Exclusive Contracts and Demand Foreclosure*

David Spector†

July 2011

Abstract

A firm may induce some customers to sign exclusive contracts in order to deprive a rival of the minimum viable size, exclude it from the market, and enjoy increased market power. This strategy may result in socially inefficient exclusion even if the excluded firm is present at the contracting stage and can make counteroffers. In addition, allowing for breach penalty clauses decreases firms’ incentives to exclude rivals, because such clauses allow a firm to use customers as a conduit for the transfer of another firm’s profits.

*Forthcoming, Rand Journal of Economics. I am grateful to Severin Borenstein, Bernard Caillaud, Jeff Ely, Joseph Farrell, Bruno Jullien, Zvika Neeman, Debraj Ray, Patrick Rey, Daniel Rubinfeld, Jean Tirole and Thibaud Vergé for their comments on an earlier draft, as well as to two anonymous referees.

†Paris School of Economics and CNRS. Address: Paris School of Economics, 48, boulevard Jourdan, 75014 Paris, France (email: spector@pse.ens.fr).
1 Introduction

This article aims to clarify the circumstances under which a firm may sign exclusive contracts with some of its customers in order to exclude a rival, even though exclusion reduces social welfare. This clarification is important both for its own sake and from the viewpoint of antitrust policy. Indeed, the legal treatment of exclusive dealing is responsive to the twists and turns of economic theory. For example, the so-called Chicago critique, which purported to prove that the reasonings underpinning the traditional hostility toward exclusive dealing were flawed, seems to have induced U.S. Courts to progressively soften their handling of exclusivity clauses.\footnote{See Wiley (1998) and Gilbert (2000) for a survey of the legal treatment of exclusive contracts and an economic discussion.}

The recent literature on the use of exclusive contracts as a means to inefficiently exclude competitors relies on the combination of two features, namely contract incompleteness and the absence of some affected agents (often, the excluded firm) from the contracting game. From a theoretical viewpoint, there is a need to better understand the respective roles of these two assumptions. For an empirical viewpoint, the assumption that the firm targeted by the exclusionary strategy is absent at the contracting stage is at odds with much of the relevant case law.

Accordingly, the main contribution of this article is to show that exclusive contracts may result not only in inefficient entry-deterrence, but also in the inefficient eviction of a competitor already present in the market and able
to respond to exclusive contracts with counteroffers of its own. It is also shown that breach penalty provisions make inefficient entry deterrence less profitable and inefficient eviction impossible, because they facilitate transfers between agents and thus Coasian bargaining.

**Relation to the literature**

The first analysis of exclusive contracts emanated from the "Chicago school" and dismissed the view that these contracts could be used by a firm in order to exclude a rival and increase its market power. The Chicago school argument is simply that if such exclusion is socially inefficient, the payment which the excluding firm needs to make in order to "bribe" customers into agreeing to exclusivity would exceed the gain from deterring entry or inducing exit\(^2\) Exclusive dealing must then have other, possibly procompetitive motives (see, e.g., Marvel, 1982, and Segal and Whinston, 2000b).

The "post-Chicago" literature has identified several circumstances under which socially harmful exclusive contracts may arise.

Matthewson and Winter (1987), for example, showed that a manufacturer may profitably impose exclusivity on a local retailer in order to foreclose a rival in a local market, and that this outcome may be (but need not be) socially harmful (see also Comanor and Frech, 1985, for a related analysis). But this result breaks down if nonlinear pricing is feasible (see the discussion

in BW and O’Brien and Shaffer, 1997).

On the other hand, various articles have shown that exclusivity clauses may facilitate profitable entry deterrence or competitors’ eviction. Their common theme is that inefficient exclusion may occur when some adversely affected parties (a potential entrant, or future customers) are absent at the contracting stage, and thus cannot make the payments to other parties which are necessary for Coasian bargaining to take place and lead to efficiency.

The seminal article in this branch of the literature is Rasmusen et al. (1991, henceforth, RRW), complemented by Segal and Whinston (2000a, henceforth SW) - these articles are the ones most closely related to ours. They show that, if increasing returns make a minimum scale of operation necessary for profitable entry, an incumbent can achieve full exclusion cheaply by exploiting the lack of buyers’ coordination, or by discriminating between buyers. The idea is that even if buyers as a whole lose when entry is deterred, entry deterrence can be profitable because the excluding firm does not need to bribe all buyers into signing an exclusivity agreement. It only needs to have some of them sign exclusive contracts, just enough to deprive the potential entrant of the minimum viable scale. The incumbent can then fully exploit its market power vis-à-vis all potential customers, including those who did not sign an exclusive contract and whose consent was not bought. The entrant’s need for a minimum scale of operation thus generates a contracting externality across customers which the incumbent can exploit\(^3\). In this case,

\(^3\)For a systematic treatment of contracting with externalities, see Segal (1999, 2003)
the Chicago critique breaks down because the excluding firm does not need to compensate the loss suffered by all buyers, but only that suffered by some of them. A coordination failure among buyers may lead to the same result. Variants of RRW consider the possibility of buyers forming coalitions (Innes and Sexton, 1994) competition among buyers (Motta and Fumagalli, 2006; Simpson and Wickelgren, 2004, 2007), and communication by potential entrants (Gerlach, 2007). But all of them stick to the assumption that the potential entrant does not take part to the contracting game.

A related idea can be found in models assuming that some adversely affected parties, other than the excluded firm, cannot take part in the contracting game. These parties can be customers in a future market (Section IV of BW), or agents whom transaction costs deter from participating to the contracting game (Gans and King, 2002). In yet one more category of articles, socially harmful exclusion may result from contracts signed between wholesalers and retailers because final consumers are left aside.

All these models assume that some adversely affected parties are absent from the contracting game. This raises two issues. The first one is theoretical. In most of the aforementioned models, contracts are restricted to

\(^4\) SW stresses that the equilibria in which exclusion is achieved by exploiting the lack of coordination among buyers are not perfectly coalition-proof.

\(^5\) Hart and Tirole (1990); Lin (1990); O’Brien and Shaffer (1993). Simpson and Wickelgren (2004) belongs to this set of articles (because the presence of downstream consumers exacerbates the inefficiency) while at the same time being a variant of RRW (because the excluded firm in their model is a potential entrant unable to participate to the contracting game).
take a rather simple form. For example, RRW and SW only consider simple exclusive contracts, ruling out both breach penalty provisions and conditional contracts. This brings the following question: is the possibility of socially harmful eviction driven by the absence of some adversely affected parties during the contracting game, or by the restrictions imposed on the nature of feasible contracts? Clearly, if these two assumptions were lifted, all affected parties could enter into very complex contracts and unhindered Coasian bargaining would induce an efficient outcome. Also, it is well known that restrictions on the contracting game can prevent participating agents from maximizing their joint surplus, especially in the presence of externalities (it is the case in RRW, see Segal 1999 for a general treatment). One contribution of this article is to check the respective importance of these two assumptions - i.e., restrictions on contracting and the absence of some affected parties - in a context that is close to the one studied in the existing literature on exclusive dealing.

This question brings us to the second issue. The aforementioned literature is at odds with the facts of much of the relevant case law, because in most cases the alleged targets of the exclusionary strategies were already present in the market and could have responded by making counteroffers.

\(^6\)Aghion and Bolton (1987) is an exception. They show that inefficient exclusion may occur if the incumbent can offer exclusive contract together with a breach penalty provision. But this result is driven by uncertainty about the potential entrant’s costs, which causes the incumbent to exclude a more efficient entrant "by mistake", because the level of the breach penalties cannot be tailored to each realization of uncertainty. Also, a section of SW considers breach penalties, but under highly specific assumptions.
This point is made by Whinston (2001, pp. 68-69) who stresses that the standard theories of exclusion cannot apply to the *US v Microsoft* case because the allegedly targeted rivals were already present in the market when the disputed exclusivity clauses were offered. Rasmusen et al. (2004) make the same point when they acknowledge that RRW does not square with the facts of the landmark *Lorain Journal* case. The same remark applies to several recent antitrust cases involving market share discounts (which can be seen as a mild form of exclusivity requirement) like *Michelin II* in Europe\(^7\) and *LePage’s* in the United States\(^8\). In other words, much of the literature is about anticompetitive entry deterrence whereas the case law calls for theories of anticompetitive eviction.\(^9\)

This article is also related to contestability theory. Its proponents (e.g., Baumol et al., 1983) argued that long-term contracts could allow potential entrants to discipline incumbents more effectively than the threat of hit-and-run entry because they are immune to rapid price responses. However, critics such as Schwartz (1986) replied that entry through long-term contracts was unlikely because of the lack of buyer coordination.

**This article’s contribution**

This article disentangles between two possible factors making inefficient

\(^7\)European Commission Decision 2002/405/EC, Michelin, 2002 O.J. (L 143).

\(^8\)3M Co. v. LePage’s Inc., U.S., No. 02-1865, 10/6/03.

\(^9\)As noted above, there exist theories of anticompetitive eviction, like BW, but they rely on the assumption that some other adversely affected party is absent from the contracting game (in BW, this party is future consumers). This assumption is certainly plausible in some cases, but not all.
exclusion possible, namely limits on the nature of possible contracts and limits on the set of agents present at the contracting stage. Section 2 presents a very general, reduced-form model, of which RRW and SW are subcases. Just like in these two articles, exclusive contracts may allow a firm to deny a rival the minimum viable scale. The main results are the following.

- The inefficient eviction of an already present competitor facing increasing returns to scale is possible when manufacturers are restricted to offering simple contracts, even though it is less likely than inefficient entry deterrence.¹⁰ In other words, the targeted firm’s ability make counteroffers may not suffice to allow it to defeat exclusionary strategies (Section 3). The reason for this possible inefficiency is that even though all parties are present at the contracting stage, direct payments between firms or between customers are impossible. When interpreted against the backdrop of the discussions of contestability theory, our findings also imply that entry is even less likely than Schwartz (1986) argued because it may in some cases be deterred despite buyer coordination and entrants’ ability to sign contracts before sinking fixed costs.

- Allowing for more complex contracts, by including breach penalty provisions, makes inefficient eviction impossible and inefficient entry deterrence less likely (Section 4). These results are driven by the fact that

---

¹⁰Throughout the article, expressions such as “A is less likely than B” mean that A is an equilibrium outcome for a smaller set of parameters than that for which B is an equilibrium outcome.
breach penalties allow a customer to become a conduit for transfers between firms. Breach penalties thus increase the scope for transfers between agents, reducing the likelihood of inefficient outcomes.

2 The model

There are two firms, labeled Firm 1 and Firm 2, and \( N \) customers, with \( N \geq 2 \). Firms’ cost structures are different, whereas customers have identical preferences.\(^{11}\)

In order to conduct the analysis at a general enough level, we specify reduced forms rather than detailed preferences and technologies.

**Customer demand**

Customers are assumed to have identical preferences. Each customer’s utility level is equal to \( U_i \) if this customer is served by Firm \( i \) alone, and \( V \) if it is served by both firms.

The assumption that customer preferences are independent deserves a discussion, because in many exclusive dealing cases, buyers are not final consumers but firms selling their output in a downstream market.\(^{12}\) However, even in this latter case, customer preferences need not be strongly interdependent. For instance, demand interdependence is unlikely to be important

\(^{11}\)The assumption that customers have identical preferences is made in RRW as well. RRW also assumes that firms are identical (except for the fact that one is incumbent), but SW relaxes this assumption when discussing breach penalties (in Section IV).

