Emissions Reduction and Profit-Neutral Permit Allocations *

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August, 2014

Abstract

This paper shows that when a regulator implements a market for permits, the number of free allowances ordinarily required to neutralize profits is low. More precisely, the impacts of three parameters are discussed: market structure, demand elasticity and the cap for emissions. This is shown in a model where firms compete "à la Cournot" and the demand function is iso-elastic. Firms use polluting technologies. The regulator implements a market for permits in order to reduce emissions. The paper determines the profit-neutral allocations and the level of reductions that a regulator could implement while it offsets profits' losses. The paper shows that in the cases of either a monopoly or a duopoly with high reductions, the regulator cannot offset the firms' losses. The model is illustrated for the two first phases of the EU-ETS. The percentage of permits which would have offset profits' losses for the first phase of EU-ETS is less than 10%.

Keywords: Pollution permits, Cournot oligopoly, Relative reduction of emissions, Profit-neutral allowances, EU-ETS.
Classification JEL : F18 ; H2 ; L13; L51; Q2.

1 Introduction

The introduction of pollution permits ordinarily decreases firms' profits. Therefore, free allowances may be given to offset profits' losses. The profit-neutral permit allocations are the

*First of all, I would like to thank Roger Guesnerie for the long discussions we had when I started writing this paper. I would like to thank Jean-Pierre Ponssard, Juan-Pablo Montero, John Quah, Robert Ritz, Jean-Marc Bourgeon, Juan Carluccio, Clémence Christin, Anna Creti, Matthieu Glachant, Guy Meunier and Jorge Zamorano-Ford for useful advice and comments. I also thank participants to the PET conference (Indiana, 2011), the Louis-André Gérard Varet conference (Marseille, 2011), JMA (Brest, 2012), AFSE annual conference (Paris, 2012), the EAERE 19th annual conference (Prague, july 2012), Swiss Society of Economics and Statistics Annual Meeting (Neuchatel, 2013), the Adres conference (Marseille, 2012), the Atlantic Workhsop on Energy and Environmental Economics (Toxa, Spain, 2014), the 41st annual congress of European Association for Research in Industrial Economics (Milan, 2014), the 29th annual congress of the European Economic Association (Toulouse, 2014), the Gate-seminar (Lyon, 2014), the lunch seminar in environmental and resource economics at Paris I (Paris, 2011) and the lunch seminar at Ecole Polytechnique (Palaiseau, 2011). Financial support is gratefully acknowledged from the Ecole Polytechnique chair EDF-Sustainable Development and Swiss Re.

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permits given for free such as the profit after regulation is equal to the initial profit. Neutralizing profits rises some redistributive issues since the consumers and the State bear the whole cost of the environmental policy. However, the possibility to offset losses means that the regulator may get the assent of firms to the environmental regulation.

The profit-neutral allowances have been mainly analyzed with general equilibrium and empirical approaches. Indeed, the environmental economic literature has focused on two main issues: (i) Which is the efficiency cost of avoiding profit losses? (ii) Which percentage of permits should be granted for free to compensate firms' losses? If fiscal distortions exist on other taxes (for instance taxes on labor or taxes on capital), the revenue of the selling of permits may be used to curb them. Thus, giving permits for free has an opportunity cost. Furthermore, the required percentage has been analyzed mainly for the whole-economy. Such approaches impede us to differentiate sectors and to analyze the sector-based determinants of profit-neutral allowances. Therefore, the paper tackles the issue in a partial equilibrium framework.

However, pollution permits ordinarily cover oligopolistic sectors such as cement, electricity or iron. This paper considers then an oligopoly subject to a market for permits and determines the conditions under which the regulator can offset profits' losses. The effect of the introduction of a market for permits on profits depends on (i) the market structure, (ii) the characteristics of demand and (iii) the percentage of reduction of emissions. (i) For instance, if competition is perfect and marginal cost is constant, the profits after the regulation remain equal to zero. However, in the monopoly case, the implementation of market for permits generates profit's losses. These two extreme cases show that the profit-neutral policy depends on market structures. (ii) The elastcicty of demand plays a double role. First, Seade (1985)[13] shows that in a Cournot framework, an increase in the marginal cost of all firms can increase all profits, if the elasticity of the slope of demand is sufficiently high. To be more intuitive, when the demand function is isoelastic, this condition is equivalent to a weak elasticity of demand. Second, when the elasticity is quite high, firms are sensible to a variation of the costs. (iii) Furthermore, the more emissions are reduced, the higher the permits price is, and the more profits are altered. This paper determines according to these three previous ingredients the profit-neutral allowances.

The closest paper to this one is Hepburn et al (2012)[6]. They examine the impact of pollution permits on equilibrium emissions, output, price, market concentration, and profits in a generalized Cournot model. They determine a formula for the number of emissions permits that have to be freely allocated to firms to neutralize the profit impact of pollution permits and show that it is related to the Herfindahl index. They consider exogenous permits price. This paper is complementary to Hepburn et al (2012)[6] since it endogeneizes the permits price and determines the profit-neutral policy according to the level of emissions reduction. From a policy viewpoint, this question is crucial since the reduction of emissions implemented depends on firms' lobbying.

In order to make the permits price endogenous, the equilibrium on the market for permits is assessed. This paper assumes that firms are price-takers in this market even if they are price-makers in the markets for products. Indeed, the EU-ETS covers several sectors and more than 11 000 plants. Firms compete "à la Cournot" in the market for products. In order to get explicit results, an isoelastic demand function is assumed.[1] Furthermore, this paper considers that the regulator reduces emissions with a reducing factor, such that the total emission after regulation is equal to the initial emission multiplied by the reducing factor. A low value of this factor denotes obviously an important reduction. The production technology considered here is more flexible and more general than the one assumed in Hepburn et al (2012)[6].[2] The paper also assumes that firms cannot abate pollution emissions otherwise by reducing production. Indeed, the goal of this paper is to determine an upper bound of free allowances for offsetting profits' losses. If abatement technology is available, firms reduce less their quantities and ordinarily suffer less from the introduction of an environmental regulation. This assumption is discussed further in the paper (section 4.3).

First of all, the paper contributes to the theoretical literature on neutral-profit allowances. This paper characterizes the cases in which free allowances are not required on the ground of profits neutrality. Otherwise, the neutral-profits percentage of permits and the neutral-profits grand-fathering are determined for any percentage of reduction of emissions. The paper shows that the number of free allowances ordinarily required to neutralize profits is low. This result is coherent with the literature. The main novelty is to highlight the constraint met by the regulator: free allowances should be lower than the permits put into circulation.[3] Hepburn et al (2012)[6] notice that in some cases an industry may receive more permits than it needs but they cannot determine these cases since they assume an exogenous permits price. The paper shows that in the cases of either a monopoly or a duopoly with high reductions, the regulator cannot offset the firms' losses. This paper determines the level of reductions that a regulator could implement while it offsets profits' losses. This result implies a crucial policy implication; the regulator may then be more ambitious when he chooses the reducing factor if its main constraint is the firms' participation. Moreover, we may also analyze the conditions under which the optimal regulation may be implemented and made acceptable from the firms' point-of-view. Another novelty relative to Hepburn et al (2012)[6] is to consider asymemetric firms and to relax the assumption of positive correlation between marginal costs and polluting factor. This paper shows that relatively the firms the more harmed by the environmental regulation are those with the lowest ratio of marginal cost over polluting factors and should be granted more free allowances. Finally, the paper also analyzes the welfare-optimal emissions reduction. As in the case of an exogenous reducing factor, acceptability is not reachable in case of monopoly. The more numerous firms are, the higher the marginal damage, under which the regulator can implement the optimal reduction of emissions and may offset profits' losses. This result reconciles efficiency with acceptability, which are ordinarily analyzed independently in the literature.

[1] The paper considers a less general framework than Hepburn et al (2012)[6] in order to make the permits price endogenous.

[2] Indeed, Hepburn et al (2012)[6] assumes that marginal cost and polluting factors are positively correlated.

[3] This condition is equivalent to the percentage of permits lower than one hundred and the grand-fathering rate lower than the reducing factor.

Secondly, the paper contributes to the literature on the implementation of pollution permits. This paper determines equilibrium emissions, output, price and profits according to the reducing factor. Usually, the literature on pollution permits focuses on the utilisation of a pollution cap instead of a reducing factor. As far as I know, this paper is the single paper considering a market for permits covering several sectors and reducing emissions from a certain factor of reduction. The approach retained here is the one followed by policy makers, who determine the percentage of reduction of emissions and fix then the pollution caps. Moreover,the paper shows that the repartition of the reduction of emissions between sectors depends on the elasticities of demand and the ratios of marginal costs over polluting factors. This result is crucial to assess how the different sectors are altered by the environmental regulation and also allows us to make the bridge between the partial equilibrium results and the empirical analysis.

Thirdly, this paper contributes also to the empirical literature on the implementation of pollution permits. Empirical studies show that in Europe no more than 50% of permits given for free is enough to get profits neutrality (Demailly & Quirion (2006)[2], Bovenberg & Goulder (2001)[1], Grubb & Neuhoff (2006)[7]). Furthermore, Goulder & al(2010)[5] considers that giving 20% is enough to neutralize the profits of all US industries. The paper considers the two first periods of the EU-ETS, focusing on the three main covered sectors: electricity, steel and cement. The framework retained allows us to consider two different ways of offestting profits'losses. The paper shows that if the policy of distribution of free allowances is sector-based, 10% of permits would have been sufficient for the two first periods of the EU-ETS to offset firms' losses. Moreover, in the cases under which the total emissions are reduced by 10% and 20%, the percentage of total allowances that the regulator should give for free is respectively equal to 9.77 and 11.33. However, if the regulator uses an uniform policy of distribution (the same grand-fathering rate), 20% of permits would then have been necessary. In the cases under which the total emissions are reduced by 10% and 20%, the percentage of total allowances that the regulator should give for free is respectively equal to 23% and 28%. These figures are far from the 99% of permits given for free during the first period and show that the lobbying power of firms is high in Europe.

The remainder of the paper is structured as follows. Section 2 presents the modeling assumptions. Section 3 focuses on the symmetric case. Section 4 presents some extensions. In section 5, the results are illustrated for the two first periods of the EU-ETS. Section 6 concludes the paper.

2 The Model

I now introduce the set-up of the model.

Firms. There are n symmetric firms competing in a market and producing an homogenous good. The production technology is polluting. Let c be the marginal cost and assume that the polluting factor is equal to one: one unit of production generates one unit of pollution.

Firms compete "a la Cournot", simultaneously choosing their quantity to maximize profits.

