Optimal Government Debt Maturity*

PRELIMINARY AND INCOMPLETE

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Abstract

This paper develops a model of optimal government debt maturity in which the government cannot issue state-contingent bonds and the government cannot commit to fiscal policy. In contrast to an environment with full commitment, there is a tradeoff between the cost of funding and the benefit of hedging. Borrowing long term provides the government with a hedging benefit since the value of outstanding government liabilities declines when short-term interest rates rise. However, borrowing long term lowers fiscal discipline for future governments unable to commit to policy, which leads to higher future short-term interest rates. Therefore, lack of commitment ex post increases the government’s cost of borrowing long term ex ante. A consequence of this tradeoff is that the slope of the yield curve is increasing in the maturity of newly issued debt. Our main theoretical result is that, as in the case of full commitment, the optimal maturity structure of government debt is tilted to the long end, but it is more flat than in case of full commitment. Our quantitative analysis shows that, in contrast to debt positions under full commitment—which involve a short-term asset position and a long-term debt position, both extremely large relative to GDP—debt positions under lack of commitment are positive at all maturities, much smaller relative to GDP, and have a nearly flat maturity.

Keywords: Public debt, optimal taxation, fiscal policy, consumption and savings

JEL Classification: H63, H21, E62, E21

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1 Introduction

How should government debt maturity be structured? Two seminal papers by Angeletos (2002) and Buera and Nicolini (2004) argue that the maturity of government debt can be optimally structured so as to completely hedge the economy against shocks. Using some examples, they show that optimal debt maturity is tilted long, with the government purchasing short-term assets and selling long-term debt. This allows the market value of outstanding government liabilities to decline when short-term interest rates rise. Moreover, quantitative exercises imply that optimal government debt positions, both short and long, are large (in absolute value) relative to GDP.

In this paper, we show that these conclusions are sensitive to the assumption that the government can fully commit to fiscal policy. In practice, a government chooses taxes and debt sequentially, taking into account its current portfolio of debt holdings at any date, as well as the behavior of future governments. We show that once the lack of commitment by the government is taken into account, it becomes costly for the government to use the maturity structure of debt to completely hedge the economy against shocks; there is a tradeoff between the cost of funding and the benefit of hedging. Our main theoretical finding is that the optimal maturity structure of government debt, while still tilted to the long end, is more flat than under full commitment. Moreover, quantitatively, the maturity structure is nearly flat with positive short-term and long-term debt positions which are significantly smaller (in absolute value) in comparison to the case of full commitment.

We present these findings in the spender-saver model of Mankiw (2000).1,2 This is an endowment economy in which some fraction of households participate in the government bond market ("saver households") while another fraction of households do not participate in the bond market ("spender households"). Spender households experience shocks to the marginal value of their consumption and the government utilizes lump sum taxes and debt in order to smooth out their consumption. In this environment, if the government issues more bonds, these bonds are purchased by the saver households who require a higher return to be induced to purchase the bonds. Thus, a natural positive relationship between debt issuance and yields emerges.

Our model features two important frictions. First, we assume that state-contingent bonds are unavailable, and that the government can only issue non-contingent bonds of all maturities. Second, we assume that the government lacks commitment to policy, so that the government dynamically chooses its policies at every date as a function of payoff relevant variables: the state of the economy and its debt position at various maturities. As is well-known from previous

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1 We use this model of public debt as opposed to the more commonly used model of Lucas and Stokey (1983) for technical reasons. As is shown in Krusell et al. (2006), introducing lack of commitment into the Lucas and Stokey (1983) model—even in the absence of shocks and long maturities—is complicated by the presence of discontinuities in equilibria, making quantitative analysis of the infinite horizon economy very challenging.

2 The main results in Angeletos (2002) and Buera and Nicolini (2004)—who use the framework of Lucas and Stokey (1983)—continue to hold in the spender-saver model. Analogously, our main results continue to hold quantitatively in the environment of Lucas and Stokey (1983) in a three period economy. Details available upon request.
work, neither of these frictions on their own lead to any inefficiency. First, the work of Angeletos (2002) and Buera and Nicolini (2004) shows that, even in the absence of contingent bonds, an optimally structured portfolio of non-contingent bonds can perfectly insulate the government from all shocks to the economy. Second, the work of Lucas and Stokey (1983) shows that, even in the absence of commitment by the government, an optimally structured portfolio of contingent bonds can perfectly induce a government without commitment to pursue the ex-ante optimally chosen policy ex post.

While each of these two frictions in isolation is irrelevant, the combination of the two leads to a non-trivial tradeoff between market completeness and commitment in the government’s choice of maturity. The more tilted is the government’s debt position towards the long end, the higher the insurance against economic shocks. However, the more flat the government’s debt position, the more committed the government is to its ex-ante optimal plan. To get a sense of this tradeoff, let us consider the benchmark cases which only feature one friction.

Suppose that the government has full commitment but only has access to non-contingent bonds. In this case, the government can use the rich maturity structure of the government bonds to create insurance claims which fully insulate it from economic shocks. Specifically, the government’s optimal choice of maturity is tilted towards the long end, and this maturity choice guarantees that the total value of outstanding government liabilities declines whenever short-term interest rates rise and the government budget becomes tighter.

Now consider the alternate benchmark case in which the government lacks commitment, but there are no shocks, so that the government does not need to worry about insurance. In this case, the ex-ante optimal policy is perfectly smooth taxation. A government today can guarantee commitment to this policy by future governments by choosing a flat maturity structure, so that the lack of commitment does not impose any additional inefficiency. A tilted debt position, however, would cause a future government to deviate from the optimal smooth path. If, for example, a future government chooses policy while holding only long-term debt, then it has an incentive to cut taxes and increase debt issuance ex post. This action increases ex-post short-term interest rates, which benefits the government by reducing the market value of its liabilities, but this hurts long-term debt holders. In contrast, if a future government chooses policy while holding only short-term debt, then it has an incentive to raise taxes and reduce debt issuance ex post. This action reduces ex-post short-term interest rates, which benefits the government by reducing the cost of rolling over short-term debt, but this hurts short-term debt holders. Therefore, only a flat maturity structure can guarantee that taxes remain smooth since the government does not have any beneficial deviations ex post in this case (i.e., any deviation’s marginal effect on the market value of long-term debt is outweighed by the marginal effect on the rollover cost of short-term debt).

Our main theoretical result is that, under non-contingent bonds and lack commitment, the optimal maturity of government debt is tilted long, but is more flat in comparison to the case of full commitment. This result emerges from the tradeoff previously described above. Full
insurance with a debt position highly tilted to the long end is too expensive for the government. Suppose that the government were to choose a highly tilted debt position ex ante. Saver households purchasing government bonds ex ante would internalize the fact that ex post, the government will have lower fiscal discipline and will choose lower taxes and higher debt (relative to under commitment), thereby diluting the claims of long-term debt holders. Saver households therefore require a higher yield ex ante for buying long-term debt relative to short-term debt (since short-term debt can be rolled over at higher interest rates). Therefore, borrowing primarily long term can be very expensive. More generally, the slope of the yield curve is increasing in the maturity of newly issued bonds, and this is because the private sector internalizes the lack of commitment by the government. Taking this fact into account, the government chooses a flatter maturity structure than under full commitment. The fact that the maturity structure is still tilted long follows from the fact that some hedging continues to be beneficial.

We additionally provide some quantitative results. We find that, in contrast to the case of full commitment, under lack of commitment, the optimal maturity structure of government debt is nearly flat, and government debt positions at the short and long end are both positive. Furthermore, we find that the absolute magnitude of these debt positions are significantly smaller when compared to the case of full commitment. More specifically, in our simulated economy, we consider an environment in which the government issues a one-year bond and a console. In our benchmark simulation, we find that under full commitment, the short-term bond is -124% of GDP the market value of the console is 165% of GDP, with annual payouts equal to 7% of GDP. In contrast, under lack of commitment, the short-term bond is 2.5% of GDP and the market value of the console is 61% of GDP, with annual payouts equal to 2.5% of GDP. Thus, the optimal maturity structure is essentially flat.

This result is intuitive and follows from the fact that completing the market—which is done under full commitment—requires very large positions relative to the size of the economy. This fact, which is also present in Angeletos (2002) and Buera and Nicolini (2004), is due to the reality that interest rates are not sufficiently volatile so as to allow full hedging with a small position. Such enormous positions, however, exacerbate the problem of lack of commitment, which means that such positions are extremely expensive to maintain. More generally, the cost of lack of commitment significantly exceeds the benefits of hedging, and for this reason, optimal policy involves a nearly flat maturity structure.

