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New Estimates of the Elasticity of Substitution of Land for Capital[*]

Abstract: We reconcile conflicting evidence on the magnitude of the elasticity of substitution of land for capital, which is a key determinant of the relationship between the price of land and the density of land use. We first compare the performance of classic estimation approaches with a new estimation procedure using a series of Monte Carlo experiments. We then apply the approaches to various real-world data sets drawn from Berlin, Chicago, and Pittsburgh. Our results indicate that many existing estimates are likely to be biased downward, and the true elasticity is likely to be closer to one than widely believed in the literature. The results suggest that a Cobb-Douglas form is a reasonable approximation to of the production function for housing.

Keywords: *Elasticity of substitution, housing production function, land values, Monte Carlo simulation*
Version: *February 2014*
JEL: R20, R30

1. Introduction

The elasticity of substitution between land and capital is a critical parameter in the production function for housing. As the primary determinant of the extent to which housing producers respond to land prices by intensifying land use, the elasticity of substitution influences building densities and the overall spatial structure of cities. High elasticities of substitution lead to intense land use – tall buildings on small lots. Locations where the price of land is high are likely to be occupied by firms in industries with production pro-

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* We thank the participants of the 2013 Urban Economics and Public Finance conference hosted by the Lincoln Institute of Land Policy, especially Sebastien Bradley, Leah Brooks, Tom Davidoff and Chris Redfearn. We also thank the Lincoln Institute and especially Joan Youngman and Semida Munteanu for generous funding of the data collection. We are also grateful to the Center for Metropolitan Studies, especially Dorothee Brantz, for hosting a team of researchers during the project work. Kristoffer Moeller and Sevrin Weights are acknowledged for their great contribution to designing and coordinating the compilation of the data set. Philip Boos, Aline Delatte, Nuria-Maria Hoyer Sepulvedra, Devika Kakkar, Rene Kreichauf, Maike Rackwitz, Lea Siebert, Stefan Tornack, Tzvetelina Tzvetkova provided excellent research assistance.

cesses for which capital is readily substituted for land. Despite its importance, estimates of the elasticity of substitution between land and capital are surprisingly scarce, relatively old, and not entirely uniform.

Empirical research on the housing production function dates back at least to Muth (1964, 1971) and, thus, is almost as long as the history of modern urban economics. However, the estimates produced in this literature are not particularly consistent and vary from less than 0.5 to greater than unity. Some studies report values as low as around 0.4, including Arnott & Lewis (1979); Dowall & Treffeisen (1991); Fountain, (1977a); Jackson, Johnson, & Kaserman, 1984 (1984); Polinsky & Ellwood (1979); Rosen, (1978). Studies reporting elasticities of greater than one include Clapp (1980); McDonald (1979); and Epple, Gordon, and Sieg (2010). Thus, there not only is no consensus on the magnitude of this important parameter, there is disagreement whether expenditures on land tend to increase or decrease as the price of land rises.

Traditional estimation approaches to estimating the elasticity of substitution between land and capital are plagued by measurement error. The most common approach is to regress some measure of the log of capital spent per unit of land on the log of a measure of land value per unit of land (e.g. Clapp, 1979; Fountain, 1977b; Koenker, 1972 among many others). Because capital is not observed directly, its value is typically measured as the difference between a property's sale price and a measure of the price of land. Unfortunately, land prices are also not readily observed, and moreover the approach results in the proxy appearing on both sides of the estimating equation. The measurement error will generally lead to a downward bias in the estimate of the elasticity of substitution. Consistent with this expectation, one of the few studies with accurate land values found a relatively high elasticity of substitution, and could not reject a unitary elasticity (Thorsnes, 1997). A possible alternative to accurate land values is to use an instrumental variable (IV) approach to correct for measurement error. However, good instruments are not necessarily easy to find in practice.

In this paper, we attempt to reconcile the conflicting evidence on the elasticity of substitution between land and capital by taking advantage of a new approach to the housing production function developed by Epple, Gordon, and Sieg (2010), which we will refer to as the "EGS" approach. EGS use duality theory to derive an estimation strategy that requires

data on only three variables, land values, lot sizes, and housing values. We argue that their approach lends to an estimation strategy that is less sensitive to measurement error problems than the classic approach. Building on their approach, we present various estimation strategies that can be used to obtain accurate estimates of the elasticity of substitution. We argue that much of the difference between earlier studies that produced relatively low estimates and more recent evidence pointing to larger values can be attributed to measurement error problems. We further demonstrate that the true elasticity of substitution of land for capital is likely closer to unity than long suspected in the literature.

This finding is important because it suggests that the Cobb-Douglas functional form is a reasonable approximation to the housing production function. This finding lends support to studies of housing and land markets that assume a Cobb-Douglas functional form for theoretical convenience (see e.g. Combes, Duranton, & Gobillon, 2013 for a recent example). If the share of capital and land in the production of space are approximately constant, there may also be less need for a separate treatment of land in the macroeconomic production function. Our results therefore also lend support to the macroeconomic literature that typically treats land as a component of capital.[1] Finally, a unity elasticity of substitution greatly facilitates the approximation of land values and the implementation of a Georgian land value tax, which is often argued to be less distortionary than a property tax.[2]

We proceed in two steps. First, we compare the traditional estimation technique to various derivatives of a new estimation approach in a Monte Carlo study. The main purpose of this exercise is to evaluate the sensitivity of the estimation methods to measurement error in land prices under laboratory conditions using a stylized data set. Second, we apply the traditional and new methods to a number of real-world data sets. We provide new estimates of the elasticity of substitution using our preferred new approach. We then compare

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[1] The macroeconomic literature has typically focused on the estimation of the elasticity of substitution of capital for labor without explicitly considering land (Gollin, 2002; Kaldor, 1961; Raurich, Sala, & Sorolla, 2012; Revankar, 1971). Valentinyi & Herrendorf (2008) consider land as a separate input factor, but assume a Cobb Douglas function.

[2] See McDonald (1981) for a theoretical discussion of the link between property price and land value. See Dye and England (2010) for a recent summary of the benefits of a land value tax.

the estimates to the results derived from the traditional estimation procedure to assess the degree of bias in existing estimates.

We take advantage of two new data sets for the study, along with the residential and commercial data sets for the Pittsburgh metropolitan area previously used by EGS. The data sets are from Chicago and Berlin. For Chicago, we are able to exploit two alternative sources of land prices. We use a 1990 cross-section of land values from by Olcott's Land Values Blue Book of Chicago, a source that has been used extensively by academic researchers, including Berry (1976), Kau and Sirmans (1979), McDonald and Bowman (1979), McMillen ((1996), McMillen and McDonald (2002), Mills (1969), and Yeates (1965), among others. As an alternative, we also use a sample of vacant land sales to measure land prices for Chicago. For Berlin, our data set comprises all transactions of developed properties for 1990-2010, including the sale price, date of sale, lot size, and an assessed value of land. Thus, our data sets include both actual prices of vacant land (for Chicago) and assessed values (for Berlin, Chicago, and Pittsburgh). The results are remarkably similar across data sets, all of which point to approximately unitary elasticities of substitution.

The remainder of the paper is organized as follows. Section 2 introduces the traditional and the new estimation approach. Section 3 presents the results of a Monte Carlo study of a stylized data set. Sections 4 and % present the data and the analysis of the real-world data sets. Section 6 offers some conclusions.

2. Estimating the Elasticity of Substitution

The starting point for most studies of the elasticity of substitution is a simple concave production function of degree one for housing (H) with two inputs, land (L) and capital (K). Normalizing the price of capital to unity, zero profits imply that $pH = K + RL$, where p is the unit price of housing and R is land rent per unit. By definition, the elasticity of substitution between land and capital is:

$$\sigma = \frac{dlog(K/L)}{dlogR} \tag{1}$$

2.1 Traditional Studies

The early empirical literature is summarized in McDonald (1981). Most studies assume a constant elasticity of substitution (CES) production function. The most general form of the CES production function is obtained by substituting the first-order conditions for profit maximization into equation (1). The result implies a simple linear relationship between the log of the capital to land ratio and the log of land rent:

$$log\left(\frac{K}{L}\right) = c + \sigma logR \tag{2}$$

where c is a constant. However, K is not directly observable when estimating a production function for housing. A potential solution to this problem is to use the zero-profit condition to obtain the value of K implied by the sale price of housing and land rent:

$$log\left(\frac{pH - RL}{L}\right) = c + \sigma logR \tag{3}$$

Equation (3) has served as the estimating equation in several early studies (e.g. Clapp, 1979; Fountain, 1977b; Koenker, 1972).

