Impact of trade on marginal income tax rates with alternative* δ

How does trade with optimal policies affect the pollution-inequality nexus?

From (21), we know that the impact of trade liberalization on inequalities (as measured by the ratio of net incomes $\frac{I_u}{I_s}$) is determined completely by the variations of μ^s (the cost of redistribution) when trade barriers decrease. The shadow cost of the incentive compatibility constraint is strictly decreasing in p^*, because of the Stolper-Samuelson effect: the wage ratio $\frac{w^u}{w^s}$ increases when trade stimulates the sector making intensive use of unskilled labour, which decreases the information rent necessary to prevent skilled workers from mimicking. This remains true whatever the source of the externality, as shown in Figure 9.

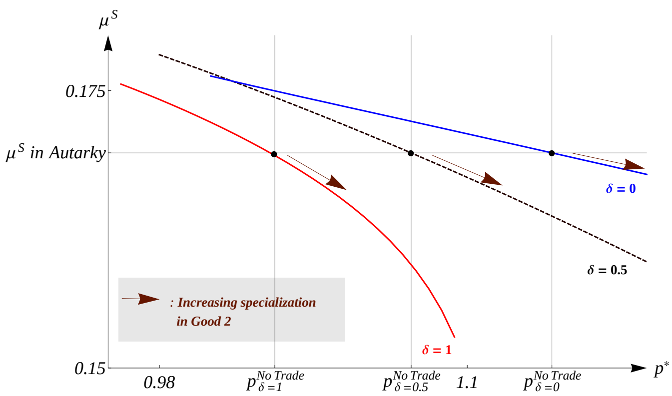

Figure 9: *Impact of trade liberalization on the cost of redistribution, under alternative assumptions on the source of emissions*

[12]Recall that in Autarky, the source of the externality does not matter as argued above.

A decreasing value for μ^s thus involves an increasing ratio of net incomes $\frac{I_u}{I_s}$: in other words, optimal redistributive policies do not modify the expected outcome of trade liberalization in terms of income inequalities: they unambiguously increase when the sector making intensive use of unskilled labour shrinks and the economy imports more of the good 2, but decrease for exporters of good 2.

Looking at the possible outcomes of trade liberalization in terms of both pollution and inequalities leads to the following results, as presented in Figure 10. Note also that the source of the externality does not matter in autarky.

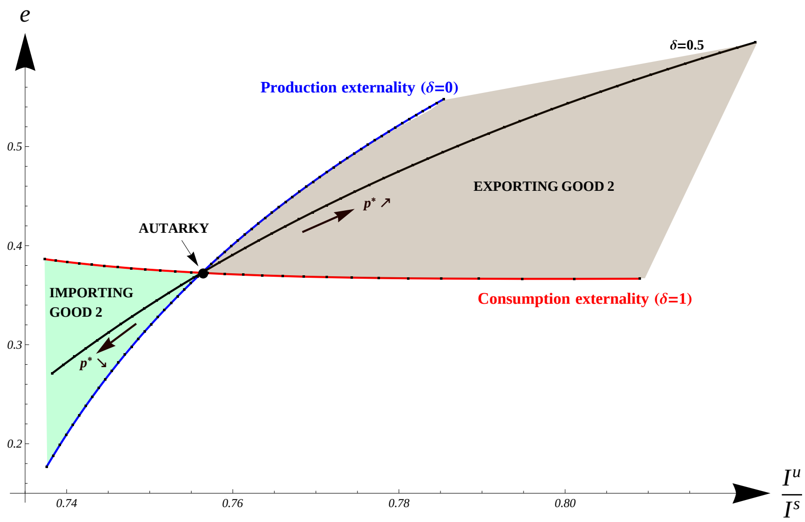

Figure 10: *The pollution-inequality nexus : importance of the source of emissions*

While exporters of Good 2 always face a decrease in income inequalities from trade liberalization, the consequences on pollution will depend on income effects, and except for a pure production externality where emissions necessarily increase, they depend on parameters. The simulations presented here show that the nature of the externality can have strong implications on the pollution-inequality nexus.

5.4. Simulating a technological shock: decreasing emission intensity

Suppose an exogenous change in the intensity of polluting emissions β, for example an exogenous technological progress allowing each unit of the polluting good produced or consumed to cause less emissions. The issue at stake is how does this influence the economy?

First, a change in the intensity of emissions has a direct consequence on the economy's comparative advantage: for a given world price, a decrease in β results in more production of the polluting good y_2 (as it becomes cleaner) and less production of good 1; consumption of both goods increases. Eventually, for an important decrease in β , an economy that used to be a net importer of good 2 can become a net exporter.

The impact of a diminishing intensity of emissions on pollution is not monotonic: total emissions are given by $e = \beta\left[\delta c_2 + (1-\delta)y_2\right]$ where both c_2 and increase y_2 when β decreases. This involves that for very high initial levels of β, technological progress allowing a diminution in per-unit emissions may paradoxically result in more pollution. Since the dirty sector is intensive in unskilled labour, and production y_2 increases with a fall in β, this tends to increase w^u. Our simulations indicate that the cost of redistribution μ^s decreases with a fall in β.

