

# Understanding the monetary transmission mechanism in the United Kingdom: The role of nominal and real rigidities\*

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## Abstract

This paper aims to contribute to our understanding of the monetary transmission mechanism in the United Kingdom by estimating two Dynamic Stochastic General Equilibrium models and assessing the role of nominal and real rigidities within them. We first obtain an empirical representation of the monetary transmission mechanism in the United Kingdom and then estimate the models by minimising the difference between this representation and its model equivalents. We find that both models can explain the data reasonably well without relying on undue amounts of price and wage stickiness.

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# 1 Introduction

Most monetary policy makers focus on achieving price stability: typically defined as low and stable inflation. But in order to achieve price stability, it is important to understand what the dynamics of prices (and inflation) are, what drives them and, perhaps most importantly, how monetary policy fits into this, ie, how the monetary transmission mechanism works. For many years, the traditional Phillips Curve provided the standard framework for understanding inflation dynamics. But this is a reduced-form approach and could mislead as a result. This led to the development of models with explicit microfoundations of optimising behaviour, imperfect competition, and ‘sticky’ prices at the microeconomic level. The most popular of these models has been the Calvo pricing model (based on Calvo (1983)), where individual companies have an exogenous probability of being able to change prices in any given period. Because of this fixed probability, companies who are changing their prices have to consider what future prices are (and will be) optimal in case they don’t get the chance to change prices again for some time. This intuition results in a derived ‘New Keynesian’ Phillips Curve (NKPC) that relates inflation this period to expected inflation in the next period, and to the deviation of real marginal cost from trend.

But, although the NKPC is a useful framework for thinking about the monetary transmission mechanism and how various shocks might affect inflation, it cannot be used to provide quantitative predictions unless it forms part of a general equilibrium model. And then, it is important that the key parameters within the general equilibrium model are estimated rather than simply calibrated as only if we do this can we assess the uncertainty around the parameters themselves and, hence, predictions generated by the model. This has led to a number of recent papers in which authors have used different techniques to estimate dynamic stochastic general equilibrium (DSGE) models based around the NKPC. For example, Smets and Wouters (2003), Smets and Wouters (2007) and Harrison and Oomen (2008) use Bayesian techniques

to estimate a New Keynesian model on data from the euro area, the United States and the United Kingdom, respectively. Gertler *et al.* (2008) also used Bayesian techniques and US data to estimate a New Keynesian model with search and matching in the labour market. The key advantage of Bayesian techniques are that, in theory, they can provide a complete description of the data generating process and, so, allow you to test hypotheses within the DSGE models, evaluate their relative performance against each other, and use them to run forecasts. Against this, the parameter estimates seem to be driven by the priors and the choice of priors will also affect model comparisons. (See del Negro and Schofheide (2008).)

These problems motivate an alternative approach. Initiated by Rotemberg and Woodford (1997), the ‘minimum distance’ approach has been widely used to assess the empirical performance of DSGE models. For example, Christiano *et al.* (2005) investigate the role of various nominal and real frictions in explaining the inflation inertia and persistence in US data. Amato and Laubach (2003) analyzes the welfare implications of various interest rate rules using an estimated model with sticky wages and prices. Boivin and Giannoni (2006) examine the change in the effectiveness of the monetary policy in the United States for the pre- and post-Volcker periods. Meier and Muller (2006) quantify the role of financial frictions in the transmission of monetary policy shocks. The idea of this approach is to obtain values of the parameters so that the model matches as closely as possible those features of the data in which you’re particularly interested. An added advantage over the Bayesian approach is that parameters estimated by the minimum distance method tend to be more robust.

In this paper, we use the minimum distance approach to estimate two DSGE models using UK data. In particular, we are interested in estimating the parameters of our models so as to match as closely as possible the responses of variables to a monetary policy shock. This is motivated by our particular interest in understanding the monetary transmission mechanism

and how monetary policy makers can set interest rates so as to achieve their (implicit or explicit) inflation target.

The two models we consider are those of Smets and Wouters (2003) and Gertler *et al.* (2008). The Smets and Wouters model has become a ‘work-horse’ DSGE model and has been estimated using Bayesian methods on both US and euro-area data. By estimating it using a minimum distance approach on UK data we can assess how similar the United Kingdom is to the United States and the euro area and where differences may lie, eg, in the degree of nominal price and wage rigidity. We can also compare our estimates with those of di Cecio and Nelson (2007) who estimate the same model but use a smaller vector autoregression (VAR), so matching the response of fewer variables to monetary policy movements than us, and a less recent dataset.

A long tradition in monetary economics, starting with Phillips (1958), has assigned labour market frictions and, in particular wage-setting frictions, a central role in inflation dynamics but in the Smets and Wouters (2003) model, as is the case for most models based on the NKPC framework, the labour market is modelled as a spot market with no realistic distinction being made between heads and hours.<sup>1</sup> This motivates consideration of a model in which the labour market is modelled more explicitly within the New Keynesian framework. One such example of this is the model of Gertler *et al.* (2008) which appends a variant of the Mortensen and Pissarides (1994) model of search and matching frictions to the New Keynesian framework. So, we estimate this model in the current paper. Again we compare our results with those of Gertler *et al.* in order to get a feel for how different the UK labour market might be to that in the United States and where any differences might lie. Finally, by comparing our results for both models, we can assess the importance of explicitly modelling unemployment for understanding the monetary transmission mechanism; in particular, once you’ve

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<sup>1</sup>In Smets and Wouters (2003), workers are assumed to have market power with the result that there is a difference between the amount of labour supplied in equilibrium and the amount that would be supplied if this distortion were not there.

controlled for total hours worked/employment, does unemployment/labour market tightness give you any additional information about the effects of monetary policy movements on inflation?

The paper is structured as follows. We first use a structural vector autoregression (SVAR) approach to obtain an empirical representation of the monetary transmission mechanism, ie, how a monetary policy change affects some important macroeconomic variables in the United Kingdom. We then discuss the two models we are going to estimate before moving on to discuss the estimation strategy. In brief, our aim is to obtain values for the parameters of the two models that enable them to replicate the empirical representation of the monetary transmission mechanism we found in Section 2. After discussing our estimation strategy, we present our results before concluding.

## 2 Monetary Transmission in UK

We estimate a nine-variable VAR in order to identify the effects of a monetary policy shock on macroeconomic variables in the United Kingdom. Our estimation period starts in the second quarter of 1979, when Margaret Thatcher became Prime Minister. DiCecio and Nelson (2007) find that the break date on a VAR similar to ours is located between 1977-1981 and they argue that 1979:2 constitutes an important monetary and government policy regime change. Of course, there have been subsequent changes in the UK monetary policy regime; indeed, Nelson (2003) identifies regimes lasting from 1979-1987, 1987-1990 and 1992-1997. But, given the problems with estimating a VAR on a short sample, we chose to follow di Cecio and Nelson (2007) and assume that these different monetary policy regimes were all compatible with the same implied policy reaction function.

In their work, di Cecio and Nelson (2007) estimated a six-variable VAR including real GDP, real consumption, real investment, labour productivity,

the treasury bill rate and retail price inflation. To these variables, we added capacity utilisation, the relative price of investment goods and the real wage.<sup>2</sup> This left us with a similar list to Altig *et al.* (2005), although they also included the money supply.<sup>3</sup> We also took care to use variables in the VAR that were consistent with their model counterparts. So, we used consumption spending on non-durables per head of population for consumption,  $c$ . Investment,  $I$ , was defined as business investment plus consumption spending on durables per head of population. Output was defined as the sum of these two series ( $y = c + I$ ). Our series for inflation used the implied output deflator, given our definition of output. We calculated our real wage series by dividing the nominal private-sector wage per worker by this deflator. We calculated the relative price of investment goods implied by our investment and output series. To these series we added private-sector employment per head of population – dividing our output measure by this variable to create a measure of productivity – capacity utilisation, and the Bank of England’s policy rate. All series were detrended using a linear trend.

In summary, we estimate a VAR with the following nine variables :

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<sup>2</sup>Given that the Gertler *et al.* (2008) model we consider is designed specifically to model frictions in the labour market, it would seem particularly important to include real wages and employment within our set of variables. Indeed, it could be argued that we should also include the unemployment rate, but this variable has no analogue within the Smets and Wouters (2003) model, so we leave it out.

