Tests of the Rational Expectations Hypothesis

By Michael C. Lovell*

This paper reviews evidence from a number of empirical studies challenging the validity of the received hypothesis of rational expectations. The analysis does not rest primarily on new empirical evidence, but instead on evidence from a number of studies; some recent, some unpublished, many of older vintage. I demonstrate that the cumulative empirical evidence does not establish that the received doctrine of rational expectations dominates alternative hypotheses about expectations. After reviewing the models and the evidence, I express my qualms about the dangers stemming from the categorical acceptance of the rational expectations hypothesis as "stylized fact" to the exclusion of alternatives in empirical investigations, in theoretical research, and, most important, in policy analysis.

Is it appropriate to test the rational expectations hypothesis at the micro level? It can be said that the force behind the rational expectations argument had nothing to do with claims concerning its empirical validity. Thomas Sargent argues:

Research in rational expectations and its dynamic macroeconomics has a momentum of its own. That momentum stems from the logical structure of rational expectations as a modeling strategy, the questions that it invites researchers to face, and the standards that it imposes for acceptable answers to those questions. [1982, p. 382]

Sargent does not claim that the assumption that expectations are rational is realistic; he does not claim that the momentum of rational expectations derives from direct empirical evidence.¹

Edward Prescott has argued that the rational expectations hypothesis is not amenable to direct empirical test: "Like utility, expectations are not observed, and surveys cannot be used to test the rational expectations hypothesis. One can only test if some theory, whether it incorporates rational expectations or, for that matter, irrational expectations, is or is not consistent with observations" (1977, p. 30).

It seems to me that it may be a mistake to argue that we can divide variables into those that are observable and those that are not. After all, utility can be measured up to a linear transformation; measuring sales expectations, while not easy, may be no more difficult than trying to measure economic profit. A theory that claims to have a strong microeconomic foundation should be amenable to testing with micro data. Observe that over the last several decades a number of economists—from Franco Modigliani and Owen Sauerlander (1955) to Otto Eckstein, Patricia Mosser, and Michael Cebry (1984) —has found that survey observations on expectational variables can be of assistance in the empirical modeling of economic behavior and econometric forecasting.

¹While the momentum of rational expectations may derive from the fact that John Muth's rational expectations hypothesis provides a fundamental extension to the classical economic paradigm, this explanation does not account for the fact that Muth's contribution lay dormant for a number of years, including a period in which Muth and Robert Lucas were colleagues at Carnegie-Mellon; and while Thomas Sargent was exposed to the concept of rational expectations while at Carnegie-Mellon in 1967, he did not pursue the concept at the time (see Aljo Klamer, 1983, p. 61). The momentum of rational expectations may well derive as much or more from appreciation of its forceful policy implications as from its intellectual contribution to classical theory per se.

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It has long been argued that theories should be judged on their predictive ability rather than on the basis of the validity of their simplifying assumptions, which must of necessity be false. In a recent paper Robert Lucas asserts: “Any model that is well enough articulated to give clear answers to the questions we put to it will necessarily be artificial, abstract, patently ‘unreal’” (1980, p. 696).

In explaining how confidence in the validity of a model’s predictions can be earned, Lucas goes on to state:

...individual responses can be documented relatively cheaply, occasionally by direct experimentation, but more commonly by means of the vast number of well-documented instances of individual reactions to well-specified environmental changes made available “naturally” via censuses, panels, other surveys, and the (inappropriately maligned as “casual empiricism”) method of keeping one’s eyes open.

[p. 696]

Lucas’s statement raises the issue of whether evidence on individual behavior from surveys, censuses, and so forth is admissible only for the testing of the predictions of a theory, or whether they can and should be used to test the validity of the rational expectations hypothesis itself. My own view is that the appropriate realm for empirical research should not be demarcated in terms of the dichotomy between assumptions and predictions—I think that direct testing of the rational expectations hypothesis is an appropriate and worthwhile activity. In order to be able to claim that a theory is based on firm micro foundations requires more than the derivation of propositions from the assumption that economic agents maximize, however esthetically pleasing such derivations may be; a theory that is said to be based on micro foundations should survive empirical testing at the level of the individual decision making unit. To the extent that the survey evidence supports the hypothesis of rational expectations, results derived under that assumption, policy impossibility theorems, and so forth, will be both more interesting and more demanding of serious attention.

This paper examines the evidence. After reviewing in the next section the features distinguishing Muth’s theory of rational expectations from alternative models, I examine in Section III the weight of the evidence accumulated to date from a variety of empirical studies.

I. On the Structure of Expectations

The rational expectations hypothesis is only one of a variety of strategies that have been used by researchers in modeling expectations; many were developed in the 1950’s. I first look at the alternatives and compare the rival models; then I shall look at the weight of the accumulated empirical evidence provided by a number of studies.

