The cyclical behavior of the unemployment, job finding and separation rates

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Abstract

In this paper, we determine a particular calibration that makes the search and matching model able to replicate the unemployment volatility with the right contributions for the job finding and separation rates. We also point out a central mechanism of the model: introducing cyclical separations amplifies the volatility of the job finding rate. This mechanism has two implications. First, the value of the employment opportunity cost necessary to reproduce all the volatilities is lower, and more realistic, than in the model with exogenous separations. Secondly, the model with endogenous separations does not require real wage rigidities to solve the unemployment volatility puzzle.

JEL Codes: E24, J63, J64.

Keywords: Endogenous separation, Unemployment volatility, Matching.
1 Introduction

The “unemployment volatility puzzle”, following the well-known contribution of Shimer (2005), embodies the inability of the Mortensen-Pissarides (henceforth MP) class of models to replicate the volatility of the unemployment rate in the US. Empirically, the standard deviation of the unemployment rate is explained for an half by the standard deviation of the job finding rate, and for the other by that of the separation rate (Fujita and Ramey (2009), Elsby and al.(2010)), even though this proportion is still debated. However, the first attempts to solve this puzzle narrowed on the canonical version of the matching model, in which the separation rate is purely exogenous and constant. In his paper, Shimer (2005) targeted the volatility of the job finding rate but this solution reproduces only half of the unemployment standard deviation. Hagedorn and Manovskii (2008) targeted the unemployment variability in their calibration of the worker’s opportunity cost of employment. In this case, all the unemployment volatility stems from the job finding, which clearly overstates the job finding standard deviation.

Since the seminal work of Mortensen and Pissarides (1994), the cyclical behavior of the separation rate has received an increasing attention. Mortensen and Nagypál (2007), Pissarides (2009) and Fujita and Ramey (2012) notably introduce an additional margin on which the firms adjust the number of their jobs. In all these papers, the volatility of the separation rate is restituted but the job finding variability (and thus the unemployment volatility) is always far below its empirical counterpart. Moreover, the model fails in reproducing the Beveridge curve, i.e. the highly negative correlation between the unemployment and vacancy rates.

In this paper, we determine a calibration that makes the search and matching model with endogenous separations capable to replicate simultaneously the volatility of the unemployment, job finding and separation rates, as well as the Beveridge curve. The strategy followed is close to the one applied by Hagedorn and Manovskii (2008) for the constant separation rate model: the value of some key parameters is chosen to match the various standard deviations and the unemployment/vacancy correlation. Particularly, the opportunity cost of employment is calibrated to reproduce the job finding standard deviation.

We also highlight a central mechanism of the model with endogenous separations: introducing cyclical separations amplifies the volatility of the job finding rate. Intuitively, since firms have the ability to adjust employment through job separations, we may expect that they would adjust less on job creations. The job finding rate would therefore be less volatile and we should find some trade-off between the job finding and separation rates variabilities. On the contrary, we stress the existence of an amplification mechanism, operating through the profit of the firm, which makes the job finding response rise with the introduction of cyclical separations. This amplification is an increasing and convex function of the opportunity cost of employment.
The amplification mechanism has two implications. First, the value of the opportunity cost of employment (around 85% of labor productivity) required to replicate the standard deviation of the job finding rate is lower, and more plausible, than for the model with constant separations. Secondly, the search and matching model with endogenous separations delivers the right standard deviations for a real wage determined by the symmetric Nash bargaining. The amplification mechanism therefore makes the model able to solve the unemployment volatility puzzle without resting on any real wage stickiness.

The rest of the paper is organized as follows. In the next section, we present the model with endogenous separations. We also describe the amplification mechanism and how this mechanism depends on the opportunity cost of employment. In the third section, we find a calibration that replicates the right standard deviations for the unemployment, job finding and separation rate, as well as the correlation between the unemployment and vacancy rates. We also illustrate the amplification. Section 4 concludes.