One problem with the traditional approach is that land rents are not readily observable. If R is measured with error, attenuation bias is likely to cause the estimated value of σ to be biased downward. Although instrumental variable estimation procedures can produce unbiased estimates, good instruments for R are not necessarily easy to identify.

2.2 Epple, Gordon, and Sieg (2010) Approach

EGS use duality theory to develop the implications for the relationship between the housing production function and two potentially observable variables, housing value per unit of land ($v \equiv pH/L$) and land rent (R). Using the relatively general assumption of a concave, constant returns to scale production function and a competitive construction sector, they show that land value in equilibrium is a function of housing value per unit of land as long as v is a monotonic function of p. The general relationship is:

$$R = f(v) \tag{4}$$

EGS use the no-profit condition to express the capital-land ratio as a function of v:

$$\frac{K}{L}(v) = v - R(v) \tag{5}$$

Thus, there is no need to attempt to measure the capital-land ratio directly; all that is necessary is to estimate the relationship between land rent and *v*.

EGS suggest using a polynomial function or a nonparametric estimation procedure to estimate equation (4). The emphasis of the empirical section of their paper is the estimation of the supply function of housing rather than the elasticity of substitution of land and capital. Their discussion suggests a two-stage procedure for estimating the elasticity of substitution: (1) estimate equation (4) directly using a flexible functional form or using nonparametric methods, (2) use the implied estimates for the capital-land ratio to estimate the elasticity of substitution. Since the objective in stage 2 is to estimate $dlog(K/L)/dlogR$, the implied second-stage regression is:

$$\log\left(\frac{\widehat{K}}{L}\right) = log\big(v - \hat{R}(v)\big) = \kappa + \sigma log\hat{R}(v) \tag{6}$$

Empirically, an advantage of this approach is that $R(v)$ can be derived from the relationship between land values and property price per land unit, which implies that problems related to measurement error and omitted variables are likely to be less serve than is the case with traditional methods.

Equation (6) imposes a restriction of a constant elasticity of substitution. This restriction need not be imposed, however. Equation (6) can be estimated using flexible functional forms or nonparametric methods. The estimated values of σ then vary by *R*.

Equation (6) does not include any information that is not already contained in equation (5). Both equations summarize relationships between *v* and *R*; it is only the functional form specification in equation (6) that implies a constant elasticity. The estimated relationship $\log\left(\frac{\hat{K}}{L}\right) = log\big(v - \hat{f}(v)\big)$ implies that

$$\hat{\sigma} = \frac{\hat{f}(v)}{v - \hat{f}(v)}\left(\frac{1}{\hat{f}'(v)} - 1\right) \tag{7}$$

The combination of the levels and derivatives of $\hat{f}(v)$ is all the information needed to estimate the elasticity of substitution. The estimates in equation (7) vary by observation.

The average of the estimates will be close to the estimated value of σ from equation (6) if the production function truly is CES.

The estimated functional form in equation (4) does not have to be linear. For example, a logarithmic relationship may provide a better fit:

$$logR = g(\log(v)) \tag{8}$$

The estimated capital-land ratio is then $\frac{\hat{K}}{L} = v - exp(\hat{g}(v))$, and the implied elasticity of substitution is:

$$\hat{\sigma} = \frac{1}{v - exp(\hat{g}(v))}\left(\frac{v}{\hat{g}'(v)} - exp(\hat{g}(v))\right) \tag{9}$$

Nonparametric estimates of equation (8) provide the necessary predictions of both levels and derivatives that are required to estimate the elasticity of substitution.

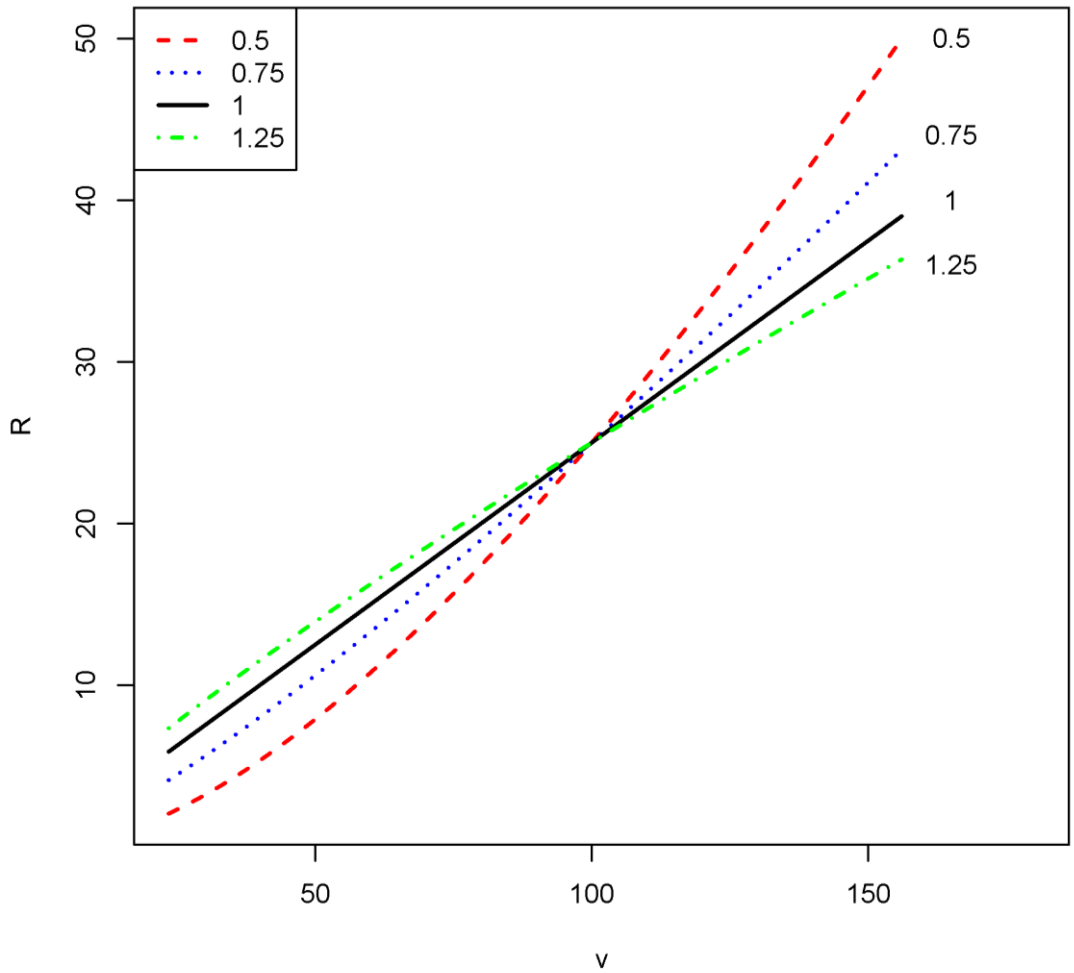
EGS suggest using a polynomial function for the linear specification of equation (4) because a flexible parametric function allows the constraint $f(0) = 0$ to be imposed easily. In contrast, nonparametric estimators use a series of averages or local regressions to approximate an unknown function at each data point. If the range of the data does not include the origin, there is no need to impose this constraint on nonparametric estimates, and the fit of the function will not be improved by imposing the constraint. Thus, there is no reason to restrict estimation to a linear dependent variable; a logarithmic specification such as equation (8) can work well in a nonparametric setting.

3. Monte Carlo Study

The EGS approach can be implemented in a variety of ways. A flexible parametric functional form or a nonparametric estimator can be employed to estimate the relationship between R and v or between simple transformations such as log(R) and log(v). The elasticity of substation can then be estimated directly using the implied expressions for $dlog(K/L)/dlogR$ as shown in equations (7) and (9), or it can be estimated using a second-stage linear regression as in equation (6). In this section, we report the results of a limited Monte Carlo study that compares alternative methods of estimating the elasticity of substitution between land and capital in a stylized data set.