Finally, Figure (11) shows that decreasing the intensity of polluting emissions may allow a reduction in pollution and in inequalities, provided the initial level of β is not too high: thus *"green" technological progress may be desirable for more than just environmental objectives.*

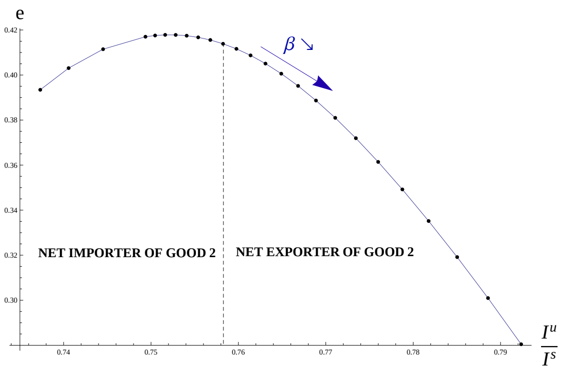

Figure 11: Impact of diminishing intensity of emissions on the pollution-inequality nexus

5.5. Simulating a change in the regulator's objective: variation in the Pareto weights

Because the optimal policy is defined for given Pareto weights in the government's objective, it is interesting to look at the consequences of a political shock (e.g. a change in the government's objective after elections) in this small open economy. In this simulation set, we fix the world price at 1.1 which is larger than the autarky price, so that the open economy is exporting the polluting good on the world market. We then vary the social weight λ^u from 0.5 to 1. This change expresses an increasing willingness of the government to redistribute from the skilled to the unskilled. This scenario corresponds under autarky to a shrinking economy. Indeed, all productions (and consumptions for both skilled and unskilled) are decreasing as well as pollution. The reason is that in this economy redistribution is socially costly and this inefficiency increases with the willingness to redistribute. The cost of redistribution as measured by the shadow price of incentive compatibility constraints actually increases in λ^u, as well as the cost of public funds ν. It is worth noting that the polluting sector size is decreasing although the subsidy t_{EW} increases with λ^u, which indicates that the government seeks to stimulate the polluting sector in order to favor the unskilled wage level.

Looking at the income tax structure under autarky, we notice that there is a raising discrepancy between the negative marginal tax for skilled people (intended to stimulate their labor supply) which raises in absolute value and the positive marginal tax intended for unskilled people which raises with λ^u. Globally, after tax incomes (as well for wages and before tax incomes) inequality is reduced as the desire to redistribute increases.

By contrast, in an open (and polluting good exporting) economy, the polluting sector is still shrinking whereas imports in sector 1 are decreasing as the production of numeraire increases. This causes also pollution to decrease with λ^u, but the pollution level in the open economy is always larger than under autarky. In short, the expansion of pollution emissions following a trade liberalization for an exporting country comes from a larger increase in production than the decrease in the consumption of the polluting good.

Note also that the pigovian parts of the indirect tax system are decreasing with λ^u which can be explained by a simultaneous decrease in the shadow price of pollution (ρ) and an increase in the cost of public funds (ν).

6. Conclusion

Several points deserve particular attention in our analysis. First, we have shown that the optimal mixed tax system does not differ fundamentally under a situation of autarky and of open trade: the formula are the same, even though the equilibrium levels naturally differ. This contrasts with previous findings that trade liberalization constrains the form of optimal redistributive policies as prices become exogenous. Second, the targeting principle holds true: the level of the externality does not modify the expression of marginal income tax rates.

The externality is optimally addressed using indirect taxes on production and consumption (with a pigovian term reflecting the marginal damage arising from either source). Both producer and consumer taxes may include a redistributive term aimed at alleviating the cost of redistribution. These terms may conflict with pigovian terms : for example if the polluting sector is intensive in unskilled labour, a public regulator will optimally both subsidize the sector for redistributive objectives and tax it for environmental reasons. Using simulations, we show that this effect is even greater when the economy becomes an exporter of the polluting good after trade liberalization - even though the wages of the unskilled workers naturally tend to increase: trade in this case exacerbates the polarization between environmental and redistributive objectives.

Simulations based on a simple specification (additive separable preferences, Cobb-Douglas technologies, linear externality function) shed some light on several recurrent questions in the literature. Results suggest that: *(i)* optimal redistributive policies do not cancel the polarization of skilled vs. unskilled interests to trade liberalization : inequalities increase when the sector using unskilled labour intensively suffers from increased imports, and conversely, just as predicted in the standard Stolper-Samuelson framework ; *(ii)* the impact of trade on pollution is not trivial ; except for the special case of a pure-production externality, where Copeland and Taylor's result (on the composition effect as the only driver of an increase/decrease in emissions) holds true in our framework with redistributive policies and endogenous factor supplies, predictions depend on several crucial variables like the share of consumption in total emissions, the composition effect and the importance of income effects. Besides, to our knowledge, this paper is the first one to consider the consequences of trade liberalization on the *level* of all domestic taxes adjusted optimally to maximize a weighted sum of skilled and unskilled worker's utility.