This paper is connected to several literatures. As discussed, we build on the work of Angeletos (2002) and Buera and Nicolini (2004) by introducing lack of commitment. In this regard, our

This prediction is consistent with findings in the empirical finance literature (e.g., Greenwood and Vayanos, Forthcoming). Note that this prediction is not due to the presence of segmented markets (e.g., Greenwood et al., 2010)—in our setting savers have access to debt of all maturities—but rather it is due to the lack of commitment to fiscal policy.

Additional work explores government debt maturity while continuing to maintain the assumption of full commitment. Shin (2007) explores optimal debt maturity when there are fewer debt instruments than states. Faraglia et al. (2010) explore optimal debt maturity in environments with habits, productivity shocks, and capital. Lustig et al. (2008) explore the optimal maturity structure of government debt in an economy with nominal
work is related to that of Arellano and Ramanarayanan (2012) and Aguiar and Amador (2013), but in contrast to this work, we ignore the possibility of default and focus purely on commitment to taxation and debt issuance. Our work is also complementary to that of Arellano et al. (2013), but in contrast to this work, we ignore the presence of nominal frictions and the lack of commitment to monetary policy. In this regard, our work is most applicable to economies in which the risks of default and surprise inflation are not salient. Moreover, it should be noted that the absence of these risks in our framework implies that the maturity structure of government debt is not tilted towards the short end as it is in this other work.

More broadly, our paper is also tied to the literature on optimal fiscal policy which explores the role of non-contingent debt and lack of commitment. A number of papers have studied optimal policy under full commitment but non-contingent debt, such as Barro (1979) and Aiyagari et al. (2002). As in this work, we find that optimal taxes respond persistently to economic shocks, though in contrast to this work, this persistence is due to the lack of commitment by the government as opposed to the ruling out of long-term government bonds. Other work has studied optimal policy in settings with lack of commitment, but with full insurance (e.g., Debortoli and Nunes, 2013 and Krusell et al., 2006). We depart from this work by introducing long-term debt, which in a setting with full insurance implies that the lack of commitment friction no longer introduces any inefficiencies.

Our paper proceeds as follows. In Section 2, we describe our baseline model. In Section 3, we provide our main theoretical result using a three-period example. In Section 4, we provide our main quantitative results. Section 5 concludes and the Appendix provides additional results not included in the text.

2 Model

2.1 Economic Environment

We consider an environment analogous to the spender-saver model of Mankiw (2000). Some fraction of households called "spenders" live hand-to-mouth so that their consumption equals their disposable income. Another fraction of households called "savers" participate in the government bond market and can trade government bonds of all maturities. In every period, the government issues bonds and chooses lump sum taxes applied uniformly to all households. Spender households are subject to a shock to their marginal utility of consumption. As such, the purpose of fiscal policy in this framework is to effectively transfer resources across household types. For example, if spenders’ marginal utility of consumption is high (low), the government can reduce rigidities.

5See also Farhi (2010) and Shin (2007).
6Chari and Kehoe (1993a,b) and Sleet and Yeltekin (2006) also consider the lack of commitment under full insurance, though they focus on settings which allow for default. Alvarez et al. (2004) consider problems of commitment in an deterministic environment with long-term debt where the possibility of surprise inflation arises.
(raise) taxes to increase (decrease) spender’s consumption, and it can finance this change in taxes by issuing more (fewer) bonds purchased by saver households.

More formally, consider an economy with discrete time periods \( t = \{0, 1, \ldots \} \) and a stochastic state \( \theta_t \in \Theta \) which follows a first-order Markov process. \( \theta_0 \) is given. There is a continuum of mass 1 of households. Mass \( \lambda \in (0, 1) \) of households are spenders with preferences

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t \theta_t u^p (c_t^p), \quad \beta \in (0, 1),
\]

where \( c_t^p \) represents the consumption of spenders at \( t \) and \( u^p(\cdot) \) is increasing and weakly concave. Spender households have a constant endowment \( y^p \) and are subject to lump sum taxes \( \tau_t \geq 0 \), so that their consumption in every period \( t \) satisfies

\[
c_t^p = y^p - \tau_t. \tag{2}
\]

Mass \( 1 - \lambda \) of households are savers with preferences

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u^r (c_t^r)
\]

where \( c_t^r \) represents the consumption of savers and \( u^r(\cdot) \) is increasing and weakly concave. Saver households have a constant endowment \( y^r \) are subject to lump sum taxes \( \tau_t \), and can trade in the government bond market. A saver household enters every period \( t \) with a portfolio of bonds \( \{b_{t-1}^{t+j}\}_{j=0}^{\infty} \), where \( b_{t-1}^{t+j} \leq 0 \) represents a bond purchased at \( t-1 \) which matures at \( t+j \). At date \( t \), bonds \( b_{t-1}^{t} \) mature, saver households can buy or sell \( t+j \) maturing bonds at price \( q_{t+1}^{t+j} \), and they choose their consumption \( c_t^r \) subject to their budget constraint:

\[
c_t^r = y^r - \tau_t + \sum_{j=1}^{\infty} q_{t+1}^{t+j} \left( b_{t-1}^{t+j} - b_t^{t+j} \right) + b_{t-1}^t. \tag{4}
\]

The government enters every period \( t \) with a portfolio of government bonds \( \{B_{t-1}^{t+j}\}_{j=0}^{\infty} \), where \( B_{t-1}^{t+j} \geq 0 \) represents a bond sold at \( t-1 \) which matures at \( t+j \). Following the realization of \( \theta_t \), the government repays its immediate liabilities \( B_{t-1}^t \), it buys or sells \( t+j \) maturing bonds at price \( q_{t+1}^{t+j} \), and it chooses lump sum taxes subject to the government budget constraint:

\[
\tau_t = \sum_{j=1}^{\infty} q_{t+1}^{t+j} \left( B_{t-1}^{t+j} - B_t^{t+j} \right) + B_{t-1}^t. \tag{5}
\]

The economy is closed, so that the bonds issued by the government must be purchased by
the saver households, implying that for all \( t \) and \( t + j \):

\[
B_{t}^{t+j} = (1 - \lambda) b_{t}^{t+j}.
\] (6)

Equations (2), (4), (5), and (6) imply the resource constraint

\[
\lambda c_{t}^{p} + (1 - \lambda) c_{t}^{r} = \lambda y^{p} + (1 - \lambda) y^{r}.
\] (7)

The initial level of bonds \( \{ B_{j=1}^{j=1} \}^{\infty}_{j=1} \) is given.

It is useful to note that a key friction in this environment is the absence of state-contingent debt, since the value of outstanding debt \( B_{t}^{t+j} \) is independent of the realization of \( \theta_{t+j} \). If state contingent bonds were available, then at any date \( t \), the government would own a portfolio of bonds \( \{ \{ B_{t}^{t+j}(\theta^{t+j}) \}_{\theta^{t+j} \in \Theta^{t+j}} \}_{j=0}^{\infty} \), where the face value of each bond payout at date \( t + j \) would depend on the realization of a history of shocks \( \theta^{t+j} \in \Theta^{t+j} \). In our discussion, we will refer back to this complete market case.

### 2.2 Political Environment

The government has preferences

\[
\psi \mathbb{E} \sum_{t=0}^{\infty} \beta^{t} \theta_{t} w^{p}(c_{t}^{p}) + (1 - \psi) \mathbb{E} \sum_{t=0}^{\infty} \beta^{t} w^{r}(c_{t}^{r}),
\]

so that \( \psi \in [0, 1] \) corresponds to the relative weight that it assigns to spender households. The government cannot commit to policies. More specifically, in every period, nature determines \( \theta_{t} \), the government chooses policies \( \{ \tau_{t}, \{ B_{t}^{t+j} \}_{j=1}^{\infty} \} \), and households choose their consumption given policies and bond prices.

### 2.3 Markov Perfect Competitive Equilibrium

A Markov Perfect Competitive Equilibrium corresponds to a stochastic consumption and debt sequence \( \{ c_{t}^{p}, c_{t}^{r}, \{ b_{t}^{t+j} \}_{j=1}^{\infty} \}_{t=0}^{\infty} \), a stochastic policy sequence, \( \{ \tau_{t}, \{ B_{t}^{t+j} \}_{j=1}^{\infty} \}_{t=0}^{\infty} \), and a stochastic bond price sequence \( \{ q_{t}^{t+j} \}_{j=1}^{\infty} \}_{t=0}^{\infty} \). This sequence must be competitive, so that it satisfies the following conditions:

1. \( \{ c_{t}^{p} \}_{t=0}^{\infty} \) is determined by the spender budget constraint (2).

