Monetary Policy Uncertainty and Macroeconomic Performance: An Extended Non-Bayesian Framework

Daniel Laskar*

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Abstract

The existing literature has shown that less political uncertainty, or more central bank transparency, may worsen macroeconomic performance by raising the nominal wage. We extend this analysis to a non-Bayesian framework, where there is some aversion to ambiguity. We show that the result found in the literature under the Bayesian approach does not hold when the distance from the Bayesian case is large enough, or when a reduction in "Knightian uncertainty" is considered. Then, less uncertainty, or more transparency of the central bank, does not raise the nominal wage and, as a consequence, macroeconomic performance is not worsened (and is in general strictly improved).

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1 Introduction

The existing literature (Sorensen (1991) and Grüner (2002)) has shown that less uncertainty on the objectives of policymakers may worsen macroeconomic performance by raising the nominal wage in the labor market. The

*PSE (Paris School of Economics, CNRS UMR8545) and CEPREMAP. Address: CEPREMAP-PSE, 142 rue du Chevaleret, 75013 Paris, France. Ph: 33 (0)1 40 77 84 08. Fax: 33 (0)1 44 24 38 57. E-mail: laskar@pse.ens.fr
argument relies on the analysis of a game between a monopoly labor union and a central bank. The labor union sets the nominal wage before the central bank chooses its monetary policy. As the weight the central bank attach to its inflation objective relatively to its unemployment objective is not known to the labor union, this creates some uncertainty on how the central bank reacts to the nominal wage. It is then shown that more uncertainty decreases the level of the nominal wage chosen by the labor union. As a consequence, this may improve macroeconomic performance defined in terms of unemployment and inflation. The result implies that less political uncertainty might be harmful. Alternatively, it may be interpreted as an argument against too much transparency of the central bank.

The argument was developed in the standard Bayesian framework of expected utility maximization. However, some insufficiencies of this bayesian approach have been pointed out. In particular, the Ellsberg paradox (Ellsberg (1961)) has underlined the existence of some "aversion to ambiguity". Therefore, more recently, new approaches have been proposed which can take into account such an aversion to ambiguity, and which encompass the Bayesian approach as a special case.

The purpose of this paper is to re-evaluate the previous argument by using such an extended framework of decision under uncertainty. The central issue we consider consists in asking how the macroeconomic equilibrium changes when the amount of uncertainty varies, and to examine whether less uncertainty is beneficial or not. Therefore, we need an approach which makes

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1This is indeed an extreme case which simplifies the analysis and underlines more clearly the results. A number n of labor unions can easily be introduced in the model, which seems more suitable when empirical applications are considered (see Grüner et al. (2005)).

2In the recent short theoretical review contained in the first part of Dincer and Eichengreen (2007), this kind of argument is listed as one of the existing arguments against more transparency of the central bank.

3Classical references are Gilboa and Schmeidler (1989) and Schmeidler (1989). A survey of some economic applications can be found in Mukerji and Tallon (2004). Knight (1921) had already made a distinction between a situation of "risk" when there exists objective probabilities and a situation of "uncertainty" when such probabilities do not exist.

4Schipper and Winschel (2004) have also examined the effect of "Knightian uncertainty" in a similar model of interaction between a labor union and a central bank. But this uncertainty is modelled as strategic uncertainty in a Nash game, which constitutes a different approach. First, these authors consider an equilibrium where the two players play simultaneously, while, in the literature cited and in the present paper, it is essential for the argument that an equilibrium where the labor union plays first, is considered. Second, in Schipper and Winschel (2004), in accordance with the literature on ambiguous games, uncertainty is modelled as Knightian uncertainty on the strategy of the other player, which leads to an equilibrium which generalizes the Nash equilibrium. Here, and in the literature cited, uncertainty is on a parameter of the model.
explicit the prior information available to the decision maker, and which allow us to compare and evaluate different prior informations, while at the same time taking into account the fact that the decision maker may have an aversion to ambiguity. Gajdos et al. (2004) provides such an approach. The uncertainty is represented both by a "central probability distribution" and by some "imprecision of information" around this central distribution. This leads to a criterion where the decision maker maximises a weighted average of two terms: the expected value under the central distribution, on the one hand; and the minimum of the expected utility under the available information, on the other hand.

While in the Bayesian framework less uncertainty means a lower value of the variance of the probability distribution, in the present extended framework we can consider different kinds of reduction in uncertainty. When we consider either a decrease in the variance of the central distribution, or a global decrease in uncertainty, we find results which are qualitatively similar to those obtained in the literature under the Bayesian approach, provided that the distance from the Bayesian case is not too large. Thus, in this sense, the results obtained in the literature can be generalized. However, when we consider a decrease in the imprecision of information (which we may call a decrease in "Knightian uncertainty"), i.e. a shrinking of the set of the possible distributions around the central distribution, we get the opposite result that less uncertainty tends to lead to a lower nominal wage and to improved macroeconomic performance. Furthermore, when the distance from the Bayesian case becomes large enough, even in either of the first two cases of a lower variance of the central distribution or of a reduced global uncertainty, the nominal wage does not rise and therefore macroeconomic performance is in general also improved (and at least never worsened). Therefore, in all these last cases, the result obtained in the literature under the Bayesian approach does not hold. Less political uncertainty, or more transparency of the central bank, does not raise the nominal wage and is never harmful. It is in general even strictly beneficial in these cases.

The framework of analysis, which uses a model similar to that of Grüner (2002), but extend it to a non-Bayesian case, is presented in section 2. The determination of the equilibrium nominal wage is considered in section 3. The effect of less uncertainty on the nominal wage is studied in section 4. The "welfare" effects, i.e. the effects on "macroeconomic performance", are examined in section 5. Section 6 concludes.
2 Framework of analysis

2.1 Model

The model is the same as in Grüner (2002)\(^5\). There is a unique labor union in the labor market and we consider a two-stage game between this labor union and the central bank. The labor union fixes the nominal wage at the beginning of the period, and the central bank subsequently determines the inflation rate. The equations are the following:

\[ U^{CB} = -\psi \pi^2 - u^2 ; \quad \psi > 0 \]  
\[ U^{LU} = w - \pi - \frac{A}{2} u^2 ; \quad A > 0 \]  
\[ u = a(w - \pi) ; \quad a > 0 \]

\( U^{CB} \) and \( U^{LU} \) represent the utility functions of the central bank (CB) and the labor union (LU), respectively. The CB tries to stabilize both (log) inflation and unemployment around some desired levels, with a relative weight given by \( \psi \) (\( \pi \) and \( u \) being the gap variables between (log) inflation and unemployment and their desired levels, respectively). The LU wants to stabilize (log) unemployment around the same level as the CB (with a weight given by \( A \)), and to increase the (log) real wage \( w - \pi \), where \( w \) is the (log) nominal wage variable (we use the normalization \( p_{-1} = 0 \), where \( p_{-1} \) is the (log) price level of the previous period, which implies \( \pi = p \)). After the nominal wage has been fixed, employment is determined by labor demand. This leads to equation (3), where unemployment is an increasing function of the real wage\(^6\).

The equilibrium of the game is solved backward. In the second stage, once \( w \) has been fixed by the LU, the CB chooses \( \pi \) which maximizes \( U^{CB} \) under the constraint (3). This leads to the reaction function \( \pi = bw \), where \( b = \frac{a^2}{a^2 + \psi} \). Using the notation \( \gamma \equiv 1 - b \), we get

\[ \gamma = \frac{\psi}{a^2 + \psi} \]  

We have \( 0 < \gamma < 1 \). The CB reaction function can then be written:

\[ w - \pi = \gamma w \]

\(^5\)This model is itself taken from the existing literature analyzing the interaction of labor unions and the central bank, and its implications for monetary policy and the macroeconomic equilibrium (Cukierman and Lippi (1999)).

\(^6\)Note that by a suitable normalization of the nominal wage variable, the constant term in equation (3) can always be taken equal to zero.
In the first stage, the LU, when choosing $w$, takes into account the reaction function (5) of the CB. Using (2), (3) and (5) we can write $U^{LU}$ as the following function of $\gamma$ and $w$:

$$U^{LU}(\gamma, w) = \gamma w - \frac{Aa^2}{2} \gamma^2 w^2$$

(6)

As $\psi$ is not known with certainty by the LU, $\gamma$ is also uncertain. We now have to be more explicit on how this uncertainty on $\gamma$ will be taken into account.

2.2 Criterion under uncertainty

As indicated in the introduction, we will use an enlarged framework to decision under uncertainty. For, the standard Bayesian approach of expected utility maximization\(^7\) has been shown not to be compatible with some observed behavior of decision under uncertainty. In particular, Ellsberg (1961) has underlined that individuals exhibit some "aversion to ambiguity". One of the experiments involves asking individuals to bet on the color of a ball drawn from an urn. There are two urns. Urn A contains 50 black balls and 50 red balls; while urn B contains 100 balls, which can be either black or red, but with unknown proportion. The results of the experiment indicate that individuals are indifferent between betting on black or on red, but that they prefer to bet on the color of a ball drawn from urn A rather than from urn B. Such a behavior cannot be explained in the Bayesian expected utility framework.

