Looking Ahead versus Looking Back:
Revisiting the Carbon Tax*

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This version, December 2011

Abstract

We propose an alternative tax on CO₂ emissions, taxing countries according to their emissions *stocks* rather than *flows*. Under standard assumptions, such a retrospective tax leads to the first-best emissions pattern. Moreover, the tax rate is based only upon *observed* data rather than on forecasts of future damage, circumvents the discounting issue altogether, is more robust to errors in the parameters of the model, and implements gradual tax payments while immediately curbing emissions. Finally, the optimal retrospective tax is not only cheaper than the optimal tax on flows, but also sets the stage for an explicit debate regarding historical emissions.

Keywords: Pigou Tax, Global Warming, Discounting, Climate Ramp, Retrospective Tax, Stock Tax.

JEL classification code: Q54, H23.

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*The authors wish to thank Geir Asheim, Yann Bramoullé, Georges Dionne, Paul Ekins, Jérémy Laurent-Luchetti, Peter Hartley, Marcus Hermann, Marc Santugini, Jens Steffek, Bruno Versael, Mirko Wiederholt, Ralph Winkler and participants at the 4th World Congress of Environmental and Resource Economists, at the Society for Economic Design 2011 meeting, 12th Global Conference on Environmental Taxation, the Montreal Resource and Environmental Economics Workshop, the University of Bern and Oeschger Center for Climate Change Research, Université Laval, Rice University and the University of Texas at Austin. Financial support from the SSHRC is gratefully acknowledged.

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1 Introduction

Regarding the climate change issue, a tax on greenhouse gas emission flows—also dubbed \textit{carbon tax}—is arguably the most prominent instrument put forth to correct "the greatest market failure the world has seen" (Stern, 2008). As per the familiar Pigovian argument, the optimal carbon tax rate charges emitters the marginal damage of their emissions. However, given the high persistence of CO$_2$ in the atmosphere, the effects of carbon emitted today will be felt over periods of time spanning several generations. To account for this fact, a direct application of the Pigovian argument is the following: the optimal tax rate should be set equal to the sum of expected marginal damage of carbon emitted today \textit{discounted} over the duration of its lifetime in the atmosphere. Unfortunately, this seemingly innocent formulation comes with two major caveats: First, correctly anticipating the path of damage caused by today’s emissions over hundreds of years is a heroic, and highly uncertain, exercise. Second, discounting future damage begs the difficult question of choosing an appropriate discount rate, which is the topic of strong ethical disagreements.

We propose an alternative tax on greenhouse gas emissions that is free of both above-mentioned caveats: its rate is computed relying neither on predictions of future damage nor on the choice of a discount rate. Specifically, the scheme we propose taxes \textit{past} cumulated emissions at a rate equal to \textit{current marginal damage}. Intuitively, while the standard scheme taxes emission flows with the understanding that they are responsible for future environmental damage, the retrospective tax turns the problem on its head: it taxes emissions \textit{stocks} with the understanding that they are responsible for the damage incurred \textit{today}. After all, climate change is a stock problem, not a flow problem. Hence, because it mirrors the physics of the climate externality more closely, the retrospective tax ends up exhibiting more desirable properties than the traditional one.

First of all, the retrospective tax provides the same incentives as the standard tax. Under standard assumptions, we show that it can lead to the same emissions pattern as the standard tax and therefore may also achieve the first-best Pigovian outcome (Proposition 1).\footnote{Carbon dioxide (CO$_2$) is the most abundant greenhouse gas in the earth’s atmosphere after water vapor, and is the leading contributor to anthropogenic radiative forcing (the so-called \textit{greenhouse effect}). Nonetheless, the arguments put forth here hold for most types of greenhouse gases.} The equivalence comes from the agents’ anticipation that they will have to pay tomorrow the contribution of today’s
emission flow to tomorrow’s damage, via what will remain of today’s emission flow in tomorrow’s stock. Under the retrospective tax, emitting CO₂ is akin to "emitting" debt to be repaid over time. As a result, while both taxes lead to the same pattern of CO₂ emissions, the pattern of tax payments differs markedly from that of the standard tax:

The difference in the patterns of tax payments leads to at least two remarkable properties. A first one is that there is a sense in which the retrospective tax is cheaper than the standard one: at any point in time, the payments made for a ton of CO₂ emitted today are less than those paid under the standard tax (Proposition 2). This is due to the fact that the standard tax requires an up-front payment while the retrospective tax requires emitters to pay their dues in installments. The mere promise of future tax payments is indeed enough to discipline current behavior. In other words, although both tax burdens are identical in the very long run, that of the retrospective tax will likely be easier to incur than paying the standard tax burden up front, much like mortgage financing facilitates access to homeownership.

Another remarkable property due to the pattern of the retrospective tax payments is that it provides an answer to the so-called "ramp/springboard" implementation debate. Put succinctly, the question there is whether a tax policy should be phased in (or "ramped up") so as to avoid disrupting the economy, or implemented immediately in full force (like a "springboard") because reducing emissions cannot wait. We show that the retrospective tax combines the benefits of both. On the one hand, by spreading out the tax burden over time, tax payments are effectively ramped up. On the other hand, because the retrospective tax implements the same emissions pattern as the full-blown standard tax, it effectively acts as a springboard (Proposition 3).