2. \( \{ c_{t}^{r}, \{ b_{t}^{t+j} \}_{j=1}^{\infty} \}_{t=0}^{\infty} \) maximizes saver welfare (3) subject to the saver budget constraint (4).

3. \( \{ \tau_{t}, \{ B_{t}^{t+j} \}_{j=1}^{\infty} \}_{t=0}^{\infty} \) satisfies the government budget constraint (5).
4. (6) is satisfied so that markets clear.

In addition, the government must optimally choose its preferred policy at every date. In this environment, government policies at any date $t$ can be determined by backward induction. Specifically, at every date $t$, the government chooses policies as a function of the state $\theta_t$ and the portfolio of debt $\{B_{t-1}^{j+k}\}_{t=0}^{\infty}$ with which it enters the period. It takes into account that its choice affects future debt and thus affects the policies of future governments. Saver households rationally anticipate these future policies, and their expectations are in turn reflected in current bond prices.

3 Three-Period Example

Given the complexity of solving for Markov Perfect Competitive Equilibrium, we focus our theoretical analysis on a simple three-period example. This example highlights the tradeoffs between insurance and commitment which emerge in this framework. Let $t = \{0, 1, 2\}$ and let the shock process satisfy

$$\theta_0 \geq 1 + \delta,$$
$$\theta_1 = \{1 - \delta, 1 + \delta\} \text{ with equal probability, and}$$
$$\theta_2 = 1$$

for some $\delta \in [0, 1)$ representing the volatility of the shock. In addition, let us assume that $\lambda = 1/2$ so that groups have equal size, and that $\psi = 1$ so that the government only values the welfare of the spenders. Finally, let us assume that $u^p(c^p_t) = c^p_t$ and $u^r(c^r_t) = \log c^r_t$, and that initial debt positions are zero.

In this environment, the government has financing needs at date 0 since spenders have high marginal utility of consumption at that date. The government can finance these needs with short-term and with long-term debt, and in doing so, it must take into account that it faces some risk at date 1. In particular, if $\theta_1 = 1 + \delta$, it will again have some financing needs at date 1, and it could potentially face a rollover crisis if interest rates are high in that period.

Given that there is only a single realization of uncertainty at date 1, we refer to an allocation in this setting as $\alpha$, where

$$\alpha = \left\{ c^j_0, c^j_1(\theta_1), c^j_2(\theta_1) \right\}_{\theta_1=1-\delta,1+\delta}^{\theta_1=1-\delta,1+\delta}_{p,r},$$

where consumption at dates 1 and 2 depend on the realization of $\theta_1$.

Our main result is that there is a tradeoff between insurance and commitment in this framework and that this tradeoff emerges purely from the interaction between limited commitment and market incompleteness. In order to make this case, we begin by considering the two benchmark
cases of full commitment and no uncertainty. After we establish that no inefficiency emerges in these two settings, we show how the interaction of the two frictions leads to a tradeoff.

3.1 Equilibrium under Full Commitment

We begin by considering the problem of the government under full commitment. To this end, note that optimality on the side of the savers implies that bond prices must satisfy

\[ q_1^0 = \beta \mathbb{E} \left[ \frac{1}{c_1^r (\theta_1)} \right] c_0^r, \quad q_2^0 = \beta^2 \mathbb{E} \left[ \frac{1}{c_2^r (\theta_1)} \right] c_0^r, \quad \text{and} \quad q_1^2 (\theta_1) = \beta \left[ \frac{1}{c_1^r (\theta_1)} \right] c_1^r (\theta_1) \forall \theta_1, \]  

which represent the Euler equation for all traded bonds, and where \( q_1^2 (\theta_1) \) represents short-term interest rates conditional on \( \theta_1 \)'s realization. Combining constraints (4), (5), and (6) with (8), we achieve the following conditions:

\[
\begin{align*}
0 &= \left( y^r - c_0^r \right) + \beta \mathbb{E} \left( y^r - c_1^r (\theta_1) \right) + \beta^2 \mathbb{E} \left( y^r - c_2^r (\theta_1) \right), \quad \text{and} \quad (9) \\
0 &= \left( y^r + B_0^1 - c_1^r (\theta_1) \right) + \beta \left( y^r + B_0^2 - c_2^r (\theta_1) \right) \forall \theta_1. \quad (10)
\end{align*}
\]

Standard arguments (e.g., Lucas and Stokey, 1983) imply that any stochastic consumption allocation \( \alpha \) and debt policy \( \{B_0^1, B_0^2\} \) which satisfy (7), (9), and (10) can be implemented as a competitive equilibrium, and that any competitive equilibrium must satisfy (7), (9), and (10). This observation allows us to use the primal approach by choosing \( \alpha \) and \( \{B_0^1, B_0^2\} \) directly to maximize social welfare.

Thus, using (7) to substitute in for \( c_0^r \) in the government’s objective, the problem of a government with full commitment can be written as

\[
\min_{\alpha, B_0^1, B_0^2} \left\{ \theta_0 c_0^r + \beta \mathbb{E} [\theta_1 c_1^r + \beta c_2^r] \right\} \text{ s.t. } (9) \text{ and } (10).
\]  

Proposition 1 (full commitment) Under full commitment and non-contingent bonds, the solution to (11) is

\[
\begin{align*}
c_t^r &= \frac{y^r \mathbb{E} \left[ \sum_{t=0,1,2} \beta^t \theta_t^{1/2} \right]}{\theta_t^{1/2} + \beta + \beta^2} \forall t, \quad (12) \\
B_0^1 &= -y^r, \quad \text{and} \quad (13) \\
B_0^2 &= y^r \left( \frac{1 + \beta \mathbb{E} \left[ \sum_{t=0,1,2} \beta^t \theta_t^{1/2} \right]}{1 + \beta + \beta^2} - 1 \right), \quad (14)
\end{align*}
\]

and constraint (10) is slack.
Corollary 1 (irrelevance of market incompleteness) Under full commitment, real allocations and welfare when bonds are state-contingent coincide with those when bonds are non-contingent.

Proposition 1 states that in the solution to the government’s problem under commitment, the level of consumption of the savers and the spenders depends only on the state of the economy $\theta_t$. More specifically, when $\theta_t$ is high, the consumption of the spenders is also high, so that taxes are low. Therefore, at date 2, taxes are independent of the history of shocks.

Corollary 1 states that the solution to the government’s problem is the exact same as in an economy in which state-contingent bonds are available. To see why, note that under complete markets, constraint (10) can be ignored, since the debt liabilities of the government at date 1 can depend on the realization of $\theta_1$, thus making (9) the only necessary constraint that must be satisfied so as to guarantee the satisfaction of all dynamic budget constraints of the government. Moreover, one can verify that the solution to (11) which ignores (10) also yields (12). This implies that the constraint of incomplete markets does not impose any additional inefficiency in an environment with commitment.

This result is similar to that of Angeletos (2002) and Buera and Nicolini (2004), and it follows from the fact that there are as many debt maturities as the number of shocks, so that the space of uncertainty can be fully spanned. Moreover, we find that the types of debt positions that the government takes in order to insure itself are similar as in their work. The government takes a short asset position and a long debt position, so that the structure of government debt issuance is very tilted. The reason such a tilted debt position is optimal is that the value of the net debt position of the government declines when the spending needs of the government are high. More specifically, when $\theta_1 = 1 + \delta$, spending needs are high so that taxes are low and the consumption of the spenders is high. Given that there are finite resources, the consumption of the savers must in turn be low, so that short-term interest rates between dates 1 and 2 are high. These high interest rates imply that the price of outstanding long-term debt is low, leaving more resources for the government to spend at date 1.

Moreover, note that the quantitative magnitudes of the debt positions chosen in this stylized example can in principle be very large. For example, if the endowment of the savers $y_s$ exceeds that of the spenders $y_p$, then the short asset position of the government $-B_{10}$ and the long debt position $B_{20}$ both exceed the total output in the economy. The observation that the debt positions required to complete the market can be large in our simple example is consistent with the results in Angeletos (2002) and Buera and Nicolini (2004) who argue that these positions are quantitatively large—and perhaps excessively so—in a fully dynamic environment.