Thus, some new approaches to uncertainty which can take into account such an aversion to ambiguity have been developed\(^8\). In the present paper we will more particularly use the approach of Gajdos et al. (2004), which seems to be particularly convenient for our purpose. For we want to examine the effect of a change in the prior information toward less uncertainty, and

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\(^7\) The classical references are Savage (1954) and Anscombe and Aumann (1963).

\(^8\) Two of them have more particularly retained attention. One is the the multiple prior model of Gilboa and Schmeidler (1989), where, instead of having a unique prior probability distributions, the decision maker have a set of possible probability distributions, and use a maximin of utility approach. The other is developed in Schmeidler (1989). Individuals continue to maximize some form of expected utility but use a "Choquet integral" with respect to a non-additive probability (or "capacity"). Both approaches contain the Bayesian expected utility approach as a special case. In the multiple prior approach, this occurs when the set of prior is reduced to a unique probability distribution. In the capacity-Choquet integral approach, the Bayesian case is obtained when the capacity is an additive probability distribution, the Choquet integral becoming in that case identical to the usual integral.
to see whether such a change toward less uncertainty is beneficial or not. Therefore we need an approach which makes explicit the prior information available to the decision maker, and which allows us to compare and evaluate different prior informations\(^9\). Such an approach can be found in Gajdos et al. (2004). In their framework, the prior information is characterized by \([\mathcal{P}, C]\), where \(C\) is a "central probability distribution" around which there is some "imprecision of information", which is expressed by the fact that all probability distributions contained in the set \(\mathcal{P}\) are considered as possible. Then the preference of the decision maker concerns both the decision (or "act") \(f\) and the prior information \([\mathcal{P}, C]\): it is a binary relation on couples \((f, [\mathcal{P}, C])\).

The axiomatic approach, which in particular contains an axiom of "aversion toward the imprecision of information", leads to a criterion which consists in a weighted average of two terms: the expected value under the central distribution \(C\), on the one hand; and the minimum of the expected utility under the (convex hull of the) set \(\mathcal{P}\) of all possible probability distributions, on the other hand (see Gajdos et al. (2004) Theorem 2 p.661)\(^{10}\). In the present model, this implies that, for a given prior information \([\mathcal{P}, C]\), the LU chooses \(w\) which maximizes \(\Omega_{LU}\) given by

\[
\Omega_{LU} = \theta \left[ \min_{P \in \text{co}(\mathcal{P})} E_{\mathcal{P}U^{LU}} \right] + (1 - \theta) E_{CU^{LU}}
\]

where \(P\) and \(C\) are probability distributions on \(\gamma\), and \(E_P\) and \(E_C\) represent the corresponding expected value operators. The minimum in (7) is taken on the convex hull \(\text{co}(\mathcal{P})\) of the set \(\mathcal{P}\) of all possible probability distributions\(^{11}\). Coefficient \(\theta\), which belongs to \([0, 1]\), represents the degree of aversion to the imprecision of information of the LU.

To simplify, we will define the set \(\mathcal{P}\) of all possible distributions in the following parametrical way. Let \(S \subseteq [0, 1]\) be the support of the central distribution \(C\), and let \(\mathcal{R}(S)\) be the set of all probability distributions with support \(S\). Then, we consider the set \(\mathcal{P}\) defined by \(\mathcal{P} = \{(1 - \delta) C + \delta P' : P' \in \mathcal{R}(S)\}\), where \(\delta\) is a given parameter satisfying \(0 \leq \delta \leq 1\). This means that any possible distribution belonging to \(\mathcal{P}\) can be written as a weighted average of the central distribution \(C\) and of a distribution with support \(S\) (which

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\(^9\)In the axiomatics developed in Gilboa and Schmeidler (1989), or in Schmeidler (1989), the information available to the decision maker is not made explicit.

\(^{10}\)A criterion which involves a weighted average between an expected utility under a given distribution and a minimum of expected utilities under a set of alternative distributions, can also be obtained under the "capacity-Choquet integral" approach (see Eichberger and Kelsey (1999)).

\(^{11}\)Note that the central distribution is assumed to belong to the convex hull \(\text{co}(\mathcal{P})\) of \(\mathcal{P}\).
implies that the support of any possible distribution is also $S$\textsuperscript{12}. Parameter $\delta$ determines the size of the set $\mathcal{P}$ and is an indicator of the imprecision of the information. In the case $\delta = 0$, we have $\mathcal{P} = \{C\}$, and the set $\mathcal{P}$ is reduced to the central distribution $C$. In the opposite extreme case $\delta = 1$, we have $\mathcal{P} = \mathcal{R}(S)$, and the set $\mathcal{P}$ becomes identical to the set of all probability distributions with support $S$.

From this definition of $\mathcal{P}$, for any $P$ belonging to $\mathcal{P}$ we get $E_P = (1 - \delta) E_C + \delta E_{P'}$, where we have $P' \in \mathcal{R}(S)$. Note also that $\mathcal{P}$ thus defined is a convex set\textsuperscript{13}, which implies $co(\mathcal{P}) = \mathcal{P}$. This gives

$$\min_{P \in co(\mathcal{P})} E_P U^{LU} = \delta \min_{P \in \mathcal{R}(S)} E_P U^{LU} + (1 - \delta) E_C U^{LU}.$$ Substituting this expression into (7), we get, as a criterion, the maximisation of $\Omega^{LU}$ given by:

$$\Omega^{LU} = \alpha \left[ \min_{P \in \mathcal{R}(S)} E_P U^{LU} \right] + (1 - \alpha) E_C U^{LU} \quad (8)$$

where we have

$$\alpha = \theta \delta \quad (9)$$

From (8), we see that, by defining $\mathcal{P}$ in such a way, the minimization part of $\Omega^{LU}$ can be taken on the set $\mathcal{R}(S)$ of all probability distributions with support $S$ (while in (7) it was taken on the (convex hull of the) set $\mathcal{P}$ itself). Note, however, that coefficient $\alpha$ which appears in (8) is equal to $\theta \delta$ (while in (7) we had $\theta$ instead). Thus, coefficient $\alpha$ is an increasing function both of the imprecision of information, represented by parameter $\delta$, and of the aversion toward the imprecision of information, represented by parameter $\theta$.

The standard Bayesian approach is obtained as a special case when we have $\alpha = 0$. For, from (8), the criterion then becomes equivalent to the maximization of the expected utility under the central distibution $C$. From (9), this case occurs either when there is no imprecision of information ($\delta = 0$), or when there is no aversion toward the imprecision of information ($\theta = 0$). Parameter $\alpha$ can be considered as representing the distance from the Bayesian case. In the opposite extreme case $\alpha = 1$ (which requires that we have both $\theta = 1$ and $\delta = 1$), we have a maximin criterion on the set $\mathcal{R}(S)$ of all probability distributions with support $S$.

Let $\gamma_0$, where $0 < \gamma_0 < 1$, be the mean of the central distribution $C$. To simplify the analysis, we will further assume that the support $S$ of $C$ is a symmetric interval around $\gamma_0$. We have $S = [\gamma_0 - \mu, \gamma_0 + \mu]$, where $\mu > 0$

\textsuperscript{12}Such a definition is similar to the one used in Epstein and Wang (1994), where the set of probability distributions considered is defined by "contamination" from a given distribution.

\textsuperscript{13}From the definition of $\mathcal{P}$ we can easily see that if $P_1$ and $P_2$ belong to $\mathcal{P}$, then, for any $\xi$ satisfying $0 \leq \xi \leq 1$, $\xi P_1 + (1 - \xi) P_2$ also belongs to $\mathcal{P}$.
is a parameter which represents the length of that support. As we have $0 < \gamma < 1$, parameter $\mu$ has to satisfy the inequalities $\mu < \gamma_0$ and $\mu < 1 - \gamma_0$.

3 Nominal wage under uncertainty

The nominal wage chosen by the LU under uncertainty is the value $\hat{w}$ of $w$ which maximizes $\Omega^{LU}$ given by (8), where $\mathcal{R}(S)$ is the set of all probability distributions with support $S = [\gamma_0 - \mu, \gamma_0 + \mu]$.

As a first step, let $\bar{w}(\gamma)$ be the nominal wage chosen by the LU under certainty when $\gamma$ is known. Maximizing $U^{LU}$ given (6) gives

$$\bar{w}(\gamma) = \frac{1}{Aa^2\gamma}$$

(10)

Let $\sigma_\gamma^2 > 0$ be the variance of the central distribution $C$. Note that we must have $\sigma_\gamma^2 \leq \mu^2$. Then, the nominal wage $\hat{w}$ chosen by the LU under uncertainty is given by the following proposition:

**Proposition 1** Consider the threshold value $\alpha^*$, which satisfies $0 < \alpha^* \leq 1$, and is given by

$$\alpha^* = \frac{\sigma_\gamma^2}{\mu(\gamma_0 - \mu) + \sigma_\gamma^2}$$

(11)

In the case $\alpha < \alpha^*$, we have

$$\hat{w} = \frac{1}{Aa^2\alpha(\gamma_0 - \mu)^2 + (1 - \alpha)(\gamma_0^2 + \sigma_\gamma^2)} (\gamma_0 - \alpha \mu)$$

(12)

In that case, we have $0 < \hat{w} < \bar{w}(\gamma_0)$, and therefore uncertainty reduces the nominal wage. However, the larger $\alpha$ is, the smaller this reduction is. For we have $\frac{\partial \hat{w}}{\partial \alpha} > 0$.