Regarding the trade-pollution-inequalities nexus, our simulations show that the source of the externality (production vs consumption) is a crucial parameter. With a production externality, trade will result in an increase in emissions and a decrease in inequalities for an exporter of the polluting good (and conversely for an importer) ; with a consumption externality, trade will decrease both pollution and inequalities for an exporter of a polluting good (and conversely for an importer). Finally, we show that a technological progress allowing for a decrease in the intensity of emissions also unambiguously decreases the cost of redistribution, and that trade may reinforce this effect.

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Appendix

Appendix A. Proof that $y_2 = \sum_{i=s,u} \pi^i l^i \frac{\partial w^i}{\partial p}$

The zero profits conditions for both sectors are

$$\begin{aligned} p &= C_2(w^s, w^u) \\ 1 &= C_1(w^s, w^u) \end{aligned}$$

Hence, $y_1 + py_2 = C_2(w^s, w^u)y_2 + C_1(w^s, w^u)y_1$. Differentiating totally w.r.t. p, we obtain

$$\begin{aligned} \left(y_2 + p\frac{\partial y_2}{\partial p} + \frac{\partial y_1}{\partial p}\right) dp &= \left(\frac{\partial C_2}{\partial w^s}\frac{\partial w^s}{\partial p}y_2 + \frac{\partial C_2}{\partial w^u}\frac{\partial w^u}{\partial p}y_2 + C_2\frac{\partial y_2}{\partial p} + \frac{\partial C_1}{\partial w^s}\frac{\partial w^s}{\partial p}y_1 + \frac{\partial C_1}{\partial w^u}\frac{\partial w^u}{\partial p}y_1 + C_1\frac{\partial y_1}{\partial p}\right) dp \\ &= \left(\frac{\partial w^s}{\partial p}\left(L_2^s y_2 + L_1^s y_1\right) + \frac{\partial w^u}{\partial p}\left(L_2^u y_2 + L_1^u y_1\right) + C_2\frac{\partial y_2}{\partial p} + C_1\frac{\partial y_1}{\partial p}\right) dp \end{aligned}$$

because $\frac{\partial C_k}{\partial w^i} = L_k^i$ following Sheppard's lemma. Also from the envelop theorem, we have

$$p\frac{\partial y_2}{\partial p} + \frac{\partial y_1}{\partial p} = C_2\frac{\partial y_2}{\partial p} + C_1\frac{\partial y_1}{\partial p} = 0.$$

Hence, we have finally for any dp,

$$\left(y_2 - \sum_{i=s,u} L^i \frac{\partial w^i}{\partial p}\right) dp = 0.$$

Appendix B. Proof of Proposition 1

Derivating the Lagrangean (6) with respect to labor income yields to:

$$\frac{\partial \mathcal{L}}{\partial I^i} = (\lambda^i + \mu^i)\frac{\partial V^{ii}}{\partial I} - \mu^j\frac{\partial V^{ji}}{\partial I} - \nu\pi^i + \nu\pi^i(q-p^*)\frac{\partial c_2^i}{\partial I} + \rho\pi^i\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial I} = 0 \tag{B.1}$$

with $i, j = s, u$ and $i \neq j$. Also the first-order condition w.r.t. the consumption price q is:

$$\frac{\partial \mathcal{L}}{\partial q} = \sum_{i=s,u}\left(\lambda^i + \mu^i\right)\frac{\partial V^{ii}}{\partial q} - \sum_{\substack{i,j=s,u \\ i\neq j}} \mu^i\frac{\partial V^{ij}}{\partial q} + \nu\sum_{i=s,u}\pi^i c_2^i + \nu\sum_{i=s,u}\pi^i(q-p^*)\frac{\partial c_2^i}{\partial q} + \rho\sum_{i=s,u}\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial q}\right] = 0 \tag{B.2}$$

By Roy's identity, we have

$$\begin{aligned} \frac{\partial V^{ii}}{\partial q} &= -c_2^i\frac{\partial V^{ii}}{\partial I} \\ \frac{\partial V^{ij}}{\partial q} &= -c_2^{ij}\frac{\partial V^{ij}}{\partial I}. \end{aligned}$$

Introducing these terms in (B.2), we finally obtain:

$$-\sum_{i=s,u}\left(\lambda^i+\mu^i\right)c_2^i\frac{\partial V^{ii}}{\partial I}+\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i c_2^{ij}\frac{\partial V^{ij}}{\partial I}+\nu\sum_{i=s,u}\pi^i c_2^i+\nu\sum_{i=s,u}\pi^i(q-p^*)\frac{\partial c_2^i}{\partial q}+\rho\sum_{i=s,u}\left[\pi^i\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial q}\right]=0 \tag{B.3}$$