All of the Monte Carlo experiments are derived from the implied relationship between land rent, R, and housing value per unit of land, v. For the first set of Monte Carlo experiments, the relationship between R and v implies a constant elasticity of substitution. Equation (7) implies that the elasticity of substitution will be constant if $\frac{R}{v-R}(f'(R)-1)=\sigma$. A function that meets this condition is $v=R+\gamma R^{\sigma}$. Figure 1 illustrates how the shape of the function varies in σ. We allow the elasticity of substitution to vary from 0.25 to 1.25 in increments of 0.25 for the Monte Carlo experiments. This set of experiments should be most favorable to the traditional estimation procedure in which $\log\big((PH-RL)/L\big)=\log(v-R)$ is regressed on $\log(R)$ using either OLS or IV estimation procedures. This set of experiments should also be favorable to the two-stage version of the EGS approach, in which first-stage estimates of $R=f(v)$ or $logR=g(\log(v))$ are followed by regressions of $log\big(v-\hat{f}(v)\big)$ on $log\hat{R}(v)$. For these sets of CES experiments, we draw R from a uniform distribution ranging from 2 to 50. We then set $v=R+\gamma R^{\sigma}$ for σ = 0.25, 0.50, 0.75, 1.0, and 1.25. Since $E(R)=25$ and $\gamma=\frac{v-r}{r^{\sigma}}$, we set $\gamma=\frac{75}{25^{\sigma}}$ to ensure that $Ev\approx 100$.

Fig. 1. CES Functional Forms at Alternative Elasticities



The second set of experiments allows for variable elasticities of substitution (VES). The base functions are based on the locally weighted regression (LWR) estimates of $R = f(v)$ and $logR = g(\log(v))$ using the established EGS (residential) data set. To ensure that the data sets for these experiments do not have sparse regions, we set v for the VES experiments to a series of values with equal increments ranging from the smallest to the largest values of v in the EGS data set: $\min(v)$ = 0.15 and $\max(v)$ = 366.62. These sets of experiments will be more favorable to the EGS approach in which the elasticity is calculated directly from equations (7) or (9). The number of observations is 1000 for all experiments. The number of replications of each experiment is also 1000.

We use measurement error in land rents as the basis for variation across 1000 iterations of each experiment. We draw errors randomly from a normal distribution with a constant variance to define the "measured" values of log(R): $u_m \sim N(0, (1-\rho_{rm})^2 var(R)/\rho_{rm}^2)$, where ρ_{rm} is the desired correlation between the true and measured values of $logR$, i.e.,

$E\{cor(logR, logR_m)\} = \rho_{rm}$, where $logR_m = logR + u_m$. We set $\rho_{rm} = 0.8$ for all experiments.

Whereas EGS estimation approaches only require information on v and R, traditional OLS and IV estimation also requires a direct measure of lot size, L. Although an assumption of constant returns to scale implies that $cov\left(\frac{K}{L}, R\right) < 0$, it has no direct implications for $cov(R, L)$. For the Monte Carlo experiments, we impose the reasonable assumption that $cor(logR, logL)$ = -0.80, and use this assumption to draw errors from a normal distribution to imply $E(logL) = \log(10{,}890)$: $logL = \log(10{,}890) - \left(\log(R) - mean(\mathrm{R})\right) + u_r$, where $u_r \sim N\left(0, \frac{1-.8^2}{.8^3} var(\log(R))\right)$. These values imply that lot sizes are close to a quarter of an acre (10,890 square feet) on average. With this variable observed, the dependent variable for traditional OLS and IV estimation is simply $\log(v - R)$ at the true values of R, and $\log(v - R_m)$ at the observed or "measured" values.

The final variable needed for the Monte Carlo analysis is an instrument for $logR_m$. We construct instruments, Z, such that $cor(Z, logR)$ = .8 on average. A classic instrument is also uncorrelated with the measurement error in $logR_m$. In practice, there is no guarantee that this condition is met. Thus, we also conduct experiments in which $cor(Z, u_m) =$ .25 and $cor(Z, u_m) =$ .50.

For each of the EGS methods, we use two methods to construct estimates of the average elasticity of substitution from the estimated relationship between either $logR_m$ and $\log(v)$ or R_m and v. In the "single-stage" version, we construct trimmed means of equations (7) for the linear models and (9) for the logarithmic models, discarding the lowest and highest 5% of the estimated values. The "regression" version is simply the estimated coefficient from a regression of $log\left(v - \hat{R}\right)$ on $log\hat{R}(v)$. We also experiment with approximations of the relationship between R and v in logs and levels as well as using polynomial and LWR approximations. The sequence in which we run our Monte Carlo analysis is defined in detail in Table 1:

Tab. 1. Monte Carlo Study Design

Traditional Methods	
1-1)	OLS: Regress $\log(v - R_m L)$ on $logR_m$
1-2)	IV: Regress $\log(v - R_m L)$ on $logR_m$
1-2-1)	Instrument Z for $logR_m$. with $cor(Z, u_m) = 0$
1-2-2)	Instrument Z for $logR_m$. with $cor(Z, u_m) = 0.25$
1-2-3)	Instrument Z for $logR_m$. with $cor(Z, u_m) = 0.50$
EGS	
2-1)	Linear:
2-1-1)	Estimate $R_m = f(v)$ using a fourth-order polynomial regression
2-1-1-1)	Two-stage : Equation (4) & equation (6)
2-1-1-2)	Single-stage: equation (7)
2-1-2)	Estimate $R_m = f(v)$ using LWR using a 25% window width.
2-1-2-1)	Two-stage: Equation (4) & equation (6)
2-1-2-2)	Single-stage: equation (7)
2-2)	Logarithmic:
2-2-1)	Estimate $logR_m = g(\log(v))$ using a fourth-order polynomial regression.
2-2-1-1)	Two-stage: equation (8) and
2-2-1-2)	Single-stage: equation (9) & equation (6)
2-2-2)	Estimate $logR_m = g(\log(v))$ using LWR using a 25% window width
2-2-2-1)	Two-stage : equation (9) & equation (6)
2-2-2-2)	Single-stage: equation (9)

Means and standard deviations across 1000 replications of the experiment are shown in Table 2. The results for the CES production function are shown in the first four columns. As expected, the IV estimator with a perfect instrument ($cov(Z, e) = 0$) corrects the downward bias in the OLS estimate of the substitution elasticity. However, the estimated values of σ display a significant downward bias when $cov(Z, e) > 0$. All of the EGS estimation approaches provide accurate estimates of the elasticity of substitution. The polynomial regression approaches are more accurate when the elasticity is estimated using a second-stage regression rather than as the trimmed mean of the individual values. Overall, the logarithmic specifications have the lowest standard deviations among the EGS approaches. Indeed, the standard deviations for the logarithmic versions of the EGS approach are lower than the IV procedure even when $cov(Z, e) = 0$.

The last two columns of Table 2 show estimates of the average elasticities for the VES versions of the model. The true average elasticity is 1.26 when the base function is $R = f(v)$ and it is 1.14 when the base function if $\log(R) = g(\log(v))$. The IV model with $cov(Z, e) = 0$ produces a reasonably accurate estimate of the average elasticity under the linear base specification, but it significantly overstates the average elasticity for the logarithmic model. The polynomial versions of the EGS model tend to have high variance and are particularly prone to bias when the average elasticity is estimated in a single stage

rather than using a second-stage regression. The LWR versions of the EGS model tend to be quite accurate, particularly when the average elasticity is estimated using a second-stage regression.

