Unemployment (Fears), Precautionary Savings, and Aggregate Demand

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Overview

1 Model
- interaction between goods and labor market
- precautionary savings *could* end up in productive investment

2 Algorithm: XPA
- laws of motion for aggregate variables are obtained by *explicit* aggregation of individual policy functions
- correct firm value when firm owners are heterogeneous and markets are incomplete

3 Model properties
- fear of unemployment exacerbates (dampens) downturn when nominal wages are (are not) sticky
Model: Key ingredients

1. Search frictions in labor market
2. Heterogeneous agents and incomplete markets
3. (Some) nominal wage stickiness
**Individual agent**

unemployed and employed agents

- unemployed search for work
- employed get nominal wage $W_t$
- exogenous job loss probability, $\rho_x$
- agents can invest in
  - money, $M_i,t$
  - firm ownership (equity), $q_{i,t}$
First-order conditions

\[ C_{i,t} + J_t q_{i,t} + M_{i,t} \]

\[ = e_{i,t} W_t + (1 - e_{i,t}) U_t + q_{i,t-1} (D_t + (1 - \rho_x) J_t) + M_{i,t-1} \]

\[ q_{i,t} \geq 0 \]

\[ C_{i,t} = P_t c_{i,t} \]
\[ D_t = P_t d_t \]
\[ J_t = P_t j_t \]
First-order conditions

\[ c_{i,t}^{-\nu} = \beta E_t \left[ \frac{P_t}{P_{t+1}} c_{i,t+1}^{-\nu} \right] + \zeta_0 \left( \frac{M_{i,t}}{P_t} \right)^{-\zeta_1} \]

\[ \frac{J_t}{P_t} = \beta E_t \left[ \left( \frac{c_{i,t+1}}{c_{i,t}} \right)^{-\nu} \left( \frac{D_{t+1}}{P_{t+1}} + (1 - \rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right] \]
Job/Firm creation

Standard free-entry condition:

\[ P_t \psi = \pi_{f,t} J_t \]

\[ \pi_{f,t} = \phi_o \left( \frac{\nu_t}{1 - n_{t-1}} \right)^{\phi_1 - 1} \]

\[ n_t = (1 - \rho_x) n_{t-1} + \phi_o \nu_t^{\phi_1} (1 - n_{t-1})^{1-\phi_1} \]
Existing firm

\[ D_t = P_t z_t - W_t \]
Wage setting

\[ W_t = \omega_0 z_t^{\omega_1} P_t^{\omega_2} \]

- \( \omega_1 = 0, \omega_2 = 1 \): sticky real wages
- \( \omega_1 > 0, \omega_2 = 0 \): sticky nominal wages
Equilibrium

- demand for money $= (\text{constant})$ money supply
- demand for firm ownership $= \text{number of firms}$
Algorithm

1. Correctly dealing with firm value

2. XPA
   - explicit aggregation to get aggregate variables right
   - surprisingly few state variables
Firm value

\[ \frac{J_t}{P_t} \equalant \mathbb{E}_t \left[ MRS_{i,t+1} \left( \frac{D_{t+1}}{P_{t+1}} + (1 - \rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right] \]

Which \( MRS_{i,t+1} \) to use?
Firm value

\[
\frac{J_t}{P_t} \overset{?}{=} E_t \left[ MRS_{i,t+1} \left( \frac{D_{t+1}}{P_{t+1}} + (1 - \rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right]
\]

Literature:

- representative agent: \( MRS_{t+1} = \left( \frac{c_{t+1}}{c_t} \right)^{-\nu} \)
- heterogeneous agents:
  - Krusell, Mukoyama, Sahin (2010): two assets and two outcomes for aggregate state \( \rightarrow \) use prices of the two Arrow-Debreu securities
  - dinky "solution": assume risk neutral firm manager, which is inconsistent with risk averse firm owners

This paper: Get \( J(\cdot) \) by imposing equilibrium
Solving for firm value

\[ J_t = J(s_t) \]

- solve for \( J(s_t) \) by imposing equilibrium

\[ \int_i q_{i,t} \, di = n_t \]

- LHS: demand for firm ownership from individual problem
- RHS: supply of firm ownership comes from free-entry condition
Idea behind XPA

Suppose individual policy rules are linear in *individual* state variables:

\[ k_{i,t} = \alpha_0 (s_t) + \alpha_1 (s_t) k_{i,t-1} \]

\[ \implies \text{aggregation trivial, namely} \]

\[ K_t = \alpha_0 (s_t) + \alpha_1 (s_t) K_{t-1} \]
Idea behind XPA

Suppose individual policy rules are quadratic:

\[ k_{i,t} = \alpha_0 (s_t) + \alpha_1 (s_t) k_{i,t-1} + \alpha_2 (s_t) k_{i,t-1}^2 \]

\[ \implies \text{aggregation gives} \]

\[ K_t = \alpha_0 (s_t) + \alpha_1 (s_t) K_{t-1} + \alpha_2 (s_t) K_{t-1}^2 \quad (2) \]

\[ K_{t-1} (2) = \int_i k_{i,t-1}^2 di \]

\[ \implies \text{we need a law of motion for} \ K_t (2) = \int_i k_{i,t}^2 di \]
Idea behind XPA

Approach #1:

use $k_{i,t}^2 = \left( \alpha_0 (s_t) + \alpha_1 (s_t) k_{i,t-1} + \alpha_2 (s_t) k_{i,t-1}^2 \right)^2$

$K_t(2) = \int_i k_{i,t}^2 di = \int_{i,t}^\infty \left( \alpha_0 (s_t) + \alpha_1 (s_t) k_{i,t-1} + \alpha_2 (s_t) k_{i,t-1}^2 \right)^2 di$

