

An Environmental-Economic Measure of Sustainable Development

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Abstract

A central issue in the study of sustainable development is the interplay of growth and sacrifice in a dynamic economy. This paper investigates the relationship among current consumption, sacrifice and sustainability improvement in a general context and in two canonical, stylized economies. We argue that the maximin value of utility measures what is sustainable and provides the limit to growth. Maximin value is interpreted as a dynamic environmental-economic carrying capacity and current utility as an environmental-economic footprint. The time derivative of maximin value is interpreted as net investment in sustainability improvement. It is called durable savings to distinguish it from genuine savings, usually computed with discounted-utilitarian prices.

Key words: sustainable development, growth, maximin, sustainability indicator

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1 Introduction

The term *sustainable development* describes growth toward a developed state that can be sustained for what Solow (1993) calls the very long run.

The maximum level of utility that can be sustained from a given, current economic state is the so-called maximin level of utility (Solow, 1974; Cairns and Long, 2006). This reference level depends on the economic endowments and the technology. The development paths that sustain this level are given by the solution of a maximin optimization problem.

An important criticism of applying maximin as a social objective in a poor economy is that future generations may be mired in a “poverty trap.” Poverty may be sustained. This criticism implies that the sustainable (maximin) level of utility is considered to be so low that economic development is called for. Development, or growth, entails the diversion of resources from consumption by the current generation to investment that will increase productivity in the future. For sustainable growth to occur the standard of living of the present must be reduced to an *even lower* level than that of the poverty trap. Moreover, the development path followed by the economy must be within environmental and technological constraints.

The issue is how to grow out of poverty while improving what can be sustained. The concept of *sustainability*, which has sometimes been defined as requiring that utility be no greater than the maximal sustainable utility, is not sufficient to tackle this issue. It characterizes the sustainability of current utility, but provides no information on how current decisions impact future sustainability and the development prospect.

The present paper formalizes the relationship among current consumption, sacrifice and *sustainability improvement*. Current decisions reduce the level of utility that society is *able to sustain* over time if the current maximin value decreases. Sustainability improvement is defined as non-decreasing of the current maximin value. This concept is used to characterize sustainable development paths.

We examine the conditions for a sacrifice by present generations to improve the sustainable level of utility (the maximin value). We find that, except for a non-regular case,

if the current level of utility is greater than the maximin value the current maximin value decreases. Conversely, if the level of utility is lower than the maximin value, *sustainable development* is possible, with both current utility and the sustainable level of utility of the economy increasing through time. Once utility catches up with the (dynamic) maximin value, utility can be sustained at (but not above) the maximin level then prevailing.

Our results are illustrated in two canonical models that have been prominent in the study of sustainability, the simple fishery and the *Dasgupta-Heal-Solow* (DHS) model (Dasgupta and Heal, 1974; Solow, 1974). Each addresses a fundamental issue in environmental economics. Each implies that growth is subject to environmental constraints. Open access in the fishery leads to a tragedy of the commons. The DHS model illustrates the fact that sustaining an economy may not involve a steady state. Each of open access and growth can lead to unsustainability and to a poverty trap.

Our contributions to the analysis of sustainable development stress two current indicators that, as conveyed by the words “sustainable” and “development,” look to the ability to sustain economic well-being in the very long run. In particular, we use

1. the current maximin value as the indicator of sustainability; and
2. the rate of change of the current maximin value as an indicator of sustainability improvement (if it is non-negative) or decline (if it is negative).

The maximin value is a well known indicator in a maximin program. In the present paper, we extend it outside a maximin program to apply to any trajectory, optimal or not, efficient or not. The maximin value characterizes the dynamic limit to growth. It generalizes the concept of ecological footprint. The evolution of the maximin value over time is measured by current net investment at maximin accounting prices. This investment indicator, that we call *durable savings*, can be used to measure sustainable development, providing an alternative to genuine savings, which is usually computed with discounted-utilitarian prices.

2 Maximin value and sustainability

For a vector of available capital stocks $X \in \mathbb{R}_+^n$ (including natural resources, levels of technological knowledge and other forms of comprehensive capital) and a vector of decisions within the set of feasible controls, $c \in C(X) \subseteq \mathbb{R}^p$, let utility at time t be represented by $U(X(t), c(t))$. The transition equations for the stocks are

$$\dot{X}_i(t) = F_i(X(t), c(t)), \quad i = 1, \dots, n. \quad (1)$$

Formally, the maximin value of a given economic state X is defined as

$$\begin{aligned} m(X) &\equiv \max_{c(\cdot)} \min_{s \geq t} U(X(s), c(s)) \\ \text{s.t.} \quad &X(t) = X, \\ &\dot{X}_i(s) = F_i(X(s), c(s)), \quad i = 1, \dots, n, \forall s \geq t. \end{aligned} \quad (2)$$

This is the highest level of utility that can be sustained, over all feasible paths starting from state X . By Bellman's principle the maximin value $m(X)$ depends only on the current state X and not on the vector of current decisions c .

We restrict the analysis to models for which a maximin value function is well-defined, in the sense that maximin paths actually achieve the maximin value at any time.¹

The maximin value is sometimes identified as intertemporal social welfare, but such an identification is not essential to a study of the properties of sustainability and sustainable development. We do not assume that the economy follows a maximin path, nor any other optimal or efficient path. The maximin value is defined for all states and it can be computed at any time for the current economic state. We study the evolution of the maximin value over time for any feasible vector of decisions $c \in C(X)$.

Sustainability has sometimes been defined (see, e.g., Pezzey, 1997) as requiring that utility be no greater than the maximal sustainable utility, i.e.,

$$U(X(t), c(t)) \leq m(X(t)). \quad (3)$$

¹Assuming that the maximin value can be achieved allows us to consider a “max min” problem instead of a “sup inf” problem. Mitra et al. (2013) provide conditions on the technology for the existence of a maximin solution in the Dasgupta-Heal-Solow model.

Condition (3) establishes whether the current level of utility can be sustained by comparing it to the maximin value. That value is a reference point, and not an objective. While condition (3) plays a supporting role in the formal definition of sustainability, it does not account fully for the effect of current decisions on the future conditions for sustainability, in particular of investment decisions.

Sustainable development not only depends on current utility but also on the future ability to sustain economic well-being. It entails investment choices that do not result in the decreasing of the maximin value. We argue that the condition of non-decreasing of the maximin value,

$$\frac{dm(X(t))}{dt} \geq 0, \quad (4)$$

which we call *sustainability improvement*, is more useful than condition (3) in the study of sustainable development.

Before presenting our formal analysis, some comment is in order concerning our perception of sustainability and sustainable development. Our approach has a great deal of kinship with those of Doyen and Martinet (2012) and Fleurbaey (2013), who also use maximin as a foundation for the study of sustainability and consider maximin in non-optimal economies. A part of Fleurbaey's analysis, in particular, can be considered to be complementary to ours and, though there are some significant differences, is in several ways parallel.² An important technical difference is that he considers a discrete time model with finite horizon while we consider a continuous time model in infinite time.³

²Fleurbaey (2013) presents an insightful discussion of how to incorporate the sustainability indicator into a welfare analysis.

