STATUS, MARKET POWER, AND VEBLEN EFFECTS*

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Abstract. We analyze a duopoly model of imperfect competition where firms sell conspicuous goods to vertically-differentiated consumers. These consumers care about both the intrinsic quality of the good they purchase as well as the social status conveyed by this good (namely, the social inference of their hidden type based on their purchase). Firms offer non-linear price and quality schedules that, in effect, screen consumers using a combination of two commonly observed instruments: large markups (or "Veblen effects") and upward-distortions in quality. We show that, in equilibrium, firms use an elaborate combination of these two instruments. Our work differs from previous literature in that Veblen effects and quality distortions simultaneously arise, and it also provides a setting in which their interaction can be analyzed. Finally, we study corrective taxation and find that, contrary to informal proposals, high end luxury goods need not be taxed.

1. Introduction

In markets for conspicuous goods two phenomena are frequently observed. The first is an overinvestment in quality, whereby consumers purchase goods with costly features that are hard to justify based on intrinsic value alone (see, e.g., Bagwell and Bernheim, 1996, Becker, Murphy, and Glaeser 2000, and Frank, 1985, 1999). Some simple and popular examples include sport cars capable of surpassing 300% of the speed limit, SUVs equipped for off-road combat conditions, and luxury wristwatches that are water-resistant beyond the scuba-diving world record.

The second phenomenon is the presence of large markups vis-a-vis marginal costs, which are commonly called "Veblen effects." Although it is typically difficult to obtain precise empirical estimates for markups due to the hidden nature of marginal costs, it is hardly controversial that successful luxury brands command significant premia. One need only compare Tiffany's diamond rings against chemically and physically equivalent versions at BlueNile.com, Hermes's scarves and ties against alternative brands that employ equally fine silk. Or merely ponder the prices of

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Armani cotton T-shirts and Von Dutch "trucker" hats worn by movie stars and their fans. For the case of cars, Figure 1 presents markup estimates by Feenstra and Levinsohn (1995) across all major brands and models in 1987. In this cross section, markups are on average sharply increasing in the price of the car, and reach their highest levels (around 50%) for well-known luxury brands.

What is puzzling about these large markups is that there exists a large selection of brands and luxury items that a status-seeking consumer can choose from. For example, one does not really need a Rolex and a Porsche in order to signal one's wealth, a collection of Zegna business suits and a Viking kitchen could be equally effective. In fact, Bagwell and Bernheim (1996) and Becker et al. (2000) show that in the presence of perfect competition across luxury brands, Veblen effects do not arise at all for status–seeking consumers whose preferences satisfy the standard single-crossing property. Rather, the effect of such status-seeking behavior is translated entirely into inefficiently high quality levels.

Of course, markups can be readily explained if one assumes a monopolistic supplier of conspicuous goods. However, in addition to being unrealistic for several markets of interest, this assumption would lead to an *under*-investment in quality for the standard reason that the monopolist wishes to extract information rents from the wealthiest consumers (see Mussa and Rosen, 1978, for the classical analysis, and Rayo, 2005, for a simple extension to the case in which consumers seek status in addition to intrinsic quality).

In this paper, we study a duopoly model of imperfect competition and show that both excessive quality and Veblen effects arise simultaneously. These two instruments serve as substitute strategies that allow firms to screen their consumers and the forces of imperfect competition, combined with the restrictions imposed by incentive compatibility, determine the equilibrium mix of the two.

Our setup is a hybrid between the model of Rochet and Stole (2002), where two firms located at the extremes of a Hotelling space offer nonlinear price–quality menus that target vertically differentiated consumers, and the models of Bagwell and Bernheim (1996) and Becker et al. (2000), where vertically differentiated consumers seek higher social status through purchases of conspicuous goods. We extend the work of Rochet and Stole by assuming that consumers care about the social status they enjoy when consuming a particular good in addition to their conventional interest in the intrinsic value of this good. We also extend the competitive models of Bagwell and Bernheim and Becker et al. by adding horizontal differentiation among firms.

Our setup can also be considered an extension of the monopolistic non-linear pricing models of Mussa and Rosen (1978), Maskin and Riley (1984), and Rayo (2005), with the important differences that consumers now care about social status and there is a degree of competition between firms.

Our main result concerns the equilibrium mix of the two screening instruments described above. Although both quality distortions and Veblen effects are widespread in our model, they are not employed equally at different points along the vertical spectrum. Quality distortions are maximal in the middle of the spectrum, whereas towards the high end of this spectrum, in stark contrast to the competitive case, quality distortions wane and may disappear altogether. Consequently, at the high end, Veblen effects become very large and serve as the primary screening device.

As is customary in environments in which consumers jockey for status (which, by nature, is in fixed supply), the equilibrium allocations fail to maximize aggregate surplus. The resulting inefficiencies have lead a number of authors to propose corrective taxation. In our model, the inefficiency corresponds to the upward-distortion in quality, and a corrective tax can theoretically remedy this concern. However, we show that the optimal tax schedule is by no means simple, or even monotonic, since quality distortions are non-monotonic across the vertical spectrum. In particular, since larger Veblen effects substitute for quality distortions as a screening device, they also serve as a substitute for corrective taxation. Accordingly, the need for taxation vanishes toward the high end of the spectrum where Veblen effects are intensively used by the firms.

In Sections 2-5 we present and analyze our model absent government intervention. In section 6 we study corrective taxation. Section 7 concludes. All proofs are contained in Appendix 1.

2. **Model**

Consider a unit mass of consumers who wish to purchase conspicuous goods. Each consumer is characterized by two types: a "vertical" type $\theta \in [\theta_L, \theta_H]$ (e.g., her wealth), and a "horizontal" type $x \in [0,1]$ (e.g., her tastes). Each type is private information and is independently distributed. Let $f(\theta)$ and $g(x)$ represent the marginal densities of θ and x, respectively, so that the pair (θ, x) has joint density $f(\theta)g(x)$. We assume f and g are strictly positive and smooth, and that g is symmetric around the horizontal midpoint $x = \frac{1}{2}$. Also, let $F(\theta)$ and $G(x)$ denote the associated cumulative distribution functions.

There are two firms offering conspicuous goods and each consumer will buy at most one unit of these goods. Denote each firm by $i \in \{L, R\}$. Each firm can offer multiple goods, and these goods differ according to their price dimension $p \in \mathbb{R}$ and their intrinsic quality dimension $q \in \mathbb{R}_+$. The marginal cost of producing each unit of a good with quality q is given by $c(q)$, which is increasing, differentiable, and convex in q. We also assume that the function $c(\cdot)$ is the same for both firms and $c(0) = 0$. Following standard practice in the literature on non–linear pricing, we abstract away from fixed costs.

In addition to being intrinsically valuable, conspicuous goods deliver a level of social status $s(p, q, i) \in \mathbb{R}$, which can depend on the price p and quality q of the good, as well as the identity i of the firm that supplies it. The details of how the function $s(p, q, i)$ is determined are presented below.

Consumers obtain the following utility when purchasing a unit of the conspicuous good:

$$\theta \cdot v(q, s) - p - T(x). \tag{1}$$

The term $\theta \cdot v(q, s)$ represents gross utility, which is determined by the consumer's vertical type θ, the intrinsic quality q of the good, and the status level s associated with this good. We assume that the function v is non-negative and smooth with $v_q > 0$, $v_{qq} \leq 0$, $v_s \geq 0$, and $v_{qs} \geq 0$. The term $T(x)$, on the other hand, represents a transportation cost. This cost equals $t \cdot x$ if the consumer purchased the good from firm L, and equals $t \cdot (1 - x)$ if she purchased from firm R, where $t \in \mathbb{R}_+$ is an exogenous parameter. If a consumer does not purchase a good, we assume that she obtains a reservation utility of zero, but this specific assumption will play no role in our results.

There are three stages. In the first, each firm i simultaneously offers a menu of goods $\langle p_i(\theta), q_i(\theta)\rangle_{\theta \in [\theta_L, \theta_H]}$, where the price-quality pair $p_i(\theta), q_i(\theta)$ targets consumers with vertical type θ.[1] Using the the revelation principle, we can restrict attention to menus that satisfy a standard incentive compatibility constraint, namely,

$$\theta \cdot v(q_i(\theta), s_i(\theta)) - p_i(\theta) \geq \theta \cdot v(q_i(\theta'), s_i(\theta')) - p_i(\theta') \text{ for all } \theta, \theta', \tag{IC}$$

[1]Given the additive nature of the transportation cost $T(x)$, there is no loss in conditioning the menu on the vertical type only.

where $s_i(\theta)$ and $s_i(\theta')$ denote, respectively, the levels of status associated with the price-quality pairs $p_i(\theta), q_i(\theta)$ and $p_i(\theta'), q_i(\theta')$.[2]

In the second stage, consumers simultaneously decide whether or not to purchase a good and, if so, they select their most preferred item among the two available menus. Finally, in the third stage, after purchases are made, social status is determined and payoffs are realized.