<sup>3</sup>In addition, Altig *et al.* (2005) transform their variables so as to make them stationary; their final list is: Change in the relative price of investment goods, productivity growth, inflation, capacity utilisation, total hours worked per head of population, the labour share, the shares of consumption and investment in output, the nominal interest rate and the change in the velocity of money.

$$Y_t = \begin{pmatrix} \ln(\text{output}) \\ \Delta \ln(\text{output deflator}) \\ \ln(\text{consumption}) \\ \ln(\text{investment}) \\ \ln(\text{real wage}) \\ \ln(\text{productivity}) \\ \text{capacity utilization} \\ \ln(\text{relative investment price}) \\ \text{BoE interest rate} \end{pmatrix}$$

In order to identify monetary policy shocks, we follow the identification strategy used in Christiano *et al.* (2005) and Altig *et al.* (2005). The monetary authority is assumed to operate according to a rule which takes the following form:

$$1 + rg_t = f\{\Omega_t\} + \varepsilon_t \quad (1)$$

where  $\Omega_t$  is the information set of the monetary authority as of time  $t$ . The structural VAR representation is given by:

$$A_0 Y_t = A(L) Y_{t-1} + \varepsilon_t \quad (2)$$

We estimate the reduced form VAR with the variables in  $Y_t$ . That is:

$$Y_t = B(L) Y_{t-1} + u_t \quad (3)$$

where  $u_t$  are the reduced form residuals. In order to recover the structural shocks,  $\varepsilon_t$ , we assume that the relationship between the reduced form and structural errors are given by:

$$u_t = C \varepsilon_t \quad (4)$$

where  $C$  is a lower triangular matrix. This identification strategy implies that none of the variables in our VAR respond contemporaneously to the

monetary policy shock. With this assumption, the relationship between the parameters of the reduced form and structural VAR representations is given by:

$$C = A_0^{-1} \quad B(L) = A_0^{-1}A(L) \quad (5)$$

Chart 1 displays the impulse responses (IRFs) to a one standard deviation increase in the Bank of England interest rate. The solid line is the estimated response and the shaded areas correspond to 90% confidence intervals. We summarise our results by comparing them with the effect of monetary policy shocks in the United States.<sup>4</sup> The following results are similar:

- The response of output, consumption, investment and capacity utilisation responses are hump-shaped. The peak response of output occurs five quarters after the shock.
- The inflation response is hump-shaped with a peak after two years and the effect on inflation of a monetary policy shock dies out after three years. There is also a price puzzle lasting one period, but this is not statistically significant.
- The relative investment price and real wages decrease but the effects are not statistically significant.
- The peak response of productivity is one period after the shock. Given the response of GDP, this path suggests that the adjustment in labour input occurs with a lag relative to the response of output.
- Following a monetary policy shock, the investment response is only slightly higher than the response of output. Cyclical investment is however 2.2 times more volatile than cyclical output.<sup>5</sup>

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<sup>4</sup>Our comparison takes the results in Christiano *et al.* (2005) as the ‘benchmark’ response to a monetary policy shock in the United States.

<sup>5</sup>We define cyclical investment and output as the logarithm of the quarterly investment and output series that are HP filtered with a smoothing parameter of 1600.

In Chart 2, we present the IRFs of output, inflation, productivity and real wage from rolling sample estimates of the VAR. The responses of output, and productivity are broadly stable over time. The inflation response seems to display a larger ‘price puzzle’ towards the end of the sample. The real wage also increases after a positive interest rate shock at the end of the sample, but this effect is not statistically significant.<sup>6</sup>

### 3 Theoretical Models

We analyze two small-scale DSGE models. The models are almost identical except for the functioning of the labour market. The first model, developed in Christiano *et al.* (2005) and Smets and Wouters (2003), assumes the household is a monopoly supplier of a differentiated labour service, while the second, Gertler *et al.* (2008), represents the labour market with search and matching frictions. This difference is key since it introduces ‘unemployment’ into the model in such a way as to match how unemployment is measured in the data. Shimer (2005) showed that the search model was unable to match the volatility of unemployment in the data and other papers, largely sparked by this critique, have sought to improve the modeling or calibration of the labour market in order to match better the unemployment data, e.g., Gertler and Trigari (2006), Fujita and Ramey (2005) and Yashiv (2006). But we are more interested in whether including unemployment enables us better to match the monetary transmission mechanism, ie, the responses of different variables to a monetary policy shock. The rationale for thinking it might comes from the belief that sluggish responses in labour market variables to shocks are a natural place to look for the origins of the sluggish response of inflation to shocks. In terms of the New Keynesian framework, which nests both of these models, labour market frictions will alter either aggregate

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<sup>6</sup>The standard errors of individual IRFs are available from authors upon request.

marginal cost or the firms' price-setting behaviour for given marginal cost.<sup>7</sup>

### 3.1 Smets and Wouters model

The model consists of three sectors: households, firms and a central bank. There are nominal rigidities in the goods and labour markets and real rigidities such as habit formation in consumption, investment adjustment costs and variable capital utilization.

#### 3.1.1 Households

Households consume the final good and supply differentiated labour to the firms. They are also assumed to own the capital stock and make decisions about capital accumulation and utilisation. This assumption, now standard in the business cycle literature, is necessary in order to simplify the firms' decision problem.

Each household maximises the discounted future flows of utility :

$$\max \sum_{s=0}^{\infty} \beta^s \frac{1}{1-\sigma} (C_{j,t} - \psi C_{t-1})^{1-\sigma} - \phi_h \frac{h_{j,t}^{1+\phi}}{1+\phi} \quad (6)$$

where  $C$  is consumption and  $h_{j,t}$  denotes hours per worker.  $\sigma$  is the inverse of the intertemporal elasticity of substitution and  $\psi$  determines the degree of habit persistence in consumption. Our utility function implies that the Frisch elasticity of labour supply is given by the inverse of  $\phi$ .

The representative household maximise the objective function subject to an intertemporal budget constraint:

$$C_{j,t} + I_{j,t} + R_t \frac{B_{j,t}}{P_t} = \frac{B_{j,t-1}}{P_t} + D_{j,t} \quad (7)$$

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<sup>7</sup>Thus, the labour market was seen as a source of 'real rigidities'. For an overview of the extensive literature on real rigidities more generally, see Woodford (2003).

where the household's total income ( $D_{j,t}$ ) is composed of its wage earnings ( $w_{j,t}$ ), rents on capital net of utilization costs ( $r_t^k z_{j,t} k_{j,t-1} - a(z_t) k_{j,t-1}$ ) and profits ( $\Pi_{j,t}$ ):

$$D_{j,t} = w_{j,t} h_{j,t} + r_t^k z_{j,t} k_{j,t-1} - a(z_{j,t}) k_{j,t-1} + \Pi_{j,t} \quad (8)$$

Households can vary their intensity of capital utilization, ( $z_t$ ) at a cost determined by the function  $a(z_t)$ . Each period the capital stock depreciates at rate  $\delta$  and the household undergoes investment adjustment cost ( $S(I_t, I_{t-1})$ ):

$$k_{j,t} = (1 - \delta) k_{j,t-1} + (1 - S(I_{j,t}, I_{j,t-1})) I_{j,t} \quad (9)$$

The investment adjustment cost is increasing with changes in investment. The assumption of investment adjustment costs, rather than capital adjustment costs, enables the model to capture the hump-shaped dynamics of investment.

The functional forms for adjustment costs is given by:

$$a(z_t) = \frac{a_o}{1 + \sigma_z} (z_t^{1+\sigma_z} - 1)$$

for capacity utilization and

$$S(I_t, I_{t-1}) = \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2$$

for investment adjustment. Investment adjustment costs satisfy the standard restrictions ( $S(1) = S'(1) = 0$ ) as in Christiano *et al.* (2005) and Smets and Wouters (2003) and, as the loglinear form of this equation will make clear,  $\kappa = S''(1)$  captures the effects of investment adjustment costs on the model dynamics.

The modeling of the labour market implies that members of the household may receive different wage rates. Since the objective of the paper is not the distributional issues which can emerge from heterogeneity within the

labour market, we make the simplifying assumption that there is a perfect insurance market which enables agents to ensure themselves against idiosyncratic risks. Combined with our separable utility assumption, this will result in the equalization of the marginal value of wealth across agents, and each household will be identical with respect to their consumption and asset holdings. We can therefore write the households' decision problems by solving the program of a representative household.

The household's optimal choices on bonds, consumption, capital, investment and capital utilization can be summarised by the following five equations:

$$\lambda_t = (C_t - \psi C_{t-1})^{-\sigma} \quad (10)$$

$$\lambda_t = \beta E_t \left( \lambda_{t+1} \frac{R_t}{E_t \pi_{t+1}} \right) \quad (11)$$

$$p_t^k = \beta \frac{\lambda_{t+1}}{\lambda_t} [z_{t+1} r_{t+1}^k - a(z_{t+1}) + p_{t+1}^k (1 - \delta)] \quad (12)$$

$$p_t^k \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) = 1 + p_t^k \kappa \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) - \beta \frac{\lambda_{t+1}}{\lambda_t} p_t^k \kappa \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \quad (13)$$

$$r_t^k = a'(z_t) = a_o z_t^{\sigma_z} \quad (14)$$

Equation (10) and (11) are the first order conditions for bond holdings and consumption. Equation (10) relates the marginal utility of income to both current consumption and past consumption due to the presence of habit formation in consumption in the preferences. The marginal utility of income evolves according to standard intertemporal condition in (11).

Equation (12) gives the evolution of the value of installed capital stock. The price evolves according to the standard arbitrage rule. The cost of buying one unit of capital today is equalised to the discounted return on this unit of capital, net of utilization costs and depreciation. Holding utilization constant and assuming no adjustment cost ( $p_t^k = 1$ ), this condition collapses

to:

$$\frac{R_t}{E_t \pi_{t+1}} = r_{t+1}^k + 1 - \delta$$

which states that the return on capital net of depreciation should be equal to the prevailing real interest rate.

Equation (13) is the first order condition for investment. It takes into account that one unit of investment produces an amount of capital net of investment adjustment costs and investment today will also affect investment next period. This intertemporal effect manifests itself by possible savings on future investment adjustment costs. Equation (14) determines how capacity utilization varies in response to changes in rental rate of capital and  $\frac{1}{\sigma_z}$  gives the elasticity of utilization to the rental rate of capital.

### 3.1.2 Labour Supply

Households supply differentiated labour services to the firms. Each household is a monopoly and has price-setting power. They are also subject to calvo-style nominal wage rigidities. Each period only a fraction,  $(1 - \alpha_w)$ , of households can adjust their wages. Wages that cannot be adjusted are indexed to past inflation.