³Analysis in discrete time allows Fleurbaey to introduce the consideration of overlapping generations in the traditional way. As regards the choice of the time horizon, we lean to the argument of Takayama (1974, p.446) in discussing optimal growth theory: "[T]here is one serious objection to a finite T [the horizon], however large T may be. What happens after time T ?... When we decide the size of T , we automatically decide to ignore the time after T , and such a T is arbitrarily chosen for there is no *a priori* criterion by which to choose the size of T . The general consensus among economists about this point seems to be to choose $T = \infty$ in order to avoid such an arbitrary cut-off point." Takayama's further discussion deals with some of the points raised by Fleurbaey to support his assumption, and in particular with what Fleurbaey judges to be "complications of dubious practical relevance that are solely due to what may happen in an infinite time." These technical points are kept out of our discussion, which does

Fleurbaey (2013) defines sustainability in a discrete time framework as the condition that current utility is no greater than next period maximin value, i.e., “ $U(X(t), c(t)) \leq m(X(t+1))$ ”, so that future generations are able to sustain current utility. This sustainability condition is more informative than Criterion (3), as it accounts for the effect of current decisions on the ability of future generations to sustain utility.

The counterpart condition in our continuous time framework is (a) $U(X(t), c(t)) < m(X(t))$ or (b) $U(X(t), c(t)) = m(X(t))$ and $dm(X(t))/dt \geq 0$, as stressed in Doyen and Martinet (2012). Sustainability improvement then plays a central role in the characterization of sustainability and sustainable development in the continuous time framework. Requiring sustainability improvement leads us to propose a concept of *durable savings*.

3 Sustainability and sustainability improvement

The formal juxtaposition of the criteria (3) and (4) is instructive to stress the relationship between (un)sustainability and sustainability improvement or decline. For the sake of notational simplicity, where there can be no confusion we omit the time argument in what follows. Let $M(X, c) \equiv \frac{dm(X)}{dt}|_c$ denote the change in the maximin value for given current economic decisions $c = (c_1, \dots, c_p)$. The following table summarizes the nine possible combinations of the conditions $U(X, c) \lesseqgtr m(X)$ and $M(X, c) \lesseqgtr 0$.

We start by proving the impossibility of case 1 and then characterize the two non-regular cases 2 and 3. Then, we characterize the sustainability and unsustainability of the regular case.⁴ These are done under the following three assumptions.

not utilize (but of course assumes) transversality conditions. (For a discussion of transversality conditions see Cairns and Long (2006).) We believe that it is prudent to follow the usual practice in the study of sustainability.

⁴On a *regular* maximin path, utility remains constant and equal to the maximin value over time (Burmeister and Hammond, 1977; Cairns and Long, 2006). One key assumption made by Fleurbaey (2013) is what he calls *transferability*. This assumption assures that the program is *regular*, because utility can be transferred from one time period to another, so that it is feasible to follow a development path with constant utility over time equal to the maximin level. This assumption eliminates the possibility of maximin paths with declining current utility (such as in the simple fishery). It also eliminates from

Table 1: Utility, maximin value, and net maximin investment

	$U(X, c) > m(X)$	$U(X, c) = m(X)$	$U(X, c) < m(X)$
$M(X, c) > 0$	1. Impossibility	2. Non-regularity: “Bounded utility”	6. Sustainability improvement
$M(X, c) = 0$	3. Non-regularity: “Bounded investment”	4. Regularity (or wasting)	7. Unproductive sacrifice
$M(X, c) < 0$	5. Sustainability decline due to overconsumption	8. Sustainability decline due to inadequate investment	9. Sustainability decline due to inadequate investment

Assumption 1 *The functions $F_i(X, c)$ are continuous and differentiable.*⁵

Assumption 2 *Utility $U(X, c)$ is continuous and differentiable.*

Assumption 3 *A maximin-value function $m(X)$ exists and is differentiable, i.e., $\mu_i(X) \equiv \frac{\partial m(X)}{\partial X_i}$, $i = 1, \dots, n$, exist.*

Under Assumption 3, the maximin value varies smoothly over the state space.⁶ Cairns and Long (2006, Proposition 1) show that the partial derivatives $\mu_i(X)$ are the co-state variables, or shadow-values, of a maximin problem. They depend on the state variable X and not on choice of the vector of decisions c , and are thus independent of the trajectory

consideration another type of non-regularity described in Cairns and Tian (2010) and discussed in greater generality by Doyen and Martinet (2012). Our contention is that, in the current state of knowledge, it is prudent in a general analysis to treat non-regularity as well as regularity. Such a treatment is a distinguishing feature of the present paper.

⁵Under assumption 1, along a feasible path satisfying the transition equations (1), the state trajectory $X(\cdot)$ is continuous.

⁶Consistently with Bellman’s principle, the maximin value at a state depends on the set of feasible paths starting from that state. This dependence of the maximin value on time paths is fully encompassed in the maximin value and its derivatives under our assumptions. In many places, our analysis is founded on the state dependent functions $m(X)$ and $\mu_i(X)$, allowing us to rely on the current state $X(t)$ at any point in time and not on particular time paths.

determined by the functions $F_i(X, c)$. These maximin shadow values are the accounting prices of the present paper.⁷

Definition 1 (Net maximin investment) *For a current state $X(t)$ and a vector of decisions $c(t)$, the time derivative of the maximin value measures current net maximin investment:*

$$M(X(t), c(t)) = \sum_{i=1}^n \frac{\partial m(X(t))}{\partial X_i} \dot{X}_i = \sum_{i=1}^n \mu_i(X(t)) F_i(X(t), c(t)) . \quad (5)$$

This definition of net maximin investment applies to any feasible vector of decisions $c = (c_1, \dots, c_p) \in C(X)$.

Future decisions are unpredictable and it is difficult to project the path of an economy. Our results are related to the current generation's decisions only. We make no assumptions about decisions in the future.⁸ The analysis focuses on the choices at a particular point in time, t , given the comprehensive capital stocks $X(t)$ at that date.

Among all vectors of feasible decisions $C(X(t))$ at state $X(t)$, at least one is the initial vector of decisions for a maximin path. It is useful to stress the properties of such decisions in terms of sustainability and sustainability improvement to discuss the cases in Table 1. The following characterizes the current decisions that are on a maximin path (for short, maximin decisions) from the current state.

Lemma 1 (Maximin decisions) *Any vector of decisions $c(t) \in C(X(t))$ which is such that $U(X(t), c(t)) \geq m(X(t))$ and $M(X(t), c(t)) \geq 0$ is consistent with following a max-*

⁷An important advantage of the continuous time framework is that we provide an exact condition to measure net maximin investment, which can easily be compared to the genuine savings literature. In discrete time, the conditions on net investment at shadow prices are only approximations (Fleurbaey, 2013). The approximations improve as the time horizon increases, but are never exact in a finite time framework.