A. Ferber’s Law

Robert Ferber (1953) concluded from interviews with a number of business enterprises that, in making forecasts, firms typically attempt to allow for seasonal movements by modifying the figure for the corresponding quarter of the previous year in the light of recently observed trend; that is to say, with quarterly observations Ferber’s law states:

\[ P_t = \rho_0 + A_{t-4} \left[ \rho_1 + \rho_2 \left( A_{t-1} - A_{t-5} \right) / (A_{t-5}) \right]. \]

Here \( P_t \) is the value predicted for quarter \( t \) on the basis of lagged actual values \( A_{t-4} \), where time is measured in quarterly units. Observe that expectations are simply “same-as-last-year” forecasts if \( \rho_0 = \rho_2 = 0 \) and \( \rho_1 = 1 \). However, expectations are last period’s...
experience modified by a crude seasonal adjustment factor if \( \rho_0 = 0 \) and \( \rho_1 = \rho_2 = 1 \); that is,

\[
(1') \quad P = A_{-1}(A_{-4}/A_{-5}).
\]

On the basis of his study of the Railroad Shippers Forecast data, Ferber concluded that expectations are regressive; that is, he found that \( \rho_2 \) was considerably less than one, implying that recent gains or losses since last year are not expected to persist.\(^3\) In an interesting study of inventory behavior and the production decision, John Johnston (1961) used Ferber’s equation in order to proxy out unobservable sales anticipations.

B. Adaptive Expectations (Exponential Smoothing)

The “adaptive” model of expectations formation, which stems from John Hicks’ (1939) concept of the elasticity of expectations, has been analyzed by Marc Nerlove (1964) and advocated as a practical procedure by management scientists, as in Charles Holt et al. (1960). In its most elementary form, this model may be written

\[
(2) \quad P = A_{t-1} + \lambda(P_{t-1} - A_{t-1}).
\]

Thus, the prediction is same-as-last-period if last period’s forecast turned out to be perfectly accurate. As one extreme case, if \( \lambda = 0 \), the model reduces to a naive prediction of no change; alternatively, if \( \lambda = 1 \) we continue with the same static forecast as before without revision for current error.

C. “Implicit” vs. “Rational” Expectations

An alternative to these two structural approaches is to avoid explicit modeling of the process by which expectations are generated. Rather, certain reasonable stochastic properties are hypothesized.

First, it has seemed reasonable to many investigators to hypothesize that expectations will be unbiased; that is, the expected value of the forecast error \( \epsilon \) is zero. More formally, I define

\[
(3) \quad \epsilon = P - A
\]

and impose the restriction that \( E(\epsilon) = 0 \).

Edwin Mills (1957) imposed an additional restriction in developing his concept of “implicit expectations” in connection with his fundamental empirical study of inventory behavior. Specifically, Mills conjectured that the prediction error is uncorrelated with the actual realization; with this restriction, the basic assumption of the regression model is satisfied with the anticipated variable selected as the dependent variable:

\[
(4) \quad P = \alpha_0 + \alpha_1 A + \epsilon
\]

with \( \alpha_0 = 0; \quad \alpha_1 = 1; \quad E(\epsilon) = 0. \)

On the basis of this argument, Mills used the actual realization as a proxy for the unobserved anticipated level of sales in his empirical study of inventory behavior.

John Muth (1961) pioneered a procedure that is just the opposite of Mills’ implicit expectations hypothesis. For rational expectations, Muth required that the forecast error be distributed independently of the anticipated value; that is,

\[
(5a) \quad A = \beta_0 + \beta_1 P + \epsilon
\]

with \( \beta_0 = 0; \quad \beta_1 = 1; \quad E(\epsilon) = 0. \)

For Muth, \( \epsilon \) must be uncorrelated with \( P \), the predictions; therefore, it must be correlated with \( A \), the actual realizations; hence the variance of \( A \) is larger than the variance of \( P \). All this is precisely the reverse of Mills’ implicit expectations, which have a larger variance than the actual realizations.\(^4\)

\(^3\)The same data source was also used in the pioneering Modigliani and Sauerlander study of inventory behavior. The validity of the Railroad Shipper Forecast data was questioned by Albert Hart (1960) and by me (1964: appendix).

\(^4\)The sample variance of actual realizations will exceed that of the anticipations, as required by the rational expectations hypothesis, only if \( r < b_1 \), where \( b_1 \) is the least squares estimate of \( B_1 \) in equation (5a). To verify this, observe that the regression coefficient \( a_1 = \text{cov}(a, P)/S_p^2 \) while \( b_1 = \text{cov}(a, P)/S_p^2 \); further \( r^2 = a_1 b_1 \); therefore, \( S_p/S_a = r/b_1 \).
To earn the fully rational accolade more is required; specifically:

The prediction error must be uncorrelated with the entire set of information that is available to the forecaster at the time the prediction is made.

This rationality concept might be called "sufficient expectations," for it is closely related to the statistical concept of a "sufficient estimator," which may be loosely defined as an estimator that utilizes all the information available in the sample.

One implication of this requirement is that the prediction error must be uncorrelated with historical information on prior realizations of the variable being forecast; this weak rationality condition, as it is sometimes called, implies that if lagged values of $A$ are added to the right-hand side of regression model (5a), they must appear with zero coefficients; for example, the coefficient $b_2$ in the following regression should not differ significantly from zero:

$$5b \quad A = b_0 + b_1P + b_2A_{t-1}.$$  

The "full rationality" conjecture also has a more demanding implication: it requires as a condition of "strong rationality" that any other variables known to the forecaster (for example, public information on the rate of growth of the money supply, federal deficits, and the unemployment rate) must also be uncorrelated with the forecast error.