Now, using the FOC w.r.t. I given in equation (B.1), we replace the terms $\frac{\partial V^{ii}}{\partial I}$ in (B.3) and finally obtain after rearranging:

$$\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i\left(c_2^{ij}-c_2^j\right)\frac{\partial V^{ij}}{\partial I}+\nu(q-p^*)\sum_{i=s,u}\pi^i\left(\frac{\partial c_2^i}{\partial q}+c_2^i\frac{\partial c_2^i}{\partial I}\right)+\rho\sum_{i=s,u}\pi^i\frac{\partial E}{\partial c_2}\left(\frac{\partial c_2^i}{\partial q}+c_2^i\frac{\partial c_2^i}{\partial I}\right)=0 \tag{B.4}$$

Denote $\tilde{c}_2^i$ the compensated demand (or hicksian demand) for consumer i and good 2, then the Slutsky equation tells us that

$$\frac{\partial c_2^i}{\partial q}+c_2^i\frac{\partial c_2^i}{\partial I}=\frac{\partial \tilde{c}_2^i}{\partial q}.$$

Replacing in equation (B.4), we finally obtain

$$\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i\left(c_2^{ij}-c_2^j\right)\frac{\partial V^{ij}}{\partial I}+\nu(q-p^*)\sum_{i=s,u}\pi^i\frac{\partial \tilde{c}_2^i}{\partial q}+\rho\sum_i\pi^i\frac{\partial E}{\partial c_2}\frac{\partial \tilde{c}_2^i}{\partial q}=0. \tag{B.5}$$

which establishes the expression (7) in Proposition 1.

Appendix C. Proof of Proposition 3

Derivating the Lagrangean (6) with respect to the production price p yields to:

$$\frac{\partial \mathcal{L}}{\partial p}=-\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i l^j\frac{\partial V^{ij}}{\partial l}\frac{\partial\left(\frac{w^j}{w^i}\right)}{\partial p}+\nu\left[\sum_{i=s,u}\pi^i l^i\frac{\partial w^i}{\partial p}-(p-p^*)\frac{\partial y_2}{\partial p}-y_2\right]+\rho\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial p}=0 \tag{C.1}$$

Using the fact that $\sum_{i=s,u}\pi^i l^i\frac{\partial w^i}{\partial p}=y_2$ as established in Appendix A, equation (C.1) yields to the result contained in Proposition 3.

Appendix D. Proof of Proposition 5

First, by derivating the Lagrangean (6) with respect to labor supply l^i yields to:

$$\frac{\partial \mathcal{L}}{\partial l^i}=(\lambda^i+\mu^i)\frac{\partial V^{ii}}{\partial l}-\mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}+\nu\left[\pi^i w^i+(q-p^*)\pi^i\frac{\partial c_2^i}{\partial l^i}-(p-p^*)\pi^i\frac{\partial y_2}{\partial L^i}\right] \tag{D.1}$$
$$+\rho\left[\pi^i\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\pi^i\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]=0$$

Then, using (11) and replacing in (12), we get

$$\begin{aligned}
\tau'^{i} &= 1+\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}+(q-p^*)\left[-\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I}+\frac{1}{w^i}\frac{\partial c_2^i}{\partial l}\right]\\
&= 1+\frac{q-p^*}{w^i}\frac{\partial c_2^i}{\partial l}+\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\left(1-(q-p^*)\frac{\partial c_2^i}{\partial I}\right)
\end{aligned}$$

From the first-order conditions with respect to l^i (D.1) and I^i (B.1), we obtain:

$$\begin{aligned}
(\lambda^i+\mu^i)\frac{\partial V^{ii}}{\partial I} &= \mu^j\frac{\partial V^{ji}}{\partial I}+\nu\pi^i-\nu\pi^i(q-p^*)\frac{\partial c_2^i}{\partial I}-\rho\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial I}\\
(\lambda^i+\mu^i)\frac{\partial V^{ii}}{\partial l} &= \mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\nu\left[\pi^i w^i+(q-p^*)\pi^i\frac{\partial c_2^i}{\partial l}-(p-p^*)\pi^i\frac{\partial y_2}{\partial L^i}\right]\\
&\quad -\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]
\end{aligned}$$

Hence,

$$\begin{aligned}
&\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\left[\mu^j\frac{\partial V^{ji}}{\partial I}+\nu\pi^i-\nu\pi^i(q-p^*)\frac{\partial c_2^i}{\partial I}-\rho\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial I}\right]=\mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}\\
&\qquad -\nu\left[\pi^i w^i+(q-p^*)\pi^i\frac{\partial c_2^i}{\partial l}-(p-p^*)\pi^i\frac{\partial y_2}{\partial L^i}\right]-\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]
\end{aligned}$$

