Force, Wealth, and Elections*

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Abstract

Some countries, "failed states," are unable to establish any kind of a peaceful order. A few maintain order without holding elections. Many celebrate peaceful elections in which opposition is either not allowed at all or not given a chance to win. Finally, in some countries elections are competitive and peaceful. During most of modern history, civil peace was maintained when one political force consolidated its military power and potential challengers acquiesced to elections in which they had no chance to win: there is nothing new about "electoral authoritarianism." The shadow of violence fades only when people are wealthy enough that they do not want to bear the costs of fighting to increase their incomes. The paper analyzes why civil peace is frequently difficult to establish, why most often it emerges under the dominance of one party, and why some rulers allow competitive elections and leave office when they lose.

*First draft. Comments will be most appreciated. Already this version reflects comments by students in my seminar on "Elections and Violence," Jennifer Gandhi, and Beatriz Magaloni.
1 Introduction

Some countries, "failed states," are unable to establish any kind of a peaceful order. A few maintain order without holding elections. Many celebrate peaceful elections in which opposition is either not allowed at all or not given a chance to win. Finally, in some countries, those that we identify as democracies, elections are competitive and peaceful. Consolidation of state power is often a protracted process and even when the state becomes sufficiently powerful to ward off potential challengers, elections take place under the shadow of violence. During most of modern history, civil peace was maintained when some political forces consolidated their military power to the point that potential challengers were sufficiently intimidated to acquiesce to elections in which they had no chance to win: there is nothing new about "electoral authoritarianism." The shadow of violence fades only when people are wealthy enough that they do not want to bear the costs of fighting to increase their incomes.

The purpose of this analysis is to understand why civil peace is frequently difficult to establish, why most often it emerges under the dominance of one party, and finally why some rulers allow competitive elections and leave office when they lose. Here is the core of the argument. Parties participate in elections to determine some policy \( x \) over which they have conflicting interests. Having observed the result of an election, they decide whether to respect the outcome or to try to impose their will by force. The probability that the current incumbent would be reelected is \( p \). The probability that the incumbent would prevail in a violent conflict is \( q \). Hence, political actors face two lotteries over \( x \), with different payoffs and different probabilities. The generic conclusion of this way of thinking is that outcomes of elections are obeyed only if electoral chances reflect relative military power, which means that if one party dominates militarily it must also dominate electorally. Already Herodotus (quoted in Bryce 1921: 25-26) thought that in a democracy "physical force of the citizens coincides (broadly speaking) with their voting power," while Condorcet (1986: 11) observed that in the ancient, brutal times, "for the good of peace and general utility, it was necessary to place authority where the force was." Yet the relation between military force and electoral chances becomes less important when people value less whatever they can acquire by fighting. Hence, if conflicts concern income, peace is easier to maintain in wealthier societies.

To pinpoint the role of elections in inducing peace, think somewhat differently. Two groups in society are in conflict over \( x \). They can obtain their ideal outcomes by fighting, with \( q \) regulating their probable victories. If they want to avoid violence, then given \( q \) they could agree to
a $p$ according to which they would alternate in office and thus solve the conflict peacefully. But Fearon (2006) is correct to point out that the desire to solve conflicts peacefully is not sufficient to justify elections. If everyone knows everything, then they also know the expected value of the policy outcome $x$ associated with $q$. Why then hold elections, rather than simply agree to $x = E_q(x_t), \forall t$? Moreover, if utilities are concave in $x$, then agreeing to a fixed $x$ is superior in welfare terms to policies chosen by alternating parties (Alesina 1993). Hence, additional reasons must be adduced to understand why to be peaceful conflicts must be processed by elections. Przeworski (2005) argues that $x$ could not be completely specified and rulers would use their residual power to deviate from any agreement. Fearon (2006) sees elections as a device to coordinate revolt in case the incumbent abuses his power, that is, exceeds $x = E(x_t)$.

Londregan and Vindigni (2006) think that elections are a cheap way to learn the value of $q$, which was already the view of Simmel (1950: 241-2): "because the voting individuals are considered to be equals, the majority has the physical power to coerce the minority.... The voting serves the purpose of avoiding the immediate contest of forces and finding out its potential result by counting votes, so that the minority can convince itself that its actual resistance would be of no avail." But, in the end, all these are arguments to the effect that elections are a peaceful way of processing conflicts that otherwise may have or would have been violent. As Bobbio (1984: 156) put it, "What is democracy other than a set of rules ... for the solution of conflicts without bloodshed."

The generic mechanism by which elections induce peace is that they enable inter-temporal horizons. A party that loses in a conflict may be prone to revert to force if it expects the defeat to be permanent or just indefinite, yet it may respect the result if it believes that at some fixed time it will have a chance to win. This is what elections enable: the prospects of alternation in power. This mechanism, however, works only under some conditions: the purpose of this paper is to identify them. And even if elections can be peaceful given the exogenous conditions, violence may still ensue if political actors miscalculate. Indeed, elections often are and frequently have been accompanied by violence.