D. Change Underestimation

In empirical work on the determinants of inventory investment (1961), I proxied out the unobserved expectational variables by invoking the conjecture that the predicted change is a fraction $\rho$ of observed changes:

$$6 \quad P - A_{t-1} = \rho(A - A_{t-1}) + \varepsilon.$$  

This equation is less restrictive than either implicit or rational expectations. With $\rho = 1$, this model reduces to either implicit or rational expectations, depending on whether it is conjectured that $\varepsilon$ is distributed independently of $P$ or $A$. In contrast, if $\rho = 0$ and $\varepsilon = 0$, we have naive "no change" extrapolative forecasts; with $0 < \rho < 1$, there is a systematic tendency to underestimate change, as hypothesized by J. M. Keynes: "...it is sensible for producers to base their expectations on the assumption that the most recently realized results will continue except insofar as there are definite reasons for expecting a change" (1936, p. 51).

E. Evaluation

In my judgement the choice among these alternatives is not easily resolved by a simple appeal to maximization or theoretical principle. Although closely related concepts, the choice between the mutually exclusive rational and implicit expectations models is not easy. One or the other of two opposing arguments can be advanced to rationalize the alternative formulations:

1) First, suppose the sales forecaster makes the prediction on the basis of a regression model (for example, historical sales explained by earlier values of sales and other variables); then the prediction error (at least over the sample period) will be uncorrelated with the explanatory variables. For example, suppose the forecaster fits to historical data the regression equation

$$7 \quad A = b_0 + b_1A_{-1} + b_2A_{-4} + \varepsilon$$

and uses the resulting least squares coefficients to generate predictions

$$8 \quad P = b_0 + b_1A_{-1} + b_2A_{-4},$$

the resulting prediction errors will be uncorrelated with the explanatory variables, at least over the sample period, in accordance with Muth's rationality hypothesis.

2) Alternatively, with regard to Mills' concept of implicit expectations, it was pointed out by Albert Hirsch and me (1969, pp. 73-74) that sales forecasts derived by periodically surveying a sample of reliable customers are likely to satisfy this condition; that is, a survey of a random sample of customers will yield an estimate of average sales per customer which will be subject to sampling error; the survey results will be randomly distributed about the actual popu-
lation response, as required by Mills' implicit expectations model.\footnote{This is not the only rationalization of Mills' approach. In their empirical study of the informational content of prices in dealer securities markets, it is assumed by K. D. Garbade et al. (1979, p. 52) that a dealer's observed offering price is randomly distributed about the true equilibrium price, rather than the reverse. Although they do not mention the concept, they are assuming that the offering price is an implicit rather than rational forecast of the equilibrium outcome; they do point out, however, that their assumption will not hold when a dealer wishes to adjust his inventory position in a security.}

For certain econometric purposes it facilitates matters to assume implicit rather than rational expectations. Consider the problem of estimating the standard flexible accelerator inventory model I used earlier (1961):

\begin{equation}
I_t = \delta \gamma_0 + (1 + \delta \gamma_1) X_t^e + (1 - \delta) I_{t-1} - X_t.
\end{equation}

Here $I_t$ is end of period inventories, $X_t^e$ is anticipated sales, and $X_t$ actual sales.\footnote{The flexible accelerator inventory model, frequently employed in empirical research, assumes that sales are exogenous. This assumption underlies much of the management science prescriptive literature on inventories and the production decision. The same assumption is employed in the Jorgenson neoclassical model of fixed investment. In contrast, Michael Brennan's (1959) approach of having the firm determine its inventory stocks on the basis of seasonal price movements is particularly relevant in the study of inventory holdings of agricultural commodities.} Substituting from equation (4) yields

\begin{equation}
I_t = \delta \gamma_0 + \delta \gamma_1 X_t + (1 - \delta) I_{t-1} + (1 + \delta \gamma_1) \epsilon_t.
\end{equation}

As Mills explained (1957), because the implicit expectations forecast error $\epsilon_t$ is uncorrelated with $X_t$, the application of least squares to the equation obtained by using the implicit expectations proxy will yield asymptotically unbiased parameter estimates. With rational expectations, the estimation problem is more complex. Thus an advocate of the principle of parsimony might cite Occam's Razor in support of implicit over rational expectations.

In support of the rational expectations hypothesis, it should be observed that it is precisely this concept of expectations that is required in order for the "certainty equivalence" argument of Herbert Simon (1956) and Henri Theil (1957) to go through in the derivation of optimal linear decision rules. A classic management science application has to do with the task of optimally scheduling production, as formalized in Holt et al. A firm interested in utilizing their linear decision rule procedure for production scheduling should use forecasts that satisfy the rational expectations condition that the forecast error be uncorrelated with the forecast. While it might be tempting to conjecture that this procedure may have been put into practice by the generations of graduate business school students nurtured on the production smoothing algorithm at Carnegie-Mellon's Graduate School of Industrial Administration and elsewhere, caution is required. The Holt et al. argument does not suffice to establish the adequacy of the rational expectations formulation, for quite restrictive assumptions are required in order for the certainty equivalence argument to go through. The loss function must be quadratic, and there must be no sign constraints on the decision variables (for example, negative output and inventories must be admissible); as Simon (1979) cautioned in his Nobel Laureate address, single-valued forecasts applied with linear decision rules are unlikely to suffice in more complex situations.