\(^{12}\)Demand interdependence plays a central role in Motta and Fumagalli (2006) and in the literature on slotting allowances (see, e.g., Rey et al., 2011, and the references therein).
if each customer possesses a significant degree of downstream market power because of spatial differentiation (e.g., when customers are retailers, as in LePage’s) or product differentiation (when the products sold upstream are reprocessed by customers and made into differentiated products for sale in the downstream market).\textsuperscript{13}

**Firm profits**

Firm 1’s profit is equal to $\lambda \pi_1 + \mu_1 \pi_1$ where $\lambda$ and $\mu_i$ are, respectively, the number of customers served by both firms and the number of customers served by Firm $i$ only. This is consistent with the assumption that Firm 1’s technology displays constant returns to scale.

Firm 2’s technology is assumed to be characterized by economies of scale because Firm 2 needs to sink a fixed cost $NF > 0$ in order to produce. Accordingly, and with the same notations as above, Firm 2’s profit is equal to $\lambda \pi_2 + \mu_2 \pi_2 - NF$ if $\lambda + \mu_2 > 0$ (i.e., if Firm 2 serves at least one customer), and zero otherwise.

**Assumptions about parameters**

The following assumptions about parameters are made throughout the article for $i=1$ and $i=2$:

$$\hat{\pi}_i > \pi_i \geq 0$$

\textsuperscript{13}In accordance with this claim, we use of the word ”customer” rather than ”consumer” throughout the article (at the cost of unpalatable expressions such as ”customer welfare”) to stress that the model does not only apply to settings where buyers are final consumers.
\[ V > U_i \]  \hfill (2)

\[ \pi_2 - F \geq 0 \]  \hfill (3)

\[ \text{Max}(\pi_2 + V, \tilde{\pi}_2 + U_2) - U_1 < NF. \] \hfill (4)

\[ V + \pi_1 + \pi_2 > U_2 + \tilde{\pi}_2 \] \hfill (5)

These assumptions have the following interpretation. (1) implies that each firm earns greater profits when supplying a customer alone rather than alongside the other firm; (2) implies that customer welfare is greater when each customer is served by both firms than when one firm only is active. (3) means that Firm 2’s profits under duopoly (i.e., when both firms sell to each customer) are enough to cover the fixed cost \( NF \), and (4) implies that Firm 2 cannot recover its fixed cost \( NF \) by serving a single customer, even if it leaves this single customer a utility level of only \( U_1 \) and irrespective of whether it serves this single customer alongside Firm 1 or as an exclusive supplier.\(^\text{14}\) Finally, (5) means that the total surplus of firms and consumers

\(^{14}\text{In order to understand this, notice that if Firm 2 serves a single customer that is not served by Firm 1, then the combined surplus of Firm 2 and this customer is equal to } \tilde{\pi}_2 + U_2. \text{ If Firm 2 serves a single customer that is also served by Firm 1, then the combined surplus of Firm 2 and this customer is equal to } \pi_2 + V. \text{ If Firm 2 serves a single customer, the joint surplus of these two agents is thus at most } \text{Max}(\tilde{\pi}_2 + U_2, \pi_2 + V). \text{ If this single customer is left with utility } U_1, \text{ Firm 2’s profit is at most } \text{Max}(\tilde{\pi}_2 + U_2, \pi_2 + V) - U_1.
is strictly greater when each customer is served by both firms than when Firm 1 is excluded. This assumption is made for the sake of simplicity. Since we are interested in the potential exclusion of Firm 2, we may want to consider whether Firm 2’s exclusion is efficient or not, but in order to avoid too complex results distinguishing between many different cases, we assume that Firm 1’s exclusion would be inefficient.

In the remainder of this article, exclusion will be called strictly inefficient if the following strict inequality holds:

\[ V + \pi_1 + \pi_2 - F > U_1 + \tilde{\pi}_1, \]  

(6)

and strictly efficient if the opposite inequality holds strictly. (6) simply means that the total surplus of firms and consumers is greater when each customer is served by both firms than when Firm 1 only is active.\(^{15}\)

Our model is very general because it relies on reduced forms for demand and firms profits. For example, it allows for the possibility that both firms sell differentiated goods as well as for nonlinear pricing or bargaining\(^ {16}\).

The assumption that Firm 1 faces constant returns to scale whereas Firm

---

\(^{15}\)In the remainder of this article, we occasionally mention the expression ”entry is efficient” as a synonymous for ”exclusion is inefficient”.

\(^{16}\)This reduced form corresponds, for instance, to the case where a consumer can consume at most one unit of each good, production costs are zero (except for Firm 2’s sunk cost), the utility derived from the consumption of good \(i\) alone is \(W_i\) and that from the consumption of both goods is \(W\) with \(W < W_1 + W_2\) (meaning that the goods are substitutes), and each producer bargains with each customer with a level of bargaining power such that the joint surplus from bargaining is split in proportions \(\alpha, 1 - \alpha\). In this case, \(\tilde{\pi}_i = \alpha W_i, \pi_i = \alpha(W - W_j), V = \alpha(W - W_1 - W_2) + (1 - \alpha)W, \) and \(U_i = (1 - \alpha)W_i.\)
2 faces increasing returns might seem questionable for two main reasons. First, our modeling of exclusion in the two-incumbents case, i.e., in the case where both firms can propose contracts to customers at the same time, relies on the assumption that Firm 2 can enter into contracts with potential customers before incurring fixed costs. This assumption has been criticized in the discussion of "contestability theory" in the 1980s on the grounds that in practice, customers might be reluctant to sign a contract with a firm not yet present in a market. However, as was pointed out in Baumol et al. (1983), this assumption is in many cases realistic. For instance, an airline can sell tickets for a given route before actually incurring the fixed cost of committing a plane to that route. In this example, Firm 2 need not be an entrant in the airline industry; it may well be an established airline contemplating entry on a particular route. The second possible criticism is about the extreme asymmetry between the two firms’ cost structures, i.e., the assumption that Firm 1 faces no fixed costs. This assumption is made for the sake of tractability. What matters for our results is only that, given customer preferences and both firms’ cost structures, exiting the market is never an attractive option for Firm 1.

3 Foreclosure with simple exclusive contracts

This section shows that with simple exclusive contracts, inefficient entry deterrence can occur (Proposition 1, which generalizes the main result in
RRW and SW), but also the inefficient eviction of a rival firm, even if both firms are on an equal footing as regards the contracts they can offer and the timing of moves (Proposition 2). Inefficient eviction occurs however for a smaller set of parameters than inefficient entry deterrence, because the ability to make counteroffers may allow a firm to deter customers from signing exclusive contracts proposed by a rival.

**The benchmark: one incumbent and one entrant**

In accordance with RRW and the related literature, we start by considering the following simple game.

**Stage 1.** Firm 1 may offer each customer a contract specifying that (i) the customer commits not to purchase from Firm 2; and (ii) a lump-sum transfer from Firm 1 to the customer. Firm 1 can discriminate among customers.

**Stage 2.** Customers simultaneously decide whether to sign the contracts possibly offered to them in Stage 1. The lump-sum payments corresponding to the contracts which end up being signed are made.

**Stage 3.** Observing the outcome of Stage 2, Firm 2 decides which customers it wants to serve. It cannot decide to serve a customer who signed an exclusive contract in Stage 2. However a firm can serve a customer with

---

17 Throughout the article, the words "incumbent" and "entrant" refer to a firm’s ability, or lack thereof, to offer contracts to customers early on. In contrast to another frequent use of these words, they are not construed here as meaning something about whether firms have already sunk their fixed costs.
whom it signed no contract at the previous stage. Production, transfers and consumption take place in accordance with the contracts possibly signed in Stage 2 and with Firm 2’s decisions.

This setting is a generalization of RRW and SW (with small, unessential modelling changes).

Equilibrium multiplicity is pervasive when exclusive contracts are possible (see, e.g., BW, SW, and O’Brien and Shafer, 1997). Following the existing literature, and in order to rule out situations where exclusion results only from a lack of coordination between customers or firms (a possibility arising for instance in RRW), we restrict our attention to pure-strategy perfect coalition-proof Nash equilibria (PSPCPNE), as defined in Bernheim et al. (1987).18 The reason for this restriction is not that we believe lack of coordination to be an implausible assumption. On the contrary, this perfectly plausible assumption has received a lot of (deserved) attention, so that it is now well-known that inefficient exclusion may result from the failure of buyer coordination. In this article, as the main question is whether exclusion can be an equilibrium outcome in spite of all affected parties participating in the contracting game, we want to focus on assumptions that tend to make exclusion less likely, i.e., we rule out coordination failures.

Proposition 1 shows that even when coordination failures are ruled out,
inefficient entry deterrence can be an equilibrium outcome.\footnote{Throughout this article, the expression "Firm 2 is excluded" (or, equivalently, "entry deterrence is an outcome") means that Firm 2 decides not to incur its fixed cost.}

**Proposition 1** Define $R$ as the smallest integer such that $(N-R)\pi_2 - NF < 0$. If $R(V-U_1) < N(\bar{\pi}_1 - \pi_1)$, i.e., if the joint surplus of Firm 1 and $R$ customers is strictly greater under exclusion than under no-exclusion, then Firm 2 is excluded in any PSCPNE. If the opposite inequality holds strictly, then both firms serve all customers in any PSCPNE.

**Corollary.** Entry deterrence occurs whenever entry is inefficient but also for some parameter values such that entry would be efficient.

Proposition 1 (proved in the appendix) coincides with Proposition 3 in SW if the special assumptions of RRW are made. The proof of this result is straightforward. $R$ is equal to the minimum number of customers with whom Firm 1 needs to sign an exclusive contract so as to deter Firm 2’s entry. (3) implies that $R \geq 1$. Coalition-proofness implies that for Firm 1 to deter entry, it must guarantee a utility level equal to at least $V$ to at least $R$ customers. Indeed, if this were not the case, then a coalition of all customers earning less than $V$ in the hypothetical equilibrium could decide not to sign any contract, which would cause Firm 2 to enter and yield these customers a utility level of $V$. All coalition-proof equilibria of the continuation game would thus be such that all customers left with a utility below $V$ under Firm 1’s exclusive offers would not sign the contracts offered by Firm 1,
which would lead to Firm 2’s entry. Since the provision of a utility of $V$ to a consumer signing an exclusive contract with Firm 1 requires a lump-sum transfer of $V - U_1$, the minimum cost of entry deterrence, for Firm 1, is $R(V - U_1)$. Firm 1’s profit when deterring entry at the lowest cost is thus $N\tilde{\pi}_1 - R(V-U_1)$. Deterring entry is optimal if this is greater than the profit that Firm 1 could earn by offering no contract, i.e., $N\pi_1$.

The intuition behind the result is the same as in RRW and BW. Firm 2’s fixed cost creates an externality across customers, because the signing of an exclusive contract by $R$ of them suffices to deter entry and allows the incumbent to exert its monopoly power vis-à-vis all $N$ customers. This may be profitable even though the incumbent needs to fully compensate the $R$ customers signing an exclusive contract.20

**Foreclosing an already present competitor**

In order to check whether this standard theory of exclusion carries over to the frequent case where the targeted firm is already present in the market, we assume now that both firms are present at the time when contracts can be offered to customers and that both are on an equal footing as regards the ability to offer contracts. To this end, we consider the following game, which is almost the same as the previous one, except for the assumption that both firms can offer contracts in Stage 1.