2 The model with endogenous separations and the amplification mechanism

The framework is derived from Mortensen and Pissarides (1994). Here we use the presentation of Fujita and Ramey (2012).

2.1 The framework

2.1.1 Basic structure

The model with endogenous separations sets the same assumptions than the canonical one. The only element that differs is the modelisation of the separation rate, which is no longer an exogenous probability but depends on the productivity of the match.

There are two types of agents in the economy, the workers and the firms. In every period $t$, a worker is either employed, and receives a wage $w_t$, or unemployed, and receives a flow benefit $z$ (which may include unemployment benefits, the value of leisure and home production). $z$ is also the opportunity cost of employment for the worker. A firm is either matched with a worker or posting a vacancy. In this latter case, the firm bears a cost $c$. 
The number of vacant jobs is denoted by $v_t$ while the number of unemployed workers is given by $u_t$. Since the labor force is normalized to one, $v_t$ and $u_t$ are also the vacancy and unemployment rates. We denote by $\theta_t = \frac{v_t}{u_t}$ the labor market tightness. The number of matches created every period is given by the following matching function $m(u_t, v_t) = A u_t^{\alpha} v_t^{1-\alpha}$, which is a Cobb-Douglas function with constant return to scale. We define the probability $q(\theta_t)$ of filling a vacancy by $q(\theta_t) = m(u_t, v_t) v_t = A \theta_t^{1-\alpha}$ and the job finding rate by $\theta_t q(\theta_t) = m(u_t, v_t) u_t = A \theta_t^{-\alpha}$.

There are two sources of productivity in the economy: $p_t$, the aggregate productivity (macro productivity) and $x_t$, a match specific productivity factor (micro productivity). $x_t$ is a random variable, which evolves according to the c.d.f $G(x)$. Following a common practice in the literature, every match begins at the maximum micro productivity $x_h$ and has a probability $\lambda$ of being hit by a shock each period.

There are also two sources of job separation: exogenous and endogenous. In every period, a match has an exogenous probability $s$ of being destroyed. We denote by $S_t(x_t)$ the value of the joint surplus of a match. In every period, the worker and the firm agree to continue the match if $S_t(x_t) > 0$. Instead, they separate if separation is jointly optimal, i.e. if $S_t(x_t) = 0$. This case occurs whenever the micro productivity falls under a certain level $R_t$, called the reservation productivity. In this model, the separation rate is therefore equal to $s + (1-s) \lambda G(R_t)$.

In period $t$, the value of a firm with a vacant job satisfies the following Bellman equation:

$$V_t = -c + \beta E_t[q(\theta_t) J_{t+1}(x_h) + (1 - q(\theta_t)) V_{t+1}]$$

with $\beta$ the discount factor. The firm’s value of a filled match, when the continuation of this match is chosen, is given by:

$$J_t(x_t) = p_t x_t - w_t(x_t) + \beta E_t[(1-s)(\lambda \int_{R_{t+1}}^{x_h} J_{t+1}(y)dG(y) + (1-\lambda) J_{t+1}(x_t)) + s V_{t+1}]$$

In period $t$, the unemployment value is:

$$U_t = z + \beta E_t[\theta_t q(\theta_t) W_{t+1}(x_h) + (1-\theta_t q(\theta_t)) U_{t+1}]$$

The worker’s value of a match, after continuation of this match is chosen, is:

$$W_t(x_t) = w_t(x_t) + \beta E_t[(1-s) (\lambda \int_{R_{t+1}}^{x_h} W_{t+1}(y)dG(y) + (1-\lambda) W_{t+1}(x_t)) + s U_{t+1}]$$

The joint surplus of a match is shared following the standard Nash bargaining. The value of the joint surplus is then defined by:

$$S_t(x_t) = J_t(x_t) + W_t(x_t) - V_t - U_t$$
The Nash bargaining entails the following surplus-sharing rule:

\[ W_t(x_t) - U_t = \zeta S_t(x_t) \]

\[ J_t(x_t) - V_t = (1 - \zeta) S_t(x_t) \]

with \( \zeta \) the bargaining power of the worker.

