

Anticompetitive Vertical Mergers Waves

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Abstract

We develop an equilibrium model of vertical mergers. We show that, when a wave of mergers removes all upstream firms, the competitive forces on the upstream market may collapse. Indeed, when an integrated firm supplies the upstream market, it internalizes the fact that customers lost on the downstream market can be recovered *via* the upstream market. Thus, the upstream supplier charges higher downstream prices. Its integrated rivals benefit from this behavior, they may therefore not undercut on the upstream market. This mechanism leads to anticompetitive waves of mergers.

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1 Introduction

The anticompetitive effects of vertical mergers have long been a hotly debated issue among economists. Until the end of the 1960s, the traditional vertical foreclosure theory was widely accepted by antitrust practitioners. According to this theory, vertical mergers were harmful to competition, since vertically integrated firms had incentives to raise their rivals' costs. This view was seriously challenged in the 1970s by Chicago school authors (see Posner (1976) and Bork (1978)). In a nutshell, the Chicago school criticism claimed that vertical integration could not kill the competitive pressure on the upstream market. The argument goes as follows: If the upstream market is supplied by some firms, integrated or not, at a high price, then an integrated firm has incentives to charge a slightly lower upstream price in order to capture the upstream market, since this would affect the intensity of downstream competition only marginally. Basically, that means that integrated firms have as much incentives as non-integrated ones to be tough competitors on the upstream market. If this criticism is taken seriously, vertical mergers can no longer be anticompetitive. They can even have pro-competitive effects, arising for instance from the elimination of double marginalization.

More recently, a new research agenda has developed to assess the validity of the two competing theories using modern game-theoretic tools.¹ In a recent article, Chen (2001) has shown that vertical integration does not only change the upstream pricing incentives. It may also affect the downstream pricing incentives. Indeed, when an integrated firm supplies some firms on the downstream market, it internalizes the fact that customers lost on the downstream market can be recovered on the upstream market. Therefore, it has incentives to charge higher downstream prices. We will refer to this effect as the accommodation effect. Chen shows that, if an upstream firm is more efficient than its competitors, and if downstream firms have to pay a fixed cost to switch to another upstream supplier, vertical mergers can have anticompetitive consequences, due to the accommodation effect. This effect will also play a crucial role in our model.

Most of the models in the vertical mergers literature share a common framework. In these models, the competitive pressure on the upstream market is usually higher than on the downstream market. This is modeled by assuming that there are more upstream firms than downstream firms. An additional assumption is then needed to yield equilibrium vertical mergers. For instance, Ordober et al. (1990) have to give a commitment power to the merged firm on the upstream market. Chen (2001) needs both upstream switching costs and efficiency gains. Choi and Yi (2000) assume that integrated firms can choose their inputs specifications. We assume away all these ingredients, which have been shown to lead to anticompetitive vertical mergers. Instead, we slightly alter the common framework by assuming that the competitive pressure is larger on the downstream market. We then show that downstream firms have incentives to integrate forward with pure upstream firms, until there is no more residual demand on the upstream market, so that a non-competitive equilibrium can be implemented.

¹Important contributions include Ordober et al. (1990), Salinger (1988), Hart and Tirole (1990) and Choi and Yi (2000).

More precisely, we assume that there are initially two identical upstream firms and three downstream firms. Firms compete in prices on both markets. The upstream product is homogenous, while downstream goods are differentiated. The only assumption we make on demand functions is strategic complementarity. In the first two stages, the three downstream firms can bid to acquire the upstream firms. In the subsequent stages, upstream and downstream prices are set and quantities are supplied. If no merger has occurred, Bertrand's result applies, and the upstream good is priced at its marginal cost. In the one-merger case, Chen (2001)'s result extends: Non-competitive upstream equilibria cannot exist in the absence of upstream cost differentials.

The novelty of our paper is that non-competitive upstream equilibria are likely to exist in the two-merger subgames. Indeed, assume that the upstream market is supplied by an integrated firm at a price above the marginal cost. As we said before, the upstream supplier has incentives to charge higher downstream prices, in order not to jeopardize its upstream profits. The other integrated firm benefits from this accommodating behavior. As a result, when it decides whether or not to set a lower price on the upstream market, it trades off two effects. On the one hand, undercutting on the upstream market allows it to capture the upstream profits, on the other hand, it loses the accommodation effect. There is no reason to believe that the first effect should always dominate the second one. This departure from the standard Bertrand competition framework implies that the competitive outcome may not be the only equilibrium on the upstream market. It is worth noticing that our point is robust to the Chicago school criticism. In our context, the critics would argue that the integrated firm which does not supply the upstream market unambiguously wants to undercut its integrated rival, since it would then steal all the upstream profits, without affecting the downstream outcome. Yet, with the accommodation effect, it is clear that undercutting does have an important adverse impact on the downstream outcome.

In particular, a monopoly-like outcome, i.e., a situation in which an integrated firm sets its upstream monopoly price, and its integrated rival exits the upstream market can be an equilibrium provided that the accommodation effect is strong enough. We show that these monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria, from the integrated firms' point of view. Besides, these equilibria are the only ones which do not involve weakly dominated strategies, which basically means that integrated firms have no reasons to play another equilibrium strategy. Therefore it seems reasonable to presume that integrated firms will coordinate on a monopoly-like equilibrium in every two-merger subgames. If we use this equilibrium selection criterion, it becomes clear that two-merger outcomes are the only subgame-perfect equilibria. These equilibria are strongly anticompetitive: Both upstream and downstream prices are strictly higher than in the no-merger case. This is the main result of our paper: When the accommodation effect is strong enough, downstream firms bid to acquire both upstream firms, in order to implement a non-competitive equilibrium on the upstream market, which in turn increases downstream prices. In other words, a wave of vertical mergers can occur for purely non-competitive reasons, without involving any form of efficiency gains.

The equilibrium selection criterion may not be fully satisfying. For instance, the upstream equilibrium chosen in the two-merger subgame may be affected by some out-of-the-model considerations. For instance, firms may anticipate that the competition authority would prevent them from playing a non-competitive equilibrium. Downstream firms would then not submit any bid, since there would be nothing to be gained in merging.

We can also obtain some interesting results with the following anticipation scheme. Consider that one of the three downstream firms is a *maverick*, i.e., a tough competitor which would never accept to implement a non-competitive upstream equilibrium if it became integrated. Such an aggressive behavior may come from out-of-the-model considerations. For instance, if a firm is having troubles with the competition authority, it may be reluctant to be part of a non-competitive equilibrium. The maverick would have a lot to lose if its two rival downstream firms became integrated. They would indeed charge a high upstream price, which would raise the maverick's marginal cost. If the expected loss that the maverick would incur exceeds the expected gains of its competitors, then there is a subgame-perfect equilibrium, in which the maverick acquires an upstream firm, while its competitors remain pure downstream. In this case, a vertical merger has no anticompetitive effect; actually the merger even occurs to prevent an anticompetitive mergers wave.

Finally, in order to go further, we specify the demand functions. We use a simple linear demand framework, with a substitutability parameter. First this allows us to exhibit non-competitive upstream equilibria, which proves that, with standard demand functions, the accommodation effect can indeed be strong enough to outweigh the upstream profit effect. Second, this yields interesting comparative static results on the substitutability parameter. Under high product substitutability, when the upstream supplier cuts its downstream price, it steals many customers from the pure downstream firms: The accommodation effect is strong, and non-competitive equilibria emerge on the upstream market. Conversely, when downstream goods are mostly unrelated, the accommodation effect becomes marginal: Only the competitive outcome can arise in equilibrium. In other words, there exists a tension between upstream and downstream competitiveness. A highly (resp. poorly) competitive downstream market implies a poorly (highly) competitive upstream market. This provides us with an important policy implication: Regulators should not allow vertical mergers when downstream market competition is fierce.