In the light of this argument, elections can be peaceful only when the probability that a particular party wins them bears some relation to the probability that this party would impose itself by force. Note that in this barebone sketch the military strength of the ruler is the only exogenous feature of the environment in which incumbents make decisions whether to hold an election or rule without them, whether or not to allow opposition, whether or not to give the opposition a chance to win, and whether or not to yield power if they happen to lose: exactly
the features by which we distinguish political regimes. I argue below that this environment is richer, namely, that we should distinguish the coercive power of the state from the coercive power of the particular incumbents. But this rudimentary formulation is sufficient to see that regimes are structured by the relations of physical force and the kind of elections they hold, if any, is endogenous. Regimes are autocratic when rulers can govern by force alone; they are authoritarian when rulers can muster enough force to hold elections which they are certain to win; they are democratic when rulers are compelled to tolerate a chance that they may lose and are forced to leave when they do.

The paper is structured as follows. The section that follows summarizes some historical patterns and offers some analytical intuitions. It is followed by an analysis of the conditions under which elections are peaceful when everyone shares the belief about the military prowess of the incumbent. This model is then extended to situations in which these beliefs diverge, so that some political actors may miscalculate. A brief summary and some caveats conclude the paper.

2 Some history and intuitions

One striking pattern visible in examining political history of several countries from 1788 until recently is that many experienced long periods during which successive governments were elected and completed their constitutionally specified terms, interrupted by periods during which governments, elected or not, were repeatedly overthrown by force. To get a flavor of such histories, here are some examples (Completed terms are shown by continuous lines at the value of 1; successful coups are indicated by isolated points with value of 1 or 2, period when terms were not completed but there were no coups have the value of 0.):

![Completed terms and coups in Ecuador](image)

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1 All the data used in this section are from Przeworski et al. (2007).
Here is an intuition of how such patterns may emerge. Several political actors contest political power. At some time one contender, thought by everyone to have military power $q(0)$, becomes the ruler. If the potential challengers believe that the military power of the incumbent is low, one of them revolts, seeking to overthrow the incumbent by force. What it often meant in 19th century Latin America was that a local caudillo, who called himself "general," recruited a bunch of his peones ("soldiers"), and invaded the capital or claimed independence for his region. If the incumbent is defeated, the challenger becomes the incumbent, with $q = q(0)$. But if the incumbent defends himself successfully, everyone updates upward their beliefs about his military prowess. When these beliefs pass some critical value, that is, when the potential opponents have learned that an incumbent is proficient in defending himself, they are willing to participate in elections in which they do not have much of a chance to win. A period of peace thus ensues. But this is peace supported by the threat of violence.

Hence, we should expect elections to be often non-competitive and the opposition to be sufficiently intimidated to obey their results. While some voices claim that we are currently witnessing an emergence of a qualitatively new phenomenon, "electoral authoritarianism," such regimes were the prevalent form of political organization throughout history. The idea of an official government list submitted to voters for a plebiscitary approval was present already in France under the Directorate (Crook 1996), used under Restoration, and perfected under Napoleon III (Zeldin 1958). The Spanish monarchy gained in this way such a complete control over voters that between 1876 and 1917 it was able to orchestrate a system in which governments alternated in every elections according to a pre-arranged agreement between parties. The same was true in Portugal between 1851 and 1869. Promoting government candidates was not a transgression but a duty of public officials: the French Prime Minister,
de Vilèlle, issued in 1822 a circular instructing "All those who are members of my ministry must, to keep their jobs, contribute within the limits of their right to the election of M.P.s sincerely attached to the government" (quoted in Zeldin 1958: 79). Partisan use of public administration was ubiquitous in Latin America as well as in Europe. Following Chile after 1831 (about which see Valenzuela 1995), several Latin American countries established stable systems of succession in which incumbent presidents completed their terms, faithfully obeying term limits, chose their successors and used governmental power to assure their victory at the polls. The stability of such systems of oligarchical pluralism – Chile between 1831 and 1891 and again until 1924, Nicaragua between 1856 and 1890, Brazil between 1894 and 1930, Argentina between 1897 and 1916, Uruguay between 1898 and 1932, Mexico between 1934 and 2000 – was remarkable. Indeed, in the entire history of Latin America only three incumbent presidents who presented themselves for reelection ever lost. As Halperin-Donghi (1973: 116) observed, “Among the many ways of overthrowing the government practiced in postrevolutionary Spanish America, defeat at the polls was conspicuously absent.” Nothing is new about Putinism.²

This history poses an intriguing question about civil wars in Africa, namely, whether they are exceptionally frequent or just reflect the length of period since independence and the income levels (about the role of income, see below). After all, civil wars were frequent in 19th century Latin America, and many among them were related to elections (Posada-Carbó 1994, Alonso 2000, Sabato 2008). Here is a comparison of average proportion of countries with completed electoral terms on the two continents, by years since independence.

