Environmental Uncertainties and Prevention: the Role of Ambiguity Aversion

January 13, 2014

Abstract

Environmental quality evolution and environmental policy involve uncertainties in various dimensions. These uncertainties can often not be represented by a unique, probabilistic belief and thus standard expected utility preferences representation models can not be used for the determination of optimal mitigation and adaptation decisions. Several alternatives for the expected utility model have been proposed in the literature. The aim of this paper is to determine the optimal adaptation and mitigation levels when future environmental quality and the efficiency of adaptation are ambiguous. We consider three models of preferences representation under ambiguity: the smooth ambiguity model, the α MaxMin model and the neo-additive capacities model. Hopefully, our results are quite robust to the model chosen. Ambiguity aversion drives the decision maker to undertake the economic instrument that involves less ambiguity.

Keywords: environmental risks, mitigation, adaptation, ambiguity, non expected utility

JEL code: D81, Q54

1 Introduction

In a world of uncertainty, it is vital to clarify the ways in which various kinds of uncertainties influence optimal environmental policy design. In particular, environmental issues often involve various compounding levels of uncertainty, among them: uncertainty over the underlying physical or ecological processes or/and uncertainty over technological changes that might reduce the cost of limiting the environmental damage. These sources of uncertainty apply to both the benefit and cost sides of design and evaluation policy. A striking and concrete example of this argument is global warming, where uncertainties remain on the relationship between environmental deterioration (GHG emissions levels, GHG concentration) and events like the increase in global temperature, the rise in sea level, the melting of snow, the loss of biodiversity, the spread of diseases and the like. We could break the state of scientific knowledge about climate change into two categories: broad scientific principles and detailed empirical predictions. Typically, the last report provided by the IPCC (Fifth Assessment Report, hereafter AR5) [1] (IPPC 2013) offers predictions, based on different *scenarii*.

For instance, with respect to an increase in global temperature over the 21^{th} century, the AR5 provides estimations which range from 0.3 to 4.8 celsius degree. A similar observation can be made for the expected rises in sea level over the period: from 0.26 to 0.82 meters. The wide ranges of outcomes and the uncertainties that surround them result from very sophisticated but complex computer models with many socio-economic and scientific parameters to be estimated, beyond the long time horizon considered and its natural grey areas (Heal and Kriström, 2002 and Pindyck, 2007). In particular, the *scenarii* rely on crucial technical assumptions, among which the growth rate of population, the discount rate, the growth rate of the economy, the polluting intensity of the production, but might also reflect some beliefs about future outcome that should come along with climate change, like the possible failure of the Gulf Stream system, the impact of aerosols on clouds formation or even the full effects of changes in ice sheet flows. In the same spirit, the Nordhaus-Stern controversy can also be used as a relevant illustration of wide disparate beliefs about the policy to be implemented in order to fight against climate change. Finally, an other striking example of uncertainty in environmental risks could be the link between health effects and atmospheric or/and water pollution. In most studies, results also appear to be intervals (see Koop and Tole 2004, Neidell 2004 or Seaton *et al.* 1995). A recent european

[1] IPCC: Inter-governmental Panel of experts on Climate Change

study, named APHEKOM is a relevant illustration of such uncertainties.

As economists, we are not only concerned by the scientific uncertainty rather than its induced effects on the human well-being. What matters to us is the effect of a potential environmental deterioration on agent's welfare. Some recent studies have explored the cost of environmental damages over time in terms of GDP in order to emphasize the urgent need of action (examples are also provided in AR5). In our paper, we do not focus on that involved growth effect, but we consider that agents withdraw satisfaction from the state of the environment, through amenities provided by nature and the quality of the environment it-self (agents value the opportunities of using the environment for recreative activities, but also may enjoy breathing a pure air or suffer from an increase in temperature). Now the issue to be tackled is how to deal with such an incomplete information in order, at least, to preserve individuals' utility? In order to protect the society against the uncertainties, the public authority may undertake two types of actions: mitigation and adaptation. Mitigation consists in curtailing GHG emission to lower the likelihood that bad states of nature occur while adaptation reduces severity of a bad state of nature when it occurs. As underlined by Kane and Shogren (2000), the mix policy has often been neglected in actual policy making.

Our objective is to determine the optimal environmental policy design and to explore the role played by ambiguity aversion considering two types of uncertainty. On the one hand, we consider a lack of precise probabilistic information about the occurrence of different possible events. On the other hand, the resilience capacity[2] to the potential adverse impacts of climate extremes events can also be ambiguous. Hopefully, there has been a series of theoretical advances in decision theory which provide us representations of preferences that account for ambiguity aversion (for a survey, see Etner *et ali.*, 2012). The Subjective Expected Utility model of Savage offers a preferences representation for any types of ambiguous information. However, it has been criticized as a descriptive model (see the Ellsberg paradox) as well as for its incapacity to take into account the quantity of available information. More general models have been proposed to represent preferences depending on the information structure. In this paper, we investigate three models: The Smooth Ambiguity Model (hereafter KMM, Klibanoff *et al.*, 2005), α Max Min (Ghirardato and Marinacci, 2002) and finally, the Neo-Capacity (Chateauneuf *et al.*, 2007).

[2]The resilience capacity refers to the ability of the society to adapt by their own. The two expressions will be used in the paper.

Our contribution lies in the robustness of our conclusions while we explore different models of preferences representation. In fact, we show that, under reasonable assumptions, an increase in ambiguity aversion drives the policy maker to undertake the economic instrument that involves less ambiguity. The paper is organized as follows. In section 2 we introduce the general framework and present in more details our two policy instruments. In section 3 we investigate the three models of ambiguity. Section 4 contains a brief conclusion.