Tab. 2. Monte Carlo Results: Constant Elasticity of Substitution

Estimation Procedure		Constant Elasticity of Substitution δ				$R = f(v)$	$logR = g(\log(v)$
		0.50	0.75	1	1.25	$\bar{\delta} = 1.26$	$\bar{\delta} = 1.14$
1-1	OLS	0.141	0.308	0.477	0.648	0.708	0.722
		(0.018)	(0.018)	(0.020)	(0.023)	(0.023)	(0.018)
1-2-1	IV, cor(Z, e) = 0	0.472	0.741	1.012	1.273	1.232	1.260
		(0.028)	(0.037)	(0.047)	(0.055)	(0.052)	(0.048)
1-2-2	IV, cor(Z, e) = 0.25	0.303	0.522	0.742	0.961	0.962	1.107
		(0.021)	(0.025)	(0.031)	(0.039)	(0.035)	(0.045)
1-2-3	IV, cor(Z, e) = 0.50	0.186	0.370	0.555	0.742	0.778	0.978
		(0.019)	(0.021)	(0.024)	(0.030)	(0.024)	(0.040)
2-1-1-1	1: Linear Polynomial	0.493	0.747	0.996	1.228	1.186	1.082
	2: Regression	(0.025)	(0.038)	(0.052)	(0.071)	(0.116)	(0.232)
2-1-1-2	Single-Stage Linear	0.542	0.791	1.029	1.258	1.492	1.022
	Polynomial	(0.081)	(0.089)	(0.100)	(0.122)	(0.304)	(17.672)
2-1-2-1	1: Linear LWR	0.496	0.753	1.004	1.234	1.205	1.127
	2: Regression	(0.020)	(0.028)	(0.035)	(0.041)	(0.078)	(0.024)
2-1-2-2	Single-Stage Linear	0.525	0.762	1.006	1.267	1.438	1.350
	LWR	(0.033)	(0.043)	(0.053)	(0.066)	(0.094)	(0.152)
2-2-1-1	1: Log Polynomial	0.513	0.763	1.008	1.237	1.207	1.236
	2: Regression	(0.018)	(0.024)	(0.032)	(0.038)	(0.035)	(0.035)
2-2-1-2	Single-Stage Log	0.551	0.795	1.033	1.262	1.519	1.302
	Polynomial	(0.070)	(0.077)	(0.086)	(0.102)	(0.163)	(0.073)
2-2-2-1	1: Log LWR	0.512	0.762	1.007	1.235	1.201	1.236
	2: Regression	(0.018)	(0.024)	(0.032)	(0.038)	(0.034)	(0.035)
2-2-2-2	Single-Stage Log	0.501	0.763	1.010	1.231	1.220	1.236
	LWR	(0.024)	(0.030)	(0.036)	(0.041)	(0.036)	(0.037)

Notes: Mean elasticity of substitution estimates are followed by standard deviations are in parentheses. A more detailed description of the methods used is in Table 1.

The results shown for the LWR versions of the EGS model in Table 2 all have a window size of 25% for the first-stage regressions. For the versions with the "single-stage" label, the derivatives of the functions are estimated with a different window size, 75%. Table 3 shows how the results change as the two windows sizes vary for the CES model with σ = 0.5. When the elasticity is estimated by a second-stage regression, the results are most accurate when a relatively small window size of 10% or 25% is used for the first-stage LWR estimates. Comparable results are achieved by single-stage models when the second-stage window size is relatively large – 75% or 100%. Overall, the results suggest that the LWR models are not highly sensitive to the choice of window size as along as a relatively large window size is used in for the derivatives or the elasticity is estimated using a second-stage regression.

In summary, the Monte Carlo results confirm that traditional estimates are subject to significant downward bias when the instruments are correlated with the measurement error in land rent. The EGS approach is not vulnerable to this problem because land rent serves as the dependent variable. In general, a two-stage EGS approach appears to provide the best combination of low bias and variance. Based on these results, are preferred approach involves nonparametric estimation of either $R = f(v)$ or $\log(R) = g(\log(v)$, followed by simple OLS estimation of $\log\left(\frac{\hat{R}}{L}\right) = \kappa + \sigma log\hat{R}(v)$.

Tab. 3. Alternative Window Sizes for CES Model with σ = .5

Window for Levels	Window for Derivatives	LWR, Regression (2-1-2-1)	LWR, Single Stage (2-1-2-2)	Log LWR, Regression (2-2-2-1)	Log LWR, Single Stage (2-2-2-2)
0.1	0.1	0.495	0.103	0.508	0.158
0.1	0.1	(0.020)	(0.168)	(0.018)	(0.153)
0.1	0.25		0.581		0.595
0.1	0.25		(0.189)		(0.254)
0.1	0.5		0.527		0.548
0.1	0.5		(0.069)		(0.059)
0.1	0.75		0.510		0.529
0.1	0.75		(0.044)		(0.036)
0.1	1		0.521		0.501
0.1	1		(0.033)		(0.023)
0.25	0.25	0.496	0.580	0.512	0.595
0.25	0.25	(0.020)	(0.190)	(0.018)	(0.255)
0.25	0.5		0.530		0.549
0.25	0.5		(0.069)		(0.059)
0.25	0.75		0.514		0.529
0.25	0.75		(0.044)		(0.037)
0.25	1		0.525		0.501
0.25	1		(0.033)		(0.024)
0.5	0.5	0.477	0.532	0.513	0.549
0.5	0.5	(0.025)	(0.068)	(0.018)	(0.059)
0.5	0.75		0.517		0.529
0.5	0.75		(0.043)		(0.037)
0.5	1		0.528		0.501
0.5	1		(0.033)		(0.024)
0.75	0.75	0.434	0.520	0.514	0.529
0.75	0.75	(0.035)	(0.042)	(0.018)	(0.037)
0.75	1		0.531		0.501
0.75	1		(0.032)		(0.024)
1	1	0.397	0.532	0.511	0.502
1	1	(0.030)	(0.032)	(0.018)	(0.024)

Notes: Mean elasticity of substitution estimates are followed by standard deviations are in parentheses.

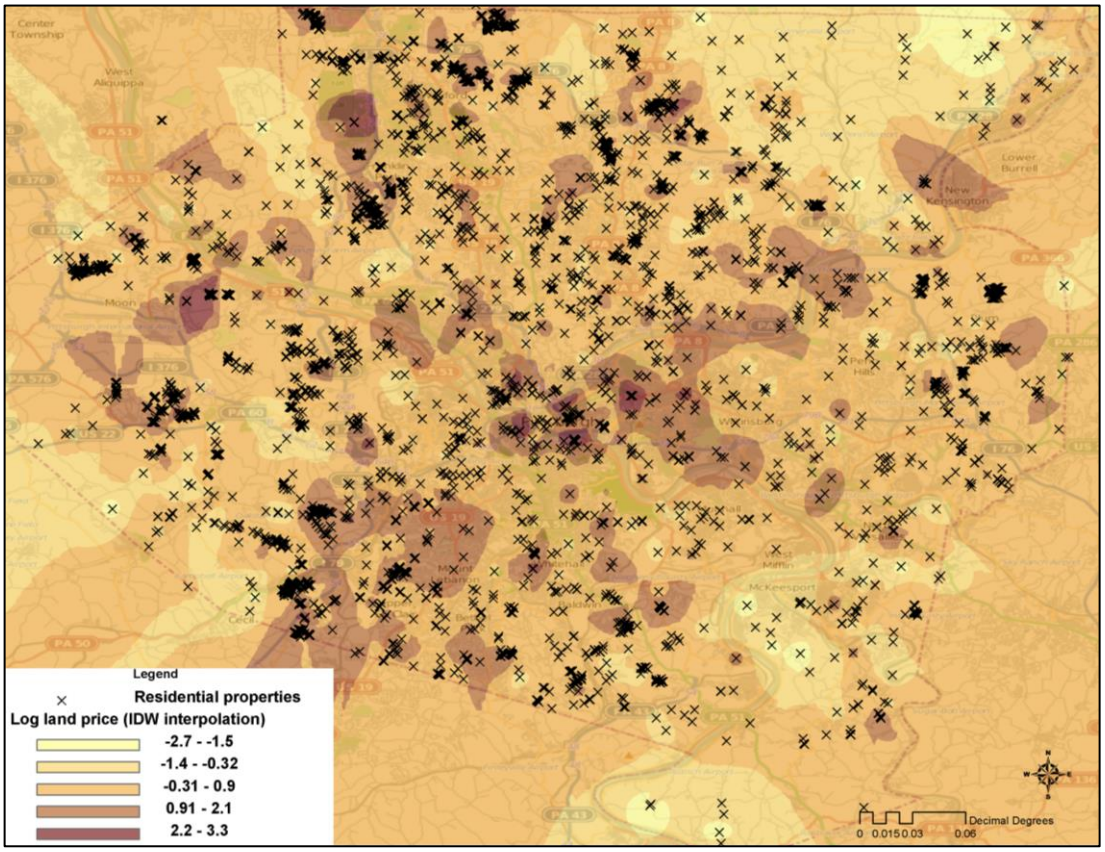
4. Data

Our data sets are drawn from three different urban areas: Berlin, Chicago, and Allegheny County, PA, which includes most of the Pittsburgh metropolitan area. In this section, we provide a discussion of the three datasets and the methods used to process the raw data where necessary. For Allegheny County, we use the EGS data as provided on the AEA website. The Berlin and Chicago data sets are both new. We have trimmed the Berlin and Chicago data to exclude outliers and implausible observations. All numbers reported below refer to the cleaned data sets.