2.1.2 Creation and destruction

The free entry condition implies that vacancies are opened as long as the expected profit is higher than the expected cost. Hence, \( V_t = 0 \) in equilibrium, which gives the job creation condition:

\[ \beta E_t J_{t+1}(x_h) = \frac{c}{q(\theta_t)} \]

Given the surplus-sharing rule, the job creation condition also reads:

\[ \beta(1 - \zeta) E_t S_{t+1}(x_h) = \frac{c}{q(\theta_t)} \]  

From the definitions of \( U_t, W_t, J_t \) and the surplus-sharing rule, the value of the joint surplus, when continuation of the match is chosen, can be expressed as:

\[ S_t(x_t) = p_t x_t - z + \beta E_t[ (1 - s)(\lambda \int_{R_t+1}^{x_h} S_{t+1}(y)dG(y) + (1 - \lambda)S_{t+1}(x_t)) - \theta_t q(\theta_t) \zeta S_{t+1}(x_h)] \]  

This equation leads to the job destruction condition. The reservation productivity corresponds to the micro productivity which cancels out the joint surplus, i.e. \( S(R_t) = 0 \). The job destruction condition is thus written:

\[ p_t R_t = z - \beta E_t[(1 - s)(\lambda \int_{R_t+1}^{x_h} S_{t+1}(y)dG(y) - \theta_t q(\theta_t) \zeta S_{t+1}(x_h)] \]  

From equations (2) and (3), \( S_t(x_t) \) also reads:

\[ S_t(x_t) = p_t(x_t - R_t) + \beta(1 - s)(1 - \lambda)E_t S_{t+1}(x_t) \]
2.2 Trade-off or amplification?

In this sub-section, we examine how the volatility of the separation rate affects the job finding standard deviation.

When endogenous separations are integrated into the canonical MP class of models, one could intuitively expect that the volatility of the separation rate would come at the expense of the vacancies and job-finding volatilities, since firms have now an additional margin on which they could adjust. Following a positive aggregate shock, firms could reduce the number of destructions and/or increase the number of vacancy creations, whereas in the constant destructions model, they are only able to adjust the number of creations.

More precisely, three effects, going in opposite directions, impact the job creation condition when endogenous separations are introduced. Recall that this condition is given by equation (1):

$$\beta(1 - \zeta)E_tS_{t+1}(x_h) = \frac{c}{q(\theta_t)}$$

and the following expression for $S_t(x_h)$ (from equation (4)):

$$S_t(x_h) = p_t(x_h - R_t) + \beta(1 - s)(1 - \lambda)E_tS_{t+1}(x_h)$$

The first two effects are related to the discounted profit of a vacancy creation (the left-hand side of this condition) while the third effect impacts the discounted cost of such a creation (the right-hand side).

Let us assume that there is a positive aggregate shock. The first consequence is a direct increase in the flow profit associated to a vacancy and then in the discounted profit. This effect, which obviously raises the number of vacancy creations and the job-finding rate, is present and quantitatively identical for separations being endogenous or exogenous. It is all the more powerful as $z$ is high. Indeed, Hagedorn and Manovskii (2008) stress that the dynamics of vacancy creations is driven by the variations of the firms’ profit in percentage terms. From equation (3), a higher value for $z$ raises the steady state value of $R$, which reduces the steady state joint surplus and then makes the profit more responsive in proportion.

The second effect is a reduction in the reservation productivity that increases the flow and discounted profit linked to a new job. This effect only stands in the endogenous separations model. The strength of this effect is also increasing in $z$: with a high value for $z$, the profit at the steady state is low and even a small decrease in the reservation productivity entails a sharp rise in the discounted profit in proportion.
At this stage, the job-finding volatility is increasing in $z$, with or without cyclical separations. This volatility is nonetheless higher in the endogenous separation framework because of the second effect. Consequently, adding some cyclical behavior in the separation rate amplifies job finding variations. This amplification is furthermore rising in $z$ since the second effect depends positively on this parameter.