⁸Our view is that the decisions of the future properly belong to the future, that the role of the present is to bequeath opportunities and not outcomes. Fleurbaey (2013) discusses an alternative proposal for accounting for what actually is expected to happen in the future as opposed to the opportunities given to the future.

imin path from the current state $X(t)$ and thus constitutes a vector of current maximin decisions.

Proof of Lemma 1 Consider a state vector $X(t)$ at time t and the associated maximin value $m(X(t))$, as well as a vector of decisions $c(t)$ such that $U(X(t), c(t)) \geq m(X(t))$ and $M(X(t), c(t)) \geq 0$. The transition equations $\dot{X} = F(X(t), c(t))$ define a state $X(t + dt)$. From that state, it is possible to sustain $m(X(t + dt))$, which, by condition $M(X(t), c(t)) \geq 0$ is greater than or equal to $m(X(t))$. As $U(X(t), c(t)) \geq m(X(t))$, there is thus a path starting from state $X(t)$ and decisions $c(t)$ sustaining $m(X(t))$. Decisions $c(t)$ are maximin decisions.

Cases 1 to 4 in Table 1 thus correspond to maximin decisions. We show in Theorem 1 that it is not possible to have both a utility level greater than the maximin value and a positive net maximin investment. This impossibility theorem is key to the discussion of the other cases in Table 1.

Theorem 1 (Maximin impossibility theorem) : For any state X , there is no vector of decisions c such that $U(X, c) > m(X)$ and $M(X, c) > 0$.

Proof of Theorem 1 Consider a state vector $X(t)$ at time t and the associated maximin value $m(X(t))$. Suppose that there exists a vector of decisions $c(t) \in C(X(t))$ such that $U(X(t), c(t)) > m(X(t))$ and $M(X(t), c(t)) > 0$. The transition equations $\dot{X} = F(X(t), c(t))$ define a state $X(t + dt)$. The condition $M(X(t), c(t)) > 0$ implies that $m(X(t + dt)) > m(X(t))$. From that state, it is possible to sustain $m(X(t + dt)) > m(X(t))$. As $U(X(t), c(t)) > m(X(t))$, there would be a path such that for all $s \geq t$, $U(X(s), c(s)) \geq \min(U(X(t), c(t)), m(X(t + dt))) > m(X(t))$, in contradiction to the definition of $m(X(t))$.

We now turn to case 2, which is non-regular. Proposition 2 states that the maximin value can increase when utility is equal to the maximin value only if it is not possible at the margin to increase the utility above the maximin value.⁹

⁹Utility is not necessarily globally bounded from above. There may be decisions such that $U(X, c) > m(X)$, but these decisions cannot be marginally close to maximin decisions, and they necessarily imply $M(X, c) \leq 0$, in accordance with Theorem 1.

Proposition 2 (Non-regularity due to locally bounded utility) : *It is possible to have both $U(X, c) = m(X)$ and $M(X, c) > 0$ only if $\frac{\partial U(X, c)}{\partial c_j} = 0$ for all $c_j \in c$, wherever these partial derivatives are defined.*

Proof of Proposition 2 Consider a state vector X and a vector of decisions c such that $U(X, c) = m(X)$ and $M(X, c) > 0$. Suppose that there is a decision $c_j \in c$ such that $\frac{\partial U(X, c)}{\partial c_j} \neq 0$ and c_j is not on the boundary of $C(X)$ so that it is possible to increase utility above $m(X)$ by marginally changing decision c_j .¹⁰ Even if $\frac{\partial M(X, c)}{\partial c_j} = \sum_{i=1}^n \mu_i(X) \frac{\partial F_i(X, c)}{\partial c_j} < 0$, by continuity of the $F_i(X, c)$ and of $U(X, c)$, there is a vector of decisions \tilde{c} such that $U(X, \tilde{c}) > m(X)$ while $M(X, \tilde{c}) > 0$. This contradicts Theorem 1.

In this case, for the given state, utility is locally bounded from above in the neighborhood of the maximin decisions considered. This corresponds to a particular case of non-regularity in maximin problems. An example has been described by Cairns and Tian (2010).¹¹

Corollary 2 *In case 2, it is not possible to increase $U(X, c)$ by decreasing $M(X, c)$ at the margin.*

Proof of Corollary 2 Obvious since $\frac{\partial U(X, c)}{\partial c_j} = 0$ for all c_j .

A main result below is that, apart from the non-regular case 2, net maximin investment cannot be positive unless current utility is lower than the maximin value. There must be a sacrifice of utility by present generations to increase the sustainable level of utility.

We now characterize another type of non-regularity. Proposition 3 states that utility can exceed the maximin value without implying a decrease in that value only if no decision c_j marginally affects net maximin investment.

¹⁰If some controls are on the boundary of the admissibility set $C(X)$, the derivatives of these controls are defined on only one side. The condition is then that the derivative is non-positive (resp. non-negative) on the right-hand (resp. left-hand) side when the control is bounded from below (resp. above).

¹¹In Cairns and Tian (2010), non-regularity arises in states for which the utility is locally bounded from above. The maximin value is equal to the maximal utility given the state vector, and the maximin path corresponds to a myopic behavior of instantaneous utility maximization. Along this path, the maximin value increases as the state evolves.

Proposition 3 (Non-regularity due to locally bounded investment) : *It is possible to have both $U(X, c) > m(X)$ and $M(X, c) = 0$ only if $\frac{\partial M(X, c)}{\partial c_j} = 0$ for all $c_j \in c$.*

Proof of Proposition 3 *Consider a state vector X and a vector of decisions c such that $U(X, c) > m(X)$ and $M(X, c) = 0$. Suppose that there is a decision $c_j \in c$ such that $\frac{\partial M(X, c)}{\partial c_j} = \sum_{i=1}^n \mu_i(X) \frac{\partial F_i(X, c)}{\partial c_j} \neq 0$ and c_j is not on the boundary of $C(X)$, so that it is possible to increase net maximin investment above zero by marginally changing decision c_j . Even if marginally changing c_j reduces utility, by continuity of the $F_i(X, c)$ and of $U(X, c)$, there is a vector of decisions \tilde{c} such that $M(X, \tilde{c}) > 0$ while $U(X, \tilde{c}) > m(X)$. This contradicts Theorem 1.*

This type of non regularity includes as a main particular case the situation in which all the elements of the sum $\sum_{i=1}^n \mu_i(X) \frac{\partial F_i(X, c)}{\partial c_j}$ are equal to zero, i.e., $\mu_i(X) = 0$ for any X_i for which $\frac{\partial F_i(X, c)}{\partial c_j} \neq 0$ for some control c_j . All the capital stocks that are locally influenced by at least one decision have no marginal contribution to the maximin value. These stocks are redundant from a maximin point of view.¹² This particular case was studied by Asako (1980).