The notion of certainty equivalence does not necessarily go through even when the loss function is quadratic. As one example, in the Tobin-Markowitz portfolio selection model more than point estimates are required; that is, the notion of certainty equivalence does not go through because the variance of the loss depends on the decision; this contrasts with the Holt et al. production scheduling model, for they assumed that the costs of uncertainty were independent of the production decision.

As a second example, consider Milton Friedman's formal demonstration (1953) of the intuitively reasonable proposition that efforts at applying macroeconomic stabilization policy should be less aggressive when the policymaker's forecasts are subject to greater error. It turns out that Friedman's
argument holds for Mills’ implicit expectations, but not for Muth’s rational expectations.\(^7\)

That certainty equivalence does not go through in quite simple circumstances when the loss function is asymmetric is illustrated by a simple every day example:

*It is best to get to the bus stop a few minutes in advance of the expected 8:00 arrival time for your bus because your loss from missing the bus by one minute is greater than the cost of waiting an extra minute — that is, your loss function is not symmetric. Some commuters plan as though the expected arrival time were 7:55; other commuters set their clocks five minutes ahead.*

Finally, it must be observed that although the concept of implicit and rational expectations are mutually exclusive, it would be a relatively simple matter to modify expectations satisfying Mills’ implicit expectations hypothesis in order to obtain transformed expectations that are rational in the sense of Muth; it is only necessary to apply a linear transformation to the implicit expectations, as with equation (9), utilizing coefficients obtained by regressing historical actual realization on the implicit expectations. However, I know of no evidence that firms customarily modify sample survey forecasts in this way.

II. Evidence

Direct evidence on these issues is provided by a variety of empirical studies, some recent but others of long standing, on the structure of expectations.\(^8\)

A. Manufacturers’ Sales and Inventory Anticipations

A rich body of *ex ante* evidence is provided by the quarterly Manufacturers’ Inven-
tory and Sales Expectations (MISE) Survey conducted by the U.S. Department of Commerce from late 1959 through 1976. Firms were asked for both short (2 month) and long (5 month) sales and inventory forecasts; they were also asked whether they regarded their inventory stocks as high, low, or about right relative to current sales. The survey evidence has been extensively analyzed by me (1967), by Hirsch and me, and by F. Owen Irvine, Jr. (1983). Hirsch and I had access to the responses of 83 firms in five industries through the fourth quarter of 1964 as well as the industry aggregates.

Hirsch and I (p. 71) reported that the sales expectations of individual firms are biased, as defined by equation (3) above, which contradicts both Muth’s rational and Mills’ implicit expectations models; some firms are perennial optimists, generally overestimating future sales, while others are perennial pessimists, usually understating sales volume. For 30 percent of the sampled firms, the mean of anticipated sales, two-months horizon, differed from the mean of actual realizations at the 5 percent level of significance. However, the overestimates of the optimistic firms roughly cancelled the underestimates of pessimistic firms so that for industry aggregates there is no bias; this offsetting of systematic error partially explains why the aggregates of anticipations data appear to be more accurate than the predictions of individual firms.

The rational expectations hypothesis asserts that the variance of actual realizations will exceed the variance of forecasts; the implicit expectations hypothesis holds the opposite. In fact, Hirsch and I (p. 74) found a mixed picture, for a sizable proportion of firms (about 35 percent) sales anticipations have a larger variance than realizations.

As a further test of the rational expectations model, Hirsch and I considered regressions of the form:\(^9\)

\[
A_t = \beta_0 + \beta_1 P_t + \beta_2 A_{t-1} + \beta_3 A_{t-4} + \epsilon.
\]

\(^7\)And the policy formulation task is still more involved when the parameters of the structure by which policy has its impact have to be estimated (compare William Brainard, 1967: Prescott, 1971).

\(^8\) This review will not cover survey evidence of professional forecasters, such as that provided by the Livingston Survey.

\(^9\) As before, \(A_t\) is the actual realization and \(P_t\) is the prediction of the outcome; \(A_{t-1}\) is last quarter’s realization and \(A_{t-4}\) is the same-period-last-year realization.
Under Muth's assumption that firms exploit efficiently all available information in making their forecasts, we should find $\beta_1 = 1$ and $\beta_2 = \beta_3 = \beta_4 = 0$. On the other hand, if firms fail to fully exploit the information on last period sales or same-period-last-year sales, these conditions will be violated.

The results Hirsch and I reported (pp. 171–77) are supportive of the rational expectations hypothesis for the durable manufacturing aggregate and for the seven component industry aggregates; for nondurables, however, $\beta_2$ is substantially less than unity for a number of industries. Further, too many of the estimated values of $\beta_3$ and $\beta_4$ have large $t$ values. And the evidence for individual firms is even more discouraging for the rational expectations hypothesis. A pooled regression was run for each of the five industries for which individual firm observations were available; almost always, the value of $\beta_2$ was significantly less than unity; equally discouraging, the remaining two regression coefficients were usually significantly different from zero, which contradicts the rational expectations hypothesis.