2 The Framework

Let us present the basic one-period model. The public decision maker, hereafter DM, derives utility from environmental quality at the end of the period, $v(E)$, which exhibits standard properties[3]. The future environmental quality is not perfectly known and we assume, for simplicity, that there are only two possible states of environmental quality at the end of the period. This assumption can be easily justified through the existence of non-linearities or "tipping points" in environmental damage functions (see Kemp, 2005 and Pindyck, 2007 among others). Indeed, the impact of pollution can be small or even negligible until a threshold value has been reached; Only then, will it be severe and substantial and the environment is highly deteriorated. Consequently, we denote by p, the probability of environmental damage (and the corresponding environmental quality, $\underline{E}$) and with a probability $(1-p)$, the environment is clean $(\overline{E})$, with $\underline{E} < \overline{E}_2$.

2.1 Risk mitigation and risk adaptation

In this context, the DM may use two different tools to reduce the environmental risk: she can either invest in *mitigation*, that consists in reducing the probabilities of environmental deterioration or she can engage in *adaptation* that consists in limiting the negative consequences of environmental degradation when it occurs. Typically, as underlined by Heal and Kriström (2002), in climate change issues, the effort of mitigation may encompass all actions that cut down the flow of GHG in the atmosphere and thus change the probability distribution over future climate states. According to the AR5 (2013), there exists a substantial potential for mitigation of global GHG emissions, that could be exploited thanks to future energy infrastructure investments, the spread of low-carbon technologies, the improvement in crop and grazing land management (to increase soil carbon

[3] v is increasing and concave and satisfies the Inada conditions.

storage), the reduction of deforestation and the like. On the other hand, expenditure in adaptation lower the damages that come along with a given climate state and thus reduce the vulnerability to environmental deterioration. Adaptation actions[4] have various drivers like economic development or poverty reduction that may reinforce initiatives taken at local or sectoral levels. For instance, strategies to fight against health status deterioration could be embedded in heat-health actions plans, improvement of emergency medical services, water resources plans or coastal defence could be implemented to limit the effects of sea level rises, etc...

Formally, we consider that the DM chooses the level of investment in mitigation and adaptation at the beginning of the period. This is mainly due to the fact that the environment is considered to be stock that reacts slowly to all implemented environmental policies, due to existing inertia in the natural process of evolution. Then, an investment of M in mitigation can transform the probability p into $p(M)$ with $p'(M) \leq 0$ and $p''(M) \geq 0$. An investment in adaptation, A, improves the bad state of environmental quality $\underline{E}$ by $\lambda g(A)$ with $g'(A) \geq 0$ and $g''(A) \leq 0$ and λ is a positive real, so that the environmental quality becomes $\underline{E} + \lambda g(A)$.

We suppose that a given amount of public expenditure, T, is spent to environmental policies, that is $T = M + A$. Consequently, the DM has to choose the trade-off between mitigation and adaptation. Let us denote by m, the part of T devoted to mitigation.

2.2 Two sources of ambiguity

Crucial to our analysis, we argue that the overall picture is, in fact, featured by ambiguity. On the one hand, it can be characterized by a lack of precise probabilistic information about the occurrence of different possible events. In fact, scientific knowledge is often not of sufficient quality to allow their description by a unique probabilistic belief although they can be better described by a set of probabilistic beliefs issued either from incomplete data sets or from conflicting expert opinions. Typically, as underlined by Lange and Treich (2008), when dealing with climate change issue, no objective probabilistic assessment exists. To that extent, the controversy between Nordhaus's recommendations (Nordhaus,

[4]We may have assumed that adaptation actions are supported at the end of the period meanwhile they reduce the potential environmental damage. Indeed, we believe that all plans of actions that aim at reducing the vulnerability of agents require time to be built and implemented so that they should be financed early and displays any effects later. For instance, it will certainly be the case of building coastal defences.

2007) and the Stern report has participated to bring forward more information but also to increase ambiguity on climate change outcome. On the other hand, the resilience capacity to the potential adverse impacts of climate extremes events can also be ambiguous. Similarly, there is no unique post-disaster recovery function. Let us for instance refer to the risk of sea level augmentation. Building coastal defences can constitute an appropriate adaptation strategy. However, there still uncertainty with respect to the size (height and length) or the building code which are necessary to really prevent from flooding. Similarly, we may think to changes in meteorological features (amount in sunshine, pluviometry etc.). Does the technology will be able to provide improved ways of irrigation ? Efficient evacuation plans and infrastructure ? Finally, when dealing with health risks, can improvements in medical scientific knowledge be sufficient to reduce harmful effects of air pollution on health in a sustainable manner, during both acute and chronic episode of pollution ?

In our simple framework, where only two states of nature exist, the set of probabilistic beliefs can be boiled down to a probability interval. The goal pursued by an environmental policy is to lower the probabilities of the bad state of nature. For instance, reducing greenhouse gas emissions reduces the probabilities of a substantial rise in air temperature or increase in sea levels. In addition, when the probabilistic information on the future state of the environment is ambiguous - described by a probability interval $\left[\underline{p},\overline{p}\right]$- the impact of such policies is no more uniquely defined. Indeed, different types of environmental policies may exhibit varied impacts on the probabilities of the set and thus the optimal effort provided to fight against environmental deterioration depend on the type of the implemented policy. More precisely, it is possible to distinguish three types of actions:*(i)* all the probabilities in the set are reduced; *(ii)* only the highest probability in the set is reduced;*(iii)* only the lowest probability in the set is reduced.