Finally, and perhaps most importantly, the retrospective tax rate is robust to errors in parameters of the climate model considered and to inaccurate knowledge of the discount rate effectively used by the countries involved. This is due to the fact that the retrospective tax asks not only that countries internalize the externality—like any Pigovian tax—but also make use of their own predictions of future climate damage and of their own discount rate (Propositions 4, 5, 6 and 7). Clearly, one cannot escape the prediction and discounting issues altogether given the dynamic nature of the problem, but we argue that the fact that the retrospective approach keeps them away from the determination of the tax rate is of critical importance in achieving international agreements. In fact, with these issues taken out of the equation, the international debate
can now focus on the heart of the matter, which is to determine the extent to which countries should be held responsible for their historical emissions. We show in Section 5 that the retrospective tax can easily be modified to ignore emissions produced prior to a chosen date, \( \tilde{t} \), by simply taking them out of the tax base. This amounts to granting countries a lump-sum credit that does not distort incentives because the tax rate is unaffected. Hence, the debate on the implementation of the retrospective tax becomes one-dimensional and amounts to deciding from what date, \( \tilde{t} \), countries should be held accountable for their historical emissions.

This work belongs to the large literature on the mitigation of externalities originated by Pigou (1920) and Baumol (1972), which points to economic instruments to discipline economic agents and act as coordination tools to correct market failures. Because the time dimension is paramount when considering issues of climate change, choices essentially concern the trade-off between consumption today and in the future (Nordhaus and Boyer, 1999), as in all optimal growth models. This trade-off is commonly described via a discount factor in the utilitarian intertemporal social welfare function to be maximized. The validity of such a social welfare function is beyond the scope of this work and we refer the reader to Roemer (2009) and references therein for eye-opening discussions. We adopt the same objective function, not because we endorse it, but because we wish to compare our results using the most widespread conceptual framework. As such, it constitutes an important benchmark by which to gauge the relative performance of the retrospective tax scheme.

The effectiveness of taxing stocks rather than flows results from a well-known rational expectations argument. In the context of local pollution, taxing stocks has been proposed since Bencheokroun and Long (1998) when polluters have market power. We build upon this argument to explore the practical implications of a tax on stocks beyond the equivalence of emissions path. In particular, the crucial question of robustness of the tax rate brought about by the lengths of time involved in the climate change context required addressing. Likewise, the place of a tax on stocks in the climate ramp-springboard debate as well as its relative cheapness had yet to be uncovered.

The intuition behind why a retrospective tax performs better in many respects than the standard carbon tax is also closely connected to the cost sharing literature. There, the central topic is designing a division of the cost of an externality between agents as a function of their contributions to it. Theoretical
results converge to the general conclusion that the most desirable sharing rules (in terms of efficiency, incentive compatibility and even fairness, though the latter is beyond the scope of this work) are the ones which best reflect the structure of the externality (see Moulin, 2002, for a thorough survey; Maniquet and Sprumont 1999; Leroux 2004, 2007). While the model used here is not one typically encountered in the cost-sharing literature, the approach is nonetheless similar in spirit.

Finally, our results also contribute to a growing debate on the timing of climate change taxation, colloquially referred to as the "ramp/springboard" debate (Nordhaus, 2008; Stern, 2009). While the latter debate rests entirely on economic arguments (which Mason, 2010, and Krugman, 2010, sum up quite well), the implementation question is also eminently political. The fact that our retrospective tax relies only on observed data, rather than on predictions of largely uncertain outcomes and on vehemently debated issues is likely to increase its political acceptance (Steffek, 2006).

The paper is organized as follows. The next section is a descriptive one delineating how the practical difficulties in determining the standard tax rate act as overwhelming barriers to the adoption of a global environmental agreement. Section 3 describes the standard conceptual model on which we shall compare both taxation schemes. The formal equivalence in terms of incentives is shown in Section 4. In Section 5, we discuss the important properties of the retrospective tax in terms of the burden it imposes. Section 6 discusses the robustness of the retrospective tax to prediction and discounting errors and compares them with that of the standard tax rate. Finally, Section 7 analyzes the case where countries may be heterogeneous in their discount rates. We show that even if this heterogeneity is ignored by the planner, the corresponding loss in welfare is only of second-order for the retrospective tax, but is of first-order magnitude for the standard tax. Section 8 concludes.

2 The failure of the standard carbon tax

Despite being well understood and accepted, initiatives in implementing a (standard) carbon tax have been carried out only sporadically, and can only be witnessed in a mere dozen countries. Moreover, what is perhaps most striking is the fact that those regions of the world that are equipped with a carbon tax differ
widely in the tax rate they impose, ranging from $13 to $150/tCO₂ (Kansas Energy Council, 2007; Environmental Tax Policy Institute, 2009). According to economic theory, this would imply that CO₂ emitted in Sweden (where emitting carbon is most expensive) is ten times more damaging than CO₂ emitted in New Zealand. This heterogeneity is unsupported by any climate model, quite to the contrary: wind patterns tend to mix all emissions together so that CO₂ concentrations in the atmosphere bear no relationship with its source. In other words, the impact of emissions is roughly the same no matter from where they originate.

Thus, we are far from the kind of global adoption of a uniform climate policy that international efforts are aiming at. Naturally, many obstacles stand in the way of any global cooperation, but we argue that an important reason for this reluctance is the lack of consensus on the level of the optimal tax rate. In fact, one may suspect that one reason why most countries are not yet on board is that they may simply be waiting for a consensual tax level to emerge from the early adopters. Consequently, providing a very clear prescription of what the tax rate should be may go a long way towards global cooperation.