²Here is how the governor of Murmansk, Jurij Jewdokimow, described the practices of his own party, Only Russia, in the municipal election of March 15, 2009: "Employees of municipal enterprises are being forces to staff mailboxes with fliers calling for voting for the 'proper' candidate. Pre-school teachers have to distribute to parents agitational materials...." (Gazeta Wyborcza, Warsaw, Poland, March 17, 2009).
If we treat completed terms as an indication of civil peace, Africa appears to have been more peaceful than Latin America during the first fifty years of independence. True, many African countries maintained peace under one-party rule, while Latin American elections tended to be contested. But these patterns suggest that consolidation of any kind of stable political regime may just take time and there may be nothing specific about Africa. Moreover, the role of ethnicity in sparking conflicts in Africa becomes doubtful. Perhaps the mechanism is the same as it was in Latin America – local "caudillos" invade the capital or proclaim independence of their region – only that in Africa local happens to coincide with "ethnic."

Why, however, periods of lasting peace would be shattered by violence, which happened in several Latin American countries after 1925 and in Africa in the late 1980s? The most frequent reason, I suspect, is that incumbents sometimes lose elections. Another may be that the partisan postures of the military change exogenously. Yet another possibility is that beliefs diverge, in particular that having won several consecutive elections the incumbent becomes overconfident about his military prowess.

The outcomes of elections are eloquent in showing that electoral defeats of incumbents have been historically rare and peaceful alternations in office even more so. Table 1 summarizes outcomes of elections in which the office of the chief executive was at stake and the subsequent events. The "incumbent" is not necessarily the same person: he or she may be a member of the same party or an otherwise designated successor. "Winner," as well, may be a person or a party. Note that if the incumbent had won, he is the winner; if the incumbent lost, the winner is someone else. "Assumed indirectly" stands for sequences in which the winner assumed office but only after someone else – the loser or a third party – held it unconstitutionally in the immediate aftermath of an
election. "Assumed," whether directly or not, indicates that the winner held office for at least one year, but not necessarily that he completed the constitutionally specified term.

Table 1: Events surrounding elections

<table>
<thead>
<tr>
<th>Incumbent</th>
<th>Winner</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>assumed</td>
<td></td>
</tr>
<tr>
<td>directly</td>
<td>indirectly</td>
<td>assume</td>
</tr>
<tr>
<td>won</td>
<td>1999</td>
<td>9</td>
</tr>
<tr>
<td>lost</td>
<td>473</td>
<td>19</td>
</tr>
<tr>
<td>total ran</td>
<td>2472</td>
<td>28</td>
</tr>
<tr>
<td>did not run</td>
<td>84</td>
<td>6</td>
</tr>
<tr>
<td>unclear</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>2571</td>
<td>37</td>
</tr>
</tbody>
</table>

Most of the time, in $2472/2648 = 0.93$ cases, results of elections were obeyed: someone won, someone lost, and the winner assumed office. Yet most striking is the frequency with which incumbents won elections: $2,103$ out of $2,648$ cases in which they ran, which gives $p = 0.79$, and $4:1$ odds of winning. In the remaining $176$ cases, $0.07$ of elections, the winner either never made it into office or had to overcome some usurper first.

These data suggest that parties are willing to obey outcomes of elections even when current incumbents enjoy an overwhelming advantage. Given the estimate of $p$, the expected tenure in office is five electoral terms, implying that partisan alternations have been infrequent. Note that partisan alternations are peaceful only if the incumbent loses and the winner assumes office, which occurred in $473/2648 = 0.18$ cases, one in $5.6$ elections. Moreover, even if the first partisan alternation in history occurred in the United States in 1801, peaceful alternations have been rare until the last quarter of the past century.
There is also strong evidence that the frequency of peaceful alternations rises steeply in per capita income. The intuitive explanation is that when incomes are higher people care less about increasing them through violence, and if the cost of violence is constant, above some income level they obey even if they lose (Benhabib and Przeworski 2006, Przeworski 2005).

3 When can elections be peaceful?

3.1 Assumptions

There is a set of potential contenders for power, \( j \in \{A, B, \ldots\} \). Contenders have preferences over policy \( x \in R \). Each contender is characterized by an ideal point, \( x^*_j \sim U(\underline{x}, \overline{x}), \underline{x} < 0 < \overline{x} \). Utilities depend on the distance between the ideal and the implemented policy, so that \( U_j(x^*_j, x) = -d(x^*_j, x) = -|x^*_j - x| \). If a violent conflict occurs, the loser also bears the cost of fighting, which is \( c > 0 \).
At some initial time, nature draws a pair of contenders, one of whom becomes the incumbent, $I$, while the other, $O$, is the opposition. The incumbent decides whether or not to hold an election. If an election is held, parties offer platforms $x_j$. Regardless of the view of electoral competition one takes, it must be true that the electoral platforms are located somewhere between their ideal points. For simplicity, I assume that parties propose platforms\(^3\)

$$x_j = \frac{x_j^*}{\bar{x} - \underline{x}}$$

Voters select governments by voting and, given the votes, some rules determine who is the winner.