Consumers. Assume an iso-elastic demand function. Let β be the elasticity of the demand. Firms face a demand given by:

$$P(Q) = \alpha Q^{-\frac{1}{\beta}} \quad with \quad Q = \Sigma_{i=1}^{n} q_i, \tag{1}$$

where α is the market size.

Assumption 1. $\beta > 1/n$. Assumption 1 says that the elasticity is higher than $\frac{1}{n}$ and it is shown below that it ensures the existence of the equilibrium.

Moreover, in order to be realistic enough, assume that the elasticity of demand is lower than 10. Notice that an iso-elastic demand function has an interesting and crucial property for the issue of how profits are altered by the implementation of a regulation. A constant elasticity demand ensures the potential profit-increasing effect of a cost increase, which appears in the general demand framework. This potential profit-increasing effect cannot occur with linear demand. Policy makers ordinarily use linear demand which prevents them from considering this potential profit-increasing effect which has generated an important strand of the IO literature.[4]

Regulation. In order to cut down pollution, the regulator implements a market for permits. A firm has to own a permit in order to pollute one unit. Firms are price-takers in the market for permits. The price of permits is denoted by σ and clears when the supply equals the demand. $Q(\sigma)$ is total production when the price of permits is equal to σ. The goal of the regulator is to reduce emmissions with a reducing factor z, such that

$$Q(\sigma) = zQ(0), \tag{2}$$

where $0 < z < 1$. The total quantity produced when the price of permits is equal to zero $Q(0)$ is the initial production before the regulation. Note that low values of z denote high reduction. The number of permits put into circulation is equal to $zQ(0)$.

3 Profit-neutral allocations in a symmetric case

The regulator distributes free allowances ε_i to firm i and auction the remaining permits. Profits may be written as the sum of the profits in the market for products and the gain due to free allowances.

$$\pi_i(\sigma) = (p(Q) - c - \sigma)\, q_i + \varepsilon_i \sigma.$$

Since allowances are grandfathered, they are only a lump-sum transfer from the regulator to the firms. They do not affect the firms' decisions, because they have no effect on the firms'

[4]See a survey of this literature in Meunier & Nicolaï.

marginal production cost. However, free allowances do increase the firms' profits. The profit-neutral allowances ϵ_i^N are defined as the amount of free allowances that would level out the firms' profits with or without the environmental regulation: $\pi_i(0) = \pi_i(\sigma) + \epsilon_i^N \sigma$.

I demonstrate that the effect of the implementation of a market for permits on profits is of second order whereas the effect of free allowances on profit is of first order. Secondly, the amount of profit-neutral allowances, i.e. the amount of free allowances that would level out the firms' profits with or without the environmental regulation, is determined.

Let me first deal with the profit in the market for products. The perceived marginal cost is equal to the sum of the marginal cost of production and the price of permits, since the polluting factor is equal to one. At symmetric equilibrium, all firms produce the same. The quantities produced, the product price and the mark-up rate are given by:

$$q_i(\sigma) = \frac{1}{n}\left(\frac{\alpha(\beta - 1/n)}{\beta(c+\sigma)}\right)^{\beta}, \qquad p(\sigma) = \frac{c+\sigma}{1-1/(n\beta)}, \qquad \frac{p(\sigma)-c-\sigma}{c+\sigma} = \frac{1}{n\beta - 1}.$$

Quantity decreases with the price of permits and the marginal cost. Furthermore, the passthrough decreases with the number of firms and the elasticity. Moreover, the mark-up decreases with the elasticity β and the number of firms. Since the mark-up rate is constant, the mark-up increases with the price of permits. When the elasticity is sufficiently high ($\beta > 1$), production increases with demand elasticity while the price decreases with this latter. When the elasticity increases, firms reduce the price of products since consumers' demand decreases and consequently firms increase the production.

Let me introduce the equilibrium on the market for permits. The aggregate demand for permits is equal to the total amount of permits firms' need and have not been granted for free, that is, $Q(\sigma) - \Sigma_{i=1}^n \varepsilon_i$. The total supply is the amount of permits that the regulator is ready to sell, that is, $zQ(0) - \Sigma_{i=1}^n \varepsilon_i$. Thus, the perfectly competitive permits market clears when supply equals demand, or:

$$Q(\sigma) - \Sigma_{i=1}^n \varepsilon_i = zQ(0) - \Sigma_{i=1}^n \varepsilon_i \Leftrightarrow Q(\sigma) = zQ(0).$$

Note that the price of permits is independent of the way in which permits are distributed. Free allowances decrease the supply and the demand in the same way. Thus grand-fathered free allowances modify neither the firms' decisions, nor the price of permits.

Lemma 1. *The equilibrium price of permits does not depend on the market structure and increases with the marginal cost and the reducing factor. It is equal to:*

$$\sigma = (z^{-\frac{1}{\beta}} - 1)c. \tag{3}$$

The price of permits decreases with the reducing factor z and increases with the marginal cost. A rise of z denotes that the environmental policy is less stringent and induces an increase in the supply. A rise of the marginal cost generates a lower initial production. The lower the initial quantity is, the higher the cost to reduce one unit is. For the same reasons, the price of permits increases with demand elasticity since the initial production decreases with the latter. Notice that reducing the total production with a factor z is equivalent to implementing a marginal cost equal to $z^{-\frac{1}{\beta}}$ multiplied by the initial marginal cost.[5] The price of permits is equal to the difference between this targeted marginal cost and the initial marginal cost.

The price of permits does not depend on the market structure since the reduction of production is relative and firms are symmetric.[6] Note that the individual gain of firms due to free allowances do not depends on the number of firms. Replacing the permit price by its value, profits may be written as a function of the reducing factor as:

$$\pi_i(z) = \left(\frac{1}{n}\right)^{\beta+1} \left(\frac{\alpha}{\beta}\right)^{\beta} (n\beta - 1)^{\beta-1} \left(z^{-\frac{1}{\beta}} c\right)^{1-\beta}. \tag{4}$$

The following Lemma indicates how profits are altered by the implementation of a market for permits.

Lemma 2. *The introduction of a market for permits leads to:*

(i) The profit increases when the elasticity of demand is weak (< 1), and it decreases otherwise.

(ii) When the elasticity of demand is high (> 1), the profit 's losses for a firm decreases with the number of firms. (the profit's losses $\propto 1/n^2$)

(iii) When the elasticity of demand is high (> 1), the total profits' losses decreases with the number of firms. (the total profit's losses $\propto 1/n$)

Point (i) says that reducing the total production with a reducing factor z is equivalent to multiplying profits by $z^{1-\frac{1}{\beta}}$. Profits increase when the demand elasticity is weak (< 1), and decrease otherwise. This phenomenon is well known from Seade (1985)[13]: when the elasticity is sufficiently low, implementing a tax on production leads to increasing profits.[7] The tax or the price of permits helps firms to coordinate themselves to increase prices. Moreover, the effect on the mark-up prevails on the effect on quantities. In such a case, free allowances should not be given on the ground of the profits neutrality.

Point (ii) says that when the elasticity of demand is sufficiently high (> 1), profits decrease when competition is imperfect. Free allowances are then required to offset losses. For instance, with perfect competition, profits are equal to zero. In the case of a monopoly, the profit decreases with the implementation of the market for permits. The profit 's losses for a firm decrease with the number of firms. The larger the number of firms is, the lower individual profits are and the lower losses are. In fact, the losses are high all the more initial profit is high. Thus, the profit-neutral policy depends on market structure.

[5] The demand function is equal to $P(Q) = \alpha Q^{-\frac{1}{\beta}}$. Thus, in order to reduce production until $zQ(0)$ the price of products should be multiplied by $z^{-\frac{1}{\beta}}$. Since the product price is linear with the marginal cost, the latter sould be equal to $z^{-\frac{1}{\beta}}$ multiplied by the initial marginal cost.

[6] If a cap is implemented instead of a reducing factor, the price of permits would depend on market structures.

[7] According to Seade (1985)[13], profits increase when the elasticity of the slope of demand is higher than two if marginal cost is constant. Note that the elasticity of the slope of demand is constant with an iso-elastic demand function and equal to the inverse of the elasticity.

Point (iii) says that the total profits' losses decrease with the number of firms. Such result is well known in the literature and explains why firms have incentives to go into a cartel. For the same reason as previously, the losses are high all the more initial profits are high.

From now on, let me focus on the second part of the profit, which is the gain due to free allowances. Consider only the case in which the elasticity of demand is sufficiently high ($\beta > 1$) and profits decrease with the price of permits. The profit-neutral allowances level out the firms' profits with or without the environmental regulation(ϵ_i^N).[8] Let $\gamma_p = \frac{n\epsilon_i^N}{Q(\sigma)}$ be the neutral-profits ratio of free allowances.[9] Let $\gamma_{gf} = \frac{n\epsilon_i^N}{Q(0)}$ be the neutral-profits grand-fathering rate.[10] The characteristics of the profit-neutral policy are given by the following proposition.

Proposition 1. *If the elasticity is high (> 1), in order to keep the profits at their levels without regulation, the ratio of free allowances over permits(γ_p) and the grand-fathering rate (γ_{gf}), are given by:*

$$\gamma_p = \frac{1}{n\beta - 1}\left(\frac{z^{-1} - 1}{z^{-\frac{1}{\beta}} - 1} - 1\right), \qquad \gamma_{gf} = \frac{1}{(n\beta - 1)}\left(\frac{1 - z^{1-1/\beta}}{z^{-1/\beta} - 1}\right).$$

Let me first deal with the required ratio of free allowances over permits. The profit-neutral percentage of permits decreases with the reducing factor, $\frac{\partial \gamma_p}{\partial z} < 0$. A rise of the reducing factor denotes that the environmental policy is less stringent and consequently the price of permits is lower. The ratio of free allowances over permits should be higher to offset losses. The profit-neutral percentage of permits decreases with the number of firms. The profits' losses decrease with the number of firms (part two of Lemma 2) while the gain due to free allowances is independent of the number of firms (Lemma 1). Therefore, when firms increase, the required allowances decrease.

Let me give some magnitude orders on the percentage of permits required. When the number of firms is ten, the percentage of reduction equal to 5% and the demand elasticity equal to 2, it is sufficient to give 5.5% of the permits put into circulation for free. Ceteris paribus, when the percentage of reduction is equal to 20%, 5.88% of permits are required. However, for this percentage of reduction,[11] 37.27 % of permits should be given for free in the case of a duopoly and the same demand elasticity.