The third effect is related to the discounted cost of vacancy posting. The unemployment rate falls following a positive aggregate shock, which entails an increase in the market tightness, a decrease in the probability for firms to fill a vacancy and then an increase in the expected duration of vacancy posting. The resulting rise in the discounted cost of vacancy posting reduces the incentive to create jobs and thus job finding fluctuations. Nevertheless, this effect is stronger in the endogenous separations model, since the fall in unemployment following the positive shock is higher than in the constant destructions model: this effect implies that introducing cyclical separations comes at the expense of job finding variations and illustrates the intuitive trade-off between the job finding and separation rates volatilities.

Whether endogenous separations amplify or reduce the job finding volatility depends on the relative strength of the second and third effects. The value of $z$ is prominent in this comparison since the quantitative impact of the second effect crucially rests on this parameter while the third effect is independent from it. For low values for $z$, the second effect is weak and should be dominated by the third effect. In this case, the job finding volatility is lower in the endogenous separations model than in the constant separations model, implying the intuitive trade-off. Alternatively, with higher values for $z$, the second effect is strong and should more than offset the third effect. The job-finding volatility is now amplified by endogenous separations.

3 Calibration strategy and the amplification in practice

3.1 Calibration

The empirical series come from the CPS\footnote{This data was constructed by Robert Shimer. For additional details, see Shimer (2007) and his webpage http://sites.google.com/site/robertshimer/research/flows.}. The unemployment, separation and job finding rates were calculated for the 1976 - 2010 period. These monthly data were seasonnally adjusted and then HP filtered. The mean unemployment, job finding and separation rates
are respectively 0.061, 0.017 and 0.305. The standard deviations of those rates are given in Table 2.

In order to calibrate the model with cyclical separations, we follow the strategy initiated by Hagedorn and Manovskii (2008) for the model with constant separations: the values of some key parameters are determined to replicate the standard deviations of the unemployment, job finding and separation rates, as well as the correlation between vacancy and unemployment rates. Three parameters are critical to get the right volatilities and the Beveridge curve: the opportunity cost of employment $z$, the standard deviation of specific shocks $\sigma_x$ and their arrival rate $\lambda$.

We have argued above that the opportunity cost of employment has a strong impact on the variations of the job finding rate. A high value for $z$ implies a high volatility for this rate, since the firm’s profit is more responsive in proportion. We therefore select the value of $z$ so as to reproduce the empirical standard deviation of the job finding rate. We obtain a value of 0.86 for this parameter: the search and matching model with endogenous separations generates the right volatility of the job finding rate for an opportunity cost of employment representing 86% of labor productivity. This value is between the evaluation of Mortensen and Nagypál (2007) (0.73) and the value found by Hagedorn and Manovskii (2008) for the model with exogenous separations (0.96). The evaluation of Mortensen and Nagypál includes unemployment benefits and the value of leisure forgone amounting to 30% and 43% of labor productivity, respectively. Recall that $z$ could also include the value of home production. The value we find for the model with cyclical separations means that home production would represent 13% of labor productivity.

The standard deviation of specific shocks has a strong impact on the variations of the separation rate. Indeed, $\sigma_x$ determines the size of specific shocks. A high value for this parameter means that specific shocks are large. The movements of the reservation productivity are therefore magnified, making the separation rate more volatile. We thus choose the value of $\sigma_x$ to replicate the empirical standard deviation of the separation rate. We obtain a value of 0.035 for $\sigma_x$, which is close to Fujita and Ramey (2012).