Corollary 3 *In case 3, it is not possible to increase $M(X, c)$ above zero by reducing utility at the margin.*

Proof of Corollary 3 *Obvious since $\frac{\partial M(X, c)}{\partial c_j} = 0$ for all c_j .*

If the two types of non-regularity are ruled out, maximin decisions belong to case 4 and are regular, as stated in the following proposition.¹³

Proposition 4 (Regularity) *For a state vector X and a vector of maximin decisions c , if there is a decision c_j such that $\frac{\partial U(X, c)}{\partial c_j} \neq 0$ and a decision c_k such that $\frac{\partial M(X, c)}{\partial c_k} \neq 0$, then the vector of decisions c necessarily satisfies $U(X, c) = m(X)$ and $M(X, c) = 0$.*

¹²An even more restrictive case is when all the maximin shadow values are equal to zero at the considered state. This is the case in the simple fishery or in the Ramsey (1928) model when the single capital stock is above the golden rule level.

¹³The regular case 4 is the one in which there is “transferability” of utility as discussed by Fleurbaey (2013). Case 4 could also occur if there is “waste” in non-regular cases, in the sense that potential maximin investment is “wasted” (case 2) or potential utility is “wasted” (case 3).

Proof of Proposition 4 Consider a state vector X and any associated vector of maximin decisions c . One has $U(X, c) \geq m(X)$ and $M(X, c) \geq 0$ (Lemma 1). It is not possible to have $U(X, c) > m(X)$ and $M(X, c) > 0$ (Theorem 1). If there is a decision $c_j \in c$ such that $\frac{\partial U(X, c)}{\partial c_j} \neq 0$, one cannot have $U(X, c) = m(X)$ and $M(X, c) > 0$ (Proposition 2). If there is a decision $c_k \in c$ such that $\frac{\partial M(X, c)}{\partial c_k} \neq 0$, one cannot have $U(X, c) > m(X)$ and $M(X, c) = 0$ (Proposition 3). One necessarily has $U(X, c) = m(X)$ and $M(X, c) = 0$.

Regularity has been understood as the ability “to spread” utility equally over time (Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006). The two types of non-regularity arise if there is a restriction on current spreading (at a particular current state). The restriction in case 2 is that current utility cannot be increased given the current state by reducing the current positive net maximin investment (Corollary 2). The restriction in case 3 is that net maximin investment cannot be increased given the current state by marginally reducing current utility (Corollary 3). These two conditions allow the deducing of conditions for regular current maximin decisions: regular maximin decisions must be able to influence both current utility and current net maximin investment. Corollary 4 formalizes this property. The condition derived is related to the concept of “eventual productivity” (Asheim et al., 2001).

Corollary 4 For a state vector X , if for any vector of maximin decisions c there are a decision c_j such that $\frac{\partial U(X, c)}{\partial c_j} \neq 0$ and a decision c_k such that $\frac{\partial M(X, c)}{\partial c_k} \neq 0$, then $\frac{\partial U(X, c)}{\partial c_k} \neq 0$ and $\frac{\partial M(X, c)}{\partial c_j} \neq 0$, and it is possible to smooth the current utility to the maximin value.

Proof of Corollary 4 Assume that $\frac{\partial U(X, c)}{\partial c_j} \neq 0$ and $\frac{\partial M(X, c)}{\partial c_j} = \sum_{i=1}^n \mu_i(X) \frac{\partial F_i(X, c)}{\partial c_j} = 0$. By continuity of the $F_i(X, c)$ and $U(X, c)$, it would be possible to increase current utility (by changing decisions c_j and c_k) and the maximin investment (by changing decision c_k) to define a vector of decisions \tilde{c} such that $U(X, \tilde{c}) > m(X)$ and $M(X, \tilde{c}) > 0$. This contradicts Theorem 1. (A similar argument holds if $\frac{\partial M(X, c)}{\partial c_k} \neq 0$ and $\frac{\partial U(X, c)}{\partial c_k} = 0$.) Decision c_j thus satisfies $\frac{\partial U(X, c)}{\partial c_j} \frac{\partial M(X, c)}{\partial c_j} \neq 0$. The product cannot be strictly positive (again, by Theorem 1). Therefore, if $\frac{\partial U(X, c)}{\partial c_j} > 0$, one has $\frac{\partial M(X, c)}{\partial c_j} < 0$, and vice versa. It is possible to increase (decrease) current utility and decrease (increase) maximin investment at the margin of maximin decisions.

We now characterize case 5 in which sustainability decline is due to overconsumption. Except in the non-regular case 3, realizing a utility greater than the maximin value necessarily reduces this value, i.e., comes at the cost of reducing the maximum sustainable level.

Proposition 5 (Sustainability decline due to overconsumption) : *If there is a control c_j such that $\frac{\partial M(X,c)}{\partial c_j} \neq 0$, then $U(X,c) > m(X) \Rightarrow M(X,c) < 0$.*

Proof of Proposition 5 *A direct consequence of Theorem 1 and Proposition 3.*

We now characterize sustainability improvement (case 6). Except in the non-regular case 2, to increase the maximin value ($M(X,c) > 0$), there must be a sacrifice of utility by the current generation ($U(X,c) < m(X)$). This is stated in part *i*) of Theorem 6. This condition is not sufficient, however. The sacrifice results in a sustainability improvement only if the applied decisions result in a positive net maximin investment, as stated in part *ii*) of Theorem 6, which rules out case 3.¹⁴

Theorem 6 (Sustainability improvement) :

i) If, for a state vector X and vector of decisions c , there is a decision $c_j \in c$ such that $\frac{\partial U(X,c)}{\partial c_j} \neq 0$, then $M(X,c) > 0 \Rightarrow U(X,c) < m(X)$.

ii) Let a vector of maximin decisions for state X be denoted by $c^m(X) = (c_1^m, \dots, c_p^m)$. If there is a decision c_j such that, on an open interval I containing c_j^m , one has $\frac{\partial U(X, (c_1^m, \dots, c_j, \dots, c_p^m))}{\partial c_j} \neq 0$ and $\frac{\partial M(X, (c_1^m, \dots, c_j, \dots, c_p^m))}{\partial c_j} \neq 0$, then there are decisions \tilde{c} by which $U(X, \tilde{c}) < m(X)$ and $M(X, \tilde{c}) > 0$ on that interval. The result holds also if the two signs are reversed.

Proof of Theorem 6 *i) A direct consequence of Theorem 1 and Proposition 2.*

ii) We demonstrate that is it possible to deviate from a maximin path by reducing current utility and increasing maximin investment.

Consider a vector of maximin decisions $c^m(X) = (c_1^m, \dots, c_p^m)$ for which there is a decision c_j such that, on an open interval I containing c_j^m , one has $\frac{\partial U(X, (c_1^m, \dots, c_j, \dots, c_p^m))}{\partial c_j} \neq 0$

¹⁴A sacrifice cannot increase the maximin value in case 3 (e.g., in a fishery), as stated in Corollary 3.

and $\frac{\partial M(X, (c_1^m, \dots, c_j, \dots, c_p^m))}{\partial c_j} \neq 0$. In particular, $\frac{\partial U(X, c^m)}{\partial c_j} \neq 0$ and $\frac{\partial M(X, c^m)}{\partial c_j} \neq 0$. According to Proposition 4, c^m is a vector of regular maximin decisions and satisfies $U(X, c^m) = m(X)$ and $M(X, c^m) = 0$. Moreover, from the proof of Corollary 4, $\frac{\partial U(X, c^m)}{\partial c_j} \frac{\partial M(X, c^m)}{\partial c_j} < 0$.