Household specific labour services are aggregated to final labour input by the following technology:

$$L_t = \left( \int_0^1 (h_{j,t})^{1/(1+\lambda_w)} di \right)^{1+\lambda_w}$$

and the demand for  $j$ th labour services is:

$$h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{-(1+\lambda_w)/\lambda_w} L_t$$

where  $W_t$  is the aggregate wage index. The relationship between indi-

vidual and aggregate wage is:

$$W_t = \left( \int_0^1 (W_{j,t})^{-1/\lambda_w} di \right)^{-\lambda_w}$$

Wages which are not adjusted optimally are indexed to past level of inflation. The optimal decision of a household that adjust its wage implies that the optimal wage,  $W_t^*$ , is given by:

$$\frac{W_t^*}{P_t} E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left( \frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+s}/P_{t+s-1}} \right) \frac{h_{j,t+s} \lambda_{t+s}}{1 + \lambda_w} = E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s h_{j,t+s} \phi_h h_{j,t+s}^{\phi}$$

In order to interpret this equation, it is useful to define marginal rate of substitution between consumption and labour:

$$mrs_t = \frac{U_t^l}{\lambda_t} = \frac{\phi_h h_{j,t}^{\phi}}{(C_t - \psi C_{t-1})^{-\sigma}}$$

In the absence of nominal rigidities, the optimal wage equation collapses to:

$$\frac{W_t^*}{P_t} = (1 + \lambda_w) mrs_{j,t}$$

where the wage is given as a markup over the marginal rate of substitution. With nominal rigidities households take into account the possibility that the  $mrs$  can change and sets its wage as a markup over the weighted sum of future marginal rates of substitution.

Given the definition of the wage index, the aggregate wage evolves according to:

$$W_t^{-1/\lambda_w} = \alpha_w (W_{t-1} \bar{\pi}_{t-1}^{\gamma_w})^{-1/\lambda_w} + (1 - \alpha_w) (W_t^*)^{-1/\lambda_w}$$

### 3.1.3 Production sector

In order to simplify the problem of firms and in particular to separate the pricing and hiring decisions, it is useful to divide the production sector in two sectors. First, retailers combine the differentiated goods to the final good and sell it to the household. They operate in a perfectly competitive market. Wholesale firms operates in a monopolistically competitive market and produce using capital and labour. They are subject to nominal rigidities.

**Retailers** Retailers combine differentiated intermediate goods  $(y_j, t)$  according to a constant return to scale technology:

$$y_t = \left( \sum_0^1 (y_{j,t})^{1/(1+\lambda_p)} dj \right)^{1+\lambda_p} \quad (15)$$

where  $y_t$  is the final consumption good and  $(1 + \lambda_p)/\lambda_p$  is the elasticity of substitution between intermediate goods. Cost minimization yields the following demand for each differentiated good where the demand for each intermediate good depends negatively on its relative price:

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-(1+\lambda_p)/\lambda_p} y_t$$

where  $P_t = \left( \sum_0^1 (P_{j,t})^{-1/\lambda_p} dj \right)^{-\lambda_p}$  is the price of the final good and  $P_{j,t}$  is the price of intermediate good  $j$ .

**Wholesale firms** Wholesale firms produce differentiated goods in a monopolistically competitive market. They produce according to following technology:

$$y_{i,t} = (z_{i,t} K_{i,t-1})^\alpha (h_{i,t})^{1-\alpha} \quad (16)$$

Wholesale firms rent capital and labour in competitive markets. Cost minimization implies the following relationship between marginal cost and

the real wage and rental rate of capital.

$$mc_t = \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha} \left( \frac{r_t^k}{\alpha} \right)^\alpha$$

and in a symmetric equilibrium all the firms have the same capital-labour ratio, which yields:

$$\frac{z_t K_{t-1}}{L_t} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t^k}$$

The wholesale firms are subject to nominal rigidities *a la* Calvo (1983). Each period only a fraction  $(1 - \alpha_p)$  of them are allowed to change their prices. The probability of being allowed to change price is independent of the pricing history of firms. When given the chance to adjust its price, a firm reoptimises it in order to maximise its discounted future flow of profits, while non-optimising firms index their prices to past inflation.

The problem facing a price setting firm is:

$$\max_{p^*} E_t \sum_{s=0}^{\infty} \beta^s \alpha_p^s \lambda_{t+s} \left[ \left( \frac{p_t^*}{P_t} \frac{(P_{t-1+s}/P_{t-1})^{\gamma_p}}{P_{t+s}/P_t} - mc_{t+s} \right) y_{j,t+s} \right] \quad (17)$$

where  $\gamma_p$  determines the degree of indexation. The term in brackets gives the period by period profit of the firm, by taking into account that the firm will be able to update its price with inflation indexation. The first terms takes into account that firms discount future profits by  $\beta$  but also by  $\alpha_p^s$ , as this gives the horizon during which its price won't be reoptimised. Finally, since the firms are owned by households, the profits are multiplied by the marginal value of income to express this value in utility terms.

As all the firms face the same problem, in the symmetric equilibrium they all choose the same price. The solution to this maximization programme is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_p^s \lambda_{t+s} y_{t+s}^i \left( \frac{p_t^*}{P_t} \frac{(P_{t-1+s}/P_{t-1})^{\gamma_p}}{P_{t+s}/P_t} - (1 + \lambda_p) p_{t+s}^w \right) = 0 \quad (18)$$

Given the definition of the price index, the overall price level in each period can be expressed as a weighted sum of newly optimised prices and old prices updated by the past inflation:

$$P_t^{-1/\lambda_p} = \alpha_p (P_{t-1} \pi_{t-1}^{\gamma_p})^{-1/\lambda_p} + (1 - \alpha_p) (P_t^*)^{-1/\lambda_p} \quad (19)$$

### 3.1.4 Resource Constraint and Monetary Policy

Monetary policy is assumed to follow a variant of the Taylor (1993) rule with interest rate smoothing:

$$R_t = R_{t-1}^{\rho_r} \left( \frac{1}{\beta} \left( \frac{\pi_{t+1}}{\pi} \right)^{\rho_\pi} \left( \frac{y_t}{y} \right)^{\rho_y} \right)^{1-\rho_r} \exp(\varepsilon_t)$$

where  $\varepsilon_t$  is a monetary policy shock, assumed to be normally distributed with mean zero and variance  $\sigma_m^2$ .

Finally, the market for final goods clears in every period:

$$y_t = c_t + i_t + a(z_t)K_{t-1}$$

## 3.2 Gertler, Sala and Trigari model

The Gertler *et al.* (2008) model is similar to the Smets and Wouters model except that labour market is characterised by search and matching frictions. As we said earlier, this allows us to assess whether or not the impact of unemployment on the monetary transmission mechanism is important in terms of enabling the model to fit better the data.

### 3.2.1 Households

Our modeling of the labour market implies that some members of the household will be unemployed. Using the same arguments as for the Smets and Wouters (2003) model, we assume that household members pool their income

and there is complete consumption insurance.

The representative household maximises the discounted future flows of utility:

$$\max \sum_{s=0}^{\infty} \beta^s \frac{1}{1-\sigma} (C_t - \psi C_{t-1})^{1-\sigma} \quad (20)$$

subject to an intertemporal budget constraint:

$$C_t + I_t + R_t \frac{B_t}{P_t} = \frac{B_{t-1}}{P_t} + D_t \quad (21)$$

where the household's total income ( $D_t$ ) is composed of its wage earnings of working members ( $w_t$ ), unemployment benefits of its unemployed members ( $b_t$ ), rents on capital net of utilization costs ( $r_t^k z_t k_{t-1} - a(z_t) k_{t-1}$ ) and profits ( $\Pi_t$ ):

$$D_t = w_t n_t + (1 - n_t) b_t + r_t^k z_t k_{t-1} - a(z_t) k_{t-1} + \Pi_t \quad (22)$$

As a result, equations (10)-(14) fully describe the household's optimal decisions in this model.

### 3.2.2 Labour Market

The formation of a job is a costly and time consuming process. In order to create a productive job, firms must post vacancies,  $v_t$ , and workers must look for jobs. The number of new matches each period is determined by a matching function which relates new matches to existing vacancies and unemployed workers:

$$m_t = a_m u_t^{\sigma_u} v_t^{1-\sigma_u} \quad (23)$$

As Hall (2005) points out, fluctuations in labour market flows are mainly driven by job creation. So we abstract out of job destruction decisions by assuming that, in each period, a fixed part of existing jobs are exogenously

destroyed at rate  $1 - \rho_n$ . Employment evolves according to:

$$n_t = \rho_n n_{t-1} + m_t \tag{24}$$

The evolution of employment makes clear that new matches become productive within the same period. Accordingly, each period unemployment is given by:

$$u_t = 1 - \rho_n n_{t-1}$$

It is also useful to define transition probabilities. The probability for a firm to find a worker,  $q_t$ , and the probability for a worker to find a job,  $s_t$  are given by:

$$q_t = \frac{m_t}{v_t} \tag{25}$$

$$s_t = \frac{m_t}{u_t} \tag{26}$$

Our timing assumption implies successful matches become productive immediately. Most matching models are calibrated to one quarter implying a rather long period between the realization of a shock and the adjustment of employment. Other work, e.g., Blanchard and Gali (2006) and Ravenna and Walsh (2007) in addition to Gertler *et al.* (2008), has assumed that successful matches become productive immediately citing empirical evidence in favour of this.<sup>8</sup> Since Gertler *et al.* (2008) assume that firms cannot alter their labour input via changes in hours, then they need to allow firms to adjust at the extensive margin to shocks. Furthermore the more pronounced reaction of unemployment to shocks will mean that labour market tightness responds more to shocks and this, in turn, will lead to greater pressure on wages in response to shocks.

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<sup>8</sup>US data suggests that more than 60 % of unemployment spells take less than 14 weeks. Source: Bureau of Labor Statistics, ‘Labor Force Statistics from the Current Population Survey’.