The rest of the article is organized as follows. Section 2 describes the model. We solve the upstream price competition subgames in section 3. The main result of the article, namely the existence of anticompetitive vertical mergers wave, is stated in section 4. Section 5 presents the comparative statics results on downstream substitutability. In section 6, we conclude by discussing our assumptions and giving some policy recommendations.

2 The model

Two upstream firms, $U1$ and $U2$, provide a homogeneous input to three downstream firms, $D1$, $D2$ and $D3$, which produce a differentiated good. Firms compete in prices in both markets. The input is produced under constant returns to scale, and the constant marginal cost m is the same for both upstream firms. The downstream technology is also constant returns to scale: for each unit of the final good, one unit of the intermediate input is needed.

The downstream demand addressed to firm Di , $i = 1, 2, 3$, is $q_i(p_1, p_2, p_3)$, where p_j denotes Dj 's price. The demand addressed to a firm is decreasing in its own price, $\frac{\partial q_i}{\partial p_i} < 0$, and increasing in the other firms' prices, $\frac{\partial q_i}{\partial p_j} > 0$, for $i \neq j$ in $\{1, 2, 3\}$. In line with the vertical mergers literature, we also assume that the demands are symmetric. Formally, firm Di 's demand can be written as $q_i = q(p_i, p_{-i})$, where, as usual, p_{-i} denotes the prices of Di 's rivals.

Let π_i denote Di 's profit. In all the configurations studied in this article, we make the following assumptions:

- (i) firms' best responses on the downstream market are unique and defined by first-order conditions $\frac{\partial \pi_i}{\partial p_i} = 0$,
- (ii) there exists a unique Nash equilibrium on the downstream market,
- (iii) as is standard in the literature, prices are strategic complements: for all $j \neq i$ in $\{1, 2, 3\}$, $\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0$.

Assumption (i) together with (iii) implies the classical result that the best response function of a firm is increasing in its rivals' prices. Combining (ii) with (iii), we also get that the unique downstream equilibrium is stable.² Finally, for the sake of clarity we normalize all downstream costs to 0.

We now describe the four-stage game played by the firms. In the first stage, the three downstream firms can bid to acquire upstream firm $U1$. If a merger has occurred, the remaining pure downstream firms can counter it by bidding to integrate forward with $U2$ in stage 2. In the third stage, the firms owning an upstream division announce the prices at which they are ready to supply any pure downstream firm.³ Pure downstream firms then choose from which upstream producer to purchase. Downstream prices are set in the fourth stage. Pure downstream firms are allowed to switch to another upstream supplier at zero cost in stage 4, once downstream prices have been determined.⁴ We assume that they will do so if this strictly increases their profits. To avoid trivial

²See Vives (1999), p. 54.

³The internal transfer price for vertically integrated firms is set equal to the upstream marginal cost.

⁴In Chen (2001)'s model, a positive switching cost was necessary for vertical mergers to have anticompetitive effects. Our zero switching cost assumption means that we are in the least favorable position if we are willing to get non-competitive equilibria.

situations, we also consider that a firm decides to merge if it is strictly preferred. This would obviously be the case whenever mergers involve transaction costs. We look for the subgame-perfect Nash equilibria of this game.

3 Equilibrium analysis

3.1 Upstream-downstream equilibrium with no merger

When no merger has taken place, both upstream firms compete head-to-head to attract downstream competitors. Since downstream firms can switch to another supplier at zero cost, the only credible decision for downstream firms is to choose the cheaper provider in stage 3. In such a framework, the Bertrand result clearly applies, driving upstream prices to the marginal cost m , and implying that both upstream firms make zero profits. The downstream firms' profits are then given by $\pi_i = (p_i - m)q_i$. After downstream prices are determined, by symmetry, the three downstream firms earn the same profit, denoted by π^* .

Proposition 1. *When no merger has taken place, upstream competition yields the Bertrand outcome.*

3.2 Upstream-downstream equilibrium with one merger

Assume that a downstream firm, say $D1$, merged with $U1$ in stage 1, and denote the merged firm by $U1 - D1$. Then it is not clear *a priori* whether or not price competition on the upstream market will drive prices to the marginal cost. Indeed, contrary to the no-merger case, both the identity of the upstream supplier and the price it proposes matter for downstream firms. The behavior of an integrated firm on the downstream market will depend on whether or not it provides the intermediate input to a pure downstream firm.

To make that point, let us show that if both $U1 - D1$ and $U2$ make the same upstream offer $w > m$, then both pure downstream firms strictly prefer purchasing from the vertically integrated competitor. Assume that $D2$ buys from $U2$, and consider the choice of firm $D3$. If it purchases from $U2$ the profits are given by

$$\begin{aligned}\pi_1 &= (p_1 - m)q_1 \\ \pi_2 &= (p_2 - w)q_2 \\ \pi_3 &= (p_3 - w)q_3.\end{aligned}$$

If it purchases from $U1 - D1$ the profits are

$$\begin{aligned}\pi_1 &= (p_1 - m)q_1 + (w - m)q_3 \\ \pi_2 &= (p_2 - w)q_2 \\ \pi_3 &= (p_3 - w)q_3.\end{aligned}$$

The only difference between these two situations is the presence of the term $(w - m)q_3$ in $U1 - D1$'s profit function. Moving from the former to the latter situation, $U1 - D1$'s first-order condition shifts from

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} = 0 \quad (1)$$

to

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} = 0. \quad (2)$$

Then its best response function moves upwards, since $w > m$ and $\frac{\partial q_3}{\partial p_1} > 0$. Intuitively, when $U1 - D1$ supplies $D3$ on the upstream market, it tends to charge higher prices on the downstream market, since it internalizes the fact that any customer lost on the downstream market may be recovered *via* the upstream market. Since prices are strategic complements, its rivals have incentives to raise their prices as well, implying further price increases. At the end of the day, all downstream prices are higher, and in particular those of $D3$'s rivals. This implies that $D3$ is unambiguously better off when it accepts $U1 - D1$'s offer rather than $U2$'s. In a nutshell, choosing the integrated firm as an upstream supplier creates favorable conditions for a less competitive outcome on the downstream market.

As pointed out by Chen (2001), the vertical foreclosure literature has largely emphasized that an integrated firm has incentives to preserve its downstream profit through a soft upstream pricing. The reasoning we have just made shows that the reverse mechanism also exists, namely, an integrated firm has incentives to preserve its upstream profit through a soft downstream pricing. In the following we will refer to this mechanism as the *accommodation effect*.

Obviously the accommodation effect would still be at work if $D2$ were purchasing from $U1 - D1$. Hence, if both upstream firms propose the same upstream price $w > m$, $D3$ prefers strictly purchasing from the integrated competitor. By symmetry we get the same result for $D2$. This discussion is summarized in the following lemma:

Lemma 1. *If $U1 - D1$ and $U2$ announce the same upstream price $w > m$, then both pure downstream firms purchase from the integrated firm.*

We now show that the upstream market cannot be supplied at a price $w > m$ in equilibrium. From now on we use the following terminology: a firm *matches* (*respectively undercuts*) when it sets a price equal to (*resp.* lower than) its rival's price.