Because $\frac{\partial U(X, (c_1^m, \dots, c_j, \dots, c_p^m))}{\partial c_j} \neq 0$ on the interval and $U(X, c)$ is continuous in c_j , $U(X, c)$ is also monotone in c_j on the interval. The same holds for $M(X, c)$. Since c_j has an opposite effect on $U(X, c^m)$ and $M(X, c^m)$ and the functions are monotone, this opposite effect holds on the whole interval. By choosing $\tilde{c}_j - c_j^m > 0$ if $\frac{\partial U(X, c^m)}{\partial c_j} < 0$ and $\tilde{c}_j - c_j^m < 0$ if $\frac{\partial U(X, c^m)}{\partial c_j} > 0$, one can define a vector of decisions $\tilde{c} = (c_1^m, \dots, \tilde{c}_j, \dots, c_p^m)$ such that $U(X, \tilde{c}) < m(X)$ and $M(X, \tilde{c}) > 0$. A reversal of the sign of $\tilde{c}_j - c_j^m$ entails that $U(X, \tilde{c}) > m(X)$ and $M(X, \tilde{c}) < 0$.

According to Theorem 6, in the regular case it is possible to improve sustainability (to increase $m(X)$ over time) by reducing utility. This is not a sufficient condition, however, as the resources freed up by utility reduction have to be reinvested so as to increase the maximin value; i.e., net maximin investment must be positive. Depending on the sacrifice of utility, there may be many different vectors of decisions for which $M(X, c) > 0$. The notion of sustainability improvement does not rely on any definition of efficiency.¹⁵

The remaining three cases in Table 1 correspond to current wasting of consumption or investment. The following Proposition follows directly from Theorems 1 and 6 and Propositions 2 to 5.

Proposition 7 (Deficient decisions) :

i) Unproductive sacrifice (Case 7): A sacrifice of current utility with respect to the maximin sustainable level, $U(X(t), c(t)) < m(X(t))$, does not result in current sustainability improvement if investment decisions are such that $M(X(t), c(t)) = 0$ (including

¹⁵The assumption that $\frac{\partial U(X, c^m)}{\partial c_j} \neq 0$ rules out non-regular case 2 in part ii) of Theorem 6. If $\frac{\partial U(X, c^m)}{\partial c_j} = 0$, net maximin investment can be positive ($M(X, c^m) > 0$) without decreasing utility in Case 2. The result on sustainability improvement may, however, hold even for this non-regular case if $\frac{\partial U(X, \tilde{c})}{\partial c_j} \neq 0$ for $\tilde{c}_j \in I - \{c_j^m\}$. A sacrifice of utility makes it possible to increase net investment more than the non-regular maximin decision, i.e., $M(X, \tilde{c}) > M(X, c^m) > 0$.

the non-regular case 3).

ii) Sustainability decline due to inadequate investment (Case 8): Current decisions may result in a reduction of the maximin value, $M(X(t), c(t)) < 0$, even if utility is equal to the maximin value, $U(X(t), c(t)) = m(X(t))$. Current decisions yield a sustainable utility but current investment results in a sustainability decline. In this case, the current utility will not be sustained, whatever future decisions are, as any path starting from $X(t + dt)$ can at best sustain $m(X(t + dt)) < m(X(t)) = U(X(t), c(t))$.

iii) Sustainability decline due to inadequate investment (Case 9): A sacrifice of current utility with respect to the maximin sustainable level $U(X(t), c(t)) < m(X(t))$ results in sustainability decline if investment decisions are such that $M(X(t), c(t)) < 0$. In this case, from continuity, the current utility may still be sustainable from state $X(t + dt)$ if $U(X(t), c(t)) < m(X(t + dt)) < m(X(t))$.

Case 8 (part *ii*) of Proposition 7) corresponds to the definition of “unsustainability” provided by Fleurbaey (2013), but he considers that Case 9 (part *iii*) of Proposition 7) “achieves sustainability in a dubious way,” as “ $m(X(t + 1))$ ” is at least equal to $U(X(t), c(t))$, even if there is a sustainability decline. We consider Case 9 to be unsustainable, because we take sustainable development to mean that current decisions do not reduce the maximin value, even when current utility is sustainable. The outcome of both cases 8 and 9 is driven by (poor) investment choices that are harmful to future generations and lead to $M(X, c) < 0$ while $M(X, c) \geq 0$ is possible. In this assessment, there is no link with the sustainability criterion (3), which compares current utility to current maximin value.

Therefore, criterion (3) is suggestive but is not adequate to describe sustainable development. It focuses on current utility without considering the effects of current investment decisions on the future. It fails to characterize sustainability decline when net maximin investment is negative.

In non-regular cases, and more importantly in economies in which some resources can be wasted, criterion (3) can be misleading. In a non-efficient or non-optimal setting,

the sign of $M(X, c)$ indicates sustainability improvement (or decline). Sustainability improvement always implies that criterion (3) holds. The reverse is not true. The failure of criterion (3) does not imply a negative net maximin investment ($M(X, c) < 0$) and sustainability decline in the non-regular case 3. Current decisions that waste resources (in particular in terms of investment) may, however, result in a decline of sustainability even when current utility can be sustained ($U(X(t), c(t)) \leq m(X(t))$). The evolution of the maximin value over time is a measure of sustainable development.

4 (Un)sustainability and sustainability improvement in two canonical economies

4.1 The Fishery

The simple fishery model involves one renewable resource stock S and one economic decision, the fishing effort $E \geq 0$. This model illustrates cases 3, 4, 5 and 6, including regular and non-regular cases.¹⁶

The natural rate of growth of the stock is given by $S(1 - S)$ and the consumption (catch level) by $C = SE$. The evolution of the stock is then given by $\dot{S} = S(1 - S) - SE$. The highest sustainable level of consumption is called the “maximum sustainable yield” (MSY); its value is $C_{MSY} = \max_S [S(1 - S)] = \frac{1}{4}$. The associated stock is $S_{MSY} = \frac{1}{2}$ and the level of effort is $E_{MSY} = \frac{1}{2}$.

If the initial stock S_0 is less than S_{MSY} , the maximin criterion (2) prescribes a constant harvest, $C(t) = S_0(1 - S_0)$. If the initial stock is greater than S_{MSY} , the maximin value is C_{MSY} . The maximin value is thus given by

$$m(S) = \begin{cases} S_{MSY}(1 - S_{MSY}) & \text{if } S > S_{MSY}, \\ S(1 - S) & \text{if } S \leq S_{MSY}. \end{cases}$$

Consider a harvesting schedule with four time intervals which correspond with the conditions of cases 3, 5, 4 and 6, respectively. Let $S(0) = 1$. For simplicity, let the fishing

¹⁶As a single decision determines consumption (the catch) and investment (the growth rate of the stock) simultaneously, we cannot use this model to illustrate cases 7–9 with the appropriate wasting.

effort be constant within each interval.¹⁷ The four intervals are defined as follows and depicted in Fig. 1.