### 3.2.3 Wholesale Firms

The production function of wholesale firms is given by:

$$y_{i,t} = (z_t K_{i,t-1})^\alpha n_{i,t}^{1-\alpha} \quad (27)$$

It is useful to define the hiring rate,  $x_{i,t}$ :

$$x_{i,t} = \frac{q_t v_{i,t}}{n_{i,t-1}} n_{i,t-1} \quad (28)$$

and the firm pays a quadratic adjustment cost of hiring given by:

$$LAC = \frac{\kappa}{2} x_{i,t}^2 n_{i,t-1} \quad (29)$$

The firm maximises its value defined by:

$$F_{i,t} = p_t^w y_{i,t} - w_{i,t} n_{i,t} - \frac{\kappa}{2} x_{i,t}^2 n_{i,t-1} - r_t^k k_{t-1} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} F_{i,t+1} \quad (30)$$

The first order condition for capital:

$$r_t^k = p_t^w \alpha \frac{y_t}{z_t k_{t-1}} \quad (31)$$

Our assumption of labour adjustment cost implies that it is equivalent for the firm to set the number of vacancies or the hiring rate. Then, vacancy posting condition is given by:

$$\kappa_t x_{i,t} = p_t^w (1 - \alpha) \frac{y_t}{n_t} - w_{i,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{2} x_{i,t+1}^2 + \rho \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \kappa x_{i,t+1} \quad (32)$$

Because of the assumption that workers start working within the period, in the vacancy posting condition, the relevant productivity and wage are those of the current period:

The value of an additional worker will determine the surplus of the firm when entering into the bargaining process.  $J_{i,t}$  is defined as the value of a new worker at time  $t$ :

$$J_{i,t} = p_t^w(1 - \alpha)\frac{y_t}{n_t} - w_{i,t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{2} x_{i,t+1}^2 + \beta E_t \frac{n_{t+1}}{n_t} \frac{\lambda_{t+1}}{\lambda_t} J_{i,t+1} \quad (33)$$

### 3.2.4 Workers

$V_{i,t}$  and  $U_t$  are defined to be the value of being employed at firm  $i$  and the value of being unemployed, respectively.  $V_{i,t}$  is given by:

$$V_{i,t} = w_{i,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [\rho V_{i,t+1} + (1 - \rho)U_{t+1}] \quad (34)$$

A worker value depends on the current wage plus the discounted future value of being employed and unemployed, weighted by their respective probabilities.

As there is a wage dispersion across firms, define  $V_{x,t}$  as the average value of employment conditional on being a new worker at time  $t$ :

$$V_{x,t} = \int_0^1 V_{i,t} \frac{x_{i,t} n_{i,t-1}}{x_t n_{t-1}} di \quad (35)$$

The idea is that the workers don't know the exact level of wages in each firm. Since there is no directed search in the sense that workers can not choose to look for high wages firms, Gertler *et al.* (2008) argue that, as the contract differentials, due to nominal rigidities, are transitory, the gain from directed search may not be large. Then  $U_t$  is given by:

$$U_t = b_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [s_{t+1} V_{x,t+1} + (1 - s_{t+1})U_{t+1}] \quad (36)$$

with  $b_t$  representing unemployment benefits:

$$b_t = bk_{t-1}$$

In our estimation, we estimate,  $\bar{b}$ , the flow value of unemployment relative to the flow value of a worker to the firm at the steady state, defined as:

$$\bar{b} = \frac{bk}{p^w m pl + \beta \frac{\kappa}{2} x^2}$$

The value for the worker of finding a job relative to his value when unemployed is given by:

$$H_{i,t} = V_{i,t} - U_t \quad (37)$$

$$H_{x,t} = V_{x,t} - U_t \quad (38)$$

### 3.2.5 Nash Bargaining and Wage dynamics

The model departs from the standard Nash Bargaining framework by assuming that each period a firm has a fixed probability  $(1 - \lambda_w)$  that it may re-negotiate the wage. Otherwise, the firms index their wages to past inflation. As we don't have trend growth or inflation, the indexation rule is given by:

$$W_t^n = W_{t-1}^n \pi_{t-1}^{\gamma_w} \quad (39)$$

The nominal contract wage,  $W_t^{n*}$ , is chosen to solve:

$$\max H_{i,t}^\eta J_{i,t}^{1-\eta} \quad (40)$$

s.t

$$W_{i,t+j}^n = \begin{cases} W_{i,t+j-1}^n \pi_{t+j-1}^{\gamma_w} & \text{with probability } \lambda_w \\ W_{i,t+j}^{n*} & \text{with probability } 1-\lambda_w \end{cases} \quad (41)$$

The first order condition is given by:

$$\eta \frac{\partial H_{i,t}}{\partial W_{i,t}^n} J_{i,t} + (1 - \eta) \frac{\partial J_{i,t}}{\partial W_{i,t}^n} H_{i,t} = 0 \quad (42)$$

The marginal values of the worker's and firm's surplus with respect to the real wage,  $\Delta_t = \frac{\partial H_{i,t}}{\partial W_{i,t}^n / p_t}$  and  $\Sigma_t = -\frac{\partial J_{i,t}}{\partial W_{i,t}^n / p_t}$ , are given by:

$$\Delta_t = 1 + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\rho \lambda_w) \frac{p_t}{p_{t+1}} \pi_t^{\gamma_w} \Delta_{t+1} \quad (43)$$

and

$$\Sigma_{i,t} = 1 + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \lambda_w \frac{p_t}{p_{t+1}} \pi_t^{\gamma_w} \Sigma_{i,t+1} \quad (44)$$

Then the first order condition for wages can be rewritten as:

$$\chi_{i,t} J_{i,t} = (1 - \chi_{i,t}) H_{i,t} \quad (45)$$

with

$$\chi_{i,t} = \frac{\eta}{\eta + (1 - \eta) \Sigma_{i,t} / \Delta_t} \quad (46)$$

All the details of the real wage derivation are in Gertler *et al.* (2008). The negotiated wage is given by:

$$\Delta_t w_t^* = w_{i,t}^o + \rho \lambda \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Delta_{t+1} w_{t+1}^* \quad (47)$$

As the firms may not be able to renegotiate the wage, the current real wage depends also on the expectations of future wages.

The target real wage  $w_{i,t}^o$  is given by:

$$w_{i,t}^o = \chi (p_t^w m p l_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{2} x_{i,t+1}^2) + (1 - \chi) (b_t + s_{t+1} \beta E_t \frac{\lambda_{t+1}}{\lambda_t} H_{x,t+1}) + \Phi_{i,t} \quad (48)$$

$$\Phi_{i,t} = \phi_{i,t} - \rho\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \phi_{i,t+1} \quad (49)$$

$$\phi_{i,t} = (\chi_{i,t} - \chi)(J_{i,t} + H_{i,t}) \quad (50)$$

The target wage is similar to the wage under flexible Nash bargaining. Omitting the last term, the wage is a weighted sum of worker's contribution to the firm and worker's outside options.

## 4 Estimation

We evaluate the model following the minimum distance estimation strategy developed in Rotemberg and Woodford (1997), Christiano *et al.* (2005), Boivin and Giannoni (2006) and Meier and Mueller (2005). As Smets and Wouters (2003) stress, this strategy helps to focus on empirical properties that the model has been developed to explain. The objective is to minimise the difference between empirical and model based impulse responses.

Formally, define  $\hat{J}$ , the vector containing the empirical impulse responses resulting from our VAR estimation and  $J(\theta)$  the vector of theoretical impulse responses of the DSGE model where the vector  $\theta$  contains the parameters we are looking to estimate.

$$L = \min_{\theta} [(\hat{J} - J(\theta))' W^{-1} (\hat{J} - J(\theta))'] \quad (51)$$

where  $W$  is a diagonal weighting matrix which contains the variance of estimated impulse responses. This weighting matrix gives more weight to more precisely estimated impulse responses and ensures that the resulting model-based impulse responses lies within the estimated confidence intervals.

## 4.1 Smets and Wouters

Following di Cecio and Nelson (2007), we use the minimum distance approach to estimate the following vector of parameters:

$$\theta = \{\psi, \gamma_p, \gamma_w, \alpha_p, \alpha_w, \kappa, \sigma_z, \rho_R, \rho_\pi, \rho_y, \sigma_m\} \quad (52)$$

The parameters which are not estimated can be inferred either from the steady state relationships or from the microeconomic studies. In particular, we fix the discount rate,  $\beta$  to 0.99 implying a steady state annual nominal interest rate of about 4%. We fix  $\alpha = 0.36$  and  $\delta = 0.025$ , values commonly used in the literature, including by di Cecio and Nelson. We adopt a log utility function ( $\sigma = 1$ ) and set  $\phi = 1$  so that the Frisch elasticity of labour supply is equal to unity. Finally, the wage mark-up,  $\lambda_w$ , is set to 0.5 following Smets and Wouters (2003) and the price mark-up,  $\lambda_p$ , is set to 0.2 following the results reported in Macallan *et al.* (2008).

Table 1 presents the estimated values for the parameters in our benchmark model using information contained in all the empirical impulse response functions (IRFs). Figure 3 displays the empirical IRFs and the IRFs from the model obtained using the estimated parameter values. Our model does well in explaining the dynamic responses of macroeconomic variables in the United Kingdom to a monetary policy shock: all the model IRFs lie within the 90% confidence intervals apart from the response of productivity, which is too persistent. This result suggests the need for a better specification of the labour market in the model and helps motivate our estimation of the Gertler *et al.* (2008) model below.