Firms obtain a payoff equal to their monetary profits. Let $\Omega_i(\theta) \equiv p_i(\theta) - c(q_i(\theta))$ denote the markup firm i obtains from each consumer with vertical type θ, and let $D_i(\theta)$ denote the fraction of consumers of type θ that purchase from this firm. Accordingly, firm i's profits are given by

$$\Pi_i \equiv \int_{\theta_L}^{\theta_H} \Omega_i(\theta) D_i(\theta) dF(\theta). \tag{2}$$

We say that a pair of menus $\langle p_i(\theta), q_i(\theta) \rangle_{\theta \in [\theta_L, \theta_H]}$, for $i = L, R$, constitutes an equilibrium if, given that consumers behave optimally, and given the rule for allocating status (described below), no firm can unilaterally benefit from selecting an alternative menu.

Throughout, we focus on symmetric equilibria with full market coverage in which both firms follow the same strategy, consumers are split 50-50 between firms (a consequence of the symmetric distribution of the horizontal type x), and each consumer purchases a single unit from one of the firms.

2.1. **Social Status.** In its most general form, we assume that the status function $s(p, q, i)$ is determined as follows. Let $\mathcal{C} \in [\theta_L, \theta_H] \times [0, 1]$ denote a generic subset of consumers, and let $\varphi(\mathcal{C})$ denote an arbitrary exogenous function that maps $\mathcal{C}$ into $\mathbb{R}$. In addition, let $\mathcal{C}^*(p, q, i)$ denote the subset of consumers who, in equilibrium, purchase a good with price p and quality q from firm i. Given φ, we assume that

$$s(p, q, i) = \varphi(\mathcal{C}^*(p, q, i)).$$

In other words, the status level conveyed by a particular good is an arbitrary function of the subset of consumers who, in equilibrium, purchase this particular good.

A concrete formulation of interest, which we employ throughout the paper, is when status corresponds to the Bayesian posterior belief of a consumers' vertical

[2]As in Rochet and Stole (2002), given the linearity of the transportation cost, the firms' menus cannot screen along the horizontal type x.

type θ (e.g., her expected wealth) based on the specific conspicuous good she purchased. Formally, this case is represented by setting

$$\varphi(\mathcal{C}) = E\left[\theta \mid \mathcal{C}\right] \equiv \frac{\int_\theta \int_x \theta \cdot \mathbf{1}_{\{(\theta,x)\in\mathcal{C}\}} dF(x) dG(\theta)}{\int_\theta \int_x \mathbf{1}_{\{(\theta,x)\in\mathcal{C}\}} dF(x) dG(\theta)},$$

where $\mathbf{1}_{\{(\theta,x)\in\mathcal{C}\}}$ is an indicator function specifying whether consumer (θ, x) is in $\mathcal{C}$.

A simple special case of this formulation arises when, in equilibrium, any two consumers that differ in their vertical type purchase goods that differ in either p, q, or i. In this case, θ is fully revealed in equilibrium, and every consumer enjoys a status level equal to her true vertical type.

Finally, when characterizing an equilibrium, we must also specify how beliefs are determined off the path of play. We address this point below when focusing on particular classes of equilibria.

3. **Separating Equilibria**

We begin our analysis by studying "separating equilibria" in which firms offer menus $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ such that the price-quality pair $p_i(\theta)$, $q_i(\theta)$ always differs across θ. Accordingly, every consumer perfectly reveals her vertical type θ when selecting the pair $p_i(\theta)$, $q_i(\theta)$, and therefore she enjoys status $s = \theta$.

Formally, a symmetric equilibrium is a pair of menus $\langle p_i^*(\theta), q_i^*(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ ($i = L, H$) such that: (1) every pair $p_i^*(\theta), q_i^*(\theta)$ delivers a status level $s_i = \theta$, and (2) given optimal consumer behavior, no firm can gain from deviating to an alternative incentive compatible menu $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ under which each new price-quality pair $p_i(\theta), q_i(\theta)$ delivers a status level of θ.[3]

For future reference, let $S(q, \theta) \equiv \theta \cdot v(q, \theta) - c(q)$ denote the net surplus created when a consumer with type θ receives quality q and status level θ. Also define the first-best quality for type θ, denoted by $q^{FB}(\theta)$, as the value of q that maximizes $S(q, \theta)$ (so that the derivative $S_q(q^{FB}(\theta), \theta)$ equals zero).

3.1. **Special Cases.**

Our results are best understood in relation to three well-known special cases of the model that have been studied in the literature:

[3]In addition, firms should not gain from deviating to a new schedule that pools types, and therefore delivers status levels that potentially differ from θ. However, as will become clear below, in order to characterize separating equilibria, it suffices to restrict to the subset of deviations where no pooling takes place.

Hotelling (1929). The simplest case arises when status does not affect utility and there is only one vertical type ($\theta_L = \theta_H$). Under these assumptions, our model reduces to Hotelling's model of spatial competition with linear transportation costs and fixed location firms. In equilibrium, firms only offer one type of good with first-best quality level q^{FB}, and this good is sold at a price equal to the marginal production cost $c(q^{FB})$ plus a markup equal to $\tau \equiv \frac{t}{g(1/2)}$, which is proportional to the marginal transportation cost t.

Rochet and Stole (2002). Consider the case in which status does not affect utility but consumers now have different vertical types ($\theta_L < \theta_H$). As shown by Rochet and Stole, the unique symmetric equilibrium with full market coverage is such that every consumer receives first-best quality $q^{FB}(\theta)$ and the price for each good equals the marginal production cost $c(q^{FB}(\theta))$ plus a markup that is constant across types. This constant markup guarantees that the first-best quality schedule is incentive compatible. As in the Hotelling model, the markup equals τ. Notice that this result would also arise if firms could directly observe θ and discriminate based on this type.[4]

Bagwell and Bernheim (1996), and Becker, Murphy, and Glaeser (2000). Now suppose status affects utility ($v_s > 0$) and consumers are vertically differentiated, but the marginal transportation cost t is zero. In this case, the horizontal type x is immaterial and we obtain a model of perfect competition that is simply a continuous-type version of the two-type signaling models of Bagwell and Bernheim, and Becker, Murphy, and Glaeser. Because of perfect competition, firms earn zero profits and prices must equal marginal costs. This means that all markups are zero and, because of the status motive, the first-best schedule $q^{FB}(\theta)$ is no longer incentive compatible: consumers with types lower than θ_H would imitate their peers with higher types merely to increase their social status. As a result, all consumers save for the lowest types θ_L must consume more than first-best quality so they can separate from the types below them. We proceed to characterize the resulting quality schedule, denoted $q_{BB}(\theta)$, which is a useful benchmark for our analysis.

Let $V(\theta) \equiv \theta \cdot v(q(\theta), \theta) - p(\theta)$ denote the optimized payoff for a consumer with type θ given arbitrary incentive-compatible schedules $p(\theta)$ and $q(\theta)$. From the envelope theorem (e.g., Myerson, 1981, Milgrom and Segal, 2002), $V(\theta)$ must grow with

[4]Rochet and Stole also analyze the case in which the market is not fully covered, which leads to a general under-provision of quality for a subset of types, and first-best quality for the rest.

θ at a rate equal to the partial derivative of the consumers' utility (1) with respect to their true type, namely,

$$V'(\theta) = v(q(\theta), \theta). \tag{3}$$

Moreover, under perfect competition, markups are zero ($p(\theta) = c(q(\theta))$), and therefore the consumer's payoff $V(\theta)$ equals net surplus $S(q(\theta), \theta)$. Combining this fact with equation (3), we obtain a differential equation for the equilibrium schedule $q_{BB}(\theta)$:

$$\theta \cdot v_s(q_{BB}(\theta), \theta) = q'_{BB}(\theta) \cdot |S_q(q_{BB}(\theta), \theta)| \, . \tag{4}$$

Since consumers with the lowest type consume first-best quality ($q_{BB}(\theta_L) = q^{FB}(\theta_L)$), we obtain the required the initial condition for this differential equation. An example of the resulting quality distortions is presented in panel (a) of Figure 2.