Formally, when the true probability of damage, denoted p, is not known with certainty, in the Subjective Expected Utility model (Savage, 1954), agents' preferences are represented by an expected utility with respect to one probability of environmental damage, $q \in \left[\underline{p},\overline{p}\right]$, chosen according to their beliefs as follows:

$$V(m) = q(mT)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + \left(1 - q(mT)\right)\overline{v}$$

where $\overline{v} = v\left(\overline{E}\right)$

Second, when the efficiency of adaptation, denoted by λ, is ambiguous, in the

Subjective Expected Utility model, preferences are represented as follows:

$$W(m) = p(mT)v\left(\underline{E} + \rho g\left((1-m)T\right)\right) + \left(1 - p(mT)\right)\overline{v},$$

where ρ is the DM beliefs with respect to λ.

3 Preferences representation

Following the recent literature, we investigate three models of preferences representation: The Smooth Ambiguity Model (hereafter KMM, Klibanoff *et al.*, 2005), the α Max Min (Ghirardato, Marinacci 2002), and the Neo-Capacities model (Chateauneuf *et al.* 2007).

3.1 The Smooth Ambiguity Model

Consider first the case of an ambiguity on the probability p. The probability p is sensitive to some parameter θ whose true value is unknown.

For a given θ, the utility writes:

$$U(m,\theta) = p(mT,\theta)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + \left(1 - p(mT,\theta)\right)\overline{v} \tag{1}$$

Let us denote by ϕ, the function representing attitude to ambiguity, the utility function is:

$$V^{KMM}(m) = \phi^{-1}\left(\int_{\underline{\theta}}^{\overline{\theta}} \phi\left(p(mT,\theta)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + \left(1 - p(mT,\theta)\right)\overline{v}\right) dF(\theta)\right) \tag{2}$$

The optimal condition for an interior solution is:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'\left[U(m,\theta)\right] \times \frac{\partial U(m,\theta)}{\partial m} dF(\theta) = 0 \tag{3}$$

with

$$\frac{\partial U(m,\theta)}{\partial m} = T\left\{\frac{-\partial p(mT,\theta)}{\partial m}\left[\overline{v} - v\left(\underline{E} + \lambda g\right)\right] - \lambda g' v'\left(\underline{E} + \lambda g\right)\right\}$$

The optimal level of mitigation, m, is given by a trade-off between its marginal benefits and costs. Observe the expression of $\frac{\partial U(m,\theta)}{\partial m}$. The first term is positive and represents the marginal benefit of an investment in mitigation. The second term is negative and represents the opportunity cost of an investment in mitigation and not in adaptation.
Now, let us study the effect of an increase in ambiguity aversion represented by the concavity of function ϕ.

Proposition 1 *In the case of an ambiguous probability of environmental damage. If* $\dfrac{\partial p(mT,\theta)}{\partial \theta}$ *and* $\dfrac{\partial^2 p(mT,\theta)}{\partial m \partial \theta}$ *have the same sign, an increase in ambiguity aversion incites the decision maker to decrease the level of mitigation and increase the level of adaptation.*

Proof. Proof is in appendix ■

We obtain a similar result as Alary, Gollier and Treich (2011). When the probability of environmental damage is ambiguous, the ambiguity averse DM prefers to invest more in adaptation for which the effects are clear. Indeed, if the ambiguity aversion increases, the marginal cost of ambiguity increases and to compensate it, the DM will increase the marginal benefit of adaptation. Note that this result is does not depend on risk aversion or prudence.

Consider now the case of an ambiguity on the adaptation efficiency λ which is sensitive to some parameter θ whose true value is unknown.

For a given θ, the utility writes:

$$H(m,\theta) = p(mT)v\left(\underline{E} + \lambda(\theta)g\left((1-m)T\right)\right) + (1-p(mT))\,\overline{v} \tag{4}$$

Let us denote by ϕ, the function representing attitude to ambiguity, the utility function is:

$$W^{KMM}(m) = \phi^{-1}\left(\int_{\underline{\theta}}^{\overline{\theta}} \phi\left(p(mT)v\left(\underline{E} + \lambda(\theta)g\left((1-m)T\right)\right) + (1-p(mT))\,\overline{v}\right) dF(\theta)\right) \tag{5}$$

The optimal condition for an interior solution is:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'\left[H(m,\theta)\right] \times \frac{\partial H(m,\theta)}{\partial m} dF(\theta) = 0 \tag{6}$$

We obtain a very similar condition on the optimal level of mitigation, m, resulting from a trade-off between its marginal benefits and costs. The effect of an increase in ambiguity aversion represented by the concavity of function ϕ is given by the following proposition.

Proposition 2 *In the case of an ambiguous efficiency of adaptation. If the Relative Risk Aversion, RRA is lower than 1, an increase in ambiguity aversion incites the decision maker to increase the level of mitigation and decrease the level of adaptation .*

Proof. Proof is in appendix ■

In our case of multiplicative risk, a RRA lower than 1 means that the DM is more sensitive to a reduction in the mean final wealth than a harm disaggregation (Eeckhoudt *et al.*, 2009). In that case, if the ambiguity aversion increases the marginal cost of ambiguity increases and to compensate it, the DM will increase the marginal benefit of mitigation.