Allegheny County, Pennsylvania

Although the Pittsburgh data set was used by EGS to illustrate their approach to estimating housing production functions, their emphasis was not on the elasticity of substitution. The data set contains assessed values of developed properties. The assessments include overall property value, along with land value and area of each lot. All property and land was valued in a major assessment exercise in 2001. To determine the site value the assessors used a mix of comparable vacant land sales and residual land option methods. Since the EGS data set includes housing units that were built after 1995 it is reasonable to assume that the assessed housing stock was relatively new. Their data set contains 6,362 residential units, which are depicted in Figure 2. The data set also contains 992 commercial properties with a lot area of at least 10,000 square feet, which are all located in the downtown area. However, a map cannot be presented for the commercial properties because the geographic coordinates are missing. A more detailed description of the data is provided by EGS.

Fig. 2. Residential Property Transactions and Land Value in Allegheny County



Notes: Data provided by EGS. (log) land prices are spatially interpolated from properties included in the EGS data set (black crosses) using inverse distance weights (IDW).

Chicago, Illinois

We have merged our Chicago data sets from various sources. First, we use information on property characteristics stem from the 2003 assessment roll provided by the Cook County Assessor's Office. The data set covers the full cross-section of small residential properties, including information on lot size and construction date. Second, a property transaction data set that was provided by the Illinois Department of Revenue featuring transaction dates and sales prices as well as a number of indicators to identify "non-arm's length" sales (see Daniel P. McMillen & O'Sullivan, 2013 for details). Both data sets can be merged via a unique parcel identifier (PIN) and geo-referenced using a detailed electronic map showing the boundaries of these parcels. We then add two distinct measures of pure site value to the this data set

The first land price measure is a digitized version of the 1990 edition of *Olcott's Blue Books of Chicago*. Olcott's Blue Books provide front-foot land values estimates for Chicago,and many of its suburbs in the form of detailed printed maps. Olcott's Blue Books are a reputa-

ble source from an established assessment company that stayed in business for more than 80 years. Smaller samples of Olcott's land values have previously been used in such studies as Berry (1976), Kau and Sirmans (1979), McDonald and Bowman (1979), McMillen ((1996), McMillen and McDonald (2002), Mills (1969), and Yeates (1965). This project is the first to draw from a completely digitized version of the 1990 edition, which was one of the last editions published before the Olcotts retired.[3] The Olcott's data were coded for 330 x 330 feet tracts that closely follow the Chicago grid street structure.

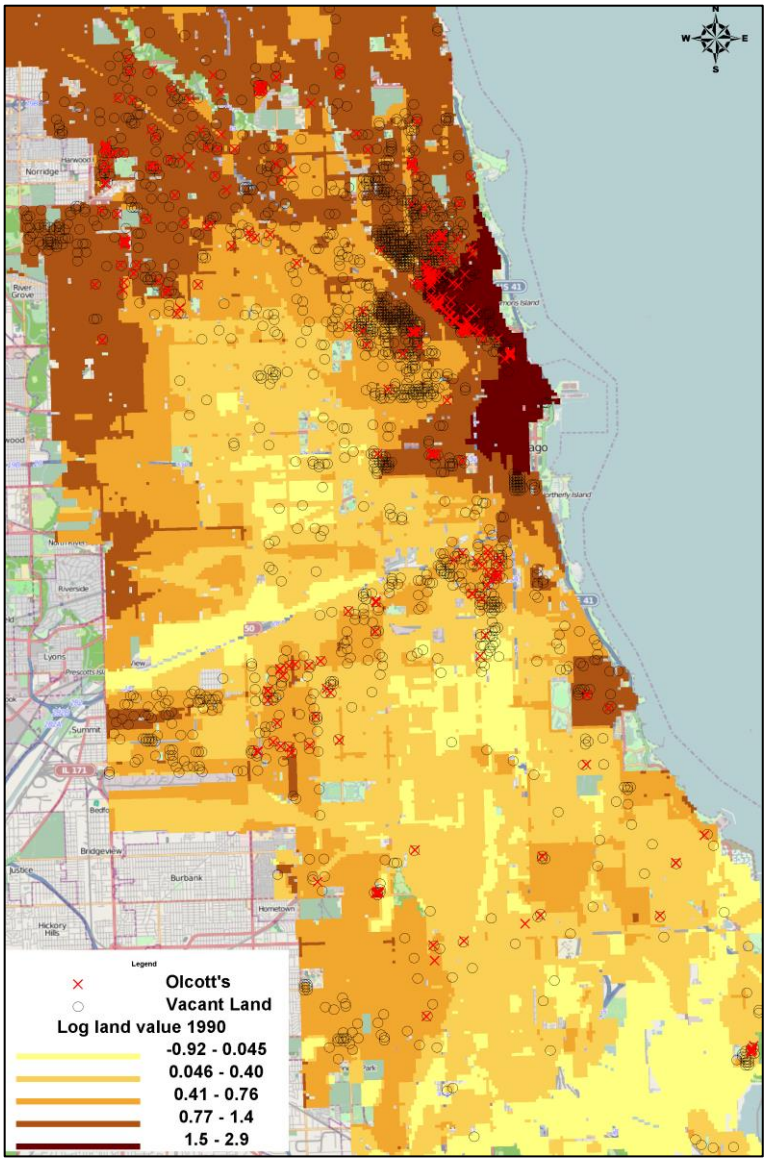
The next step is to merge the Olcott's data with the transactions file. Most of Chicago's housing stock is relatively old, dating from the 1950s and earlier. To assure that the capital stock reflects current market conditions rather than the those from the time the homes were built, we restrict the set of transactions to properties that were built within 1986 and 1994, i.e., a 5-year window around time covered by the Olcott's data. This restriction to new construction leaves us with 414 transactions. Transaction prices for this sample were adjusted to 1990 levels using a repeated sales price index.

The second land price measure is based on vacant land sales. An advantage of using actual sales of vacant land is that they represent true transaction prices rather than assessed values. Potential disadvantages are that the sample of vacant land sales may not be representative of the overall market, and the locations of the land sales are not the same as the locations of the sales of new homes. To match two sets of locations, we run a series of locally weighted regression of (log) vacant land sales prices on distance to the CBD, the geographic coordinates, and the year of a vacant land transaction. We run one LWR for each developed property *j* for which a pure site value needs to be predicted. In each iteration we weight all observed vacant land transactions *i* with the following kernel weight $w_{ij} = \exp\left(-\tau^2 D_{ij}^2\right) \times \exp(-\tau^2 T_{ij}^2)$, where *D* is the geographic distance between a property transaction *i* and a vacant land sale *j*, and *T* is the time distance simply defined as the absolute difference between the sales years ($T_{ij} = |T_i - T_j|$). We set τ to 0.5, which ensures that only vacant land sold in close proximity and at a similar point in time receives a significant weight in the LWR. This approach provides estimated land values that are tailored specifically to the location and date of sale for each transaction of newly built property. As our

[3] Funding for the digitization project was generously provided by the Lincoln Institute of Land Policy.

full set of transactions data for vacant land covers 1983-2011 rather than just 1990, our LWR estimates allow us to match land prices to a much larger set of newly built properties: the data set includes transactions of 3,576 properties that were no more than 5 years old at the time of sale. The locations of the properties represented in the two resulting data sets is shown in Figure 3. The LWR land value estimates are also shown in this figure.