The L.H.S. of equation (4) represents the marginal utility of status, namely, type θ's marginal willingness to pay to defend her position in the social ranking. The R.H.S., on the other hand, measures the resources spent defending this position in the ranking. Recall that, under first best quality, the derivative $S_q(q^{FB}(\theta))$ equals zero. In contrast, when $q_{BB}(\theta) > q^{FB}(\theta)$, the derivative $S_q(q_{BB}(\theta), \theta)$ is negative. Thus, the term $|S_q(q_{BB}(\theta), \theta)|$ measures waisted surplus per unit of quality. The term $q'_{BB}(\theta)$, on the other hand, is the additional quality that type θ must consume relative to marginally lower type in order to keep her place in the ranking. Accordingly, the product $q'_{BB}(\theta) \cdot |S_q(q_{BB}(\theta), \theta)|$ is the marginal waste of surplus necessary to keep this place in the ranking. This waste can be interpreted as the price of a marginal unit of status.

Crucially, since firms obtain no markups, the entire price of status is paid through a destruction of surplus, as opposed to a mere monetary transfer. We return to this point below.

Before proceeding with the analysis, it is worth mentioning two additional related models in which the market is not covered:

Mussa and Rosen (1978) and Rayo (2005). The non-linear pricing model of Mussa and Rosen (also studied in detail by Maskin and Riley, 1984) can be interpreted as the extreme case in which the transportation cost t is infinity and status does not affect utility. In this case, firm L has full monopoly power over consumers with horizontal type $x = 0$, firm R has full power over consumers with type $x = 1$, and no other horizontal types consume in equilibrium. The result is that each monopoly offers a quality schedule that is lower than first-best for all types except for the highest type, who is the only to receive first best. As shown by Rochet and Stole (2002), such under-investment in quality shrinks as τ falls, and disappears altogether as soon as the market becomes fully covered.

Rayo, on the other hand, considers the monopolistic case in which status also enters utility. In his model, consumers are charged premium prices for higher quality because they implicitly purchase status as well. Nevertheless, quality remains distorted downward as in Mussa and Rosen. Thus, in contrast to the competitive case of Bagwell and Bernheim (1996) and Becker et al. (2000), the price consumers pay for status now takes the form of a pure money transfer to the monopolist, rather than an upward distortion of quality.

4. **Imperfect Competition Preliminaries**

We now return to the case in which the market is fully covered, but we now assume that status enters utility ($v_s > 0$), which extends Rochet and Stole (2002), and we also assume that competition is imperfect ($t > 0$), which extends Bagwell and Bernheim (1996), and Becker et al. (2000).

For any given incentive-compatible menu $\langle p_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ offered by firm i, let $V_i(\theta) \equiv \theta \cdot v(q_i(\theta), \theta) - p_i(\theta)$ denote the gross payoff assigned to type θ ignoring transportation costs. Conditional on $q_i(\theta)$, this identity provides a one-to-one mapping between $p_i(\theta)$ and $V_i(\theta)$. Thus, following standard practice, we can assume that firms directly offer menus of the form $\langle V_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ while sending prices to the background. Expressed in terms of this new menu, the incentive constraint (IC) is described by two joint conditions:

$$V_i'(\theta) = v(q_i(\theta), \theta) \text{ for all } \theta, \text{ and} \qquad (i)$$

$$v(q_i(\theta), \theta) \text{ is non-decreasing in } \theta. \qquad (ii)$$

As mentioned above, the first condition is derived from the envelope theorem, whereas the second condition is a simple generalization of the standard monotonicity constraint for the case in which status enters utility.[5]

Moreover, using the identity $p_i(\theta) \equiv c(q_i(\theta)) + \Omega_i(\theta)$ (where $\Omega_i(\theta)$ is the markup extracted from type θ), the envelope condition (i) can be expressed as:

$$\theta \cdot v_s(q_i(\theta), \theta) = q_i'(\theta) \cdot |S_q(q_i(\theta), \theta)| + \Omega_i'(\theta). \qquad (5)$$

[5]Formally, condition (i) must only hold for a full-measure subset of the type space. But directional derivatives must exist for V at all points, and these derivatives must satisfy the directional equivalent of condition (i).

This differential equation generalizes (4) for the case in which markups are no longer zero, and is central to the interpretation of our results. As in (4), the L.H.S. is the marginal willingness to pay for status. In equilibrium, this willingness to pay must be translated into an actual cost borne by the consumers. But from the R.H.S., we learn that such cost can now take a richer form. A firm can either induce consumers to purchase an excessively high quality leading to wasted surplus $(q_i'(\theta) \cdot |S_q(q_i(\theta), \theta)| > 0)$, or it can extract an increasing markup $(\Omega_i'(\theta) > 0)$ so that status is effectively purchased through a higher transfer to the firm, or any combination of the two.

In what follows, we refer to the difference $\Omega_i(\theta) - \tau$ as type θ's Veblen effect, which corresponds to the markup experienced by a consumer above and beyond the benchmark level of Hotelling (1929) and Rochet and Stole (2002) in the absence of a status motive. This terminology is borrowed from Bagwell and Bernheim (1996) where $\tau = 0$ and Veblen effects are defined as gross markups.

Definition 1. *When a consumer with type θ purchases from firm i, we say that she experiences a **Veblen effect** equal to $\Omega_i(\theta) - \tau$, and a **marginal Veblen effect** equal to $\Omega_i'(\theta)$.*

It turns out that, in equilibrium, firms will use a non–trivial combination of Veblen effects and quality distortions, with the optimal mix depending on the position occupied by each consumer in the social ranking.

Lemma 1 describes a basic feature of the equilibrium markups:

Lemma 1. *In any symmetric equilibrium, the average markup charged by firm i is equal to τ:*

$$\int_{\theta_L}^{\theta_H} \Omega_i(\theta) dF(\theta) = \tau. \tag{6}$$

In other words, the average Veblen effects are zero.

Proof. See Appendix 1. □

This result follows from a particular strategy available to the firms. Each firm can change the entire payoff schedule $V_i(\theta)$ by a constant amount ε while keeping the quality schedule fixed (which means that all prices are simultaneously increased or decreased by ε). Since this change is constant across θ, the new schedule remains incentive compatible. This change leads to the same fundamental trade-off present in Hotelling's model: a higher markup for each θ is traded-off against a lower demand. Moreover, since the elasticity of demand for each type is the same as in Hotelling, this trade-off is governed by the same parameter τ.

This result is useful because, for any given quality schedule $q_i(\theta)$, it allows us to pin down the precise markup required for each type, as dictated by the incentive constraint. In particular, rearranging equation (5), we obtain the marginal Veblen effect required for each type:

$$\Omega_i'(\theta) = \theta \cdot v_s(q_i(\theta), \theta) - q_i'(\theta) \cdot |S_q(q_i(\theta), \theta)| \,. \tag{5'}$$

Given $q_i(\theta)$, this differential equation determines the markup schedule $\Omega_i(\theta)$ up to a constant, and this constant is in turn is determined by (6).

5. Quality Distortions and Veblen Effects

We begin by characterizing "monotonic" equilibria in which the monotonicity constraint (ii) does not bind. Following conventional methodology, we obtain these equilibria by allowing firms to solve the relaxed optimization problem where only the envelope condition (i) is imposed (as opposed to the full (IC) constraint that also requires (ii)), and then verifying that the resulting equilibrium in fact satisfies (ii). Theorem 1 characterizes such equilibria:

Theorem 1. *Consider a symmetric separating equilibrium with full market coverage. If this equilibrium is monotonic, it must satisfy the following properties:*

- **a.** *For all intermediate types $\theta \in (\theta_L, \theta_H)$ quality is distorted upward ($q^*(\theta) \geq q^{FB}(\theta)$), and for both extreme types $\theta = \theta_L, \theta_H$ quality is first-best.*
- **b.** *Markups are, on average, strictly increasing. Namely, for any intermediate type $\widehat{\theta} \in (\theta_L, \theta_H)$,*

$$E\left[\Omega^*(\theta) \mid \theta \in [\widehat{\theta}, \theta_H]\right] > \tau > E\left[\Omega^*(\theta) \mid \theta \in [\theta_L, \widehat{\theta}]\right].$$

- **c.** *All types in a neighborhood of θ_H experience positive marginal Veblen effects. Moreover, as θ converges to θ_H, these marginal Veblen effects converge to the marginal utility of status $\theta \cdot v_s(q^*(\theta), \theta)$.*

Proof. See Appendix 1. □

Theorem 1 tells us that, in equilibrium, firms use an elaborate combination of quality distortions and Veblen effects to screen across types. In contrast to the perfectly competitive cases of Bagwell and Bernheim (1996) and Becker et. al (2000), quality distortions vanish at the high end. Instead, firms use large marginal

Veblen effects as a substitute screening device. In addition, markups are always increasing on average, which means that there is an implicit cross-subsidy from the high types to the low types. This cross-subsidy serves as an implicit market device through which high types, in effect, purchase status from their low-ranking peers. In fact, this implicit market is created by the firms as a way to increase overall efficiency and capture some of the rents.