The incumbent chooses the probability with which he will win the election, $p_I$. This probability depends on the platforms $\{x_I, x_O\}$ but also on a set of instruments, $e_I$, discussed below, that are controlled by the incumbent. Hence, $p_I = p(x_I, x_O; e_I)$.

Once the outcome of the election is revealed, each contender decides whether to obey its result or to use force to impose his ideal policy ("revolt" or "rebel" or simply "fight."). If the election is not followed by a revolt, the winning platform is realized and a new election is held. If a violent conflict occurs, the winner implements his ideal policy and may or may not hold another election. If no election takes place, revolts occur with the periodicity of elections. The loser of a revolt returns to the pool of contenders (or disappears: it makes no difference as long as the number of contenders is sufficiently large) and a new contender takes his place.

Each incumbent is characterized by the military capacity $q_I$, which is probability that the incumbent defeats any challenger in the eventuality of a revolt. This capacity is not observed but everyone, including the incumbent, believes that it is $q$, which need not equal $q_I$. Each new incumbent who enters power by force is believed to have $q(0)$, not necessarily the same for different entrants. Each time the current incumbent defeats a new challenger, all contenders, including the incumbent, update their beliefs according to

$$q(n + 1) \geq q(n),$$

\(^3\)I also assume that $x_I \neq x_O$: platforms do not converge and elections make a difference. If they do converge, $p$ plays no role in decisions whether to obey or revolt. Hence, this model is more general than those of Przeworski (2005) and Benhabib and Przeworski (2006). In turn, it is shown below that as long as the platforms bear some monotonic relation to the ideal points, the specifics of the electoral equilibrium do not matter for the qualitative conclusions.
where \( n \) is the number of challengers the incumbent already defeated. The specific updating rule does not matter at this point.\(^4\)

Everyone is forward-looking but myopic, in a specific sense that actors do not anticipate that their beliefs may change. As a consequence, when calculating the expected value of revolt, the contenders do not consider the possibility that if the current incumbent defeats successive revolts, elections may occur without revolts.

### 3.2 Equilibrium

Let \( j \in \{I, O\} \) indicate the current and \( k \in \{I, O\} \) the hypothetical incumbency status. The strategy of each contender is an action \( a^k_j \in \{\text{Obey}, \text{Rebel}\} \), \( \{a, r\} \) for short, and if an election is held, the probability with which each incumbent wins the election, \( p_I^j \). Hence, the strategy is \( \{a^I_j, a^O_j; p_I^j\} \): the action of the current incumbent or the current opposition as the incumbent as well as the opposition and the probability with which the current incumbent and the current opposition win elections as incumbents.

The equilibrium is characterized as follows:

1. The current incumbent \( I \) knows the value of \( p_I^I \) for which the current opposition \( O \) is indifferent between obeying and rebelling. Let this value be \( \max p_I \). This value depends, in turn, on the maximum value of \( p_O^I \) that would make the current incumbent indifferent between obeying and rebelling if the current opposition would become the incumbent, \( \max p_O \). Note that the current opposition cannot commit itself not to use \( \max p_O \), which is the best reply to any \( p_I \). Given that the best reply to any \( p_O = \max p_I \), this part of the equilibrium is \( \{\max p_I, \max p_O\} \). Because the functions \( \max p_I(p_O) \) and \( \max p_O(p_I) \) are linear, this equilibrium is unique, although not necessarily bounded by \((0, 1)\).

2. While by construction the opposition is indifferent between obeying and rebelling when \( p_I^I = \max p_J \), these values are not sufficient to guarantee that the incumbent obeys. Each incumbent compares the values of obeying and rebelling given the actions of the current opposition as the opposition and as the eventual incumbent. These comparisons determine the action \( a^I_j \). If the incumbent rebels given \( p_I^I = \max p_J \), no elections are held.

3. The equilibrium consists of two pairs of actions \( \{a^I_j, a^O_j\} \) that are best replies to each other and, if elections are held, of probabilities with which incumbents win elections \( \{\max p_I, \max p_O\} \). Hence, the equilibrium is characterized by \( \{(a^I_j, a^O_j), (a^O_j, a^I_j); (\max p_I, \max p_O)\} \).

\(^4\)For example, let \( q \sim \text{Beta}(a, b; n) \) and let the updating process be Bayesian, with the expected value \( E(q|n) = (a + n)/(a + b + n) \).
Definition 1 An equilibrium is peaceful if at least one contender obeys in both incumbency statuses.

Definition 2 An equilibrium is electoral if both parties obey in both incumbency statuses.