In the case $\alpha \geq \alpha^*$, uncertainty has no effect: we have $\hat{w} = \bar{w}(\gamma_0)$.

See Appendix 1 for the proof. Proposition 1 indicates that uncertainty reduces the nominal wage but that this effect is dampened when we are not in the Bayesian case $\alpha = 0$. And the greater $\alpha$ is (i.e. the greater the departure from the Bayesian case is), the smaller the decrease in the nominal wage due to uncertainty is. The effect of uncertainty even disappears when $\alpha$ becomes greater or equal than some threshold value $\alpha^*$. For, in that case, $\hat{w}$ becomes always equal to $\bar{w}(\gamma_0)$, which is the value of the nominal wage which would occur if $\gamma$ were known to be equal to $\gamma_0$ with certainty.
These results can be explained more intuitively in the following way. Let \( \hat{w}_0 \) be the solution in the Bayesian case \( \alpha = 0 \), where the LU maximizes the expected value of its utility under the central distribution. We have \( 0 < \hat{w}_0 < \bar{w}(\gamma_0) \), which is in accordance with the result found in the literature that more uncertainty implies a lower nominal wage. In the case \( \alpha > 0 \), according to (8), the LU also considers what happens in the worst case. When we have \( 0 < w < \bar{w}(\gamma_0) \), the worst case occurs when \( \gamma = \gamma_0 - \mu \) with probability one. For, the real wage, which, from (5), is equal to \( \gamma w \), is then the furthest apart from the value that maximizes \( U_{LU} \), which, according to (6), is equal to \( \frac{1}{\gamma_0 - \mu} \). As a consequence, when \( \alpha \) starts increasing from zero, the optimal wage \( b_w \) becomes a weighted average of \( b_w^0 \) and \( w(\gamma_0 - \mu) \).

Formally, (12) can be written:

\[
\hat{w} = (1 - \kappa) \hat{w}_0 + \kappa \bar{w}(\gamma_0 - \mu) \tag{13}
\]

where \( \kappa \), which satisfies \( 0 \leq \kappa \leq 1 \), is given by

\[
\kappa = \frac{\alpha (\gamma_0 - \mu)^2}{\alpha (\gamma_0 - \mu)^2 + (1 - \alpha) (\gamma_0^2 + \sigma^2)} \tag{14}
\]

and where, from (12) with \( \alpha = 0 \), and from (10), we get

\[
\hat{w}_0 = \frac{1}{A a^2} \frac{\gamma_0}{\gamma_0^2 + \sigma^2} ; \quad \bar{w}(\gamma_0 - \mu) = \frac{1}{A a^2} \frac{1}{\gamma_0 - \mu} \tag{15}
\]

As, from (10), we have \( \bar{w}(\gamma_0 - \mu) > \bar{w}(\gamma_0) > \hat{w}_0 \), this raises \( \hat{w} \) above \( \hat{w}_0 \). The greater \( \alpha \) is, the greater the weight \( \kappa \) given to \( \bar{w}(\gamma_0 - \mu) \), and therefore the higher \( \hat{w} \) is. When \( \alpha \) reaches some threshold value \( \alpha^* \), then \( \hat{w} \) becomes equal to \( \bar{w}(\gamma_0) \). However, \( \hat{w} \) cannot go above \( \bar{w}(\gamma_0) \). The reason is that when we have \( w > \bar{w}(\gamma_0) \), then the worst case changes: it is now obtained for \( \gamma \) equal to its greatest value \( \gamma_0 + \mu \). As, from (10), we have \( \bar{w}(\gamma_0 + \mu) < \bar{w}(\gamma_0) \), a value of \( \hat{w} \) greater than \( \bar{w}(\gamma_0) \) can never be a solution. Therefore, for values of \( \alpha \) greater than \( \alpha^* \), the optimal nominal wage \( \hat{w} \) stays at the level \( \bar{w}(\gamma_0) \), where both \( \gamma = \gamma_0 - \mu \) and \( \gamma = \gamma_0 + \mu \) are worst cases. The discontinuity in the worst cases at \( \bar{w}(\gamma_0) \) allows \( \bar{w}(\gamma_0) \) to be a solution when we have \( \alpha > \alpha^* \).

### 4 Effect on the nominal wage of less uncertainty

We will now consider the issue of how a lower amount of uncertainty affects the nominal wage. In the Bayesian framework of Sorensen (1991) and
Grüner (2002), reduction in uncertainty means a decrease in the variance of the prior distribution, and the result obtained was that less uncertainty has the unfavorable effect of increasing the nominal wage. The present enlarged framework allows us to make a distinction between different kinds of reduction in uncertainty. For the amount of uncertainty around $\gamma_0$ is now characterized by the three parameters $(\sigma^2_{\gamma}, \mu, \delta)$. We can therefore examine different cases, depending on which parameter(s) is (are) concerned by the change in uncertainty we consider.

We will consider three cases. The first is a decrease in the variance of the central distribution alone. The second is a "global" reduction in uncertainty, where both the standard deviation of the central distribution and the length of the support of the set of possible distributions decrease proportionally. In these two cases, it will be shown that the nominal wage either increases or stays unchanged. Therefore, the results obtained in these two cases may be seen as some generalization of the results obtained in the literature under the Bayesian approach. In the third case, the central distribution is unchanged but there is a decrease in the imprecision of information around the central distribution (this case could be labelled a decrease in "Knightian uncertainty"). We will show that, on the contrary, such a reduction in uncertainty leads to a lower nominal wage.

In the notations we use, we will make explicit the dependence of the endogenous variables on the uncertainty parameters $(\sigma^2_{\gamma}, \mu, \delta)$. Thus, from proposition 1, we will write $\hat{\omega}(\sigma^2_{\gamma}, \mu, \delta)$ and $\alpha^*(\sigma^2_{\gamma}, \mu)$, where the dependence of $\hat{\omega}$ on $\delta$ goes through its dependence on $\alpha$ in (12) and from the equality $\alpha = \theta\delta$, given by (9).

**4.1 Decrease in the variance of the central distribution**

We first consider the case where the variance of the central distribution $\sigma^2_{\gamma}$ decreases while the other two parameters $\mu$ and $\delta$ stay constant. We get the following result:

**Proposition 2** When the variance of the central distribution decreases from $\sigma^2_{\gamma,1}$ to $\sigma^2_{\gamma,2} < \sigma^2_{\gamma,1}$:

- When initially we have $\alpha < \alpha^*(\sigma^2_{\gamma,1}, \mu)$, the nominal wage (strictly) increases: we have $\hat{\omega}(\sigma^2_{\gamma,1}, \mu, \delta) < \hat{\omega}(\sigma^2_{\gamma,2}, \mu, \delta)$.

- When initially we have $\alpha \geq \alpha^*(\sigma^2_{\gamma,1}, \mu)$, the nominal wage is unchanged: we have $\hat{\omega}(\sigma^2_{\gamma,1}, \mu, \delta) = \omega(\sigma^2_{\gamma,2}, \mu, \delta) = \omega(\gamma_0)$.

See appendix 2 for the proof. Proposition 2 indicates that as long as $\alpha$ is smaller than the threshold value $\alpha^*(\sigma^2_{\gamma,1}, \mu)$ for the initial situation, then
any decrease in the variance of the central distribution raises the nominal wage\textsuperscript{14}. When $\alpha$ is greater or equal to this threshold value, a decrease in this variance has no effect. This result constitutes a first generalization to the case $\alpha \neq 0$ of the result obtained in the literature under the Bayesian approach (i.e. only in the case $\alpha = 0$).

\subsection*{4.2 Global decrease in uncertainty}

We now consider the case where, starting from an initial situation, uncertainty declines "globally": both the standard deviation of the central distribution $\sigma_\gamma$ and the length of the support of possible distributions, given by $\mu$, decrease proportionally, parameter $\delta$ being unchanged\textsuperscript{15}. Therefore, we consider the effect of a decrease in both $\sigma_\gamma^2$ and $\mu$, while the ratio $\lambda \equiv \frac{\sigma_\gamma^2}{\mu^2}$ stays constant\textsuperscript{16}.