We provide intuition for theorem 1 using two hypothetical benchmarks. First, suppose firms offered first-best quality schedules. In this case, the second term on the R.H.S. of ($5'$) would disappear, and therefore all types experience positive Veblen effects equal to $\theta \cdot v_s(q^{FB}(\theta), \theta)$. The result is an increasing markup schedule that crosses τ for some intermediate type (see panel (b) of Figure 2).[6] But from Hotelling's logic, firms would deviate away from this allocation by rotating the markup schedule clockwise so that each markup becomes closer to its ideal level τ.

In particular, from the equality $\Omega(\theta) = S(q(\theta), \theta) - V(\theta)$ we learn that rotating the markup schedule $\Omega(\theta)$ clockwise (making it flatter, see panel (b) of Figure 2) can be achieved by rotating the value schedule $V(\theta)$ counterclockwise (making it steeper, see panel (c) of Figure 2). In order to do so, the firm would need to increase $V'(\theta)$, which from the envelope equation $V_i'(\theta) = v(q_i(\theta), \theta)$, amounts to increasing quality beyond first-best. Moreover, notice that distorting the quality of intermediate types can have a large effect in terms of rotating $V(\theta)$ counterclockwise, but this effect shrinks for types that are closer to the extremes and, in fact, fully vanishes for θ_L and θ_H.

In addition, this strategy comes at a cost. Whenever quality is distorted, surplus $S(q(\theta), \theta)$ is reduced. As a result, the sum $V(\theta) + \Omega(\theta)$ must fall, which means that the firm either loses customers (if $V(\theta)$ falls), or loses profits per customer (if $\Omega(\theta)$ falls), or both. In equilibrium, this loss must be traded off against the benefit described above. But since the benefit vanishes for the extreme types, their quality will not be distorted at all.

As a second hypothetical benchmark, suppose firms offered the perfectly-competitive quality schedule $q_{BB}(\theta)$ characterized by (4). In this case, the two terms on the R.H.S. of ($5'$) would exactly cancel each other out. As a result, all marginal Veblen effects would become zero and the markup would equal τ across types (the Hotelling

[6] In contrast, in Rochet and Stole (2002), firms can simultaneously offer first-best quality and charge the ideal markup τ for every type because, absent the status motive, a constant markup is automatically incentive compatible.

ideal). In fact, this arrangement constitutes an equilibrium when $\tau = 0$. However, once firms gain market power, they can capture a fraction of the surplus created for each type. As a result, they have reason to replace at least part of quality distortions embedded in $q_{BB}(\theta)$ with cross-subsidies among consumers, since this implicit market mechanism expands the overall pool of surplus from which profits are drawn.

Proposition 1 considers "non-monotonic" equilibria in which the monotonicity constraint (ii) binds for some subset of types:

Proposition 1. *Consider a symmetric separating equilibrium with full market coverage. If the monotonicity constraint does not bind for the highest type θ_H, this equilibrium must satisfy all properties described in theorem 1.*

On the other hand, if the monotonicity constraint does bind for the highest type, this equilibrium must satisfy the properties described in theorem with the following exceptions:

1. *Quality is distorted upward for the highest type.*

2. *All types in a neighborhood of θ_H experience marginal Veblen effects that are strictly larger than the marginal utility of status $\theta \cdot v_s(q^*(\theta), \theta)$.*

The intuition behind this result is as follows. Recall that the solution to the relaxed problem prescribes first-best quality for both extreme types, and quality strictly larger than first-best for all other types. As a result, the monotonicity constraint will always have slack at the low end of the interval. However, precisely because the quality distortion is corrected for θ_H, it can in fact be the case that the relaxed schedule is no longer monotonic at the high end.

In this case, in order to meet the monotonicity constraint, the quality level for θ_H will no longer be driven down to first-best (as claimed in part **1** of the proposition). In addition, over the range of types for which this constraint happens to bind, the quality schedule $q_i(\theta)$ will acquire the lowest possible slope consistent with a non-decreasing function $v(q_i(\theta), \theta)$ (from (ii')). But since the partial derivative $v_s(q_i(\theta), \theta)$ is positive, this minimum feasible slope $q_i'(\theta)$ happens to be negative (with decreasing quality being compensated with increasing status). As a result, from the envelope condition $(5')$, the marginal Veblen effect becomes strictly larger than the marginal utility for status over the relevant range (as claimed in part **2** of the proposition).

Finally, if the monotonicity constraint happens to bind over an interval of intermediate types, the relaxed schedule must be ironed over the relevant range. This ironing, however, will not change the fact that quality is distorted above the first-best.

6. Corrective Taxation

The inefficiencies arising from status seeking suggest a role for government intervention. Here, we consider the use of corrective taxation. We discuss two cases. First, under the hypothetical assumption that production costs $c(q)$ are observable, we consider taxes that are directly imposed on these production costs. Since this instrument attacks the direct source of the inefficiency (i.e., over–investment in quality), the first best can indeed be achieved. Nevertheless, we show that given the non–monotonic nature of the quality distortions, the optimal tax-schedule will not have a conventional shape.

Second, we consider the more realistic case in which taxes are imposed over prices instead of over costs. Since such a policy instrument does not only affect the quality distortions (our target), but they also alter the efficient Veblen effects, achieving first-best with this instrument may not be feasible. This conclusion casts doubt over informal proposals that luxury goods should be heavily taxed.

6.1. Taxes on Production Costs.

Suppose that whenever a firm produces a good with quality q, in addition to incurring the cost $c(q)$, it is required to make a tax payment equal to $\alpha(q)$. In this case, the firm's problem is identical to the original problem except for the fact that it now faces a higher effective cost function given by:

$$\widetilde{c}(q) \equiv c(q) + \alpha(q).$$

The goal is to find a function $\alpha(q)$ such that the equilibrium quality that arises under the new cost $\widetilde{c}(q)$ corresponds to the first best.

Let $\widetilde{\Omega}(\theta) \equiv p(\theta) - \widetilde{c}(q)$ denote the firm's after–tax markup, which equals the gross markup minus the tax: $\Omega(\theta) - \alpha(q)$. We refer to $\widetilde{\Omega}(\theta)$ as the firm's (after–tax) Veblen effect and to $\Omega(\theta)$ as the gross Veblen effect experienced by consumers. From the envelope condition ($5'$), in order for the tax schedule to implement the first-best ($S_q(q_i(\theta), \theta) = 0$) we require that:

$$\Omega_i'(\theta) = \widetilde{\Omega}'(\theta) + \alpha'(q(\theta))q'(\theta) = \theta \cdot v_s(q_i(\theta), \theta), \tag{7}$$

where $\alpha'(q)$ denotes the marginal tax on quality.

The relationship in (7) implies that the marginal utility for status must be fully translated into monetary transfers (as opposed to quality distortions), and these transfers must either go to the firm (through a positive marginal Veblen effect $\widetilde{\Omega}'(\theta)$) or to the government (through the marginal tax rate $\alpha'(q)$). In other words, taxes are necessary only insofar as the firms do not impose sufficiently high marginal Veblen effects $\widetilde{\Omega}'(\theta)$ to begin with, and the optimal marginal tax $\alpha'(q)$ precisely supplements the firm's marginal Veblen effects in such a way that the gross Veblen effects experienced by the consumers equal their full marginal utility for status.

The following corollary of theorem 1 describes the optimal tax schedule.

Corollary 1. *Suppose the marginal tax schedule $\alpha(q)$ implements the first-best quality schedule under an equilibrium with full market coverage. Then, for all θ, the marginal tax $\alpha'(q)$ is such that:*

- **a.** *For both extreme types, $\alpha'(q^{FB}(\theta)) = 0$.*
- **b.** *For all interior types, $\alpha'(q^{FB}(\theta)) = \frac{1}{\tau f(\theta)} v_q(q^{FB}(\theta), \theta) \int_0^\theta \left[\tau - \widetilde{\Omega}(z)\right] f(z)dz$, which is positive from Theorem 1(b).*

Proof. See Appendix 1. □

This result tells us that only the quality sold to the interior types must be taxed in the margin. The reason is that, from theorem 1, in any equilibrium with a monotonic quality schedule, the firms are tempted to impose quality distortions (and low marginal Veblen effects) only for these interior types (and the first-best quality schedule is, by assumption, monotonic). Panel (b) of Figure 3 depicts the optimal tax schedule for the special case in which $f(\theta)$ is uniformly distributed on $[1, 2]$, $v(q, s) = q + s$, and $\widetilde{c}(q) = \frac{1}{2}q^2$.