Fig. 3. Property Transactions and Land Values in Chicago



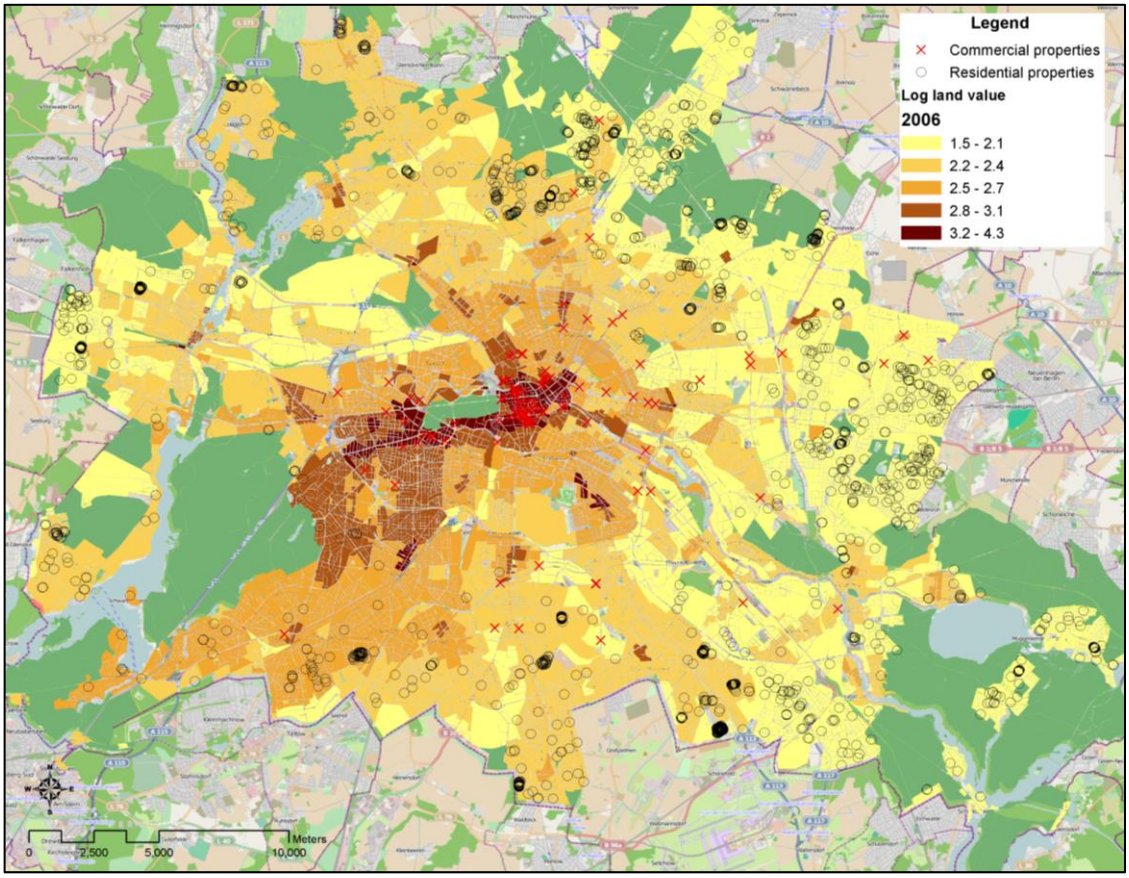
Notes: Log land values (in the back ground) were digitized from the 1990 Olcott's blue book edition. Properties in the Olcott data set (red crosses) were transacted between 1986 and 1994. Properties in the vacant land data set were transacted between 1983 and 2008.

Berlin, Germany

The Berlin data set was provided by the local Committee of Valuation Experts (Gutachterausschuss fuer Grundstueckswerte). We were provided with a full record of trans-

actions of developed properties that occurred between 1990 and 2010. The data set includes all the information we require for the analysis, including the transaction price and date, the lot size, the year of construction as well as an assessed land value (Bodenrichtwert) computed by the Committee. Valuation committees have been commissioned to produce similar assessed land values for more than fifty years in every municipality across the country. Transaction data and assessed land values provided by the Berlin Committee of Valuation experts have been used e.g. by Ahlfeldt (2011) and Ahlfeldt et al (2012) who provide more detailed information on the data. Figure 4 plots the spatial distribution of 5,466 residential and 273 commercial properties, which by the time of their transaction were not older than five years and are considered in the analysis.

Fig. 4. Property Transactions and Land Values in Berlin



Notes: Log land values (in the background) were digitized from the 2006 land value map published by the the local Committee of Valuation Experts (Gutachterausschuss fuer Grundstueckswerte). Properties in both displayed data sets were transacted between 1990 and 2010.

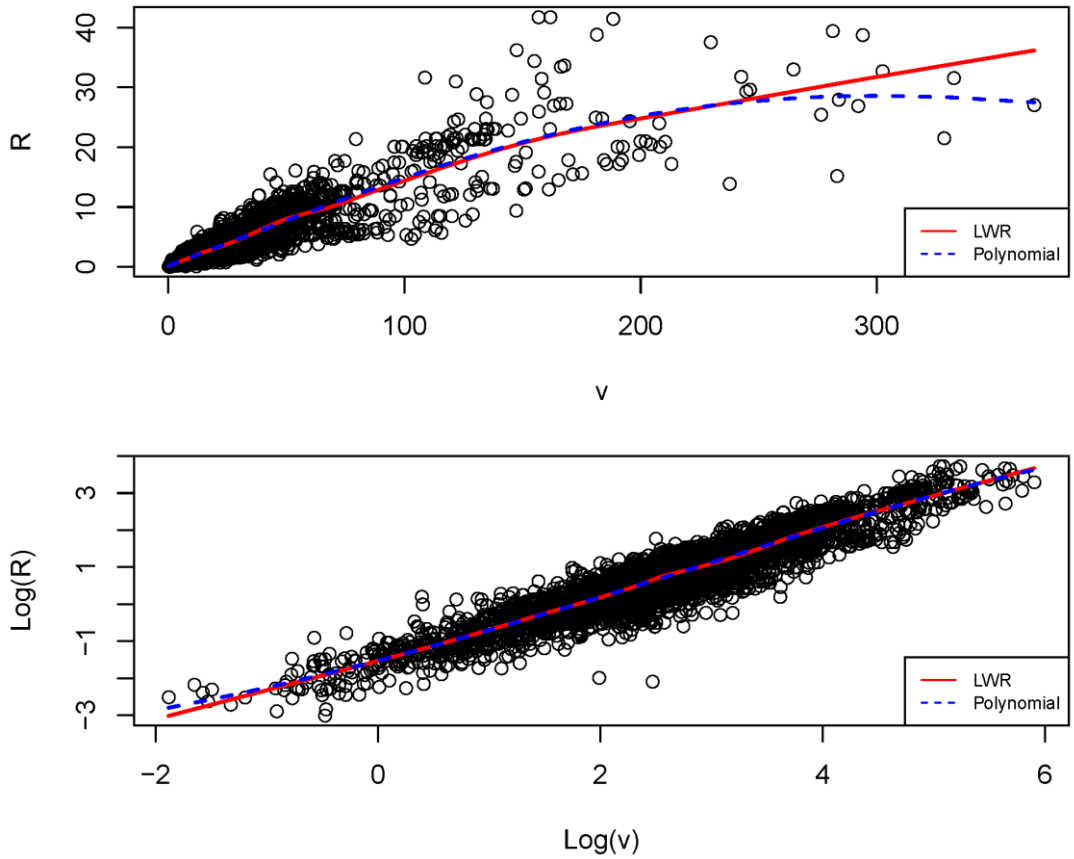
5. Empirical results

We begin our empirical analysis with a detailed presentation of results for the Pittsburgh data set. We then compare the results across the three data sets.

5.1 Pittsburgh

Figure 5 presents the raw data, nonparametric, and fourth-order polynomial estimates of equations (4) and (8) for residential Allegheny County data set. Note the sparseness of the data at low values of v, which is evidence favoring the logarithmic specification. The implied origin for the lower panel of Figure 1 is $v = -\infty$, which clearly is far from the lower limit of the data. The top panel of Figure 5 shows that the data are heteroskedastic in a linear specification. The sparseness of the data at high values of v under the linear specification of the dependent variable can cause polynomials and nonparametric estimates to behave erratically as they chase observations in sparse areas. Figure 5 suggests that a logarithmic specification is the preferred econometric model.