Our estimate for the habit formation parameter is somewhat higher than others (e.g., Altig *et al.* (2005), di Cecio and Nelson (2007), Christiano *et al.* (2005), Fuhrer (2000) and Harrison and Oomen (2008)) and it implies a substantial role for the backward looking behaviour in consumption. The lower bound fixed for the parameter  $\sigma_z$  is binding, which means that the

Table 1: Estimated Parameter Values for the Smets and Wouters (2003) model

Parameter		Value
$\psi$	Habit formation in consumption	0.88 (0.03)
$\gamma_p$	Degree of price indexation	1.00 (-)
$\gamma_w$	Degree of wage indexation	1.00 (-)
$\alpha_p$	Probability of not being able to reset prices	0.63 (0.57)
$\alpha_w$	Probability of not being able to reset wages	0.77 (0.12)
$\kappa$	Elasticity of investment adjustment costs	7.15 (4.42)
$\sigma_z$	Elasticity of capacity utilisation costs	0.00 (-)
$\rho_R$	Persistence parameter in Taylor rule	0.79 (0.14)
$\rho_\pi$	Coefficient on inflation in Taylor rule	1.48 (1.71)
$\rho_y$	Coefficient on output in Taylor rule	0.28 (0.68)
$\sigma_m$	Standard deviation of monetary policy shock	0.0015 (0.0002)

elasticity of capital utilisation with respect to the rental rate of capital tends toward infinity. This finding is in line with the previous estimation results for the United States, though completely out of line with that of di Cecio and Nelson. The reason for this difference is that, in addition to the empirical IRFs used by di Cecio and Nelson in their estimation, we have also sought to match the empirical IRF of capacity utilisation. Our estimates indicate that we match this response quite well. But, to do this we need the elasticity of utilisation to the rental rate to be infinite, compared with di Cecio and Nelson's estimate of zero. As Christiano *et al.* (2005) points out, variable capital utilization helps the model to match observed inflation persistence by lowering the elasticity of rental rate to monetary policy shocks. Our estimate for the investment adjustment cost is higher than in the United States and the euro area but lower than the previous UK estimate. The reason for this result is that, although investment is more volatile than output at business cycle frequencies, we find, in line with di Cecio and Nelson, that the investment response after a monetary policy shock is not large.

Our results for the parameters governing the nominal side of the economy contrast with previous UK estimates. First, our estimates indicate that wage rigidities are more important than price rigidities. According to the estimated values the average duration of wages is almost a year whereas the average duration of prices is only just over eight months. This is in line with the recent survey evidence on price durations reported in Greenslade and Parker (2008) and the results of Christiano *et al.* (2005) and Harrison and Oomen (2008), who estimate slightly higher wage rigidity than price rigidity for the United States and the United Kingdom, respectively. But, in strong contrast with our results, Smets and Wouters (2003) and di Cecio and Nelson (2007) estimate higher price rigidities for the euro area and the United Kingdom, respectively. In particular, di Cecio and Nelson find that, for the 1979Q2-2005Q4 period, there is no nominal wage rigidity while they estimate nominal rigidities in the goods market to be very high, with an av-

erage price duration of three and a half years. Again, we are trying to match the IRF for the real wage, in addition to those IRFs matched by di Cecio and Nelson; it is likely that this explains the different result we obtain for the extent of nominal wage rigidity. Finally,  $\gamma_p$  and  $\gamma_w$  are estimated to be equal to the upper bound of one, implying full indexation both in the goods and the labour market. This finding contrasts with the estimates of Smets and Wouters (2003), Smets and Wouters (2007) and Groth *et al.* (2006) who find much lower values for the euro area, United States and United Kingdom, respectively.

Our results confirm earlier findings in the United States, euro area and the United Kingdom that monetary policy exhibits high interest rate smoothing.<sup>9</sup> The parameters governing the response of the central bank to inflation and output are not very precisely estimated. This is likely a result of the changes in monetary policy regime over our sample period which we discussed earlier. According to our point estimate, monetary policy did adhere to the Taylor principle in the United Kingdom over the period as a whole, as we found the parameter governing the response of the Central Bank to inflation expectations to be greater than one.

## 4.2 Gertler Sala Trigari

The Gertler *et al.* (2008) model differs from the Smets and Wouters (2003) model only in its specification of the labour market. In this case the vector of parameters we wish to estimate is given by:

$$\theta = \{\psi, \gamma_p, \gamma_w, \alpha_p, \alpha_w, \kappa, \sigma_z, \rho_R, \rho_\pi, \rho_y, \bar{b}, \eta, \sigma_m\} \quad (53)$$

Again, we were not able to estimate all the parameters of the model so, following Gertler *et al.* (2008), we used other evidence to set these parameters. Given the lack of direct evidence on the parameters governing labour market

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<sup>9</sup>See, e.g., Clarida *et al.* (1998) and Nelson (2003).

flows, we had to calculate these parameters using UK labour market data.

Specifically, we estimated a matching function for the 2001-2008 period in order to infer about the elasticities of the matching function with respect to unemployment and vacancies. Our estimation takes a standard approach, described in Petrongolo and Pissarides (2001). We estimate a loglinear matching function where the dependant variable is outflows from unemployment. In theory, the matching function gives the number of new hires in terms of workers looking for jobs and vacancies. However, the data on unemployment may not reflect the real number of job searchers, as some workers may go from inactivity to activity without declaring themselves as unemployed. But as in Blanchard and Diamond (1990), we assume that, for United Kingdom, the unemployment rate measured by those claiming unemployment benefit may be a good proxy for all job seekers. We also report our estimates using unemployment measured by the Labour Force Survey (LFS).

As in Blanchard and Diamond (1990), we estimate the following equation using OLS :

$$\ln(M_t) = \alpha_1 + \alpha_2 \ln(U_t) + \alpha_3 \ln(V_t) + \alpha_4 Trend + \varepsilon_t \quad (54)$$

We use monthly data and our estimation period covers 2001:6-2008:6.

Table 2 presents our estimation results. The estimated elasticities of matches with respect to unemployment and vacancies are significant and positive. We also find a small but negative coefficient for the time trend, which implies a decrease over time in the efficiency of the matching technology. As Pissarides and Petrongolo (2001) point out, the estimated weight on unemployment is higher in the United Kingdom than in the United States. This finding is in line with previous matching function estimates of Pissarides et al. (1986) and Burda and Wyplosz (1994) for the United Kingdom. One noticeable point is that the estimated values are sensitive to the measure of unemployment we use in the estimation. The LFS unemployment measure is always higher than claimant count unemployment and it also yields higher

Table 2: Estimated Parameter Values: Matching Function

Parameter	Claimant Count		LFS	
	Unrestricted	Restricted	Unrestricted	Restricted
$\alpha_1$	-0.46 (0.74)	-0.47 (0.02)	-0.31 (0.38)	-0.52 (0.02)
$\alpha_2$	0.55 (0.13)	0.55 (0.05)	0.68 (0.07)	0.72 (0.05)
$\alpha_3$	0.44 (0.15)		0.23 (0.09)	
$\alpha_4$	-0.0004 (0.0001)	-0.0004 (0.0001)	-0.00011 (0.0001)	-0.00011 (0.0001)
CRS test's P-value	0.99		0.57	$R^2$
$R^2$	0.41	0.42	0.63	0.64

estimates for the elasticity of matches to unemployment. We are however very close to the 0.5-0.7 range considered in the literature.

Most of the empirical studies conclude that a constant return to scale matching function describes the data well. We also tested this assumption. The restriction that the elasticities sum up to 1 is not rejected only when we use claimant count unemployment. When we re-estimate the model imposing constant return to scale, the estimated values remains the same. We can therefore confidently set the parameter  $\sigma_u$  in our estimation to some value between 0.5 and 0.7.

The other two parameters that we can get from data are  $s$ , the probability of finding a job for an unemployed worker and  $\rho$ , the ratio of surviving jobs at each period (or one minus the separation rate).

We calculate the probability for an unemployed worker to find a job by dividing unemployment outflows by unemployment. This calculation yields a value of 0.55 for  $s$ , implying an average duration of unemployment of approximately 5 months. The unemployment series are not, however, consistent with our model as we are not modeling the labour market participation decision explicitly. In reality, the time needed to find a job may be a bit higher. Therefore, we adopt a lower value for  $s$  and check the robustness of our results for different values.

To calibrate the job separation rate, we use data on unemployment inflows. This calculation indicates that each quarter, the inflow to unemployment is just 1% of total employment. This would imply a value of 99% for  $\rho = 0.99$ . Since, we don't have data on workers leaving a job and going to inactivity, we revise our calculation upward. In our simulation, we follow Gertler *et al.* (2008) and set  $\rho$  to 0.95. Finally, our steady state, when  $s = 0.5$  and  $\rho = 0.95$  imply that the steady state unemployment rate is 9.1%. This value is higher than what we observe in the data. Our model, however, doesn't explicitly model the participation decision, *i.e.*, the possible transitions from inactivity to employment or unemployment. Our higher unemployment rate can be seen as a result of this difference between the model and the data.

Table 3 presents the estimated values for the parameters in our model. Figure 4 displays the empirical IRFs and the IRFs from the model obtained using the estimated parameter values. The model again does well in explaining the dynamic responses of macroeconomic variables in the United Kingdom to a monetary policy shock. But, it seems to do less well at explaining the response of productivity to the shock than the Smets and Wouters (2007) model; essentially, the search frictions result in the produc-

tivity response to the shock being dampened, though it is more persistent. Given that the difference between the two models relates to how the labour market is modelled, this result is a little disappointing.

In terms of the parameter estimates for the Gertler *et al.* (2008) model, they are similar to those for the Smets and Wouters (2007) model. In this case, the degree of wage indexation is estimated to be less than unity and the elasticity of capital utilisation costs is estimated to be greater than zero, though these differences are not significant.

