Ambiguity and Coordination in a Global Game Model of Financial Crises

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Models of financial crises: problem of coordination between the participants in the financial markets, because of complementarities between their actions. This may lead to multiple equilibria of the corresponding game, for some intermediate range of the levels of "fundamentals": currency crises (coordination between speculators); liquidity crises (coordination between creditors); bank runs (coordination between depositors).

"Global games" (Carlsson and van Damme (1993)): instead of assuming that the level of fundamentals is perfect knowledge, players only receive an imperfect signal. Then, the equilibrium is always unique, even if the uncertainty of the signal becomes small.

This has been applied to financial crises: exchange rate crises (Morris and Shin (1998)); bank runs (Goldstein and Pauzner (2005), Rochet and Vives (2004)); debt and creditors (Morris and Shin (2004)).
Comparative statics. One issue which can be considered: **effect of uncertainty on the occurrence of the crisis.** In the literature, no clear-cut results: this may depend on the model and, in a model, on the value of some parameter of the model.

Usually, this literature uses a standard expected utility approach to uncertainty. But, this rules out **ambiguity aversion** (Ellsberg (1961)): decision makers often prefer situations with known probabilities to situation with unknown probabilities. There are more recent approaches to uncertainty which introduce ambiguity (Gilboa and Schmeidler (1989) and Schmeidler (1989)): Ambiguity aversion would imply that, for each decision taken, the decision maker is "pessimistic" and therefore gives more weight to the bad states of the world.
I will use such an approach and I will highlight a new channel through which uncertainty can affect the occurrence of a financial crisis: I will underline that more ambiguity reduces the amount of perceived coordination between market participants (players). First, I will consider a model of debt financing with several creditors.

There are a few analyses of ambiguity in global games, with applications to models of financial crises (Ui (2009) and Kawagoe and Ui (2010). But this channel through perceived coordination was not present because of their implicit assumption that the distribution of the signals is the same for all the players.
Model

Morris and Shin (2004). A group of creditors financing a project through a collaterized debt have to decide whether to roll over their loans or not. Two player version, as in Kawagoe and Ui (2010).

Two actions: R (to roll over the loan) or N (to not roll over the loan).

Payoffs:

- If a player plays N, he gets the value of the collateral $\lambda$, $0 < \lambda < 1$;

- If a player plays R, he gets 0 if the project fails, and gets 1 if the project succeeds. The project always succeeds if "fundamentals" $\theta$ are strong enough ($\theta > 1$), and always fails if fundamentals are weak enough ($\theta < 0$).
In the intermediate case $0 \leq \theta \leq 1$, the project succeeds if both creditors play R, but fails otherwise:

\[
\begin{array}{cc}
  R & N \\
  R & 1, 1 & 0, \lambda \\
  N & \lambda, 0 & \lambda, \lambda
\end{array}
\]

If $\theta$ were common knowledge, there would be two Nash equilibria: one without a financial crisis (R,R); and one with a financial crisis (N,N), due to a lack of coordination between creditors.
But each player only receives an imperfect signal on $\theta$:

$$x_i = \theta + \xi_i$$

(1)

Furthermore, it is assumed that the distribution of $\xi_i$ is not known: its mean $\mu_i$ is ambiguous:

$$x_i = \theta + \mu_i + \varepsilon_i$$

(2)

The signal can therefore be biased (Cheli and Della Posta (2007)), and there is ambiguity on the value of the mean (or "bias") $\mu_i$. We have $\mu_i \in [\underline{\mu}, \bar{\mu}]$, and each player has a corresponding maxmin criterion of expected utility. We allow the distributions of the signals received by the players to be different, by allowing $\mu_1 \neq \mu_2$. 
Define the ambiguity parameter

\[ \eta \equiv \frac{1}{2}(\bar{\mu} - \mu) \]  

(3)

\( \theta \) is uniformly distributed on \([-\delta, 1 + \delta]\); and \( \varepsilon_i \) is uniformly distributed on \([-\gamma, \gamma]\). It is assumed that we have \( 0 < \gamma \leq \frac{\delta}{2} \) (which will make the prior on \( \theta \) non informative when we consider the equilibrium condition) and (to simplify the analysis) \( \gamma \leq \frac{1}{2} \).