As mentioned in the introduction, the formulation of the standard carbon tax rests on the following interpretation of the Pigovian argument: the optimal tax should be set equal to the sum of expected marginal damage of carbon emitted today discounted over the duration of its lifetime in the atmosphere. We now explain why this seemingly natural extension actually introduces two insurmountable obstacles to a seamless and accurate implementation. First, correctly anticipating the path of damage caused by today’s emissions over time seems highly doubtful given the durations involved. Indeed, it has been argued that certain greenhouse gases, including CO₂, remain in the atmosphere for several hundreds of years, if not more (Archer, 2005; Archer and Brovkin, 2008). Therefore, environmental damage far off in the future being highly uncertain, one may question the accuracy of the expected marginal damage upon which the tax rate is to be based. It should also be noted that computing expected damage far off in the future implies understanding more than just the meteorological effects of high carbon concentrations on the climate. It also requires taking our technological adaptability into account, which is far from obvious. Finally, it requires making explicit forecasts about future emissions patterns. These forecasting assumptions, especially the latter, are far from innocent and require a sharp understanding of the interplay between climate change and technological
Second, discounting future damage begs the difficult question of what is an appropriate rate by which to discount welfare far off in the future (Lind, 1990; Henderson and Bateman, 1995). Sadly, this choice is the topic of strong disagreements, as illustrated by the famous Nordhaus v. Stern debate, and is unlikely to become consensual anytime soon. Briefly presented, the debate pits the view that the relevant discount rate should be the average rate of return on capital (about 6% in Nordhaus 2007; 4% in Nordhaus, 2008), because it is the rate at which society can save for the future, against the view that the future should be discounted at a rate that is close to zero, because the welfare of generations to come is as legitimate as that of our own (Stern, 2006, 2008; Weitzman 1998). Once again, at the root of the problem are the very slow decay rates of greenhouse gases (CO₂, in particular) and the vast lengths of time they imply, thereby impacting several—if not many—generations.

As we show formally in the remainder of the paper, the retrospective tax addresses these issues simultaneously by neither requiring knowledge of future damage nor the adoption of a discount factor to set the optimal tax rate. Thus, the latter is likely to be much less subject to interpretation than that of the standard tax, which can only facilitate international discussion towards the adoption of a global harmonized tax.

3 The model

Consider a set of countries \( j = 1, ..., N \) that derive (a flow of) private benefits, \( B_j^t \left( X_j^t \right) \), as a function of their emissions, \( X_j^t \), where the superscript \( j \) denotes the country and the subscript \( t \) indicates the (discrete) time period considered. Greenhouse gas emissions accumulate in the atmosphere and decay at a rate we denote by \( \gamma \in (0, 1) \). The concentration in the atmosphere at date \( T \) writes as follows:

\[
Z_T = \sum_{t=0}^{T} \gamma^{T-t} X_t,
\]

\(^3\text{By contrast, computing current marginal damage as does the retrospective tax, while remaining a difficult exercise, is far more reasonably attainable. Indeed, establishing climate evolution relies on observing key meteorological indexes. Of particular interest is the estimation of trends in the temperature of the tropical troposphere. We refer the reader to Douglass et al (2008), Santer et al (2008) and McKitrick et al (2010) for further details.}
where $X_t := \sum_{j=1}^{N} X_t^j$ denotes the global flow of emissions in period $t$. The stock of emissions, $Z_t$, is a public bad resulting in (a flow of) private damage, $D_t^j$. We consider damage to be possibly stochastic and we write:

$$D_t^j \equiv D_t^j (\theta_t, Z_t),$$

(2)

where $\theta_t$ is a random variable of known distribution.

Therefore, each period, the global net flow of benefits writes:

$$W_t = \sum_{j=1}^{N} \left[B_t^j \left(X_t^j\right) - D_t^j (\theta_t, Z_t)\right].$$

(3)

Let $\beta$ denote the common discount factor with which each country (or its planner) discounts future welfare. Obviously, it is also the discount factor the (global) social planner must use in discounting social welfare. We assume that the global planner aims to achieve efficiency by maximizing global discounted welfare as measured by:

$$W = \sum_{t=0}^{+\infty} \beta^t W_t = \sum_{t=0}^{+\infty} \beta^t \sum_{j=1}^{N} \left[B_t^j \left(X_t^j\right) - D_t^j (\theta_t, Z_t)\right].$$

(4)

Finally, we shall work under the benchmark assumption that each country’s marginal impact on its own damage is small relative to the global marginal damage it causes:

$$E_t \left[\sum_{s=t}^{+\infty} (\gamma^*)^{s-t} \frac{\partial D_t^j}{\partial Z_s}\right] \ll E_t \left[\sum_{s=t}^{+\infty} (\gamma^*)^{s-t} \frac{\partial D_s}{\partial Z_s}\right].$$

(5)

This assumption is equivalent to the benchmark Pigovian assumption of competitive markets.

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\footnote{In fact, for the purposes of this work, only knowledge of the mean damage function is required because risk neutrality is assumed throughout.}
4 Equivalence of the two taxes

4.1 The standard tax: Looking ahead

Consider a stream of global emissions \((X_t)_{t=0}^{\infty}\). Looking ahead, the expected marginal damage of one unit of emissions at date \(t = 0\) writes:

\[
E_0 \left[ \sum_{t=0}^{+\infty} \beta^t \left( \sum_{i=1}^{N} \frac{\partial D^i_t}{\partial X_0} \left( \theta_t, \sum_{s=0}^{t} \gamma^{t-s} X_s \right) \right) \right] = E_0 \left[ \sum_{t=0}^{+\infty} (\gamma\beta)^t \frac{\partial D_t}{\partial Z_t} \right],
\]

where \(D_t = \sum_{i=1}^{N} D^i_t\) denotes the flow of global damage of period \(t\). The optimal global emissions path is thus implicitly defined by the following first-order conditions:

\[
\frac{\partial B^j_t}{\partial X^j_t} = E_t \left[ \sum_{s=t}^{+\infty} (\gamma\beta)^{s-t} \frac{\partial D_s}{\partial Z_s} \right],
\]

for each country, \(j\), at every time period, \(t\). We recover the standard result according to which one can implement the optimal emissions path by setting a Pigovian tax on emission flows—the \(X^j_t\)'s—equal to the expected global marginal damage.