Within this model, we estimate two additional parameters relative to the Smets and Wouters (2007) model: the bargaining power of workers and the flow value of being unemployed. We estimate the workers' bargaining power to be equal to 0.9 and the flow value of unemployment to be 0.66, close to the values estimated for the United States in Gertler *et al.* (2008).<sup>10</sup> The high bargaining power implies that wages are closely related to productivity, that is, the contribution of the worker to the firm. Our estimated value of unemployment benefit flows is higher than one would expect given the unemployment benefit replacement ratios we see and close to the higher bound in the literature. However, Hall (2005) argues that the right way of thinking about this parameter is to think of it as unemployment benefits *plus* the utility gained from being able to enjoy leisure. As such, he argues that a value of about 0.7 seems appropriate for this parameter and our estimate of 0.66 is close to this.

### 4.3 Understanding the Estimation Results

In this subsection, we evaluate the contribution of our estimated parameter values to the understanding the dynamics of inflation. We do this by using the Gertler *et al.* (2008) model with all the parameters – other than that whose contribution we wish to understand – set at their estimated values.

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<sup>10</sup>As is the case in Gertler *et al.* (2008), we are not able to identify separately both parameters.

Table 3: Estimated Parameter Values (Gertler et al. (2008))

Parameter		Value
$\psi$	Habit formation in consumption	0.91 (0.03)
$\gamma_p$	Degree of price indexation	1.00 (-)
$\gamma_w$	Degree of wage indexation	0.90 (0.33)
$\alpha_p$	Probability of not being able to reset prices	0.67 (0.33)
$\alpha_w$	Probability of not being able to reset wages	0.78 (0.20)
$\kappa$	Elasticity of investment adjustment costs	8.78 (5.88)
$\sigma_z$	Elasticity of capacity utilisation costs	0.05 (0.46)
$\rho_R$	Persistence parameter in Taylor rule	0.80 (0.15)
$\rho_\pi$	Coefficient on inflation in Taylor rule	1.78 (1.56)
$\rho_y$	Coefficient on output in Taylor rule	0.27 (0.77)
$\eta$	Workers' bargaining power	0.90 (1.15)
$b$	Flow value of being unemployed	0.66 (4.67)
$\sigma_m$	Standard deviation of monetary policy shock	0.0014 (0.0002)

Chart 5 displays, the red lines represent the implied IRFs from our benchmark estimation. The blue lines correspond to the IRFs when we set some parameters to extreme values while keeping all the other parameters at their estimated values.

We first consider the impact of price rigidities and inflation persistence. The first graph on the first row displays the response of inflation when we set price rigidities to zero. In this case, as might be expected, the response of inflation is higher on impact and inflation comes back to its steady state quickly. The second graph shows that when there is no indexation. Inflation drops just after the monetary policy shock and then increases monotonically, again as expected. The combination of sticky prices and indexation helps to generate the small and delayed effect of monetary policy shocks on inflation.

The last graph in the first row and the first one in the second row show how the persistence of marginal cost affects inflation dynamics. When there are no wage rigidities at all, marginal cost and, hence, inflation are more volatile than in the data. The same is true if we increase the elasticity of utilisation to the rental rate of capital.

Finally, the last two graphs show that if the flow value of being unemployed is higher, the response of wages to a monetary policy shock decreases. This is the well-known result of Hagedorn and Manowski (2005), who show that this particular calibration enables the model to generate increased volatility in unemployment and, so, better match this feature of the data. With wages not responding as much to a monetary policy shock, real marginal cost will not respond as much. This, in turn, lowers the response of inflation to the shock.

## 5 Conclusion

In this paper, we used the minimum distance approach to estimate the DSGE models of Smets and Wouters (2003) and Gertler *et al.* (2008) using UK

data. This was motivated by our interest in understanding the monetary transmission mechanism and how monetary policy makers can set interest rates so as to achieve their (implicit or explicit) inflation target and a belief that labour market frictions and, in particular wage-setting frictions, play a central role in inflation dynamics.

We first used a structural vector autoregression (SVAR) approach to obtain an empirical representation of the monetary transmission mechanism, ie, how a monetary policy change affects some important macroeconomic variables in the United Kingdom. We found that output, consumption, investment and capacity utilisation all fell in response to the shock and that the responses of all these variables were hump-shaped. The peak response of output occurs five quarters after the shock. Inflation rose on impact (though this rise was not statistically significant) before falling to a trough two years after the shock. The effect on inflation of the shock dies out after three years. The relative price of capital and real wages fell in response to the shock, but these effects were not statistically significant. The peak response of productivity was one period after the shock. Given the response of output, this result suggested that the adjustment in labour input occurs with a lag relative to the response of output.

In terms of the models, we found that both were able to explain reasonably well the dynamic responses of the macroeconomic variables we considered in the United Kingdom to a monetary policy shock. In addition, they were able to do this without relying on excessive degrees of price or wage stickiness. In particular, wages were set about once a year and prices about every eight months, both in line with survey and other evidence. Having said that, the results implied a large degree of indexation in price and wage setting. It is not clear that this result is in line with our intuition for what actually happens in the United Kingdom.

Unfortunately, neither model was able to satisfactorily explain the response of productivity. An implication of this is that they were unable to

explain the response of employment, given that they could explain the response of output. This result is particularly disappointing in the case of the Gertler *et al.* (2008) model, given that the big difference between this model and that of Smets and Wouters (2007) is that it takes seriously modelling the frictions contained in the labour market and, so, you might expect it to match better the responses of labour market variables to shocks.

These results leave us with a big question: given that the Smets and Wouters (2007) model was able to explain the monetary transmission mechanism – in the sense of matching the implied impulse response functions – fairly well, what is the role, if any, of search and matching frictions and unemployment in the monetary transmission mechanism? We leave finding an answer to that question to future research.

## 6 Annex: Log-linear models

### 6.1 Smets and Wouters model

- Marginal Utility of consumption

$$\widehat{\lambda}_t = -\sigma \frac{1}{1-\psi} (\widehat{c}_t - \psi \widehat{c}_{t-1}) \quad (55)$$

- Euler Equation

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + r_t - E_t \widehat{\pi}_{t+1} \quad (56)$$

- IS consumption

$$\widehat{c}_t = \frac{\psi}{1+\psi} \widehat{c}_{t-1} + \frac{1}{1+\psi} E_t \widehat{c}_{t+1} - \frac{1-\psi}{(1+\psi)\sigma} (r_t - E_t \widehat{\pi}_{t+1}) \quad (57)$$

- Capital

$$\widehat{p}_t^k = E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \frac{r^k}{r^k + 1 - \delta} \widehat{r}_{t+1}^k + \frac{1 - \delta}{r^k + 1 - \delta} \widehat{p}_{t+1}^k \quad (58)$$

- Investment

$$\widehat{i}_t = \frac{1}{1 + \beta} \widehat{i}_{t-1} + \frac{\beta}{1 + \beta} E_t \widehat{i}_{t+1} + \frac{1}{\kappa} \frac{1}{1 + \beta} \widehat{p}_t^k \quad (59)$$

- Capital accumulation

$$\widehat{k}_t = (1 - \delta) \widehat{k}_{t-1} + \delta \widehat{i}_t \quad (60)$$

- Capital utilization

$$\widehat{z}_t = \frac{1}{\sigma_z} \widehat{r}_t^k \quad (61)$$

- Wage Phillips Curve (Indexation to price inflation)

$$\widehat{\pi}_t^w - \gamma_w \widehat{\pi}_{t-1} = \beta E_t (\widehat{\pi}_{t+1}^w - \gamma_w \widehat{\pi}_t) + \frac{(1 - \beta \alpha_w)(1 - \alpha_w)}{\alpha_w (1 + \frac{1 + \lambda_w}{\lambda_w} \phi)} (\widehat{mrs}_t - \widehat{w}_t) \quad (62)$$

- Definition of real wage

$$\widehat{w}_t = \widehat{w}_{t-1} + \widehat{\pi}_t^w - \widehat{\pi}_t \quad (63)$$

- Definition of MRS

$$\widehat{mrs}_t = \phi \widehat{l}_t - \widehat{\lambda}_t \quad (64)$$

- Production function

$$\widehat{y}_t = (1 + \lambda_p) (\alpha \widehat{k}_{t-1} + \alpha \widehat{z}_t + (1 - \alpha) \widehat{L}_t) \quad (65)$$

- NKPC

$$\widehat{\pi}_t = \frac{\beta}{1 + \beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1 + \beta\gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1 + \beta\gamma_p} \frac{(1 - \beta\alpha_p)(1 - \alpha_p)}{\alpha_p} \widehat{mc}_t \quad (66)$$

- Definition of marginal cost

$$\widehat{mc}_t = \alpha \widehat{r}_t^k + (1 - \alpha)(\widehat{w}_t + r_t) \quad (67)$$

- Capital labour ratio or labour demand

$$\widehat{L}_t + \widehat{w}_t = \widehat{z}_t + \widehat{k}_{t-1} + \widehat{r}_t^k$$

- Monetary policy

$$i_t = \rho_r i_{t-1} + (1 - \rho_r)(\rho_\pi \pi_{t+1} + \rho_y y_t) + \varepsilon_t \quad (68)$$

- Real interest rate

$$f_t = r_t - E_t \widehat{\pi}_{t+1} \quad (69)$$

- Resource constraint

$$\widehat{y}_t = \frac{c}{y} \widehat{c}_t + \frac{i}{y} \widehat{i}_t + r^k \frac{k}{y} \widehat{z}_t$$