3.2 Equilibrium under Full Insurance

We now consider another benchmark which is the problem of the government under full insurance in an environment in which there is no government commitment. To simplify the discussion, we
let $\delta = 0$ so that there is no uncertainty at date 1, and we assume at that $\theta_0 > 1$ so that there are financing needs for the government at date 0.\footnote{An analogous exercise can be performed in the case in which there are shocks but state contingent bonds are available. We consider the deterministic environment to simplify the discussion.}

In order to solve this problem in the absence of government commitment, we utilize backward induction. Given $B_0^1$ and $B_0^2$, the government at date 1 chooses policies to solve

$$
\min_{c_1^r, c_2^r, B_1^1, B_1^2} \{c_1^r + \beta c_2^r\} \quad \text{s.t.} \\
0 = \left( \frac{y^r + B_0^1 - c_1^r}{c_1^r} \right) + \beta \left( \frac{y^r + B_0^2 - c_2^r}{c_2^r} \right).
$$

(16)

where we have utilized analogous reasoning as in the construction of (11) to substitute in for the objective function and write the implementability condition (16). The solution to (15) yields

$$
c_t^r (B_0^1, B_0^2) = \left( \frac{y^r + B_0^t}{\theta_t} \right)^{1/2} \frac{1}{1 + \theta_t} \sum_{t=0,1,2} \beta^{t-1} \theta_t^{1/2} (y^r + B_0^t)^{1/2} \quad \text{for } t = 1, 2.
$$

(17)

Now consider the problem of the government at date 0. As we previously discussed, the absence of any risk implies that constraint (10) is redundant and implied by the satisfaction of (9). Thus, the date 0 government solves

$$
\min_{\alpha, \{B_0^1, B_0^2\}_{t \in \Theta}} \{\theta_0 c_0^r + \beta E \theta_1^r \} \quad \text{s.t.} \quad (9) \text{ and } (17),
$$

(18)

where (17) takes into account that the government internalizes its impact on the decisions of the future government.

**Proposition 2 (full insurance)** In a deterministic economy with $\delta = 0$ and no-commitment, the solution to (18) is

$$
c_t^r = \frac{y^r}{\theta_t^{1/2}} \frac{E \left[ \sum_{t=0,1,2} \beta^t \theta_t^{1/2} \right]}{1 + \theta_t + \theta_t^2} \quad \forall t
$$

(19)

$$
B_0^1 = B_0^2 = y^r \left( \frac{E \left[ \sum_{t=0,1,2} \beta^t \theta_t^{1/2} \right]}{1 + \theta_t + \theta_t^2} - 1 \right),
$$

(20)

and constraint (17) is slack.

**Corollary 2 (irrelevance of lack of commitment)** In a deterministic economy, real allocations and welfare when the planner has commitment coincide with those when the planner lacks commitment.
Proposition 2 characterizes the solution, and shows that it has the feature that policies are smooth from date 1 onward since there are no shocks. As stated in Corollary 2 the allocation in a deterministic economy and no commitment is identical to that under full commitment. This result is similar to that of Lucas and Stokey (1983) who show that even in an environment with no commitment, the lack of commitment imposes no additional efficiency if there are as many debt instruments as there are decision-making nodes for the government. In this particular environment, one can see that a flat maturity structure (i.e., a maturity structure in which short debt issuance equals long debt issuance) implies that the solution under commitment can be implemented in the absence of commitment.

To gain an insight as to why a flat debt maturity structure provides full commitment, consider a counterfactual scenario in which all date 0 government borrowing is long, so that  \( B_0^1 = 0 \) and  \( B_0^2 > 0 \). Under full commitment, it would be possible to implement the optimum with such a maturity structure, since the government at date 1 can perfectly commit to repurchasing a fixed amount of debt and issuing a fixed amount of short-term debt so as to keep (19) satisfied.

Under limited commitment, however, this is no longer the case, since the ex-post optimal consumption at date 1 is defined according to (17), which only coincides with (19) if debt is chosen optimally according to the optimal policy (20). More specifically, if all date 0 borrowing is long, a comparison of (17) relative to (19) given that  \( B_0^1 = 0 \) and  \( B_0^2 > 0 \) implies that the date 1 government will tilt consumption of spenders much more so into date 1 relative to date 2. Intuitively, the date 1 government prefers to deviate from the commitment solution by issuing more short-term debt than the date 0 government would prefer. The date 1 government does this since doing so increases short-term interest rates at date 1, and this happens because the increased bond issuance must be purchased by the savers who reduce their date 1 consumption relative to their date 2 consumption. This increase in interest rates translates into a reduction in the value of outstanding date 2 liabilities, which benefits the date 1 government. From the date 0 government’s perspective, however, this deviation is costly, since it is perfectly anticipated by the saver households at date 0 who now require a higher premium for lending long. These saver households realize that short-term interest rates at date 1 are going to be higher than anticipated under full commitment, and they therefore require a higher interest rate at date 0 to compensate them for purchasing long-term versus short-term bonds, since short-term bonds could be rolled over at date 1 at this higher interest rate.

An analogous argument holds if instead all date 0 borrowing is short, so that  \( B_0^1 > 0 \) and  \( B_0^2 = 0 \). In this case, the date 1 government will tilt consumption of spenders much more so towards date 2 relative to date 1. It does this since the implied reduction in short-term debt issuance at date 1 reduces short-term interest rates at date 1, which lowers the cost of rolling over short-term debt for the date 1 government. From the date 0 government’s perspective, however, this deviation is costly, since it is perfectly anticipated by the saver households at date 0 who now require a higher premium for lending short. This effect occurs because savers realize that short-term interest rates at date 1 are going to be lower than anticipated under full
commitment, and they therefore require a higher interest rate at date 0 to compensate them for short-term relative to long-term lending.

As such, a flat maturity structure fixes these incentive problems by making any ex-post deviation by the date 1 government not profitable. More specifically, if the government issues more short-term debt ex post, any benefit it achieves by diluting long-term debt is fully outweighed by the additional cost of rolling over short-term debt. Analogously, if the government issues less short-term debt ex post, any benefit it achieves by reducing the cost of rolling over short-term debt is outweighed by the additional cost of buying back long-term debt. The private sector anticipates that the government cannot deviate ex post, and ex-ante, the flat maturity structure minimizes the funding costs of the government.

3.3 Equilibrium under Limited Commitment and Incomplete Markets

Propositions 1 and 2 illustrate that the constraints of non-contingent debt and no commitment by the government on their own impose no efficiency loss for the economy. In the case of non-contingent debt, full insurance can be achieved by a government choosing a heavily imbalanced debt position which is tilted towards the long end. In the case of no commitment but full insurance, the current government can induce future governments to choosing its prefered policy by choosing a flat debt maturity structure. We now consider an environment in which the two frictions interact and we show that this leads to a tradeoff between insurance and commitment.

As in the previous subsection, we characterize the equilibrium by backward induction. By analogous reasoning as in the construction of (15), the government at date 1 solves the following problem:

\[
\min_{c_1^r,c_2^r,B_1^1} \{ \theta_1 c_1^r + \beta c_2^r \} \]

s.t.
\[
0 = \left( \frac{y^r + B_0^1 - c_1^r}{c_1^r} \right) + \beta \left( \frac{y^r + B_0^2 - c_2^r}{c_2^r} \right),
\]

and the solution implies that consumption at dates 1 and 2 satisfy:

\[
c_t^r (\theta_1, B_0^1, B_0^2) = \left( \frac{y^r + B_0^t}{\theta_t} \right)^{1/2} \sum_{t=1,2} \beta^{t-1} \theta_t^{1/2} \left( y^r + B_0^t \right)^{1/2} \frac{1}{1 + \beta} \text{ for } t = 1, 2. \tag{21}
\]

Substitution of the above solution into the government’s objective function implies that the expected continuation value to the government from date 1 onward can be written as

\[
V (B_0^1, B_0^2) = -\frac{1}{1 + \beta} E \left( \sum_{t=1,2} \beta^{t-1} \theta_t^{1/2} (y^r + B_0^t)^{1/2} \right)^2.
\]

12
Moreover, substitution of (21) into (9) implies that consumption at date 0 satisfies
\[
c_0^r(B_0^1, B_0^2) = \frac{y^r}{1 + \beta + \beta^2 - y^r - \frac{\beta}{1 + \beta} \sum_{t=1,2} \beta^{t-1} q_t^{1/2} (y_t + B_0^t)^{-1/2}}.
\]
It thus follows that one can write period zero bond prices as a function of short debt issuance \(B_0^1\) and the long debt issuance \(B_0^2\):

\[
q_0^1(B_0^1, B_0^2) = \beta c_0^r(B_0^1, B_0^2) \mathbb{E} \left[ \frac{1}{c_1^r(B_0^1, B_0^2)} \right]
\]
and
\[
q_0^2(B_0^1, B_0^2) = \beta^2 c_0^r(B_0^1, B_0^2) \mathbb{E} \left[ \frac{1}{c_2^r(B_0^1, B_0^2)} \right].
\]

Using this notation, and substituting (2), (4), (5), and (6) into the government’s objective function, the date 0 problem of the government can be written as
\[
\max_{B_0^1, B_0^2} \left\{ \theta_0(q_0^1(B_0^1, B_0^2) B_0^1 + q_0^2(B_0^1, B_0^2) B_0^2) + \beta V(B_0^1, B_0^2) \right\}.
\]
The first term in the objective function captures the benefit at date 0 from borrowing whereas the second term captures the cost from date 1 onward of repaying the debt, taking into account the possibility of facing financing needs again at date 1. To get a sense of the tradeoffs faced by the government, it is useful to see how the bond prices \(q_0^1(B_0^1, B_0^2)\) and \(q_0^2(B_0^1, B_0^2)\) depend on the level of bond issuance at date 0.