The arrival rate of specific shocks has an impact on both the vacancy and unemployment rates. A high value for $\lambda$ means that the probability for a new match to be hit by a negative specific shock is high, which reduces the expected duration of this match. Hence, the incentive to post vacancies is also reduced. At the same time, a high value for $\lambda$ implies that specific shocks are weakly persistent. This creates an incentive for firms to dampen job separations following a negative specific shock. The separation and unemployment rates are consequently less volatile. By affecting both vacancy and unemployment rates, $\lambda$ has an impact on the Beveridge curve. We select the value that allows the model to deliver the empirical correlation between those rates and find a value of 0.1 for $\lambda$. Again, this value is close to Fujita and Ramey (2012) and represents a mean waiting time of three months between two shocks.
The remaining parameters are chosen in a standard way. The aggregate productivity is simulated with an AR(1) process \( p_t = \mu_p p_{t-1} + \sigma_p \), with \( \mu_p = 0,9895 \) and \( \sigma_p = 0,0034 \). This is the process retained by Hagedorn and Manovskii (2008) and followed by Fujita and Ramey (2012). The discount factor is equal to \( \beta = \frac{1}{1+r} \), with \( r = 0,04/12 \). This corresponds to an annual interest rate of 4%, consistent with the data. The wage bargaining is assumed to be symmetric, an assumption often made in the literature. This implies a value of 0.5 for \( \zeta \). The elasticity of the matching function \( \alpha \) is set to 0.5, in the range determined by Petrongolo and Pissarides (2001). The efficiency parameter of the matching function \( A \) is determined in order to replicate the empirical level of the job finding rate. The cost of posting a vacancy ensures that the job creation always holds at sample mean, for a value of the tightness equal to 0,72 at steady-state (Pissarides (2009)). \( G(x) \) is assumed to be a log normal distribution function, of zero mean. The highest match-specific productivity \( x_h \) is set to 1. Finally, The exogenous separation rate \( s \) is selected to replicate the empirical value of the separation rate.

Table 1 summarizes all the parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline calibration</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z )</td>
<td>0,86</td>
<td>job finding standard deviation</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>0,035</td>
<td>separation rate standard deviation</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0,1</td>
<td>Beveridge curve</td>
</tr>
<tr>
<td>( \mu_p )</td>
<td>0,98</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>( \sigma_p )</td>
<td>0,0034</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0,99</td>
<td>annual interest rate of 4%</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0,5</td>
<td>Petrongolo-Pissarides (2001)</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>0,5</td>
<td>symmetric bargaining</td>
</tr>
<tr>
<td>( \mu_x )</td>
<td>0</td>
<td>Ramey (2008)</td>
</tr>
<tr>
<td>( x_h )</td>
<td>1</td>
<td>Mortensen Nagypál (2007)</td>
</tr>
<tr>
<td>( A )</td>
<td>0,36</td>
<td>job finding rate</td>
</tr>
<tr>
<td>( c )</td>
<td>0,13</td>
<td>mean ( \theta )</td>
</tr>
<tr>
<td>( s )</td>
<td>0,015</td>
<td>separation rate</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0,72</td>
<td>CPS</td>
</tr>
<tr>
<td>( A\theta^1-\alpha )</td>
<td>0,305</td>
<td>CPS</td>
</tr>
</tbody>
</table>

Table 2: Data and results
### Standard deviations

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>unemployment</td>
<td>0.125</td>
<td>0.120</td>
</tr>
<tr>
<td>job finding</td>
<td>0.0687</td>
<td>0.0682</td>
</tr>
<tr>
<td>separation</td>
<td>0.0525</td>
<td>0.0535</td>
</tr>
<tr>
<td><strong>Correlation u/v</strong></td>
<td>- 0.95</td>
<td>- 0.95</td>
</tr>
</tbody>
</table>

#### 3.2 The amplification in practice

To illustrate how the amplification mechanism works and depends on $z$, we compare the job finding standard deviations for, on the one hand, the MP framework with endogenous separations and, on the other, the framework with a constant separation rate. Table 3 displays the standard deviations of the job finding rate for each value of $z$.