3.3 Conditions for an electoral equilibrium

Assume that $q$ characterizes the current incumbent whether he wins or loses an election, that is, as incumbent and as opposition. Also, just as a convention, let $x \leq x^*_I < \mu < x^*_O \leq \bar{x}$, where $\mu$ is the mean of $U(x, \bar{x})$.

Lemma 1 (1) If $q > q^*$, the current opposition obeys as the incumbent if the current incumbent obeys as the incumbent. (2) If $q < q^*$, the current incumbent obeys as the incumbent if the current opposition obeys as the incumbent.

Proof. All proofs are in the Appendix. ■

These threshold values depend on the discount rate, $\rho$, the range of ideal points from which the contenders are drawn, $D = \bar{x} - x$, the cost of fighting, $c$, as well as on the ideal points of the current contenders. Specifically, $\partial q^*/\partial c < 0$ and $\partial q^{**}/\partial x^*_O > 0$ while $\partial q^*/\partial c > 0$ and $\partial q^*/\partial x^*_I < 0$. It is intuitive that the threshold above which the opposition obeys is lower and the threshold below which the incumbent obeys is higher when the costs of fighting are higher. More interestingly, contenders whose ideal points are farther apart opt to fight at a wider range of beliefs about the incumbent’s power. Conversely,

Remark 1 If both the incumbent and the opposition are sufficiently moderate (their ideal points are close to the mean ideal point), they never fight.

Note that as long as the electoral platforms are some monotonic function of the ideal points no generality is lost by picking any electoral equilibrium in which platforms do not converge.

A necessary condition for both the incumbent and the opposition to obey is that $q^{**} < q^*$.

Lemma 2 There is an upper value of the range of ideal points from which contenders are drawn, $D(x^*_I) = \bar{x} - x$, such that $q^{**} < q^*$ only if the range is narrower than this value. In particular, if a defeat in a revolt is costless, $c = 0$, then $q^{**} < q^*$ only if $D(x^*_I) < \frac{2-\rho}{1-\rho}$. If $c > 0$, this range is broader.
Hence, an electoral equilibrium does not exist if the polity from which the incumbents are drawn is highly polarized and the costs of fighting are relatively low.

**Lemma 3** If the current opposition obeys as the incumbent, \( q > q^* \), a sufficiently extreme current incumbent, \( x_I > -c \), establishes a one-party system, setting \( \max p_I = 1 \), at some \( q^* < q < 1 \). Otherwise, \( \max p_I < 1 \) for all \( q^* < q \leq 1 \). The corresponding conditions for when the current opposition becomes the incumbent are that \( \max p_O = 1 \) at some \( 0 < q < q^* \) if \( x_O > c \) and \( \max p_O < 1 \) for all \( 0 < q < q^* \) otherwise.

As this Lemma shows, electoral equilibria need not be competitive.

**Definition 3** An electoral equilibrium is competitive if \( \max p_j < 1 \) for both parties.

To remind, a non-competitive electoral equilibrium is not the same as a peaceful equilibrium in which one party rebels and the other obeys. Even in one-party systems, the incumbent gets only the value of his electoral platform, while if the incumbent rules by force he gets his ideal policy. Hence, the incumbent may rebel even if he is sufficiently strong to maintain a one-party rule.

**Proposition 1** If \( q \) does not depend on outcomes of elections and if \( q^* < q < q^* \), incumbents implement \( \{ \max p_L, \max p_R \} \) and both parties obey whether they are the incumbent or the opposition. The electoral equilibrium is always competitive if both parties are moderate. Otherwise, there exists some \( q^* < q < q^* \) for which at least one party establishes a one-party system.

Particularly interesting is the state \( q < q^* < q^* \), in which the current opposition rebels when it becomes the incumbent. If the incumbent defeats some number of consecutive revolts, the challengers learn that he is stronger than they had thought. Then the process moves to the state \( q^* < q < q^* \), in which election results are obeyed. Note that if \( q^* < q(0) < q^* \), they are always obeyed.

To provide an intuition, here is an example.

**Example 1** Suppose \( q^* = 0.54, q(0) = 0.4, q^* = 0.75 \), the incumbent is \( A \), the opposition \( B \). \( A \) holds an election and loses. Because \( q(0) < q^* \), \( B \) rebels as the incumbent. Suppose that \( A \) defeats \( B \), \( q \) increases to
\( q(1) = 0.5^5 \) and a new contender, \( C \), emerges to take \( B \)'s place. Given \( x^*_C \), if \( C \) wins an election, a revolt occurs again. Now \( C \) defeats \( A \), becomes the new incumbent, with beliefs about his power reverting to \( q(0) \), and \( D \) is the new opposition. This process continues until the incumbent \( L \) defeats two successive contenders and when the current challenger is \( R \), \( q \) rises to \( q(2) = 0.55 > q^{**} \). Given \( q(2) \), an election occurs with \( \max p_L = 0.8, \max p_R = 0.76 \) and everyone obeys. The incumbent \( L \) wins \( 1/(1 – \max p_L) = 5 \) successive elections. Since no new information is generated by these elections, the opposition \( R \) obeys if it wins.