**Lemma 1 (bond prices)** \(q_0^1(B_0^1, B_0^2)\) and \(q_0^2(B_0^1, B_0^2)\) are continuously differentiable functions with the following properties:

1. **(debt issuance and yields)** \(q_0^1(B_0^1, B_0^2)\) and \(q_0^2(B_0^1, B_0^2)\) decrease in both \(B_0^1\) and \(B_0^2\),

2. **(debt issuance and yield curve)** \(q_0^1(B_0^1, B_0^2) / q_0^2(B_0^1, B_0^2)\) decreases in \(B_0^1\) and increases in \(B_0^2\), and

3. **(debt maturity, yields, and yield curve)** \(q_0^1(B_0^1, B_0^2)\) decreases and \(q_0^2(B_0^1, B_0^2)\) increases if \(B_0^1\) increases and \(B_0^2\) decreases so as to keep \(c_0^r(B_0^1, B_0^2)\) constant.

The first part of Lemma 1 states that interest rates at all maturities increase in government borrowing at all maturities. To get a sense of the intuition for this result, note that from (21), the consumption of the savers at dates 1 and 2 increases with government borrowing at all maturities. This is because an increase in government borrowing represents an increase in the future wealth of savers. By constrast, this increase in consumption for savers at dates 1 and 2 implies a reduction in their consumption at date 0 according to equation (9). Intuitively,
The savers must be reducing their date 0 consumption in order to purchase the newly issued government bonds. As such, from (22) and (23), this implies that the yields at all maturities must rise.

The second part of Lemma 1 states that the slope of the yield curve, here parameterized by the ratio of the short bond price to the long bond price, is decreasing in short debt issuance and increasing in long debt issuance. In other words, although all bond prices are decreasing in debt issuance by the first part of the lemma, the extent to which they do so depends in part on how the maturity structure of government debt is also changing. This result builds on the fact that the private sector at date 0 prices the bonds taking into account the actions of the government at date 1. From (21), while an increase in short-term debt $B^1_0$ raises savers’ consumption at dates 1 and 2, it increases date 1 consumption by more than date 2 consumption. The rationale is that the government ex post at date 1 sees a benefit to limiting short-term debt issuance and tilting savers’ consumption towards date 1, which reduces short-term interest rates at date 1. In contrast, an increase in $B^2_0$ raises savers’ consumption at date 2 by more than at date 1. The reason is that the government ex post at date 1 sees a benefit to increasing short-term debt issuance and tilting savers’ consumption towards date 2, which increases short-term interest rates at date 1.

The third part of this lemma generalizes this result to a case in which the maturity structure is altered while keeping total debt issuance constant. In addition, the third part of the lemma also determines the movements in the prices of long and short-term bonds. It basically implies that, holding total borrowing fixed, a government effectively changes the relative interest it pays for bonds of different maturities by altering the maturity structure. So in other words, even though issuing a more tilted maturity structure can provide more insurance, it is also more costly since it entails a much higher interest rate on long-term bonds.

To get a sense mathematically of the second and third results in Lemma 1, define

$$\kappa = \frac{y^r + B^2_0}{y^r + B^1_0}.$$  \hspace{1cm} (25)

$\kappa$ parameterizes the maturity of the issued bonds at date 0, with higher values of $\kappa$ denoting a longer maturity of public debt. Substitution into (22) and (23) implies that

$$q^1_0 \left( B^1_0, B^2_0 \right) = \frac{1}{\beta} \kappa^{1/2} \mathbb{E} \left[ \frac{\theta^{1/2}_1}{\theta^{1/2}_1 + \beta \kappa^{1/2}} \right] / \mathbb{E} \left[ \frac{1}{\theta^{1/2}_1 + \beta \kappa^{1/2}} \right],$$ \hspace{1cm} (26)

so that the effective slope of the yield curve depends only on $\kappa$ and not on the total level of debt issuance. Moreover, one can show that that this ratio increases in $\kappa$.

Consider for example the government’s debt maturity in the case of full commitment described in Proposition 1. In this case, $\kappa = \infty$ since $B^1_0 = -y^r$, so that debt maturity is maximally tilted. If a government without commitment were to attempt this maturity struc-
ture, it would imply that $q_1^0 (B_{0,1}, B_{0,2}) / q_0^2 (B_{0,1}, B_{0,2}) = \infty$, so that the yield curve is maximally tilted. Intuitively, saver households anticipate zero consumption and infinite short-term interest rates at date 1, and by no arbitrage, this means that the ratio of long-term rates to short-term rates at date 0 is infinite.

Consider instead if the government’s debt maturity were chosen to be flat as in the case of complete markets described in Proposition 2 with $\kappa = 1$. In this scenario, $q_1^0 (B_{0,1}, B_{0,2}) / q_0^2 (B_{0,1}, B_{0,2})$ is finite, and with a magnitude which depends on the variance of $\theta_1$. For examples, if $\delta = 0$ so that there is no uncertainty regarding the value of $\theta_1$, the yield curve is effectively flat with $q_1^0 (B_{0,1}, B_{0,2}) / q_0^2 (B_{0,1}, B_{0,2}) = \beta^{-1}$.

We now provide the main result of the paper.

**Proposition 3 (limited commitment and incomplete markets)** Under limited commitment and incomplete markets, the solution to (24) satisfies $\kappa \in (1, \infty)$.

Proposition 3 states that in the presence of limited commitment and incomplete markets, the optimal debt maturity chosen by the government is tilted towards the long end, so that $B_{0,2} > B_{0,1}$, but not to the same extent as under full commitment. So in other words, the optimal maturity $\kappa$, is chosen to be in between that chosen under full insurance (i.e., $\kappa = 1$) and full commitment (i.e., $\kappa = \infty$). Intuitively, if $\kappa = 1$, the marginal value of hedging achieved by tilting the maturity structure towards the long end exceeds the additional marginal cost of financing generated by increasing the term structure of interest rates. If instead $\kappa = \infty$, the cost of limited commitment is extremely high, since saver households anticipate immiseration at date 1, which makes short-term interest rates at date 0 to be negative infinity and therefore implies that borrowing more short term is cheaper on the margin for the government.

**4 Quantitative Exercise**

In this section, we consider the quantitative implications of our model in an infinite horizon economy. In such an economy, the government at every date $t$ makes a fiscal policy decision as a function of the state $\theta_t$ and the entire portfolio of outstanding bond holdings $\left\{ B_{t+j} \right\}_{j=0}^{\infty}$. Since the state space is infinite, this problem is very complicated to compute. As such, we reduce the set of tradeable bonds in a manner analogous to the work of Woodford (2001) and Arellano et al. (2013). Namely, we consider an economy with two types of bonds: a one-period bond and a perpetuity with decaying coupons.\(^8\)

Let $b_{t-1,1}^S \geq 0$ denote the value of the one-period bond purchased by the savers at $t - 1$. Moreover, let $b_{t-1,1}^L \geq 0$ denote the value of the per period coupon associated with the perpetuity purchased by the savers at $t - 1$. It follows then that the budget constraint of the saver households

---

\(^8\)None of the results of the three-period economy change if we constrain the set of tradeable bonds to a one-period bond and a perpetuity.
(4) is now replaced by

\[ c^*_t = y^* - \tau_t - q_t^S b^S_t + q_t^L (\gamma b^L_{t-1} - b^L_t) + (b^S_{t-1} + b^L_{t-1}) \].

(27) takes into account that at date \( t \), saver households receive a flow payoff \((b^S_{t-1} + b^L_{t-1})\) from their portfolio of one-period bonds and perpetuities; they purchase one-period bonds \( b^S_t \) at price \( q_t^S \); and they exchange their non-decayed perpetuities \( \gamma b^L_{t-1} \) for new perpetuities \( b^L_t \) at price \( q_t^L \), where \( \gamma \in (0, 1) \) is the decay rate. The government’s budget constraint (5) is analogously replaced with

\[ \tau_t = -q_t^S B^S_t + q_t^L (\gamma B^L_{t-1} - B^L_t) + (B^S_{t-1} + B^L_{t-1}) \],

and the market clearing condition (6) is replaced with

\[ B^S_t = (1 - \lambda) b^S_t \quad \text{and} \quad B^L_t = (1 - \lambda) b^L_t. \]  

Note that according to this formulation, a scenario with \( \gamma = 0 \) is equivalent to an economy with only one-period bonds, and a scenario with \( \gamma = 1 \) is equivalent to an economy in which consoles are available.