Why do firms fail to exploit fully the information in their own sales history in formulating their forecasts of future sales volume? One possibility, pointed out by Hirsch and me (p. 177), is that while it may be true that a decision maker who knew the parameters of equation (11) could improve the accuracy (as measured by the root mean square error) of the raw forecasts with an appropriate linear transformation, departures from the orthogonality conditions imposed by Muth may arise because the decision maker has not accumulated enough historical evidence to obtain precise estimates of the parameters of equation (11).

Hirsch and I (pp. 181–85) concluded that in empirical work the most appropriate assumption to make about expectations when anticipations are not directly observable depends on the level of aggregation. At the firm level Ferber's law and the exponential smoothing model both yield a better estimate of anticipated sales than is provided by the actual realization proxy. For industry aggregates, however, it is better to use actual sales as a proxy for anticipations rather than to assume that expectations are generated either by Ferber's law or by exponential smoothing. This discrepancy arises because the cancelling of offsetting forecasting errors of individual firms makes aggregate anticipations much more accurate predictors of aggregate realizations, and conversely. But Hirsch and I also found that the assumption that predicted changes are proportional to actual changes, the relaxation of the assumption of rational expectations restrictions on equation (11), does better than either Ferber's law or exponential smoothing in predicting short sales anticipations.

B. Further Tests Based on the MISE Survey

In his more recent study, Irvine utilizes the data for the entire seventeen years over which the Survey was conducted, 1959 through 1976, but only for the durable, nondurable, and total manufacturing aggregates rather than disaggregated to the industry or the firm level; as a result, he could not investigate the tendency observed by Hirsch and me for some firms to be perennial optimists and others perennial pessimists. Focusing on the short sales forecast aggregates, Irvine finds that there was a two-and-one-half year sequence of sales underprediction beginning in late 1972, which might arise because firms did not adequately allow for the dramatic upward sweep of inflation in pricing projected sales volume—this could be interpreted as a protracted transitional period in which business firms were slowly learning about a change in structure. For the pre-OPEC period, 1961–72, his tests revealed no evidence inconsistent with the hypothesis that the one-period-ahead sales forecasts of durable manufacturing are fully rational. But he found for the nondurable manufacturing aggregates, as had Hirsch and I, that expectations are not fully rational in that they do not appropriately incorporate information on seasonality and the rate of growth of the money supply.

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which may appear significant if the forecaster fails to exploit systematic seasonal movements. The data were not seasonally adjusted.
C. Five Pittsburgh Firms

Muth (1985) tested alternative theories of expectations on monthly data spanning the period 1957–70 for five Pittsburgh-based business firms, two of which were steel producing, two metal fabricating, and one an electric utility. For each individual firm Muth had observations on anticipated production for three successive months plus information on realized production, deliveries, new orders, the order backlog, and total inventories. The data suggest that some firms are perennial optimists while other firms are perennial pessimists, as was the case with the firms studied by Hirsch and me.\(^{10}\) In the majority of cases, the variance of anticipations is larger than the variance of the realizations, which is inconsistent with the rational expectations hypothesis.\(^{11}\) Muth also found his data inconsistent with a number of alternative structural expectations models, with the possible exception of the expectations revision model of Hicks and of David Meisselman (1962). As explained later in this paper, Muth was led by the negative empirical evidence to substantially modify his original model of rational expectations.

D. Price Expectations

Inflationary expectations are of obvious interest in their own right; they are also of special interest in the study of inventory behavior for two reasons. First, in attempting to measure the real rate of interest, a component of inventory carrying cost, it is necessary to take into account the price changes expected by business firms; second, in times of unanticipated inflation it is particularly useful to be able to decompose errors in anticipating sales volume into errors in predicting real sales volume and errors in estimating sales price.\(^{12}\)

Two studies by Frank de Leeuw and Michael McKelvey (1981, 1984) exploit the evidence on the price expectations of business firms provided by the responses to the year-end survey of Business Expenditures on Plant and Equipment conducted by the Bureau of Economic Analysis since 1970. They report (1981, p. 302) that the expected price changes are somewhat more accurate than simply forecasting the same rate of inflation as last year; specifically, Theil’s U-statistic, the ratio of the root mean square error of the observed forecast over the root mean square error that would be made by a forecaster who always predicted the same inflation rate as last year, averages out to about 77 percent over the decade of the 1970’s, ranging from a most impressive 58 percent for textiles to a tie at 100 percent for food and beverages. They found that the two rounds of OPEC price hikes caused major errors in the anticipated prices of goods and services sold, the first in 1974 and the second in 1981. The expected percent change in price in 1974 was only 5.3 percent while the actual hike was 16 percent; but for 1975 the expected 8.8 percent inflation fell just short of the actual 8.9 percent. This suggests that at least part of the two-and-one-half-year sequence of underprediction of nominal sales reported by Irvine can be attributed to unanticipated inflation.