### 3.4 Other equilibria

Suppose that \( q > q^* \) or \( q < q^{**} \), so that either the current incumbent rebels or the current opposition would rebel as the incumbent. If the opposition obeys when the current incumbent rebels, the current opposition gets \( U_O(x^*_I)/(1 – \rho) \); if the current incumbent obeys in opposition when the current opposition rebels as the incumbent, the current incumbent would get \( U_I(x^*_O)/(1 – \rho) \). If they rebel, they get their expected values of revolt.

**Lemma 4** If costs of defeat are relatively high and if the rebelling incumbent is relatively moderate, the opposition accepts the rule of the incumbent.

The intuition is that if either contender rebels in opposition and is defeated, someone else may revolt and replace the current ruler. If the current ruler is moderate, it may be better for the opposition to tolerate this ruler than to risk being defeated at the cost \( c \).

Now, suppose that the current incumbent knows that he will obey if the current opposition rebels as the incumbent. Will the current incumbent obey as the incumbent?

**Lemma 5** If the costs of defeat are relatively high and if the current opposition is relatively moderate, the current incumbent obeys if the current opposition would rebel as the incumbent and the current incumbent would obey in opposition. Otherwise, the current incumbent rebels as the incumbent.

We can now establish the conditions when equilibria are peaceful and violent.

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5This will be true if \( q \sim Beta(2, 3; n) \), generating posteriors \( E(q|n) = (2 + n)/(2 + 3 + n) \).
Proposition 2 If the power of the incumbent is very high or very low, \( q > q^* \) or \( q < q^{**} \), and (1) if the party that rebels is moderate, equilibrium is peaceful, (2) if the party that rebels is more extreme, the equilibrium is violent.

This proposition says that if one of the parties is militarily strong and moderate in terms of its policy preferences, it imposes its rule by the mere threat of force without having to apply it, because the opposition does not want to engage in a potentially costly fight that may result in a more extreme ruler taking power. If the current incumbent is more extreme, however, then the opposition takes the risk of rebelling if the incumbent rebels.

3.5 Partisanship of the military

The analysis above was based on the assumption that the military power of the incumbent is not affected by outcomes of elections. But one can easily imagine that an electoral defeat changes the actual relations of physical force and the beliefs about them.

Let the first time when an election does not generate a revolt be \( t = T \), so that the incumbent holds this election when everyone believes that \( q = q_T > q^{**} \). If the incumbent wins, there is no reason for beliefs to change and the opposition obeys. But suppose that he loses. What are the contenders to believe about their respective military strength? This question matters because there may be situations in which elections are peaceful only if the same party always wins, so that partisan alternation in office cannot be peaceful.

Consider two extreme possibilities. In one, beliefs do not change: in all subsequent elections not followed by a revolt everyone continues to believe that the original incumbent’s strength is \( q_T \). At the other extreme, all winners of elections believe to have the same coercive power. Hence, if the original incumbent entered with \( q_T \) and is defeated, everyone believes that the power of the new government is \( q_T \).

The first situation would hold if the repressive apparatus is known to be completely partisan, standing behind the same party in or out of office. The second case would occur if the coercive forces are known to be perfectly neutral, supporting all winners of elections equally. Let \( s \) be the, common knowledge, loss of power that an incumbent suffers from a defeat in the election, so that when he loses he believes that his strength in opposition is \( q_T - s \), while the former opposition believes that its power an as incumbent is \( 1 - (q_T - s) \). If the military are completely partisan, \( s = 0 \). In turn, if they are completely neutral, the defeated incumbent knows that the power of the new government must be \( q_T \) and, because it
must be true that \(1 - (q_T - s) = q_T\), \(s = 2q_T - 1\). Hence, \(0 \leq s \leq 2q_T - 1\) characterizes the partisan neutrality of the coercive forces (or, if they are divided along partisan lines, the balance of force between them).

The loss of coercive power in the face of electoral defeat may occur because such events unify citizens against rulers who seek to remain in power (as in Fearon 2006). An alternative mechanism is that electoral defeats induce splits within the ruling block (Magaloni 2007). When a ruler holds an election and is unable to secure a victory, members of the coercive apparatus must envisage the possibility that the ruler may fall and they may be held individually responsible for of repression. Hence, they become hesitant to take this risk (Przeworski 1988): after General Pinochet lost the 1989 plebiscite to extend his term, the other members of the Chilean junta refused to override the result by force (Barros 2002).

Suppose the military are unconditionally partisan, \(s = 0\). The incumbent loses an election and is now in opposition, believing that the military power of the electoral winner is \(1 - q_T\). It may well be that \(1 - q_T < q^* < q_T < q^*\), so that a rebellion ensues. In turn, suppose that the military are perfectly constitutionalist, \(s = 2q_T - 1\). Then the defeated incumbent knows that the military power of the electoral winner is \(q_T\) and he obeys. In turn, the former opposition knows that its power as the incumbent is \(q_T\) and it also obeys. There is thus a minimum level of military non-partisanship, \(s^*\), such that partisan alternations are peaceful if \(s \geq s^*\).