## 6.2 Gertler, Sala and Trigari model

- Household's FOC

$$\widehat{c}_t = \frac{\psi}{1 + \psi} \widehat{c}_{t-1} + \frac{1}{1 + \psi} E_t \widehat{c}_{t+1} - \frac{1 - \psi}{(1 + \psi)\sigma} (r_t - E_t \widehat{\pi}_{t+1}) \quad (70)$$

$$\widehat{p}_t^k = E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \frac{r^k}{r^k + 1 - \delta} \widehat{r}_{t+1}^k + \frac{1 - \delta}{r^k + 1 - \delta} \widehat{p}_{t+1}^k \quad (71)$$

$$\widehat{i}_t = \frac{1}{1 + \beta} \widehat{i}_{t-1} + \frac{\beta}{1 + \beta} E_t \widehat{i}_{t+1} + \frac{1}{\kappa} \frac{1}{1 + \beta} \widehat{p}_t^k \quad (72)$$

$$\widehat{z}_t = \frac{1}{\sigma_z} \widehat{r}_t^k \quad (73)$$

$$\widehat{k}_t = (1 - \delta) \widehat{k}_{t-1} + \delta \widehat{i}_t \quad (74)$$

- Unemployment

$$u_t = -\frac{n}{u} \widehat{n}_{t-1} \quad (75)$$

- Matching

$$\widehat{m}_t = \sigma_m \widehat{u}_t + (1 - \sigma_m) \widehat{v}_t \quad (76)$$

- Employment

$$\widehat{n}_t = \rho \widehat{n}_{t-1} + (1 - \rho) \widehat{m}_t \quad (77)$$

- Vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_{t-1} \quad (78)$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \quad (79)$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \quad (80)$$

- Market Tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \quad (81)$$

- Production Function

$$\widehat{y}_t = \alpha \widehat{z}_t + \alpha \widehat{k}_{t-1} + (1 - \alpha) \widehat{n}_t \quad (82)$$

- Capital demand

$$\widehat{r}_t^k = \widehat{p}_t^w + \widehat{y}_t - \widehat{z}_t - \widehat{k}_{t-1} \quad (83)$$

- Vacancy posting condition (also gives marginal cost)

$$(\kappa x) \widehat{x}_t = p^w \text{mpl}(\widehat{p}_t^w + \widehat{mpl}_t) - w \widehat{w}_t + (\kappa x) \beta E_t x_{t+1} + (\kappa x) (\rho + x/2) \beta (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) \quad (84)$$

- Marginal product of labour

$$\widehat{mpl}_t = \widehat{y}_t - \widehat{n}_t \quad (85)$$

- Phillips Curve

$$\widehat{\pi}_t = \frac{\beta}{1 + \beta \gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1 + \beta \gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1 + \beta \gamma_p} \frac{(1 - \beta \alpha_p)(1 - \alpha_p)}{\alpha_p} \widehat{mc}_t \quad (86)$$

- Bargaining weights

$$\widehat{\chi}_t = -(1 - \chi)(\widehat{\Sigma}_t - \widehat{\Delta}_t) \quad (87)$$

$$\widehat{\Delta}_t = \rho \lambda_w \beta E_t (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma \widehat{\pi}_t + \widehat{\Delta}_{t+1}) \quad (88)$$

and

$$\widehat{\Sigma}_t = (1 - \rho \lambda_w \beta) E_t \widehat{x}_{t+1} + \lambda_w \beta E_t (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma \widehat{\pi}_t + \widehat{\Sigma}_{t+1}) \quad (89)$$

- Target wage

$$w\widehat{w}_t^o = \varphi_{mpl}(\widehat{p}_t^w + \widehat{mpl}_t) + \varphi_x E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \varphi_b \widehat{b}_t + \varphi_\lambda E_t (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \varphi_\chi (\widehat{\chi}_t - \beta(\rho - s) E_t \widehat{\chi}_{t+1}) \quad (90)$$

$$\varphi_{mpl} = \chi p^w mpl(w)^{-1}$$

$$\varphi_x = \chi \beta (1 - \rho) (\kappa x w)^{-1}$$

$$\varphi_s = \chi \beta s (\kappa x w)^{-1}$$

$$\varphi_b = (1 - \chi) bk(w)^{-1}$$

$$\varphi_\chi = \frac{\chi}{(1 - \chi)} (\kappa x w)^{-1}$$

- Real wage

$$\widehat{w}_t = \gamma_b (\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1}) + \gamma_o \widehat{w}_t^o + \gamma_f (\widehat{w}_{t+1} - \widehat{\pi}_{t+1} + \gamma \widehat{\pi}_t) \quad (91)$$

$$\gamma_b = (1 + \tau_2) \phi^{-1}$$

$$\gamma_f = (\rho \beta - \tau_1) \phi^{-1}$$

$$\phi = (1 + \tau_2 + \varsigma + \rho \beta - \tau_1)$$

$$\tau_1 = \varphi_s \Gamma (1 - \rho \lambda_w \beta)$$

$$\tau_2 = [\varphi_x \lambda_w - \varphi_\chi (1 - \chi) (1 - \rho) \Psi \Delta^{-1}] (\kappa x)^{-1} \Sigma w (1 - \rho \lambda_w \beta)$$

$$\Gamma = (1 - \eta (1 - \rho) \Psi) \eta^{-1} (\kappa x)^{-1} \Sigma w$$

$$\begin{aligned}\gamma_o &= \varsigma \phi^{-1} \\ \phi &= (1 + \tau_2 + \varsigma + \rho\beta - \tau_1) \\ \varsigma &= (1 - \lambda_w)(1 - \rho\lambda_w\beta)\lambda_w^{-1} \\ \tau_2 &= [\varphi_x\lambda_w - \varphi_\chi(1 - \chi)(1 - \rho)\Psi\Delta^{-1}](\kappa x)^{-1}\Sigma w(1 - \rho\lambda_w\beta) \\ \Gamma &= (1 - \eta(1 - \rho)\Psi)\eta^{-1}(\kappa x)^{-1}\Sigma w \\ \Psi &= \beta\lambda_w^2/(1 - \lambda_w\beta)\end{aligned}$$

- Monetary Policy

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\rho_\pi \pi_{t+1} + \rho_y \hat{y}_t) + \varepsilon_t \quad (92)$$

- Real interest rate

$$\hat{f}_t = r_t - \pi_t \quad (93)$$

- Resource Constraint

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + r^k \frac{k}{y}\hat{z}_t + \frac{\kappa x^2 n}{2y}(2\hat{x}_t + \hat{n}_{t-1}) \quad (94)$$