Rearranging

$$\begin{aligned}
&\nu\pi^i\left[\left(1-(q-p^*)\frac{\partial c_2^i}{\partial I}\right)\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}+w^i+(q-p^*)\frac{\partial c_2^i}{\partial l}\right]=\\
&\qquad \mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}+\nu(p-p^*)\pi^i\frac{\partial y_2}{\partial L^i}-\mu^j\frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\\
&\qquad\qquad +\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\rho\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial I}-\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]
\end{aligned}$$

Note that the term between brackets on the left-hand side is equal to $w^i\tau'^i$. Hence, we obtain that:

$$\begin{aligned}
\tau'^{i} &= \frac{1}{w^i}(p-p^*)\frac{\partial y_2}{\partial L^i}+\frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ii}}{\partial l}\frac{\frac{\partial V^{ji}}{\partial I}}{\frac{\partial V^{ii}}{\partial I}}\right]\\
&\quad +\frac{1}{\nu w^i}\left[\rho\frac{\partial E}{\partial c_2}\left(\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\frac{\partial c_2^i}{\partial I}-\frac{\partial c_2^i}{\partial l}\right)-\rho\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]
\end{aligned}$$

Recall that $p-p^*=\frac{\rho}{\nu}\frac{\partial E}{\partial y_2}+t_{EW}$. Hence, we have finally:

$$\begin{aligned}
\tau'^{i} &= \frac{t_{EW}}{w^i}\frac{\partial y_2}{\partial L^i}+\frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ii}}{\partial l}\frac{\frac{\partial V^{ji}}{\partial I}}{\frac{\partial V^{ii}}{\partial I}}\right]\\
&\quad +\frac{1}{\nu w^i}\left[\rho\frac{\partial E}{\partial c_2}\left(\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\frac{\partial c_2^i}{\partial I}-\frac{\partial c_2^i}{\partial l}\right)\right]
\end{aligned}$$

Note that the terms corresponding to the production externalities have disappeared while the terms due to the presence of consumption externalities still remain. Indeed, only consumption externalities matter when computing the effective marginal tax that consumers face.

Denoting[13]

$$\left.\frac{dc_2^i}{dl}\right|_{dV^i=0} = -\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I} + \frac{\partial c_2^i}{\partial l},$$

we can rewrite the effective marginal tax as follows:

$$\tau'^i = \frac{t_{EW}}{w^i}\frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j} - \frac{\partial V^{ii}}{\partial l}\frac{\frac{\partial V^{ji}}{\partial I}}{\frac{\partial V^{ii}}{\partial I}}\right] - \frac{\rho}{\nu w^i}\frac{\partial E}{\partial c_2}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}$$

which establishes the first part of the Proposition.

Now, the marginal rate of income taxation is given by:

$$\begin{aligned}
T'^i &= \tau'^i + (q-p^*)\left[\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I} - \frac{1}{w^i}\frac{\partial c_2^i}{\partial l}\right] \\
&= \tau'^i - \frac{(q-p^*)}{w^i}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0} \\
&= \tau'^i - \frac{1}{w^i}\left[t_{NS} - \frac{\rho}{\nu}\frac{\partial E}{\partial c_2}\right]\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}
\end{aligned}$$

Replacing τ'^i, we obtain

$$\begin{aligned}
T'^i &= \frac{t_{EW}}{w^i}\frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j} - \frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right] \\
&\quad -\frac{\rho}{\nu w^i}\frac{\partial E}{\partial c_2}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0} - \frac{1}{w^i}\left[t_{NS} - \frac{\rho}{\nu}\frac{\partial E}{\partial c_2}\right]\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}
\end{aligned}$$

and

$$T'^i = \frac{t_{EW}}{w^i}\frac{\partial y_2}{\partial L^i} + \frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j} - \frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right] - \frac{t_{NS}}{w^i}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}$$

which establishes the second part of the Proposition.

[13]Indeed, note that by using (11), we have $\frac{dc_2^i}{dl} = \frac{\partial c_2^i}{\partial I}w^i(1-T'^i) + \frac{\partial c_2^i}{\partial l} = -\frac{\frac{\partial V^i}{\partial l}}{\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I} + \frac{\partial c_2^i}{\partial l}$.

Appendix E. Expressions with our specification

From perfect competition and constant returns to scale, unit costs in each sector are equal to the producer price, and we obtain the following expression for wages and the wage ratio:

$$
\begin{aligned}
w^s(p) &= p^{-(\frac{1-\sigma_1}{\sigma_1-\sigma_2})}\left[\frac{K_1^{1-\sigma_1}}{{K_2}^{1-\sigma_2}}\right]^{\frac{1}{\sigma_1-\sigma_2}} \\
w^u(p) &= p^{\frac{\sigma_1}{\sigma_1-\sigma_2}}\left[\frac{K_2^{\sigma_1}}{{K_1}^{\sigma_2}}\right]^{\frac{1}{\sigma_1-\sigma_2}} \\
\Omega(p) &= \frac{w^u(p)}{w^s(p)} = p^{\frac{1}{\sigma_1-\sigma_2}}\left[\frac{K_2}{K_1}\right]^{\frac{1}{\sigma_1-\sigma_2}}
\end{aligned}
$$