As we have $0 < \sigma_\gamma^2 \leq \mu^2$, we have $0 < \lambda \leq 1$. We get the following result:

**Proposition 3** Consider a global decrease in uncertainty: while parameter $\delta$ is unchanged, the variance of the central distribution decreases from $\sigma_{1,1}^2$ to $\sigma_{2,1}^2 < \sigma_{1,1}^2$, and the length $\mu$ of the support decreases from $\mu_1$ to $\mu_2 < \mu_1$, in such a way as to keep the ratio $\lambda \equiv \frac{\sigma_{2,1}^2}{\mu_2^2}$ constant. Then:

- When initially we have $\alpha < \alpha^*(\sigma_{1,1}^2, \mu_1)$, the nominal wage (strictly) increases: we have $\hat{w}(\sigma_{2,1}^2, \mu_1, \delta) < \hat{w}(\sigma_{1,1}^2, \mu_2, \delta)$.
- When initially we have $\alpha \geq \alpha^*(\sigma_{1,1}^2, \mu_1)$, the nominal wage is unchanged: we have $\hat{w}(\sigma_{2,1}^2, \mu_1, \delta) = \hat{w}(\sigma_{1,1}^2, \mu_2, \delta) = \bar{w}(\gamma_0)$.

See appendix 3 for the proof. Proposition 3 gives the same qualitative results as proposition 2 and constitutes a further generalization of the result obtained in the literature under the Bayesian approach\textsuperscript{17}. We see that in

\textsuperscript{14}According to the interpretation in terms of a weighted average given by (13), we can see from (14) and (15) that a decrease in $\sigma_\gamma^2$ raises both $\hat{w}_0$ and the weight $\kappa$. Both effects contribute to a higher value of $\hat{w}$ in the case $\alpha < \alpha^*$.

\textsuperscript{15}According to the definition of the set of the possible distributions $\mathcal{P}$ we have taken, then, with parameter $\delta$ unchanged, the support and standard deviation of the possible distributions are also reduced proportionally. It is therefore justified to consider that this case is a "global" reduction in uncertainty.

\textsuperscript{16}As an example, take the case where we reduce the value of $\mu$ when $C$ is the uniform distribution with support $[\gamma_0 - \mu, \gamma_0 + \mu]$. Then $\sigma_\gamma$ decreases proportionally, and we have $\lambda = \frac{1}{3}$. In the same way, a decrease in $\mu$ in the case of a symmetric triangular distribution with support $[\gamma_0 - \mu, \gamma_0 + \mu]$ would correspond to a global reduction in uncertainty with $\lambda = \frac{1}{6}$.

\textsuperscript{17}The qualitative results of proposition 3 cannot be deduced by simply considering the separate qualitative effects on $\hat{w}_0$, $\bar{w}(\gamma_0 - \mu)$ and $\kappa$ given in the weighted average interpretation of (13). From (14) and (15) we can see that $\hat{w}_0$ is raised by a smaller value
order to determine the effect of a decrease in the variance of the central distribution on the nominal wage, it does not matter qualitatively whether the length $\mu$ of the support of the possible distributions is held constant, or alternatively is decreased proportionally to the standard deviation $\sigma_\gamma$ of the central distribution.

4.3 Decrease in the imprecision of information

Alternatively, we could keep constant the central distribution and examine what happens when the set of possible distributions gets smaller around the central distribution. This corresponds to a decrease in parameter $\delta$, holding $\sigma_\gamma^2$ and $\mu$ constant, and can be interpreted as a decrease in the imprecision of information\textsuperscript{18}, or in "Knightian uncertainty". From (9), when we have $\theta \neq 0$, this lowers parameter $\alpha$. According to proposition 1, this reduces the nominal wage, or leaves it unchanged. More precisely, we get:

**Proposition 4** Consider a decrease in the imprecision of information around the central distribution, given by a lower value of parameter $\delta$: we have $\delta_1 > \delta_2 \geq 0$. In the non-bayesian case $\theta \neq 0$, we get:

- When after the reduction in uncertainty we have $\theta \delta_2 < \alpha^*(\sigma_\gamma^2, \mu)$, then the nominal wage is (strictly) lower: we have $\hat{w}(\sigma_\gamma^2, \mu, \delta_1) > \hat{w}(\sigma_\gamma^2, \mu, \delta_2)$.
- When after the reduction in uncertainty we have $\theta \delta_2 \geq \alpha^*(\sigma_\gamma^2, \mu)$, then the nominal wage is unchanged and stays at its value under certainty: we have $\hat{w}(\sigma_\gamma^2, \mu, \delta_1) = \hat{w}(\sigma_\gamma^2, \mu, \delta_2) = \overline{w}(\gamma_0)$.

In the Bayesian case $\theta = 0$ the nominal wage is always unchanged.

See appendix 4 for the proof. According to proposition 4, a decrease in uncertainty that consists in a reduction in the imprecision of information gives an effect on the nominal wage which tends to be opposite to that obtained in the two previous cases of reduction in uncertainty considered. It is therefore also opposite to the effect emphasized in the literature. For, when it is changed, the nominal wage is now lower instead of higher. Thus, if less political uncertainty, or more transparency of the central bank, mainly takes the form of reduced imprecision of information (i.e. of less "Knightian uncertainty"), the nominal wage will never be increased: it will be reduced, or at most unchanged.

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\textsuperscript{18} As the central distribution is unchanged, Gajdos et al. (2004), p.652 call it a "center preserving decrease in imprecision of information".
5 Effect on macroeconomic performance

5.1 Criterion

In order to examine whether it would be beneficial to have less uncertainty, through less political uncertainty or more transparency of the central bank, we have to go further than the effect on the nominal wage: we should consider its effect on "macroeconomic performance" (or its "welfare effect"). For that, following Sorensen (1991), we will assume that the utility function $U_{MP}$ used to evaluate macroeconomic performance is that of the central bank where the weight $\psi_0$ corresponds to the expected value $\gamma_0$. We take

$$U_{MP} = -\psi_0 \pi^2 - u^2$$  \hspace{1cm} (16)

where, from (4), $\psi_0$ is given by $\psi_0 = \frac{\gamma_0 - a^2}{1 - \gamma_0}$. Then, using (3) and (5), we get

$$U_{MP} = -a^2 \left[ \frac{\gamma_0}{1 - \gamma_0} (1 - \gamma)^2 + \gamma^2 \right] w^2$$  \hspace{1cm} (17)

As $\gamma$ is uncertain, the criterion that society uses to evaluate macroeconomic performance should take into account this uncertainty. As we have indicated in section 2, the approach to uncertainty we consider, with its implied criterion, allows us to evaluate and compare different prior informations. Thus, for such a comparison, we will use a criterion similar to (8) which is

$$\Omega_{MP} = \alpha \left[ \min_{P \in \mathcal{R}(S)} E_P U_{MP} \right] + (1 - \alpha) E_C U_{MP}$$  \hspace{1cm} (18)

where, as in (9), $\alpha$ is equal to $\theta \delta$, and where $\mathcal{R}(S)$ is the set of all probability distributions with support $[\gamma_0 - \mu, \gamma_0 + \mu]$. We assume that society has the same information $[P, C]$ as the labor union, given by the same parameters $(\gamma_0, \sigma^2_\gamma, \mu, \delta)$. For simplicity, we also assume that society has the same aversion toward imprecision of information parameter $\theta$ as the labor union.

As before, we want to consider the effect of a decrease in uncertainty, where this uncertainty is characterized by the three parameters $(\sigma^2_\gamma, \mu, \delta)$. Thus, define $\hat{\Omega}_{MP} (\sigma^2_\gamma, \mu, \delta)$ as the value of $\Omega_{MP}$ under $(\sigma^2_\gamma, \mu, \delta)$ when the nominal wage $w$ is equal to the level $\hat{w} (\sigma^2_\gamma, \mu, \delta)$ chosen by the labor union. Then, the uncertainty of the prior information characterized by $(\sigma^2_{\gamma,2}, \mu_2, \delta_2)$ is (strictly) preferred to that characterized by $(\sigma^2_{\gamma,1}, \mu_1, \delta_1)$ if and only if we have $\hat{\Omega}_{MP} (\sigma^2_{\gamma,2}, \mu_2, \delta_2) > \hat{\Omega}_{MP} (\sigma^2_{\gamma,1}, \mu_1, \delta_1)$ (with indifference in case of equality).
We can give a more explicit expression to \( \hat{\Omega}^{MP}(\sigma^2, \mu, \delta) \). First, consider the term \( \min_{P \in \mathcal{R}(S)} E_P U^{MP} \) in (18). From (17), this is equivalent to \( \max_{P \in \mathcal{R}(S)} \left[ \frac{\gamma_0}{1-\gamma_0} (1 - \gamma)^2 + \gamma^2 \right] \). As the term into brackets is a quadratic convex function of \( \gamma \), the maximum can be attained only on the set of the two extreme distributions where we have \( \gamma = \gamma_0 - \mu \) with probability one, or \( \gamma = \gamma_0 + \mu \) with probability one. It can be seen that \( \gamma = \gamma_0 - \mu \) and \( \gamma = \gamma_0 + \mu \) actually always give the same value to \( \frac{\gamma_0}{1-\gamma_0} (1 - \gamma)^2 + \gamma^2 \). Thus, replacing \( \gamma \) by \( \gamma_0 - \mu \), or equivalently by \( \gamma_0 + \mu \), in (17) we get

\[
\min_{P \in \mathcal{R}(S)} E_P U^{MP} = -a^2 \left( \gamma_0 + \frac{\mu^2}{1-\gamma_0} \right) w^2
\]