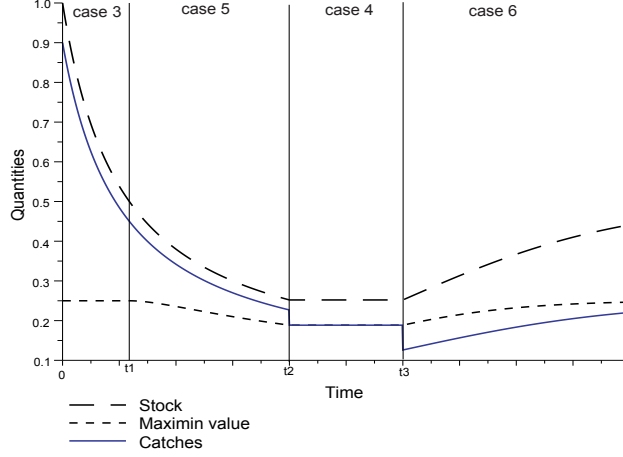


Figure 1: (Un)sustainability and sustainability improvement in the fishery

- (Case 3): The first interval is characterized by a constant fishing effort $E_0 > E_{MSY}$ ($E_0 = 0.9$ in Fig. 1) and by a fish stock $S(t) > S_{MSY}$. Consumption is $C(t) = E_0 S(t) > C_{MSY}$. The stock declines over time to S_{MSY} at the end of the interval. As long as $S(t) > S_{MSY}$, one has $\frac{dm(S)}{dS} = 0$. The maximin value of the stock is zero at the margin and the maximin value remains constant at the MSY level. In this non-regular case, consuming more than the maximin value does not reduce this value (Proposition 3). At the end of the interval, t_1 , $S(t_1) = S_{MSY} = \frac{1}{2}$ and $m(S(t_1)) = \frac{1}{4}$.

- (Case 5): The second interval begins at t_1 , where the stock declines below S_{MSY} . The effort level is kept constant at E_0 and the stock keeps decreasing. Once the MSY

¹⁷Along a constant-effort path with effort $E_0 \in]0, 1[$, consumption at time t is given by $C(t) = E_0 S(t)$ and the dynamics of the resource by $\dot{S}(t) = S(t)(1 - E_0 - S(t))$. The stock evolves as $S(t) = \left[\frac{1}{1-E_0} + \left(\frac{1}{S_0} - \frac{1}{1-E_0} \right) e^{-(1-E_0)t} \right]^{-1}$. The stock tends toward a limit $S_\infty = 1 - E_0$ if the effort is maintained.

stock is overshoot, $\frac{dm(S)}{dS} > 0$. The maximin value decreases as the stock decreases. This interval illustrates the sustainability decline due to overconsumption described in Proposition 5. It corresponds to the “tragedy of the commons” for a fishery in open access.

- (Case 4): At the beginning of the third time interval, t_2 , a limitation of the fishing effort is implemented to maintain the stock at $S(t_2)$. On the interval, the trajectory follows the maximin path. Net investment is $\dot{S}(t) = 0$ (the catch equals natural growth). The catch stays constant at $m(S(t_2))$. This part of the path illustrates sustainability as described in Proposition 4. If the catch is low, this part of the program corresponds to a poverty trap.
- (Case 6): On the last time interval, beginning at t_3 , a recovery strategy is adopted. Effort is set at $E(t) = E_{MSY}$, which for $S(t) < 1/2$ is less than the maximin level of effort $E^{mm} = 1 - S(t)$. The stock size increases toward S_{MSY} and the maximin value increases toward $m(S_{MSY})$. Consumption is $C(t) = \frac{1}{2}S(t) < m(S(t))$. Moreover, $C(t_3) < C(t_2)$: the catch is initially less than the level of the poverty trap. An initial sacrifice is required for sustainability improvement. Consumption increases toward C_{MSY} as the stock increases. This part of the path illustrates sustainability improvement as described in Theorem 6.

Interval 1 is part of a maximin path whatever the level of effort: the maximin path is not unique. On intervals 1, 2 and 4, the levels of effort need not be constant. (On interval 3, of course, the level of effort is endogenous.) All that is required in intervals 1 and 2 is that the level of effort remains greater than $1/2$. The times t_2 and t_3 are arbitrary but t_2 determines the stock size in interval 3. In interval 4, effort could have been chosen in the interval $[1/2, E^{mm}(S(t))]$; the stock would tend to a limit $S_\infty < S_{MSY}$. The paths of $C(t)$ and $S(t)$ would be determined by these choices.

4.2 The Dasgupta-Heal-Solow model

The DHS model can be used to illustrate cases 4, 5, and 6, as well as case 9 on sustainability decline due to inadequate investment. Consider a society that has stocks of a nonrenewable resource, S , and of a manufactured capital good, K , at its disposal. It produces output (consumption c and investment \dot{K}) by using the capital stock and depleting the resource stock at rate $\dot{S}(t) = -r(t)$, according to a Cobb-Douglas production function,

$$c + \dot{K} = F(K, r) = K^\alpha r^\beta, \text{ with } 0 < \beta < \alpha, \text{ and } \alpha + \beta \leq 1.$$

If the discounted-utility criterion with a constant, positive discount rate is applied to this economy, consumption decreases asymptotically toward zero (Dasgupta and Heal 1974, 1979). Analysis of how consumption can be sustained requires a different approach.

For given levels of the capital and resource stocks, Solow (1974) and Dasgupta and Heal (1979) show that the maximin consumption is given by

$$m(S, K) = (1 - \beta) (\alpha - \beta)^{\frac{\beta}{1-\beta}} S^{\frac{\beta}{1-\beta}} K^{\frac{\alpha-\beta}{1-\beta}}. \quad (6)$$

This maximin value is the highest level of consumption that the economy can sustain for the long term. Sustaining consumption at this level requires that investment in manufactured capital offset the depletion of the resource (Hartwick, 1977).

To illustrate the interplay of consumption, investment and sustainability improvement, we choose a feasible trajectory and study the evolution of the maximin value along that trajectory. The path depicted in Fig. 2 is composed of four time intervals, corresponding to the conditions of cases 5, 9, 6 and 4, respectively. Each interval is characterized by an illustrative consumption pattern and extraction rule. For simplicity, we consider a constant rate of change of consumption in each interval.