**Proposition 0** The optimal tax rate, \(\tau_t\), on emissions flows is:

\[
\tau_t = E_t \left[ \sum_{s=t}^{+\infty} (\gamma\beta)^{s-t} \frac{\partial D_s}{\partial Z_s} \right].
\]

**Proof.** Under assumption (5), a country will evaluate its net benefits as:

\[
B^j_t = \sum_{i=0}^{+\infty} \beta^i \left[ B^j_t \left( X^j_t \right) - \tau_t X^j_t \right],
\]

and will choose an emissions stream \(\left( X^j_t \right)_{t=0}^{\infty}\) such that:

\[
\frac{\partial B^j_t}{\partial X^j_t} = \tau_t = E_t \left[ \sum_{s=t}^{+\infty} (\gamma\beta)^{s-t} \frac{\partial D_s}{\partial Z_s} \right],
\]

for all \(t\). □

This tax, to which we refer to as the "standard tax" throughout this work, allows for decentralization of the social optimum. Expression (7) clearly states the well-known fact that the standard tax rate, \(\tau_t\), is a discounted sum of
expected marginal damages, over an infinite horizon.

4.2 The retrospective tax: Looking back

Consider instead taxing countries according to the stock of greenhouse gas they emitted over time (rather than to their emission flows). More precisely, assume that, in any period $t$, country $j$ is required to pay $\mu_t Z^j_t$, where

$$Z^j_t = \left( \sum_{s=0}^{t} \gamma^{t-s} X^j_s \right)$$

(8)

is the fraction of the current stock for which country $j$ is responsible at time $t$. Under such a tax scheme, emitting greenhouse gases is akin to "emitting" financial debt, decaying at rate $\gamma$, to be repaid over time.

Under rational expectations, a country will evaluate its net benefits as:

$$B^j_t = \sum_{t=0}^{+\infty} \beta^t \left[ B^j_t \left( X^j_t \right) - \mu_t Z^j_t \right],$$

(9)

and will choose an emissions stream $\left( X^j_t \right)_{t=0}^{+\infty}$ such that:

$$\frac{\partial B^j_t}{\partial X^j_t} = E_t \left[ \sum_{s=t}^{+\infty} \beta^{s-t} \mu_s \frac{\partial Z^j_t}{\partial X^j_t} \right] = E_t \left[ \sum_{s=t}^{+\infty} (\gamma \beta)^{s-t} \mu_s \right],$$

(10)

for all $t$. Hence, setting the retrospective tax rate to $\mu_t = \frac{\partial D_t}{\partial Z_t}$, for all $t$, makes Expression (10) identical to the planner’s first-order condition, (6). In words, this alternative taxation scheme decentralizes the same (efficient) decisions as the standard tax.

**Proposition 1** The optimal tax rate, $\mu_t$, on emission stocks is:

$$\mu_t = \frac{\partial D_t}{\partial Z_t}.$$ 

(11)

This optimal tax on stocks decentralizes the same emissions path as the standard Pigovian tax.

Observe that the retrospective tax rate, $\mu_t$, does not depend on the discount factor, $\beta$, nor on the decay parameter, $\gamma$. Moreover, rather than requiring the planner to take expectations over uncertain outcomes, the retrospective tax rate
is computed only on the basis of the observations of current marginal damages and the tax base only on the basis of past emissions.

The intuition behind the equivalence is as follows. Because agents are forward looking and anticipate what their payments will be in the future, they internalize these future payments in today’s decisions. In other words, under the retrospective tax, agents not only internalize the externality (as is the case in the standard tax, and in all Pigovian taxes), but also the temporality of the impacts of emissions on climate. More simply put, because agents are forward looking, the tax does not have to be.

As an important aside, the retrospective tax achieves much more decentralization than the standard taxation scheme, because it befalls on each country to accurately know its own discount factor as well as make predictions regarding future climate damage\(^5\). In this respect, one may argue that the retrospective tax espouses the spirit of the Pigovian principle even better than the standard one.

5 Discussions

Proposition 1 establishes that the two taxes generate equivalent incentives. Yet, there are several dimensions along which the two schemes are not identical. We now turn to three appealing properties of the retrospective tax. The first two relate to the timing of the tax payments while the last discusses how the retrospective tax may significantly simplify the international debate.

A cheaper tax

A striking feature of the retrospective tax is that there is a sense in which it is cheaper than the standard tax. More precisely, even though it implements the same emissions path as the standard tax, it does so by exacting lower tax payments per ton of GHG emitted at any given time period. This is a consequence of the forward-looking nature of countries: the promise of future tax payments acts as a device to call on countries to reduce their emissions today. Because these tax payments are not levied today but only in the future, they allow one to achieve the allocation target more cheaply.

\(^5\)This latter point does not prohibit the fact that climate damage predictions could be entrusted to a single intergovernmental authority like, say, the IPCC. For a thought-provoking proposal involving prediction markets, we refer the reader to Hsu (2011).
**Proposition 2** At any given time, the expected tax burden per ton of emissions is less under the retrospective tax than under the standard tax.