where $K_1 = A_1\sigma_1^{\sigma_1}(1-\sigma_1)^{1-\sigma_1}$ and $K_2 = A_2\sigma_2^{\sigma_2}(1-\sigma_2)^{1-\sigma_2}$. The expression of output in sector 1 and 2 is then:

$$
\begin{aligned}
y_1(l^s, l^u, p) &= \frac{K_1}{\sigma_1-\sigma_2}\left[(1-\sigma_2)\Omega(p)^{\sigma_1-1}\pi^s l^s - \sigma_2\Omega(p)^{\sigma_1}\pi^u l^u\right] \\
y_2(l^s, l^u, p) &= \frac{K_2}{\sigma_1-\sigma_2}\left[-(1-\sigma_1)\Omega(p)^{\sigma_2-1}\pi^s l^s + \sigma_1\Omega(p)^{\sigma_2}\pi^u l^u\right]
\end{aligned}
$$

Replacing $\frac{\partial y_2}{\partial p}$ and $\frac{\partial \Omega(p)}{\partial p}$ in the expression of the producer subsidy and simplifying yields

$$
\begin{aligned}
t_{EW} &= \frac{\mu^s}{\nu}\frac{\left[\Omega(p)l^u\right]^{1+1/\xi}}{\frac{\partial y_2}{\partial p}}\frac{\partial \Omega(p)}{\partial p} \\
&= \frac{\mu^s}{\nu} l^{u^{1+1/\xi}} \frac{\left[\Omega(p)\right]^{3-\sigma_2+1/\xi}}{(1-\sigma_1)(1-\sigma_2)\pi^s l^s + \sigma_1\sigma_2\pi^u l^u\Omega(p)}\frac{\sigma_1-\sigma_2}{K_2}
\end{aligned}
$$

Appendix F. The case of autarky (not to be published, for reviewers only)

This appendix is devoted to examining the optimal fiscal policy for a closed economy. In this setting, the program of the government can be written as follows:

$$\max_{I^i, l^i, q, p, e} \sum_{i=s,u} \lambda^i V^i(q, I^i, l^i, e)$$

s.t.

$$V^i(q, I^i, l^i, e) \geq V^i(q, I^j, \frac{w^j l^j}{w^i}, e) \quad \forall i, j = s, u, \; i \neq j$$

$$\sum_{i=s,u} \pi^i (w^i l^i - I^i) + (q - p) \sum_{i=s,u} \pi^i c_2^i(q, I^i, l^i, e) \geq 0$$

$$e = E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) \qquad \forall i, j = s, u$$

$$y_2(p, \pi^s l^s, \pi^u l^u) \geq \sum_{i=s,u} \pi^i c_2^i(q, I^i, l^i, e)$$

As before, let μ^i, ν and ρ be the Lagrange multipliers of the type-i workers' incentive-compatibility constraint, the budget constraint and the pollution definition, respectively. We also denote η the multiplier of the resource constraint stipulating that total production should exceed total consumption. The corresponding lagrangean is

$$\begin{aligned}\mathcal{L} = &\sum_{i=s,u} \lambda^i V^i(q, I^i, l^i, e) + \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \left[V^i(q, I^i, l^i, e) - V^i(q, I^j, \frac{w^j l^j}{w^i}, e) \right] \\ &+ \nu \left[\sum_{i=s,u} \pi^i \left[(w^i l^i - I^i) + (q - p) c_2^i(q, I^i, l^i, e) \right] \right] \\ &+ \rho \left[E(\pi^s c_2^s(q, I^s, l^s, e) + \pi^u c_2^u(q, I^u, l^u, e), y_2(p, \pi^s l^s, \pi^u l^u)) - e \right] \\ &+ \eta \left[y_2(p, \pi^s l^s, \pi^u l^u) - \sum_{i=s,u} \pi^i c_2^i(q, I^i, l^i, e) \right]\end{aligned} \tag{F.1}$$

Deriving the Lagrangian (F.1) with respect to respectively incomes and consumer price, we obtain the following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial I^i} = (\lambda^i + \mu^i) \frac{\partial V^{ii}}{\partial I} - \mu^j \frac{\partial V^{ji}}{\partial I} - \nu \pi^i + \left(\nu (q - p) + \rho \frac{\partial E}{\partial c_2} - \eta \right) \pi^i \frac{\partial c_2^i}{\partial I} = 0 \tag{F.2}$$

with $i, j = s, u$ and $i \neq j$, and

$$\frac{\partial \mathcal{L}}{\partial q} = \sum_{i=s,u} \left(\lambda^i + \mu^i \right) \frac{\partial V^{ii}}{\partial q} - \sum_{\substack{i,j=s,u \\ i \neq j}} \mu^i \frac{\partial V^{ij}}{\partial q} + \nu \sum_{i=s,u} \pi^i c_2^i + \nu (q - p) \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial q} + \rho \sum_{i=s,u} \pi^i \left[\frac{\partial E}{\partial c_2} \frac{\partial c_2^i}{\partial q} \right] - \eta \sum_{i=s,u} \pi^i \frac{\partial c_2^i}{\partial q} = 0. \tag{F.3}$$