(19)

Second, from (17), we obtain \( E_C U^{MP} = -a^2 \left( \gamma_0 + \frac{\sigma^2}{1-\gamma_0} \right) w^2 \). Substituting this expression and (19) into (18), and replacing \( w \) by \( \hat{w}(\sigma^2, \mu, \delta) \), we get

\[
\hat{\Omega}^{MP}(\sigma^2, \mu, \delta) = -a^2 Q(\sigma^2, \mu, \delta) \left[ \hat{w}(\sigma^2, \mu, \delta) \right]^2
\]

(20)

where \( Q(\sigma^2, \mu) \) is given by

\[
Q(\sigma^2, \mu) = \gamma_0 + \frac{\alpha \mu^2 + (1 - \alpha)\sigma^2}{1-\gamma_0}
\]

(21)

5.2 Effect of a decrease in uncertainty

Equation (20) indicates that there are two channels through which a reduction in uncertainty can affect macroeconomic performance. The first goes through the nominal wage \( \hat{w}(\sigma^2, \mu, \delta) \). As we have \( \hat{w}(\sigma^2, \mu, \delta) > 0 \), from (20) this implies that a rise in the nominal wage is unfavorable, while a decrease is beneficial. In the last section we have seen that the sign of this effect depends on the kind of reduction in uncertainty we consider. The second channel goes through coefficient \( Q(\sigma^2, \mu, \delta) \). From (21), this coefficient is a non decreasing function of \( \sigma^2 \) and \( \mu \). Therefore, when it exists, the effect going through this second channel can only be favorable.

5.2.1 Decrease in the variance of the central distribution

As before, we first consider the effect of a lower variance of the central distribution \( \sigma^2 \), while the other parameters \( \mu \) and \( \delta \) are left unchanged. In the case where initially we have \( \alpha \geq \alpha^* \), according to proposition 2, the nominal wage is unchanged. As, in general, coefficient \( Q(\sigma^2, \mu, \delta) \) is reduced, this implies that macroeconomic performance is then improved. However, in the
case where initially we have \( \alpha < \alpha^* \), proposition 2 indicates that the nominal wage increases, which can counterbalance the smaller coefficient \( Q(\sigma^2, \mu, \delta) \).

We will consider a marginal decrease in \( \sigma^2 \). Macroeconomic performance is worsened if we have \( \frac{\partial \Omega^{MP}}{\partial \sigma^2} > 0 \), and improved if we have \( \frac{\partial \Omega^{MP}}{\partial \sigma^2} < 0 \). We get:

**Proposition 5** Consider a marginal decrease in the variance \( \sigma^2 \) of the central distribution \( C \), holding \( \mu \) and \( \delta \) constant.

- In the case where initially we have \( \alpha < \alpha^*(\sigma^2, \mu) \), then \( \frac{\partial \Omega^{MP}}{\partial \sigma^2} \) has the same sign as \( Z \) given by

\[
Z = 2\gamma_0 - 3\gamma_0^2 + \sigma^2 \gamma + (\mu^2 - \sigma^2 \gamma + 2\gamma_0 \mu) \alpha
\]  

(22)

Therefore, macroeconomic performance may be either worsened or improved, depending on the parameters of the model. When we have \( Z > 0 \) it is (strictly) worsened, while when we have \( Z < 0 \) it is (strictly) improved (with no effect on macroeconomic performance when \( Z = 0 \)).

- In the case where initially we have \( \alpha \geq \alpha^*(\sigma^2, \mu) \), we have \( \frac{\partial \Omega^{MP}}{\partial \sigma^2} < 0 \) when we have \( \alpha \neq 1 \), and consequently macroeconomic performance is then (strictly) improved. In the extreme case \( \alpha = 1 \), which occurs when we have both \( \theta = 1 \) and \( \delta = 1 \) (and which corresponds to the maximin case) we have \( \frac{\partial \Omega^{MP}}{\partial \sigma^2} = 0 \), and macroeconomic performance is then unchanged.

See appendix 5 for the proof. Proposition 5 underlines that when \( \sigma^2 \) decreases, it is possible that macroeconomic performance deteriorates. This was the result emphasized by Sorensen (1991) in the bayesian case. Such a possibility is now seen to hold also in the non-Bayesian case \( \alpha \neq 0 \), and proposition 5 has explicited the conditions (which are that both inequalities \( \alpha < \alpha^*(\sigma^2, \mu) \) and \( Z > 0 \) are satisfied) under which macroeconomic performance worsens.

But proposition 5 also implies that, for some values of the parameters of the model, macroeconomic performance may also improve. First, it says that this could occur even in the Bayesian case \( \alpha = 0 \) studied in the literature. For, from (22), in that case the inequality \( Z > 0 \) becomes equivalent to \( \gamma_0 (2 - 3\gamma_0) + \sigma^2 > 0 \). When we have \( \gamma_0 \leq \frac{2}{3} \), we always have \( Z > 0 \), which means that reduced uncertainty always worsens macroeconomic performance. However, in the case \( \gamma_0 > \frac{2}{3} \), we have the opposite result if \( \sigma^2 \) is not too large, i.e. if we have \( \sigma^2 < \gamma_0 (3\gamma_0 - 2) \) : then, macroeconomic performance is improved. We can try to relate this result to the findings
obtained by Sorensen (1991) under the Bayesian approach\textsuperscript{19}. This author also found that reduced political uncertainty may either improve or worsen macroeconomic performance. However, in Sorensen (1991), this ambiguity of the result was entirely due to the presence of shocks to the unemployment equation. But in the present analysis we do not have such shocks. In that case, the analysis of Sorensen (1991) would actually imply that reduced uncertainty always worsens macroeconomic performance\textsuperscript{20}. Thus, there is a discrepancy between Sorensen's result and ours. However, we can note that Sorensen made the simplifying assumption that on average the relative weight to inflation stabilization relatively to unemployment given by the policymaker is equal to one, and also that the coefficient of the real wage in the unemployment equation is equal to one. In our model, this would mean that we have $\psi_0 = a = 1$, and therefore, from (4), $\gamma_0 = \frac{1}{2}$. But when we have $\gamma_0 = \frac{1}{2}$, the inequality $\gamma_0 \leq \frac{2}{3}$ is satisfied. As we have just pointed out, we are precisely in a case where we always have $Z > 0$ under the Bayesian approach, and where therefore a decrease in $\sigma^2_\gamma$ always worsens macroeconomic performance. Thus, under the same simplifying assumption as the one made by Sorensen (1991), the discrepancy between the result of Sorensen (1991) and the result found in the present analysis in the Bayesian case, disappears\textsuperscript{21}. Finally, proposition 5 indicates that when the distance from the bayesian case is large enough, i.e. when we have $\alpha \geq \alpha^*(\sigma^2_\gamma, \mu)$, then macroeconomic performance is always improved by a lower variance of the central distribution (and strictly so, except in the special maximin case $\alpha = 1$ where it is unchanged). Therefore, the result obtained in the literature that a lower variance might be beneficial, does not hold anymore when the distance from the Bayesian case, given by parameter $\alpha$, becomes large enough (i.e. equal or larger than the threshold value $\alpha^*$). From (9) this would tend to be the case when either the aversion toward the imprecision of information, given

\textsuperscript{19}Grüner (2002) examined the effect of lower uncertainty on the variance inflation, but did not explicitly considered its welfare effect.

\textsuperscript{20}See equation (14) p. 379 in Sorensen (1991). In the absence of unemployment shocks, this equation always implies that macroeconomic performance is worsened by a decrease in the variance of political uncertainty.

\textsuperscript{21}However, whether we would also find an ambiguous result in the model of Sorensen (1991) if this simplifying assumption was removed, as in the present analysis, is an open issue. For the model of Sorensen (1991) is slightly different from the one (taken from Grüner (2002)) we use. First, instead of (2), the labor union tries to minimize the deviation of the real wage from a desired level which is too high (and also therefore creates unemployment in the certainty case). Second, the exogenous variance is that of the relative weight $\psi$ between inflation and unemployment in the central bank utility function, while in the present analysis it directly concerns the coefficient $\gamma$ of the reaction function of the central bank.
by parameter $\theta$, or the imprecision of information, given by parameter $\delta$, or both, are sufficiently large.

5.2.2 Global decrease in uncertainty

Consider now the case of a global reduction in uncertainty where both $\sigma^2_{\gamma}$ and $\mu$ decrease while the ratio $\lambda \equiv \frac{\sigma^2_{\gamma}}{\mu^2}$ is held constant. In section 4 we have obtained that this also leads to a higher nominal wage in the case where initially we had $\alpha < \alpha^*$. Therefore for the same reasons as in the case of a decrease in $\sigma^2_{\gamma}$ alone, macroeconomic performance may also be either worsened or improved. We obtain:

**Proposition 6** Consider a marginal global decrease in uncertainty where both $\sigma^2_{\gamma}$ and $\mu$ decrease while the ratio $\lambda \equiv \frac{\sigma^2_{\gamma}}{\mu^2}$ is held constant.