First, it is obvious that $U2$ undercuts its rival whenever the upstream price it proposes is higher than the marginal cost. The integrated firm has actually even more incentives to replace its rival on the upstream market. Assume that $U2$ makes an offer $w > m$. Then $U1 - D1$ prefers matching its rival, rather than do nothing and exit the upstream market. If $U1 - D1$ matches, it gets positive upstream profits. It also becomes less aggressive on the downstream market, since it knows that a customer lost on the downstream market can be recovered on the upstream market. At the end of the day, with strategic complementarity, downstream prices increase, making everybody better

off. Using the taxonomy of Fudenberg and Tirole (1984), supplying the downstream market allows the integrated firm to become a “fat cat” on the downstream market, which relaxes downstream competition. To summarize, when the vertically integrated firm matches its rival, it benefits both from a profit-stealing effect and from the accommodation effect, which unambiguously raises its profits. As a result, this firm will never give up the upstream market, as long as the price of its rival is above the marginal cost.

Besides, the upstream market cannot be supplied at a price $w < m$. Clearly, firm $U2$ would never make any offer below its marginal cost, since this would generate losses. The same result holds for the integrated firm, which would make upstream losses and suffer from a negative strategic effect.⁵ We can now conclude this discussion with the following proposition:

Proposition 2. *When exactly one merger has taken place, upstream competition yields the Bertrand outcome.*

When only one merger has occurred, the standard Bertrand logic is clearly at work. Neither of the potential suppliers will abandon the upstream market as long as the price is above the marginal cost. In equilibrium, all firms purchase inputs at the same price m , just as in the no-merger case. Obviously there are no upstream profits to be gained in equilibrium. Since downstream competition is not modified with respect to the previous section, the three downstream firms earn the same profits: $\pi_1 = \pi_2 = \pi_3 = \pi^*$. This is Chen (2001)’s result: when there are no efficiency gains to be realized through a merger (i.e., no upstream cost differentials), a single vertical merger cannot create anticompetitive effects in equilibrium.

3.3 Upstream-downstream equilibrium with two mergers

In this section we assume that two vertical mergers have taken place. To clarify, let us suppose, without loss of generality, that $D1$ has merged with $U1$, and $D2$ has merged with $U2$, giving birth to two integrated firms $U1 - D1$ and $U2 - D2$. Since the two integrated firms are identical, if both of them propose the same upstream price, the remaining pure downstream firm $D3$ is indifferent between purchasing from either of them. We deal with this case using the usual subgame perfection tools. In any other situation, $D3$ purchases from the cheaper provider, since, in the absence of switching costs, it cannot commit to purchasing from the more expensive one. From now on, we assume, without loss of generality, that $D3$ buys inputs from $U1 - D1$ at price w .

⁵Notice that the statement that the integrated firm would never propose an upstream price lower than the marginal cost is not as obvious as it seems. For $U1 - D1$, announcing a price $w < m$ is bad for two reasons. First this would obviously generate upstream losses. Second, it would induce the integrated firm to behave more aggressively on the downstream market in order to minimize its upstream sales. With strategic complementarity, its rivals would lower their prices in turn, adding a negative strategic effect to the trivial upstream profit effect. However, if prices were strategic substitutes, the rivals would rather increase their prices, a behavior which the integrated firm likes. In other words, under strategic substitutability, supplying the upstream market at a very low price would provide a way for the integrated firm to display a “lean-and-hungry look”.

Downstream equilibrium In the following, we assume that the upstream price w is such that all firms supply a positive quantity on the downstream market. Basically, this defines a range of values $]w, \bar{w}[$ for w . The profit functions can be written as:

$$\begin{aligned}\pi_1 &= (p_1 - m)q_1 + (w - m)q_3 \\ \pi_2 &= (p_2 - m)q_2 \\ \pi_3 &= (p_3 - w)q_3\end{aligned}\tag{3}$$

For all w , we define $\{p_i(w)\}_{i=1,2,3}$, the set of equilibrium downstream prices which prevails when the upstream market is supplied by $U1 - D1$ at price w . As assumed before, these prices solve the following set of first-order conditions:

$$\begin{aligned}q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} &= 0 \\ q_2 + (p_2 - m)\frac{\partial q_2}{\partial p_2} &= 0 \\ q_3 + (p_3 - w)\frac{\partial q_3}{\partial p_3} &= 0\end{aligned}\tag{4}$$

We also denote by $\pi_i(w)$ the equilibrium profits when the upstream price is w .

Comparing the first-order conditions of both integrated firms, it seems clear that the upstream supplier has less incentives to cut its price on the upstream market. Indeed, it internalizes the fact that, when it increases its downstream price, some of the customers it loses will purchase from the pure downstream firm $D3$, which increases its upstream revenues. In other words, the accommodation effect is still at work. In equilibrium, the upstream supplier should therefore charge a higher downstream price than its integrated rival. The following lemma summarizes this insight.

Lemma 2. *If $w > m$, then*

$$p_1(w) > p_2(w).$$

As already emphasized in the one-merger case, the identity of the upstream supplier matters for downstream competition. In particular, Lemma 2 makes it clear that the downstream profits made by the integrated firms are likely to differ. We know that the upstream supplier tends to be less aggressive on the downstream market, since it does not want to jeopardize its upstream revenues. The integrated rival clearly benefits from this accommodative behavior. As a result it should earn more downstream profits than the upstream supplier. The following lemma confirms this intuition:

Lemma 3. *If $w > m$, then the upstream supplier earns strictly less downstream profits than its integrated rival.*

Proof. Let $w > m$.

$$\begin{aligned}(p_1(w) - m)q_1(p_1(w), p_2(w), p_3(w)) &< (p_1(w) - m)q_1(p_1(w), p_1(w), p_3(w)) \text{ since } p_1(w) > p_2(w) \\ &= (p_1(w) - m)q_2(p_1(w), p_1(w), p_3(w)) \text{ by symmetry} \\ &< (p_2(w) - m)q_2(p_1(w), p_2(w), p_3(w)) \text{ by revealed preference}\end{aligned}$$

□

An important consequence of this result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market, on the other hand, its integrated rival benefits from larger downstream profits thanks to the accommodation effect. It may well be the case that the latter effect is strong enough to outweigh the upstream profit effect and make $U2 - D2$ earn more total profits than its rival. Having said that, it becomes clear that the usual logic of Bertrand competition may not work here, namely an integrated firm may not always want to undercut its integrated rival on the upstream market. This opens the door to potential supra-competitive outcomes on the upstream market, in which the input would be non-cooperatively priced above its marginal cost.

Upstream equilibrium We now start investigating whether non-competitive outcomes can arise on the upstream market. To make things as clear as possible, let us state clearly what an upstream equilibrium is. First of all, we have to define the strategy space over which integrated firms can set their upstream prices. Basically both integrated firms can make an offer which would allow the pure downstream firm to be active on the downstream market, i.e., $w \in]\underline{w}, \bar{w}[$. They can also choose not to make any offer, which we will denote by $w = +\infty$. If both firms exit the upstream market, they both earn the same profit $\pi_1(+\infty) = \pi_2(+\infty)$.⁶ Therefore, the strategy space for each firm is $S =]\underline{w}, \bar{w}[\cup \{+\infty\}$. Then a pair of upstream offers $w_1 \leq w_2$, $w_1, w_2 \in S$, is part of an equilibrium if, and only if, no integrated firm wants to set another price and be the upstream supplier,

$$\pi_i(w_1) \geq \max_{w < w_j} \pi_1(w), \quad \{i, j\} = \{1, 2\},$$

and the upstream supplier does not want to exit the upstream market and let its integrated rival be the upstream supplier,

$$\pi_1(w_1) \geq \pi_2(w_2).$$

This set of necessary and sufficient conditions may well characterize many equilibria. The following proposition states that the competitive outcome is always an equilibrium.