$\implies K_t(3) \text{ and } K_t(4) \text{ become state variables, etc.}$
Idea behind XPA

Approach #2:

approximate $k_{i,t}^2$ with

$$k_{i,t}^2 = \tilde{\alpha}_0 (s_t) + \tilde{\alpha}_1 (s_t) k_{i,t-1} + \tilde{\alpha}_2 (s_t) k_{i,t-1}^2$$

which gives

$$K_t (2) = \tilde{\alpha}_0 (s_t) + \tilde{\alpha}_1 (s_t) K_{t-1} + \tilde{\alpha}_2 (s_t) K_{t-1} (2)$$

$\implies$ set of state variables does not increase
Implementation

- Individual problem is solved accurately with a global method and piecewise linear policy functions
- For aggregation a linear approximation of this nonlinear policy function is used
State variables

• Individual state variables
  • cash on hand: $q_{t-1} (D_t + (1 - \rho_x) J_t) + M_{i,t-1}$
  • employment status

• Aggregate state variables
  • aggregate productivity
  • number of firms = equity shares
Precautionary savings

How to get precautionary savings in a model?

• typically done through $\Delta \beta$
• this paper through $\Delta$unemployment
Typical precautionary savings story

Households want to save more

- \[\implies\] demand for consumption \(\downarrow\) & prices do not adjust
- \[\implies\] demand for labor \(\downarrow\), etc.

Where do savings end up?

- typically not allowed to end up in investment because
  - there is no physical investment
  - or incorrect discounting of firm profits
Precautionary savings in this paper

We do have something like the standard channel:

- unemployment $\uparrow \implies$ demand for money $\uparrow$
- $\implies P_t \downarrow \implies$ real profits $\downarrow$ (because of sticky nominal wages)
- $\implies$ firm/job creation $\downarrow$

- but in this paper !!!
Precautionary savings in this paper

We do have something like the standard channel:

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- but in this paper !!!
- precautionary savings could end up in productive investment since $MRS_{i,t} \uparrow$ when precautionary savings $\uparrow$
Precautionary savings and productive investment

- This paper: investment in firm/job creation could ↑ when precautionary savings ↑

Reasons why it could ↓:

- agents less willing to hold firm equity when profits ↓
- agents less willing to hold risky assets when unemployment ↑
Idiosyncratic risk & investment portfolio

- simple example
Idiosyncratic risk & investment portfolio

\[
\max_{c_1,c_2,m,a} \ c_t^{1-\nu} + \beta c_{t+1}^{1-\nu}
\]

s.t.

\[
c_1 = y_1 - m - a
\]

\[
c_2 = y_2 + m(1 + r_m) + a(1 + r_a)
\]

\[
y_1 = E[y_2] = 1
\]

\[
r_a = \begin{cases} 
+0.060 \text{ with prob. } \frac{1}{2} \\
-0.039 \text{ with prob. } \frac{1}{2}
\end{cases}, \quad E[r_a] > r_m
\]
Case 1 no idiosyncratic risk

- no idiosyncratic risk: $y_2 = 1$
- $m = -0.0408$ and $a = 0.0408$
- no savings, $m + a = 0$
  realizations of $r_a$ chosen to get this outcome
Case 2 idiosyncratic risk

- \( y_2 = 1 \) when \( r_a \) takes on high value
- \( y_2 \in \{0, 2\} \) \( E y_2 = 1 \) when \( r_a \) takes on low value
  - higher spread in recession
  - but mean not affected (for transparency)

- not surprisingly, \( m + a \uparrow \) to 0.226
Case 2 idiosyncratic risk

- $y_2 = 1$ when $r_a$ takes on high value
- $y_2 \in \{0, 2\}$ $E y_2 = 1$ when $r_a$ takes on low value
  - higher spread in recession
  - but mean not affected (for transparency)

- not surprisingly, $m + a \uparrow$ to 0.226
- but $m \uparrow$ to 9.2872 and $a \downarrow$ to -9.0610
Model properties

1. Model 1: no nominal wage stickyness
   precautionary savings *dampen* downturn

2. Model 2: with nominal wage stickyness
   precautionary savings *worsen* downturn
No nominal wage stickyness

- productivity ↓
- \( \rightarrow \) profits ↓ \( \rightarrow \) firm value ↓ \( \rightarrow \) unemployment ↑
- \( \rightarrow \) precautionary savings ↑
  - \( \rightarrow \) demand for firm ownership may ↑ \( \rightarrow \) unemployment ↓
  - \( \rightarrow \) demand for money ↑ \( \rightarrow \) \( P \) ↓ \( \not\rightarrow \) \( \Delta \) profits since nominal wages adjust
No nominal wage stickyness

Precautionary demand for $M$ reduces price increase
No nominal wage stickyness

Precautionary savings has small upward effect on firm value
**No nominal wage stickyness**

Precautionary savings has small upward effect on employment
With nominal wage stickyness

- productivity ↓
- \( \rightarrow \) profits ↓ \( \rightarrow \) firm value ↓ \( \rightarrow \) unemployment ↑
- \( \rightarrow \) precautionary savings ↑
  - \( \rightarrow \) demand for firm ownership may ↑ \( \rightarrow \) unemployment ↓
  - \( \rightarrow \) demand for money ↑ \( \rightarrow \) \( P \) ↓ \( \rightarrow \) profits ↓ unemployment ↑ \( \rightarrow \) downward spiral
With nominal wage stickyness

Precautionary demand for $M$ strongly reduces prices
With nominal wage stickyness

Precautionary demand for $M$ strongly reduces firm value
With nominal wage stickyness

Precautionary demand for $M$ strongly reduces firm value