- In the case where initially we have $\alpha < \alpha^*(\sigma^2_{\gamma}, \mu)$, the sign of $\frac{\partial \Omega_{MP}}{\partial \mu}$ depends on the parameters of the model. Therefore, macroeconomic performance may be either worsened or improved. In the case of a small amount of uncertainty $\mu$, then, at the first order, $\frac{\partial \Omega_{MP}}{\partial \mu}$ can be shown to have the same sign as $X$ given by

$$X = (2 - 3\gamma_0) \lambda \mu - \gamma_0 (1 - \gamma_0) \alpha$$

(23)

Therefore, (in the case of a small amount of uncertainty), a global reduction in uncertainty (strictly) worsens macroeconomic performance when we have $X > 0$, and (strictly) improves it when we have $X < 0$ (with no effect on macroeconomic performance when we have $X = 0$).

- In the case where initially we have $\alpha \geq \alpha^*(\sigma^2_{\gamma}, \mu)$, we have $\frac{\partial \Omega_{MP}}{\partial \mu} < 0$, and consequently macroeconomic performance is then always strictly improved.

See appendix 6 for the proof. We obtain qualitative results which are similar to the case of a reduction in $\sigma^2_{\gamma}$ alone. Nonetheless, and although we will not put too much emphasis on it, there are two differences which it may be worthwhile to note. First, in the case $\alpha \geq \alpha^*(\sigma^2_{\gamma}, \mu)$, where the nominal wage is unchanged, macroeconomic performance is always strictly improved (even in the maximin case $\alpha = 1$).

Second, in the case $\alpha < \alpha^*(\sigma^2_{\gamma}, \mu)$, the condition under which macroeconomic performance is worsened is different in the non-Bayesian case $\alpha \neq 0^2$.

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\(^{22}\)In the general case, the condition is not easy to interpret, and we have given it explicitly only in the case of small uncertainty.

\(^{23}\)In the Bayesian case $\alpha = 0$, the criterion becomes the expected utility under the central distribution and therefore parameter $\mu$ has no effect in itself. As a consequence, starting
Parameter $\alpha$ does not appear to have the same qualitative role for the issue considered. In particular, from (22), we see that $Z$ is an increasing function of $\alpha$, while from (23) we get that $X$ is a decreasing function of $\alpha$.

One of the origins of these differences lies in the different effects that the two kinds of reduced uncertainty have on the favorable channel going through a lower coefficient $Q$. It can be seen that parameter $\alpha$ has effects of opposite signs on this channel in the two cases considered: from (21) we get
\[
\frac{\partial}{\partial \alpha} \left( \frac{dQ(\sigma^2,\mu,\delta)}{d\sigma^2} \right) = -\frac{1}{1-\gamma_0} < 0
\]
in the first case, and
\[
\frac{\partial}{\partial \alpha} \left( \frac{dQ(\sigma^2,\mu,\delta)}{d\mu} \right) = \frac{2\mu(1-\lambda)}{1-\gamma_0} \geq 0
\]
in the second case. Therefore, when $\alpha$ gets larger, this favorable channel gets smaller in the case of a decrease in $\sigma^2$ alone, while on the contrary it gets larger in the case of a global decrease in uncertainty.

5.2.3 Decrease in the imprecision of information

We consider a decrease in parameter $\delta$, holding $\sigma^2$ and $\mu$ constant. We have seen that when there is less imprecision of information, then the nominal wage is either lowered or unchanged (proposition 4). As a consequence, macroeconomic performance is improved, or at most unchanged in some special cases. More precisely, we get:

**Proposition 7** In the non bayesian case $\theta \neq 0$, a decrease in the imprecision of information (a lower value of $\delta$), always (strictly) improves macroeconomic performance when we have $\sigma^2 \neq \mu$. In the special case $\sigma^2 = \mu$, macroeconomic performance is (strictly) improved whenever, according to proposition 4, the nominal wage is strictly lower; and is left unchanged when the nominal wage stays the same.

In the Bayesian case $\theta = 0$, a decrease in the imprecision of information has no effect on macroeconomic performance.

See appendix 7 for the proof. This result indicates that when we consider a change in the imprecision of information (or in "Knightian uncertainty"), we obtain a result which is opposite to the one emphasized in the literature under the bayesian approach. A reduction in this kind of uncertainty always improve macroeconomic performance (or leave it unchanged in some special from the same initial situation where we have $\sigma^2 = \lambda \mu^2$, the two cases of a decrease in $\sigma^2$ (alone, and associated to a proportional decrease in $\mu$), should lead to the same condition for having a worsened economic performance. This appears in the conditions given in propositions 5 and 6. For in the case $\alpha = 0$, (22) gives $2\gamma_0 - 2\gamma_0^2 + \lambda \mu^2 > 0$. When $\mu$ is small, $\lambda \mu^2$ is a second order term and therefore, at the first order, the condition becomes $\gamma_0(2 - 3\gamma_0) > 0$. From (23), this is actually identical to the condition $X > 0$ in the case $\alpha = 0$. 

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cases, as in the Bayesian case $\theta = 0$). In general, this occurs both through a reduced nominal wage, and through the favorable effect that less uncertainty has for a given nominal wage (through a lower coefficient $Q$ in (20)). Thus, less political uncertainty, or more transparency of the central bank, would be beneficial if it mainly takes the form of a lower amount of Knightian uncertainty, i.e. of a reduction in the imprecision of information.

6 Conclusion

The existing literature has developed the argument that less political uncertainty, or more transparency of the central bank, may worsen macroeconomic performance by raising the nominal wage. The argument relies on the analysis of the game between a labor union and the central bank. The labor union is assumed to be uncertain about the utility function of the central bank, which creates an uncertainty in the way the central bank reacts to the nominal wage set by the labor union. As the labor union (which plays first in the game considered) takes into account this reaction function of the central bank when it sets its nominal wage, less uncertainty has been shown to increase the nominal wage. This in turn could worsen macroeconomic performance.

The underlying analysis was done under the traditional Bayesian framework of expected utility maximization. In the present paper, we have extended this analysis to a non-Bayesian framework, which takes into account what has been called an "aversion to ambiguity" in the literature. We have taken from the existing literature a version of this non-Bayesian approach which allows us to explicitly introduce the information available to the decision maker and to evaluate and compare different prior informations. The information available to the decision maker then consists of two parts: first, a central distribution on the relevant parameter; second, some imprecision around this central distribution.

We have found that the results obtained under the Bayesian approach are not necessarily valid when we consider this extended approach. Thus, less uncertainty does not always lead to higher wages. On the contrary, the nominal wage is even lower in some cases. In that respect, a crucial distinction concerns the type of decrease in uncertainty we consider. Under the Bayesian approach, a reduction in uncertainty consists of a lower variance of the probability distribution considered. Under the present extended approach, the amount of uncertainty of the information available is characterized by three parameters: the variance of the central distribution; the length of the support of the probability distributions; and a coefficient characterizing the imprecision of information. Thus, we have considered three types of decreases in
uncertainty. The first consists in a lower value of the variance of the central distribution, keeping the other two parameters unchanged. The second is a global decrease in uncertainty, where both the standard deviation of the central distribution and the length of the support decrease proportionally, while the imprecision of information parameter stays the same. In the third, the central distribution is kept unchanged, and there is a decrease in the imprecision of information. This last kind of reduction in uncertainty could also be labelled a decrease in "Knightian uncertainty".

In the first two cases of decreases in uncertainty, we have shown that the results found under the Bayesian approach in the literature could still be valid when the distance from the Bayesian case is not too large: a lower value of the variance of the central distribution, or a lower amount of global uncertainty, always lead to a higher nominal wage, and consequently may worsen macroeconomic performance. However, when the distance from the Bayesian case becomes larger than some threshold value, the results obtained under the Bayesian approach do not hold anymore. In that case, the nominal wage is unchanged and stays equal to the value it would have under certainty. As a consequence, the effect on macroeconomic performance of reduced uncertainty is in general strictly beneficial, and at least can never be harmful.

Furthermore, when we consider the third type of uncertainty reduction, i.e. a decrease in the imprecision of information (or in "Knightian uncertainty"), we find that, on the contrary, the nominal wage is either reduced or unchanged. This implies that macroeconomic performance never deteriorates: it is strictly improved in general, and unaffected in some special cases, as in the Bayesian case where there is no aversion toward the imprecision of information. Thus, the result found in the literature under the Bayesian approach that less uncertainty increases the nominal wage and consequently may worsen macroeconomic performance, never holds when we consider a reduction in "Knightian uncertainty".