De Leeuw and McKelvey report on a number of pooled industry cross-section time-series regressions testing the rational expectations hypothesis. When they regressed the actual rate of increase in the sales price (\(p_t\)) on the anticipated change (\(p_{t+1}\)) over the period 1971–80, the regression suggested by equation (5a) above, they obtained

\[
\begin{align*}
(12) \quad p_t &= -0.112 + 1.345p^t_{t+1} \\
\bar{R}^2 &= .304. \\
(1.12) & \quad (.155)
\end{align*}
\]

An F-test of the joint hypothesis \(\beta_0 = 0.0\) and \(\beta_1 = 1.0\) yields a highly significant F-

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10. Of 24 bias t-ratios, 12 were greater than 2 in magnitude; all but 4 were greater than unity.
11. In half of 28 cases, the variance of the forecast was larger than the variance of the realization.
12. The evidence considered here, in contrast to the Livingston Survey data analyzed by such writers as John Carlson (1975), concerns the expectations of business firms and consumers rather than professional forecasters.
statistic of 12.2. With capital goods prices, the results were also inconsistent with the rational expectations hypothesis, but with slope coefficients significantly less than unity. Equally serious, de Leeuw and McKelvey present evidence suggesting that firms do not fully exploit publicly available information on the lagged rate of growth of the money supply and capacity utilization.

In the follow-up study (1984), de Leeuw and McKelvey worked with both individual firm data and with data grouped in order to mitigate the problem of errors in the variables. They generally found that regressions of the form (5a) violate the rationality hypothesis, the estimates of \( \beta_1 \) being substantially less than unity on both grouped and firm disaggregated data. Expected price change is determined by a variety of variables, including lagged expected rates of inflation and recently observed changes in the rate of price change. They are able to conclude, however, that expectations may not be subject to long-run bias (i.e., \( E(\varepsilon) = 0 \) in equation (3) in the long run).

A study by Edward Gramlich (1983) based on a quite distinct body of price expectations data provided similar results. Using timeseries derived from the University of Michigan household survey data, he found a slope coefficient of 1.222, only slightly flatter than the sales price slopes reported by de Leeuw and McKelvey (equation (12) above).

E. Wage Expectations

Jonathan Leonard (1982) has analyzed data on employers’ wage expectations provided by the Endicott survey on average starting wages for inexperienced college graduates. Data for eight occupational categories are collected from a sample of 170 large and medium-sized corporations. Expectations appear to be biased downward, for in each of the eight occupational categories employers underestimate the wages they will have to pay new recruits. The slope coefficients in equation (5a) are closer to the value of unity than those obtained in most other studies; nevertheless, he finds in contradiction to the hypothesis of rational expectations that the F-test is significant in five of eight occupational categories. He concludes that firms underestimate wages because they underestimate demand; forecast errors are not explained by mismeasurements of inflation or by either the expected or the unanticipated money supply change.

F. Data Revisions

Business analysts and economic forecasters are confronted with a confusing sequence of flash, preliminary, provisional and revised numbers for each observation of interest. It has been observed by a number of writers, most notably Arnold Zellner (1958) and Rosanne Cole (1970), that preliminary data on GNP and other economic indicators deviate systematically from subsequent revisions. A review of the evidence suggests that the official preliminary data are not always rational predictors of the revised time-series that eventually appear. Such departures are of importance if economic agents utilizing official information sources as part of the information set in making their own projections are induced to make forecasts that fail to satisfy the rational expectation hypothesis.

Evidence that preliminary X-11 seasonally adjusted money supply growth is subject to systematic error has appeared in a number of studies. As a result, a revised X-11 ARIMA seasonal adjustment procedure was adopted in 1982. That the revised seasonal adjustment procedure yields preliminary data that

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13 Leaving out the 1974 OPEC shock year yields results that are even more damaging to the rational expectations hypothesis.

14 While revisions in official data series do occasionally earn comment, the possibility of improving preliminary figures through appropriate linear transformation when they are not rational forecasts of the revision has apparently escaped attention from professional economists and economic analysts. However, it is reported that Edward J. Hyman, chief economist at the New York securities firm of Cyrus J. Lawrence Inc., attempts to forecast the business cycle by tabulating the direction of revision of a number of economic indicators (Wall Street Journal, June 6, 1984, p. 35).

15 See the literature cited by Scott Hein and Mack Ott (1983, pp. 19).
do not constitute rational predictions of the revised data is suggested by a regression presented by Scott Hein and Mack Ott:

\[
(13) \quad M1^{\text{revised}} = 3.149 \\
\quad (1.480) \\
\quad + 0.581 M1^{\text{preliminary}} \\
\quad (0.126) \\
R^2 = 0.681; \quad D-W = 2.26.
\]

This is precisely the form of the rational expectations equation (5a) above, with the preliminary data on the rate of growth of the money supply \( M1^{\text{preliminary}} \) serving as the predictor of the revised figure \( M1^{\text{revised}} \). As Hein and Ott point out, the coefficient on the preliminary growth rate of the money supply deviates significantly at the 5 percent level from unity; equally serious, the intercept deviates significantly from 0.

Hein and Ott do not mention certain interesting implications of their results. First of all, rejecting either one of these two null hypotheses would suffice to establish that the preliminary data are not rational forecasts of the revised seasonally adjusted data. In contrast, a similar regression shows that preliminary nonseasonally adjusted data provide a rational forecast of the revised seasonally unadjusted rate of money supply growth. It is also of interest to observe that if the regression is run in the opposite direction, in accordance with Mills' implicit expectations assumption of equation (4) above, the regression coefficient is 1.17; since this is rather close to unity, the new X-11 ARIMA procedure appears to be generating implicit rather than rational preliminary forecasts of the revised rate of money supply growth.\(^{16}\)

Evidently, the problem of achieving rational forecasts by learning from prior experience has not been solved by the many statisticians who have been working on the problem of seasonal adjustment—and if the statisticians are slow learners, it is hard to believe that individual business enterprises devoting fewer resources to the problem are likely to learn from their own historical experience how to achieve rational sales forecasts within a reasonable time frame. To the extent that preliminary seasonally adjusted money supply growth rates are taken seriously by economic agents, departures from rationality may result from the failure of the estimates to satisfy Muth's concept of rational expectations. The greater variance of the preliminary data, the numbers that everyone looks at, may help to explain the excessive variability in financial markets that has been observed by Robert Shiller (1978).