---

20It is possible for entry to be deterred in equilibrium for a reason independent of the externality across customers, i.e., even if $R = N$. This is the case if $V + \pi_1 < U_1 + \tilde{\pi}_1$, which is equivalent to Firm 2’s duopoly profit being greater than its contribution to social welfare.
Stage 1. Both firms offer as many contracts as they wish (possibly none) to each customer. They can discriminate across customers. A contract mentions a lump-sum transfer and, possibly an exclusivity clause prohibiting the customer from purchasing from any other firm. A firm signing a contract with a customer commits to serving this customer in Stage 3.21

Stage 2. Customers simultaneously decide whether to sign one of the contracts possibly offered to them in Stage 1. The lump-sum payments corresponding to the contracts which end up being signed are made.

Stage 3. Observing the outcome of Stage 2, both firms simultaneously decide which customers they want to serve. A firm cannot serve a customer who signed an exclusive contract with the other firm in Stage 2 (assuming such a contract has been offered and accepted). However a firm can serve a customer with whom it did not sign a contract at the previous stage, and it is obliged to serve customers with whom it is bound by a contract signed in Stage 2.

Proposition 2 (proved in the appendix) establishes the conditions for inefficient exclusion to occur in equilibrium. We start by defining $L$ as the smallest integer such that $(N - L)\max(\pi_2, \hat{\pi}_2 + U_2 - V) + V - U_1 - NF < 0$. Assumptions (3) and (4) imply that $1 \leq R \leq L \leq N - 1$. This definition means that $(N - L)$ is the largest number of customers whom Firm 2 cannot

21The assumption that a firm can commit not to exit by signing a contract with a customer is relaxed below. Relaxing this assumption dramatically changes the results, but the possibility of inefficient eviction in equilibrium remains.
profitably serve if it must guarantee a utility level of $U_1$ to one of them and a utility level of $V$ to the remaining $(N - L - 1)$, while being precluded from serving the other $L$ customers.

**Proposition 2** 1. If exclusion is strictly inefficient and $L(V - U_1) + N(\pi_2 - F) > N(\hat{\pi}_1 - \pi_1)$ then there exists a PSPCPNE in which no firm is excluded, and there exists no PSPCPNE in which either firm is excluded. 2. If exclusion is strictly inefficient and $L(V - U_1) + N(\pi_2 - F) < N(\hat{\pi}_1 - \pi_1)$, then there exists a PSPCPNE in which no firm is excluded, as well as a a PSPCPNE in which Firm 2 is excluded, but there exists no PSPCPNE in which Firm 1 is excluded. 3. If exclusion is strictly efficient, then there exists a PSPCPNE in which Firm 2 is excluded, and no PSPCPNE in which Firm 2 is not excluded.

**Corollary.** The inefficient eviction of an already present competitor is possible in a PSPCPNE, even though (i) it occurs for a set of parameters smaller than that for which inefficient entry deterrence occurs in a PCPNE, and (ii) when eviction would be inefficient, there always exists a PSPCPNE without exclusion.

The intuition behind Proposition 2 is as follows. Since discrimination is allowed, Firm 1 needs only $L$ customers to sign an exclusive contract in order to evict Firm 2. But as Firm 2 can make offers at the same time as Firm 1, it can attempt to bribe some customers so as to deter them from signing exclusive contracts with Firm 1, and it is willing to pay up to its loss from being excluded, i.e., $N(\pi_2 - F)$. In order to lure $L$ customers into an exclusive
contract, Firm 1 thus needs to leave them an aggregate surplus of at least $LV + N(\pi_2 - F)$, i.e., it must pay them a total of $L(V - U_1) + N(\pi_2 - F)$. Exclusion occurs in a PSPCPNE only if this amount, which is the cost that Firm 1 needs to pay to evict Firm 2, is less than the difference between monopoly and duopoly profits, i.e., $N(\tilde{\pi}_1 - \pi_1)$. In other words, the choice for the "coalition" formed by Firm 1 and $L$ customers is between exclusion on the one hand and on the other hand no-exclusion combined with the appropriation of Firm 2’s surplus.\textsuperscript{22} The parties whose combined surplus is maximized are thus the two firms and $L$ customers. This is more than in the incumbent-entrant case (one firm and $R$ customers), which is why exclusion is less likely. But this is less than all agents (($N - L$) customers are left out of the calculation), which is why inefficient exclusion can still be an equilibrium outcome. The underlying reason is that the contractual environment considered in Proposition 2 does not allow for these ($N - L$) customers to bribe the $L$ "privileged" customers so as to induce them not to sign the exclusive contracts proposed by Firm 1.

In order to better understand the logic underlying the above result (and in particular the existence of two very different PSPCPNE), it may be worthwhile to analyze a variant of the above game, in which firms offer contracts sequentially rather than simultaneously (i.e., Stage 1 is subdivided into two periods, during which one firm, and then the other, offer contracts, and cus-

\textsuperscript{22}This description is meant to provide a general intuition but should not be taken as a statement about equilibrium payoffs (see the proof for details).
tomers choose in Stage 2). Proposition 3 (proved in the appendix) shows that whereas this change in assumptions does not alter the results for parameter values such that the above game has one PSPCPNE only, it leads to the selection of a single equilibrium in the case where the above game has two PSPCPNE. Intuitively, in a sequential variant of the above game, the first mover chooses the equilibrium it likes best. If Firm 2 moves first, it can easily fend off exclusion attempts, simply by offering attractive enough contracts that allow it to commit not to exit later. Conversely, if Firm 1 moves first, it can offer very attractive exclusive contracts to $L$ customers, which Firm 2 cannot profitably match (if the inequality $L(V - U_1) + N(\pi_2 - F) < N(\tilde{\pi}_1 - \pi_1)$ holds).

**Proposition 3** Consider the same game as above, except for the assumption that Firms 1 and 2 offer their contracts sequentially rather than simultaneously. The following results hold. 1. The game has at least one PSPCPNE. 2. If exclusion is strictly inefficient and $L(V - U_1) + N(\pi_2 - F) > N(\tilde{\pi}_1 - \pi_1)$ then irrespective of which firm offers contracts first, exclusion does not occur in any PSPCPNE. 2. If exclusion is strictly inefficient and $L(V - U_1) + N(\pi_2 - F) < N(\tilde{\pi}_1 - \pi_1)$, then no firm is excluded in any PSPCPNE in the variant where Firm 2 offers contracts before Firm 1, and Firm 2 is excluded in all PSPCPNE in the variant where Firm 1 offers contracts before Firm 2.

Another reason for considering sequential games is that they are better adapted to considering the case where Firm 2 cannot commit not to exit. In this case (considered below), the simultaneous-move game does not have a pure-strategy equilibrium for all parameter values, whereas each of the two variants of the sequential-move game (depending on which firm moves first) has a subgame-perfect pure strategy equilibrium.

21
2. 3. *If exclusion is strictly efficient, then irrespective of which firm offers contracts first, Firm 2 is excluded in all PSPCPNE.*

**Variant: assuming that firms cannot commit to remain in the market**

The existence of equilibria such that Firm 2 is not excluded, in Propositions 2 and 3, results from the assumption that Firm 2 can commit to remain in the market by signing even a single contract with a customer. This strong assumption is questionable and one may want to assess how the above results are affected if one removes it, assuming instead that in Stage 3 of the above game, Firm 2 may decide to exit and serve no customer (and thus not to pay any fixed cost), even if it signed contracts with some of them in Stage 2. It turns out, however, that if one makes this assumption, the above game has no pure-strategy equilibrium for some parameter values. We thus focus on sequential games (which imply the existence of pure-strategy subgame-perfect equilibria, and therefore of PSPCPNE), considering in turn the case where Firm 1 moves first (Proposition 4) and the case where Firm 2 does (Proposition 5). For technical reasons, we assume that a firm can propose at most one contract per customer.\(^{24}\)

---

\(^{24}\)This assumption is meant to keep the model tractable. If Firm 2 cannot commit to remain in the market and each customer can be offered many contracts, very complex coordination issues arise because, when wondering which contract (if any) it should sign, each customer would have to make conjectures about whether each possible contract choice, combined with other customers’ decisions, will induce Firm 2 to remain in the market or to exit. Assuming that each firm offers at most one contract per customer reduces the resulting complexity.
As Propositions 4 and 5 show, whether socially inefficient exclusion occurs in equilibrium depends on the timing of moves, with the last mover being advantaged: if Firm 2 moves after Firm 1, it can defeat any eviction attempt by offering the "right" contracts to the customers targeted by Firm 1’s "divide and conquer" strategy (Proposition 4). Conversely, if Firm 2 moves first, then for some parameter values, it may be unable to prevent Firm 1 from evicting it.\textsuperscript{25} The contrast between these results and Proposition 3 is discussed below.

**Proposition 4** Consider the same game as above, except for the assumptions that (i) Firm 1 offers contracts before Firm 2, and (ii) Firm 2 can decide to exit even after signing contracts with some of them (in which case the lump-sum transfers provided for in these contracts are refunded). If Firm 2’s exclusion is inefficient, then it does not occur in any PSPCPNE.

At first glance, Proposition 4 may seem surprising, especially in the light of Proposition 3. Proposition 4 means that if signing a contract no longer commits Firm 2 to stay in the market and Firm 2 can offer contracts after Firm 1, it becomes more difficult (in fact, altogether impossible) for Firm 1 to exclude it. This apparent paradox has a simple explanation: if signing a contract with even a single customer commits Firm 2 to remain in the market, a customer expecting Firm 2 to sign a contract with at least one other customer can guarantee itself a utility level $V$ simply by signing no contract with either firm. Under this assumption, all customers (except one, at most)

\textsuperscript{25}This second-mover advantage results from the same mechanism as in "Colonel Blotto" games.
have a utility level equal to or greater than $V$ in any equilibrium without exclusion. This limits Firm 2’s ability to extract value from its customers so as to cover its fixed costs and defeat Firm 1’s eviction attempts. On the contrary, if Firm 2 cannot commit to remain in the market, customers may have an interest in subsidizing it in order to deter it from exiting. This increases Firm 2’s ability to induce its customers to contribute financially to defeat Firm 1’s eviction attempts.\footnote{However, Firm 2’s second-mover advantage is not big enough to allow it to avoid exclusion when exclusion is efficient. For instance, if $\pi_2 + V \geq \tilde{\pi}_2 + U_2$, Firm 1 can earn $\pi_1 + (U_1 + \tilde{\pi}_1) - (V + \pi_1 + \pi_2 - F)$ per customer, by offering each customer an exclusive contract together with a lump-sum transfer $V - U_1 + \pi_2 - F$, thereby inducing Firm 2’s exclusion. If Firm 2’s exclusion is efficient, this is greater than Firm 1’s maximal profit compatible with the absence of exclusion of Firm 2, namely $\pi_1$ per customer. Therefore, under this assumption, Firm 2 is excluded in any sequential equilibrium of the game in which Firm 1 moves first. The same reasoning holds if $\pi_2 + V < \tilde{\pi}_2 + U_2$.}

We address below the opposite timing, i.e., Firm 2 moving before Firm 1. For the sake of simplicity, we restrict our attention to the case where the following conditions hold: we assume that Firm 2’s eviction would be socially inefficient and that the joint surplus of Firm 2 and a single customer is greater when the customer also deals with Firm 1 than when it does not ($\pi_2 + V \geq \tilde{\pi}_2 + U_2$). These restrictions are legitimate because the goal of Proposition 5 is not to solve the game completely, but rather to show that, for some parameter values, inefficient eviction occurs in all PSPCPNE.