In order to get the standard deviation of the job finding rate for the model with a constant separation rate, we set $s$ at 0.017 and $\lambda$ at 0. In this case, the match specific productivity is drawn at the first period and does not change thereafter, given that the probability to be hit by a match-specific productivity shock is now equal to zero.

<table>
<thead>
<tr>
<th>$z$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.86</th>
<th>0.9</th>
<th>0.96</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.009</td>
<td>0.011</td>
<td>0.014</td>
<td>0.016</td>
<td>0.021</td>
<td>0.023</td>
<td>0.030</td>
<td>0.068</td>
<td>0.093</td>
<td>0.255</td>
</tr>
<tr>
<td>(2)</td>
<td>0.009</td>
<td>0.011</td>
<td>0.013</td>
<td>0.015</td>
<td>0.018</td>
<td>0.019</td>
<td>0.023</td>
<td>0.056</td>
<td>0.078</td>
<td>0.211</td>
</tr>
<tr>
<td>(1) - (2)</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.004</td>
<td>0.007</td>
<td>0.012</td>
<td>0.015</td>
<td>0.043</td>
</tr>
</tbody>
</table>

(1) : job finding standard deviation in the endogenous separation framework  
(2) : job finding standard deviation in the exogenous separation framework

For a worker’s opportunity cost below 0.4, the job finding standard deviations for the two frameworks are quite similar, meaning that the second and third effects depicted in Section 2.2 cancel each other out. In this range of values for $z$, introducing a cyclical behavior for the separations neither amplifies nor reduces the job finding volatility.

From $z = 0.4$, the job finding standard deviation is higher for the endogenous separations model than for the exogenous one. The second effect dominates the third effect. Introducing
cyclical separations now amplifies the job finding response to aggregate productivity shocks. As shown in Table 3, this amplification is increasing and convex in the value of \( z \). For \( z = 0.4 \), the standard deviations difference is only equal to 0.001 while for \( z = 0.86 \), this difference amounts to 0.012: for this value of \( z \), the model with endogenous separations produces a volatility of the job finding rate 22% higher than the model with a constant separation rate. As \( z \) approaches 1, the amplification becomes considerable: at \( z = 0.96 \), the job finding standard deviation difference amounts to 0.043, which is two thirds the empirical value (0.0687).

This amplification has two consequences. First, the value of \( z \) required to replicate the job finding standard deviation for the model with endogenous separations is lower than for the model with a constant separation rate. For this latter model, Hagedorn and Manovskii (2008) find that the required value for \( z \) equals 0.96. Introducing cyclical separations thus enables the search and matching model to restitute the job finding volatility for a lower, and more realistic, value of the opportunity cost of employment. Secondly, our calibration does not imply any real wage stickiness. The elasticity of the real wage with respect to productivity is equal to 0.96. Therefore, the amplification makes the model with endogenous separations able to reproduce the various standard deviations and then to solve the unemployment volatility puzzle without resting on real wage rigidities. This is another difference with the model with exogenous separations: even with their extreme value for \( z \), Hagedorn and Manovskii (2008) need sharp wage rigidities to solve this puzzle.

4 Conclusion

In this paper, we have followed a strategy close to Hagedorn and Manovskii (2008) by determining a calibration that makes the MP framework replicate the unemployment standard deviation with the right contributions of the job finding and separation rates, and the Beveridge curve. We have also stressed a central mechanism of the model with endogenous separations: introducing cyclical separations amplifies the volatility of the job finding rate. This mechanism dominates the intuitive trade-off between job finding and separation rates variabilities. The amplification mechanism has two implications. First, the required value for the opportunity cost of employment in the model with cyclical separations is lower, and more plausible, than in the model with exogenous separations. Secondly, the framework with endogenous separations solves the unemployment volatility puzzle with the symmetric Nash bargaining, i.e. without resting on real wage rigidities.
References