Proposition 3. *When two mergers have taken place, the Bertrand outcome on the upstream market is an equilibrium.*

As common sense suggests, there always exists an upstream equilibrium in which the input is priced at its marginal cost. This is the intuitive outcome for any economist familiar with price competition in homogenous products. It is worth noting that, if firms coordinate on this equilibrium, their downstream prices and operating profits will

⁶We will not have much to say as to whether the eviction of the pure downstream competitor can arise in equilibrium. This is indeed part of a more general issue, since this would involve to analyze the way downstream demands are modified when a competitor is squeezed. This, of course, is beyond the scope of this article.

be the same as in the no-merger and one-merger cases. As a result, if firms anticipate in stage 1 that the Bertrand outcome will arise on the upstream market, no merger will take place, since there would not be anything to be gained in merging. In the following, we show that other upstream equilibria may exist.

Let us define $w_m = \arg \max_{w \in S} \pi_1(w)$, the monopoly upstream price. w_m is the price that firm $U1 - D1$ would charge if $U2 - D2$ had to exit the upstream market for exogenous reasons. In other words, the input would be priced at w_m if there were no competitive pressure on the upstream market. For the sake of clarity, we assume that this price is unique.⁷ We show in appendix that $w_m > m$. The next proposition introduces a particular type of non-competitive equilibrium.

Proposition 4. *When two mergers have taken place, the pairs of upstream offers $(w_m, +\infty)$ and $(+\infty, w_m)$ are upstream equilibria if, and only if,*

$$\pi_1(w_m) \leq \pi_2(w_m). \quad (5)$$

We call them monopoly-like outcomes. From the integrated firms' point of view, monopoly-like equilibria, when they exist,

- *Pareto-dominate all other equilibria,*
- *are the only equilibria involving no weakly dominated strategies.*

Condition (5) means that the accommodation effect must be large enough to make the upstream supplier earn less profits than the integrated firm which is inactive on the upstream market. When this condition holds, there exists an equilibrium in which one integrated firm supplies the upstream market at the monopoly upstream price, while the other prefers to stay outside the upstream market. In other words, a monopoly-like outcome exactly replicates the virtual situation in which one of the integrated firm has exogenously exited the upstream market. Although both integrated firms are free to make upstream offers, the competitive pressure can totally collapse in equilibrium because of the accommodation effect.

Notice that a particular kind of monopoly-like equilibrium arises when $w_m = +\infty$: the eviction of the pure downstream firm. In that case, the existence condition is necessarily satisfied (with an equality).

It is also worth understanding how our results on the potential anticompetitive effects of vertical mergers differ from Chen's. In Chen's model, vertical mergers can be anticompetitive because the remaining downstream firm prefers buying from the integrated firm at a high price, rather than purchase the input from a pure upstream firm which prices it at its marginal cost. The downstream firm accepts to do so, because it anticipates that the integrated firm will behave less aggressively if it supplies

⁷This assumption implies no loss of generality. Defining w_m as $\max\{\arg \max_{w \in S} \pi_1(w)\}$, our results would still hold if $\pi_1(\cdot)$ reached its maximum for several values of w .

the upstream market. In our model, when two mergers have occurred, there is no pure upstream firm to supply the market. The non-competitive outcome is sustainable, not because the pure downstream firm does not want to buy from a cheaper supplier (actually it could not commit to do so because of the zero switching cost assumption), but rather because nobody wants to undercut.

We would like to emphasize that, although other equilibria exist,⁸ the monopoly-like equilibria are the most likely to arise in the real world. To begin with, when they exist, monopoly-like equilibria Pareto-dominate all other equilibria from the integrated firms' standpoint. As a result, it seems rather likely that they will do their best to coordinate on one of these equilibria. Moreover, all other equilibria involve weakly dominated strategies. It seems highly implausible that a rational player would ever play such a strategy. To see why, let us wonder whether an integrated firm has incentives to play, say, the Bertrand equilibrium. If its integrated rival plays the Bertrand equilibrium, then it is indifferent between charging m or a higher upstream price. However, if its integrated rival offers a price higher than m , it strictly prefers charging w_m rather than m . In words, playing the Bertrand strategy never beats playing w_m .

4 Equilibrium vertical mergers

In this paper, we solve a sequential four-stage game, which can feature multiple equilibria in stage 3. As a result the actions taken in stage 1 will depend on firms' anticipations. Hence we shall not deliver clear-cut predictions on the equilibrium outcome. This, however, should not be seen as a weakness of our model. Indeed, using a simple equilibrium selection criterion, we characterize the situations in which mergers waves are the only equilibria of the game. By contrast, if, one has good reasons not to believe in that criterion, the presence of multiple subgame-perfect equilibria allows us to provide useful discussions on the impact of extra-model features on the selected equilibrium.

4.1 Equilibrium vertical mergers wave

First we establish our main result regarding two-merger equilibria:

Theorem 1. *If $\pi_1(w_m) \leq \pi_2(w_m)$ and integrated firms*

- *do not play weakly dominated strategies on the upstream market*

⁸Due to the high degree of generality of our model, these other equilibria are a bit trickier to characterize than the monopoly-likes. Here is a short list of what we know about them. All other equilibria are symmetric (monopoly-like equilibria are symmetric if and only if $w_m = +\infty$) and feature an upstream price lower than w_m . In particular, the Bertrand outcome is a symmetric equilibrium. Once again, for non-competitive symmetric equilibria to exist, the accommodation effect has to be strong enough to offset the upstream profit effect, so that the upstream supplier earns as much total profits as the integrated firm which does not supply the upstream market. Section 5 exhibits examples of monopoly-like and symmetric equilibria with linear downstream demand functions.

- *or do not play equilibria that are Pareto-dominated by another equilibrium,*

then there are two mergers in equilibrium.

These two-merger equilibria are the novelty of this paper. As we have already emphasized, monopoly-like outcomes are likely to emerge when two mergers have occurred. Therefore, anticipating a non-competitive upstream market after a wave of mergers, downstream firms all bid to get a share of the profits created by the mergers.

This result sheds light on the old debate between the traditional market-foreclosure proponents and the Chicago School. The traditional market-foreclosure theory argues that vertical mergers can occur for non-competitive reasons, since integrated firms have incentives to raise their rivals' costs. The Chicago School criticism replies that this view forgets the competitive pressure on the upstream market. An integrated firm can always gain by undercutting its integrated rivals on the upstream market by a small amount. By doing so, it gets all the upstream profits, while affecting the downstream outcome only marginally. As a result, there can be no non-competitive equilibria on the upstream market, and, of course, vertical mergers cannot have anticompetitive effects.

We show that the Chicago School argument is logically flawed. When upstream competition takes place between vertically integrated structures, undercutting by a small amount on the upstream market does change the downstream outcome a great deal. The tradeoff between the upstream profit effect and the accommodation effect may remove the incentives of an integrated firm to undercut, giving rise to non-competitive equilibria on the upstream market. This implies that mergers can occur for strongly anticompetitive reasons, namely eliminating all pure upstream firms in order to implement a non-competitive upstream outcome.

4.2 Zero and one merger: Tough competition

Although we believe that monopoly-like equilibria are the most plausible outcomes on the upstream market, extra-model considerations may modify that presumption. For instance, if, for some reason, a firm is willing to drive some of its rivals out of business, or if a firm is having troubles with the competition authority, this should have an impact on the anticipated upstream equilibria in some subgames, hence on the number of mergers in equilibrium.

No-merger equilibria Since the Bertrand outcome is always an equilibrium of the upstream market, there always exist an anticipation scheme that leads to an equilibrium with no merger.