Like the required percentage of permits, the profit-neutral grand-fathering rate decreases with the number of firms. However, note that the profit-neutral grand-fathering rate increases

[8]

$$\epsilon_i^N = \frac{1}{(n\beta - 1)}\left(\frac{1 - z^{1-1/\beta}}{z^{-1/\beta} - 1}\right) q_i(0) = (\frac{1}{n})^{\beta+1}(\frac{\alpha}{\beta})^{\beta}(n\beta - 1)^{\beta - 1}\left(\frac{1 - z^{1-1/\beta}}{z^{-1/\beta} - 1}\right) c^{-\beta}.$$

[9] The percentage of permits freely given which neutralizes profits is equal to 100 multiplied by γ_p.

[10] Since firms are symmetric, $\gamma_{gf} = \frac{\epsilon_i^N}{q_i(0)}$ and $\gamma_p = \frac{\epsilon_i^N}{q_i(\sigma)}$.

[11] Note that the European Commission plans to reduce by 20% emissions in 2020 according to the level of emissions in 1990. It is given by z=0.8.

with the reducing factor,

$$\frac{\partial \gamma_{gf}}{\partial z} = \gamma_p + z\frac{\partial \gamma_p}{\partial z} > 0.$$

The profit-neutral grand-fathering rate is equal to the reducing factor multiplied by the ratio of free allowances over permits which offset losses. Thus, two effects should be considered when the regulator increases the reducing-factor: (i) increasing the reducing factor induces an increase of the number of permits and consequently an increase of the grand-fathering rate ($\gamma_p \frac{\partial z}{\partial z} > 0$), (ii) increasing the reducing factor induces a decrease of the profits'losses and consequently a decrease of the percentage of permits to give for free ($z\frac{\partial \gamma_p}{\partial z} < 0$). The first effect is of first order while the second is of second order. For this reason, profit-neutral allowances increase with the reducing factor, since they are equal to the inital production multiplied by the profit-neutral grand-fathering rate.

Assume that the grand-fathering rate should be lower than the reducing factor and the ratio of free allowances over permits (γ_p) should be lower than one. Otherwise, firms receive more free allowances than there are permits into circulation. Then, firms have no incitations to reduce pollution and the price of permits is equal to zero. Both conditions are equivalent.[12] When the constraint is not respected, the regulator can not fully offset losses. I determine the conditions underwhich the compensation is possible. From the previous results, the following proposition is deduced:

Proposition 2. *Let $\overline{z}(\beta, n)$ be the reducing factor that the regulator can reach giving all permits for free and neutralizing profits. For each (z, β, n), if $z < \overline{z}(\beta, n)$, the offsetting is not possible. The threshold, for $\beta < 10$ is such that:*

(i) $\frac{\partial \overline{z}}{\partial \beta} > 0$ *and* $\frac{\partial \overline{z}}{\partial n} < 0$.

(ii) When n=1, $\overline{z}(\beta, 1) > 1$.

(iii) When n=2, $0.3 > \overline{z}(\beta, 2)$.

(iv) When $n > 2$, $\overline{z}(\beta, n) < 0.2$.

The regulator cannot offset the losses generated by the implementation of a market for permits in the case of a monopoly. The grand-fathering rate and the percentage of free allowances which offset losses are respectively higher than the reducing factor and one hundred percent. The losses suffered by a monopoly are too large to be compensated.[13] The threshold $\overline{z}(\beta, n)$ decreases with the number of firms. For instance, in the case of a duopoly, for very high reductions of emissions, the regulator cannot fully compensate firms. This result comes from the following two effects: The sum of profits' losses decreases with the number of firms (point (iii) of Lemma 2) while the gain due to free allowances is independent of the number of firms (Lemma 1). Therefore, the more firms are, the higher reductions of emissions may be implemented.

[12]In fact, $\frac{\varepsilon_i}{q_i(0)} < z \Leftrightarrow \frac{\varepsilon_i}{q_i(\sigma)} < 1$.

[13]Hepburn, Quah & Ritz (2012)[6] considers the case of the monopoly and shows that it receives more free allowances it needs permits.

From a policy point of view, this proposition is crucial. Even when the reduction is high, the regulator can offset losses fully when the sectors are not in a monopoly or a duopoly context. Thus, the regulator may be more ambitious when he chooses the reducing factor even if he wants to get the firms' assent. This result has a clear policy implication for policy-mahers who refusee to implement a market for permits anticipating that they will not get the firms' assent. This proposition shows that, except for monopoly, reductions of emissions of 70 percent may be acceptable from the point of view of firms, if all permits are given for free.

Let me show how the distribution of free allowances implemented in Europe (all permits are given for free) generates profits for firms according to elasticities and market structures. Let $z_{\pi_{FA}} = \frac{\pi(\sigma)+q_i\sigma}{\pi(0)}$ be the profit-altering factor when all permits are given for free. It is equal to:

$$z_{\pi_{FA}} = z(1 + n\beta(z^{-1/\beta} - 1)).$$

From the previous equation, the following proposition is deduced.

Corollary 1. *The profit-altering factor, when all permits are given for free, increases with the number of firms.*

The profit-altering factor in such a case increases with the number of firms. The effect on the profit in the market for products (negative in most cases) decreases with the number of firms. The effect on the gain due to free allowances does not depend on the market structure since the price of permits is independent of the number of firms. It comes from the low value of the profit before the implementation. Furthermore, the value of free allowances does not depend on market structures. Corollary 1 allows me to discuss the windfall profits theory. Sijm, Neuhoff & Chen (2006)[14], Grubb & Neuhoff (2006)[7] and Demailly & Quirion (2006)[1] show that the implementation of the EU-ETS has induced a profit increasing effect. They explain this phenomenon focusing on the pass-through and the gain due to free allowances. Corollary 1 shows that the role of the market structure is crucial since it explains the ratio of the value of free allowances over initial profit. Furthermore, the competitive sectors benefit in proportion more than others.

4 Extensions

Until now, I have focused on the regulation of one sector and I have assumed there is no market power on the market for permits. I have implicitely assumed that the different sectors may be analyzed independently. In the following part, I am going to analyze the validity of this assumption. I consider a regulation covering several sectors. Furthermore, I analyze the robustness of the results of the previous section and I assume asymmetric firms. In both cases, due to the different asymetries, the profit-neutral percentage of permits is not anymore an instrument but a consequence. The regulator may use as instrument the grand-fathering rate. Moreover, the paper focuses on two different policies of distribution: an individualized policy for each category and an uniform distribution of free allowances for all firms.

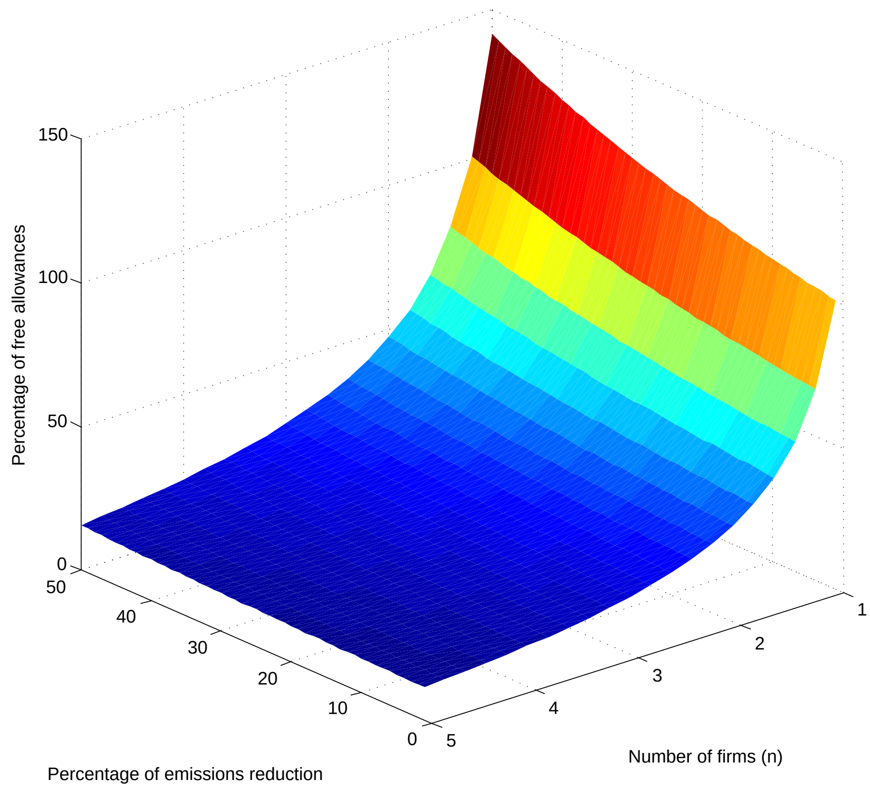

Figure 1: The percentage of free allowances over permits for $\beta = 2$.

4.1 Multi-sector market for permits

Consider, from now on, two sectors called A and B submitted to the same environmental regulation and included in the same market for permits. Inside each sector, all firms are symmetric. A sector j is characterized by the elasticity β_j, size of demand α_j, marginal cost c_j, number of firms n_j and polluting factor f_j. The goal of the regulator is to reduce global emissions by a factor z, such that $f_A Q_A(\sigma) + f_B Q_B(\sigma) = z\left(f_A Q_A(0) + f_B Q_B(0)\right)$. Firms are price-takers on the market for permits. They take into account the price of permits σ. The perceived marginal cost is equal to the sum of the marginal cost of production and the price of permits, since the polluting factor is equal to one. Let $z_A = \frac{Q_A(\sigma)}{Q_A(0)}$ and $z_B = \frac{Q_B(\sigma)}{Q_B(0)}$ be respectively the induced emission-reducing factors of sector A and sector B.

At the symmetric equilibrium, the quantities of a firm i of the sector j produced and the price of the product for the same sector are given by:

$$q_{ij}(\sigma) = \frac{1}{n_j}\left(\frac{\alpha_j(\beta_j - 1/n_j)}{\beta_j(c + f_j\sigma)}\right)^{\beta_j}, \qquad p_j(\sigma) = \frac{c_j + f_j\sigma}{1 - 1/(n_j\beta_j)}. \tag{5}$$

As in Section 3, production decreases with the price of permits and the marginal cost. The passthrough decreases with the number of firms and the elasticity. Moreover, the mark-up decreases with the elasticity and the number of firms.[14] The mark-up increases with the permits price and the polluting factor while the production decreases with the permits price and the polluting factor. An increase of the polluting factor is equivalent to an increase of the permits price. When the elasticity increases (and is higher than one), firms reduce the price of products since the demand decreases and consequently they increase the production.