The results we obtain under the present extended framework, draw our attention to two kinds of considerations, which should lead to some additional questions and research. First, these results emphasize that when the distance from the Bayesian case becomes too large (i.e. larger than some threshold value which depends on the other parameters of the model), less uncertainty, whatever its type, improves macroeconomic performance. Therefore, the result emphasized in the literature under the Bayesian approach (i.e. that macroeconomic performance may on the contrary be worsened), requires that we are not too far from the Bayesian case. If we try to apply the argument to the issue of central bank transparency, for example, this would require that we try to estimate how far, in a given situation, we are from the Bayesian
Second, the results also emphasize that a lower amount of uncertainty may have different implications, depending on the kind of decrease in uncertainty we consider. While, as in the Bayesian case, a decrease in the variance of the central distribution, or a lower amount of global uncertainty, may worsen macroeconomic performance, a decrease in "Knightian uncertainty", i.e. a decrease in the imprecision of information, on the contrary always improves it (or at most leaves it unchanged in some special cases, as in the Bayesian case). Thus, if we try to apply this analysis to the central bank transparency issue, this raises new questions. In practice, what type of decrease in uncertainty does central bank transparency lead to? For instance, does it mainly reduce proportionally all uncertainty, or does it mainly consist in reducing the imprecision of information, making the information available to the public closer to some central probability distribution? It might be that some form of transparency leads to one type, and an other form to an other type. If this happens, it would mean that a kind of central bank transparency which reduces the imprecision of information (i.e. reduces "Knightian uncertainty") might be preferable, at least according to the argument considered in the present paper.

Appendices

1. Proof of proposition 1

In (8) consider first the issue of finding the probability distribution $P$ which minimizes $E_P U^{LU}$ where $P \in \mathcal{R}(S)$ is any distribution with support $[\gamma_0 - \mu, \gamma_0 + \mu]$.

From (6), $U^{LU}(\gamma, w)$ is a quadratic concave function of $\gamma$. As a consequence, the probability distribution with support $[\gamma_0 - \mu, \gamma_0 + \mu]$ which minimizes $E_P U^{LU}$ is obtained either for the probability distribution which gives $\gamma = \gamma_0 - \mu$ with probability one, or for the probability distribution which gives $\gamma = \gamma_0 + \mu$ with probability one. We get $\gamma = \gamma_0 - \mu$ as a solution when we have $U^{LU}(\gamma_0 - \mu, w) = U^{LU}(\gamma_0 + \mu, w)$, which, using (6) and (10), can be written $2\mu A a^2 \gamma_0 w [w - \overline{w}(\gamma_0)] = 0$. Therefore, as we have $\overline{w}(\gamma_0) > 0$, the value $\gamma = \gamma_0 - \mu$ is a solution in the case $0 \leq w \leq \overline{w}(\gamma_0)$, while $\gamma = \gamma_0 + \mu$ is a solution in the cases $w \leq 0$ and $w \geq \overline{w}(\gamma_0)$ (and there is indifference between $\gamma_0 - \mu$ and $\gamma_0 + \mu$ when $w = 0$ or $w = \overline{w}(\gamma_0)$).

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In the present analysis, coefficient $a$ is an indicator of the distance from the Bayesian case. But this coefficient is itself the product of two parameters: one represents the aversion toward the imprecision of information (parameter $\theta$); and the other represents the amount of imprecision of information (parameter $\delta$). Therefore both aspects would have to be taken into account if we want to estimate this distance.
Consider the function \( \tilde{\Omega}^{LU}(\gamma, w) \) defined by

\[
\tilde{\Omega}^{LU}(\gamma, w) \equiv \alpha U^{LU}(\gamma, w) + (1 - \alpha) E_C U^{LU}(\gamma, w)
\]  

(24)

Using (6), we can see that \( \tilde{\Omega}^{LU}(\gamma, w) \) is a quadratic concave function of \( w \), which is maximized for \( \hat{w}(\gamma) \) given by

\[
\hat{w}(\gamma) = \frac{1}{2a^2} \frac{\alpha \gamma + (1 - \alpha) \gamma_0}{\sigma^2 + (1 - \alpha)(\gamma_0^2 + \sigma^2)}
\]

(25)

On the interval \([-\infty, 0]\), as we have just shown, we have \( \min_{\gamma \in \mathbb{R}(S)} E_p U^{LU} = U^{LU}(\gamma_0 + \mu) \) and therefore, from (8), we have \( \Omega^{LU} = \tilde{\Omega}^{LU}(\gamma_0 + \mu, w) \). As a consequence, on the interval \([-\infty, 0]\), the value of \( w \) which maximizes \( \Omega^{LU} \) is equal to \( \min \{ \hat{w}(\gamma_0 + \mu, 0) \} \). As we have \( \hat{w}(\gamma_0 + \mu) > 0 \), this is equal to 0. On the interval \([\gamma(\gamma_0), +\infty[\) we also have \( \min_{\gamma \in \mathbb{R}(S)} E_p U^{LU} = U^{LU}(\gamma_0 + \mu) \), and therefore the value of \( w \) which maximizes \( \Omega^{LU} \) on this interval is equal to \( \max \{ \hat{w}(\gamma_0 + \mu, 0), \gamma(\gamma_0) \} \). Using (10) and (25) we can see that we always have \( \hat{w}(\gamma_0 + \mu) < \gamma(\gamma_0) \). Therefore, on the interval \([\gamma(\gamma_0), +\infty[\), the optimal value of \( w \) is \( \gamma(\gamma_0) \). These results imply that 0 is (strictly) better than any value of \( w \) which belongs to \([-\infty, 0]\), and that \( \gamma(\gamma_0) \) is (strictly) better than any value of \( w \) which belongs to \([\gamma(\gamma_0), +\infty[\). As a consequence, the value \( \hat{w} \) which maximizes \( \Omega^{LU} \), given by (8), has to belong to the interval \([0, \gamma(\gamma_0)]\). But we have shown that on \([0, \gamma(\gamma_0)]\) we have \( \min_{\gamma \in \mathbb{R}(S)} E_p U^{LU} = U^{LU}(\gamma_0 - \mu) \), and therefore \( \Omega^{LU} = \tilde{\Omega}^{LU}(\gamma_0 - \mu, w) \). As we have \( \hat{w}(\gamma_0 - \mu) > 0 \), this implies \( \hat{w} = \min \{ \hat{w}(\gamma_0 - \mu), \gamma(\gamma_0) \} \). Using (10) and (25), we get that \( \min \{ \hat{w}(\gamma_0 - \mu), \gamma(\gamma_0) \} \) is equal to \( \hat{w}(\gamma_0 - \mu) \) in the case \( \alpha < \alpha^* \) and to \( \gamma(\gamma_0) \) in the case \( \alpha > \alpha^* \), \( \hat{w}(\gamma_0 - \mu) \) and \( \gamma(\gamma_0) \) being equal in the case \( \alpha = \alpha^* \). Finally, from (12) we get \( \frac{\partial \hat{w}}{\partial \gamma} = \frac{1}{2a^2} \frac{(\gamma_0 - \mu)(\sigma_0^2 + \gamma_0 \mu)}{\sigma^2} > 0 \), where \( D \) is the denominator which appears in (12). Proposition 1 directly follows. QED

2. Proof of proposition 2

From (11) we get \( \frac{\partial \alpha^*(\mu, \sigma_0^2)}{\partial \sigma_0^2} > 0 \) and therefore \( \alpha^*(\sigma_{0,1}^2, \mu) > \alpha^*(\sigma_{0,2}^2, \mu) \). In the case \( \alpha \geq \alpha^*(\sigma_{0,1}^2, \mu) \), this implies \( \alpha > \alpha^*(\sigma_{0,2}^2, \mu) \). Therefore, in that case, from proposition 1, we get \( \hat{w} \left( \sigma_{\gamma,1}^2, \mu, \delta \right) = \hat{w} \left( \sigma_{\gamma,2}^2, \mu, \delta \right) = \gamma(\gamma_0) \). Consider now the case \( \alpha < \alpha^*(\sigma_{0,1}^2, \mu) \). As we have \( \alpha^*(\sigma_{0,1}^2, \mu) \leq 1 \), this implies \( \alpha < 1 \), which, from (12), gives \( \frac{\partial \hat{w}(\sigma_{0,1}^2, \mu, \delta)}{\partial \sigma_0^2} < 0 \). Therefore, if \( \alpha \leq \alpha^*(\sigma_{0,2}^2, \mu) \), then proposition 1 implies that when \( \sigma_0^2 \) decreases from \( \sigma_{0,1}^2 \) to \( \sigma_{0,2}^2 \), the nominal wage strictly increases from \( \hat{w} \left( \sigma_{\gamma,1}^2, \mu, \delta \right) \) to \( \hat{w} \left( \sigma_{\gamma,2}^2, \mu, \delta \right) \), where both are less
than $\bar{w}(\gamma_0)$. If $\alpha > \alpha^*(\sigma_{\gamma,2}, \mu)$, then, from proposition 1, the nominal wage increases from $\hat{w}(\sigma^2_{\gamma,1}, \mu, \delta) \leq \bar{w}(\gamma_0)$ to $\hat{w}(\sigma^2_{\gamma,2}, \mu, \delta) = \bar{w}(\gamma_0)$. QED