6.2. **Taxes on Prices.** Now suppose that whenever a firm sells a good of price p, it is required to make a tax payment equal to $\beta(p)$. As before, the goal is to find a function $\beta(p)$ that induces the first best.

Let $\widetilde{\Omega}(\theta) \equiv p(\theta) - \beta(p(\theta)) - c(q)$ denote the firm's after–tax markup, which again equals the gross markup minus the tax: $\Omega(\theta) - \beta(p(\theta))$. The following theorem imposes necessary conditions that any tax schedule $\beta(p)$ must satisfy in order to implement the first–best.

Theorem 2. *Suppose the tax schedule $\beta(p)$ implements the first–best quality schedule under an equilibrium with full market coverage. Then, for all θ, the marginal tax $\beta'(p)$ must satisfy:*

a. $\int_{\theta_L}^{\theta_H} \beta'(p^*(\theta))f(\theta) = 0$ *(where* $p^*(\theta)$ *denotes the equilibrium price). Namely, the marginal tax* $\beta'(p)$ *averages to zero across types.*

b. *For both extreme types,* $\beta'(p^*(\theta)) = 0$.

c. *For all interior types,*

$$\beta'(p^*(\theta)) = \frac{1}{\tau\theta f(\theta)} \int_0^\theta \left[\tau - \widetilde{\Omega}(z)\right] f(z)dz - \frac{1}{\theta f(\theta)} \int_0^\theta \beta'(p^*(z))f(z)dz.$$

Proof. See Appendix 1. □

As before, if the tax is to implement first–best, only interior types must be taxed in the margin. However, part **a** tells us that the average marginal tax must be zero across consumers, which implies that some interior types must necessarily be subsidized.

Moreover, unlike the case in which quality was directly taxed, an optimal tax schedule on prices may fail to exist. The reason is that prices play a dual role in our environment. On the one hand, higher prices allow firms to charge consumers for higher quality levels. And since there is an incentive to distort quality upward, this first role implies that prices should be taxed. On the other hand, by creating Veblen effects, the use of high prices serve as a substitute screening device for quality distortions and therefore enhance efficiency. Consequently, this second role suggests that higher prices should be subsidized. Because of these two opposing goals, a tax levied on prices may not be a sufficiently rich instrument to achieve first-best.

7. **Conclusion**

We have studied the emergence of two frequently observed phenomena in markets for conspicuous goods: upward quality distortions and Veblen effects. In our model, two firms offer conspicuous goods to a heterogeneous collection of consumers with standard single-crossing preferences. This model combines elements of both screening and signaling. Namely, firms offer individually–targeted products using non-linear pricing schemes, and when purchasing these products, consumers' signal their hidden characteristics.

The firms' strategies are driven by two competing goals: (1) satisfying incentive-compatibility constraints in order to screen among different types of consumers, and (2) seeking an appropriate balance between market share and price markups. As a result, they adopt an mix of quality distortions (which attract more consumers while satisfying their incentive constraints) and cross–subsidies among consumers (which deliver an optimal balance between market share and profits per customer).

The use of cross–subsidies creates an implicit market for status, mediated the firms, in which high-ranking consumers effectively purchase status from their low-ranking peers. Unlike quality distortions, this market mechanism is an efficient way of allocating status. However, since firms are eager to gain a larger market share for high-margin consumers, quality distortions are also employed.

The novelty of our model resides in providing a rationale for the simultaneous presence of the two above phenomena under single-crossing preferences, as well as a framework for analyzing their interaction. In addition, the model uncovers clues for optimal corrective taxation. Contrary to informal prescriptions, high-end products with high markups do not require large taxes. In fact, it is precisely because of these high prices that the status competition is resolved efficiently (through cross–subsidies across consumers) as opposed to being resolved through a wasteful over-provision of quality.

8. **Appendix 1: Proofs**

We begin with a preliminary observation. Given menus $\langle V_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$, for any given θ let

$$\widehat{x}(\theta) = \frac{1}{2} + \frac{1}{2t}\left[V_L(\theta) - V_R(\theta)\right], \qquad (A1)$$

which describes the horizontal type x that is indifferent between buying from either firm.[7] Provided $\widehat{x}(\theta)$ is interior (as in the case in any symmetric equilibrium), the fractions of consumers θ that purchase from each firm are given by

$$D_L(\theta) = G(\widehat{x}(\theta)) \text{ and } D_R(\theta) = 1 - G(\widehat{x}(\theta)).$$

Proof of Lemma 1. Consider a symmetric equilibrium in which both firms offer the same menu $\langle V^*(\theta), q^*(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$, and therefore $\widehat{x}(\theta) = \frac{1}{2}$ for all θ. Accordingly, from (2), firm L's equilibrium payoff is given by

$$\begin{aligned}\Pi_L &\equiv \int_{\theta_L}^{\theta_H} \Omega_L(\theta) D_L(\theta) dF(\theta) \\ &= \int_{\theta_L}^{\theta_H} \left[S(q^*(\theta),\theta) - V^*(\theta)\right] G\left(\frac{1}{2}\right) dF(\theta),\end{aligned}$$

[7]If $\widehat{x}_i(\theta) \le 0$, every consumer with type θ would with to purchase from firm L, and the opposite occurs when $\widehat{x}_i(\theta) \ge 1$.

where the markup $\Omega_L(\theta)$ has been expressed as $S(q^*(\theta),\theta) - V^*(\theta)$. Now consider an alternative menu for firm L given by $\langle V^*(\theta)+\varepsilon, q^*(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ for some small ε (perhaps negative), which is is identical to the original menu except for the fact that all consumers are offered a payoff that is higher or lower by a constant amount ε. Since this change is constant across θ, and quality is unaffected, the new menu remains incentive compatible. The payoff obtained by firm L under this new menu becomes

$$\int_{\theta_L}^{\theta_H} \left[S(q^*(\theta),\theta) - V^*(\theta) - \varepsilon\right] G\left(\frac{1}{2}+\frac{\varepsilon}{2t}\right) dF(\theta), \tag{A2}$$

where, from equation $(A1)$, the horizontal cutoff $\widehat{x}(\theta)$ has now increased to $\frac{1}{2}+\frac{\varepsilon}{2t}$. Notice that the derivative of $(A2)$ with respect to ε evaluated at $\varepsilon = 0$ is given by

$$\int_{\theta_L}^{\theta_H} \left\{-G\left(\frac{1}{2}\right) + \left[S(q^*(\theta),\theta) - V^*(\theta)\right] g\left(\frac{1}{2}\right)\frac{1}{2t}\right\} dF(\theta). \tag{A3}$$

Since the original schedule constitutes an equilibrium, it must be the case that the new payoff $(A2)$ is maximized, with respect to ε, when $\varepsilon = 0$. But this in turn implies that $(A3)$ must be equal to zero, which is equivalent to the desired equality $\int_{\theta_L}^{\theta_H} \Omega_L(\theta)dF(\theta) = \tau$. The analysis is symmetric for firm R. □

Proof of Theorem 1. In the relaxed problem where the monotonicity constraint (ii) is ignored, each firm selects a menu $\langle V_i(\theta), q_i(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$ that maximizes profits given the opponents menu $\langle V_{-i}(\theta), q_{-i}(\theta)\rangle_{\theta\in[\theta_L,\theta_H]}$, subject to the envelope condition (i). We also relax the constraint that quality is non-negative for all types, since it will not bind in equilibrium. Without loss, we consider the problem for firm L:

$$\max_{V_L(\theta),q_L(\theta)} \int_{\theta_L}^{\theta_H} \Omega_L(\theta)G(\widehat{x}(\theta))dF(\theta) =$$

$$\max_{V_L(\theta),q_L(\theta)} \int_{\theta_L}^{\theta_H} \left[S(q_L(\theta),\theta) - V_L(\theta)\right] G(\widehat{x}(\theta))dF(\theta)$$

$$s.t.$$

$$V_L'(\theta) = v(q_L(\theta),\theta) \text{ for all } \theta. \tag{i}$$

This problem can be expressed as an optimal control problem with state variable $V_L(\theta)$ and control variable $q_L(\theta)$. Dropping the L subindex, the corresponding Hamiltonian is given by

$$H(\theta) = \left[S(q(\theta),\theta) - V(\theta)\right] G(\widehat{x}(\theta))f(\theta) + \lambda(\theta)v(q(\theta),\theta),$$

where $\lambda(\theta)$ denotes the co-state variable for the envelope equation of motion $V'(\theta) = v(q(\theta), \theta)$.