3.2 α Max Min Model

In the α max min model, the DM has imprecise information about either the true probability p of environmental deterioration or the true value of λ.
Concerning first the ambiguity on the probability p, the DM considers that it belongs to the interval $[\underline{p}, \overline{p}]$. Then, she evaluates a situation as the weighted sum of the worst and the best expected utility evaluation compatible with the probability interval.

$$\begin{aligned} V^{\alpha MM}(m) &= \alpha[\overline{p}(mT)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + (1-\overline{p}(mT))\overline{v}] \\ &+ (1-\alpha)[\underline{p}(mT)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + (1-\underline{p}(mT))\overline{v}] \end{aligned} \tag{7}$$

Similarly, when ambiguity bears upon the efficiency of adaptation expenditure, we suppose that the DM considers that λ belongs to the interval $[\underline{\lambda}, \overline{\lambda}]$ and those preferences write:

$$\begin{aligned} W^{\alpha MM}(m) &= p(mT)[\alpha v\left(\underline{E} + \underline{\lambda} g\left((1-m)T\right)\right) + (1-\alpha)v\left(\underline{E} + \overline{\lambda} g\left((1-m)T\right)\right)] \\ &+ (1-p(mT))\overline{v} \end{aligned} \tag{8}$$

In this framework, we are able to capture ambiguity attitudes through the parameter α, so that an increase in ambiguity aversion corresponds to a raise in α, risk attitudes being characterized by the properties of v.

The first order conditions, denoted by $V_m^{\alpha MM}$ and $W_m^{\alpha MM}$, are deduced from the maximization program in each configuration. We obtain:

$$\begin{aligned} V_m^{\alpha MM} &= T(\underline{v}(\lambda, m) - \overline{v}(\lambda, m))[\alpha \overline{p}'(mT) + (1-\alpha)\underline{p}'(mT)] \\ &\quad - \lambda T \underline{v}'(\lambda m) g^{'}((1-m)T)[\alpha \overline{p}(mT) + (1-\alpha)\underline{p}(mT)] = 0 \quad (9) \end{aligned}$$

The optimal level of mitigation stems from a trade-off between marginal benefits and costs. The first term accounts for marginal benefits as the probabilities of damage are reduced. This positive effect that triggers more mitigation is higher all the more that the severity of the damage is large, that is the difference between the two states of environmental quality is big. Marginal costs stem from a substitution effect that arises when the DM devotes more resources to mitigation : the effort of adaptation is consequently reduced and this represents a loss of utility when the risk occurs indeed. Let us underline that this marginal cost of mitigation is larger when the interval is wide, that is when ambiguity is high. In particular, adaptation is more and more unavoidable as the interval widens, since it implies a higher probability of risk occurrence.

On the other hand, the optimal value of mitigation when ambiguity is on the efficiency of adaptation expenditure is given by:

$$\begin{aligned} W^{\alpha MM}(m)_m &= Tp'(mT)[\alpha v\left(\underline{E} + \underline{\lambda} g\right) + (1-\alpha) v\left(\underline{E} + \overline{\lambda} g\right) - \overline{v}] \\ &\quad - Tg' \times p(mT)[\alpha \underline{\lambda} v'\left(\underline{E} + \underline{\lambda} g\right) + (1-\alpha)\overline{\lambda} v'\left(\underline{E} + \overline{\lambda} g\right)] \quad (10) \end{aligned}$$

We examine the effect of an increase in pessimism (the parameter α) on the optimal value of mitigation in both configurations. As in the KMM setting, we can show that an increase in pessimism reduces incentives to invest in mitigation, thus increasing adaptation expenditure. However, this is not always true, depending on the design of the policy. In particular, the DM may reduce uncertainty in many ways. She can either reduce symmetrically the high and the low probability of damage, keeping unchanged the size of the interval or she may change the interval size by reducing more (or less) the high probability of risk compared to the lowest probability of damage. We can claim the following proposition:

Proposition 3 *1. When the probability of damage is ambiguous, the consequences of an increase in pessimism are not clear cut and depend on the type of mitigation. If the effect of mitigation on the worst state is lower or equal than the effect on the best one, then, the DM will invest less in mitigation and more in adaptation. Else, the global effect is unclear.*

2. When the efficiency of adaptation is ambiguous, if the relative risk aversion is lower than one, an increase in pessimism incites the DM to invest more in mitigation and less in adaptation.

Proof. Proof is in appendix ■

When the probability of damage is ambiguous, contrary to KMM results, an increase in ambiguity aversion might induce a larger effort of mitigation. This is the case when the impact of mitigation on the worst state is higher or equal than the effect on the best one. If the DM is more ambiguity averse, she grants more weight to the worst state of environment. And, if the mitigation is more efficient on this worst situation, then, she is more likely to invest in mitigation.

Moreover, we can demonstrate that an exogenous increase in the technology of adaptation, as expected, tends to reduce the optimal level of mitigation[5]. Then, we explore the effect of pessimism on the optimal effort of mitigation when the efficiency of adaptation is ambiguous.

3.3 Neo-Capacity Model

The preferences of the Decision maker are written, in each configuration, as follows:

$$\begin{aligned} V^{NC}(m) &= \delta[q(mT)v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + (1-q(mT))\overline{v}] \\ &+ (1-\delta)[\beta v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) + (1-\beta)\overline{v}] \end{aligned} \tag{11}$$

and

$$\begin{aligned} W^{NC}(m) &= p(mT)\left[\delta v\left(\underline{E} + \hat{\lambda} g\left((1-m)T\right)\right) + (1-\delta)v\left(\underline{E} + \tilde{\lambda} g\left((1-m)T\right)\right)\right] \\ &+ (1-p(mT))\overline{v} \end{aligned} \tag{12}$$

[5]Formally, we differentiate equation (5) with respect to λ. Under assumption 1, the derivative is negative, $V^{M}_{m\lambda} < 0$.

with δ is the degree of confidence granted by the DM to Experts' opinions, $q(mT)$ or $\hat{\lambda}$. The parameter $\beta \in [0,1]$ accounts for the pessimism degree of the DM. $\tilde{\lambda}$ can be greater or lower than $\hat{\lambda}$ according the DM's pessimism.