**Switching strategy** \( s_k \) with switching point \( k \): play R if \( x_i > k \), and play N if \( x_i \leq k \)

We look for equilibria in switching strategies.
Proposition 1  When player $j$ follows a switching strategy, then, the worst case for player $i$ is obtained for $\mu_i = \bar{\mu}$ and $\mu_j = \underline{\mu}$.

This implies $E(x_j \mid x_i) = x_i - 2\eta$. Therefore player $i$ expects that the other player $j$ receives a signal which is lower (i.e. worse) than his/her signal by an amount which increases with the ambiguity parameter $\eta$.

Proposition 2 1. In the case $\eta < \gamma$, if we have $\lambda \neq \frac{1}{2} \left(1 - \frac{\eta}{\gamma}\right)^2$, there is a unique equilibrium in switching strategies, where the equilibrium switching point $k^*$ is

$$k^* = \bar{\mu} + g(\eta, \gamma, \lambda)$$  \hspace{1cm} (4)

where the function $g(\eta, \gamma, \lambda)$ is given by:
(i) if we have $0 < \lambda < \frac{1}{2} \left(1 - \frac{\eta}{\gamma}\right)^2$, then we have

$$g(\eta, \gamma, \lambda) = \gamma \left(1 - 2\sqrt{\left(1 - \frac{\eta}{\gamma}\right)^2 - 2\lambda}\right) - 2\eta \quad (5)$$

(ii) if we have $\frac{1}{2} \left(1 - \frac{\eta}{\gamma}\right)^2 < \lambda < 1 - \frac{\eta}{\gamma}$, then we have

$$g(\eta, \gamma, \lambda) = 1 - \gamma + 2\sqrt{\eta^2 + \gamma^2 \left(2\lambda - \left(1 - \frac{\eta}{\gamma}\right)^2\right)} - 2\eta \quad (6)$$

(iii) if we have $1 - \frac{\eta}{\gamma} \leq \lambda < 1$, then we have

$$g(\eta, \gamma, \lambda) = 1 - \gamma + 2\lambda \gamma \quad (7)$$

(and therefore does not depend on $\eta$ in this case)
In the case $\eta < \gamma$, if we have $\lambda = \frac{1}{2} \left(1 - \frac{\eta}{\gamma}\right)^2$, then any switching point $k^*$ such that $\gamma - 2\eta \leq k^* - \mu \leq 1 - \gamma$ gives an equilibrium in switching strategies.

2. In the case $\eta \geq \gamma$, then, for all $\lambda$ (where $0 < \lambda < 1$), there is a unique equilibrium in switching strategies, where the equilibrium switching point $k^*$ is given by (12) and (7) (and therefore $k^* - \mu$ does not depend on $\eta$ in this case).

Effects on the probability of a crisis

A financial crisis occurs when the investment project fails because of a problem of early liquidation by creditors. This happens in the case $0 \leq \theta \leq 1$ when at least one of the two creditors does not roll over the loan. Therefore, more ambiguity increases the probability of a crisis if and only if we have $k'^* > k^*$. 
The equilibrium switching point \( k^* \) is an increasing function of \( \lambda \). It may be increasing or decreasing with \( \gamma \) (i.e. with the variance of the shocks \( (\varepsilon_i) \)), depending on the values of the other parameters of the model.

**Effect of ambiguity:**

\((\underline{\mu}', \overline{\mu}')\) is more ambiguous than \((\underline{\mu}, \overline{\mu})\) if we have \([\underline{\mu}', \overline{\mu}'] \supset [\underline{\mu}, \overline{\mu}]\), which can be written: \(\underline{\mu}' \leq \underline{\mu}\) and \(\overline{\mu}' \geq \overline{\mu}\), with at least one strict inequality. Note that this implies \(\eta' > \eta\).