From the Maximum Principle, the solution is characterized by the following Hamiltonian system. For all θ,

$$\frac{\partial}{\partial q} H(\theta) = 0,$$

$$V'(\theta) = v(q(\theta), \theta), \text{ and}$$

$$\lambda'(\theta) = -\frac{\partial}{\partial V} H(\theta),$$

together with the transversality condition $\lambda(\theta_H) = 0$. In a symmetric equilibrium where $\widehat{x}(\theta) = 1/2$ for all θ, this system becomes

$$S_q(q(\theta), \theta) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta), \theta), \tag{A4}$$

$$\Omega'(\theta) = \theta \cdot v_s(q(\theta), \theta) + q'(\theta) \cdot S_q(q(\theta), \theta), \text{ and} \tag{A5}$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\Omega(\theta)}{\tau}\right] f(\theta), \tag{A6}$$

where $(A4)$ and $(A6)$ are derived from differentiating $H(\theta)$, setting $G(\widehat{x}(\theta)) = 1/2$, and rearranging terms, and $(A5)$ is derived from (i) using the equality $\Omega(\theta) = S(q(\theta), \theta) - V(\theta)$.

On the other hand, we can express the transversality condition $\lambda(\theta_H) = 0$ as follows:

$$\lambda(\theta_H) = \lambda(\theta_L) + \int_{\theta_L}^{\theta_H} \lambda'(\theta) d\theta = 0.$$

Moreover, from $(A6)$, Lemma 1 is equivalent to the equality $\int_{\theta_L}^{\theta_H} \lambda'(\theta) d\theta = 0$. Using the above equation, this equality delivers $\lambda(\theta_H) = \lambda(\theta_L) = 0$, which combined with $(A4)$ implies that $S_q(q(\theta_H), \theta_H) = S_q(q(\theta_L), \theta_L) = 0$. Accordingly, quality is first-best for both extreme types (as claimed in part **a** of the theorem).

On the other hand, an inspection of the system $(A4) - (A6)$ reveals that $\lambda(\theta)$ must be strictly positive for all interior values of θ. Suppose, contrary to this claim, that $\lambda(\theta_0) = 0$ for some $\theta_0 \in (\theta_L, \theta_H)$. Then, it follows from $(A4)$ that $S_q(q(\theta_0), \theta_0) = 0$, and from $(A5)$ that $\Omega'(\theta_0) = \theta_0 \cdot v_s(q(\theta_0), \theta_0) > 0$. Moreover, we require that $\lambda'(\theta_0) = 0$ and $\lambda''(\theta_0) > 0$ (otherwise, λ would become negative for values close to θ_0, which is impossible). When combined with $(A6)$, these conditions imply, respectively, that $\Omega(\theta_0) = \tau$ and $\lambda''(\theta_0) = -\frac{1}{2}\frac{\Omega'(\theta_0)}{\tau} f(\theta_0) > 0$. But the last inequality is impossible given the previous observation that $\Omega'(\theta_0)$ is strictly positive.

Given that $\lambda(\theta) > 0$ for all $\theta \in (\theta_L, \theta_H)$, it follows from $(A4)$ that $S_q(q(\theta), \theta) < 0$ for all such types. As a result, quality is distorted upward. This observation completes the proof for part **a**.

For part **b**, on the other hand, notice that for any interior $\widehat{\theta}$, we have $\int_{\theta_L}^{\widehat{\theta}} \lambda'(\theta) d\theta = \lambda(\widehat{\theta}) > 0$ (recall that $\lambda(\theta_L) = 0$). From $(A6)$, this inequality implies that $\int_{\theta_L}^{\widehat{\theta}} \frac{1}{2} \left[1 - \frac{\Omega(\theta)}{\tau}\right] f(\theta) d\theta > 0$, which is equivalent to

$$\tau > E\left[\Omega^*(\theta) \mid \theta \in [\theta_L, \widehat{\theta}]\right].$$

Moreover, from lemma 1 we have $E\left[\Omega^*(\theta) \mid \theta \in [\theta_L, \theta_H]\right] = \tau$, which combined with the above equality implies that

$$E\left[\Omega^*(\theta) \mid \theta \in [\widehat{\theta}, \theta_H]\right] > \tau,$$

therefore completing the proof for part **b**.

We now turn to part **c**. From the transversality condition ($\lambda(\theta_H) = 0$), as θ converges to θ_H, $\lambda(\theta)$ converges to zero. Moreover since $q(\theta)$ converges to $q^{FB}(\theta)$ from above, the derivative $q'(\theta)$ must remain bounded around θ_H. Thus, from $(A4)$, $S_q(q(\theta), \theta)$ converges to zero, and so does the product $q'(\theta) \cdot S_q(q(\theta), \theta)$. Therefore, it follows from $(A5)$ that the marginal Veblen effect $\Omega'(\theta)$ converges to the positive function $\theta \cdot v_s(q(\theta), \theta)$, as desired. □

Proof of Proposition 1. Once the monotonicity constraint (ii) is introduced, firm L's problem becomes:

$$\max_{V_L(\theta), q_L(\theta)} \int_{\theta_L}^{\theta_H} \left[S(q_L(\theta), \theta) - V_L(\theta)\right] G(\widehat{x}(\theta)) dF(\theta)$$

$$s.t.$$

$$V_L'(\theta) = v(q_L(\theta), \theta) \text{ for all } \theta, \text{ and} \qquad (i)$$

$$v(q_i(\theta), \theta) \text{ is non-decreasing in } \theta. \qquad (ii)$$

This problem can be expressed as an optimal control problem with state variables $V_L(\theta)$ and $q_L(\theta)$, and control variable $q_L'(\theta)$. Dropping the L subindex, the corresponding Hamiltonian is given by

$$H(\theta) = \left[S(q(\theta), \theta) - V(\theta)\right] G(\widehat{x}(\theta)) f(\theta) + \lambda(\theta) v(q(\theta), \theta) + \mu(\theta) q'(\theta),$$

where, as before, $\lambda(\theta)$ denotes the co-state variable for the envelope equation $V'(\theta) = v(q(\theta), \theta)$, and $\mu(\theta)$ now denotes the co-state variable for the motion of $q(\theta)$.

In addition, the monotonicity constraint (ii) implies that $\frac{\partial}{\partial\theta}v(q(\theta),\theta) \geq 0$, and therefore

$$q'(\theta) \geq -\frac{v_s(q(\theta),\theta)}{v_q(q(\theta),\theta)}. \qquad (ii')$$

Let $\gamma(\theta)$ denote the Lagrange multiplier for this constraint.

The solution is now characterized by following system:

$$\frac{\partial}{\partial q}H(\theta) = -\mu'(\theta),$$

$$-\mu(\theta) = \gamma(\theta),$$

$$V'(\theta) = v(q(\theta),\theta), \text{ and}$$

$$\lambda'(\theta) = -\frac{\partial}{\partial V}H(\theta),$$

together with the transversality conditions $\lambda(\theta_H) = 0$ and $\mu(\theta_L) = \mu(\theta_H) = 0$. In a symmetric equilibrium where $\widehat{x}(\theta) = 1/2$ for all θ, this system implies

$$S_q(q(\theta),\theta) = -\frac{2\lambda(\theta)}{f(\theta)}v_q(q(\theta),\theta) - \mu'(\theta), \qquad (A4')$$

$$\Omega'(\theta) = \theta \cdot v_s(q(\theta),\theta) + q'(\theta) \cdot S_q(q(\theta),\theta), \text{ and} \qquad (A5)$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\Omega(\theta)}{\tau}\right]f(\theta), \qquad (A6)$$

where the only difference with respect the system $(A4) - (A6)$ analyzed in the proof of theorem 1 is the term $-\mu'(\theta)$, which appears on the R.H.S. of $(A4')$.

This system has the same properties as the original system with the exception that, in regions where the monotonicity constraint binds, the quality schedule $q(\theta)$ changes at the negative rate $q'(\theta) = -v_s/v_q$ (from (ii')). Nevertheless, following a standard ironing argument, the quality schedule remains distorted away from the first-best for all interior types $\theta \in (\theta_L, \theta_H)$. The co-state variable $\lambda(\theta)$, on the other hand, follows the same general behavior as before: it equals zero for both extreme types and is positive for all interior ones.