- (Case 5): At first let consumption be greater than the maximin value ($c_0 > m(S_0, K_0)$) and decrease at a constant rate $\gamma > 0$, so that $c(t) = c_0 e^{-\gamma t}$. Extraction is determined such that production is equal to consumption. Investment in manufactured capital, \dot{K} , is zero but the resource stock is depleted; therefore,

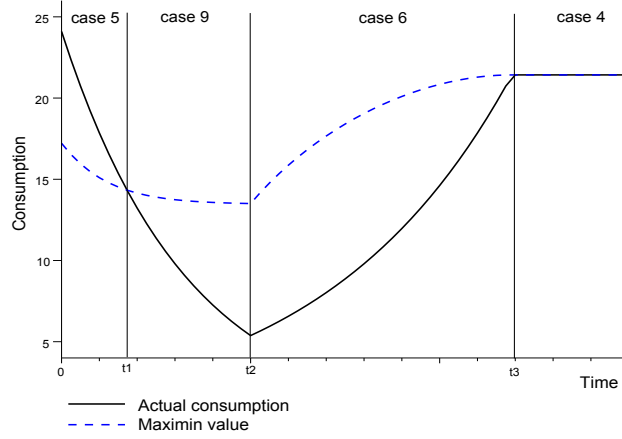


Figure 2: Sustainability and unsustainability in the DHS model

net investment $\frac{dm(S,K)}{dt} = M((S, K), (r, c))$ is negative.¹⁸ The maximin value decreases in accordance with Proposition 5. This program is inefficient: the same consumption path could be followed with a higher level of net investment, so that the maximin value would not fall so fast. But overconsumption alone would also have led to a decrease of the maximin value, i.e., to sustainability decline due to overconsumption.

- (Case 9): The second time interval starts once consumption decreases below the maximin value, at t_1 . The consumption and extraction decisions are unchanged. Consumption is lower than the maximin value, but still net maximin investment $M((S(t), K(t)), (r(t), c(t)))$ is negative and the maximin value continues to decrease. This interval illustrates Proposition 7, where sustainability decline is not related to overconsumption but to inadequate investment.
- (Case 6): At the beginning of the third interval, t_2 , consumption has reached a low

¹⁸We thus have the feedback rule $r(c, K) = c^{1/\beta} K^{-\alpha/\beta}$. As there is no investment in manufactured capital, we can express the extraction as an open-loop decision, $r(t) = c_0 e^{-\gamma t} K_0^{-\alpha/\beta}$.

level, $c(t_2) = c_0 e^{-\gamma t_2}$. A decision is made to follow a growth path while improving what can be sustained. The consumption pattern changes; it is now defined by a positive growth rate g , so that $c(t) = c(t_2) e^{g(t-t_2)}$. As long as $c(t) < m(S(t), K(t))$, net investment can be positive. The extraction rule is modified so that production is sufficient to have a positive net investment.¹⁹ With this positive level of net investment, the maximin value increases, resulting in a sustainability improvement in accordance with Theorem 6. Consumption growth can be maintained as long as consumption remains below the maximin value.

- (Case 4): The fourth time interval starts once consumption has caught up with the maximin value, at t_3 . To avoid the unsustainability of interval 1, the consumption pattern must change from the constant-growth path to the maximin path with consumption constant at $c^m(t) = m(S(t_3), K(t_3))$. Net maximin investment $M((S(t), K(t)), (r(t), c(t)))$ is nil. Extraction and investment in capital are determined by the maximin solution. This is a sustainable path as described in Proposition 4.

At any time, the society can choose to follow a regular maximin path with a maximin value determined by the stocks at that time, or to deviate from it. We have examined some particular cases that are illustrative. On intervals 1, 2 and 3, society deviates from the maximin paths at each point of the intervals. On intervals 1 and 2, “degrowth” (negative growth) results in sustainability decline. On interval 2, the maximin value decreases even though consumption is lower than the sustainable level. On interval 3, the maximin value increases even though consumption is growing at a constant, positive rate. Growth may be sustainable.

¹⁹In Fig. 2, the extraction rule is arbitrarily defined so as to maximize $M((S, K), (r, c))$ given the current stock levels and current consumption. It is in fact the feedback extraction rule of a maximin program. This extraction rule is not efficient, however, as the Hotelling rule is not satisfied. This is not an issue, here, as the choice of the extraction rule is purely illustrative. The question of the “efficiency” of sustainability improvement requires different concepts than those discussed in this paper and will be addressed in future research.

The analysis of these two models illustrates that the maximin value can be used as an indicator of sustainability and of sustainability improvement, even when sustaining utility by following the maximin path is not a policy objective. What is sustainable in the long-run is eventually defined by the maximin value, which is dynamic and depends on investment.

5 A perspective on sustainable development and its measurement

The findings of the theorems, propositions and corollaries, which hold at any point on a path being followed by an economy (sustainable or not, efficient or not) can be used to provide a perception of sustainability. We begin by defining sustainable development.

Definition 2 (Sustainable development) *The path of the economy exhibits sustainable development at time t if $M(X(t), c(t)) \geq 0$ and $\frac{dU(X(t), c(t))}{dt} \geq 0$.*

5.1 Sustainable growth paths and “degrowth” paths

Departures from the maximin path have implications for the maximin value. Theorem 6 shows that, apart from the non-regular case 3, so long as utility is less than the maximin value, the maximin value can be increased. It is possible to choose the vector of decisions c such that both utility and the maximin value increase. In the DHS model, for example, a deviation downward from the maximin consumption can allow for growth at a parametric rate through investment (d’Autume and Schubert, 2008). Asheim *et al.* (2007) show that it is possible, with what they call quasi-arithmetic growth, for the maximin value to increase indefinitely and for the consumption level to approach it asymptotically. It is also possible for the utility level to catch up with the maximin value in finite time, as illustrated in subsection 4.2. Once $U(X(t), c(t)) = m(X(t))$, the only sustainable program is for current utility to remain equal to the maximin value forever. Such paths can be considered to be sustainable-development paths.

Also possible is what may be called “degrowth to sustainability” from an initial utility level that is greater than the maximin value. So long as the utility level remains above the maximin level, the latter decreases. The utility level can be decreased until it reaches the maximin level and thereafter held constant at the maximin level. Degrowth to sustainability consists of decreasing utility until it reaches the maximin value; for example, it would have resulted in interval 1 of Figure 2 if the society had chosen to follow the maximin path from t_1 .

Obviously, on inefficient paths there is scope to reduce the inefficiency. Llavador *et al.* (2011) find that sustainable consumption for the USA was higher than actual consumption in 2000. A possible reason is inefficiency. For them, the long-term solution is to address the inefficiency, not necessarily to invest more in the present. Our result in Case 9 stresses that the two issues are linked.

We have not stressed technological progress, which is often viewed as a major source of continuing improvement in the human condition. In the general model of the present paper, endogenous or exogenous technological progress can be introduced by defining stocks of knowledge or R&D among the n states. Investments in the associated stocks then have maximin prices.²⁰

5.2 Practical implications for sustainability accounting based on investment at maximin shadow values

The maximin indicator is a very-long-run indicator of what is sustainable, of the sort that Solow (1993) seeks. At least two other indicators have been proposed to evaluate sustainability, namely the ecological footprint and genuine savings.