**Proposition 5** Consider the same game as above, except for the assumptions that (a) Firm 2 offers contracts before Firm 1, and (b) Firm 2 can decide to exit even after signing contracts with some of them (in which
case the lump-sum transfers provided for in these contracts are refunded).

Then, if (i) exclusion is socially inefficient, (ii) $\pi_2 + V \geq \bar{\pi}_2 + U_2$, (iii) 
$$(V - U_1) \left(1 + \text{Int} \left( N - \frac{NF}{\pi_2+V-U_1} \right) \right) < N(\bar{\pi}_1 - \pi_1) \text{ and}$$
$$(iv) (\pi_2 + V - U_1) \left(1 + \text{Int} \left( N - \frac{NF}{\pi_2} \right) \right) < N(\bar{\pi}_1 - \pi_1),$$
then Firm 2 is excluded in all PSPCPNE.

The explanation for the possibility of inefficient eviction when the targeted firm moves first and cannot commit not to exit is that by moving second, the excluding firm (Firm 1)’s "divide and rule" strategy is immune to the type of targeted counter-offers that are available to the targeted firm when it moves second.

Propositions 4 and 5, compared with Propositions 2 and 3, show that equilibrium outcomes are sensitive to the assumptions made about firms’ ability to commit not to exit the market. However, Proposition 5 also shows that the possibility of inefficient eviction taking place in equilibrium exists irrespective of whether firms are able to commit not to exit.

4 More complex contracts: breach penalty clauses

Sufficiently complex contracts could allow agents to make unrestricted transfers between themselves, leading to the maximization of total surplus. We show in this section that one specific type of "complexification", namely, the
possibility of including breach penalty clauses in exclusive contracts, goes a long way towards ruling out inefficient exclusion. Considering breach penalty clauses makes sense both from an empirical and a theoretical viewpoint. Empirically, such clauses are not infrequent in the real world (although there are in most jurisdictions limits as to the permissible level of these penalties, which we do not consider in this article). Theoretically, as pointed out by Aghion and Bolton (1987), breach penalty clauses may allow an incumbent and a buyer to jointly extract an entrant’s rent, and they are thus an instrument facilitating transfers between agents. We show in this section that, as breach penalties allow a customer to become a conduit for transfers between firms, their availability decreases the likelihood of inefficient outcomes in the entrant-incumbent case (Proposition 6) and makes them impossible in the two-incumbents case (Proposition 7).

**The incumbent-entrant case**

We modify the incumbent-entrant game as follows to allow Firm 1 to include breach penalty clauses in the contracts it offers.

**Stage 1.** Firm 1 may offer each customer a contract specifying that (i) the customer commits not to purchase from Firm 2; (ii) a lump-sum transfer from Firm 1 to the customer; and (iii) possibly a penalty which the customer must pay to Firm 1 if it breaches the exclusivity requirement. Firm 1 can discriminate across customers.
Stage 2. Customers simultaneously decide whether to sign the contracts possibly offered to them in Stage 1.

Stage 3. Observing the outcome of Stage 2, Firm 2 may decide to offer some customers a lump-sum payment in exchange for these customers breaching the exclusive contract signed with Firm 1.

Stage 4. Customers who signed an exclusive contract in Stage 2 leaving them the possibility to breach, and who were offered by Firm 2 to breach it in Stage 3, decide whether to accept Firm 2’s offer. A customer breaching an exclusive contract signed in Stage 2 receives the lump-sum payment proposed by Firm 2 in Stage 3, and pays Firm 1 the penalty provided for as per the breached contract.

Stage 5. Firm 2 decides which customers it wants to serve. It cannot decide to serve a customer who signed an exclusive contract in Stage 2 and did not breach it in Stage 4. Each customer’s utility level and each firm’s profit is then determined according to Firm 2’s choices in Stage 5. Lump-sum payments or breach penalties provided for in the various contracts are added to or subtracted from firms’ profits and customers’ welfare level.

Proposition 6 below shows that if exclusive contracts can include breach penalty provisions, then Firm 2’s exclusion occurs only if the joint surplus of Firm 1, Firm 2, and $R$ customers, is greater under exclusion than under competition.\(^{27}\)

\(^{27}\)Proposition 6 is equivalent to Proposition 5 in SW. Its proof, which follows the same
Proposition 6 If breach penalties are possible, then inefficient entry deterrence occurs in equilibrium if and only if the joint surplus of both firms and R customers is greater under exclusion than under no exclusion: (i) if $RU_1 + N\pi_1 > RV + N\pi_1 + N(\pi_2 - F)$, then Firm 2 is excluded in any PSPCPNE; and (ii) if $RU_1 + N\pi_1 < RV + N\pi_1 + N(\pi_2 - F)$, then both firms serve both customers in any PSPCPNE.

Corollary. When breach penalty clauses can be included in contracts, entry deterrence occurs whenever entry is inefficient as well as for some parameter values such that entry would be strictly efficient. But the set of parameter values for which entry deterrence is an equilibrium outcome is smaller than in the case when breach penalties are not allowed.

The reason why entry deterrence is less likely when breach penalty clauses are possible is that the condition $RU_1 + N\pi_1 > RV + N\pi_1 + N(\pi_2 - F)$ is met for a smaller set of parameter values than the condition for the occurrence of entry deterrence established in Proposition 1 ($RU_1 + N\pi_1 > RV + N\pi_1$). This is not surprising, because the last term in the right-hand side of the former inequality, $N(\pi_2 - F)$, is Firm 2’s profit when it is not excluded, which Firm 1 can appropriate thanks to the use of breach penalties. Breach penalties indeed facilitate the transfer of an entrant’s rent to the incumbent, by making the customers signing a contract and then breaching it a conduit.

28 However, under the special assumptions of RRW, the entrant’s post-entry duopoly profit is zero, so that allowing for breach penalties has no impact on the likelihood of exclusion.

28
for this transfer. This extraction of the entrant’s rent makes entry deterrence less attractive.\textsuperscript{29} According to the same logic as in the two-incumbents case without breach penalties, the ”coalition” formed by the incumbent and $R$ customers can now appropriate the entirety of the entrant’s profit, and exclusion takes place only if it maximizes the joint surplus of both firms and $R$ customers. This result that breach penalties make exclusion less likely may seem at odds with Aghion and Bolton’s (1987) result that breach penalty provisions may cause inefficient exclusion. But there is in fact no contradiction. Their result is entirely driven by informational asymmetries, which sometimes cause the incumbent to exclude by mistake, having set too high a penalty which was not meant to exclude the entrant but rather to appropriate its profit.

Aghion and Bolton’s (1987) result sheds light on the limits of our reasoning: informational asymmetries about costs, which are presumably very frequent, decrease firms’ ability to use breach penalty clauses as a way to induce transfers from potential entrants. But such clauses may nevertheless

\textsuperscript{29}Marx and Shaffer (2004) note that prohibiting below-cost pricing could reduce the feasibility of such rent-shifting. This is because the equilibrium contracts in the rent-shifting, no-exclusion equilibria are such that the incumbent signs an exclusive contract with each customer, involving a low (possibly below-cost) price together with a high breach penalty (which is paid in equilibrium). In our model, prohibiting loss-making sales (assuming that potential breach penalty revenues are not taken into account when assessing whether a sale is loss-making) would make rent-shifting impossible if $\bar{\pi}_1 + U_1 - V < 0$. Prohibiting loss-making sales would also make inefficient exclusion impossible for some parameter values, for instance, in the setup of Proposition 2, if $\bar{\pi}_1 + U_1 - V - \frac{N}{F}(\pi_2 - F) < 0$. This finding also holds in RRW: in that article, the incumbent’s equilibrium strategies in the discriminatory equilibrium (which is the only coalition-proof exclusionary equilibrium) involve loss-making sales to the ”privileged” customers, so as to guarantee them the same utility level as the one they would enjoy absent exclusion.
in some cases decrease the likelihood of inefficient exclusion.

However, the availability of breach penalty provisions does not eliminate the possibility of inefficient entry deterrence, because the condition for entry deterrence to occur, $RU_1 + N\hat{\pi}_1 > RV + N\pi_1 + N(\pi_2 - F)$, may be satisfied together with the condition for such exclusion to be inefficient ($NU_1 + N\hat{\pi}_1 > NV + N\pi_1 + N(\pi_2 - F)$). This is because the condition for exclusion to occur takes into account the surplus of only $R$ customers (the minimum number whose acquiescence to exclusivity Firm 1 must obtain in order to deter Firm 2’s entry), in addition to both firms’ profits, leaving aside $(N - R)$ customers.

**The two-incumbents case**

The question we are interested in is whether this benchmark result carries over to the two-incumbents case, which is addressed below under the same contractual assumptions. Accordingly, we consider the following game.

Stage 1. Both firms choose whether to offer contracts to each customer. A firm can offer a given customer no contract, one contract or several contracts to choose from. A contract may be exclusive or non-exclusive. An exclusive contract between Firm $i$ and a customer may include a breach penalty clause specifying (i) a payment to be made to Firm $i$ should the customer breach the exclusivity requirement while still dealing with Firm $i$; and (ii) a payment to be made to Firm $i$ should the customer breach the exclusivity requirement by not dealing at all with Firm $i$.\textsuperscript{30} Discrimination across customers is allowed.\textsuperscript{30}

\textsuperscript{30}For the sake of tractability, we do not allow for breach penalties in non-exclusive

\textsuperscript{30}
A firm may offer more than one contract to each customer.

Stage 2. Customers simultaneously decide whether to sign one of the contracts possibly offered to them in Stage 1.

Stage 3. Each firm may offer contracts to customers who signed in Stage 2 an exclusive contract with the other firm.\footnote{The assumption that a firm may offer a contract in Stage 3 only to customers who previously signed an exclusive contract with the other firm is made for tractability. The results would not change under more general assumptions (e.g., if both firms were allowed to offer contracts to all customers in Stage 3, irrespective of the actions taken in previous stages).}

Stage 4. Customers simultaneously decide whether to sign the contracts possibly offered to them in Stage 3, taking into account the obligations imposed upon them by the contracts signed in Stage 2.

Stage 5. Observing the outcome of Stages 2 and 4, both firms simultaneously decide which customers they want to serve. A firm is obliged to serve a customer who signed (and did not breach) a contract it proposed, and it cannot serve a customer who signed an exclusive contract with the other firm in Stage 2 unless this contract has been breached. It is free to serve or not to serve customers with whom it is not bound by a contract, and who are not bound by an exclusive contract signed with the other firm.

Proposition 7, proved in the appendix, shows that if both firms make of-
fers simultaneously and contracts may include provisions for breach penalties, then inefficient exclusion cannot occur in a PSPCPNE.

**Proposition 7** If both firms can offer contracts simultaneously and exclusive contracts can include provisions for breach penalties, then (i) there exists PSPCPNE for all parameter values; (ii) if exclusion is strictly inefficient, then it does not occur in all PSPCPNE; and if exclusion is strictly efficient, then it occurs in all PSPCPNE. In addition, both firms earn zero profits in any PSPCPNE.