It is straightforward to see that in this environment, optimal portfolio allocation decisions by savers imply that the Euler equations from the three-period model (8) are now replaced with:

\[ q_t^S = \beta E \left[ \frac{u'^r(c^r_{t+1})}{u'^r(c^r_t)} \right] \quad \text{and} \quad q_t^L = \beta E \left[ \frac{u'^r(c^r_{t+1})}{u'^r(c^r_t)} \right] \left(1 + \delta q_t^L \right). \]  

Moreover, (27), (28), and (29) can be combined to yield:

\[ c^*_t = y^* + \frac{\lambda}{1 - \lambda} \left[ -q_t^S B^S_t + q_t^L (\gamma B^L_{t-1} - B^L_t) + (B^S_{t-1} + B^L_{t-1}) \right]. \]  

Note that resource constraint (7) together with (30) and (31) imply that the choice of taxes \( \{\tau_t\}_{t=0}^\infty \) can be ignored since a stochastic sequence \( \{c^r_t, c^r_{t+1}, q_t^S, q_t^L\}_{t=0}^\infty \) is uniquely pinned down by a stochastic policy sequence \( \{B_{t-1}\}_{t=0}^\infty \) where \( B_{t-1} = (B^S_{t-1}, B^L_{t-1}) \).

We assume that \( \theta_t = \{1 + \delta, 1 + \delta\} \), with \( \Pr \{\theta_{t+1} = \theta_t\} = \rho \). As such, analogous arguments to those of the three-period model imply that in this environment, there is generically no in-efficiency due to the absence of contingent bonds if the government has full commitment. The argument is analogous to that of Angeletos (2002) and Buera and Nicolini (2004). Namely, since there are two states of the world and two securities, so the entire space of uncertainty can be spanned.

Now let us consider the case of lack of commitment and focus on an equilibrium in which the government does not commit but instead chooses policies sequentially. In a Markov Perfect Equilibrium, at every date \( t \), the government chooses \( B_t \) given the state \( (\theta_t, B_{t-1}) \), the behavior of households, and the policies of future governments. More specifically, a Markov Perfect Equi-
equilibrium consists of four functions: $V(\theta_t, B_{t-1})$, $h^B(\theta_t, B_{t-1})$, $h^C(\theta_t, B_{t-1})$, and $h^L(\theta_t, B_{t-1})$ which satisfy the following conditions:

1. $V(\theta_t, B_{t-1})$ satisfies

$$V(\theta_t, B_{t-1}) = \max_{c_t^r, c_t^l, B_t} \{\psi \theta_t u^p(c_t^r) + (1 - \psi) u^s(c_t^l) + \beta \mathbb{E}[V(\theta_{t+1}, B_t) | \theta_t]\}$$ (32)

s.t. (7), (30), (31),

$$c_{t+1}^r = h^C(\theta_{t+1}, B_t),$$ and $$q_{t+1}^L = h^L(\theta_{t+1}, B_t).$$ (33)

2. $h^B(\theta_t, B_{t-1})$ corresponds to the value of $B_t$ which maximizes the objective in $V(\theta_t, B_{t-1})$ given $h^C(\theta_t, B_{t-1})$ and $h^L(\theta_t, B_{t-1})$.

3. $h^C(\theta_t, B_{t-1})$ corresponds to the value of $c_t^r$ which satisfies (30) and (31) given $B_t = h^B(\theta_t, B_{t-1})$, $c_{t+1}^r = h^C(\theta_{t+1}, h^B(\theta_t, B_{t-1}))$, and $q_{t+1}^L = h^L(\theta_{t+1}, h^B(\theta_t, B_{t-1}))$.

4. $h^L(\theta_t, B_{t-1})$ corresponds to the value of $q_t^L$ which satisfies (30) given $c_t^r = h^C(\theta_t, B_{t-1})$, $c_{t+1}^r = h^C(\theta_{t+1}, h^B(\theta_t, B_{t-1}))$, and $q_{t+1}^L = h^L(\theta_{t+1}, h^B(\theta_t, B_{t-1}))$.

As it holds in the three-period economy, we focus on an equilibrium with differentiable policy functions. To get a sense of the Markov Perfect Equilibrium, consider a deterministic economy. In this case, the optimal policy under commitment requires a smooth path of consumption for spenders and savers. Analogous arguments to those made in our three-period example as well as in the work of Lucas and Stokey (1983) imply that in this circumstance, the optimal policy under commitment can be implemented under lack of commitment with the government issuing debt with a flat maturity structure. In our environment with a one-period bond and a long-term perpetuity, this is only possible if $\gamma = 1$, so that the perpetuity does not depreciate. Given this fact, and given that we are interested in looking at inefficiencies which arise only from the interaction of incomplete markets and lack of commitment, we focus our quantitative exercise on the case with $\gamma = 1$. Our benchmark simulation makes the following parametric assumptions. We let $\beta = 0.96$ so that a period is interpreted as representing a year, with a riskless rate of 4% in a deterministic

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9 We solve for the Markov Perfect Equilibrium by looking for an equilibrium in which the functions $V$, $h^B$, $h^C$, and $h^L$ are all differentiable. Our solution approach finds a fixed point using an iteration which utilizes first order conditions using these functions. We cannot prove that this Markov Perfect Equilibrium is unique, though our iterative procedures always generate the same policy functions independently of our initial guesses in the iteration.

10 This is always true if initial debt liabilities do not have an infinite maturity.

11 One can make such an argument using backward induction, using a finite horizon economy with $T$ periods as $T \to \infty$.

12 If $\gamma < 1$, then analogous arguments to those of Debortoli and Nunes (2013) – who analyze a deterministic economy with a one-period bond – imply that the government debt positions are driven towards zero. In Appendix Table A-1, we provide a simulation of an economy in which $\gamma = 0.5$ and show that this is the case. We also provide an example with a one-period and a two-period bond and show that the same result holds.
We consider CRRA preferences with \( u^p(c^p) = \left( \frac{(c^p)^{1-\sigma^p} - 1}{(1 - \sigma^p)} \right) \) and \( u^r(c^r) = \left( \frac{(c^r)^{1-\sigma^r} - 1}{(1 - \sigma^r)} \right) \). To start, we assume the same individual and social preferences as in the three-period model with \( \sigma^p = 0 \), \( \sigma^r = 1 \), and \( \psi = 0 \). We will show that our main conclusions are robust to alternative choices of \( \sigma^r \), \( \sigma^p \), and \( \psi \). Based on evidence in the of Survey of Consumer Finances, we assume that \( \lambda = 0.4 \), so that 40% of households are hand to mouth and we let \( y^p = 0.5y^r \) so that these households have 50% of the per capital income of the saver households.\(^{13}\) We choose endowments so that the aggregate endowment is normalized to 1. We choose initial conditions \( B^{S}_{1} = 0.1625 \) and \( B^{L}_{1} = 0.0175 \) to match the maturity structure of government debt in the United States from 1988 to 2013.\(^{14}\) Finally, we focus on the case where shocks are i.i.d. so that \( \rho = 0.5 \), and choose the variance of the shock process \( \delta = 0.1 \) so that our benchmark simulation generates the same standard deviation in net borrowing as in the United States over the same sample period. We perform several robustness checks to show that our results are also not driven by a particular choice for \( \delta \).

\[ \text{Table 1: Statistics under Baseline Parameter Values} \]

<table>
<thead>
<tr>
<th>Variables</th>
<th>Statistics</th>
<th>No Commitment</th>
<th>Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-Term Debt</td>
<td>Mean</td>
<td>0.025</td>
<td>-1.240</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.014</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.945</td>
<td>–</td>
</tr>
<tr>
<td>Long-Term Debt (annuity)</td>
<td>Mean</td>
<td>0.025</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.014</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.945</td>
<td>–</td>
</tr>
<tr>
<td>Long-Term Debt (market value)</td>
<td>Mean</td>
<td>0.608</td>
<td>1.654</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.347</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.898</td>
<td>-0.009</td>
</tr>
<tr>
<td>Total Debt</td>
<td>Mean</td>
<td>0.633</td>
<td>0.414</td>
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<td></td>
<td>Std. Dev</td>
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<td>0.080</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.901</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

Our main results from the benchmark simulation are in Table 1 where we compare various quantities in our benchmark simulation in the case of commitment to the case of no commitment. Under full commitment, in steady state, the average value of short-term bonds, \( B^{S} + B^{L} \), is -124% of GDP, and the average market value of long-term bonds, \( q^{L}B^{L} \), is 165% of GDP, with annuity payouts, \( B^{L} \), equal to 7% of GDP.\(^{15}\) In contrast, under lack of commitment, in steady state, the average value of short-term bonds is 2.5% of GDP, and the average market value of

\(^{13}\)We calibrate these parameters to match the percentage of families that saved and their income relative to non-savers as shown in the Survey of Consumer Finances (2012). Campbell and Mankiw (1989, 1990) and the literature discussed in Mankiw (2000) use different methodologies and find similar estimates.