Proposition 5. *There is an equilibrium with no merger.*

This proposition stresses the importance of anticipations. Even if non-competitive equilibria exist on the upstream market, it may be that no merger occurs, due to anticipations of tough competition.

One-merger equilibria Most of the models in the vertical mergers literature feature a unique equilibrium with one merger. So it seems rather natural to wonder whether one-merger outcomes can arise in equilibrium in our framework. Actually we cannot exclude that possibility, though we have not been able to find any kind of demand function satisfying sufficient conditions for one-merger equilibria. In the following we give a set of conditions under which equilibria with one merger may exist.

First of all, it is obvious that a one-merger outcome cannot be an equilibrium if there are no non-competitive equilibria on the upstream market. Assume now that non-competitive equilibria exist, and consider the following anticipation scheme: if $D1$ merges with $U1$ and $D2$ merges with $U2$, then a non-competitive equilibrium is implemented ; in any other two-merger subgame, firms anticipate the Bertrand equilibrium to be played. Then, firm $D3$ knows that if both $D1$ and $D2$ win the auction, its cost will be raised, and its profit will shrink. Consequently it is in its interest to avoid this two-merger outcome, by taking part in the first stage or second stage auction. The relevant question is then whether $D3$ is able to win one of these two auctions. To answer this, we need to compare $D3$'s net loss from not merging, with its rival's net gain from merging. In the following we denote by π_i^{NC} the profit of firm Di when the non-competitive equilibrium is implemented on the upstream market. Straightforward calculations show that $D3$ wins the second stage auction if and only if $\pi^* - \pi_3^{NC} \geq \pi_2^{NC} - \pi^*$. If this inequality is satisfied, it is clear that $D1$ does not want to merge in stage 1, since it knows that $D2$ will not be able to acquire $U2$ in the next stage. This gives us a first sufficient condition for the existence of one-merger equilibria:

$$\pi^* - \pi_3^{NC} \leq \pi_2^{NC} - \pi^*. \quad (6)$$

Assume now that condition 6 holds. Then we know that $D3$ cannot prevent $D2$ from winning the auction if a merger did occur in stage 1. Therefore $D1$ has incentives to bid to acquire $U1$; and $D3$ has incentives to bid as high as it can to prevent this acquisition. Making the same surplus comparison as we did before, we show that $D3$ wins the auction if and only if

$$\pi^* - \pi_3^{NC} \geq \pi_1^{NC} - \pi^*, \quad (7)$$

namely, if $D3$ has more to lose from the $D1 - U1$ merger than $D1$ has to win. Therefore, with such an anticipation scheme, the one-merger outcome can be an equilibrium if and only if both conditions 6 and 7 hold. Until now, we have not been able to find any specification of the demand functions satisfying these inequalities. However we have not been able to prove that these conditions never hold either. The best we can do is to consider these one-merger equilibria as a theoretical possibility.

Proposition 6. *There may be an equilibrium with exactly one merger.*

Although this result seems anecdotal, it has an interesting interpretation. In standard antitrust parlance, firm $D3$ is a maverick competitor: it will never accept to implement a non-competitive equilibrium. There can be a lot of good reasons for

assuming such a behavior: for instance $D3$ may have troubles with the competition authority, which may induce it to behave aggressively on the upstream market to prove its attachment to competition; or $D3$ may want to maintain the image of a tough competitor for extra model marketing motivations. We show that if the maverick can be sufficiently harmed by the implementation of a non-competitive upstream outcome, namely if condition 7 holds, it can acquire one of the pure upstream firm, which restores the competitiveness of the upstream market. In other words, the potential maverick becomes an effective maverick, by preventing an anticompetitive wave of mergers. In most of the vertical mergers literature, one-merger equilibria are implemented for anticompetitive reasons, and therefore have anticompetitive effects. On the contrary, in our model, one-merger equilibria exist for competitive reasons, and therefore have pro-competitive effects.

To conclude this discussion on one-merger equilibria, it is worth pointing out that there exists other conditions which guarantee the existence of one-merger equilibria. Basically one can find other anticipations schemes which may lead to a one-merger outcome.⁹ We choose not to discuss them further, since, as for conditions 6 and 7, we have not been able to find a set of demand functions satisfying them, and their practical implications are less intuitive.

5 Example: Linear demands

To give further characterizations of the different upstream equilibria which may arise, we need to specify the downstream demand functions. The following specification, much used in the horizontal merger literature (see Deneckere and Davidson (1985)), allows to compute the different equilibria, without imposing cumbersome calculations:

$$q_i(p_i, p_{-i}) = 1 - p_i - \gamma(p_i - \bar{p}), \quad (8)$$

where $\bar{p} = \frac{p_1 + p_2 + p_3}{3}$ is the average price on the downstream market and $\gamma \geq 0$ is a substitutability parameter. When γ goes to 0, the different products become unrelated, and firms become monopolies. When γ approaches infinity, goods become perfect substitutes and the downstream market becomes competitive. This kind of demand function is convenient when it comes to changing the number of firms on the downstream market, namely it allows to make clear predictions regarding the total foreclosure of pure

⁹For instance, consider the following anticipations. If firm $D1$ wins the first auction, and if $D2$ or $D3$ win the subsequent auction, a non-competitive upstream equilibrium is implemented, with $D1-U1$ being the upstream supplier. In any other two-merger subgame, firms play the Bertrand equilibrium. Notice that $D2$ and $D3$ have a lot to lose if they let $D1$ win the first auction. Indeed, the second stage auction creates two losers: the firm which remains unintegrated loses because it purchases the input at a higher price; the auction winner loses because it has to make too high a bid. As a result they have incentives to prevent the first stage merger. They can do so if and only if the losses they incur following the first merger are higher than the gains captured by firm $D1$, or, formally, if condition 7 holds. Notice that with such an anticipation scheme, condition 6 does not have to be satisfied.

downstream firms. Basically, when some firm, say $D3$, is squeezed from the downstream market, we just need to redefine the downstream average price: $\bar{p} = \frac{p_1 + p_2}{2}$.

For the sake of simplicity, we normalize the upstream costs to 0. Since the assumptions on demand and profit functions we made in section 2 are trivially satisfied, we can apply our previous results to solve the model in this special case. We know from Propositions 1 and 2 that the input is priced at its marginal cost in the no-merger and one-merger cases. Profits are then given by $\pi_i = (p_i - m)q_i$ for all i . Solving the set of first-order conditions, we get the equilibrium downstream prices $p_i = \frac{3}{6+2\gamma}$ as well as the equilibrium profits $\pi_i = \frac{3(3+2\gamma)}{4(3+\gamma)^2}$.

The following lemma exhibits the different upstream equilibria in the two-merger case:

Lemma 4. *Consider the linear demand case. If $\gamma > \underline{\gamma} \simeq 40.9736$, there exist exactly four upstream equilibria:*

- *two monopoly-like equilibria, $(w_m, +\infty)$ and $(+\infty, w_m)$,*
- *one symmetric equilibrium, (w_s, w_s) with $w_s > m$,*
- *the Bertrand-like equilibrium (m, m) .*

Else, the Bertrand-like outcome is the only upstream equilibrium.

Several things are worth noting here. First of all, the total eviction of the pure downstream competitor never arises in equilibrium. If some integrated firm makes no upstream offer, its rival prefers setting $w = w_m$ and achieve a monopoly-like outcome rather than do nothing. This decision trades off several effects: on the one hand, supplying the pure downstream firm introduces a new competitor on the downstream market, which may steal customers from both integrated firms; on the other hand it allows the upstream supplier to capture some upstream profits and to relax downstream competition through the accommodation effect. The lemma above states that the latter effects always dominate the former, for any value of the substitutability parameter.