Using the Roy's identity in (F.3), we obtain:

$$-\sum_{i=s,u}\left(\lambda^i+\mu^i\right)c_2^i\frac{\partial V^{ii}}{\partial I}+\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i c_2^{ij}\frac{\partial V^{ij}}{\partial I}+\nu\sum_{i=s,u}\pi^i c_2^i+\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)\sum_{i=s,u}\pi^i\frac{\partial c_2^i}{\partial q}=0. \tag{F.4}$$

Combining (F.4) and (F.2) and rearranging with the help of the Slutsky equation, we get:

$$\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)\sum_{i=s,u}\pi^i\frac{\partial\tilde{c}_2^i}{\partial q}=\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i\left(c_2^j-c_2^{ij}\right)\frac{\partial V^{ij}}{\partial I}. \tag{F.5}$$

Now deriving the lagrangian (F.1) with respect to production price p yields to:

$$\frac{\partial\mathcal{L}}{\partial p}=-\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i l^j\frac{\partial V^{ij}}{\partial l}\frac{\partial\left(\frac{w^j}{w^i}\right)}{\partial p}+\nu\left[\sum_{i=s,u}\pi^i l^i\frac{\partial w^i}{\partial p}-\sum_{i=s,u}\pi^i c_2^i\right]+\left(\rho\frac{\partial E}{\partial y_2}+\eta\right)\frac{\partial y_2}{\partial p}=0$$

Extracting η from this equation yields to the following expression of the shadow price of the market constraint:

$$\eta=-\rho\frac{\partial E}{\partial y_2}-\nu t_{EW}. \tag{F.6}$$

Replacing the value of η in (F.5), we then obtain the value of the optimal commodity tax:

$$q-p=-\frac{\rho}{\nu}\left(\frac{\partial E}{\partial c_2}+\frac{\partial E}{\partial y_2}\right)+t_{NS}-t_{EW}.$$

Last, the optimal value of the shadow price of the environmental constraint ρ can be obtained from the first-order condition with respect to e, that writes as follows:

$$\frac{\partial\mathcal{L}}{\partial e}=\sum_{i=s,u}\lambda^i\frac{\partial V^{ii}}{\partial e}+\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i(\frac{\partial V^{ii}}{\partial e}-\frac{\partial V^{ij}}{\partial e})+\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)\sum_{i=s,u}\pi^i\frac{\partial c_2^i}{\partial e}-\rho=0$$

Replacing in this equation the term $\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)$ by using (F.5), we obtain an expression of ρ :

$$\rho=\sum_{i=s,u}\lambda^i\frac{\partial V^{ii}}{\partial e}+\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i(\frac{\partial V^{ii}}{\partial e}-\frac{\partial V^{ij}}{\partial e})+\nu t_{NS}\sum_{i=s,u}\pi^i\frac{\partial c_2^i}{\partial e}.$$

Now we characterize the income and effective marginal tax rates. Under autarky, the total tax paid by a consumer-worker of type i is defined by:

$$\tau(w^i l^i)=T(w^i l^i)+(q-p)c_2^i(q,I^i,l^i,e)$$

and the marginal effective tax rate is:

$$\tau'^i\equiv\tau'(w^i l^i)=T'(w^i l^i)+(q-p)\left[\left(1-T'(w^i l^i)\right)\frac{\partial c_2^i}{\partial I}+\frac{1}{w^i}\frac{\partial c_2^i}{\partial l}\right] \tag{F.7}$$

Recalling that the marginal rate of income taxation is given by (11) and replacing in (F.7), we have

$$\begin{aligned}\tau'^i &= 1+\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}+(q-p)\left[-\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I}+\frac{1}{w^i}\frac{\partial c_2^i}{\partial l}\right]\\ &= 1+\frac{q-p}{w^i}\frac{\partial c_2^i}{\partial l}+\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\left(1-(q-p)\frac{\partial c_2^i}{\partial I}\right).\end{aligned}$$

Now, deriving the lagrangian (F.1) with respect to l^i, we get the following first-order conditions (for $i,j=s,u$ and $i\neq j$):

$$\begin{aligned}(\lambda^i+\mu^i)\frac{\partial V^{ii}}{\partial l} &= \mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\nu\left[\pi^i w^i+(q-p)\pi^i\frac{\partial c_2^i}{\partial l}\right]\\ &\quad -\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]-\eta\pi^i\left[\frac{\partial y_2}{\partial L^i}-\frac{\partial c_2^i}{\partial l}\right].\end{aligned}\tag{F.8}$$

Recall also that from the first-order conditions w.r.t. incomes (F.2), we have:

$$(\lambda^i+\mu^i)\frac{\partial V^{ii}}{\partial I}=\mu^j\frac{\partial V^{ji}}{\partial I}+\nu\pi^i-\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)\pi^i\frac{\partial c_2^i}{\partial I}.\tag{F.9}$$