\(^{14}\)This is done by applying a 4% discount rate to determine the value of initial annuities. These initial conditions roughly match the US statistics with a total level of debt of 60% of GDP, out of which 28% is short-term debt and 72% is long-term debt.

\(^{15}\)We define short-term bonds at \( t \) as representing the value of all payouts to savers due at \( t \) and long-term bonds as the value of payouts to savers due in periods after \( t \).
long-term bonds is 61% of GDP, with annuity payouts equal to 2.5% of GDP. Thus, the optimal maturity structure is essentially flat.\footnote{Our numerical analysis does suggest that the steady state distribution of the one-period bond and the console depends on the initial levels of debt. This is consistent with the case of the deterministic economy (in which the full commitment solution can be enforced) where the steady state admits zero one-period bonds and a constant level of the console which depends on initial debt. Regardless of the initial conditions the maturity structure is essentially flat.} We find some additional interesting contrasting features between the two cases. In the case of full commitment, the market value of total debt essentially follows an i.i.d. process, like the underlying shock process, and this is because full insurance is possible. In contrast, under lack of commitment, the market value of total debt is persistent, and this is because the government actively manages its debt positions in response to shocks since it cannot achieve perfect insurance. Finally, note that the total value of debt is higher under no commitment versus full commitment. This follows from the fact that the optimal maturity structure is flat, and in this regard, having higher levels of debt allows for more insurance since the value of the net liabilities of the government then fluctuate more.

Figures 1 and 2 perform some robustness checks on our main result. Figure 1 displays the level of short-term bonds and the annuity value in the long-term bonds under full commitment. Panel (a) varies the risk aversion of the spender households $\sigma^p$; panel (b) varies the risk aversion of the saver households $\sigma^s$; panel (c) varies the welfare weight on the spender households $\psi$; and panel (d) varies the volatility of the shock $\delta$. In all cases, the main conclusion from previous research is confirmed: The maturity structure of government bonds is heavily tilted towards the long end and the value of debt positions are large relative to GDP. Figure 2 performs the same exercise under lack of commitment and finds that our main conclusions are robust. In all cases, the optimal maturity structure under no commitment is close to flat and positions are small relative to GDP.

Figures 1 and 2 also display some interesting comparative statics. Panels (a) and (b) show that increasing the risk aversion of either the spender or saver households essentially decreases the need for insurance and therefore decreases the overall size of debt positions and the tilting of the maturity structure in both cases of full commitment and lack of commitment. If spender households are more risk averse, then this increases the government’s desire to smooth the spender’s consumption and therefore smooth deficits. As a consequence, the value of the government’s liabilities should fluctuate less, implying that smaller positions are required. If saver households are more risk averse, then bond prices respond more to shocks, and this implies that smaller debt positions are required to achieve insurance.

Panel (c) shows that that changing the welfare weight so that savers enter more evenly into the welfare function increases overall debt positions and the tilting of the maturity structure in both the cases of full commitment and lack of commitment. This is because in this circumstance, larger debt and higher tilting leads to a higher cost of hedging. These higher interest rates correspond to a higher level of welfare for saver households who are now weighted more heavily in the welfare program.
Figure 1: Debt Positions – Commitment

Notes: The figure shows short-term debt, \(B^S + B^L\) and long-term debt, \(B^L\), as a function of parameters in the model with commitment. Panel (a) varies the risk aversion of the spender households \(\sigma^p\); panel (b) varies the risk aversion of the saver households \(\sigma^r\); panel (c) varies the welfare weight on the spender households \(\psi\); and panel (d) varies the variance of the shock \(\delta\).

Panel (d) explores the impact of changing the variance of the shock process. An increase in \(\delta\) increases the level and tilting of debt towards the long end under both cases of full commitment and lack of commitment, and this is because it increases the desire for insurance, since the benefit of having volatile surpluses increases. In all cases, the slope of the maturity structure remains relatively flat.\(^{17}\)

In sum, these numerical exercises confirm that our conclusions from the three-period economy are robust. Debt positions under full commitment are large and heavily tilted to the long end, but this is not true under lack of commitment. Moreover, under lack of commitment, the optimal maturity structure is nearly flat. This result is intuitive and follows from the fact that completing the market—which is done under full commitment—requires very large positions relative to the

\(^{17}\)In the extreme case where the slope is the most tilted with \(\delta = 0.4\), the standard deviation of net borrowing is about 28%, while in the US data 1988-2013 it is about 3.5%.
Figure 2: Debt Positions – No Commitment

Notes: The figure shows short-term debt, \( B^S + B^L \) and long-term debt, \( B^L \), as a function of parameters in the model with lack of commitment. Panel (a) varies the risk aversion of the spender households \( \sigma^p \); panel (b) varies the risk aversion of the saver households \( \sigma^r \); panel (c) varies the welfare weight on the spender households \( \psi \); and panel (d) varies the variance of the shock \( \delta \).

size of the economy. This fact, which is also present in Angeletos (2002) and Buera and Nicolini (2004), is due to the reality that interest rates are not sufficiently volatile so as to allow full hedging with small position. Such enormous positions, however, exacerbate the problem of lack of commitment, which means that such positions are extremely expensive to maintain. More generally, the cost of lack of commitment significantly exceeds the cost of volatility, and for this reason, optimal policy involves a nearly flat maturity structure.\footnote{The conclusion that the welfare benefit of smoothing economic shocks is small is more generally tied to the insight in Lucas (1987).}
5 Conclusion

The current literature on optimal government debt maturity concludes that the government should fully insulate itself from economic shocks. This full insulation is accomplished by choosing a maturity heavily tilted towards the long end with a short-term asset position and a long-term debt position, both extremely large relative to GDP. In this paper, we show that these conclusions strongly rely on the assumption of full commitment by the government. Once lack of commitment is taken into account, then full insulation from economic shocks becomes impossible; the government faces a tradeoff between hedging and the cost of funding. Borrowing long term provides the government with a hedging benefit since the value of outstanding government liabilities declines when short-term interest rates rise. However, borrowing long term lowers fiscal discipline for future governments unable to commit to policy, which leads to higher future short-term interest rates. We show through a series of exercises that the optimal debt maturity structure under lack of commitment—which still somewhat tilted to the long end—is nearly flat, with the government actively managing its debt in response to economic shocks.

While our quantitative exercise suggests that a government should be able to achieve the optimal allocation under no commitment with a flat maturity, it is useful to note that, in practice, government debt is often tilted towards the short end. For example, in the case of the United States, between 1988 to 2013, 28% of outstanding liabilities had a maturity below one year. A natural question concerns whether this observed behavior is simply suboptimal and can be improved by policy reform, or if this policy choice is a response to an additional friction other than lack of commitment which make it beneficial for the government to issue more short-term bonds than long-term bonds. An interesting avenue for future research is to consider the interaction of the lack of commitment friction with additional economic frictions (such as credit frictions in financial markets, for example) and to explore what these frictions imply for the optimal maturity structure of government debt.
References


Appendix

A-1 Proofs

Proof of Proposition 1

Let us assume and later verify that constraint (10) does not bind. It is straightforward to show that the solution to (11) which ignores (10) admits $q_t^r$ which satisfies (12). One can check that $B_0^1$ and $B_0^2$ which satisfy (13) and (14) satisfy (10), which verifies that the constraint does not bind.

Proof of Proposition 2

Consider a deterministic economy with $\delta = 0$. Let us assume and later verify that constraint (17) does not bind. It is straightforward to show that the solution to (18) which ignores (17) admits $c_t^r$ which satisfies (19). One can check that $B_0^1$ and $B_0^2$ which satisfy (20) satisfy (17), which verifies that the constraint does not bind.

Proof of Lemma 1

We begin by proving part (i). From (21), $c_t^r (\theta_1, B_0^1, B_0^2)$ is strictly increasing in $B_0^1$ and $B_0^2$ for $t = 1, 2$. This implies from (9) that $c_0^r$ must be strictly decreasing in $B_0^1$ and $B_0^2$. From (22) and (23), this implies that $q_0^1 (B_0^1, B_0^2)$ and $q_0^2 (B_0^1, B_0^2)$ are decreasing in both $B_0^1$ and $B_0^2$.