The reason why both firms earn zero profits if breach penalty provisions are allowed is that these clauses allow Firm $i$ to "force" Firm $j$ to transfer its profits to the "coalition" formed by Firm $i$ and some customers, by offering some customers an exclusive contract together with a breach penalty provision that will cause Firm $j$ to pay a lot so as to induce breach. When both firms can do that, competition results into each firm "forcing" the other one to transfer its entire profits to customers. This mechanism allows Firm 2 to defeat any eviction attempt by Firm 1, because Firm 2 could now offer some customers tempted by Firm 1’s exclusive offer more than its entire no-exclusion profit ($N(\pi_2 - F)$). It could also offer these customers a means (in the form of a breach penalty provision) of extracting Firm 1’s profit, by tailoring the contract so as to leave Firm 1 with no better choice than paying a large amount in exchange for breach. As is proved in the appendix, this logic implies that inefficient exclusion cannot occur in equilibrium - and that both
firms earn zero profits. The real-world implications of this result are probably less clear. Consider for instance the case where Firm 2 needs to sink some R&D or marketing costs before being able to offer contracts to customers. Proposition 4 means that in equilibrium, both firms will earn just enough to cover the costs incurred after signing the contracts. If Firm 1 has no credible way of committing not to "hold up" the entrant ex post, Firm 2 expects that it will not be able to recover the pre-contracting R&D or marketing costs, and it thus deterred from entering. The relevance of Proposition 4 thus depends on the possibility of incurring all fixed costs after signing contracts. It may apply to an already existing airline considering entry into a new route, but probably less to a firm without any foothold in the market in which it contemplates entry.

5 Conclusion

The main result of the article is that if too complex contracts are ruled out, exclusive contracts may cause socially inefficient eviction, and not only entry deterrence, even though the former is less likely than the latter. Such inefficient eviction is all the more likely that (i) the potentially evicted firm faces high fixed costs, (ii) the incumbent can discriminate across customers, and (iii) a large enough share of the surplus brought about by the presence of the potentially evicted firm accrues to customers (rather than to the firm). This result provides a theoretical basis for exclusive contracts being
subject to serious antitrust scrutiny, including in cases where all potentially
affected parties, including the potentially excluded firms, can take part in
the contracting game.\textsuperscript{32}

\textsuperscript{32}Much of the literature, including this article, finds results that could apply equally to
exclusivity clauses and to quantity-forcing clauses. Distinguishing between the two should
be a focus of future research.
6 Appendix

Proof of Proposition 1

First step: if \( RU_1 + N\pi_1 > RV + N\pi_1 \), then exclusion occurs in all PSPCPNE.

We assume that \( RU_1 + N\pi_1 > RV + N\pi_1 \). In a hypothetical no-exclusion equilibrium, Firm 1’s profit would be \( N\pi_1 \). Now consider the following strategy for Firm 1: offer \( R \) customers an exclusive contract against a payment equal to \( V - U_1 + \varepsilon \), where \( \varepsilon \) is strictly positive and small, and no contract to the remaining \((N - R)\) customers. If all \( R \) customers (the "privileged" customers) accept Firm 1’s offer, Firm 2 is deterred from entering and the surplus of each privileged customer is thus \( U_1 + (V - U_1 + \varepsilon) = V + \varepsilon \), which is greater than the utility level \( V \) any of them would earn if it rejected the offer, triggering Firm 2’s entry. Firm 1’s profit is thus \( N\pi_1 - R(V - U_1 + \varepsilon) \), which is strictly greater than \( N\pi_1 \) if \( \varepsilon \) is small enough. Therefore, offering an exclusive contract which is taken up by all privileged customers allows Firm 1 to increase its profit with respect to a hypothetical equilibrium in which Firm 2 is not excluded. This implies that exclusion occurs in equilibrium. More precisely, one can check that the only PSPCPNE of this game is such that Firm 1 offers \( R \) customers an exclusive contract against a payment equal to \( V - U_1 \), which all \( R \) customers accept, thus deterring Firm 2 from entering.

Second step: if \( RU_1 + N\pi_1 < RV + N\pi_1 \), then exclusion does not occur
in any PSPCPNE.

We assume that $RU_1 + N\hat{\pi}_1 < RV + N\pi_1$. By offering no exclusive contract, Firm 1 can obtain a profit equal to $N\pi_1$. We consider now a hypothetical PSPCPNE involving Firm 2’s exclusion. The definition of $R$ implies that in such an equilibrium, at least $R$ customers sign an exclusive contract with Firm 1. Then, we claim that in any such PSPCPNE, the utility level of at least $R$ customers is no smaller than $V$, and that each customer’s utility level is above $U_1$. By signing no contract at all, any customer is certain to be served at least by Firm 1, which yields a surplus equal to $U_1$. This implies that each customer’s equilibrium surplus is greater than or equal to $U_1$. Then, assume that strictly fewer than $R$ customers have an equilibrium utility level below $V$. This means that some customers (hereafter, ”Group A”) accept to sign with Firm 1 an exclusive contract yielding them a utility level below $V$ and that if all such customers refused Firm 1’s offers, fewer than $R$ customers would sign an exclusive contract, so that Firm 2 would in the following stage decide to serve all the customers not bound by an exclusivity clause. This observation implies that the hypothetical equilibrium is not a PSPCPNE, because in the continuation game starting after Firm 1’s offers, there exists another equilibrium in which all customers in Group A reject Firm 1’s offer, thus inducing Firm 2’s entry and ending up with a utility level of $V$, greater than in the originally hypothesized equilibrium. Second, the fact that in any PSPCPNE involving exclusion, the utility level of at least $R$ customers is greater than or equal to $V$, whereas the utility
level of all customers is greater than or equal to $U_1$ implies that in such an equilibrium, Firm 1’s profit is no greater than $N\hat{\pi}_1 - R(V - U_1)$, which is smaller than $N\pi_1$ by assumption. Therefore, Firm 1 could increase its profit by offering no contract at all and earning $N\pi_1$, so that exclusion does not occur in a PSPCPNE.

**Proof of Proposition 2**

**Part 1.** If exclusion is strictly inefficient, then there exists a PSPCPNE in which no firm is excluded.

**Step 1.** Assuming exclusion to be strictly inefficient, we show hereafter that there exists a subgame perfect equilibrium such that (i) the equilibrium of the continuation subgame starting in Stage 2 is coalition-proof, (ii) Firm $i$ (for $i = 1$ and $i = 2$) offers each customer an exclusive contract with a lump-sum transfer $t_{ei} = \hat{\pi}_i - \pi_i + Max[0,(\hat{\pi}_j + U_j) - (\pi_j + V)]$ and a non-exclusive contract with a lump-sum transfer of $t_{ni} = Max[0,(\hat{\pi}_j + U_j) - (\pi_j + V)]$ (with the notation $\{i; j\} = \{1; 2\}$); and (iii) customers choose to accept both firms’ non-exclusive offers.

First, these transfers are such that each customer is better off accepting both non-exclusive offers (which yields a payoff $V + t_{n1} + t_{n2}$) than Firm $i$’s exclusive offer (which yields a payoff $U_i + t_{ei}$), and is indifferent between these two options if $t_{nj} > 0$. Accepting both firms’ non-exclusive offers is thus a weakly dominant strategy and thus defines a PSPCPNE of the continuation game. Second, consider firms’ actions in period 1. If the
contracts offered by Firm $j$ are as described above, the cheapest way for Firm $i$ to induce a customer to be served by both firms in equilibrium involves offering a non-exclusive contract with the smallest positive lump-sum transfer required to make the customer indifferent between accepting both firms’ non-exclusive contracts and accepting Firm $j$’s exclusive contract. This minimal lump-sum transfer is equal to $Max[0; U_j + t_ej - V - t_{nj}] = Max[0; (\hat{\pi}_j + U_j) - (\pi_j + V)] = t_{ni}$. Similarly, the cheapest way for Firm $i$ to induce a customer to purchase from Firm $i$ only in equilibrium involves offering an exclusive contract with the smallest positive lump-sum transfer required to make the customer indifferent between accepting Firm $i$’s exclusive contract, and either of the two contracts offered by Firm $j$. This minimal lump-sum transfer is equal to $Max[0; U_j + t_ej - U_i; V + t_{nj} - U_i] = Max[0; \hat{\pi}_j - \pi_j + Max[0; (\hat{\pi}_i + U_i) - (\pi_i + V)];(\hat{\pi}_j + U_j) - (\pi_j + V)] \geq t_{ei}$. But, as $\hat{\pi}_i - t_{ei} = \pi_i - t_{ni}$, Firm $i$’s maximal profit, given Firm $j$’s offers, can be obtained by offering a non-exclusive contract with a lump-sum payment equal to $t_{ni}$. Firm $i$’s best response to Firm $j$’s actions is thus such that Firm $i$ offers a non-exclusive contract with a lump-sum payment of $t_{ni}$, and an exclusive contract with a lump-sum payment smaller than or equal to $Max[0; U_j + t_ej - U_i; V + t_{nj} - U_i]$. Offering a non-exclusive contract with a lump-sum payment of $t_{ni}$, and an exclusive contract with a lump-sum payment of $t_{ei}$ is thus a best response to Firm $j$’s actions. This proves that the actions described above define a subgame-perfect equilibrium. This also proves that there exists a PSPCPNE involving no exclu-
sion, and that in this PSPCPNE, Firm $i$’s profit is greater than or equal to $N (\text{Min}[\pi_i, (\pi_1 + \pi_2 + V) - (\hat{\pi}_j + U_j)] - F_i)$, which is strictly positive is exclusion is strictly inefficient (with the notation $(F_1, F_2) = (0, F)$). Indeed, in any hypothetical exclusionary subgame perfect equilibrium, one of the firms earns zero, and is thus worse off than in the non-exclusionary equilibrium described above, which implies that there exists at least one non-exclusionary PSPCPNE.

**Step 2.** The equilibrium described above is a PSPCPNE.

Proof. This is equivalent to showing that there exists no subgame-perfect equilibrium such that the continuation subgame starting in period 2 is a PSPCPNE, and such that (i) each customer is served by both firms and (ii) for one firm at least, say firm $i$, the equilibrium profit is strictly greater than $N (\text{Min}[\pi_i, (\pi_1 + \pi_2 + V) - (\hat{\pi}_j + U_j)] - F_i)$. Assume first that $(\hat{\pi}_j + U_j) \leq (\pi_j + V)$ and that Firm $i$’s equilibrium profit is greater than $N(\pi_i - F_i)$. This means that at least one customer signs a non-exclusive contract with Firm $i$ involving a strictly positive payment to Firm $i$, which is impossible because this strategy would be dominated by not signing any contract with Firm $i$. Assume now that $(\hat{\pi}_j + U_j) > (\pi_j + V)$ and that Firm $i$’s equilibrium profit is strictly greater than $N[(\pi_1 + \pi_2 + V) - (\hat{\pi}_j + U_j) - F_i]$. This means that at least one customer receives a lump-sum payment from Firm $i$ which is strictly lower than $(\hat{\pi}_j + U_j) - (\pi_j + V)$. But if this is the case, then faced with Firm $i$’s equilibrium non-exclusive offer and a hypothetical exclusive offer
proposed by Firm $j$ together with a lump-sum transfer equal to $\pi_j - \pi_j - \varepsilon$
for small enough $\varepsilon$, this customer would choose the latter. Therefore, by
offering such an exclusive contract, Firm $j$ could increase its profit by at
least $\varepsilon$ (the reason for this is that in equilibrium the non-exclusive contracts
picked by customers necessarily involve nonnegative transfers from firms to
customers). This contradicts the assumption that both customers are served
by both firms in equilibrium.

**Part 2.** If exclusion is strictly efficient, then there exists no PSPCPNE in
which no firm is excluded.

To prove this, notice that it is shown in Step 2 of Part 1 above that in a
hypothetical PSPCPNE such that no firm is excluded, Firm $i$’s profit cannot
exceed $N (\min[\pi_i, (\pi_1 + \pi_2 + V) - (\hat{\pi}_j + \hat{U}_j)] - F_i)$. If exclusion is strictly
efficient then this expression is strictly negative, implying that at least one
firm loses money in equilibrium, which is impossible because each firm could
decide to sell to no customer and earn zero profits.