**Proof.** The value at date $T \geq t$ of all retrospective tax payments associated with the (flow of) emissions, $X_t$ is:

$$V_{T,retro}(X_t) = \sum_{s=t}^{T} \beta^{s-T} (\mu_s \gamma^{s-t} X_t) = \beta^{t-T} \left[ \sum_{s=t}^{T} (\gamma \beta)^{s-t} \frac{\partial D_x}{\partial Z_x} \right] X_t.$$  

Meanwhile, the same value for standard tax payments writes:

$$V_{T,std}(X_t) = \beta^{t-T} E_t \left[ \sum_{s=t}^{+\infty} (\gamma \beta)^{s-t} \frac{\partial D_x}{\partial Z_x} \right] X_t > E_t [V_{T,retro}(X_t)].$$

Hence, in expected terms, the tax burden per ton of emissions is less under the retrospective tax. ■

A perhaps more mundane intuition for this result is the following. The standard tax asks that agents pay up front the entire (expected and discounted) damage each ton of GHG will cause over time. By contrast, the retrospective tax requires emitters to pay their dues in installments dictated by the contribution of each ton of GHG to the (current) damage of each period. Therefore, while both tax burdens are identical in the very long run, that of the retrospective tax will likely be easier to incur than paying the standard tax burden up front, much like mortgage financing facilitates access to homeownership.

**A built-in "climate change ramp"**

Beyond the issue of discounting, another important debate regarding the implementation of a carbon tax has to do with the timing of the implementation. Again, two polar opinions have emerged. On the one hand, those in favor of a gradual tax increase (e.g., Nordhaus, 2009) contend that it is more cost-effective to keep encouraging investments that exhibit a higher yield than climate mitigation technologies until technological improvements render the latter cheap enough to be worth diverting assets. This gradual increase is commonly referred to as "ramping up" the climate change policy. On the other hand, advocates of a sudden, "springboard" policy insist on taking immediate action to curb emissions. They argue that the magnitudes of the stakes, and the high level of uncertainty on the concentrations thresholds before climate change unravels,
forbid resorting to casual cost-benefit analyses (Stern, 2009; see also Mason, 2010, for a more detailed account of the carbon ramp/springboard debate).

The economic debate briefly presented above illustrates only one of the facets of the implementation issue. No less crucial is the difficult question of political acceptability. In this respect, one may argue that a climate change ramp seems more palatable, as it promises fewer disruptions to the functioning of the economy than a "springboard" shock. Therefore, it seems that the best of both worlds would be a policy that would curb emissions immediately, all the while phasing in tax payments.

Strikingly, the retrospective tax fits the bill on both counts. Tax payments start out low, to rise at the pace of stock accumulation. In other words, the retrospective tax exhibits a built-in climate change ramp. In addition, we have established the equivalence between the retrospective tax and the full-fledged, standard tax in terms of emissions patterns (Proposition 1). Therefore, the effect of the retrospective tax on emissions is as immediate as that of a full-fledged standard tax. Lastly, from a political standpoint, the fact that the tax rate increases with observed damage is also very appropriate. Indeed, when faced with the undisputable evidence of the harm caused by climate change, the concern of political actors for climate policy will likely grow proportionately (Steffek, 2006). All in all, the retrospective tax has the potential to answer these three (ecological, economic and political) concerns simultaneously.

Formally, the fact that the retrospective tax is less disruptive to the economy can be illustrated by the following Proposition.

**Proposition 3** At any date, the value of cumulative tax payments is always less under the retrospective tax than under the standard tax.

**Proof.** Under the optimal retrospective tax, the expected value at date zero of
the cumulative tax payments up to date $T$ reads as follows:

$$E_0 \left[ \sum_{t=0}^{T} \beta^t \mu_t Z_t \right] = E_0 \left[ \sum_{t=0}^{T} \beta^t \frac{\partial D_t}{\partial X_t} \sum_{s=0}^{t} \gamma^{t-s} X_s \right]$$

$$= E_0 \left[ \sum_{s=0}^{T} \beta^s \left( \sum_{t=s}^{T} \frac{\partial D_t}{\partial X_t} (\beta \gamma)^{t-s} \right) X_s \right]$$

$$= E_0 \left[ \sum_{s=0}^{T} \beta^s E_s \left( \sum_{t=s}^{T} \frac{\partial D_t}{\partial X_t} (\beta \gamma)^{t-s} \right) X_s \right]$$

$$< E_0 \left[ \sum_{s=0}^{T} \beta^s E_s \left( \sum_{t=s}^{\infty} \frac{\partial D_t}{\partial X_t} (\beta \gamma)^{t-s} \right) X_s \right] = \sum_{s=0}^{T} \beta^s \tau_s X_s,$$

as was to be proved. ■

The intuition of the result follows from of Proposition 2: Because the expected value of the tax burden corresponding to each ton emitted is less under the retrospective tax, the same inequality holds for cumulative tax payments.

**Historical responsibility**

Another issue relates to the philosophical and political stance regarding historical emissions that occurred prior to the awareness of anthropogenic climate change. Indeed, the concern over global warming takes hold after some one hundred and fifty years of heavy emissions patterns by the now-developed countries. Therefore, the question of responsibility countries may have over these "pre-awareness" emissions is paramount to the implementation of any consensual climate policy.

The standard tax and the retrospective tax differ markedly in their accounting for past emissions. By looking only at the future, the standard tax implicitly assumes that historical emissions are nobody’s responsibility and, as a result, completely dilutes the environmental burden of early emissions to all countries, irrespective of their historical emissions patterns. By contrast, the retrospective tax holds countries responsible for their historical emissions, by construction. Hence, should the retrospective tax ever be put forth in international discussions, one may expect developed countries to favor the standard tax, while developing countries (which are low historical emitters, typically) might lean towards a retrospective tax.