Let \(\mu_1\) and \(\mu_2\) be the true given biases of the signals received by player 1 and player 2, respectively. Then, we will compare \((\underline{\mu}, \overline{\mu})\) to \((\underline{\mu}', \overline{\mu}')\), where \((\underline{\mu}', \overline{\mu}')\) is more ambiguous than \((\underline{\mu}, \overline{\mu})\), and where we have \(\underline{\mu} \leq \mu_1 \leq \overline{\mu}, \underline{\mu} \leq \mu_2 \leq \overline{\mu}\), and \(\underline{\mu}' \leq \mu_1 \leq \overline{\mu}'\) and \(\underline{\mu}' \leq \mu_2 \leq \overline{\mu}'\).
Two channels

\[ k^* = \bar{\mu} + g(\eta, \gamma, \lambda) \]  

The first channel goes through the highest bias \( \bar{\mu} \). The reason is that, in the worst case, we have \( \mu_i = \mu \). As we have \( \mu_i' \geq \mu \), this makes \( k^{l*} \geq k^* \). Thus, as more ambiguity generally raises the highest possible bias, each player interprets less favorably any message received: a lower values for the fundamentals \( \theta \) is inferred.

The second channel goes through the function \( g(\eta, \gamma, \lambda) \), which, in cases 1-(i) and 1-(ii) of Proposition 2, depends on the ambiguity parameter \( \eta \). In these cases, we have \( \frac{\partial g(\eta, \gamma, \lambda)}{\partial \eta} > 0 \), which makes \( k^{l*} > k^* \). Thus, through this second channel, more ambiguity also increases the probability of a crisis.
The two channels go into the same direction. As a consequence, we find that more ambiguity increases the probability of a financial crisis. This makes more ambiguity harmful and, therefore, gives an argument for more transparency.

The second channel is the channel I want to emphasize. It is due to having, in the worst case, $\mu_i = \mu$ and $\mu_j = \mu$. Therefore this channel would disappear in the case without ambiguity ($\overline{\mu} = \mu$ and therefore $\eta = 0$), but also under ambiguity ($\eta > 0$), if we had assumed (as it is done in the literature on global games with ambiguity) that the probability distributions of the signals were the same for all the players, which would have imposed the constraint $\mu_i = \mu_j$. This is why this channel was not present in the literature.
This channel comes from having each player expect (in the worst case) that the other player receives a signal which is worse than his/her signal, and would therefore have a lower probability to play R. Therefore, more ambiguity makes each player expects that, when R is played, there is a lower amount of coordination with the other player. This makes each player less willing to play R. This raises $k^*$ and therefore increases the probability of a crisis.
Figure 1:
Figure 3:
There are two reasons why ambiguity increases the probability of a crisis:

(i) The payoff to action R depends on $\theta$ while the payoff to N does not depend on $\theta$. More ambiguity increases the incentive to choose the action (N) which does not depend on $\theta$.

(ii) The payoff to action R increases with the coordination of players while the payoff to N does not depend on the coordination of players. More ambiguity decreases the amount of perceived coordination, which reduces the incentive to choose the action (R) which depends on coordination.
Two remarks on that:

1. Both effects are of the same sign in the model considered, but this may not be necessarily so in other games.

2. In an exchange rate crisis model, where players are speculators, these two effects would also be of the same sign, but this sign would be opposite to the sign it has in the present model of a liquidity crisis in rolling over a debt. The reason is that it would be the payoff to the action "to attack the currency" which would depend both on the state of fundamentals and on the coordination of players, while the payoff to the action "not to attack the currency" would not depend on these variables. For the two previous reasons (and, in particular, because it reduces the perceived coordination between speculators), more ambiguity would therefore give more incentive to play the action "not to attack the currency". As a consequence, more ambiguity would on the contrary always decrease the probability of a crisis. Thus, for exchange crises, this would tend to make more transparency harmful.
Complementary paper: Daniel Laskar (2013), "Ambiguity, Pessimism, Optimism and Financial Crises in a Simple Global Game Model":