Besides, this linear demand example provides useful insights on the relationship between downstream differentiation and non-competitive outcomes on the upstream market. Surprisingly, lemma 4 tells us that a highly competitive downstream market (high γ) implies a poorly competitive upstream market, and vice versa. The intuition is the following: when an integrated firm decides whether it should undercut its rival on the upstream market or not, it balances the upstream profits to be gained with the accommodation effect to be given up. If the accommodation effect is strong, namely, if the upstream supplier has to behave very smoothly on the downstream market in order not to hurt its upstream profits, an integrated firm would probably choose not to undercut. This happens in particular if the degree of horizontal differentiation on the downstream market is low, meaning that the upstream supplier has to raise its prices a lot so that the pure downstream firm can survive.

As a corollary, and assuming that integrated firms do not play weakly dominated strategies, we get:

Proposition 7. *Consider the linear demand case. A wave of mergers occurs if, and only if, $\gamma \geq \underline{\gamma}$.*

6 Discussion and concluding remarks

This final section discusses some of our assumptions and gives a few policy recommendations.

Strategic interactions We have assumed that downstream prices are strategic complements, in line with the vertical mergers literature. This is however not a crucial assumption to obtain our accommodation effect, which is the key mechanism leading to anticompetitive mergers waves. On the contrary, we argue that strategic substitute prices would be even more favorable to the existence of monopoly-like equilibria. Let us informally explain why, by considering the downstream competition stage when two mergers have occurred and the upstream market is supplied at a non-competitive price. We have seen that the upstream supplier has incentives to raise its downstream price to improve its upstream profit. The subsequent behavior of its integrated rival depends on the nature of the strategic interaction. If prices are complements, the integrated rival best-responds by raising its downstream price, which reduces the gap between equilibrium downstream prices and weakens the accommodation effect. By contrast, if prices are substitutes, the integrated rival lowers its price, which enlarges the gap between equilibrium downstream prices and strengthens the accommodation effect.

Quantity competition The accommodation effect requires that the upstream supplier can raise the demand addressed to the pure downstream firm through its downstream pricing. One may then wonder whether this effect hinges on the assumption of price competition on the downstream market. Indeed, if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the accommodation effect is still at work. To summarize, the question is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can affect its rivals' quantities.

Two-part tariffs We have considered that upstream offer could only include a variable part. Let us now show that the existence of non-competitive equilibria on the upstream market with two vertically integrated firms extends to the case of two-part upstream tariffs (w, F). The equilibrium upstream variable part maximizes the joint profit

of the upstream supplier and the downstream firm, i.e., $w_{tp} = \arg \max_w \pi_1(w) + \pi_d(w)$.¹⁰ Besides, strategic complementarity implies $w_{tp} > m$. The fixed fee F must be such that, first, the integrated firm which does not supply the upstream market does not want to undercut, and, second, the remaining downstream firm is willing to accept that tariff. If the former constraint binds, then $F = \pi_2(w_{tp}) - \pi_1(w_{tp}) > 0$ and both integrated firms earn $\pi_2(w_{tp}) > \pi^*$. If the latter constraint binds, then $F = \pi_3(w_{tp})$ and $\pi_2(w_{tp}) > \pi_1(w_{tp}) + F > \pi^*$. This implies that a wave of vertical mergers always creates additional profits for the merging firms. The possibility for two-part tariffs on the upstream market softens even more the competitive pressure. This result is reminiscent of Bonanno and Vickers (1988).

Competition policy Our paper provides a new reason why vertical mergers can occur for anticompetitive motives. The goal of the merging firms is to remove all pure upstream firms. Indeed, this strengthens the vertical interactions, which softens the competitive forces on the upstream market. Contrary to the standard foreclosure theory, downstream behavior is used to raise upstream profits, not the opposite.

The main policy recommendation of our paper is the following. The upstream market remains competitive as long as there are pure upstream firms. Therefore, when there is a single pure upstream firm, one should not allow it to merge with a downstream or an integrated firm. In that sense, vertical mergers waves harm consumers. One can prevent such an outcome by forbidding the last merger of the wave, namely, the one that removes the last pure upstream firm.

The illustration we provide in section 5 provides another message to competition authorities. There is a tension between upstream and downstream competitiveness, as the same forces that strengthen the latter soften the former. Therefore, one should not systematically favor head-to-head competition on the downstream market, since it might pave the road for non-competitive outcomes on the upstream market in the case of a vertical mergers wave.

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A Appendix

To ease the proofs of all lemmas and proposition, we begin by stating the following technical lemma.

Lemma 0. *Consider our three-player game. If the best response function of at least one firm shifts upwards, then all equilibrium downstream prices increase strictly.*

Proof. Assume that firm i 's best response shifts upwards. This happens if, and only if, the first derivative of its profit with respect to its price shifts upwards. For all j , let us denote by $\pi_j^{(0)}(\cdot)$ (respectively $\pi_j^{(1)}(\cdot)$) the profit of firm j before (resp. after) the marginal profit shift. The game $(\mathbb{R}; \pi_j^{(k)}(\cdot), k = 0, 1; j = 1, 2, 3)$ is strictly supermodular. For all j , $\pi_j^{(k)}(p_j, p_{-j})$ has increasing differences in (p_j, k) , and $\pi_i^{(k)}(p_i, p_{-i})$ has strictly increasing differences in (p_i, k) . Besides we assumed that every configuration analyzed in this paper yields a unique downstream equilibrium. Supermodularity theory (see Vives (1999), p35) tells us that this equilibrium is strictly increasing in k . \square

Proof of Lemma 1. We have already seen that the best response function of firms $D3$ and $U1 - D1$ shift upwards when $D3$ chooses $U1 - D1$ rather than $U2$ on the upstream market. Using Lemma 0, we deduce that all downstream prices are higher when $D3$ chooses $U1 - D1$. All we need to do now is prove that this price hike actually makes $D3$ better off. Assume that $D2$ purchases input from one of the upstream firms, and

denote with superscript 1 (*respectively* 2) the outcome variables when firm $U1 - D1$ (*resp.* $U2$) supplies $D3$ at price w . Then,

$$\begin{aligned}
\pi_3^{(2)} &= (p_3^{(2)} - w)q_3(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \\
&< (p_3^{(2)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(2)}) \text{ since } p_i^{(1)} > p_i^{(2)} \text{ for all } i \\
&< (p_3^{(1)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \text{ by revealed preference} \\
&= \pi_3^{(1)}.
\end{aligned}$$

This implies that $D3$'s dominant strategy is to purchase from $U1 - D1$. By symmetry, this also holds for $D2$. \square

Proof of Proposition 2. Let us restate the above discussion in a more formal (and rigorous) way.