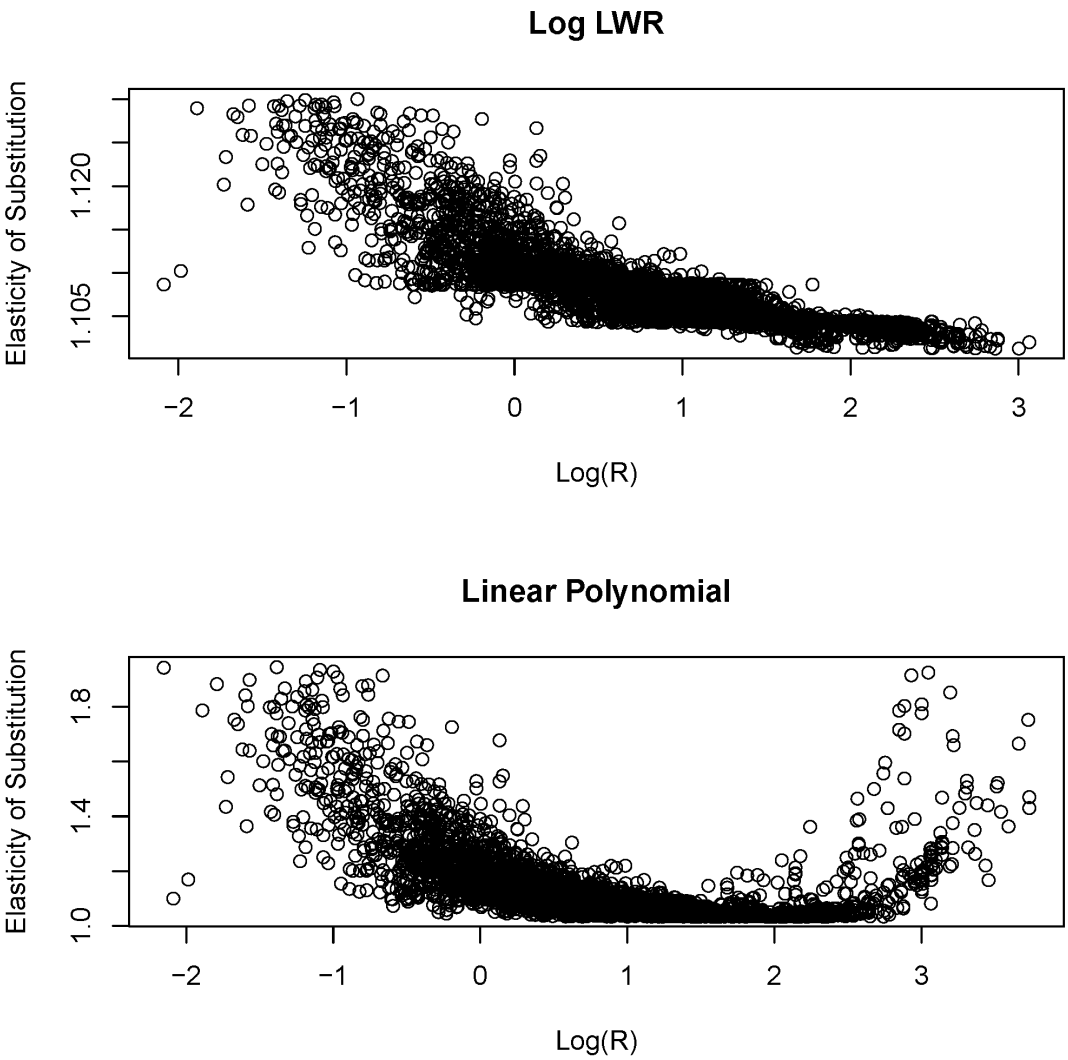
Fig. 5. Raw Data and Estimated Functions



Although all of the models appear to fit the data reasonably well, they can have much different implications for estimates of the elasticity of substitution. Figure 6 shows the elasticity estimates for the nonparametric model with a logarithmic dependent variable and the fourth-order polynomial with a linear dependent variable. The nonparametric estimates are the set of predictions from a series of LWR with a window size of 25% for the levels and 100% for the derivatives. Different windows are used for the levels and slopes because a standard result in the literature on nonparametric estimation is that derivatives require more smoothing than levels. The chosen window sizes performed well in the Monte Carlo results reported in Table 3. Both the nonparametric and polynomial estimates imply that the elasticity of substitution between land and capital tends to fall with land rent up to a value of about $R = \exp(2) = 7.39$. Beyond this value, the polynomial estimates begin to rise with land rent. The turning point coincides with the point in Figure 5 where the polynomial land rent function begins to decline with v. As a downward-sloping land rent function is a violation of the conditions for a well-behaved production function, it is clear that the nonparametric model is preferred to the polynomial. In general, polyno-

mial functions tend to have trouble with endpoints of functions, particularly when the data are relatively sparse in these regions.

Fig. 6. Elasticity of Substitution Estimates



The top panel of Figure 5 suggests that land rents are a concave function of v. The shape of the function has direct implications for the elasticity of substitution. Figure 1 shows how shape of the constant returns to scale function $v = R + \gamma R^{\sigma}$ varies with σ. A linear relationship holds if $\sigma = 1$, while a concave relationship is implied by $\sigma > 1$, and $\sigma < 1$ implies a convex function.

This relationship is confirmed by the estimates of σ implied by alternative estimation procedures applied to the EGS data. A simple regression of $log\left(\frac{pH-RL}{L}\right) = \log(v - R)$ on $\log(R)$ produces an estimate of 0.947. Since this classic CES estimate is likely subject to attenuation bias, we replace $\log(R)$ with the predicted values from LWR of $\log(R)$ on the

geographic coordinates of each property. This two-stage estimation procedure produces a value of 1.09 for σ.

The EGS estimates are remarkably similar to the two-stage estimates from the classic procedure. Table 4 shows the results from using LWR estimates with a 25% window and a fourth-order polynomial for the regression of R on v in levels and logs. The results also vary by the method used to calculate the elasticity from the first-stage regression. The column labeled "regression" shows the results from a regression of $log\left(v - \hat{f}(v)\right)$ on $log\hat{R}(v)$. The remaining columns show the results from direct calculation of equations (7) and (9). The columns vary depending on whether the elasticities are trimmed of the smallest and largest values. Our preferred estimation procedure for the first stage is LWR regressions of log(R) on log(v) (2-2-2). These results all imply an average elasticity of about 1.11. The results for the fourth-order polynomial regression for log(R) on log(v) are nearly identical to the LWR estimates. Linear specifications for the dependent variable tend to produce larger estimates for the elasticity of substitution. However, the results from Figure 5 suggest that this tendency toward higher average elasticities is explained by the tendency toward high values of σ in regions where R is implied to fall with v. As these regions violate the conditions implied by economic theory, it is safe to conclude that the average elasticity of substitution is approximately 1.11 in the EGS data set.

Tab. 4. Estimated Elasticities of Substitution

		Regression (Two-stage)	Mean Elasticity (Single-stage)	Mean Elasticity, 1% - 99% Percentiles	Mean Elasticity, 5% - 95% Percentiles
2-1-1	4th-Order Poly. R on v	1.175	1.228	1.140	1.110
2-1-2	LWR, R on v	1.132	1.234	1.216	1.204
2-2-1	4th Order Poly., Log(R) on log(v)	1.119	1.104	1.100	1.093
2-2-2	LWR, log(R) on log(v)	1.119	1.109	1.108	1.108

Notes: A more detailed description of the methods used is in Table 1.