On the market for permits, the aggregate demand for permits is equal to the total amount of permits firms need and have not been granted for free. Thus, the perfectly competitive permits market clears when supply equals demand, or:

$$\begin{aligned} & f_A Q_A(\sigma) + f_B Q_B(\sigma) = z\left(f_A Q_A(0) + f_B Q_B(0)\right) \\ & \Rightarrow f_A Q_A(0) * (z_A - z) + f_B Q_B(0) * (z_B - z) = 0. \end{aligned}$$

The price of permits is independent of the ways permits are distributed. One reducing factor is lower than the whole economy reducing factor whereas the other is higher. One sector makes proportionally more reduction than the other. The approach here is different from section 3. The goal is not to determine the price of permits but to determine the sector-based reductions. The sector-based reducing factor of the sector j are given by the following expression.

$$z_j = \frac{Q_j(\sigma)}{Q_j(0)} = \left(\frac{1}{\frac{f_j}{c_j}\sigma + 1}\right)^{\beta_j}.$$

Notice that firms choose the level of reduction of the production in order to equalize the

[14] The mark-up is equal to $\frac{c_j + f_j\sigma}{n_j\beta_j - 1}$.

price of permits and the marginal abatement cost.[15] The marginal abatement cost is equal to $(z_j^{-\frac{1}{\beta_j}} - 1)\frac{c_i}{f_j}$. It increases with the marginal cost of production and decreases with the elasticity of demand and the polluting factor. When the elasticity of demand increases or the marginal cost of production decreases, the production increases. The higher the production is, the lower the cost of abating the first unit is. When the polluting factor increases or the marginal cost of production increases, emmissions increase. Furthermore, the higher the initial emissions are, the lower the cost of abating the first unit is. The marginal abatement cost decreases with the sector-based reducing factor. Thus, at exogenous price of permits, the lower the ratio of marginal cost over polluting factor is, the lower the sector-based reducing factor is. However, all firms make decisions based on the same price for permits. Therefore, each reducing factor may be rewritten as a function of the second one.

$$\sigma = (z_A^{-\frac{1}{\beta_A}} - 1)\frac{c_A}{f_A} = (z_B^{-\frac{1}{\beta_B}} - 1)\frac{c_B}{f_B} \Rightarrow z_B = ((z_A^{-\frac{1}{\beta_A}} - 1)\frac{c_A f_B}{c_B f_A} + 1)^{-\beta_B}.$$

There is equality between both costs of reducing emissions. Thus, the order between sector-based reducing factors is only given by the comparison between both elasticities and both ratios marginal cost over polluting factor. From the previous results, the following proposition is deduced.

Proposition 3. *When $\beta_A = \beta_B$, $c_A = c_B$ and $f_A = f_B$,*

$$\frac{\partial z_A}{\partial \beta_A} < 0, \qquad \frac{\partial z_A}{\partial c_A} > 0, \qquad \frac{\partial z_A}{\partial f_A} < 0.$$

Since there is no abatement technology, a multi-sector market for permits is equivalent to several independent markets for permits with different reducing factors. One reducing factor is lower than the whole economy reducing factor whereas the other is higher. When both elasticities are the same, the sector which has the lowest ratio of marginal cost over polluting factor reduces in proportion the more emissions. The marginal abatement cost for this sector is the highest since initial emissions are the highest. Thus, in order to equalize both marginal abatement costs, this sector applies a lower sector-based reducing factor. When both ratios are the same, the sector which has the higher elasticity reduces in proportion the more emissions since initial production is higher.

The neutral-profit policy for each sector is given by the following corollary.

Corollary 2. *The grand-fathering rate (γ_{gf_j}) required to neutralize profits in sector j is,*

$$\gamma_{gf_j} = \frac{1}{f_j}\frac{1}{(n_j\beta_j - 1)}\left(\frac{1 - z_j^{1-1/\beta_j}}{z_j^{-1/\beta_j} - 1}\right),$$

where z_j is the sector-based reducing factor.

[15] Since there is no abatement technology, the abatement cost is the cost to reduce production.

In this case, it is possible to give more free allowances than permits to a monopoly if the other sectors are oligopolistic and if the number of initial emissions in the other sectors is sufficiently high. An uniform distribution (the same grand-fathering rate) generates transfer across sectors, from the more polluting to the less polluting.

4.2 Production asymmetries within the sectors

Consider two categories of firms 1 and 2 respectively at the number of n_1 and n_2. They are characterized by different marginal production costs respectively c_1 and c_2 and different polluting factor respectively f_1 and f_2. Assume that $\frac{c_1}{f_1} < \frac{c_2}{f_2}$. Firms face the following demand:

$$P(Q) = \alpha Q^{-\frac{1}{\beta}} \quad with \quad Q = \Sigma_{i=1}^{n_1} q_i + \Sigma_{j=1}^{n_2} q_j, \tag{6}$$

where α and β are the size of demand and the elasticity of demand. Assume that the elasticity of demand is defined as

$$\frac{1}{n_1 + n_2} < \beta < min(\frac{c_2}{(c_2 - c_1)n_1}, \frac{c_1}{(c_1 - c_2)n_2}).$$

The lower constraint ensures the existence of equilibrium. The upper constraint is so that the firms with the higher marginal production cost have an incentive to produce strictly positive quantities before the introduction of environmental regulation. It leads to a small number of firms with lower marginal production cost. Otherwise, the other firms would not be profitable enough to produce. As in Section 3, each firm maximizes its profit taking into account the marginal cost of production and the price of permits due to the implementation of the market for permits. Production quantities are then given by:[16]

$$\begin{aligned} q_{i,1} =& (\frac{\alpha}{\beta})^{\beta}((n_2 + n_1)\beta - 1)^{\beta} \frac{\beta n_2(c_2 + f_2\sigma) + (1 - n_2\beta)(c_1 + f_1\sigma)}{(n_1(c_1 + f_1\sigma) + n_2(c_2 + f_2\sigma))^{\beta+1}}, \\ q_{i,2} =& (\frac{\alpha}{\beta})^{\beta}((n_2 + n_1)\beta - 1)^{\beta} \frac{\beta n_1(c_1 + f_1\sigma) + (1 - n_1\beta)(c_2 + f_2\sigma)}{(n_1(c_1 + f_1\sigma) + n_2(c_2 + f_2\sigma))^{\beta+1}}. \end{aligned}$$

Notice that individual quantities are not monotonic with the price of permits. However, total quantity is monotonic and decreases with the price of permits.[17]

Firms are price-takers in the market for permits. Assume that the auctioned price of permits is such that the rest of the permits that have not been offered and the net demand for permits are equal. As previously, the price of auctioned permits is independent of the distribution policy of free allowances and the equilibrium is defined by $Q(\sigma) = zQ(0)$. The price of permits is equal to:

$$\sigma = (z^{-\frac{1}{\beta}} - 1)\frac{n_1 c_1 + n_2 c_2}{n_1 f_1 + n_2 f_2}. \tag{7}$$

[16] All the efficient firms produce the same quantity $q_{i,1}$. All the inefficient firms produce the same quantity $q_{i,2}$.

[17] The total amount is equal to $(Q_1 + Q_2)(\sigma) = (\frac{\alpha}{\beta})^{\beta} \frac{((n_1+n_2)\beta-1)^{\beta}}{(n_1c_1+n_2c_2+(n_1f_1+n_2f_2)\sigma)^{\beta}}$.

Total production depends on the average abatement marginal cost and therefore the price of permits increases with it. The larger the initial production is, the higher the decrease of production generated is. Thus, the price of permits is lower when production is initially high. As in the symmetric case, the price of permits decreases with the industry emission-reducing factor z. Indeed, an elevation of z denotes a relaxation of the regulation.[18] However, each category has a different marginal abatement cost. For the same level of reduction of production, the marginal abatement cost of the inefficient firms is higher than the one of the efficient firms. In fact, the higher individual initial production is, the cheaper the reduction induced by the implementation of a market for permits is.

At the equilibrium in the market for permits, marginal abatement costs are equal. Firms are then differently altered by the implementation of a market for permits. Let $z_i = \frac{Q_i(\sigma)}{Q_i(0)}$ be the emission-reducing factor of the quantities produced by the firm of type i. They are equal to:

$$z_1 = z\left(1 + \frac{(\beta(n_1+n_2)-1)n_2(f_2c_1 - c_2f_1))}{(\beta n_2c_2 + (1-n_2\beta)c_1)(n_1f_1+n_2f_2)}(1 - z^{\frac{1}{\beta}})\right),$$
$$z_2 = z\left(1 + \frac{(\beta - \frac{1}{n_1+n_2})n_1(f_1c_2 - c_1f_2))}{(\beta n_2c_2 + (1-n_2\beta)c_1)(n_1f_1+n_2f_2)}(1 - z^{\frac{1}{\beta}})\right).$$

From these values, the following Lemma is deduced.

Lemma 3. *The emission-reducing factor of firms 1 is lower than the emission-reducing factor of firms 2.*

For the same level of reduction of production, the marginal abatement cost of the firms 2 is higher than the one of the firms 1. At the equilibrium in the market for permits, the marginal abatement cost of each category are equal. Consequently, the firms 1 reduce more in proportion than the firms 2 do, ($z_1 < z_2$), since their marginal abatement cost is lower. For the same reason, as previously, the higher individual initial production is, the cheaper the reduction induced by the implementation of a market for permits is.

As for quantities, profits are differently altered. Let $z_{\pi_i} = \frac{\pi_i(\sigma)}{\pi_i(0)}$ be the profit-altering factor of the category i. It i is equal to:

$$z_{\pi_i} = z^{-1-\frac{1}{\beta}} z_i^2. \tag{8}$$

The profit-altering factor may be decomposed in two effects: one on production and the other on the mark-up.[19] From equation (7), the conditions under which the environmental regulation leads to a profit-increasing effect without giving permits for free are given by the following proposition.

Proposition 4. *There exist two thresholds $\underline{\beta}_1$ and $\underline{\beta}_2$, with $\underline{\beta}_1 < 1 < \underline{\beta}_2$.*

[18] As in the symmetric case, decreasing the total quantity in a proportion z is equivalent to increasing the price of permits in proportion $z^{-\frac{1}{\beta}}$ and then the average marginal costs in the same proportions. In fact, the price is a linear function of the average marginal costs

[19] The increased mark-up is given by $z^{-1-\frac{1}{\beta}} z_i$. That the quantities by z_i.

(i) If $\beta < \underline{\beta}_1$, then all firms benefit from the environmental regulation. On the ground of the profits neutrality criterion, no free allowances should be given.