- Unemployment benefits

$$\hat{b}_t = \hat{z}_t + \hat{k}_{t-1} \quad (95)$$

## 7 Charts

Figure 1: Impulse responses to Monetary Policy Shock

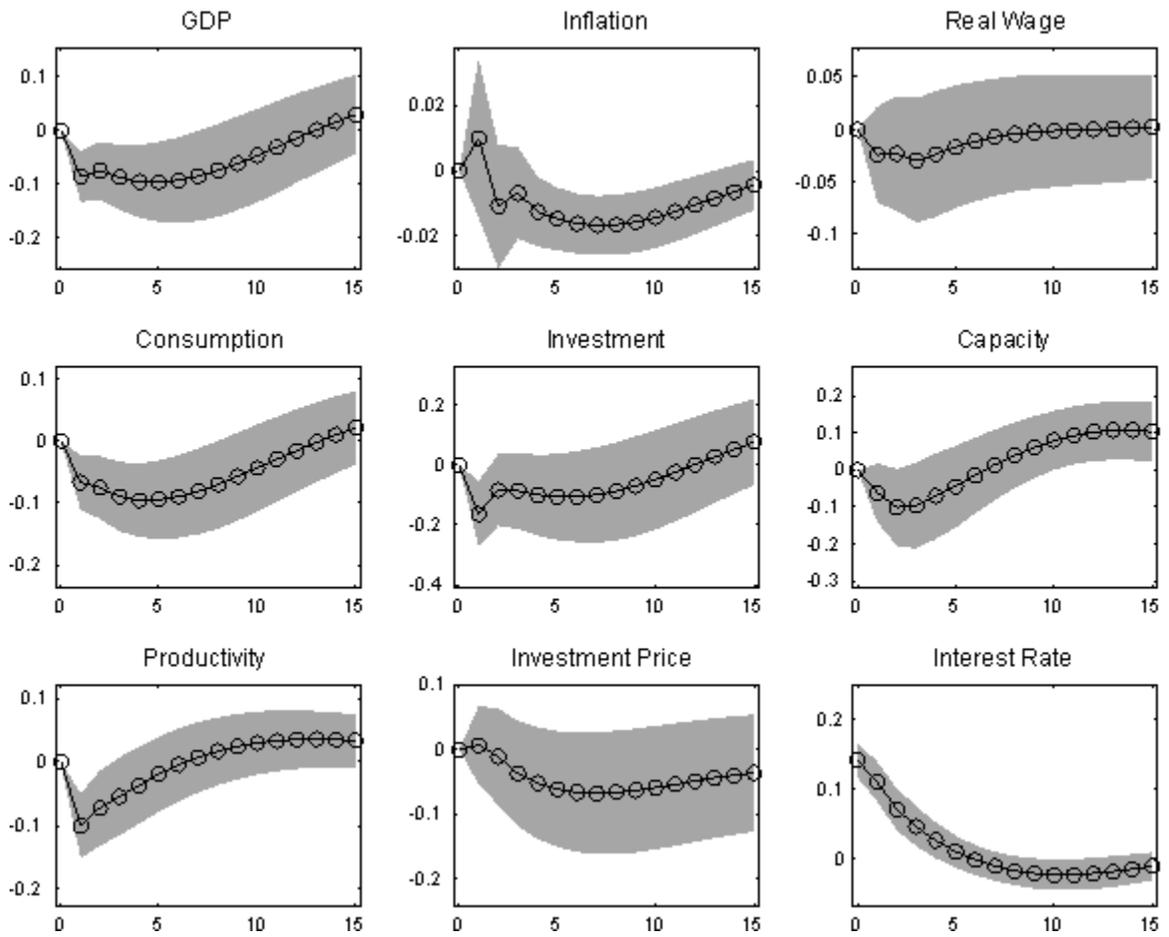


Figure 2: Recursive VAR estimates: Rolling Samples

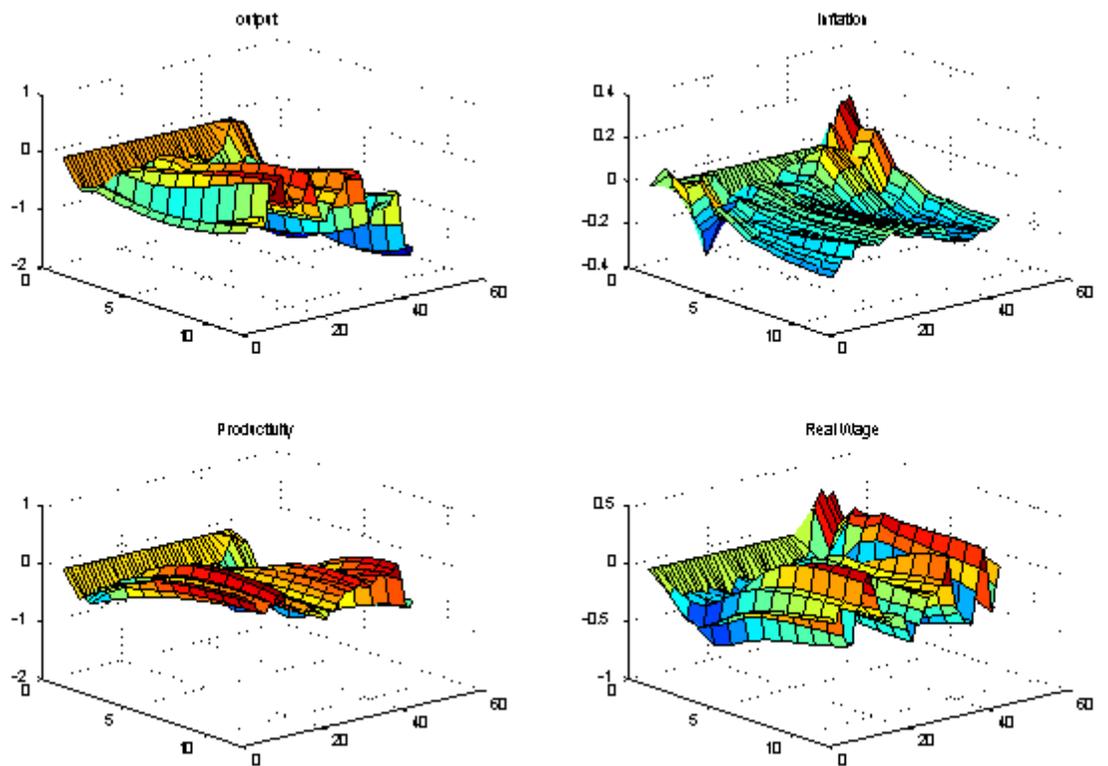


Figure 3: Estimation results: Smets and Wouters (2003) model

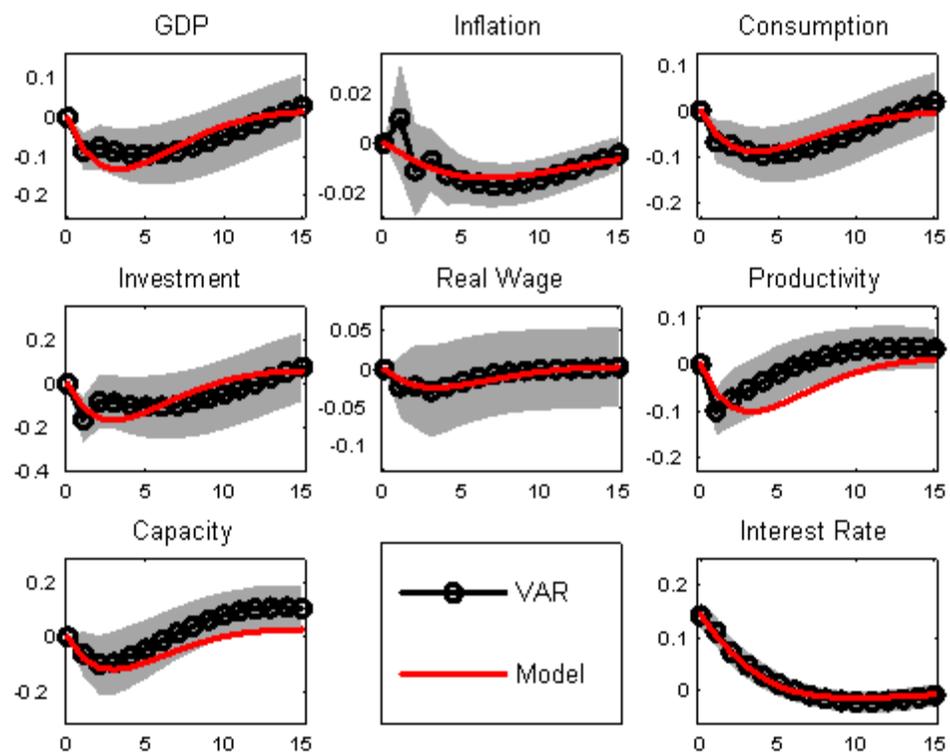


Figure 4: Estimation Results: Gertler *et al.* (2008) model

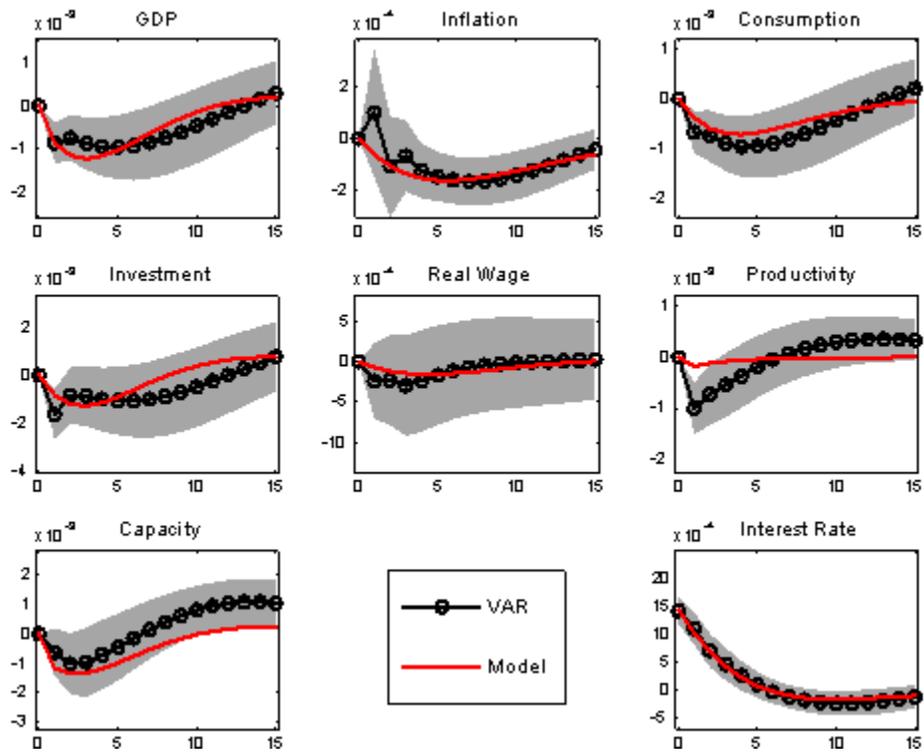
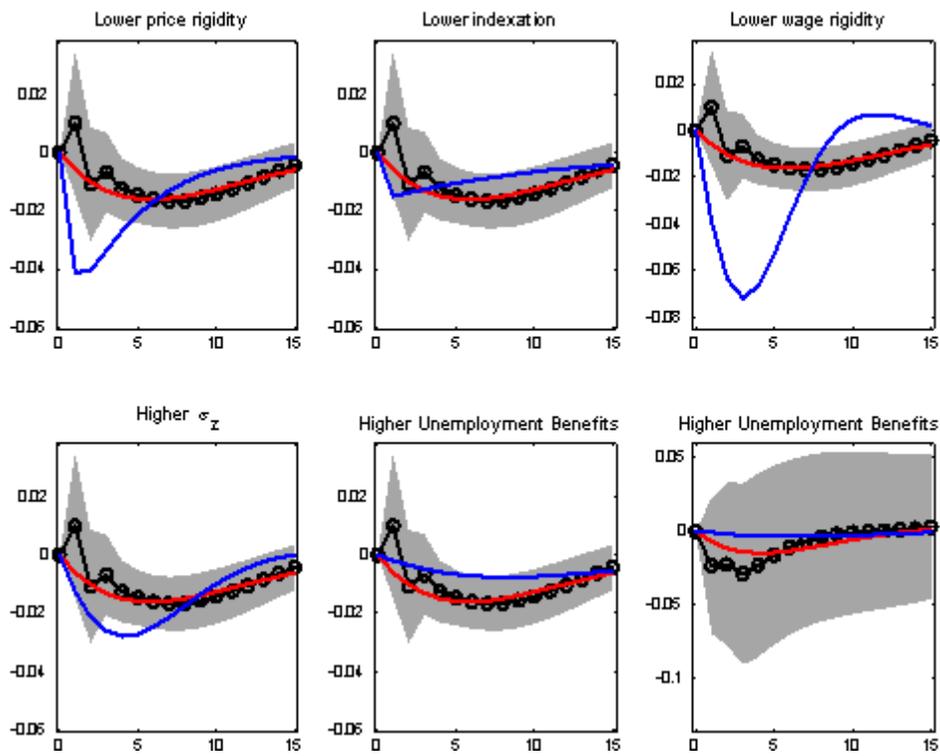


Figure 5: The role of estimated parameters on inflation dynamics



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