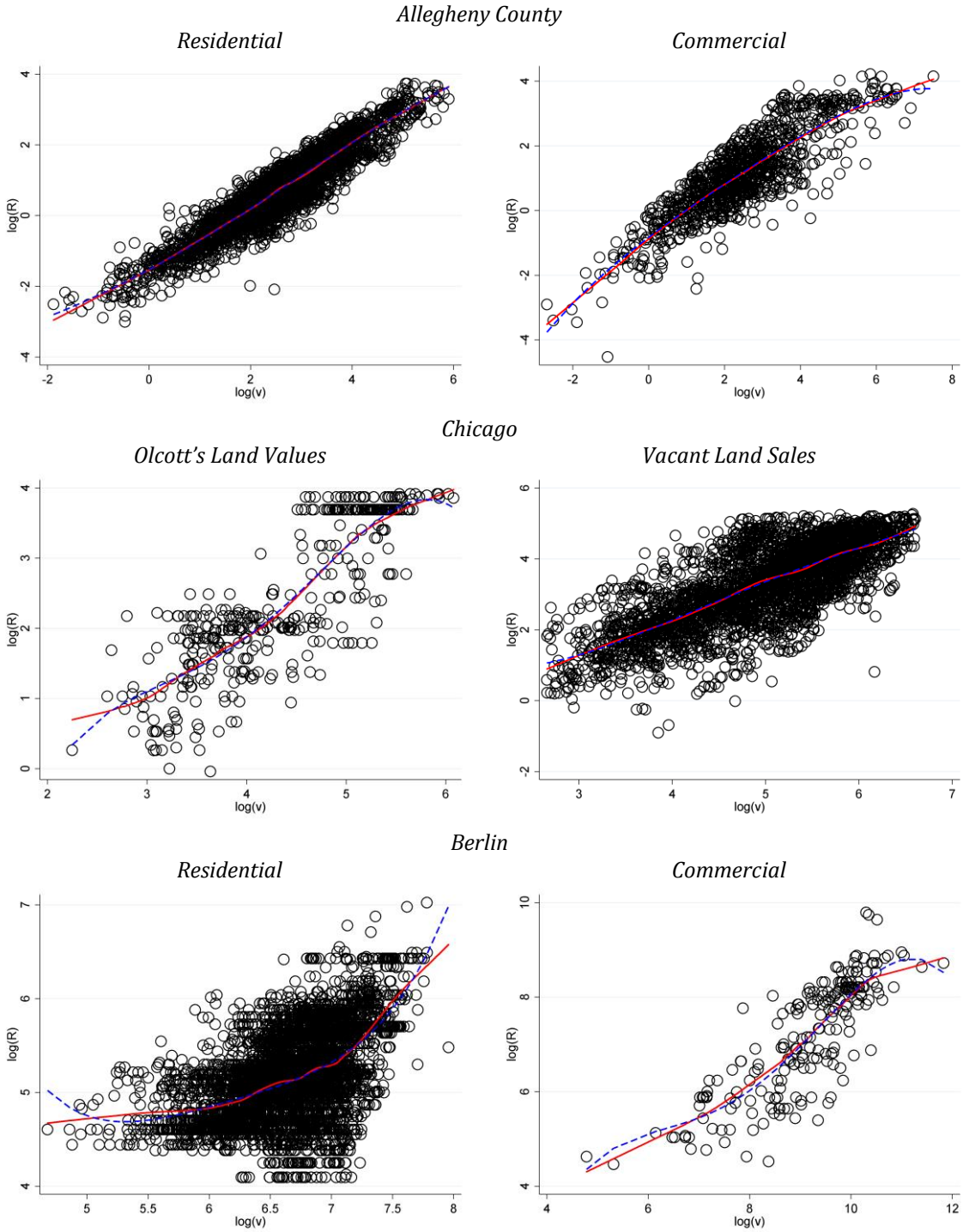
5.2 Comparative Analysis

The advantage of exploring within city variation when estimating the elasticity of substation is that the price of non-land factors can be assumed to be constant. Variation in non-land costs poses empirical challenges when using data from multiple cities (e.g. Albouy & Ehrlich, 2012; Arnott & Lewis, 1979; Muth, 1969). Confining the analysis to a single city may pose a problem of external validity if the results do not generalize to other metropol-

itan areas. One way to address this limitation is to apply the same research design to different contexts and to compare the results.

Figure 6 plots the functional relationship between log(R) and log(v) for our six data sets available for Berlin, Chicago, and Pittlsburgh. All (red) LWR fitted lines are monotonically increasing as predicted by theory. Most fitted lines point to an approximately log-linear relationship. Significant curvature is evident in the commercial Allegheny County and residential Berlin data. There is significantly more dispersion in the Chicago and Berlin data sets than in the Allegheny County data. This result is not surprising given that only the latter are based entirely on assessments of both property and land values.

Fig. 7. Raw Data and Estimated Functions: All Data Sets



Notes: Red solid (blue dashed) lines are LWR (4th order polynomial) fits. LWR use a bandwidth of 0.25 except of the Chicago Olcott's and Berlin Commercial data sets where 0.75 is used. The Allegheny County data sets are borrowed from EGS.

Table 5 presents four estimates of the elasticity of substitution of land for capital for each of the six data sets. The first two columns refer to the traditional approach implemented using OLS (1-1) and IV (1-2). To make the comparison as simple and intuitive as possible

we use distance to the CBD as an IV in all models. The only exception is the commercial Allegheny County model, for which we use zip code dummies, which is the only spatial information provided by EGS. The last two columns show results derived from two-stage variants of the EGS approach using LWR in levels (2-1-2-1) and logs (2-2-2-1) in the first stage. Based on the results of the Monte Carlo study we view the results in the last column as our preferred estimates.

Table 5 reveals some degree of variation in the elasticity estimates across data sets, which may indicate that the substitutability of land and capital to some extent depends on the institutional setting. More striking, perhaps, is the significant increase in the estimated elasticity when an IV or the new estimation approach is used. The arithmetic mean of the OLS estimates is 0.65, which is within the range of the classic literature summarized by McDonald (1981).[4] This value is also within the range of most studies employing a cross-city comparison approach (e.g. Albouy & Ehrlich, 2012; Arnott & Lewis, 1979; Muth, 1969). Using a distance to CBD as the instrument for a traditional IV approach increases the mean elasticity to above unity. Using the EGS approach the mean elasticity increases further to 1.25. Not surprisingly, the differences between the traditional and the new approach are larger for Chicago and Berlin, where the scatter plots indicate significantly more dispersion. On average, the OLS estimates of the elasticity of substation are about half the values of our preferred estimates.

Among the data sets analyzed, the Chicago vacant land data set deserves particular attention. Most studies estimating the elasticity of substitution rely on assessed land values because the pure site value of a developed property is difficult to observe. One of the potential problems with the use of assessed values in a study of the elasticity of substitution is that assessors may implicitly assume that land represents a constant share of property value. If so, the assessment process itself implies a unitary elasticity of substitution regardless of the true elasticity. The advantage of the Chicago vacant land data set is that that it is built entirely on observed market prices: the measure of land price is based on actual sales of vacant land, and the overall property value is drawn directly from transactions of newly constructed properties. Thus, the data collection method does not itself

[4] See Dowall and Treffeisen (1991) for a more recent example.

predetermine the functional relationship of between property prices and land prices. It is, therefore, particularly interesting that the preferred EGS estimates using this data set are close to unity.

Tab. 5. Classic vs. new σ-Estimates

		Classic approach		EGS Approach	
Data set	Obs.	OLS (1-1)	IV (1-2)	LWR (2-1-2-1)	Log LWR (2-2-2-1)
Allegheny County Residential	6362	0.95***	1.36***	1.13***	1.13***
		(0.01)	(0.04)	(0.00)	(0.00)
Allegheny County Commercial	992	0.93***	1.29***	1.44***	1.44***
		(0.04)	(0.37)	(0.01)	(0.01)
Chicago Residential Olcott	414	0.60***	0.85***	0.95***	0.91***
		(0.02)	(0.04)	(0.01)	(0.00)
Chicago Residential Vacant Land	3576	0.43***	0.88***	1.02***	0.97***
		(0.01)	(0.03)	(0.00)	(0.00)
Berlin Residential	5466	0.286***	1.186***	1.731***	1.834***
		(0.020)	(0.083)	(0.023)	(0.029)
Berlin Commercial	273	0.732***	0.903***	1.222***	1.202***
		(0.054)	(0.074)	(0.027)	(0.029)
Mean		0.65	1.08	1.25	1.25

Notes: See for details on the estimation procedures Table 1. The Allegheny County data sets include 2001 cross sections of properties constructed in 1995 or later. The Chicago Olcott's sample includes transactions from 1986-1994. The Chicago Vacant Land sample matches spatially interpolated vacant land sales prices to property transactions from 1983 to 2008 The Berlin samples include transactions from 1990-2012. Chicago and Berlin data sets include only properties with structures that by the time of transaction were not older than five years. The instrument is distance to the CBD in all models expect the Allegheny County commercial data set, where zip code dummies are used (the only spatial information provided by EGS). LWR in the EGS first-stages use a window size of 0.25 except for the Chicago Olcott's and Berlin Commercial data sets where 0.75 is used. Standard errors are in parenthesis. *** indicates significance at the 1% level.

6. Conclusion

Our contribution helps to reconcile conflicting evidence on one of the most fundamental parameters in urban economics, the elasticity of substitution of land for capital, which governs the relationship between the price of land and density of land use. Studies using classic estimation approaches tend to provide estimates around 0.5, which implies that expenditures on land increase as the price of land increases. New estimates provided by Epple, Gordon and Sieg (2010), however, point to an elasticity around or slightly above unity, which implies that expenditure shares remain roughly constant and developers respond to increasing land prices by densification at a rate much higher than commonly expected. We present Monte Carlo experiments that demonstrate that the classic approach suffers from downward bias in in the presence of measurement error in land prices while the new approach produces more accurate estimates. We further demonstrate that the

differences between the classic and the new estimates are not necessarily driven by geographic or institutional features of the study areas and persist when applied to the alternative data sets. Across a selection of six independent real world data sets we find an OLS downward bias of close to 50% on average. We conclude that the housing production function is likely closer to the convenient Cobb-Douglas form than long believed in the literature.

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