Before proving part (ii), we prove part (iii). Let $B_0^1$ increase and $B_0^2$ decrease so as to keep $c_0^r (B_0^1, B_0^2)$ constant. From (9), it must be that either $c_0^1 (\theta_1, B_0^1, B_0^2)$ and $c_0^2 (\theta_1, B_0^1, B_0^2)$ are both unchanged, or that one quantity increases whereas the other quantity decreases. Suppose that $c_0^1 (\theta_1, B_0^1, B_0^2)$ weakly decreases so that $c_0^2 (\theta_1, B_0^1, B_0^2)$ weakly increases conditional on $\theta_1$. From (21), this would require that $(y_t + B_0^1) / (y_t + B_0^2)$ weakly decreases conditional on $\theta_1$, leading to a contradiction. Therefore, $c_0^1 (\theta_1, B_0^1, B_0^2)$ strictly increases and $c_0^2 (\theta_1, B_0^1, B_0^2)$ strictly decreases conditional on $\theta_1$, implying from (22) and (23) that $q_0^1 (B_0^1, B_0^2)$ decreases and $q_0^2 (B_0^1, B_0^2)$ increases.

We now move to proving part (ii). Note that (21) combined with (22) and (23) implies that

$$
\begin{align*}
\frac{q_0^1 (B_0^1, B_0^2)}{q_0^2 (B_0^1, B_0^2)} &= \frac{\mathbb{E} \left[ \frac{1}{c_0^1 (\theta_1, B_0^1, B_0^2)} \right]}{\mathbb{E} \left[ \frac{1}{c_0^2 (\theta_1, B_0^1, B_0^2)} \right]} \\
&= \kappa^{1/2} \left( \frac{\mathbb{E} \left[ \frac{\theta_1^{1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right]}{\mathbb{E} \left[ \frac{1}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right]} \right),
\end{align*}
$$

where $\kappa$ is defined in (25). (A–1) implies that $q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2)$ depends only on $\kappa$,
and not on the value of \( c_0^1 (B_0^1, B_0^2) \). Suppose it were the case that \( q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2) \) weakly increases if \( B_0^1 \) increases for some \((B_0^1, B_0^2)\). This would imply from \((A - 1)\) that \( q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2) \) is weakly decreasing in \( \kappa \) for the associated \((B_0^1, B_0^2)\) (since an increase in \( B_0^1 \) decreases \( \kappa \)). However, from part (iii) proved above, starting from the associated \((B_0^1, B_0^2)\), any increase in \( B_0^1 \) and decrease in \( B_0^2 \) chosen so as to keep \( c_0^1 (B_0^1, B_0^2) \) constant must strictly decrease \( q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2) \) and decrease \( \kappa \) (by definition). Therefore, \( q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2) \) in \((A - 1)\) cannot be weakly decreasing in \( \kappa \) for the associated \((B_0^1, B_0^2)\), since this a contradiction. Analogous reasoning implies that \( q_0^1 (B_0^1, B_0^2) / q_0^2 (B_0^1, B_0^2) \) is increasing in \( B_0^2 \).

**Proof of Proposition 3**

Note that (24) can be rewritten as a function of \( B_0^1 \) and \( \kappa \):

\[
\min_{B_0^1, \kappa} \left\{ \theta_0 y^r \left[ 1 + \beta + \beta^2 - y^r \frac{1}{1 + \beta} \frac{1}{y^r + B_0^1} \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{-1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right) \right] \right\}^{-1}
\]

which yields the following first order conditions:

\[
B_0^1 : \theta_0 [c_0^1 y^2] \frac{\beta}{1 + \beta} \frac{1}{y^r + B_0^1} \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{-1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right) = \frac{\beta}{1 + \beta} \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right)^2
\]

\[
\kappa : -\theta_0 [c_0^1 y^2] \frac{\beta}{1 + \beta} \frac{1}{y^r + B_0^1} \frac{d}{d \kappa} \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{-1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right) = \frac{\beta}{1 + \beta} \frac{d}{d \kappa} \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right)^2
\]

which imply that the optimal debt maturity \( \kappa \) necessarily satisfies

\[
- \frac{d}{d \kappa} \log \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{-1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right) = \frac{d}{d \kappa} \log \mathbb{E} \left( \frac{\theta_1^{1/2} + \beta \kappa^{1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \right)^2.
\]

We first prove that the optimal value of \( \kappa \) does not equal \( \infty \). The value of the objective to minimize in \((A - 2)\) equals \( \infty \) if \( \kappa = \infty \), whereas it equals a finite number for any other positive level of \( \kappa \). Therefore, the maximizer cannot be \( \kappa = \infty \).

We now show that the solution also cannot admit \( \kappa \leq 1 \). Suppose that this were the case. Define \( H (\theta_1, \kappa) = \theta_1^{1/2} + \beta \kappa^{1/2}, F (\theta_1, \kappa) = \frac{(1 + \kappa)}{2} \theta_1^{1/2} + \beta \kappa^{1/2} \) and \( G (\theta_1, \kappa) = \frac{\theta_1^{1/2} + \beta \kappa^{1/2}}{\theta_1^{1/2} + \beta \kappa^{1/2}} \) and let us expand \((A - 3)\) to achieve:

\[
\mathbb{E} \left[ \frac{1}{H (\theta_1, \kappa)} \frac{F (\theta_1, \kappa)}{G (\theta_1, \kappa)} \right] = \mathbb{E} \frac{H (\theta_1, \kappa)}{H (\theta_1, \kappa)^2}.
\]
Note that since $H(\theta_1, \kappa)$ has positive variance, one can use Jensen’s inequality to simplify the right hand side of (A – 4) to achieve:

$$
\frac{\mathbb{E}[H(\theta_1, \kappa)]}{\mathbb{E}[H(\theta_1, \kappa)^2]} < \frac{1}{\mathbb{E}[H(\theta_1, \kappa)]} < \mathbb{E}\left[\frac{1}{H(\theta_1, \kappa)}\right]. \tag{A-5}
$$

Moving to the left hand side of (A – 4), note that $\frac{1}{H(\theta_1, \kappa)}$ are both decreasing in $\theta_1$ and therefore have positive covariance, so that

$$
\mathbb{E}\left[\frac{1}{H(\theta_1, \kappa)}F(\theta_1, \kappa)\right] \geq \mathbb{E}\left[\frac{1}{H(\theta_1, \kappa)}\right]\mathbb{E}[F(\theta_1, \kappa)]. \tag{A-6}
$$

Moreover, note that $F(\theta_1, \kappa)/G(\theta_1, \kappa) \geq 1$ for every single realization of $\theta_1$ since $\kappa \leq 1$, implying that

$$
\mathbb{E}[F(\theta_1, \kappa)]/\mathbb{E}[G(\theta_1, \kappa)] \geq 1 \tag{A-7}
$$

(A – 6) and (A – 7) imply that

$$
\mathbb{E}\left[\frac{1}{H(\theta_1, \kappa)}\right] \leq \frac{\mathbb{E}\left[\frac{1}{H(\theta_1, \kappa)}F(\theta_1, \kappa)\right]}{\mathbb{E}[G(\theta_1, \kappa)]}. \tag{A-8}
$$

However, equations (A – 4), (A – 5), and (A – 8) are not compatible with one another.
### A-2 Different Bond Maturities

Table A-1: Statistics with Different Bond Maturities.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Statistics</th>
<th>(\gamma = 0.5)</th>
<th>Two Maturities</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>No Com</td>
<td>Com</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0.010</td>
<td>-1.240</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.063</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.863</td>
<td>-</td>
</tr>
<tr>
<td>Short-Term Debt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-Term Debt (annuity)</td>
<td>Mean</td>
<td>0.017</td>
<td>1.626</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.100</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.670</td>
<td>-</td>
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<tr>
<td>Long-Term Debt (market value)</td>
<td>Mean</td>
<td>0.033</td>
<td>3.005</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.187</td>
<td>0.072</td>
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<tr>
<td></td>
<td>Autocorr.</td>
<td>0.663</td>
<td>-0.009</td>
</tr>
<tr>
<td>Total Debt</td>
<td>Mean</td>
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<td>1.765</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
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</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>0.751</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

Notes: The table shows key statistics for alternative market structures. The first and second column refer to the case with a one-period bond and a long-term perpetuity with \(\gamma = 0.5\). The third and fourth column refer to the case with a one-period bond and a two-period bond.