However, the retrospective approach can be adapted to accommodate a view where pre-awareness emissions are deemed irrelevant. A simple modification
consists in deleting emissions prior to some date \( \bar{t} \) from the tax base, so that country \( j \) pays \( \mu_j Z_i^j \), where \( \bar{Z}_i^j = \sum_{s=t}^{\bar{t}} \gamma^{t-s} X_i^j \), while the tax rate is kept unchanged. Countries are no longer held responsible for their pre-awareness emissions but, the tax rate being unaffected by this accounting decision, this adapted retrospective tax still implements the first-best emissions pattern. This amounts to granting a lump-sum credit to countries and forgiving their liability for emissions having occurred before \( \bar{t} \). By contrast it is difficult to argue that the standard tax could be so easily adapted to take past emissions into account should the international debate settle on some considerations of historical emissions.

6 Robustness of the retrospective tax rate

We now further explore the advantages the retrospective tax. Obviously, of two policies implementing the same outcome, the more robust one is to be preferred because it is more likely to be optimal than the one that is overly sensitive to errors in its defining parameters. Moreover, having stressed the importance of political acceptability, we further argue that the former is also more likely to be embraced. This is especially true in the presence of such high uncertainty. In fact, given the inertia of the political process, an "optimal" policy that would need to be adjusted with the release of every new climatic or economic study is unlikely to be adopted. We show that, for all these reasons, the retrospective tax is to be preferred to the standard carbon tax.

To appreciate more readily the difference between the two approaches, let us assume constant marginal damage:

\[
E_t \left[ \frac{\partial D_s (\theta_s, Z_s)}{\partial Z_s} \right] = \frac{\partial D}{\partial Z},
\]

for any \( s > t \geq 0 \).

Under Expression (12), the standard tax rate (7) rewrites as follows:

\[
\tau = \frac{1}{1 - \gamma \beta} \frac{\partial D}{\partial Z}.
\]

Under the same assumptions, the condition (10) for the retrospective tax to
implement the social optimum remains the following:

$$\mu = \frac{\partial D}{\partial Z}. \quad (14)$$

It immediately appears that the standard tax rate is sensitive to the value—and possible estimations errors—of the decay rate, $$\gamma$$, and of the discount factor, $$\beta$$, whereas the retrospective tax rate is completely insensitive to the two. We detail below the possible magnitudes of this sensitivity.

**Robustness to errors on the decay rate, $$\gamma$$**

Computing the elasticity of the tax rates with respect to the decay rate is straightforward, and yields the following proposition:

**Proposition 4** *The elasticity of the standard tax rate with respect to the decay rate is:*

$$\frac{\gamma \partial \tau}{\tau \partial \gamma} = \frac{\gamma \beta}{1 - \gamma \beta}.$$

*Meanwhile that of the retrospective tax rate is zero:*

$$\frac{\gamma \partial \mu}{\mu \partial \gamma} = 0.$$

In practice, the slow decay of carbon in the atmosphere implies a value of $$\gamma$$ that is close to one. Similarly, the concern for future generations imply a discount factor that is also very close to one (between 0.96 and 1, according to the current debate on discounting). As a result, the product $$\gamma \beta$$ is likely to be close to 1 as well. A conservative numerical example\(^{6}\) taking $$\gamma \beta = 0.9$$ yields an elasticity of 9, meaning that a 5% error in $$\gamma$$ (usually, a fairly acceptable margin of error) implies a 45% correction on the standard tax rate.

Meanwhile, the retroactive tax rate is unaffected.

**Robustness to climate model specifications**

Clearly, the climate model we consider is an overly simplified one, because we assume a constant decay rate of carbon, $$\gamma$$. A constant decay rate implies the

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\(^{6}\)If $$\beta$$ is interpreted as directly linked to the interest rate (*i.e.* $$\beta = 1/(1+r)$$), a value of $$\gamma \beta = 0.9$$ would correspond to $$r$$ larger than 11% and an coefficient $$\gamma$$ associated to a half-life of CO2 persistence in the atmosphere of less than 100 years. Smaller interest rates or longer persistence would result in a value of $$\gamma \beta$$ closer to one and, in turn, to a larger elasticity of the standard tax rate.
existence of an infinite carbon sink, which is consistent neither with evidence nor with climate theories (see Perman et al, 2003). However, this does not matter for our purposes, because any climate model would yield the same conclusion. Indeed, suppose a general stock accumulation (and decay) function, 
\[ Z_t = F_t(X_0, X_1, ..., X_t) \] for all \( t \geq 0 \), and denote \( \Gamma_{s,t} = \frac{\partial P_s}{\partial X_t} \) for all \( s \geq t \). The conclusions of Proposition 1 are unaffected:

**Proposition 5** The equivalence between taxing flows and taxing stocks holds for any stock accumulation functions \( (F_t)_{t=1, \ldots, \infty} \). Moreover, regardless of the specification of \( (F_t)_{t=1, \ldots, \infty} \), the equivalent retrospective tax rate remains

\[ \mu_t = \frac{\partial D_t}{\partial Z_t}. \]

**Proof.** The optimality condition (6) rewrites as follows:

\[ \frac{\partial B^j_t}{\partial X^j_t} = E_t \left[ \sum_{s=t}^{+\infty} \beta^{s-t} \Gamma_{s,t} \frac{\partial D_s}{\partial Z_s} \right]. \]

It follows that the optimal standard tax rate writes \( \tau_t = E_t \left[ \sum_{s=t}^{+\infty} \beta^{s-t} \Gamma_{s,t} \frac{\partial D_s}{\partial Z_s} \right] \), whereas the equivalent retrospective tax rate is still \( \mu_t = \frac{\partial D_t}{\partial Z_t} \).  