G. Government Forecasts

Consider two forecasts provided by the federal government: budget revenue, and EPA mileage estimates. Are these forecasts rational? If not, consumers of such forecasts may make decisions that are inconsistent with the rationality postulate.

Each January the Treasury Department estimates tax receipts for the coming fiscal year. An examination of data assembled for the period 1963–78 by the Congressional Budget Office (1981) reveals that while the estimates are often imprecise, they are not significantly biased.\(^{17}\) Specifically, actual revenue averaged \$218.67 billion over this period, just slightly in excess of the average estimate of \$217.32 billion; while some of the forecast errors are of substantial magnitude, the standard deviation of the forecast error being \$9.22 billion, the slight underestimate of \$1.35 is not significant; there is no reason to conclude that the tax revenue estimates are subject to systematic bias. Further, regressing the actual realization on the forecast yields Actual Revenue (\(AR\))

\[
(14) \quad AR = -1.041 + 1.009 \text{Forecast} + e \\
\quad (6.315) \quad (0.028) \\
R^2 = 0.99; \quad D-W = 2.004.
\]

\(^{16}\) In contrast, similar regressions reported by Hein and Ott for the rate of growth of the nonseasonally adjusted money supply appear to be compatible with both the rational and the implicit expectations hypothesis. Seasonally adjusted unemployment rates may not be rational forecasts of the revised series.

\(^{17}\) Although the Congressional Budget Office examined the accuracy of the forecasts, they did not test for rationality.
The slope coefficient differs insignificantly from unity; the Treasury forecasts satisfy the orthogonality requirement imposed by Muth's theory of rational expectations.\textsuperscript{18}

The Environmental Protection Agency publishes auto mileage estimates that are designed to help car buyers compare the relative fuel efficiency of different models. Do these predictions, based on stationary 23-minute exhaust emission dynamometer simulations conducted by the manufacturers on prototype models, constitute rational forecasts of the mileage that purchasers will realize under actual driving conditions? My comparison of the EPA estimates with Consumer Union (CU) on the road experience involving a mix of city driving, expressway driving, and driving on a 195-mile test trip revealed that contrary to conventional wisdom, the published EPA estimates for 1984 were not subject to significant optimistic bias.\textsuperscript{19} However, the (between model) variance of the EPA forecasts substantially exceeded the mileage experienced by CU test drives, implying implicit rather than rational expectations. Further, the regression of the realization on the forecast had a slope deviating significantly from the value of unity implied by the rational expectations hypothesis:\textsuperscript{20}

\begin{equation}
AveGal = 7.952 + 0.693\ EPA + e;
\end{equation}

\begin{equation}
\bar{R}^2 = 0.74.
\end{equation}

Evidence that the EPA forecasts do not fully incorporate the information set is provided by the following regression:\textsuperscript{21}

\begin{equation}
AveGal = 22.008 - 0.002\ Weight
\end{equation}

\begin{equation}
- 2.760\ stan/auto + 3.280\ G/D
\end{equation}

\begin{equation}
+ 0.415\ EPA + e
\end{equation}

\begin{equation}
\bar{R}^2 = 0.82.
\end{equation}

Thus the EPA mileage estimates fail the rational expectations test in that they do not fully incorporate all the information available at the time the prediction is made. To the extent that consumers rely on the EPA fuel estimates, their purchase decisions will not be guided by rational expectations. While the last regression reveals that the EPA forecasts are not rational,\textsuperscript{22} it also shows that these forecasts do contain information that would contribute to improved prediction if it were used in conjunction with the other variables in the regression.

IV. Implications—Should the Facts Be Allowed to Spoil a Good Story?

My survey of a number of empirical studies of expectations is not supportive of the commonly invoked rational expectations hypothesis. Quite the contrary, if the cumulative evidence is to be believed, we are compelled to conclude that expectations are a rich and varied phenomenon that is not adequately captured by the concept of rational expectations; while the predictions of some forecasters may be characterized as rational, in other instances the assumption of rationality is clearly violated. Nevertheless, there are, I think, two important reasons that can be advanced for suspending judgement on the

\textsuperscript{18}Running this regression in the opposite direction reveals that the data are also compatible with Mills' concept of implicit expectations. When the regression was run in terms of percentage change from the preceding year, the regression results were consistent with the rational expectations concept.

\textsuperscript{19}See my paper (1984). The shift in 1985 to the mandatory reporting of city and expressway estimates is likely to have introduced an optimistic bias that had been absent when auto stickers presented a single overall mileage figure that was based entirely on the city driving simulation.

\textsuperscript{20}AveGal is the actual experience on CU road tests; EPA is the published EPA estimate.