If $U1 - D1$ supplies the market at $w > m$, then $U2$ clearly wants to undercut. Assume now that $U2$ supplies the market at $w > m$, and let us show that $U1 - D1$ would be made strictly better off by matching. If it matches, we know from Lemma 1 that both downstream firms will purchase from it. Thus its first-order condition on the downstream market shifts from $q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} = 0$ to $q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\left(\frac{\partial q_2}{\partial p_1} + \frac{\partial q_3}{\partial p_1}\right) = 0$. In other words its best response function shifts upwards, whereas all other best responses remain unaffected. We can now use Lemma 0, which states that all downstream prices are higher when $U1 - D1$ matches. In the following sequence of inequalities, we denote with superscript M (*respectively* \emptyset) the outcome variables when firm $U1 - D1$ matches (*resp.* does nothing):

$$\begin{aligned}
\pi_1^{(\emptyset)} &= p_1^{(\emptyset)} q_1(p_1^{(\emptyset)}, p_2^{(\emptyset)}, p_3^{(\emptyset)}) \\
&< p_1^{(\emptyset)} q_1(p_1^{(\emptyset)}, p_2^{(M)}, p_3^{(M)}) \text{ using Lemma 0} \\
&< p_1^{(\emptyset)} q_1(p_1^{(\emptyset)}, p_2^{(M)}, p_3^{(M)}) + (w - m)\left(q_2(p_1^{(\emptyset)}, p_2^{(M)}, p_3^{(M)}) + q_3(p_1^{(\emptyset)}, p_2^{(M)}, p_3^{(M)})\right) \\
&\quad \text{since } w > m \\
&< p_1^{(M)} q_1(p_1^{(M)}, p_2^{(M)}, p_3^{(M)}) + (w - m)\left(q_2(p_1^{(M)}, p_2^{(M)}, p_3^{(M)}) + q_3(p_1^{(M)}, p_2^{(M)}, p_3^{(M)})\right) \\
&\quad \text{by revealed preference} \\
&= \pi_1^{(M)},
\end{aligned}$$

thus firm $U1 - D1$ clearly prefers matching rather than do nothing.

We now prove that the upstream market cannot be supplied at a price below the marginal cost. It is obvious that, if $U2$ supplied the market at $w < m$, it would deviate and set a higher price. Assume now that $U1 - D1$ is the upstream supplier at $w < m$, and let us show that it strictly prefers exiting the upstream market. Let w' the price of its upstream rival. Obviously $w' > w$. When $U1 - D1$ exits the upstream market, the best responses of the three firms shift upwards; this can be easily seen by checking out the first-order conditions, recalling that $w' > w$. Using Lemma 0, we deduce that all downstream prices go up when the integrated firm exits the upstream market. In

the following sequence of inequality, we denote with superscript 1 (*respectively* 2) the outcome variables when the upstream market is supplied by $U1 - D1$ at w (*resp.* by $U2$ at w'):

$$\begin{aligned}
\pi_1^{(1)} &= p_1^{(1)} q_1(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + (w - m) \left(q_2(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \right) \\
&< p_1^{(1)} q_1(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \text{ since } w < m \\
&< p_1^{(1)} q_1(p_1^{(1)}, p_2^{(2)}, p_3^{(2)}) \text{ by Lemma 0} \\
&< p_1^{(2)} q_1(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \text{ by revealed preference} \\
&= \pi_1^{(2)}.
\end{aligned}$$

Thus $U1 - D1$ prefers exiting the upstream market rather than supplying it at $w < m$. \square

Proof of Lemma 2. Define $B_1(p_2, p_3, w)$ (*respectively* $B_2(p_1, p_3, w)$) firm $U1 - D1$ (*resp.* $U2 - D2$)'s best response when the upstream market is supplied by $U1 - D1$ at price w . Strategic complementarity implies that these best response functions are increasing. Differentiating the first two equations of system (4), we easily show that, B_1 is increasing in w , and B_2 does not depend on w . Besides, when $w = m$, the upstream profit term disappears, and $B_1(., ., m) = B_2(., ., m)$.

Let $w > m$ and assume, by contradiction, that $p_1(w) \leq p_2(w)$. Then,

$$\begin{aligned}
p_1(w) &= B_1(p_2(w), p_3(w), w) \text{ by definition} \\
&> B_1(p_2(w), p_3(w), m) \text{ since } w > m \\
&= B_2(p_2(w), p_3(w), m) \text{ since } B_1(., ., m) = B_2(., ., m) \\
&= B_2(p_2(w), p_3(w), w) \text{ since } B_2(., ., w) = B_2(., ., m) \\
&\geq B_2(p_1(w), p_3(w), w) \text{ since } p_1(w) \leq p_2(w), \text{ and by strategic complementarity} \\
&= p_2(w),
\end{aligned}$$

which is a contradiction. \square

Proof of Proposition 3. Assume that both integrated firms set the same upstream price $w = m$.

Obviously, an upward deviation would not be strictly profitable, since $\pi_1(m) = \pi_2(m)$.

Let us now analyze the consequences of a downward deviation. To do so, we need to prove that the equilibrium downstream prices are increasing in w . Let $w < w'$. Looking at the first order conditions on the downstream market, it is clear that the best responses of the upstream supplier and the pure downstream firm shift upwards when the upstream price rises from w to w' . Using Lemma 0, we deduce that $p_i(w') > p_i(w)$ for all i .

Assume now that some upstream firm deviates from the Bertrand-like outcome by setting an upstream price $w < m$. This clearly makes this firm worse off, since the

prices of its competitors decrease, and it makes upstream losses (by a standard revealed preference argument). \square

Proof of $w_m > m$. In the proof of Proposition 3, we have shown that $\pi_1(w) < \pi_1(m)$ for $w < m$, which implies that $w_m \geq m$. Moreover, taking the first derivative of $\pi_1(\cdot)$ for $w = m$, we get, using the envelope theorem:

$$\frac{d\pi_1}{dw}\Big|_{w=m} = (p_1 - m) \left(\frac{dp_2}{dw} \frac{\partial q_1}{\partial p_2} + \frac{dp_3}{dw} \frac{\partial q_1}{\partial p_3} \right) + q_3.$$

This derivative is strictly positive, since we know from the proof of Proposition 3 that the downstream prices are increasing in w . \square

Proof of Proposition 4. Assume that $\pi_1(w_m) \leq \pi_2(w_m)$ and let us show that $(w_m, +\infty)$ is an equilibrium. Clearly, firm $U1-D1$ does not want to set another price, by definition of w_m . In addition, since $\pi_1(w_m) \leq \pi_2(w_m)$, and again by definition of w_m , firm $U2-D2$ does not want to undercut or match its rival.

If $\pi_1(w_m) > \pi_2(w_m)$, then monopoly-like outcomes cannot be equilibria, since the integrated firm which does not supply the upstream market would rather undercut its rival.

To show that monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria, we proceed in two steps. First we show that $\pi_2(\cdot)$ is upward-sloping ; second, we prove that asymmetric equilibria are necessarily monopoly-like outcomes.

Taking the first derivative of $\pi_2(\cdot)$ w.r.t. w , and using the envelope theorem, we get:

$$\frac{d\pi_2}{dw} = (p_2 - m) \left(\frac{\partial q_2}{\partial p_1} \frac{dp_1}{dw} + \frac{\partial q_2}{\partial p_3} \frac{dp_3}{dw} \right),$$

which is strictly positive, since downstream prices are increasing in w (see the proof of Proposition 3).

Let $w_1 \neq w_m$ and $w_2 > w_1$, and assume, by contradiction, that (w_1, w_2) is an asymmetric equilibrium. Then $w_2 \leq w_m$, otherwise the upstream supplier would rather deviate and set $w_1 = w_m$. If $\pi_1(w_1) > \pi_2(w_1)$, firm $U2-D2$ has a strictly profitable deviation: setting $w = w_1 - \epsilon$. If $\pi_1(w_1) \leq \pi_2(w_1)$, then $\pi_1(w_1) < \pi_2(w_2)$ (since $\pi_2(\cdot)$ is upward-sloping, and $w_1 < w_2$) and firm $U1-D1$ has a strictly profitable deviation: setting $w = w_2 + \epsilon$. In both cases we get a contradiction. This proves that, if an equilibrium is asymmetric, it has to be a monopoly-like outcome.