Note that the complete insensitivity of the retrospective tax rate to the climate model contrasts with the potentially much more complex expression for the standard tax rate. Hence, if anything, the simplifying assumption of a constant decay rate hides additional advantages attached to taxing stocks rather than flows.

**Robustness to errors on the discount factor, \( \beta \)**

A similar calculation to the one above yields the following:

**Proposition 6** The elasticity of the standard tax rate with respect to the discount factor is:

\[ \frac{\beta}{\tau} \frac{\partial \tau}{\partial \beta} = \frac{\gamma \beta}{1 - \gamma \beta}. \]

Meanwhile that of the retrospective tax rate is zero:

\[ \frac{\beta}{\mu} \frac{\partial \mu}{\partial \beta} = 0. \]
The numerical example of the previous section can be reused to illustrate how dramatically sensitive the traditional tax rate is to the discount rate. In light of this analysis, it is not surprising that countries balk at the idea of implementing a climate tax today, when the debate on discounting is still raging with no end in sight.

By contrast, the complete insensitivity of the retrospective tax rate confers a definitive advantage to taxing stocks. This advantage is twofold and is due to the fact that, to set $\mu$, the planner needs neither knowledge of the countries’ common discount factor (informational advantage), nor that they have reached an agreement on it (procedural advantage). In fact, the retrospective tax rate does not even require countries to have settled the ongoing debate on what is the appropriate view on discounting the welfare of future generations.

**Robustness to errors on the marginal damage**

It is clear from Expression (7) that the elasticity of the standard tax rate with respect to changes in the expected marginal damage is equal to one: a 1% change in the expected marginal damage results in a 1% change in the tax rate. Compared to the sensitivities previously computed, an elasticity of 1 seems reasonable, even desirable. In spite of this reassuring observation, the retroactive tax still fares better in this dimension as well. In fact, once again, the retrospective tax rate is entirely unaffected by such variations, because it does not rely on any expectations of future damage. From a policy standpoint, this robustness implies that delaying implementation until climate forecasts around the world reach a consensus is unwarranted.

**Proposition 7** The elasticity of the standard tax rate with respect to prediction errors is one, while that of the retrospective tax rate is zero.

Note that this does not say that the retrospective tax rate is necessarily error-free. However, these errors are attached to the *estimation* of current marginal damage ($\partial D_t/\partial Z_t$), rather than *prediction* errors of future marginal damage $E_t[\partial D_s/\partial Z_s]$, all $s \geq t$.

We have shown the advantages of the retrospective tax in terms of robustness. While the tax rate is utterly insensitive to changes in the decay rate, the discount factor and climate predictions, that is not to say that the outcome of the policy itself is unaffected. In fact, the equivalence shown in Section 4
establishes that the outcomes of the two taxes are identical in terms of emission decisions. Therefore, they will be equally sensitive to changes in the parameters of the model. Nonetheless, we argue that the robustness of the policy instrument (the tax rate) under the retrospective tax is likely to make it more easily agreed upon from an implementation standpoint.

7 Heterogenous discount factors

Countries’ social planners may differ in their valuation of future prospects for several reasons, including differences in life expectancy. If the discount factors differ across countries, so that country j’s discount factor is now denoted \( \beta_j \), the associated global social welfare problem, which becomes,

\[
\max_{\{X_t^j\}_{t=T}^{T+\infty}} E_T \left\{ \sum_{t=T}^{T+\infty} W_t \left( X_t^j \right) \right\} = \max_{\{X_t^j\}_{t=T}^{T+\infty}} E_T \left\{ \sum_{t=T}^{T+\infty} \beta_j^{t-T} \left[ D_t^j \left( \theta_t, Z_t \right) - D_t^j \left( X_t^j \right) \right] \right\},
\]

(15)
displays time-inconsistency.\(^7\) Indeed, the solutions of (15), computed at date \( T \), for the decisions \( X_t^j \) to be taken in the future \( (t > T) \) differ from those computed at date \( T+1 \). In words, a global planner will revise what he/she considers to be the socially optimal policy period after period.

Yet, it is still possible to define an efficient allocation by equalizing the marginal benefits of emission flows to expected global marginal damage, for each country and in each period:

\[
\frac{dD_t^j \left( X_t^j \right)}{dX_t^j} = \sum_{j=1}^{N} \sum_{s=t}^{\infty} \left( \gamma \beta_j \right)^{s-t} \left[ \frac{\partial D_t^j}{\partial Z_s^j} \right].
\]

(16)

Notice that Expression (16) naturally defines an optimal tax rate on emission flows, \( \tau_t^* = \sum_{j=1}^{N} \sum_{s=t}^{\infty} \left( \gamma \beta_j \right)^{s-t} \left[ \frac{\partial D_t^j}{\partial Z_s^j} \right]. \)

In addition, it is easy to show that this efficient allocation can also be decentralized by means of taxation on emission stocks. Taxing stocks requires a taxation rate which is country specific:

\[
\mu_{s,t}^j = (1 - \gamma \beta_j) \tau_t^*.
\]

---

\(^7\) As shown by Jackson and Yariv (2011), this is a necessary feature of collective dynamic choice problems.
Obviously, the fact that the optimal retrospective tax rate is country-specific calls for strategic manipulations. Nonetheless, note that the elicitation of the $\beta_j$'s is also necessary for the computation of the optimal tax rate on flows, $\tau^*_t$, as defined in (16), and is thus also vulnerable to manipulation.