**Proposition 3** If the military are sufficiently neutral politically, \(s \geq q^* + q_T - 1 \equiv s^*\), peaceful elections are an absorbing state. If the first electoral incumbent does not revolt when he loses and if \(s\) remains at \(s \geq s^*\), all subsequent alternations are peaceful.

**Example 2** Suppose the incumbent \(L\) loses an election when \(q_T = 0.55\). Then if the military continue to support the defeated incumbent, \(s < s^* = 0.09\), the incumbent rebels. If, however, the military are sufficiently neutral, \(s > 0.09\), the incumbent accepts the defeat and if he wins any subsequent election, so does the defeated party.

Partisanship of the coercive apparatus thus places a wedge between the coercive power of the state and the repressive capacity of the particular incumbents. States may be highly effective in organizing and monopolizing the coercive force, yet those who wield this force may be non-partisan, constitutionalist, so that the control over the repressive apparatus rests in the hands of elected civilian governments: such states are characterized here by a high \(q\) and a high \(s\). For example, General
Fidel Ramos in the Philippines supported Corazon Aquino after President Ferdinand Marcos was caught at having committed fraud in the election of 1985 and competitive elections ensued. In turn, other states may be able to utilize powerful repressive forces for partisan purposes: they have a high $q$ and a low $s$. An example is the Russian army executing the order of President Boris Yeltsin to shell the parliament. But some states are just coercively weak. "Failed states" are precisely those whose military power can be easily contested: they have a low $q$ and thus necessarily a low $q - s$. In such states the electoral incumbents cannot prevent armed challenges even if the military forces are partisan, simply because the state as an organization cannot muster enough physical force.

3.6 Income dependence

Suppose that the utility functions $U_j(x^*_j, x; y, c)$ are of the form $-d_j(x^*_j, x)/y - \delta c$, where $\delta$ indicates whether a contender lost in a violent conflict. Intuitively, this assumption says that the distance to the ideal point matters less in countries with higher per capita income, $y$.

**Proposition 4** Given $U_j(x^*_j, x; y, c) = -d_j(x^*_j, x)/y - \delta c$, $q^* = q^*(y)$ and $q^{**} = q^{**}(y)$, where $\partial q^*/\partial y > 0$ and $\partial q^{**}/\partial y < 0$. Moreover, $\lim_{y \to \infty} q^* = 1$ and $\lim_{y \to \infty} q^{**} = 0$.

As income increases, the incumbent obeys when he has more military power and the opposition obeys when it believes that the incumbent has less military power. When income is sufficiently high, both obey regardless of their military power. Note that because both thresholds depend on $x^*_j$, some contenders may obey while others would rebel, at the same income level. But the shadow of violence fades in economically developed societies.

Here is a story. There was an election in Costa Rica in 1948, when that country had per capita income of about $1,500 (1985 PPP, from PWT5.6). The election was technically tied: the two candidates received almost the same number of votes and there were widespread allegations of fraud, so that it was impossible to determine who in fact did win. It was not clear who should decide, but the Congress took it upon itself to declare as the winner the candidate who officially received somewhat fewer votes. A civil was ensued, in which about 3000 people were killed. At another time, there was an election in another country. The election was technically tied: the two candidates received almost the same number of votes and there were widespread allegations of fraud, so that it was impossible to determine who in fact did win. It was not clear who
should decide, but the Supreme Court, appointed in part by one of the
candidates father, took it upon itself to declare as the winner the candi-
date who officially received somewhat fewer votes. Then everyone drove
home in their SUVs to cultivate their gardens. They had SUVs and
gardens because this country has per capita income of about $20,000.

4 Why elections may be violent even if they need not be?

4.1 A general framework
As in many models of inefficient conflict, elections may not be peaceful
when beliefs differ. While Londregan and Vindigni (2006) argue that
votes can be used to estimate the military strength, this can be true
only if the vote count is perceived as accurate. And there are reasons
to believe that often it is not. While the logical implications of differing
beliefs may be easy to intuit, the mechanisms that generate violence
merit scrutiny.

Elections must inextricably follow some rules that regulate who can
vote, whether voting is direct or indirect, secret or public, compulsory
or voluntary, how votes are aggregated, and so on. And rules affect out-
comes. Even minute details, such as the color of ballots, location of the
polling places, or the day of the week when voting takes place can affect
the result. Hence, elections are inextricably manipulated. Manipulation,
however, can be more or less blatant. Somehow we feel that carving
electoral districts in the form of a salamander is excessive, while making
districts nicely square does not raise anyone’s eyebrows. Manipulation
is a matter of degree, which is represented here by \( m \). Think of \( p_I(v, m) \)
as the probability that an incumbent who enjoys support of \( v \) percent
of the citizens given \( \{x_I, x_O\} \), wins the election having manipulated the
rules to the extent \( m \).