(ii) If $\underline{\beta}_1 < \beta < \underline{\beta}_2$, the profits of the firms 2 increase whereas the profits of the firm 1 decrease. Free allowances should be given only to the firms 1 on n the ground of the profits neutrality criterion.

(iii) If $\beta > \underline{\beta}_2$, then all firms suffer from the environmental regulation. In order to keep the profits of each firm at their levels without regulation, the regulator should give for free more allowances to firms 1 than to the firms 2, $\epsilon_1^N > \epsilon_2^N$.

In the symmetric case, profits increase with environmental regulation if the elasticity is lower than one, and decreases otherwise. When two categories of firms exist, two thresholds should be taken into account. The first one is lower than in the symmetric case while the second is higher. Since firms 1 reduce more in proportion their production than firms 2 do, the lowest threshold concerns the firms 1. When the profits of firms 1 increase then other firms benefit from the environmental regulation.[20]

The firms 1 reduce proportionally more their emissions than the firms 2 do. Furthermore, the profits of firms 1 before the implementation are higher than the profits of firms 2. Thus, the absolute losses suffered by the firms 1 are higher than that beard by the firms 2. Therefore, firm 1 should be given more free allowances than an inefficient. Thus, free allowances should mainly be given to the most polluting and the biggest firms. The distribution of free allowances shall be determined by calculating for each category the grand-fathering rate.

4.3 Production adjustment and abatement technologies

In the previous section, I demonstrate that even without abatement technology, the profits neutrality criterion requires few permits. I analyze in this section the robustness of the model's assumptions. In other words, can these results be interpreted as an upper bound for offsetting the profits' losses? This analysis assumes immediate adjustment of production and lack of abatement technologies. On the first hand, I ignore the capital which is not used in the short term. So there is an argument to compensate beyond what is suggested here, at least in the short term, since in the medium term, the adjusted capital and the argument underlying the simulations find a strong relevance. On the other hand, when abatement is available, profits without abatement are ordinarily higher than profits with abatement. However, various abatement technologies exist and let me analyze this issue according to the type of abatement technologies. Requate (2005) [11] surveys various abatement technologies. I will focus on two cases: end-of pipe abatement and cleaner production.

Following Christin et al (2013), consider the case of end-of-pipe abatement, which includes capture and storage systems, pollution filters and clean development mechanisms. Firms reduce emissions once goods are produced. Therefore, abatement decisions are independent from production decisions. Firms abate if and only it is profitable. Thus, end-of-pipe abatement

[20] $z_{\pi_1} > 1 \Rightarrow z_{\pi_2} > 1$.

may be considered as a second firms' activity, which produces permits. In such a case, profits with abatement are compulsory higher than without. The results found previously are then an upper-bound for offsetting.

Consider now cleaner production. Thus, firms may use an abatement technology to reduce emissions which modifies both the polluting factor and the marginal cost of production. In that case, the cleaner a technology is the higher its marginal cost is. As in Meunier & Ponssard (2012)[9], assume that the unit cost of production and abatement for a firm i is equal to:

$$c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2, \tag{9}$$

where f_i is the resulting polluting factor of firm i, f_0 is the initial polluting factor and γ the cost parameter of abatement. The left hand of the above expression represents the marginal cost $c_i(f_i)$ of production and the other one is the unit cost related to abatement. Keep the assumptions of isoelastic demand function and symmetry.

The profit of a firm i may be written as follows:

$$\pi_i(q_i, f_i) = P(Q)q_i - \left(f_i\sigma + c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2\right) q_i. \tag{10}$$

Indeed, firms buy permits ($f_i q_i \sigma$) and bear the cost of production and abatement (($c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2)q_i$)) while they earn the revenue of sales ($P(Q)q_i$). Firms compete "a la Cournot" and maximize profits by choosing productions. The first order conditions of profits satisfy

$$P(Q) + P'j(Q)q_i = f_i\sigma + c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2. \tag{11}$$

Derivating the First Order condition with respect to σ, I get:

$$P'q_i' = f_i - [1 - (1 + \frac{1}{\beta})\frac{1}{n}]P'Q'. \tag{12}$$

By summing the First Order Condition, I obtain:

$$nP + P'Q = \sum_i \left(f_i\sigma + c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2\right). \tag{13}$$

Taking the derivative of this equation with respect to σ, I get:

$$P'Q' = nf_i/(n - 1/\beta). \tag{14}$$

Since I focus on the effect of permits price on the firms' profits, I analyze the derivative of the function π_i with respect to σ, and I obtain:

$$\frac{\partial \pi_i}{\partial \sigma} = q_i[P'(Q' - q_i')] - f_i. \tag{15}$$

By replacing (12) and (14) on (15), I get:

$$\frac{\partial \pi_i}{\partial \sigma} = q_i f_i \left[\frac{1-\beta}{n\beta - 1} \right]. \tag{16}$$

Profits increase with permits price when the demand elasticity is weak (< 1), and decrease otherwise. These results are the same as the ones found previously. In other words, the presence of cleaner production does not change qualitatively the efffect on profits. Thus, the paper's results seem to be a good approximation of the upper bound for offsetting profits'losses.

4.4 Optimal regulation

Until now, the paper has focused on an exogenous factor of reduction. Let analyze how to endogenize the emissions reduction to make it welfare-optimal. We then consider a linear damage function of the pollution, where λ is the marginal damage. The regulator takes into account the firms' profits (π), the net consumers' surplus (CS), the environmental damage (λE) and the value of the permits sold ($E\sigma$). The social welfare function is then defined as:

$$W = CS + \Sigma_{i=1}^{n} \pi_i - \lambda E + \sigma E.$$

Profits are equal to the sales minus the production costs and the costs to purchase permits. Thus, the welfare is the sum of the gross consumers' surplus minus the production costs and the environmental damages and may be rewritten as:

$$W = \int_0^{Q^*} P(Q) dQ - cQ^* - \lambda Q^*$$

Since production depends on the reducing factor, we can then rewrite the welfare as a function of the reducing factor z:

$$W = \int_0^{zQ(0)} \alpha Q^{-1/\beta} dQ - czQ(0) - z\lambda Q(0)$$

By maximizing the welfare according to the reducing factor, I deduce the optimal redcing factor, which is equal to:

$$\begin{aligned} z^{opt} &= \left(\frac{\alpha}{c+\lambda}\right)^{\beta} \frac{1}{Q(0)} && (17) \\ &= \left(\frac{\alpha}{c+\lambda}\right)^{\beta} \left(\frac{\beta c}{\alpha(\beta - 1/n)} \right)^{\beta} && (18) \\ &= \left((1 + \frac{\lambda}{c})(1 - 1/(n\beta)) \right)^{-\beta} && (19) \end{aligned}$$

The optimal factor of reduction depends on the marginal damage, the number of firms, the marginal cost of production, the elasticity of demand. When the marginal damage increases, obviously the optimal reducing factor decreases. Indeed, if pollution becomes more harmful, it is optimal to reduce more pollution. When the number of firms increases, the optimal reducing factor decreases. The higher the number of firms is, the lower the distortion in the demand side is. Moreover, the correction of environmental externality induces an increase of the exercise of the market power. Indeed, to correct market-power the regulator shoud subsidize production and then to fix a lower factor of reduction as in the case without imperfect competition. When the marginal cost of production increases, the optimal reducing factor increases. A low marginal cost of production corresponds to a high initial pollution. Therefore, optimally the regulator makes the environmental regulation more strict when the marginal cost of production increases.

From equation (3) and equation (19), the optimal permit price is given by:

$$\sigma^{opt} = \lambda(1 - 1/(n\beta)) - c/(n\beta)$$

The optimal permit price increases with the marginal damage, the number of firms and the elasticity of demand while it decreases with the unit production cost. When the elasticity of demand is high and the unit cost low, initial production is high which requires an important permits price to reduce the induced damage. The following lemma compares the optimal pollution permit price with the Pigovian tax, i.e., the marginal damage.

Lemma 4. *The optimal permit price is lower than the marginal damage.*

This result is standard and well known in the literature.[21] Under perfect competition, the optimal permit price is equal to the Pigovian tax. However, under imperfect competition, the regulator should implement a lower permit price to decrease the market power of firms in the market for products.

From equation (19), I can rewrite Proposition 2 and determine the conditions under which the regulator can implement the optimal reducing factor and make the environmental acceptable according to firms point-of-view.

Proposition 5. *Let $\overline{\lambda}(\beta, n)$ be the marginal damage under which the regulator can implement the optimal reduction of emissions and may offset profits' losses. For each (λ, β, n), if $\lambda > \overline{\lambda}(\beta, n)$, the offsetting is not possible. The threshold, for $\beta < 10$ is such that:*

(i) When n=1, $\overline{\lambda}(\beta, 1) < 0$.

(ii) When n=2, $\overline{\lambda}(\beta, 2) = \frac{1}{1-1/(2\beta)}((0.3)^{-1/\beta} - 1)c$.

(iii) When $n > 2$, $\overline{\lambda}(\beta, n) = \frac{1}{1-1/(n\beta)}((0.2)^{-1/\beta} - 1)c$.

As in the case of an exogenous reducing factor, acceptability is not reachable in case of monopoly. The more numerous firms are, the higher the marginal damage, under which the regulator can implement the optimal reduction of emissions and may offset profits' losses, is. This proposition reconciles efficiency with acceptability, which are ordinarily analyzed independently.

[21] For instance, Barnett (1980) showed in presence of market-power the optimal tax or permits price is lower than the marginal damage.

5 Discussion, illustration and policy implications

This section discusses the relevance of this paper in view of the free allowances distributed in Europe.

5.1 Summary of the EU free allowances process

The EU free allowances process is exhaustively detailed in Ellerman et al (2010)[4]. In the two first phases (2005-2007 and 2008-2012), the determination of free allowances is very close: all sectors are uniformly treated and approximately all permits were given for free. However, in the third phase (2013-2020), sectors are differentiated and auctioning is introduced.

Phase 1 (2005-2007) and phase 2 (2008-2012) During these two periods, the distribution of permits is decentralized. Each member state is allowed to auction up to 5 per cent of their totals in the first periods and up to 10 per cent in the second. In other words, the minimum of free allowances for the two phases are 95 and 90 percent respectively. Notice that in the first phase only four member states have chosen to give all permits for free. For the second phase, eight countries auctioned permits. Germany has distributed for free 91 per cent of permits. According to Ellerman et al(2010)[4] in Europe 99.87 and 97 per cent of permits have been given for free respectively for phase 1 and phase 2. The distribution of permits is uniform across sectors and grand-fathered.