**Part 3.** If $L(V - U_1) + N(\pi_2 - F) < N(\hat{\pi}_1 - \pi_1)$ (which is the case whenever
exclusion is strictly efficient, but also for some parameter values such that
exclusion is strictly inefficient) then there also exists a PSPCPNE in which
the inefficient exclusion of Firm 2 occurs.

**Step 1.** We first show that in any PSPCPNE, Firm 1 serves all customers.
Assume that there exists an equilibrium in which one customer at least,
say customer $a$, is not served by Firm 1. In this hypothetical equilibrium, customer $a$ is necessarily served by Firm 2, because otherwise Firm 1 could increase its profit by making an exclusive offer against a lump-sum payment of zero (for instance): customer $a$ would accept this offer (because it would yield a positive utility level, $U_1$) and such an offer, being exclusive, could not affect Firm 2’s decision whether to incur its fixed cost. This means that in the hypothetical equilibrium under consideration, customer $a$ signs an exclusive contract with Firm 2, against some lump-sum payment $t_{e2}$. But the inequality $U_2 + t_{e2}^a \geq \text{Max} \{\hat{\pi}_1 + U_1; \pi_1 + V\}$ must hold, because otherwise Firm 1 could increase its profit by offering customer $a$ an exclusive contract against a lump-sum transfer equal to $U_2 + t_{e2}^a - U_1 + \varepsilon$ (in the case where $\hat{\pi}_1 + U_1 > \pi_1 + V$) or a non-exclusive contract against a lump-sum transfer equal to $U_2 + t_{e2}^a - V + \varepsilon$ (in the case where $\hat{\pi}_1 + U_1 \leq \pi_1 + V$), which would be chosen by customer $a$ and yield Firm 1 a strictly positive profit if $\varepsilon$ is small enough. Therefore, if both firms changed their offers to customer $a$ (while leaving their offers to all other customers unchanged) by offering the contracts described in Part 1 (i.e., Firm $i$ would offer consumer $a$ an exclusive contract with a lump-sum transfer $t_{ei} = \hat{\pi}_i - \pi_i + \text{Max}[0, (\hat{\pi}_j + U_j) - (\pi_j + V)]$ and a non-exclusive contract with a lump-sum transfer of $t_{ni} = \text{Max}[0, (\hat{\pi}_j + U_j) - (\pi_j + V)]$, with the notation $\{i, j\} = \{1, 2\}$), the resulting set of actions would still be a PSPCPNE (because, by doing so, Firm 1 would not affect Firm 2’s willingness to incur the fixed cost $NF$, already incurred in the hypothetical equilibrium), Firm 1’s profit would increase by $\text{Min}\{\pi_1, (\pi_1 + \pi_2 + V) - (\hat{\pi}_2 + U_2)\}$ (which
is strictly positive because of (5) and Firm 2’s profit would increase by at least \((\pi_1 + \pi_2 + V) - (\pi_1 - U_2)\). This means that the equilibrium originally considered is not a PSPCPNE.

**Step 2.** If \(L(V-U_1) + N(\pi_2 - F) < N(\pi_1 - \pi_1)\), then there also exists a PSPCPNE in which the inefficient exclusion of Firm 2 occurs.

**Case 1:** \(\pi_2 + V \geq \pi_2 + U_2\). We show that there exists an equilibrium in which Firm 1 offers \(L\) customers an exclusive contract together with a lump-sum payment equal to \(\frac{N}{L}(\pi_2 - F) + V - U_1\), Firm 2 offers the same \(L\) customers a non-exclusive contract together with a lump-sum payment equal to \(\frac{N}{L}(\pi_2 - F)\), and these customers choose to accept Firm 1’s exclusive contract. Clearly, whatever the other customers’ actions are, each of these \(L\) customers is indifferent between both contracts and accepting either makes him better off than accepting none. Offering less to any of the \(L\) ”privileged” customers would not be in Firm 1’s interest, because doing so would cause at least one of these customers to accept Firm 2’s non-exclusive offer, inducing Firm 2 to serve all the other \((N - L)\) customers in Stage 3, and causing Firm 1’s profit to fall by at least \((N - L)(\pi_1 - \pi_1) + ((\pi_1 - \pi_1 - (\frac{N}{L}(\pi_2 - F) + V - U_1)) > (1 - \frac{1}{L})(N - L)(\pi_1 - \pi_1) \geq 0\). Conversely, Firm 2 cannot avoid being excluded. Offering a more generous non-exclusive contract by increasing by \(\varepsilon > 0\) the lump-sum payment offered to each of the \(L\) ”privileged” customers would cause losses, because the total lump-sum payment would exceed \(N(\pi_2 - F)\). Also, the inequality \(\pi_2 + U_2 < \pi_2 + V\) implies that providing a ”privileged”
customer with a given utility level is more expensive for Firm 2 using an exclusive than a non-exclusive contract. Finally, Firm 2 cannot avoid being excluded by selling only to the \((N - L)\) customers who are offered no contract by Firm 1. In a hypothetical equilibrium of the continuation game in which Firm 2 sells only to such customers, at most one customer gets a utility level strictly below \(V\), because if two or more such customers did, this would mean that they signed a contract with Firm 2 in Stage 2 (otherwise they would get \(V\) in Stage 3) and that any of these customers could increase its utility level by not signing a contract with Firm 2 in Stage 2, because this would not affect Firm 2’s decision to incur its fixed cost, and would thus yield \(V\) in Stage 3. Therefore, in a hypothetical equilibrium of the continuation game in which Firm 2 sells only to the \((N - L)\) customers who are offered no contract by Firm 1, it must leave a utility level of \(V\) to at least \((N - L - 1)\) customers and of \(U_1\) to at most one customer. Firm 2’s profit thus cannot be greater than \((N - L)\pi_2 + V - U_1 - NF < 0\). This proves that the aforementioned actions define an equilibrium, which is also a PSPCPNE of the subgame starting in period 2. Also, Firm 1’s profit in this equilibrium, at \(N\tilde{\pi}_1 - N(\pi_2 - F) - L(V - U_1)\), is greater than its level in the non-exclusive PSPCPNE (where it is equal to \(N\pi_1\) if \(\tilde{\pi}_2 + U_2 < \pi_2 + V\)), which proves the existence of a PSPCPNE in which Firm 2 is excluded.

Case 2: \(\tilde{\pi}_2 + U_2 > \pi_2 + V\). We show that there exists a PSPCPNE in which Firm 1 offers \(L\) customers an exclusive contract together with a lump-sum payment equal to \(\frac{N}{L}(\tilde{\pi}_2 + U_2 - F) - \frac{N-L}{L}V - U_1\), Firm 2 offers the same \(L\)
customers an exclusive contract together with a lump-sum payment equal to 
\[ \frac{N}{L}(\hat{\pi}_2 + U_2 - F) - \frac{N-L}{L}V - U_2, \]
and in equilibrium these \( L \) customers choose to accept Firm 1’s exclusive contract. First, notice that these \( L \) customers are indifferent between Firm 1’s and Firm 2’s offers, and that both yield them a utility level equal to 
\[ V + \frac{N}{L}(\hat{\pi}_2 + U_2 - V - F) > V + \frac{N}{L}(\pi_2 - F) \geq V, \]
i.e., more than the utility they would get by accepting no contract. In order to prove that Firm 2’s strategy is a best response, notice that in order to avoid exclusion at the lowest possible cost, Firm 2 has two possibilities. The first one would be for Firm 2 to serve only the \( (N - L) \) customers not offered an exclusive contract by Firm 1. For the reasons explained in the proof of Case 1, this would require Firm 2 to leave a utility level of \( V \) to at least \( (N - L - 1) \) customers and of \( U_1 \) to at most one customer. Firm 2’s profit thus cannot be greater than \( (N - L)(\hat{\pi}_2 + U_2 - V) + V - U_1 - NF < 0 \) if it chooses such a response. The second possibility is for Firm 2 to try to serve all customers. This requires Firm 2 to offer these \( L \) customers an exclusive contract (this is because the inequality \( \hat{\pi}_2 + U_2 > \pi_2 + V \) implies that exclusion is jointly optimal for any Firm 2 - customer pair) together with a transfer yielding each of them at least the same utility as it would get by accepting Firm 1’s offer, i.e., a transfer equal to 
\[ \frac{N}{L}(\hat{\pi}_2 + U_2 - F) - \frac{N-L}{L}V - U_2. \]
But in any subgame-perfect equilibrium such that the \( L \) ”privileged” customers sign a contract with Firm 2, all the other \( (N - L) \) customers know that they can get a utility of at least \( V \) simply by signing no contract. Thus, the most profitable way for Firm 2 to serve these \( (N - L) \) customers is by
offering each of them an exclusive contract against a transfer of $V - U_2$.

Thus, given Firm 1’s offers, Firm 2 cannot obtain a profit greater than

$$N\pi_2 - L\left(\frac{N}{L}(\tilde{\pi}_2 + U_2 - F) - \frac{N-L}{L}V - U_2\right) - (N - L)(V - U_2) - NF = 0$$

by choosing this second type of response. Neither of the two types of response discussed above would yield a strictly positive profit for Firm 2, given Firm 1’s actions. Firm 2’s strategy as described above is thus a best response. Conversely, Firm 1’s strategy is a best response to Firm 2’s, because the transfer offered to the $L$ ”privileged” customers is the smallest one inducing them not to choose to accept Firm 2’s offer, and not serving one of the $L$ ”privileged” customers would cause Firm 1’s profit to fall by $(N - L)(\tilde{\pi}_1 - \pi_1) + \tilde{\pi}_1 - \left(\frac{N}{L}(\tilde{\pi}_2 + U_2 - F) - \frac{N-L}{L}V - U_1\right) > \pi_1 + (1 - \frac{1}{L})(N - L)(\tilde{\pi}_1 - \pi_1) > 0$. Finally, Firm 1’s profit is greater than that it would earn in the non-exclusive PSPCPNE described in Part 1 of the proof. The difference between the former and the latter is indeed equal to

$$[N\tilde{\pi}_1 - (N(\tilde{\pi}_2 + U_2 - F) - (N - L)V - LU_1)] - N[(\pi_1 + \pi_2 + V) - (\tilde{\pi}_2 + U_2)] = N(\tilde{\pi}_1 - \pi_1) - [L(V-U_1) + N(\pi_2 - F)],$$

which is strictly positive by assumption. This proves the existence of a PSPCPNE in which Firm 2 is excluded.

**Part 4.** If $L(V - U_1) + N(\pi_2 - F) > N(\tilde{\pi}_1 - \pi_1)$, then there exists no PSPCPNE in which the inefficient exclusion of Firm 2 occurs.