To circumvent the issue of elicitation of the $\beta_j$'s, consider a tax on stocks whose rate ignores the possible heterogeneity in the countries’ discount factors, so that the emissions stocks $Z^1_t$ are taxed according to the same uniform rate as before:

$$\tilde{\mu}_t = \frac{\partial D_t}{\partial Z_t}. \quad (18)$$

Now, denote by $\mathcal{W}_t(\mu_t)$ be the level of social welfare associated with the taxation vector $\mu_t = (\mu_{1,t}, \mu_{2,t}, ..., \mu_{n,t})$. By definition,

$$\frac{\partial \mathcal{W}_t}{\partial \mu_{j,t}}|_{\mu_t=\mu^*} = 0,$$

where $\mu^*$ is defined in (17). Hence, a Taylor expansion of $\mathcal{W}_t(\mu_t)$ around $\mu = \mu^*$, shows that the difference $\mathcal{W}_t(\tilde{\mu}_t) - \mathcal{W}_t(\mu^*_t)$ is second-order with respect to differences in $\mu_{j,t}$ and, hence, to differences in $\beta_j$. Therefore, the retrospective approach allows one to completely circumvent the problems associated with the elicitations of the countries’ discount factors only at the cost of a second-order efficiency loss, while the standard approach, taxing flows, cannot. In fact, such errors would have a striking first-order effect on any reasonable averaging of the standard tax rate, because it would at least require aggregate information on the $\beta$’s.

8 Concluding remarks

There are several issues which, although beyond the scope of the paper, would have to be carefully addressed should one wish to implement a retrospective tax.

The first one relates to distributional aspects. While the retrospective tax implements the same emissions path as the standard tax, the distribution of welfare seems to differ between the two. Indeed, the reader will have noticed that countries will pay close to nothing in the first periods under the retrospective tax, with the tax burden increasing over time, whereas the tax burden under the standard tax is not so blatantly unbalanced. This observation may lead one to
believe that the standard tax is fair to all generations while the retrospective tax unduly favors the early ones. Such a reasoning is mistaken, however, because tax revenues do not vanish.

Nonetheless, one must keep in mind that the intergenerational fairness issue stems largely from the fact that later generations will suffer more climate damage than earlier ones. Thus, unless the standard tax—or the retrospective tax, for that matter—is also attached to huge intergenerational transfers, it fails miserably on the grounds of intergenerational equity.

Our concern here was only one of efficiency because it is the main motivating argument for the standard tax, the benchmark instrument of climate change policy. However, we do not consider the fairness issue to be of secondary importance. In fact, in a companion paper (Billette de Villedum and Leroux, 2011), we analyze the set of lump-sum transfers that may be carried out under the retrospective tax to alleviate the spatial equity problem (i.e., the fact that the countries most affected by climate change are typically not the ones having emitted the most). We show that transfers are more easily implemented under a retrospective tax scheme because it readily allows to take the historical responsibility of countries into account.

Secondly, while taxing stocks seems manageable at the country level, it is not immediately clear how countries should ask its constituents—businesses and consumers—to foot the bill. Of interest is the fact that the retrospective tax does not conflict with the subsidiary principle, by which a central authority should only perform those tasks which cannot be performed effectively at a more immediate or local level. A simple pragmatic solution, akin to the Baumol and Oates (1975) approach, could consist in setting a goal and introducing instruments—like permits or even taxes on emissions flows—for the country as a whole to meet its obligations. Note also that the lengths of time involved in the climate problem allow for much flexibility in setting the duration of an accounting period within the retrospective tax scheme. In particular, period durations may be chosen long enough to confer short-run stability to governments as well as the ability to adapt and set these instruments accordingly.

Finally, on the issue of commitment, one may wonder whether countries will really reduce emissions in practice if they are not fully and immediately held accountable for the impacts of today’s emissions. Political pressure may induce countries to favor the present generation more than is optimal. It is a valid point, and a crucial one for the retrospective tax to work. Recall that the retrospective tax asks that economic agents internalize the consequences of
their decisions on the welfare of future generations, whereas the standard tax rate already has consideration for future generations embedded in it. However, note that political pressure has much bite in the standard tax as well, and currently manifests itself by having countries setting a carbon tax rate that is either much too low or by refusing to adopt a carbon tax altogether. Hence, the commitment problem is present under both tax schemes, whether at the rate-setting and implementation stage (for the standard tax) or at the enforcement stage (for the retrospective tax). In fact, this is true of any mechanism where sacrifices must be made today so as to avoid future welfare losses.

The matter is one of institutional design, and it is doubtful that implementing an optimal retrospective tax would be any more problematic than implementing the optimal standard tax currently proves to be. Moreover, the very existence of the many discussions on climate agreements are a clear sign that there is indeed a concern for future generations. As far as commitment is concerned, there is no more reason for countries not to pay their dues in the future than there is for them to ignore the issue in the present. The whole point of the paper is that we should instead take advantage of countries’ concern for the future to decentralize the optimal emissions pattern.

Because the general idea of the standard tax is now so widespread, a retroactive tax is bound to be received with some amount of skepticism. Yet, taxing stocks rather than flows solves arguably two insurmountable issues in the calculation of the carbon tax rate (forecasting and discounting) and provides an answer to the recent "ramp/springboard" debate. Moreover, from an implementation standpoint, it would constitute an important step toward the implementation of a global, consensual, and efficient climate policy. As it turns out, this very simple idea of taxing stocks rather than flows may radically transform our approach to the climate change problem. We believe it to be very promising and worthy of serious consideration.

References