Phase 3 (2013-2020) During the third phase, the distribution of permits is centralized and each member state should respect the ETS Directive approved in December 2008. Moreover, three categories of sectors are considered. The power sector would receive no free allowances from 2013. Sensitive sectors, such as cement and steel, which face a significant risk of carbon leakage could receive free allowances of up to 100 per cent of their need. Notice that, while all permits were previously grand-fathered, a new mechanism for allocation entered into effect in 2013; it combines an ex-ante lump-sum transfer based on historic output (and multiplied by a benchmark) with an ex-post adjustment of this lump-sum according to rules related to actual capacity and activity level. Other sectors would receive a free allocation of 80 per cent of their share of the cap in 2013, which would be reduced by ten percentage points each year so that free allocation would be phased out in 2020.

Conclusion The two main differences, between the two first phases and the third one, are the degree of centralization and the treatment of sectors. Indeed, the distribution is uniform in the two first periods and differentiated across sectors in the third phase. However, the grand-fathering rates are very high for the two first phases and for the sensitive sectors in the third one. Furthermore, even 80 per cent of permits at the start of the third phase are high. The difference between sectors in the third phase may be captured by the demand elasticity. Indeed, the electric demand elasticity is assumed to be weak. In presence of international competition, the demand elasticity for domestic goods is ordinarily high. Across sensitive sectors there is no difference. Cement and steel receive the same treatment. The EU free allowances process does not take into account the differences between market structures, the ratio production cost over polluting factor, and the weight of the sector in the total emissions.

5.2 Illustration for cement, electricity and steel in the EU-ETS

Three sectors are considereded: electricity, cement ans steel. All the values of the parameters are provided in Table 1 and are based on Reinaud (2004)[12], Demailly & Quirion (2008)[3] and Meunier & Ponssard (2012)[9].

Let me give some precisions about the method and the data used for the illustration:

- The electricity sector is not exposed to international competition while the two other sectors are exposed. Thus, for the stell and cement sectors, as in section 4.3, I consider two countries, home and foreign, denoted by H and F. Home represents the EU area. However, concerning the electricity sector, I use the framework retained in the section 3.

- Firms may use an abatement technology to reduce emissions. Thus, firms may reduce emissions by reducing production and abating. I use the specifications of section 4.4, such that the unit cost of production and abatement for a firm i is equal to:

$$c_i(f_i) + \frac{\gamma}{2}(f_0 - f_i)^2,$$

  where f_i is the resulting polluting factor of firm i, f_0 is the initial polluting factor and γ the abatement cost parameter.

- I calculate production and abatement in the two cases; non exposed and exposed to international competition. The analytical results are given in the appendix.

- Electricity generation requires the use of much diverse technologies than in other industries. Therefore Demailly & Quirion (2008)[3] use average values for this sector. However, they consider the competition as perfect and assume that prices are equal to production costs. I retain then 47 as price of electricity. However, for the cost of electricity, I retain the cost determined by the ministery of economics in France for some coal plants, which is equal to 37. I do not retain an average cost but the cost of an intermediary technology: the combined cycle gas.

- Steel production is heterogenous and Reinaud (2004)[12] distinguishes the BOF and EAF routes for steel making. Demailly & Quirion (2008)[3] aggregate the data by summing them, weighted by their shares in total production capacity of EU and non EU countries.

- Elasticities (absolute value) correspond to values over the short-term for observed prices, with the understanding that these elasticities are difficult to estimate in practice. According to Meunier & Ponssard (2012)[9], electricity has a demand elasticity equal to

Sectors	**Electricity**	**Steel**	**Cement**
Market Size (α)	3600	200	250
	Meunier & Ponssard (2012)		
Elasticity (β)	0,4	0,6	0,5
	Meunier & Ponssard (2012)		
Price (p)	47	313	64
	Demailly (2008)	Reinaud (2004)	
Unit cost (c^H)	37	247	46,8
	Ministère de l'Economie (2003)	Reinaud (2004)	
Initial polluting factor (f_0)	0.37	1.3	0.7
	Meunier & Ponssard (2012)		
Impact over unit cost (c')	0,5	4,5	1,5
	Meunier & Ponssard (2012)		
Abatement cost parameter (γ)	1017	115	315
	Calculation		
Transport cost (τ)	Not relevant	31	25
		Meunier & Ponssard (2012)	
Unit cost of foreign firms (c^F)	Not relevant	247	35
		Meunier & Ponssard (2012)	
Market Structure (n)	$n = \frac{1}{\beta(1-c/p)}$	$n^F = 1$ and $n^H = \frac{P^B(1-\beta)+\beta(c^F+\tau)}{\beta(P-c^H)}$	
	12	7	7

Table 1: Data for the parameters and calibration of the model

0,4. The demand elasticities of the steel and cement sectors are assumed to be equal to 0,6 and 0,5 respectively.

- In order to determine the abatement cost parameter in each sector, I calculate the optimal polluting factor; I use the two values of polluting factor calculated by Meunier & Ponssard (2012)[9] for $\sigma = 0$ and $\sigma = 30$. I determine the value of abatement cost parameter by making a linear interpolation.[22]

- Market structures are determined indirectly from market prices (BAU) and unit costs by reversing the Cournot solution in a context without regulation and abatement costs (i.e., the number of firms is adjusted to fix on the observed prices and does not correspond to the number of firms observed). For the electicity production, I calculate the number of firms so that $n = 1/\beta(1 - c/p)$, and we round off to the higher unit. However, for the two other sectors, I calculate $n^H = [(p(1-\beta) + \beta(c^F)]/\beta(p - c^H)$, when $n^F = 1$. In the simulation, I consider the set of foreign firms to be one exporter, i.e., the number of foreign firms is equal to one ($n^{BF} = 1$). This asumption means that there is a single and relative big exporter.
- I assume that the emissions are reduced by five, ten and twenty per cent. The first case illustrates the emissions reduction for the first wto phases of EU-ETS. The other cases are useful to illustrate that even with higher reduction, the percentage of permits to give to firms to offset profits' losses is quite low.

Without free allowances First of all, let analyze the case without free allowances and a reduction of emissions equal to 5%. This reduction corresponds to the first two phases of EU-ETS. Notice that the permit price which reduces by 5% total emissions is equal to $\sigma = 4$, which is quite close to the permit price observed in 2012 under the EU-ETS. The three sectors are included in the same market for permits. The sector which relatively reduces the most its production is cement. Indeed, the reductions of productions in percentage are, respectively, for electricity, steel and cement equal to 1.5, 1.9 and 4.5 per cent. Section 4.1 shows that when the elasticities are the same, the sector which has the lowest ratio of marginal cost over polluting factor reduces in proportion the more emissions. Moreover, the ratio cost over polluting factor of cement sector is the lowest while the one of the steel sector is the highest. Furthermore, the elasticity in the three sectors is quite the same.

However, the sector which reduces the most its polluting factor is steel. Indeed, the abatement cost parameter of the steel sector is relatively to the two other sectors. In other words, it is cheaper to reduce polluting factor than to reduce output. The polluting factor reductions are, respectively, for electricity, steel and cement equal to 1.2, 5.6 and 2.4 per cent. The abatement cost parameter of electricity is quite high. The product prices increases relatively the most in the cement sec tor. Indeed, the cement sector is the sector which relatively reduces the most its production. The price of cement increases by 5.9%. However, the price of steel increases only by 2.9%, since firms in this sector reduce mainly the polluting factor instead of the output.

The profits of electric companies increase with the reduction of emissions. This result comes from the non exposition to international competition and the low elasticity. The Lemma 1

[22] I determine the cost parameter for abatement by the inverse of the pollution factor solution, i.e., I fix γ on the values observed from $f_i^j(\sigma = 30)$, $f_0^j(\sigma = 0)$ and the marginal cost in each sector. By Meunier & Ponssard (2011)[?]: $f_0^e(\sigma = 0) = 0,37$, $f_i^e(\sigma = 30) = 0,34$,$f_0^s(\sigma = 0) = 1,3$, $f_i^s(\sigma = 30) = 1$, $f_0^c(\sigma = 0) = 0,7$ and $f_i^c(\sigma = 30) = 0,6$.

Implementation of the market for permits			
Reduction of total emissions	-5%	-10%	-20%
Permit price (Euros)	4	9	21
Effect on quantities			
Electricity	-1.5%	-3.3%	-7.1%
Steel	-1.9%	-4.1%	-8.4%
Cement	-4.5%	-9.5%	-18%
Effect on polluting factor			
Electricity	-1.2%	-2.5%	-5.7%
Steel	-5.6%	-9%	-17%
Cement	-2.4%	-4%	-10%
Effect on price			
Electricity	+3.4%	+8.3%	+19.7%
Steel	+2.9%	+5%	+9.24%
Cement	+5.9%	+11%	+25%
Effect on profits			
Electricity	+2.4%	+5.3%	+11.8%
Steel	-1.4%	-3.4%	-8.8%
Cement	-2.3%	-5.2%	-12%
Effect on foreign emissions			
Steel	+10,88%	+22.86%	+45.52%
Cement	+57.45%	+116.11%	+215.49%
All permits for free			
Uniform policy with the grand-fathering rate=z	0.95	0.9	0.8
Effect on profits			
Electricity	+16.8%	+36.08%	+75.70%
Steel	+5.66%	+11.64%	+22.54%
Cement	+12.63%	+26.67%	+54.13%
Sector-based policy which neutralizes all profits			
Neutral-profits allowances/ initial emissions (%)			
Electricity	0%	0%	0%
Steel	+19.12 %	+20.61	+ 22.43
Cement	+14.72%	+14.69	+14.51
Percentage of total allowances given for free	8.97%	9.77%	11.33%
Uniform policy which neutralizes all profits			
Grand-fathering rate applied	0.191	0.206	0.224
Effect on profits			
Electricity	+5.45%	+8.9	+34
Steel	0%	0%	0%
Cement	0%	0.7%	11%
Percentage of total allowances given for free	20.10%	22.88%	28%

Table 2: Results of the illustration.

shows that the profit increases when the elasticity of demand is weak (< 1). Thus, the profits increase by 2.4% without free allowances. Indeed, since the pass-through rate is higher than 100%, the profits increase with the permit price increase. However, the profits in the two other sectors decrease since they are exposed to international competition and the elasticity is not sufficiently low. The cement is the sector the most harmed by the reduction of emissions. Indeed, the cement sector is the sector which relatively reduces the most its production. Let us analyze how foreign emissions are altered by the implementation of pollution permits. The foreign emissions increase mainly in the cement sector. Indeed, the foreign emissions increase by 57% while they increase by 10% in the steel sector. This result comes from the fact that cement mainly reduce its output which induces an increase of the foreign production and consequently also an increase of foreign emissions.