The FOCs are derived and given by:

$$\begin{aligned} V^{NC}(m)_m &= -T\delta q'(mT)\left[\overline{v} - v\left(\underline{E} + \lambda g\left((1-m)T\right)\right)\right] \\ &- T\left[\delta q(mT) + (1-\delta)\beta\right]\lambda g'\left((1-m)T\right)v'\left(\underline{E} + \lambda g\left((1-m)T\right)\right) \end{aligned}$$

and

$$\begin{aligned} W^{NC}(m)_m &= -Tp'(mT)\left[\overline{v} - \delta v\left(\underline{E} + \hat{\lambda} g\left((1-m)T\right)\right) - (1-\delta)v\left(\underline{E} + \tilde{\lambda} g\left((1-m)T\right)\right)\right] \\ &- Tg'p(mT) \times \left[\delta\hat{\lambda}v'\left(\underline{E} + \hat{\lambda} g\left((1-m)T\right)\right) + (1-\delta)\tilde{\lambda}v'\left(\underline{E} + \tilde{\lambda} g\left((1-m)T\right)\right)\right] \end{aligned}$$

Studying this optimal level of mitigation, we can claim the following:

Proposition 4 *1. When the probability of damage is ambiguous, the confidence degree tends to increase the effort of mitigation (and thus reduces effort of adaptation) if the DM is more pessimistic than the experts.*

2. When the efficiency of adaptation is ambiguous, if the relative risk aversion is lower than one, an increase in the confidence degree tends to decrease (increase) the effort of mitigation when the DM is more pessimistic (optimistic) than the experts.

Proof. Proof is in appendix ∎

In case of an ambiguity on p, if the DM is more pessimistic than the experts, $q(mT) - \beta \leq 0$, then, the more the DM is confident, the more the effort of mitigation is higher. But, else, the effect of confidence degree is unclear. Indeed, there are two opposite effects: an increase in the confidence degree increases the benefit of the mitigation but it increases the weight on the more pessimist scenario that tends to decrease the level of mitigation for investing more in adaptation. In case of an ambiguity on λ, if the DM is more pessimistic than the experts, $\hat{\lambda} > \tilde{\lambda}$, then, the more the DM is confident, the more the effort of adaptation is higher. Note that we need once again the condition on RRA.

Proposition 5 *1. When the probability of damage is ambiguous, the higher the subjective probability of damage, the lower the optimal level of mitigation engaged by the DM.*

2. When the efficiency of adaptation is ambiguous, if the relative risk aversion is lower than one, the higher the subjective level of efficiency (the DM becomes more optimistic) the lower the optimal level of mitigation engaged by the DM.

Proof. Proof is in Appendix ■

Indeed, in the first case, when the DM is very pessimistic she is not likely to invest in mitigation, (and thus favors adaptation) since she believes the risk has a high probability of occurrence. Consequently, it seems more appropriate to invest in adaptation and reduce the impact of environmental deterioration once the risk has realized. In the same way, when the DM is more pessimistic, she invests more in mitigation and less in adaptation.

4 Conclusion

The aim of this paper was to determine the optimal mitigation and adaptation levels when both the future state of the environment and the efficiency of adaptation are ambiguous. To represent preferences, we used three decision models under ambiguity and for each of them, we determined the impact of ambiguity aversion on optimal policy.

In case of ambiguity, it is usual to wonder which model is the more appropriate for the agents' preferences representation. Indeed, ambiguity aversion can take different forms according to the model of preferences representation. Hopefully, our results are quite robust to the model chosen. Ambiguity aversion drives the decision maker to undertake the economic instrument that involves less ambiguity. This is true under some standard assumptions. Consequently, to determine the optimal environmental policy mix, the evaluation of the degree of ambiguity aversion is crucial for public decision makers. This evaluation can de done by surveys or contingent evaluation studies. A revealed preferences approach can also be used based on passed prevention decisions. Future research can be done in two directions: one empiricaland another more theoretical. Empirical studies can try to estimate ambiguity attitudes in diffrent countries from the observed envionmental policy decisions in these countries. Theoretical studies can try to determine, in a dynamic framework, the impact of modifications of ambiguity attitudes on the optimal adaptation and mitigation policies.

5 Proofs

Proof of Proposition 1. Consider two DM with the same utility function, u, who differ in ambiguity attitude. DM2 is more ambiguity averse than DM1, that is, $\phi_2 = k \circ \phi_1$, with k is an increasing and concave function. Let us denote by m_1 and m_2, the optimal decision for respectively DM1 and DM2. $m_1 > m_2$ if and only if

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_2' \left[U(m_1, \theta)\right] \times \frac{\partial U(m_1, \theta)}{\partial m} dF(\theta) < 0 \tag{13}$$

That is

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' \left[U(m_1, \theta)\right] k'(\phi_1 \left[U(m_1, \theta)\right]) \times \frac{\partial U(m_1, \theta)}{\partial m} dF(\theta) > 0$$

As $\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' \left[U(m_1, \theta)\right] \times \frac{\partial U(m_1, \theta)}{\partial m} dF(\theta) = 0$ and ϕ_1' is strictly positive, $\frac{\partial U(m_1, \theta)}{\partial m}$ can be positive or negative for some values of θ.