Consider a PSPCPNE in which Firm 2 is excluded. It must be the case that Firm 1 signs an exclusive contract with at least $L$ customers, because
otherwise Firm 2 could profitably make offers to the more than \((N - L)\) customers not signing a contract with Firm 1, which would be accepted by them (by leaving one of these customers a utility of \(U_1\) and all other a utility of \(V\), through non-exclusive or exclusive offers, whichever is more profitable depending on the sign of \((\hat{\pi}_2 + U_2) - (\pi_2 + V)\)). Denote as \(W\) the sum of the utility levels of all \(L'\) \((L' \geq L)\) customers who sign an exclusive contract with Firm 1 in the hypothetical PSPCPNE. Firm 2’s exclusion in equilibrium implies that Firm 2 could not earn a nonnegative profit by offering these \(L\) customers contracts that they would prefer to Firm 1’s exclusive contracts while making a profit, i.e., that \(N(\text{Max}(\pi_2, \hat{\pi}_2 + U_2 - V) - F) - (W - L'V) \leq 0\), which can be rewritten \(W > N(\text{Max}(\pi_2, \hat{\pi}_2 + U_2 - V) - F) + L'V\). as the utility level of each of the \((N - L')\) customers who do not sign an exclusive contract with Firm 1 in the hypothetical PSPCPNE is exactly \(U_1\), Firm 1’s profit is equal to \(N\hat{\pi}_1 - (W - L'U_1) < N\hat{\pi}_1 - N(\text{Max}(\pi_2, \hat{\pi}_2 + U_2 - V) - F) - L'(V - U_1) \leq N\hat{\pi}_1 - N(\text{Max}(\pi_2, \hat{\pi}_2 + U_2 - V) - F) - L(V - U_1)\). It must be the case that Firm 1’s profit in a PSPCPNE with Firm 2’s exclusion is greater than Firm 1’s profit in all PSPCPNE without exclusion. But we have shown above that there exists a PSPCPNE without exclusion in which Firm 1’s profit is greater than or equal to \(N\text{Min}[\pi_1, (\pi_1 + \pi_2 + V) - (\hat{\pi}_2 + U_2)] = N[\pi_1 + \pi_2 - \text{Max}(\pi_2, \hat{\pi}_2 + U_2 - V)]\) This condition implies that \(L(V - U_1) + N(\pi_2 - F) \leq N(\hat{\pi}_1 - \pi_1)\), which proves the result.

Proof of Proposition 3.
Case 1: Firm 1 moves first

Notice first that the maximal profit that Firm 1 can obtain without causing Firm 2’s exclusion is $N (\pi_1 - \text{Max}[0, (\hat{\pi}_2 + U_2) - (\pi_2 + V)])$. The proof is the same as in Step 2 of Part 1, and in Part 2 of the proof of Proposition 2 (it carries over to the case of sequential moves).

As proved in Parts 3 and Part 4 of the proof of Proposition 2, the maximal profit that Firm 1 can earn while making exclusive offers that Firm 2 cannot improve upon (thereby causing Firm 2 to exit) is $N \hat{\pi}_1 - N(\text{Max}(\pi_2, \hat{\pi}_2 + U_2) - V) - F) - L(V - U_1)$. Therefore, it is optimal for Firm 1 to offer contracts causing Firm 2 to exit the market if $L(V - U_1) + N(\pi_2 - F) < N(\hat{\pi}_1 - \pi_1)$ (which is necessarily the case if Firm 2’s exclusion is efficient but also for some parameter values such that it is inefficient) and it is optimal not to offer such contracts (and thus not to exclude Firm 2) if the opposite (strict) inequality holds.

Case 2: Firm 2 moves first

Assume first that Firm 2’s exclusion is strictly inefficient and consider Firm 1’s optimal action assuming that Firm 2 offered each customer a non-exclusive contract together with a transfer $t_{n2} = \text{Max}[0, (\hat{\pi}_1 + U_1) - (\pi_1 + V)] + \varepsilon$, with $\varepsilon$ a small positive number. In order to exclude Firm 1, Firm 2 would have to offer each customer an exclusive contract together with a transfer leaving it indifferent between both firms’ offers, i.e., a transfer of at least $V - U_1 + \text{Max}[0, (\hat{\pi}_j + U_j) - (\pi_j + V)] + \varepsilon$. This would leave Firm 1 with
a per-customer profit smaller than or equal to $\pi_1 - V + U_1 - Max[0, (\pi_1 + U_1) - (\pi_1 + V)] + \varepsilon < \pi_1$. Therefore, Firm 1’s optimal action would be to offer no contract and Firm 2 would not be excluded. Offering the abovementioned contracts would leave Firm 2 with a per-customer profit equal to $\pi_2 - F - Max[0, (\pi_1 + U_1) - (\pi_1 + V)] - \varepsilon = Min (\pi_2 + \pi_1 + V - F - \tilde{\pi}_1 - U_1, \pi_2 - F) - \varepsilon$, which is strictly positive if exclusion is strictly inefficient and \( \varepsilon \) is small enough. Therefore, Firm 2 can avoid exclusion while earning strictly positive profits. Firm 2’s exclusion therefore does not occur in any subgame-perfect equilibrium of the game when Firm 2 moves first.

Assume now that Firm 2’s exclusion is strictly efficient. Then the reasoning presented in Step 2 of Part 1, and in Part 2 of the proof of Proposition 2 applies, leading to the result that Firm 2 is excluded in any equilibrium.

**Proof of Proposition 4.**

We assume that Firm 2’s exclusion is inefficient, i.e., $V + \pi_1 + \pi_2 - F > U_1 + \tilde{\pi}_1$.

First, notice that the maximal profit that Firm 1 can obtain without causing Firm 2’s exclusion is $max (\pi_1 - Max[0, (\pi_2 + U_2) - (\pi_2 + V)])$. The proof is the same as in Step 2 of Part 1, and in Part 2 of the proof of Proposition 2 (it carries over to the case of sequential moves, taking into account the assumption that Firm 2 cannot commit to remain in the market).

Then, assume that the game has a PSPCPNE such that Firm 2 is excluded. Let $u^1, ..., u^N$ respectively denote the various customers’ utility lev-
els in this hypothetical equilibrium (with \( u^1 \leq \ldots \leq u^N \)), and let \( W \) denote the sum of all customers’ utility levels (\( W = \sum_{k=1}^{N} u^k \)). Firm 1’s profit is equal to \( N(\hat{\pi}_1 + U_1) - W \). For each \( k \), \( u^k \) is necessarily greater than (or equal to) \( U_1 \), because otherwise customer \( k \) could strictly increase its utility level by signing no contract at all. Also, it cannot be the case that \( u^k < V \) for each \( k \) because if that were the case, then Firm 2 could increase its profit from zero to \( N\pi_2 - F \) by offering no contract at all, which would lead all customers to sign no contract with either firm. That Firm 1 prefers to offer these contracts rather than the non-exclusionary ones yielding a profit of \( N(\pi_1 - Max[0, (\hat{\pi}_2 + U_2) - (\pi_2 + V)]) \) implies that \( W \leq N(\hat{\pi}_1 + U_1 - \pi_1 + Max[0, (\hat{\pi}_2 + U_2) - (\pi_2 + V)]) \)

Assume first that \( \pi_2 + V \geq \hat{\pi}_2 + U_2 \). The inequality \( W \leq N(\hat{\pi}_1 + U_1 - \pi_1 + Max[0, (\hat{\pi}_2 + U_2) - (\pi_2 + V)]) \) implies that \( W \leq N(\hat{\pi}_1 + U_1 - \pi_1 + Max[0, (\hat{\pi}_2 + U_2) - (\pi_2 + V)]) \). Therefore, there exist \( v^1, \ldots, v^N \) such that (i) for each \( k \), \( v^k > u^k \); (ii) for each \( k \), \( v^k \neq V \); and (iii) \( N(\pi_2 + V - F) - \sum_{k=1}^{N} v^k = 0 \).

Consider now the following action by Firm 2, given Firm 1’s offers: Firm 2 offers customer \( k \) (for each \( k \) between 1 and \( N \)) a non-exclusive contract together with a lump-sum transfer \( v^k - \varepsilon \), for \( \varepsilon > 0 \) and very small. If for every \( k \), \( v^k > V \) then obviously there is a single PSPCPNE of the continuation game, such that all customers accept Firm 2’s offers. This is because the inequality \( N(\pi_2 + V - F) - \sum_{k=1}^{N} v^k = 0 \) implies that Firm 2 is induced to remain in the market after all customers accept these offers, and each customer is better off accepting these offers (if all other customers also accept
Firm 2’s offers) rather than accepting Firm 1’s or signing no contract (which yields at most $V$). If for some $k$, $v^k < V$, then the continuation game has a single PSPCPNE such that each customer accepts Firm 2’s offer. The reason is the following: For a customer $k$ such that $v^k > V$, accepting the contract is obviously a dominant strategy. For a customer such that $v^k < V$, accepting the contract if all other customers do ensures that Firm 2 will spend its fixed cost (we assume that Firm 2 spends its fixed cost whenever it is indifferent between spending it and exiting) and thus yields consumer $k$ a utility level $v^k$, whereas rejecting Firm 2’s offer induces Firm 2 to exit and yields a utility level of $U_1$ or $u^k$ (both being strictly smaller than $v^k$). This proves the existence of a PSPCPNE of the continuation game in which Firm 2 stays in the market. Therefore, the exclusion of Firm 2 cannot occur in any PSPCPNE.

**Proof of Proposition 5.**

We assume that $\pi_2 + V \geq \pi_2 + U_2$, $(V - U_1) \left(1 + Int \left( \frac{NF}{\pi_2 + V - U_1} \right) \right) < N(\pi_1 - \pi_1)$ and $(\pi_2 + V - U_1) \left(1 + Int \left( \frac{NF}{\pi_2} \right) \right) < N(\pi_1 - \pi_1)$. We show that whatever Firm 2’s offer in the first stage of the game, Firm 1’s best response is such that Firm 2 exits the market in equilibrium.

We consider a set of hypothetical equilibrium actions by Firm 1. For each customer $k$ who is offered a contract by Firm 2 in Stage 1, let $v^k$ denote this customer’s utility level if it accepts Firm 2’s contract and Firm 2 does not exit the market. Obviously, we can restrict our attention to cases where
$V_1 \leq v^k \leq \pi_2 + V$. Indeed, accepting a contract implying $v^k < V_1$ would be a strictly dominated strategy for customer $k$ because by signing no contract is can obtain either $U_1$ (in case Firm 2 exits the market) or $V > U_1$ (in case Firm 2 does not exit). Similarly, for Firm 2, offering Customer $k$ a contract such that $v^k > \pi_2 + V$ ($\geq \tilde{\pi}_2 + U_2$) necessarily leads to a lower profit than offering Customer $k$ no contract at all.

We define the following two sets, $S_1$ and $S_2$. $S_1$ comprises all customers who are offered a contract by Firm 2 such that this contract, if accepted and if Firm 2 did not exit the market, would yield a utility level strictly smaller than $V$. $S_2$ comprises all customers who are offered no contract by Firm 2, as well as those who are offered one that would yield a utility level equal to or greater than $V$, if Firm 2 did not exit. Let $N_1$ and $N_2$ denote, respectively, the cardinality of $S_1$ and $S_2$ (by construction, $N_1 + N_2 = N$).