Consider now the extreme types θ_L and θ_H. For type θ_L the monotonicity constraint cannot bind. Otherwise, $-\mu'(\theta_L) = \gamma'(\theta_L) > 0$ and, from $(A4')$, $q(\theta_L) < q^{FB}(\theta_L)$, which is inconsistent with optimal ironing.

For type θ_H, on the other hand, the monotonicity constraint might indeed bind. If so, $-\mu'(\theta_H) = \gamma'(\theta_L) < 0$ and, from $(A4')$, $q(\theta_H) > q^{FB}(\theta_H)$, as claimed in part **1** of the proposition. In addition, from (ii'), $q'(\theta)$ must be negative for all types in a neighborhood of θ_H (for which the monotonicity constraint also binds). As a result, from $(A5)$, the marginal Veblen effect $\Omega'(\theta)$ becomes strictly larger than $\theta \cdot v_s(q(\theta), \theta)$, as claimed in part **2** of the proposition.

Finally, if the monotonicity constraint does not bind for θ_H, $\mu'(\theta_H)$ equals zero. Thus, from $(A4')$, $q(\theta_H) = q^{FB}(\theta_H)$, as occurred in the relaxed problem. □

Proof of Corollary 1. Following the proof of proposition 1 (which applies for any monotonic quality allocation), the equilibrium is characterized by the system $(A4)$–$(A6)$ with $\widetilde{c}(q)$ in the place of $c(q)$, and $\widetilde{\Omega}(\theta)$ in the place of $\widetilde{\Omega}(\theta)$. This system becomes

$$S_q(q(\theta), \theta) - \alpha'(q) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta), \theta), \tag{$\widetilde{A}4$}$$

$$\widetilde{\Omega}'(\theta) = \theta \cdot v_s(q(\theta), \theta) + q'(\theta) \cdot (S_q(q(\theta), \theta) - \alpha'(q)), \text{ and} \tag{$\widetilde{A}5$}$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\widetilde{\Omega}(\theta)}{\tau}\right] f(\theta). \tag{$\widetilde{A}6$}$$

It therefore follows from the proof of proposition 1 that $\lambda(\theta)$ is zero for both extreme types and positive for all interior ones.

Moreover we require that $S_q(q(\theta), \theta) = 0$ for all θ (so that quality is first best). Thus, from $(\widetilde{A}4)$ and $(\widetilde{A}6)$ we obtain

$$\begin{aligned} \alpha'(q^{FB}(\theta)) &= \frac{2\lambda(\theta)}{f(\theta)} v_q(q^{FB}(\theta), \theta) \\ &= \frac{1}{f(\theta)} v_q(q^{FB}(\theta), \theta) \int_0^\theta \left[1 - \frac{\widetilde{\Omega}(z)}{\tau}\right] f(z) dz, \end{aligned} \tag{$A7$}$$

where the second equality follows from integrating over $(\widetilde{A}6)$ to obtain $\lambda(\theta)$. Finally, the fact that $\alpha'(q^{FB}(\theta))$ is zero from the extreme types, and positive for the interior ones, follows from $(A7)$ and the fact that $\lambda(\theta)$ has this same properties. □

Proof of Theorem 2. Based on the proof of proposition 1 (which applies for any monotonic quality allocation), once the price tax $\beta(p)$ is added, dropping the L

subindex, firm L's problem becomes

$$\max_{V(\theta),q(\theta)} \int_{\theta_L}^{\theta_H} \left[S(q(\theta),\theta) - V(\theta) - \beta(p(\theta))\right] G(\widehat{x}(\theta))dF(\theta)$$

$$\max_{V(\theta),q(\theta)} \int_{\theta_L}^{\theta_H} \left[S(q(\theta),\theta) - V(\theta) - \beta(\theta v(q(\theta),\theta) - V(\theta))\right] G(\widehat{x}(\theta))dF(\theta)$$

$$s.t.$$

$$V'(\theta) = v(q(\theta),\theta) \text{ for all } \theta, \qquad (i)$$

where $p(\theta)$ has been expressed, from the definition of $V(\theta)$, as $\theta v(q(\theta),\theta) - V(\theta)$.

Assuming $\beta(p)$ induces the first-best, the firm's equilibrium payoff is given by

$$\Pi \equiv \int_{\theta_L}^{\theta_H} \left[S(q^{FB}(\theta),\theta) - V(\theta) - \beta(\theta v(q^{FB}(\theta),\theta) - V(\theta))\right] G\left(\frac{1}{2}\right) dF(\theta),$$

Now consider an alternative menu for firm L given by such that $V(\theta)$ is unchanged, but the quality offered to each consumer is changed to a schedule $\widetilde{q}(\theta) \equiv q^{FB}(\theta) + \Delta q(\theta)$ such that

$$\theta v(\widetilde{q}(\theta),\theta) = \theta v(q^{FB}(\theta),\theta) + \varepsilon,$$

for some small ε (perhaps negative). Since $V(\theta)$ is unchanged, the new menu remains incentive compatible. The payoff obtained by firm L under this new menu becomes

$$\int_{\theta_L}^{\theta_H} \left[S(\widetilde{q}(\theta),\theta) - V(\theta) - \beta(\theta v(q^{FB}(\theta),\theta) + \varepsilon - V(\theta))\right] G\left(\frac{1}{2}\right) dF(\theta) \qquad (A8)$$

The derivative of $(A8)$ with respect to ε, evaluated at $\varepsilon = 0$, is given by

$$\int_{\theta_L}^{\theta_H} \left[S_q(q^{FB}(\theta),\theta)\Delta q(\theta) - \beta'(\theta v(q^{FB}(\theta),\theta) - V(\theta))\right] G\left(\frac{1}{2}\right) dF(\theta)$$

$$= \int_{\theta_L}^{\theta_H} \left[-\beta'(\theta v(q^{FB}(\theta),\theta) - V(\theta))\right] G\left(\frac{1}{2}\right) dF(\theta), \qquad (A9)$$

where the equality follows from the fact that $S_q(q^{FB}(\theta),\theta) = 0$.

Since the original schedule constitutes an equilibrium, it must be the case that the new payoff $(A8)$ is maximized, with respect to ε, when $\varepsilon = 0$. But this in turn implies that $(A9)$ must be equal to zero, which is equivalent to

$$\int_{\theta_L}^{\theta_H} \beta'(\theta v(q^{FB}(\theta),\theta) - V(\theta))f(\theta)d\theta$$

$$= \int_{\theta_L}^{\theta_H} \beta'(p^*(\theta))f(\theta) = 0 \qquad (A10)$$

as claimed in part **a** of the theorem.

On the other hand, following the same reasoning in lemma 1, and using part **a**, it follows that the average after-tax markup equals τ:

$$\int_{\theta_L}^{\theta_H} \widetilde{\Omega}(\theta) f(\theta) d\theta = \tau. \qquad (A11)$$

We now return to the firm's general problem. This problem can be expressed as an optimal control problem with state variable $V(\theta)$ and control variable $q(\theta)$. The corresponding Hamiltonian is given by

$$H(\theta) = [S(q(\theta),\theta) - V(\theta) - \beta(\theta v(q(\theta),\theta) - V(\theta))]\, G(\widehat{x}(\theta)) f(\theta) + \lambda(\theta) v(q(\theta),\theta),$$

The solution is characterized by the transversality condition $\lambda(\theta_H) = 0$ combined with the following Hamiltonian system, which is an extension of the system $(A4)$ – $(A6)$. For all θ,

$$S_q(q(\theta),\theta) - \beta'(\theta v(q(\theta),\theta) - V(\theta))\theta v_q(q(\theta),\theta) = -\frac{2\lambda(\theta)}{f(\theta)} v_q(q(\theta),\theta), \qquad (A12)$$

$$\Omega'(\theta) = \theta \cdot v_s(q(\theta),\theta) + q'(\theta) \cdot S_q(q(\theta),\theta), \text{ and} \qquad (A13)$$

$$\lambda'(\theta) = \frac{1}{2}\left[1 - \frac{\widetilde{\Omega}(\theta)}{\tau}\right] f(\theta) - \frac{1}{2}\beta'(\theta v(q(\theta),\theta) - V(\theta)) f(\theta), \qquad (A14)$$

Combining $(A10)$ and $(A11)$ with $(A14)$, it follows that $\int_{\theta_L}^{\theta_H} \lambda'(\theta) f(\theta) d\theta = 0$, and therefore $\lambda(\theta_L) = 0$.