$$\frac{\partial U(m, \theta)}{\partial m} = T \frac{\partial p(mT, \theta)}{\partial m} \times \left[v\left(\underline{E} + \lambda g\left((1-m)T\right)\right) - \overline{v}\right] - T\lambda g' \times p(mT, \theta) v'\left(\underline{E} + \lambda g\left((1-m)T\right)\right)$$

and

$$\frac{\partial^2 U(m, \theta)}{\partial m \partial \theta} = -T \left\{ \frac{\partial^2 p}{\partial m \partial \theta} \times \left[\overline{v} - v\left(\underline{E} + \lambda g\right)\right] + \lambda g' \times \frac{\partial p}{\partial \theta} v'\left(\underline{E} + \lambda g\right) \right\}$$

which is negative if $\frac{\partial^2 p}{\partial m \partial \theta}$ and $\frac{\partial p}{\partial \theta}$ are positive and positive if there are both negative. We suppose that there exists a $\bar{\theta}$ such that $\frac{\partial U(m_1, \bar{\theta})}{\partial m} = 0$.

Moreover,

$$\frac{\partial U(m, \theta)}{\partial \theta} = -\frac{\partial p}{\partial \theta} \left[\overline{v} - v\left(\underline{E} + \lambda g\right)\right]$$

1. If $\frac{\partial p}{\partial \theta} > 0$, $U(m,\theta)$ is a decreasing function of θ and thus, $k'(\phi_1 [U(m_1,\theta)])$ is an increasing function of θ.
   For $\theta < \bar{\theta}$, $k'(\phi_1 [U(m_1,\theta)]) < k'(\phi_1 [U(m_1,\bar{\theta})])$ and $\dfrac{\partial U(m_1,\theta)}{\partial m} > 0$, thus

$$k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} < k'(\phi_1 [U(m_1,\bar{\theta})]) \times \frac{\partial U(m_1,\theta)}{\partial m}$$

   In the same way, for $\theta > \bar{\theta}$, $k'(\phi_1 [U(m_1,\theta)]) > k'(\phi_1 [U(m_1,\bar{\theta})])$ and $\dfrac{\partial U(m_1,\theta)}{\partial m} < 0$, thus

$$k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} < k'(\phi_1 [U(m_1,\bar{\theta})]) \times \frac{\partial U(m_1,\theta)}{\partial m}$$

   We deduce that

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' [U(m_1,\theta)] \, k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} dF(\theta) < 0$$

   and $m_1 > m_2$

2. If $\frac{\partial p}{\partial \theta} < 0$, $U(m,\theta)$ is an increasing function of θ and thus, $k'(\phi_1 [U(m_1,\theta)])$ is a decreasing function of θ.
   For $\theta < \bar{\theta}$, $k'(\phi_1 [U(m_1,\theta)]) > k'(\phi_1 [U(m_1,\bar{\theta})])$ and $\dfrac{\partial U(m_1,\theta)}{\partial m} < 0$, thus

$$k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} < k'(\phi_1 [U(m_1,\bar{\theta})]) \times \frac{\partial U(m_1,\theta)}{\partial m}$$

   In the same way, for $\theta > \bar{\theta}$, $k'(\phi_1 [U(m_1,\theta)]) < k'(\phi_1 [U(m_1,\bar{\theta})])$ and $\dfrac{\partial U(m_1,\theta)}{\partial m} > 0$, thus

$$k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} < k'(\phi_1 [U(m_1,\bar{\theta})]) \times \frac{\partial U(m_1,\theta)}{\partial m}$$

   We deduce that

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' [U(m_1,\theta)] \, k'(\phi_1 [U(m_1,\theta)]) \times \frac{\partial U(m_1,\theta)}{\partial m} dF(\theta) < 0$$

   and $m_1 > m_2$

■

Proof of Proposition 2. Consider two DM with the same utility function, u, who differ in ambiguity attitude. DM2 is more ambiguity averse than DM1, that is, $\phi_2 = k \circ \phi_1$, with k is an increasing and concave function. Let us denote by m_1 and m_2, the optimal decision for respectively DM1 and DM2. $m_1 > m_2$ if and only if

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_2' \left[H(m_1, \theta)\right] \times \frac{\partial H(m_1, \theta)}{\partial m} dF(\theta) < 0 \tag{14}$$

That is

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' \left[H(m_1, \theta)\right] k'(\phi_1 \left[H(m_1, \theta)\right]) \times \frac{\partial H(m_1, \theta)}{\partial m} dF(\theta) > 0$$

As $\int_{\underline{\theta}}^{\overline{\theta}} \phi_1' \left[H(m_1, \theta)\right] \times \frac{\partial H(m_1, \theta)}{\partial m} dF(\theta) = 0$ and ϕ_1' is strictly positive, $\frac{\partial H(m_1, \theta)}{\partial m}$ can be positive or negative for some values of θ.

$$\frac{\partial H(m, \theta)}{\partial m} = -T \left\{ p'(mT) \times \left[\overline{v} - v\left(\underline{E} + \lambda g\right)\right] + \lambda(\theta) g' \times p(mT) v'\left(\underline{E} + \lambda g\right) \right\}$$

and

$$\frac{\partial^2 H(m, \theta)}{\partial m \partial \theta} = -T v'\left(\underline{E} + \lambda g\right) \lambda' \left\{ -p' \times g + g' \times p \left(1 + \lambda g \times \frac{v''\left(\underline{E} + \lambda g\right)}{v'\left(\underline{E} + \lambda g\right)} \right) \right\}$$

We assume that $1 + \lambda g \times \frac{v''(\underline{E}+\lambda g)}{v'(\underline{E}+\lambda g)}$ is positive that is $1 > RRA$ with RRA the relative risque aversion.
Under this assumption, $\frac{\partial^2 H(m,\theta)}{\partial m \partial \theta}$ is negative if and only if λ is an increasing function of θ.
We suppose that there exists a $\bar{\theta}$ such that $\frac{\partial H(m_1, \bar{\theta})}{\partial m} = 0$.