The ecological footprint has been proposed as an indicator of the environmental limit to sustainable output. It seeks to compare the level of current utilization of environmental resources (the ecological footprint) with the available flow of environmental services (the

²⁰As regards unanticipated exogenous technological change, it is not possible to include it directly in a deterministic approach. The possibility of such technological progress, however, does not invalidate our results. Such technological progress acts like manna from heaven. When occurring, there is a “jump” in the maximin value, offering room for growth by increasing the limit to growth.

ecological carrying capacity), evaluated in terms of land of a given quality. If the level of utilization is greater than the flow of available services, the society depletes the stock and is considered to be unsustainable at its current level of utilization. The ecological footprint has no explicit objective, although an implicit objective is some form of ecological sustainability. This lack of an explicit objective is what leads to the derivation of accounting prices from the (natural) constraints facing the society.²¹

Maximin analysis puts the insights of the ecological footprint on a sounder, more comprehensive footing, based not on land capacity but on “generalized capacity to produce economic well-being” (Solow, 1993). In the present paper, the idea of the footprint is made more comprehensive through the analysis of evolving environmental *and* technological constraints. The current level of utility corresponds with the environmental-economic footprint. The maximin value may be considered to be a dynamic, *environmental-economic limit* to growth. Current decisions modify this limit. In the regular case, as predicted by analyses of the ecological footprint, society faces diminishing long-run prospects, or diminishing sustainability, if utility exceeds the limit.

The indicator in Definition 1 closely resembles the genuine-savings indicator as determined from the extension of the national accounts (e.g., World Bank, 2006; Dasgupta, 2009). Genuine savings (sometimes called genuine investment) generalizes the concept of savings in the national accounts to include changes in the quantities of capital goods, especially environmental goods, that do not have market prices. It is equal to the current change in social welfare, defined to be the integral of discounted utility. An increase in this integral implies that genuine savings computed at competitive prices is positive at a given instant. Non-negative genuine-savings is sometimes considered to be an indicator of sustainability because current welfare does not decrease. For example, the World Bank (2006: 41) argues that “Economic theory tells us that there is a strong link between changes in wealth and the sustainability of development – if a country (or a household, for that matter) is running down its assets, it is not on a sustainable path. For the link to hold, however, the notion of wealth must be truly comprehensive.”

²¹Through its set of explicit trade-offs that make land the numeraire, ecological footprint analysis has implied a form of substitutability among natural and other stocks.

The issue regarding sustainability turns not solely on the assets to be included but also on the shadow or accounting prices at which investment is evaluated.²² If there is a suspicion that the market is not producing a sustainable result, the prices derived from national accounts should not be used for sustainability accounting. An increase of welfare signaled by positive genuine savings may not be lasting or durable. Rather, the genuine savings indicator can be positive along a competitive path even though consumption is not sustainable (Asheim, 1994). The welfare integral can increase at the current moment but eventually decrease, even if the environment is incorporated into optimal decisions (Dasgupta and Heal, 1979; Pezzey, 2004). Genuine savings with a discounted utility objective functional is not the long-run measure sought in considering sustainability.

According to the generalized concept of genuine savings formalized by Asheim (2007), non-negative net investment, accounted at the shadow values of a given welfare function, is associated with non-decreasing welfare at the current time. If the welfare function is denoted by $W(X)$, the associated shadow values are $\frac{\partial W(X)}{\partial X_i}$, and generalized genuine savings is defined as $\sum_{i=1}^n \frac{\partial W(X)}{\partial X_i} \dot{X}_i$. When welfare is defined as discounted utility, i.e., $W(X) \equiv V(X) = \max_{[c(\cdot)]} \int_t^\infty U(X(s), c(s)) e^{-\delta(s-t)} ds$, for a constant utility discount rate δ , the shadow values are $\frac{\partial V(X)}{\partial X_i}$, and genuine savings correspond to the usual genuine savings indicator. Maximizing discounted utility though time, however, does not require non-negative investment. There is thus no normative reason to pursue a non-negative net investment when welfare is defined as discounted utility.

In the study of maximin value, the generalized concept of genuine savings corresponds to net maximin investment as in Definition 1. Non-negative investment at maximin prices is a property of the maximin approach.²³ Pursuing non-negative investment at maximin

²²The comprehensive vector of capital stocks accounted for in the genuine savings approach is the same as the vector of capital stocks used to define the maximin value. The value of each stock is, however, different.

²³The objective of a maximin problem can mathematically be expressed as the maximization of the Hamiltonian $H(X, c, \mu) \equiv \sum_{i=1}^n \mu_i \dot{X}_i$, subject to the constraint $U(X, c) \geq m(X)$ (Cairns and Long, 2006). The maximin problem is thus tantamount to maximizing the net investment at maximin shadow values, i.e., $M(X, c)$, subject to the constraint that consumption is no less than the maximin value. By Lemma 1, maximin decisions are such that $U(X, c) \geq m(X)$ and $M(X, c) \geq 0$.

prices, even in a sub-optimal economy, is consistent with sustainability and with the optimality concept of maximin.

Genuine savings measures welfare change. It relies on assumptions about the economic path being followed and hence on the resource-allocation mechanism, be it optimal or non-optimal. In contrast, net maximin investment measures the change in the *ability to sustain* economic well-being. Given the current state X , it is independent of the resource-allocation mechanism and of the path the economy follows in the long-run.

We distinguish genuine investment, be it applied to maximized social welfare or the level of welfare generated by a resource-allocation mechanism describing the economy (Dasgupta and Mäler, 2000), from investment calculated from the maximin value by calling the latter maximin or *durable investment* (from the French term *développement durable*).

Durable investment is the indicator of the current change in sustainability. It is comprehensive investment evaluated at maximin shadow prices, along any particular path of the economy. It is the statistic that is appropriate in expressing sustainability improvement. For sustainable *development* at time t the economy must have $M(X(t), c(t)) \geq 0$ and $\frac{dU(X(t), c(t))}{dt} \geq 0$. In this case, current growth does not jeopardize the capacity of future generations to sustain utility.

6 Conclusion

Our discussion stresses a property of a growth path that is not stressed by proponents of sustainable development out of poverty. If the maximin path is not pursued, but instead some growth path is followed, then earlier generations must be deprived in order to divert toward investment the resources needed to development. Growth is possible only at a cost, and only within limits given by the technology and the environment. Otherwise, it can cause overshooting.

Our contribution to the literature on sustainability is to use the evolution of the maximin value as an indicator of sustainability and of sustainability improvement along any development path (efficient or not, optimal or not). The maximin value is a dynamic

environmental and economic indicator of the prospect for sustainable growth, which, when non-decreasing, indicates sustainability improvement.

The definition of durable savings is valid for the inefficient economies of reality. Durable savings must be evaluated at the maximin shadow values to measure sustainability improvement. How to get the “maximin prices” is a difficult question, even in simple models. The difficulty is no reason to use genuine savings with discounted utilitarian prices to measure long-term sustainability. This practice can be misleading and send an incorrect message, as genuine savings can be positive even if current utility exceeds the maximal sustainable utility and the maximin value indicator is decreasing.

The indicator of sustainability on any program, optimal or not, is the maximin value. Durable investment, the change in the maximin value, is the indicator of whether or not the level of well-being that *can* be sustained is increasing or decreasing.

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