**Case 1. $N_2 \pi_2 < NF$.**

Define $K_1 = 1 + \text{Int} \left( N_1 - \frac{NF - N_2 \pi_2}{\pi_2 + V - U_1} \right)$. By construction, $K_1 \leq N_1$. Consider the following strategy for Firm 1: offer exclusive contracts only to $K_1$ members of $S_1$, yielding each of them a utility level of $V$. Consider customer $k$ being one of these $K_1$ customers. He can obtain either $v^k$ ($< V$) by signing the contract offered by Firm 2 if Firm 2 does not exit, or $U_1$ (in case Firm 2 exits and customer $k$ signed no contract with Firm 1), or $V$ (by signing no contract if Firm 2 does not exit, or by signing the contract offered by Firm 1). Therefore, accepting the contracts offered by Firm 1 is a dominant strategy.
for all $K_1$ members of $S_1$ offered such contracts. However, if all $K_1$ members of $S_1$ offered such contracts sign them, then Firm 2’s profit-maximizing action in Stage 3 is to exit the market. This is because the revenues earned from selling to the $(N - K_1)$ remaining customers are not enough to cover the fixed cost $NF$: Firm 2 earns at most $\pi_2 + V - v^k < \pi_2 + V - U_1$ from each of the $(N_1 - K_1)$ members of $S_1$ not offered any contract by Firm 1, and at most $\pi_2$ for each of the $N_2$ members $V - U_1$, leading to total revenues smaller than $(\pi_2 + V - U_1) \left( N_1 - 1 - \text{Int} \left( N_1 - \frac{NF - N_2 \pi_2}{\pi_2 + V - U_1} \right) \right) + N_2 \pi_2 < NF$. Firm 2’s best response to the contracts offered by Firm 1 is thus to exit. Firm 1’s profit is thus equal to $N \pi_1 - K_1 (V - U_1) = N \hat{\pi}_1 - (V - U_1) \left( 1 + \text{Int} \left( N - \frac{NF}{\pi_2 + V - U_1} \right) \right) > N \pi_1$. Therefore, whatever Firm 2’s offers, Firm 1’s best response involves offering contracts that induce Firm 2 to exit.

Case 2. $N_2 \pi_2 \geq NF$.

Define $K_2 = 1 + \text{Int} \left( N_2 - \frac{NF}{\pi_2} \right)$. The inequality $(\pi_2 + V - U_1) \left( 1 + \text{Int} \left( N - \frac{NF}{\pi_2} \right) \right) < N (\hat{\pi}_1 - \pi_1)$ together with the assumption that Firm 2’s eviction is inefficient (i.e., $\pi_2 + V - U_1 > \hat{\pi}_1 - \pi_1$) implies that $K_2 < N_2$.

Consider the following strategy for Firm 1: offer exclusive contracts only to all $N_1$ members of $S_1$, yielding each of them a utility level of $V$, and offer exclusive contracts to $K_2$ members of $S_2$, yielding each of them a utility level of $\pi_2 + V$. For all customers offered these contracts, accepting them is a dominant strategy given Firm 2’s offers. But if all customers offered
such contracts sign them, then Firm 2’s profit-maximizing action in Stage 3 is to exit the market. This is because the revenues earned from selling to the \((N_2 - K_2)\) non-captive members of \(S_2\) are not enough to cover the fixed cost \(NF\) : Firm 2 earns at most \(\pi_2\) from each of them (in the case they do not choose to sign the contract possibly offered to them by Firm 2, which would yield a utility level greater than \(V\)), leading to total revenues smaller than 
\[
\pi_2 \left( N_2 - 1 - \text{Int} \left( N_2 - \frac{NF}{\pi_2} \right) \right) < NF.
\]
Firm 2’s best response to the contracts offered by Firm 1 is thus to exit. Firm 1’s profit is thus equal to

\[
N\hat{\pi}_1 - N_1(V - U_1) - K_2(\pi_2 + V - U_1)
= N\hat{\pi}_1 - N_1(V - U_1) - \left( 1 + \text{Int} \left( N_2 - \frac{NF}{\pi_2} \right) \right)(\pi_2 + V - U_1)
\geq N\hat{\pi}_1 - N_1(\pi_2 + V - U_1) - \left( 1 + \text{Int} \left( N_2 - \frac{NF}{\pi_2} \right) \right)(\pi_2 + V - U_1)
= N\hat{\pi}_1 - \left( 1 + 32 \right)(\pi_2 + V - U_1)
> N\pi_1.
\]

Therefore, whatever Firm 2’s offers, Firm 1’s best response involves offering contracts that induce Firm 2 to exit.

**Proof of Proposition 7.**

**Step 1.** We prove that in any subgame-perfect equilibrium, both firms earn zero profits. The reason is that if Firm \(i\) earned positive profits in a hypothetical equilibrium, the other firm could increase its profit by offering in stage 1 an exclusive contract together with a breach penalty clause having
the effect of inducing Firm $i$ to relinquish some of its profit in Stage 3, to its own profit.

We start by showing that it is impossible in equilibrium for a customer, say customer $a$, to make a strictly positive payment to Firm $i$ even though it is not served by Firm $i$. This could happen only if customer $a$ signed an exclusive contract with Firm $i$ together with a transfer $t$ and a breach penalty $t' > t$, before breaching it and paying the penalty, in order to sign an exclusive contract with the rival firm. But such an outcome cannot be an equilibrium, because in that case Firm $j$ could profitably deviate in Stage 1 by offering customer $a$ an exclusive contract (with no breach possibility), together with a transfer $U_i + t + (t' - t)/2$.

Assume now that Firm $i$ earns a strictly positive profit in equilibrium. This means that there exists a customer, say customer $a$, such that in equilibrium (with $t$ denoting the transfer from Firm $i$ to customer $a$) either (i) Firm $i$ is the only firm serving customer $a$, and $t < \hat{\pi}_i$, or (ii) both firms serve customer $a$, and $t < \pi_i$. Let $U^*_a$ denote customer $a$’s equilibrium surplus level. Firm $j$ could increase its profit by offering customer $a$, in Stage 1, an exclusive contract together with a transfer of $U^*_a - U_j + \varepsilon$ and a penalty for breach equal to $t + V - U_j + 2\varepsilon$ (case (i)), or $t + U_i - U_j + 2\varepsilon$ (case (ii)), with $\varepsilon > 0$ small enough. Such a contract would be accepted by customer $a$ in Stage 2 (because it would provide customer $a$ with a surplus greater than in the original equilibrium). Following this action by Firm $j$, and customer $a$’s subsequent acceptance, Firm $i$ would be better off making an offer that
customer $a$ wants to accept than not making such an offer, because it could for example increase its profit (relative to the situation in which it would not offer a contract inducing customer $a$ to breach Firm 2’s abovementioned exclusive contract) by offering customer $a$ in Period 3 a non-exclusive (case (i)) or an exclusive (case (ii)) contract together with a transfer $t + 2\varepsilon$. The payment of the breach penalty by customer $a$ would increase Firm $j$’s profit by $\varepsilon$, making Firm $j$’s deviation from the hypothetised equilibrium profitable. Notice that Firm $j$’s deviation as described above would not cause it to serve any customer it does not serve in the hypothetized equilibrium, because in the case of a customer it does not serve in that equilibrium, the deviation consists in offering an exclusive contract that is accepted only to be breached in the following stage of the game. Therefore this deviation cannot induce any firm to pay a fixed cost it would not pay in the hypothetized equilibrium.

**Step 2.** In any PSPCPNE, $\sum_{k=1}^{N} U_{k}^* \geq NMax(\pi_1 + \pi_2 - F + V, \pi_1 + U_1)$.

We start with the case in which exclusion is inefficient and we consider the following hypothetical deviation for Firm 2 in Stage 1, relative to a hypothetical PSPCPNE: offer customer $k$ an exclusive contract together with a transfer $U_{k}^* - U_2 + \varepsilon$ (for all $k$ from 1 to $N$), a clause specifying a penalty $\pi_i + V - U_2 - \varepsilon$ for breaching the exclusivity requirement, and an infinite penalty for not dealing with Firm 2 at all. Each customer would be induced to choose this contract offered by Firm 2. The reason is that, by not doing so, a customer ends up with a utility level no greater than $U_{i}^*$. In Stage
3. Firm 1’s best response involves offering each customer a non-exclusive contract together with a lump-sum transfer lying between \( \pi_1 - \varepsilon \) and \( \pi_1 \), leaving Firm 1 with a profit between 0 and \( \varepsilon \) per customer. Firm 2’s profit in any continuation subgame following the deviation would thus be equal to

\[
N(\pi_2 - F) + N(\pi_1 + V - U_2 - \varepsilon) - (\sum_{k=1}^{N} U^*_k - NU_2 + N\varepsilon) = N(\pi_1 + \pi_2 - F + V) - \sum_{k=1}^{N} U^*_k - 2N\varepsilon. 
\]

By definition of an equilibrium, the abovementioned deviation cannot be profitable for any positive \( \varepsilon \), implying that \( \sum_{i=1}^{N} U^*_i \geq N(\pi_1 + \pi_2 - F + V) \).

In the case where exclusion is efficient, we consider the following hypothetical deviation in Stage 1 for Firm 2, relative to a hypothetical PSPCPNE: offer customer \( k \) an exclusive contract together with a transfer \( U^*_k - U_2 + \varepsilon \) (for all \( k \) from 1 to \( N \)) and a clause specifying a penalty \( \hat{\pi}_1 + U_1 - U_2 - \varepsilon \) for breaching the exclusivity requirement. Each customer would be induced to choose this contract offered by Firm \( j \) because it yields a utility level \( U^*_k + \varepsilon \). In Stage 3, Firm 1’s best response involves making each customer an exclusive offer together with a lump-sum transfer lying between \( \hat{\pi}_1 - \varepsilon \) and \( \hat{\pi}_1 - F_1 \). Firm 2’s profit in any continuation subgame following its deviation away from equilibrium would thus be equal to

\[
N(\hat{\pi}_1 + U_1 - U_2 - \varepsilon) - (\sum_{k=1}^{N} U^*_k - NU_2 + N\varepsilon) = N(\hat{\pi}_1 + U_1) - \sum_{k=1}^{N} U^*_k - 2N\varepsilon. 
\]

By definition of an equilibrium, the abovementioned deviation cannot be profitable for any positive \( \varepsilon \), implying that \( \sum_{k=1}^{N} U^*_k \geq N(\hat{\pi}_1 + U_1) \).

**Step 3.** The result proved in Step 2 shows that the combined surplus of all customers in a PSPCPNE is greater than or equal to the total surplus.
in the efficient outcome (i.e., with exclusion taking place if and only if it is efficient). As firm profits are nonnegative in equilibrium, this implies that total surplus is maximized in any PSPCPNE, and thus that exclusion occurs only if it is efficient. Finally, we show below that a PSPCPNE exists.

We first consider the case in which exclusion is inefficient. In that case, the following actions form a PSPCPNE. Both firms offer in Stage 1 the same type of contract: Firm $i$ offers each customer an exclusive contract, together with (i) a transfer $\pi_1 + \pi_2 - F + V - U_i$; (ii) a clause specifying a payment for breach equal to $\pi_j - F_j + V - U_i$ ($i \neq j$), owed to Firm $i$ if the customer, having signed the contract, later decides to drop the exclusivity requirement while still dealing with Firm $i$; and (iii) a clause specifying an infinite payment to Firm $i$ if the customer later decides not to deal with it (with the convention $(F_1, F_2) = (0, F)$). In Stage 2, each customer signs one of the two such contracts offered to him (being indifferent between both). In Stage 3, if Firm $i$’s Stage 1 contract was picked by a customer, then Firm $j$ offers this customer a non-exclusive contract together with a transfer equal to $\pi_j - F_j$, which is accepted in Stage 4, causing the breach penalty to be paid to Firm $i$, as per the contract signed in Stage 2.

If exclusion is efficient, the following actions form a PSPCPNE. Both firms offer in Stage 1 the same type of contract: Firm $i$ offers each customer an exclusive contract, together with a transfer $\hat{\pi}_1 + U_1 - U_i$, and each customer picks Firm 1’s offer.
REFERENCES