Moreover,

$$\frac{\partial H(m, \theta)}{\partial \theta} = \lambda'(\theta) \times g \times T \times p(mT) v'\left(\underline{E} + \lambda(\theta) g\left((1 - m)T\right)\right)$$

1. If $\lambda'(\theta) < 0$, $H(m, \theta)$ is a decreasing function of θ and thus, $k'(\phi_1 [H(m_1, \theta)])$ is an increasing function of θ.
   For $\theta < \bar{\theta}$, $k'(\phi_1 [H(m_1, \theta)]) < k'(\phi_1 \left[H(m_1, \bar{\theta})\right])$ and $\dfrac{\partial H(m_1, \theta)}{\partial m} < 0$, thus

$$k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} > k'(\phi_1 \left[H(m_1, \bar{\theta})\right]) \times \frac{\partial H(m_1, \theta)}{\partial m}$$

   In the same way, for $\theta > \bar{\theta}$, $k'(\phi_1 [H(m_1, \theta)]) > k'(\phi_1 \left[H(m_1, \bar{\theta})\right])$ and $\dfrac{\partial H(m_1, \theta)}{\partial m} > 0$, thus

$$k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} > k'(\phi_1 \left[H(m_1, \bar{\theta})\right]) \times \frac{\partial H(m_1, \theta)}{\partial m}$$

   We deduce that

$$\int_{\underline{\theta}}^{\bar{\theta}} \phi_1' [H(m_1, \theta)] \, k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} dF(\theta) > 0$$

   and $m_1 < m_2$

2. If If $\lambda'(\theta) > 0$, $H(m, \theta)$ is an increasing function of θ and thus, $k'(\phi_1 [H(m_1, \theta)])$ is a decreasing function of θ.
   For $\theta < \bar{\theta}$, $k'(\phi_1 [H(m_1, \theta)]) > k'(\phi_1 \left[H(m_1, \bar{\theta})\right])$ and $\dfrac{\partial H(m_1, \theta)}{\partial m} > 0$, thus

$$k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} > k'(\phi_1 \left[H(m_1, \bar{\theta})\right]) \times \frac{\partial H(m_1, \theta)}{\partial m}$$

   In the same way, for $\theta > \bar{\theta}$, $k'(\phi_1 [H(m_1, \theta)]) < k'(\phi_1 \left[H(m_1, \bar{\theta})\right])$ and $\dfrac{\partial H(m_1, \theta)}{\partial m} < 0$, thus

$$k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} > k'(\phi_1 \left[H(m_1, \bar{\theta})\right]) \times \frac{\partial H(m_1, \theta)}{\partial m}$$

   We deduce that

$$\int_{\underline{\theta}}^{\bar{\theta}} \phi_1' [H(m_1, \theta)] \, k'(\phi_1 [H(m_1, \theta)]) \times \frac{\partial H(m_1, \theta)}{\partial m} dF(\theta) > 0$$

   and $m_1 < m_2$

■

Proof of Proposition 3. Let us differentiate equations (9) and (10) with respect to the pessimism degree, α.

Case 1: p is ambiguous.

$$\begin{aligned} V^{\alpha MM}_{m\alpha} &= -T\left[\overline{v} - v\left(\underline{E} + \lambda g\right)\right]\left[\overline{p}'(mT) - \underline{p}'(mT)\right] \\ &- \lambda T \underline{v}'\left(\underline{E} + \lambda g\right) g' [\overline{p}(mT) - \underline{p}(mT)] \end{aligned}$$

The second term is clearly positive, while the first term depends on the sign of the expression in brackets. Thus, we can show that if $[\overline{p}'(mT) - \underline{p}'(mT)] \geq 0$, $V^{M}_{m\alpha} < 0$. But, if $[\overline{p}'(mT) - \underline{p}'(mT)] < 0$, then the sign of $V^{M}_{m\alpha}$ is undefined. The negative sign of $[\overline{p}'(mT) - \underline{p}'(mT)]$ is a necessary condition to eventually obtain a positive derivative.

Case 2: λ is ambiguous.

$$\begin{aligned} W^{\alpha MM}(m)_{m\alpha} &= -Tp'(mT)[v\left(\underline{E} + \overline{\lambda} g\right) - v\left(\underline{E} + \underline{\lambda} g\right)] \\ &- Tg' \times p(mT)[\underline{\lambda} v'\left(\underline{E} + \underline{\lambda} g\right) - \overline{\lambda} v'\left(\underline{E} + \overline{\lambda} g\right)] \end{aligned}$$

The first term is clearly positive, while the second term depends on the sign of the expression in brackets. Thus, we can show that if $[\underline{\lambda} v'\left(\underline{E} + \underline{\lambda} g\right) - \overline{\lambda} v'\left(\underline{E} + \overline{\lambda} g\right)] \leq 0$, $W^{M}_{m\alpha} > 0$. Else, the sign of $W^{M}_{m\alpha}$ is undefined.
$[\underline{\lambda} v'\left(\underline{E} + \underline{\lambda} g\right) - \overline{\lambda} v'\left(\underline{E} + \overline{\lambda} g\right)] \leq 0$ if and only if the function $h(\lambda) = \lambda v'\left(\underline{E} + \lambda g\right)$ is increasing. That is true when $1 + \lambda g \times \frac{v''(\underline{E}+\lambda g)}{v'(\underline{E}+\lambda g)}$ is positive that is $1 > RRA$ with RRA the Relative Risk Aversion.