3. Proof of proposition 3

We replace $\sigma^2_{\gamma}$ by $\lambda\mu^2$ in (11) and (12). From (11) we get $\frac{\partial \alpha}{\partial w} > 0$. When we have $\alpha < \alpha^*$, from (12) we find that $\frac{\partial \alpha}{\partial w}$ has the sign of $B \equiv \alpha [\gamma_0^2 + \alpha \mu^2 - 2\mu\gamma_0 - (1 - \alpha)\lambda\mu^2] - 2(1 - \alpha)\lambda\mu(\gamma_0 - \alpha\mu)$. Then, using (11), the inequality $\alpha < \alpha^*$ gives $(1 - \alpha)\lambda\mu > \alpha (\gamma_0 - \mu)$. This implies $B < \alpha [\gamma_0^2 + \alpha \mu^2 - 2\mu\gamma_0 - \alpha\mu (\gamma_0 - \mu)] - 2\alpha (\gamma_0 - \mu) (\gamma_0 - \alpha\mu)$, which gives $B < -\alpha\gamma_0 (\gamma_0 - \alpha\mu)$. As we have $\mu < \gamma_0$, this implies $B < 0$. We therefore have $\frac{\partial \alpha}{\partial w} < 0$ in the case $\mu < \alpha^*$. Then, the rest of the proof proceeds exactly as in the proof of proposition 2. QED

4. Proof of proposition 4

From (12), $\hat{w}$ depends on $\delta$ through parameter $\alpha = \theta\delta$. We have $\alpha_1 = \theta\delta_1$ and $\alpha_2 = \theta\delta_2$. In the bayesian case $\theta = 0$, we have $\alpha_1 = \alpha_2 = 0$ and therefore $\hat{w}(\sigma^2_{\gamma}, \mu, \delta_1) = \hat{w}(\sigma^2_{\gamma}, \mu, \delta_2) = \hat{w}(\sigma^2_{\gamma}, \mu, 0)$ in all cases. In the non-bayesian case $\theta \neq 0$, we have $\alpha_1 > \alpha_2$. Consider first the case $\alpha_2 < \alpha^*(\sigma^2_{\gamma}, \mu)$. We have $\hat{w}(\sigma^2_{\gamma}, \mu, \delta_1) > \hat{w}(\sigma^2_{\gamma}, \mu, \delta_2)$, and if we have $\alpha_1 > \alpha^*(\sigma^2_{\gamma}, \mu)$, we get $\bar{w}(\gamma_0) = \hat{w}(\sigma^2_{\gamma}, \mu, \delta_1) > \hat{w}(\sigma^2_{\gamma}, \mu, \delta_2)$. In the case $\alpha_2 \geq \alpha^*(\sigma^2_{\gamma}, \mu)$, the inequality $\alpha_1 > \alpha_2$ implies $\alpha_1 > \alpha^*(\sigma^2_{\gamma}, \mu)$, and therefore proposition 1 gives $\hat{w}(\sigma^2_{\gamma}, \mu, \delta_1) = \hat{w}(\sigma^2_{\gamma}, \mu, \delta_2) = \bar{w}(\gamma_0)$. QED

5. Proof of proposition 5

From (20), we have

$$\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}} = -a^2 \hat{w} \left( 2Q \frac{\partial \hat{w}}{\partial \sigma^2_{\gamma}} + \hat{w} \frac{\partial Q}{\partial \sigma^2_{\gamma}} \right)$$

(26)

In the case $\alpha \geq \alpha^*(\sigma^2_{\gamma}, \mu)$, from proposition 2 we have $\frac{\partial \bar{w}}{\partial \sigma^2_{\gamma}} = 0$. Therefore, from (21) and (26) we get $\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}} = -a^2 \hat{w} \frac{1 - \alpha}{1 - \gamma_0}$. When we have $\alpha < 1$, then we get $\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}} < 0$; while in the special case $\alpha = 1$, we have $\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}} = 0$.

In the case where $\alpha < \alpha^*(\sigma^2_{\gamma}, \mu)$ is satisfied, $\hat{w}(\sigma^2_{\gamma}, \mu, \delta)$ is given by (12). Using (12), (21) and (26), we obtain after calculus $\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}} = G (1 - \alpha) Z$, where we have $G > 0$ and where $Z$ is given by (22). As we are in the case $\alpha < \alpha^*(\sigma^2_{\gamma}, \mu)$, and as we have $\alpha^*(\sigma^2_{\gamma}, \mu) \leq 1$, we necessarily have $\alpha < 1$, which implies that $\frac{\partial Q_{MP}}{\partial \sigma^2_{\gamma}}$ has the sign of $Z$. QED
6. Proof of proposition 6

From (20), we can write, as in (26):

\[
\frac{d\hat{Q}^{MP}}{d\mu} = -a^2 \tilde{w} \left( 2Q \frac{d\tilde{w}}{d\mu} + \tilde{w} \frac{dQ}{d\mu} \right)
\]  

(27)

Replacing \( \sigma^2_\gamma \) by \( \lambda \mu^2 \) in (21), we get

\[
\frac{dQ(\sigma^2_\gamma, \mu, \delta)}{d\mu} = \frac{2\mu [\alpha + \lambda (1 - \alpha)]}{1 - \gamma_0} > 0
\]  

(28)

In the case \( \alpha \geq \alpha^* (\sigma^2_\gamma, \mu) \), from proposition 3 we have \( \frac{d\tilde{w}}{d\mu} = 0 \). Then, (27), (28), \( \frac{d\tilde{w}}{d\mu} = 0 \) and \( \tilde{w} \neq 0 \) give \( \frac{d\hat{Q}^{MP}}{d\mu} < 0 \).

In the case \( \alpha < \alpha^* (\sigma^2_\gamma, \mu) \), the nominal wage \( \tilde{w} (\sigma^2_\gamma, \mu, \delta) \) is given by (12). Using (12), (21), (27) and (28), we obtain

\[
\frac{d\hat{Q}^{MP}}{d\mu} = \frac{\tilde{w}}{AD^2} H
\]  

(29)

where \( H \) is given by

\[
H = 2Q \left( \alpha D + (\gamma_0 - \alpha \mu) \frac{dD}{d\mu} \right) - D (\gamma_0 - \alpha \mu) \frac{dQ}{d\mu}
\]  

(30)

and where \( D \) is the denominator which appears in (12), which, once \( \sigma^2_\gamma \) has been replaced by \( \lambda \mu^2 \), is given by

\[
D = \alpha (\gamma_0 - \mu)^2 + (1 - \alpha) (\gamma_0^2 + \lambda \mu^2)
\]  

(31)

As we have \( \tilde{w} > 0 \), (29) implies that \( \frac{d\hat{Q}^{MP}}{d\mu} \) has the sign of \( H \). From (21), (28), (30) and (31), we can obtain an expression for \( H \) as a function of the parameters of the model. To simplify, we will actually make explicit this expression only in the case where parameter \( \mu \) is small, and therefore the amount of uncertainty is small. First, we can note that from (11) and \( \sigma^2_\gamma = \lambda \mu^2 \), we get \( \alpha^* = \frac{\lambda \mu^2}{\mu (\gamma_0 - \mu) + \lambda \mu^2} \), which, in the case where \( \mu \) is small, behaves as \( \frac{\lambda \mu}{\gamma_0} \). Therefore, the condition \( \alpha < \alpha^* \) implies that \( \alpha \) is also small, with an order of magnitude as small or smaller than \( \mu \). Using this property along with (21), (28), (30) and (31), we can approximate \( H \) at the first order. Then, we obtain \( H = \frac{2\gamma_0^2 X}{1 - \gamma_0} \), where \( X \) is given by (23). Proposition 6 follows. QED

7. Proof of proposition 7
From (9) and (21) we have $\frac{\partial Q}{\partial \delta} = \frac{\theta(\mu^2 - \sigma^2)}{1-\gamma^2} \geq 0$. As we have $\sigma^2 \leq \mu^2$, this implies $Q_2 \leq Q_1$. And according to proposition 4 we have $\hat{w}_2 \leq \hat{w}_1$. From (20) we can write

$$\hat{\Omega}_2^{MP} - \hat{\Omega}_1^{MP} = -a^2 [Q_2(\hat{w}_2 - \hat{w}_1) + \hat{w}_1(Q_2 - Q_1)]$$

(32)

The inequalities $Q > 0$, $\hat{w} > 0$, $Q_2 \leq Q_1$ and $\hat{w}_2 \leq \hat{w}_1$ then imply $\hat{\Omega}_2^{MP} \geq \hat{\Omega}_1^{MP}$.

As we have $Q > 0$ and $\hat{w} > 0$, then, from (32), the previous inequality is strict if and only if we have either $Q_2 < Q_1$ or $\hat{w}_2 < \hat{w}_1$ (or both). In the case $\theta \neq 0$, from the expression of $\frac{\partial Q}{\partial \delta}$ above, we have $Q_2 < Q_1$ if and only if we have $\sigma_\gamma < \mu$, which (as we always have $\sigma_\gamma \leq \mu$) is equivalent to $\sigma_\gamma \neq \mu$.

From (9) in the case $\theta = 0$, $\alpha$ is unchanged (and stays equal to zero). Therefore, from proposition 1, $\hat{w}$ is unchanged and, from (21), $Q$ is also unchanged. Then (20) implies that $\hat{\Omega}^{MP}$ is unchanged. QED

References