Moreover, since we require that $S_q(q(\theta),\theta) = 0$ for all θ, from $(A10)$ and $(A12)$ we obtain

$$\beta'(p^*(\theta)) = \beta'(\theta v(q(\theta),\theta) - V(\theta)) = \frac{2\lambda(\theta)}{\theta f(\theta)} \qquad (A15)$$

$$= \frac{1}{\theta f(\theta)} \int_0^{\theta} \left[1 - \frac{\widetilde{\Omega}(z)}{\tau}\right] f(z) dz - \frac{1}{\theta f(\theta)} \int_0^{\theta} \beta'(p^*(z)) f(z) dz.$$

where the last equality follows from integrating over $(A14)$ to obtain $\lambda(\theta)$. This relation delivers part **c** of the theorem.

Finally, part **c** of the theorem follows from $(A15)$ and the fact that both $\lambda(\theta_L)$ and $\lambda(\theta_H)$ are zero. □

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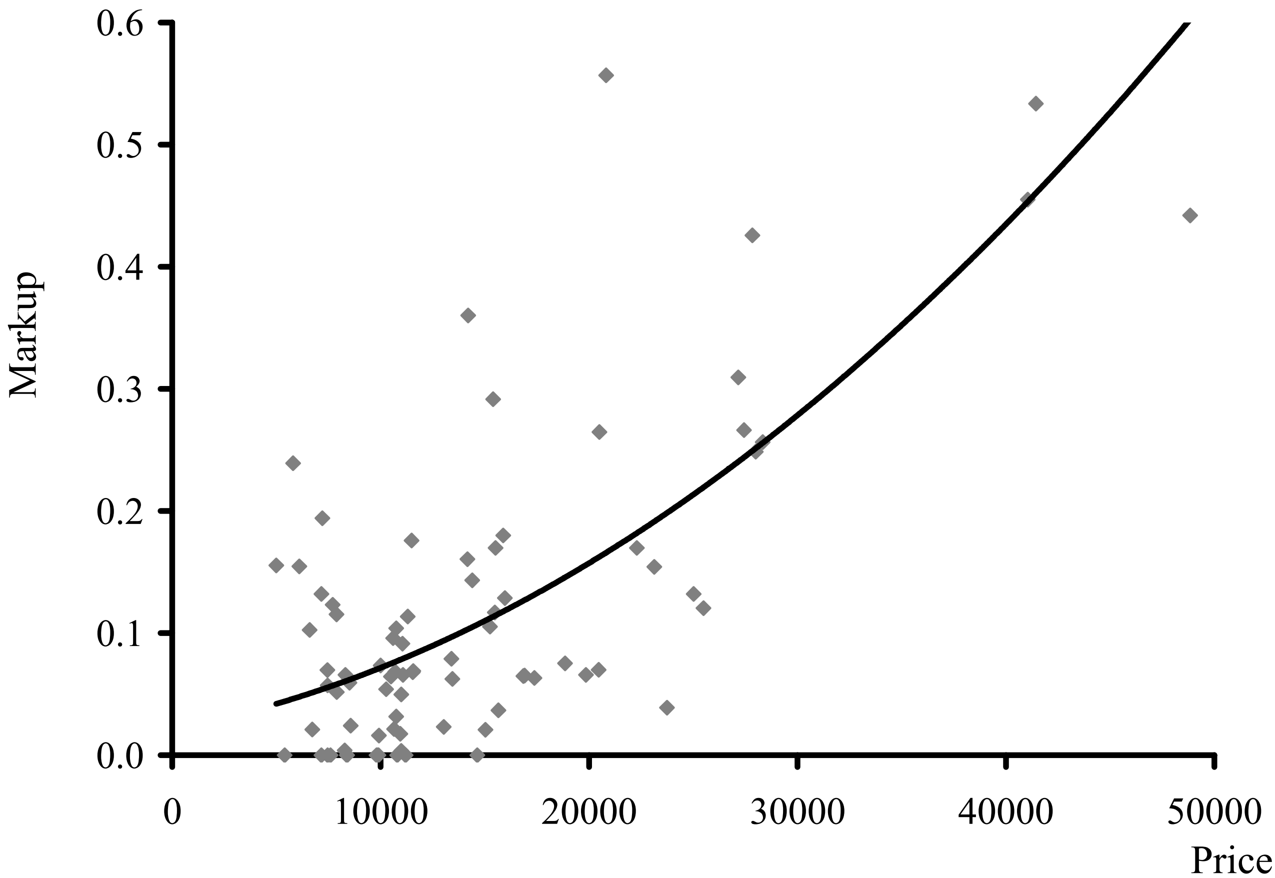
Figure 1

Markups as a percentage of Car Price



Markups as a percentage of price for 1987 car models (1987 dollars). Underlying market structure: Cournot for European cars, Bertrand for all others. Taken from Feenstra and Levinsohn (1995).

Figure 2

Quality, Markup, and Equilibrium Utility

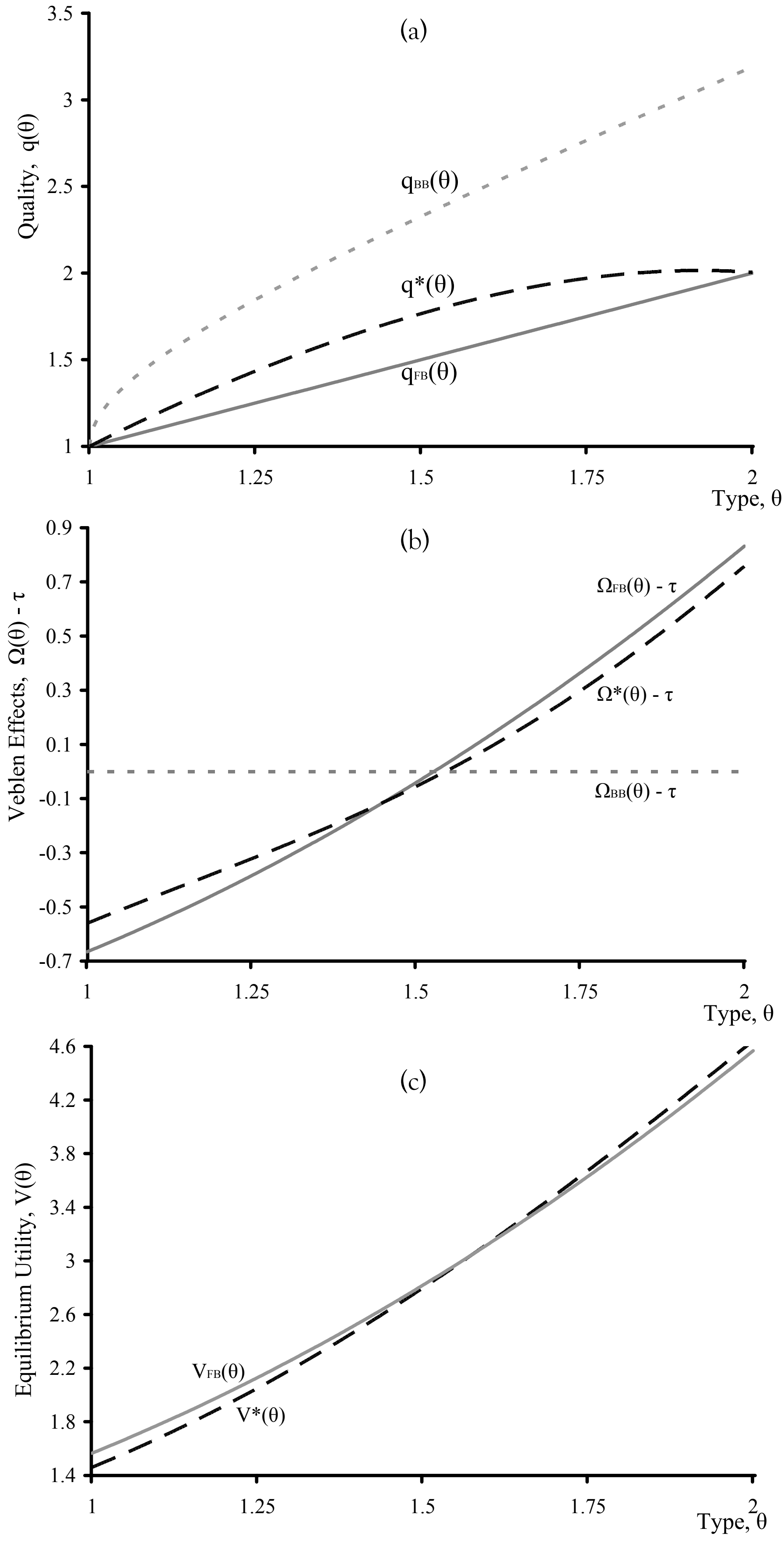


Figure 3

Tax Schedule on Quality