Combining (F.8) and (F.9), we obtain that:

$$\begin{aligned}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\left[\mu^j\frac{\partial V^{ji}}{\partial I}+\nu\pi^i-\left(\nu(q-p)+\rho\frac{\partial E}{\partial c_2}-\eta\right)\pi^i\frac{\partial c_2^i}{\partial I}\right] &= \mu^j\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\nu\left[\pi^i w^i+(q-p)\pi^i\frac{\partial c_2^i}{\partial l}\right]\\ &\quad -\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]-\eta\pi^i\left[\frac{\partial y_2}{\partial L^i}-\frac{\partial c_2^i}{\partial l}\right]\end{aligned}$$

and rearranging, we get:

$$\begin{aligned}\nu\pi^i\left[\left(1-(q-p)\frac{\partial c_2^i}{\partial I}\right)\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}+w^i+(q-p)\frac{\partial c_2^i}{\partial l}\right] &= \mu^j\left(\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right)\\ &\quad -\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]-\eta\pi^i\left[\frac{\partial y_2}{\partial L^i}-\frac{\partial c_2^i}{\partial l}\right]\\ &\quad +\left(\rho\frac{\partial E}{\partial c_2}-\eta\right)\pi^i\frac{\partial c_2^i}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\end{aligned}$$

Replacing η by its value given by (F.6) and recognizing that the term between brackets in the left hand side of the equation is precisely $w^i\tau'^i$, we obtain that:

$$\begin{aligned}\nu\pi^i w^i\tau'^i &= \mu^j\left(\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right)-\rho\pi^i\left[\frac{\partial E}{\partial c_2}\frac{\partial c_2^i}{\partial l}+\frac{\partial E}{\partial y_2}\frac{\partial y_2}{\partial L^i}\right]\\ &\quad -\left(-\rho\frac{\partial E}{\partial y_2}-\nu t_{EW}\right)\pi^i\left[\frac{\partial y_2}{\partial L^i}-\frac{\partial c_2^i}{\partial l}\right]\\ &\quad +\left(\rho\frac{\partial E}{\partial c_2}-\left(-\rho\frac{\partial E}{\partial y_2}-\nu t_{EW}\right)\right)\pi^i\frac{\partial c_2^i}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\end{aligned}$$

and rearranging, we finally get:

$$\begin{aligned}
\nu\pi^i w^i \tau'^i &= \nu t_{EW}\pi^i\left(\frac{\partial y_2}{\partial L^i}+1\right)+\mu^j\left(\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right)\\
&\quad+\pi^i\left[\rho\left(\frac{\partial E}{\partial c_2}+\frac{\partial E}{\partial y_2}\right)\left(\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\frac{\partial c_2^i}{\partial I}-\frac{\partial c_2^i}{\partial l}\right)\right]
\end{aligned}$$

hence the expression of τ'^i in the text.

The marginal rate of income taxation is given by:

$$\begin{aligned}
T'^i &= \tau'^i+(q-p)\left[\frac{\frac{\partial V^i}{\partial l}}{w^i\frac{\partial V^i}{\partial I}}\frac{\partial c_2^i}{\partial I}-\frac{1}{w^i}\frac{\partial c_2^i}{\partial l}\right]\\
&= \tau'^i-\frac{(q-p)}{w^i}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}\\
&= \tau'^i-\frac{1}{w^i}\left[-\frac{\rho}{\nu}\left(\frac{\partial E}{\partial c_2}+\frac{\partial E}{\partial y_2}\right)+\frac{\sum_{\substack{i,j=s,u\\ i\neq j}}\mu^i\left(c_2^j-c_2^{ij}\right)\frac{\partial V^{ij}}{\partial I}}{\nu\sum_{i=s,u}\pi^i\frac{\partial\tilde{c}_2^i}{\partial q}}+\frac{1}{\frac{\partial y_2}{\partial p}}\sum_{\substack{i,j=s,u\\ i\neq j}}\frac{\mu^i}{\nu}l^j\frac{\partial V^{ij}}{\partial l}\frac{\partial\left(\frac{w^j}{w^i}\right)}{\partial p}\right]\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}
\end{aligned}$$

Replacing the expression of τ'^i in this equation and simplifying, we obtain:

$$T'^i=\frac{t_{EW}}{w^i}\frac{\partial y_2}{\partial L^i}+\frac{\mu^j}{\nu\pi^i w^i}\left[\frac{\partial V^{ji}}{\partial l}\frac{w^i}{w^j}-\frac{\partial V^{ji}}{\partial I}\frac{\frac{\partial V^{ii}}{\partial l}}{\frac{\partial V^{ii}}{\partial I}}\right]-\frac{t_{NS}}{w^i}\left.\frac{dc_2^i}{dl}\right|_{dV^i=0}$$

and hence the same expression as under trade.