As a conclusion, the consumers the most harmed by the reduction of emissions are those who purchase cement. In the cases under which the total emissions are reduced by 10% and 20%, the price of cement increases respectively by 11% and 25%. Moreover, the sector which benefits from the regulation is the electricity sector. When the total emissions are reduced by 10% and 20%, the profits increase respectively 11 and 25%.

All permits for free First, consider the case in which all permits are given for free. The grand-fathering rate applied is equal to the global reducing factor. The sector which benefits the more is electricity, since the profit increases with the implementation of the market for permits and the permits given are valued to the permits price. The profits in the electricity sector increase by 16.8 per cent. This result is consistent with the windfall theory such that electricity benefits from the opening of the EU-ETS. Profits in the cement sector increase by 12.63 per cent while they increase by 5.66 per cent in steel sector. Two methods of distribution are compared: a uniform grand-fathering rate and a sector-based grand-fathering rate. The first one represents the two first phases of the EU-ETS while the second corresponds to the third phase. For both methods, the neutral-profit policy is determined.

Sector-based profit-neutral policy The sector-based policy consists to give to each sector the number of permits required to neutralize profits. For the electricity, no free allowances should be given. In the cement sector, the regulator should apply a grand-fathering rate equal to 14.72 per cent. In the steel sector, the rate to apply is higher and equal to19.12 per cent. At the global scale, only eleven per cent of permits were sufficient to neutralize all firms' profits using a sector-based distribution of permits. In the cases under which the total emissions are reduced by 10% and 20%, the percentage of total allowances that the regulator should give for free is respectively equal to 9.77 and 11.33. In these two cases, the percentage is also quite low.

Uniform profit-neutral policy The regulator offsets all firms' profits and uses an uniform policy, i.e., he applies the same grand-fathering rate to each sector. In other words, the regulator applies the higher grand-fathering rate determined in the sector-based case. Thus, the regulator uses the grand-fathering rate applied for the steel sector. The policy is such that profits' losses in the steel sector are offset and in the other sectors profits increase. The ratio of neutral-profits allowances over initial emissions used for a total emissions reduction of 5 is equal to 0.191 per cent. Firms producing cement benefit in a low proportion to this policy whereas profits in electricity sector increase by 5.45 per cent. At the global scale, if the distribution is uniform 20.10 per cent of permits would have been enough to offset losses. This figure is very low and far from the 99 per cent of permits given for free in Europe during the two first phases. In the cases under which the total emissions are reduced by 10% and 20%, the percentage of total allowances that the regulator should give for free is respectively equal to 23% and 28%.

To conclude, this illustration shows that few permits are required when the distribution of permits is sector-based in order to offset losses. Moreover the evolution of EU free allowances process, i.e. from uniform to sector-based policy prevents the regulator from giving too much permits and to decrease his revenue. Furthermore, the process turned to be centralized and the countries cannot give all for free. In a decentralized organization, countries prefer to give all for free to their own firms. For the third phase, as in our simulations, electricity producers do not receive free allowances. However, cement and stell sector receives for free all the permits they need. Thus, the number of permits given for free is always too high and both sectors should be differentiated. The simulations of 10% and 20% of reductions show that even with important reductions, the percentage of permits to grant for free in order to offset profitsŠ losses is very low.

6 Conclusion

This paper is concerned with two policy aims: to implement a market for permits and to offset firms' losses. The paper shows that a low number of free allowances is sufficient to reach these two goals. Furthermore, even when the reduction is high, the regulator can offset losses fully when the sectors are not in a monopoly context.

It is interesting to notice that the various expermiments of emission trading scheme are susbstituting grand-fathering allocation by other methods such as output based or capacity based allocation. These methods lower the effective marginal production cost but cannot achieve acceptability. However, the percentage of reduction of emissions implemented by regulators is quite still the same. This paper advocates the use of grand-fathering coupled with a significant reduction in carbon emissions.

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7 Appendix

Proof of Proposition 1:

The profit is equal to

$$\pi_i(\sigma) = q_i(\sigma)\frac{c+\sigma}{n\beta - 1}.$$

The profit-neutral allowances are given by $\varepsilon\sigma = \pi_i(0) - \pi_i(\sigma)$ and may be formulated as:

$$\varepsilon = \frac{1}{n\beta - 1}\frac{1}{\sigma}\left(q_i(0)c - q_i(\sigma)(c+\sigma)\right).$$

Moreover, $q_i(\sigma) = zq_i(0)$ and $\sigma = (z^{-\frac{1}{\beta}} - 1)c$. Then, the grand-fathering rate($\gamma_{gf} = \frac{\epsilon}{q_i(0)}$) and the ratio of free allowances over permits ($\gamma_p = \frac{\epsilon}{q_i(\sigma)}$) which are required to offset losses are:

$$\gamma_{gf} = \frac{1}{(n\beta - 1)}\left(\frac{1 - z^{1-1/\beta}}{z^{-1/\beta} - 1}\right), \qquad \gamma_p = \frac{1}{n\beta - 1}\left(\frac{z^{-1} - 1}{z^{-\frac{1}{\beta}} - 1} - 1\right). \tag{20}$$

Proof of Proposition 2

The goal of the proposition 1 is to determine when γ_p et γ_{gf} respect the constraints imposed.However, both constraints are equivalent

$$\gamma_{gf} = \frac{\varepsilon_i}{q_i(0)} < z \Leftrightarrow \gamma_p = \frac{\varepsilon_i}{q_i(\sigma)} < 1.$$

Free allowances can not exceed the number of permits in circulation. Focus on the condition relative to γ_p: When n=1, β is higher than 1 by assumption. γ_p may then be rewritten as :

$$\gamma_p = \frac{1}{n\beta - 1}\Sigma_{k=1}\beta - 1z^{(-\frac{1}{\beta})^k}.$$

γ_p is the quotient of a sum of β terms higher than 1 over $\beta - 1$ which is higher than 1. As conclusion, if n=1, then $\gamma_p > 1$. As in the previous proof,

$$\gamma_p = \frac{1}{n\beta - 1}\left(\frac{z^{-1} - 1}{z^{-\frac{1}{\beta}} - 1} - 1\right) < 1 \Leftrightarrow \left(\frac{z^{-1} - 1}{z^{-\frac{1}{\beta}} - 1}\right) < n\beta.$$

When n=2, the constraint is given by $(\frac{z^{-1}-1}{z^{-\frac{1}{\beta}}-1}) < 2\beta$ For $\beta < 10$, it corresponds to $\bar{z} = 0.253$.
When n=3, the constraint is given by $(\frac{z^{-1}-1}{z^{-\frac{1}{\beta}}-1}) < 3\beta$ For $\beta < 10$, it corresponds to $\bar{z} = 0.127$.

Since $\bar{z}$ increases with the elasticity and decreases with the number of firms, let me conclude that: when n=2, $0.3 > \overline{z}(\beta, 2)$ and when $n > 2$, $\overline{z}(\beta, n) < 0.2$.

Proof of Corollary 1

Let $z_{\pi_{FA}} = \frac{\pi(\sigma)+q_i\sigma}{\pi(0)}$ be the profit-altering factor when all permits are given for free.

The profit is equal to

$$\pi_i(\sigma) = q_i(\sigma)\frac{c+\sigma}{n\beta-1}.$$

Then, the profit-altering factor, when all permits are given for free, may be formulated as:

$$\begin{aligned}
z_{\pi_{FA}} &= \frac{\pi(\sigma)+q_i(\sigma)\sigma}{\pi(0)} = \frac{\frac{1}{n\beta-1}q_i(\sigma)(c+\sigma)+q_i(\sigma)\sigma}{\frac{q_i(0)c}{n\beta-1}}, \\
&= \frac{q_i(\sigma)}{q_i(0)} + \frac{n\beta q_i(\sigma)\sigma}{cq_i(0)}, \\
&= z + \frac{n\beta z\sigma}{c}, \\
&= z + n\beta z(z^{-\frac{1}{\beta}} - 1)
\end{aligned}$$

To conclude, the profit-altering factor, when all permits are given for free, is equal to:

$$z_{\pi_{FA}} = z(1 + n\beta(z^{-1/\beta} - 1)).$$

Proof of Corollary 2

The profit of a firm i of the sector j is equal to

$$\pi_i(\sigma) = q_i(\sigma)\frac{c+f_j\sigma}{n_j\beta-1}.$$

The profit-neutral allowances are given by $\varepsilon\sigma = \pi_i(0) - \pi_i(\sigma)$ and may be formulated as:

$$\varepsilon = \frac{1}{n\beta-1}\frac{1}{\sigma}\left(q_i(0)c_j - q_i(\sigma)(c+f_j\sigma)\right).$$

Moreover, $q_i(\sigma) = zq_i(0)$ and $c_j + f_j\sigma = (z^{-\frac{1}{\beta}} - 1)$. Thus, the neutral-profit grand-fathering rate($\gamma_{gf} = \frac{\epsilon}{f_j q_i(0)}$) is equal to:

$$\gamma_{gf} = \frac{1}{f_j}\frac{1}{(n\beta-1)}\left(\frac{1-z^{1-1/\beta}}{z^{-1/\beta}-1}\right). \tag{21}$$

Proof of Proposition 4

Consider two sectors A and B. A sector j is characterized by the elasticity β_j, size of demand α_j, marginal cost c_j, number of firms n_j and polluting factor f_j. From previous results, the quantities produced and the price of the product for the sector j are given by:

$$q_{ij}(\sigma) = \frac{1}{n_j}\left(\frac{\alpha_j(\beta_j - 1/n_j)}{\beta_j(c + f_j\sigma)}\right)^{\beta_j}, \qquad p_j(\sigma) = \frac{c_j + f_j\sigma}{1 - 1/(n_j\beta_j)}. \tag{22}$$

Let $z_A = \frac{Q_A(\sigma)}{Q_A(0)}$ and $z_B = \frac{Q_B(\sigma)}{Q_B(0)}$ be respectively the induced emission-reducing factors of sector A and sector B. The goal is not to determine the price of permits but to determine the sector-based reductions. From section 3,

$$z_j = \left(\frac{1}{\frac{f_j\sigma}{c_j} + 1}\right)^{\beta_j}.$$

All firms make decisions based on the same price for permits.

$$\begin{aligned}\sigma &= (z_A^{-\frac{1}{\beta_A}} - 1)\frac{c_A}{f_A} = (z_B^{-\frac{1}{\beta_B}} - 1)\frac{c_B}{f_B}\\ \Rightarrow z_B &= ((z_A^{-\frac{1}{\beta_A}} - 1)\frac{c_A f_B}{c_B f_A} + 1)^{-\beta_B}.\end{aligned}$$

The order between sector-based reducing factors is given by the comparison between both elasticities and both ratios marginal cost over polluting factor.