■

Proof of Proposition 4.

Let us differentiate equation (9) with respect to δ the level of trust into experts' judgements.

Case1: p is not known

$$\begin{aligned} V^{NC}(m)_{m\delta} &= -Tq'(mT)\left[\overline{v} - v\left(\underline{E} + \lambda g\left((1-m)T\right)\right)\right] \\ &- T\lambda g'\left((1-m)T\right) v'\left(\underline{E} + \lambda g\left((1-m)T\right)\right) \times \left(q(mT) - \beta\right) \end{aligned}$$

which is positive if $q(mT) - \beta \leq 0$.

Case2: λ is not known

$$\begin{aligned} W^{NC}(m)_{m\delta} &= -Tp'(mT)\left[-v\left(\underline{E}+\hat{\lambda}g\left((1-m)T\right)\right)+v\left(\underline{E}+\tilde{\lambda}g\left((1-m)T\right)\right)\right] \\ &- Tg'p(mT)\times\left[\hat{\lambda}v'\left(\underline{E}+\hat{\lambda}g\left((1-m)T\right)\right)-\tilde{\lambda}v'\left(\underline{E}+\tilde{\lambda}g\left((1-m)T\right)\right)\right] \end{aligned}$$

If $\hat{\lambda} > \tilde{\lambda}$ and $RRA < 1$, $W^{NC}(m)_{m\delta} < 0$. And if $\hat{\lambda} < \tilde{\lambda}$ and $RRA < 1$, $W^{NC}(m)_{m\delta} > 0$.
■

Proof of proposition 5. *Case1: p is not known*

$$V^{NC}(m)_{m\beta} = -T(1-\delta)\lambda g'\left((1-m)T\right)v'\left(\underline{E}+\lambda g\left((1-m)T\right)\right)$$

which is negative.

Case2: λ is not known

$$\begin{aligned} W^{NC}(m)_{m\tilde{\lambda}} &= Tp'(mT)(1-\delta)g\left((1-m)T\right)v'\left(\underline{E}+\tilde{\lambda}g\left((1-m)T\right)\right) \\ &- Tg'p(mT)\times(1-\delta)\times\left[v'\left(\underline{E}+\tilde{\lambda}g\right)+\tilde{\lambda}\times g\times v''\left(\underline{E}+\tilde{\lambda}g\right)\right] \end{aligned}$$

which is negative if $RRA < 1$. ■

References

[1] Alary, D., Gollier, Ch and N. Treich (2013) "The Effect of Ambiguity Aversion on Insurance and Self-protection", *The Economic Journal* 123, 1188-1202.

[2] Chateauneuf, A., Eichberger, J and S. Grant (2007) "Choice under uncertainty with the best and worst in mind: Neo-additive capacities", *Journal Economic Theory* 137(1), 538-567.

[3] Epstein, LG (1999) "A definition of uncertainty aversion", *Review of Economic Studies* 66, 579-608.

[4] Etner, J., Jeleva, M. and J-M. Tallon (2012) "Decision Theory under ambiguity", *Journal of Economic Surveys* 26(2), 234-270.

[5] Ghirardato, P. and M. Marinacci (2002) "Ambiguity made precise: A comparative foundation", *Journal Economic Theory* 102, 251-289.

[6] Heal, G. and B. Kriström (2002) "Uncertainty and climate change", *Environmental and Resource Economics* 22, 3-39.

[7] IPCC (2013) *Fifth assessment report.*

[8] Kane, S. and J. F. Shogren (2000) "Linking adaptation and mitigation in climate change policy", *Climate Change*, 45, 75-102.

[9] Kemp, M. (2005) "Science in culture: inventign an icon", *Nature*, 437, 1238.

[10] Klibanoff, P. and M. Marinacci and S. Mukerji (2005) "A Smooth Model of Decision Making under Ambiguity", *Econometrica* 73, 1849-1892.

[11] Koop, G. and L. Tole (2004) "Measuring the health effects of air pollution: to what extent can we really say that people are dying from bad air?", *Journal of Environmental Economics and Management* 47, 30-54.

[12] Lange, A. and N. Treich (2008) "Uncertainty, learning and ambiguity in economic models on climate policy: some classical results and new directions", *Climatic Change* 89, 7-21.

[13] Neidell, M. (2004) "Air pollution, health, and socio-economic status: the effect of outdoor air quality on childhood asthma", *Journal of Health Economics* 23, 1209-1236.

[14] Nordhaus, W. (2007) "A Review of the "Stern Review on the Economics of Climate Change"", *Journal of Economic Literature* 65, 686-702.

[15] Pindyck, R. (2007) "Uncertainty in environmental economics", *Review of Environmental Economics and Policy*, Winter.

[16] Savage, L. (1954) *The foundations of Statistics* John Wiley.

[17] Seaton, A. and Mac Nee, W. and Donaldson, K. and D. Godden (1995) "Particulate air pollution and acute health effects", *The Lancet* 345, 176-178.

[18] Schmeidler, D (1989) "Subjective probability and expected utility without additivity", *Econometrica* 57, 571-587.

[19] Stern, D (2007) *The Economics of Climate Change : The Stern Review*, Cambridge University Press.