- same kind of analysis in a simpler linear global game model with a continuum of players

\[
u_i(\theta, l, a_i) = u(\theta, l, a_i) = \begin{cases} 
\theta - 1 + l & \text{if } a_i = C \\
0 & \text{if } a_i = N 
\end{cases}
\]  

(9)

where \( l \) is the proportion of players who take action \( C \) (action "C" has a payoff which increases with \( l \), and therefore which benefits from "coordination", and correspond to rolling over the loan (R) in the previous analysis

- introduce both optimism (preference for ambiguity) and pessimism (ambiguity aversion):

\[
\Omega_{\alpha,i}(k, x_i, a_i) = \alpha \min_{M \in \mathcal{M}} \pi_M (k, x_i, a_i) + (1 - \alpha) \max_{M \in \mathcal{M}} \pi_M (k, x_i, a_i)
\]  

(10)
where $\alpha$ ($0 \leq \alpha \leq 1$) is the degree of pessimism (and $1 - \alpha$ the degree of optimism).

- **explicitly compare two models** which may represent two types of financial crises: a **liquidity crisis model with creditors as players (model above)**; and an exchange rate crisis model with speculators as players:

$$
   u_i'(\theta', l', a'_i) = u'(\theta', l', a'_i) = \begin{cases} 
   l' - \theta' & \text{if } a'_i = C' \\
   0 & \text{if } a'_i = N' 
   \end{cases}
$$  \hspace{1cm} (11)

where now $C'$ represents the action "to attack the currency"

- **consider the effect of shifts in "market sentiment"** (Cheli and Della Posta (2007)), corresponding to shifts in the biases $\mu_i$ of the signals.
Equilibrium for the first model: switching strategy with switching point $k^*$

$$
k^* = \frac{1}{2} + \mu_0 + (2\alpha - 1) \left[ \eta + \psi (2\eta) - \frac{1}{2} \right] \tag{12}$$

where $\mu_0 \equiv \frac{1}{2} (\bar{\mu} + \underline{\mu})$; $\eta \equiv \frac{1}{2}(\bar{\mu} - \underline{\mu})$; and $\psi (.)$ is the cumulative distribution function of the distribution of $\varepsilon_i - \varepsilon_j, i \neq j$.

Case of pure pessimism $\alpha = 1$ (first paper)

$$
k^* = \bar{\mu} + \psi (2\eta) \tag{13}$$

$El^*$ can be taken as an inverse index of financial difficulties:

$$
El^* = \int_0^1 \Pr (\theta + \varepsilon_j > k^* - \mu_j) \, dj \tag{14}
$$

Results:
- Ambiguity has opposite effects according to whether pessimism or optimism dominates. An increase in $\eta$ reduces the expected coordination when pessimism dominates $\left(\alpha > \frac{1}{2}\right)$, but it increases the expected coordination when optimism dominates $\left(\alpha < \frac{1}{2}\right)$.

- More pessimism decreases the expected coordination.

- If we want to know the effect on the equilibrium switching point, and on financial difficulties, we can go from one model to the other by simply replacing pessimism by optimism (i.e. $\alpha$ by $1 - \alpha$), and vice versa. A model of liquidity crises with pessimistic creditors is similar to a model of exchange rate crises with optimistic speculators, and vice versa. As a consequence, more ambiguity has opposite effects on financial difficulties in the two models of financial crises, with opposite conclusions about transparency.
- We can obtain real effects of shifts in market sentiment in a rational way: ambiguity gives some room of maneuver to have shifts in market sentiment without equal corresponding shifts in expectations. We only have to satisfy the constraints $\mu_i \in [\underline{\mu}, \bar{\mu}]$ for all $i$. 