To conclude this proof, assume that $(w_m, +\infty)$ is an equilibrium. Then we know that all other equilibria (except $(+\infty, w_m)$) are symmetric. Let (w, w) a symmetric equilibrium. Obviously, $\pi_1(w) = \pi_2(w)$, otherwise both firms would rather undercut or exit the upstream market. Then we have, by definition of w_m , $\pi_2(w) = \pi_1(w) < \pi_1(w_m) \leq \pi_2(w_m)$, which proves that the asymmetric equilibria Pareto-dominate all other equilibria. \square

Proof of Theorem 1. We first show that when two mergers have taken place, other upstream equilibria than the monopoly-like involve weakly dominated strategies. We have shown in the proof of proposition 4 that these other equilibria are of the form (w_*, w_*) , with $m \leq w_* < w_m$ and $\pi_1(w_*) = \pi_2(w_*)$. Let us show that offering $w_i = w_m$ strictly dominates offering $w_i = w_*$ for integrated firm i . If the integrated rival offers $w_j \leq w_*$, then both strategy are equivalent. If $w_* < w_j < w_m$, then offering w_m yields a payoff $\pi_2(w_j)$, which is larger than the payoff when offering w_* , $\pi_2(w_*)$, because $\pi_2(\cdot)$ is increasing. If $w_j > w_m$, then offering w_m yields a payoff $\pi_1(w_m)$, which is larger than the payoff when offering w_* , $\pi_1(w_*)$, by definition of w_m . If $w_j = w_m$, the former two cases shows that it also strictly better to offer w_m than w_* .

It remains to show that when two mergers implies a monopoly-like outcome on the upstream market, then the first two stages yields two mergers. Consider the following anticipation scheme: in any two-merger subgame, the merged firms implement equilibrium $(w_m, +\infty)$; besides, the firm merging with $U1$ sets w_m and supplies the upstream market while the firm merging with $U2$ exits the upstream market. We use the same notations as in the previous section, i.e., we denote by $\pi_1(w_m)$, $\pi_2(w_m)$ and $\pi_3(w_m)$, the profits of the upstream supplier, the integrated rival and the pure downstream firm respectively. We solve the game backward to determine the actions taken during the first two stages.

If some firm, say $D1$, won the auction in stage 1, then at least one firm wants to take part in the second stage auction. Indeed, if some firm submits no bid, its rival can capture the surplus $\pi_2(w_m) - \pi^* > 0$ by making a bid $\epsilon > 0$. Then, we show that both $D2$ and $D3$ bid $B_2 = \pi_2(w_m) - \pi_3(w_m)$ in stage 2. First we need to check that B_2 is strictly positive. This can be done by proving that $p_2(w_m) < p_3(w_m)$ using the same reasoning as in the proof of Lemma 2; then, in line with the proof of Lemma 3, we deduce that $\pi_2(w_m) > \pi_3(w_m)$, i.e. $B_2 > 0$. If any of the remaining pure downstream firms makes a bid B strictly lower than B_2 , it is in its rival's interest to bid $B + \epsilon$, with ϵ small enough, and earn a net payoff $\pi_2(w_m) - B - \epsilon > \pi_3(w_m)$. By contrast, if no merger occurred in the first stage, no downstream firm can increase its net payoff by merging with $U2$. To summarize, if some firm won the auction in stage 1, the remaining downstream firms submit positive bids B_2 ; if no firm has taken part in the stage 1 auction, no firm takes part in the subsequent auction either.

Using the reasoning we made for stage 2, we deduce that the three downstream firms bid $B_1 = \pi_1(w_m) - \pi_3(w_m)$ in stage 1, provided that $B_1 > 0$. Otherwise, one of the three downstream firms offers a small $\epsilon > 0$ and wins the first auction. We have just exhibited a set of two-merger subgame perfect equilibria. □

Proof of Lemma 4. First, using Proposition 3, we know that the Bertrand-like outcome is always an equilibrium.

We now investigate whether other equilibria exist. Let us work out the situation in which pure downstream $D3$ is totally evicted from the downstream market, namely, $w_1 = w_2 = +\infty$. Then, for $i = 1, 2$, $\pi_i = (p_i - m)q_i$. In equilibrium, downstream prices

are $p_i = \frac{2}{4+\gamma}$, and both integrated firms earn $\pi_i(+\infty) = \frac{2(2+\gamma)}{(4+\gamma)^2}$.

Assuming now, without loss of generality, that the upstream market is supplied by firm $U1 - D1$ at price w , the profit functions are given by expression 3. Solving the set of first-order conditions, we get the unique downstream equilibrium:

$$\begin{aligned} p_1 &= \frac{18+\gamma(15+w(9+5\gamma))}{2(3+\gamma)(6+5\gamma)}, \\ p_2 &= \frac{3(6+\gamma(5+w+w\gamma))}{2(3+\gamma)(6+5\gamma)}, \\ p_3 &= \frac{3(6+5\gamma)+w(18+7\gamma(3+\gamma))}{2(3+\gamma)(6+5\gamma)}. \end{aligned}$$

Plugging these expression into the profit functions, we get:

$$\begin{aligned} \pi_1(w) &= \frac{3(3+\gamma)(6+5\gamma)^2+6w(1+\gamma)(6+5\gamma)(18+\gamma(18+5\gamma))-w^2(1+\gamma)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}{4(3+\gamma)^2(6+5\gamma)^2}, \\ \pi_2(w) &= \frac{3(3+2\gamma)(6+\gamma(5+w+w\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2}, \\ \pi_3(w) &= \frac{3(3+2\gamma)(6+5\gamma-w(1+\gamma)(6+\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2}. \end{aligned}$$

$\pi_1(\cdot)$ is quadratic and concave. It reaches its maximum value $\frac{3(6+5\gamma)(54+23\gamma(3+\gamma))}{(4648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}$, for $w = \frac{3(6+5\gamma)(18+\gamma(18+5\gamma))}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4}$. Straightforward computations show that this maximum is always strictly larger than $\pi_1(+\infty)$, meaning that the total eviction of the pure downstream firm never arises in equilibrium. This gives us the value of w_m :

$$w_m = \frac{3(6+5\gamma)(18+\gamma(18+5\gamma))}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4}.$$

To be able to apply Proposition 4, we need to compare $\pi_1(w_m)$ and $\pi_2(w_m)$. After some algebra, we get that $\pi_1(w_m) - \pi_2(w_m)$ has the same sign as $\mathbb{P}(\gamma) = 648 + 1296\gamma + 864\gamma^2 + 183\gamma^3 - 5\gamma^4$. \mathbb{P} is polynomial in γ , goes to ∞ as γ goes to $+\infty$, and has only one positive root: $\underline{\gamma} \simeq 40.9736$. As a result, $\pi_1(w_m) \leq \pi_2(w_m)$ if and only if $\gamma \geq \underline{\gamma}$. We can then apply Proposition 4 to deduce that two monopoly-like equilibria exist if and only $\gamma \geq \underline{\gamma}$.

We still have to find the symmetric equilibria. To do so, we solve the equation $\pi_1(w) = \pi_2(w)$ in w . This equation has exactly two solutions: $w = 0$ (i.e., the Bertrand-like outcome) and $w = w_s = \frac{9(2+\gamma)(6+5\gamma)}{108+180\gamma+93\gamma^2+13\gamma^3}$. (w_s, w_s) is an equilibrium if and only if none of the integrated firms wants to undercut its rival on the upstream market. Since $\pi_1(\cdot)$ is quadratic and concave in w , all we need to do is check whether $w_s \leq \arg \max_w \pi_1(w) = w_m$. Straightforward computations show that $w_s \leq w_m$ if and only if $\gamma \geq \underline{\gamma}$. \square