On the market for permits, the aggregate demand for permits is equal to the total amount of permits firms need and have not been granted for free. Thus, the perfectly competitive permits market clears when supply equals demand, or:

$$Q_A(\sigma) + Q_B(\sigma) = z\left(Q_A(0) + Q_B(0)\right) \Rightarrow Q_A(0)(z_A - z) + Q_B(0)(z_B - z) = 0.$$

The value of sector-based reducing factors depends on both elasticities and both ratios marginal cost over polluting factor, market structures and the sizes of demand.

Proof of Lemma 4

The productions are equal to:

$$\begin{aligned}q_{i,1} =& (\frac{\alpha}{\beta})^{\beta}\frac{((n_2+n_1)\beta - 1)^{\beta}}{(n_1(c_1+f_1\sigma) + n_2(c_2+f_2\sigma))^{\beta+1}}\left(\beta n_2(c_2+f_2\sigma) + (1-n_2\beta)(c_1+f_1\sigma)\right),\\ q_{i,2} =& (\frac{\alpha}{\beta})^{\beta}\frac{((n_2+n_1)\beta - 1)^{\beta}}{(n_1(c_1+f_1\sigma) + n_2(c_2+f_2\sigma))^{\beta+1}}\left(\beta n_1(c_1+f_1\sigma) + (1-n_1\beta)(c_2+f_2\sigma)\right).\end{aligned}$$

The price of products is equal to

$$p = \frac{\beta(n_1c_1 + n_2c_2 + (n_1f_1 + n_2f_2)\sigma)}{((n_2+n_1)\beta - 1)}.$$

However,

$$(Q_1+Q_2)(\sigma) = (\frac{\alpha}{\beta})^{\beta} \frac{((n_1+n_2)\beta-1)^{\beta}}{(n_1c_1+n_2c_2+(n_1f_1+n_2f_2)\sigma)^{\beta}}.$$

The equilibrium in the market for permits is given by $n_1c_1+n_2c_2+(n_1f_1+n_2f_2)\sigma = (n_1c_1+n_2c_2)z^{-\frac{1}{\beta}}$ and the price of permits is equal to $\sigma = (z^{-\frac{1}{\beta}}-1)\frac{n_1c_1+n_2c_2}{n_1f_1+n_2f_2}$.

$$\begin{aligned}
\pi_{i,1} =& (\frac{\alpha}{\beta})^{\beta} \frac{((n_2+n_1)\beta-1)^{\beta-1}}{(n_1(c_1+f_1\sigma)+n_2(c_2+f_2\sigma))^{\beta+1}} \left(\beta n_2(c_2+f_2\sigma)+(1-n_2\beta)(c_1+f_1\sigma)\right)^2, \\
\pi_{i,2} =& (\frac{\alpha}{\beta})^{\beta} \frac{((n_2+n_1)\beta-1)^{\beta-1}}{(n_1(c_1+f_1\sigma)+n_2(c_2+f_2\sigma))^{\beta+1}} \left(\beta n_1(c_1+f_1\sigma)+(1-n_1\beta)(c_2+f_2\sigma)\right)^2.
\end{aligned}$$

Let z_1 be the emission-reducing factor of an efficient firm,

$$z_1 = \frac{Q_1(\sigma)}{Q_1(0)} = \frac{q_1(\sigma)}{q_1(0)}.$$

The quantity sold by an efficient firm is given by:

$$q_{i,1} = (\frac{\alpha}{\beta})^{\beta} \frac{((n_2+n_1)\beta-1)^{\beta}}{((n_1c_1+n_2c_2)z^{-\frac{1}{\beta}})^{\beta+1}} \left(\beta n_2(c_2+f_2\sigma)+(1-n_2\beta)(c_1+f_1\sigma)\right).$$

The emission-reducing factor of an efficient firm may be rewritten as:

$$z_1 = \frac{1}{(z^{-\frac{1}{\beta}})^{\beta+1}} \left(\frac{\beta n_2(c_2+f_2\sigma)+(1-n_2\beta)(c_1+f_1\sigma)}{(\beta n_2c_2+(1-n_2\beta)c_1)} \right).$$

$$\begin{aligned}
z_1 =& \frac{1}{(z^{-\frac{1}{\beta}})^{\beta+1}} (\frac{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2)z^{-\frac{1}{\beta}}}{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2)} (z^{-\frac{1}{\beta}}-1) \\
&+ \frac{1}{(z^{-\frac{1}{\beta}})^{\beta+1}} \frac{(\beta(n_1+n_2)-1)n_2(f_2c_1-c_2f_1))}{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2)} (z^{-\frac{1}{\beta}}-1)).
\end{aligned} \tag{71}$$

$$z_1 = \frac{1}{(z^{-\frac{1}{\beta}})^{\beta+1}} (z^{-\frac{1}{\beta}} + \frac{(\beta(n_1+n_2)-1)n_2(f_2c_1-c_2f_1)))(z^{-\frac{1}{\beta}}-1))}{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2)}.$$

$$z_1 = z + (\frac{(\beta(n_1+n_2)-1)n_2(f_2c_1-c_2f_1))}{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2))} (\frac{(z^{-\frac{1}{\beta}}-1)}{(z^{-\frac{1}{\beta}})^{\beta+1}}.$$

$$z_1 = z(1 + \frac{(\beta(n_1+n_2)-1)n_2(f_2c_1-c_2f_1))}{(\beta n_2c_2+(1-n_2\beta)c_1)(n_1f_1+n_2f_2)} (1-z^{\frac{1}{\beta}})).$$

By the same approach, the emission-reducing factor of an inefficient firm is given by

$$z_2 = z\left(1 + \frac{(\beta - \frac{1}{n_1+n_2})n_1(f_1c_2 - c_1f_2))}{(\beta n_2c_2 + (1-n_2\beta)c_1)(n_1f_1 + n_2f_2)}(1 - z^{\frac{1}{\beta}})\right).$$

Proof of Proposition 4

$$\pi_{i,1} = \frac{\beta n_2(c_2 + f_2\sigma) + (1-n_2\beta)(c_1 + f_1\sigma)}{(n_2+n_1)\beta - 1}q_1,$$
$$\pi_{i,2} = \frac{\beta n_1(c_1 + f_1\sigma) + (1-n_1\beta)(c_2 + f_2\sigma]}{(n_2+n_1)\beta - 1}q_2.$$

From the fisrt equation, the profit-altering factor of the firm 1 is equal to

$$z_{\pi_1} = \frac{\beta n_2(c_2 + f_2\sigma) + (1-n_2\beta)(c_1 + f_1\sigma)}{(\beta n_2c_2 + (1-n_2\beta)c_1}z_1.$$

$$z_{\pi_1} = \frac{\beta n_2(c_2 + f_2\sigma) + (1-n_2\beta)(c_1 + f_1\sigma)}{(\beta n_2c_2 + (1-n_2\beta)c_1}z_1.$$

As previously, $(\frac{\beta n_2(c_2+f_2\sigma)+(1-n_2\beta)(c_1+f_1\sigma)}{\beta n_2c_2+(1-n_2\beta)c_1}) = z_1(z^{-\frac{1}{\beta}})^{\beta+1}$. To conclude, z_{π_1} and z_{π_2} may be rewritten as:

Deduce then that $z_{\pi_1} < z_{\pi_2}$. In order to keep the profits at their levels without regulation, the regulator should give for free more allowances to efficient firms than to the inefficient. The profit-neutral allowances are equal to:

$$\begin{aligned}\epsilon_i\sigma &= \pi_i(0) - \pi_i(\sigma),\\ &= \pi_i(0)(1 - z_{\pi_i}).\end{aligned}$$

However, from previous results, $\pi_1(0) > \pi_2(0)$ and $z_{\pi_1} < z_{\pi_2}$. As a conclusion, $\epsilon_1 > \epsilon_2$.

Quantities, Emissions and Prices used for the Calibration

We present the individual and total quantities, the emissions rates and the market prices for each sector.

Table 3: Quantities, emission rate and prices in the nonexposed sector

	Nonexposed Sector
Individual Quantities	$q_i = \frac{1}{n}\left(\frac{\alpha(1-1/(n\beta))}{(c_i+f_0\sigma)+\frac{1}{2\gamma}({c_i'}^2-\sigma^2)}\right)^\beta$
Emission rate	$f_i = f_0 - \frac{1}{\gamma}(c_i' + \sigma)$
Total Quantities	$Q = \left(\frac{\alpha(1-1/(n\beta))}{(c_i+f_0\sigma)+\frac{1}{2\gamma}({c_i'}^2-\sigma^2)}\right)^\beta$
Price	$P_i(Q) = \frac{c_i+f_0\sigma+\frac{1}{2\gamma}({c_i'}^2-\sigma^2)}{1-1/(n\beta)}$

Table 4: Quantities, emission rate and prices in the nonexposed sector

	Exposed Sector
Individual Quantities	$q_i^H = (\frac{\alpha}{\beta})^\beta \frac{((n^F+n^H)\beta-1)^\beta\left(\beta n^F(c^F+\tau)+(1-n^F\beta^B)(c^H+(f_i^H-\omega f_0^H)\sigma)\right)}{(n^H(c^H+(f_i^H-\omega f_0^H)\sigma)+n^F(c^F+\tau))^{(\beta+1)}}$ $q_i^F = (\frac{\alpha}{\beta})^\beta \frac{((n^F+n^H)\beta-1)^\beta\left(\beta n^H(c^H+(f_i^H-\omega f_0^H)\sigma)+(1-n^H\beta)(c^F+\tau)\right)}{(n^H(c^H+(f_i^H-\omega f_0^H)\sigma)+n^F(c^F+\tau))^{(\beta+1)}}$
Emission rate	$f_i^H = f_0^H - \frac{1}{\gamma}({c_i'}^H + \sigma)$ $f_0^F = f_0^H$
Total Quantities	$Q^H + Q^F = (\frac{\alpha}{\beta})^\beta \frac{((n^F+n^H)\beta-1)}{(n^H(c^H+(f^H-\omega f_0^H)\sigma)+n^F(c^F+\tau))}$
Price	$P = \beta\frac{(n^H(c^H+(f^H-\omega f_0^H)\sigma)+n^F(c^F+\tau))}{((n^F+n^H)\beta^B-1)}$

Table 5: Quantities, emission rate and prices in the exposed sector