\textsuperscript{21}Weight is the vehicle gross weight in pounds, stan/auto is a dummy variable coded zero for standard and 1.0 for automatic transmission, and G/D is a dummy coded zero for gas and 1.0 for diesel power.

\textsuperscript{22}Allan Murphy and Robert Winkler (1984) imply that the U.S. Weather Bureau's Model Output Statistic procedure improves predictive accuracy by using a regression-based correction equation to improve the raw model-based forecasts.
validity of tests of the rational expectations hypothesis:

First, there is the problem of measurement error. While direct empirical studies of the validity of the rational expectations hypothesis have not generated much in the way of response from rational expectations theorists, Finn Kydland and Edward Prescott did comment as follows on the results Hirsch and I reported:

...there may be biases in their measurement of expectations, and these biases are related to lagged sales. This is not implausible, given the subtleness of the expectations concept and the imprecision of survey instruments. Further, even if there were a systematic forecast error in the past, now that the Hirsch and Lovell results are part of agents' information sets, future forecast errors should not be subject to such biases. [1977, p. 479]

Is it not conceivable that further research may reveal that much of the apparent discrepancy between the rational expectations model and the evidence currently available does indeed arise from measurement error? As is well known, the presence of measurement error in the explanatory variable means that

$$\operatorname{plim}(b) = (1 + \eta)^{-1} \beta,$$

where $\eta$ is the ratio of the variance of the measurement error to the variance of $\epsilon$, provided that the measurement error is distributed independently of the true values of the explanatory variables and $\epsilon$ (see Johnston, 1984). Thus one might cite the downward bias generated by errors in observing expectations to explain why Hirsch and I often obtained slope coefficients significantly less than unity in regressing actual on anticipated sales. But the errors of measurement argument cuts both ways. If the errors of expectations argument is to be invoked to explain why the slope is too small in these studies, then it must also make it all the more difficult to explain the too high slope estimates obtained for the price expectations data by de Leeuw and McKelvey (1981, 1984) and by Gramlich.

Second, it must also be observed that departures from rationality may be a transient phenomenon arising because economic actors are learning to adapt to a shift in regimes; in an evolving environment more complicated tests may be required in order to determine whether satisficing or optimal learning is taking place.

A. Muth’s Errors in the Variables Reformulation

While there may be a variety of arguments for resisting the implications of the empirical evidence, Muth was led by the evidence provided by his own and other empirical studies to fundamentally modify his original model (1985). In his new “errors in the variables” model, Muth relaxed a key restriction of his original rational expectations hypothesis. He specified

$$A_t = \alpha_t + \epsilon_t; \quad P_t = \alpha_t + \xi_t.$$  

Here $\alpha_t$ is the unobservable deterministic factor and $\epsilon_t$ and $\xi_t$ are unobserved stochastic disturbances subject to the restriction

$$E(\epsilon_t) = E(\xi_t) = E(\alpha_t \epsilon_t) = E(\alpha_t \xi_t) = 0.$$  

Muth’s new formulation reduces to his rational expectations model when the restriction $\sigma_\xi = 0$ holds; it reduces to Mills’ implicit expectations model when $\sigma_\epsilon = 0$.

The new model, it seems to me, can be interpreted in the following way. The stochastic term $\xi_t$ arises from a less than full understanding of underlying deterministic forces, $\alpha_t$. As in the implicit forecast model of Mills, this term could result from sampling error, as when a manufacturer relies on a sales forecast obtained from a market research survey. The other stochastic term, as in Muth’s earlier rational expectations model, reflects random developments between the time the forecast is made and the actual realization.

As Muth points out, his generalization allows the variance of the predictions to exceed the variance of the actual realizations.
This means that his new model allows the slope coefficient when realizations are regressed on anticipations to be substantially less than unity, not because of errors in measuring expectations but because of an additional random element in the process by which actual anticipations are generated.

V. Conclusions

In conclusion, it seems to me that the weight of empirical evidence is sufficiently strong to compel us to suspend belief in the hypothesis of rational expectations, pending the accumulation of additional empirical evidence. This means that at this juncture three research strategies deserve more attention than they currently receive.

1) First, it is a mistake in empirical research on such phenomena as inventories to proceed under the maintained hypothesis that expectations are rational. Instead, it should be recognized that there are several competing hypotheses. Because no single hypothesis is preeminent, the researcher must test the sensitivity of empirical results on such issues as whether interest rates influence inventory holdings by reporting what happens when alternative assumptions about the structure of expectations are considered.

2) Second, more attention needs to be given to the empirical testing of the rational expectations hypothesis against its alternatives. Unfortunately, ex ante evidence is sparse; more resources should be devoted to the collection and dissemination of survey results. It is particularly unfortunate that the Inventory and Sales Expectations Survey pioneered by Murray Foss at the Department of Commerce has been discontinued.

3) Third, it would be constructive in developing theoretical models to determine how robust policy conclusions are to departures from experimental rationality. To illustrate, one can ask whether the policy conclusions derived from a model require that individual decision makers formulate their expectations rationally, or only that the aggregate of expectations held by individuals satisfy certain rationality conditions. One can also ask whether the conclusions derived under the assumption of rational expectations also go through with alternative assumptions, such as Mills' implicit expectations, Ferber's law, or a systematic tendency to underestimate change.

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