Manipulation consists of establishing rules under which elections are
conducted. Hence, it is visible. But its consequences are not always
easy to determine. Even political scientists need to revert to simulations
in order to identify the effects of electoral systems. Consequences of
rules are a matter of judgement. In Nicaragua the opposition decided
at the last moment to boycott the election of 1984, believing that the
Sandinista government had manipulated the rules to the point that the
opposition had no chance. Yet in 1990 it won under the same rules.

\[ \text{Note that even “the opposition” can be a product of manipulation. For example,}
\text{the incumbent may allow some parties to participate and ban others (Lust-Okar 2005:}
\text{27-28).} \]
Manipulation is not the same as fraud.\textsuperscript{7} Fraud entails breaking rules, however biased they may be. Breaking into the office of the opposition party to steal its secrets is fraud because it violates a general prohibition against burglary. Buying votes constitutes fraud when it is prohibited by specific rules. So is casting votes of people whose spirits have passed to a better world. And setting rules and breaking rules are subject to different reactions. The same physical act – a campaign contribution – has different political consequences when it is permitted by law and when it is illegal: “institutional facts have some autonomy with regard to brute facts” (Sánchez-Cuenca 2003: 81-82). The technology of fraud is highly varied (Simser 2006, Lehoucq 2003), but in almost all of its forms fraudulent activity is clandestine.

Manipulation and fraud are substitutes.\textsuperscript{8} Let $f$ stand for the degree of fraud. The probability that the incumbent wins is then $p_I = p(v; m, f)$, increasing both in $m$ and in $f$. Clearly, the same probability may be generated by a lot of manipulation with little fraud or by little manipulation with extensive fraud. But the difference is that, even if their consequences may be difficult to assess, rules are explicit and public, so that they are known before the election occurs, while fraud can be detected before the election (rigging voter rolls), during (preventing some people from voting), and after (while counting). Hence, one can expect that pre-election violence occurs when the opposition believes that the incumbent gives it too little of a chance to win (For a model in which smaller parties are more inclined to use violence, see Chaturvedi 2005). In turn, while post-election violence may be a belated reaction to the defeat caused by manipulation, it is more likely to result from discovery of fraud. Note that in Magaloni’s (2007) model, if fraud is unobservable and there is more than one opposition party, one of them may allege fraud even if there was none.

\subsection*{4.2 Analysis}

Assume that the stage game has the following structure: (1) The incumbent decides whether to hold an election. Because we want to analyze the situation in which elections are be peaceful, assume that $q < q^*$, that is that he does hold them. (2) The incumbent chooses the extent of manipulation, $m$. He can choose $m$ such that $p_I = p(v; m) \leq \max p_I$ or such that $p_I = p(v; m) > \max p_I$. (2.1) If the opposition concludes that $p_I = p(v; m) \leq \max p_I$, it participates in the election and the in-

\textsuperscript{7}On the difficulties of defining fraud, see Annino (1995: 15-18).

\textsuperscript{8}To some extent they may be complements: rules may make fraud easier or more difficult to commit. In this case $p_I = p(v; m, f(m))$. But as long as $\partial p/\partial f$ is positive the results presented below stand.
incumbent chooses the degree of fraud, $f$, which again can be such that $p_I = p(v; m, f) \leq \max p_I$ or $p_I = p(v; m, f) > \max p_I$. (2.1.1) If the opposition believes that $p_I = p(v; m, f) \leq \max p_I$, it obeys the outcome. (2.1.2) If the opposition believes that $p_I = p(v; m, f) > \max p_I$, it rises in protest. (2.2) If the opposition concludes that $p_I = p(v; m) > \max p_I$, it threatens to rebel. The incumbent then has two choices: (2.2.1) Reduce $m$ and bring $p_I = \max p_I$ down to $\max p_I$, in which case the game enters into stage (2.1), or (2.2.2) Maintain $p_I > \max p_I$, and face a rebellion. Hence, electoral violence can occur either if the incumbent manipulates excessively or if he is caught having committed excessive fraud.

Suppose that beliefs about the military power of the incumbent differ: specifically, the incumbent believes that his strength is $q^* > q_T$, where $q_T$ is what the opposition believes. Consider first the manipulation stage. The incumbent chooses $p_I = \max p_I(q_I)$, which is the value of $p_I$ that would make the opposition indifferent had it believed that $q = q_I$. But if the opposition believes that $q = q_T < q_I$, the opposition rebels given $p_I = \max p_I(q_T)$. Say the opposition threatens to boycott the election. There are two questions to investigate: (1) How much is the incumbent willing to reduce thus $p_I$ and thus $m$? (2) What are the conditions for elections to be peaceful given divergent beliefs about $q$?

**Proposition 5** When beliefs about $q$ differ, $q_I > q_T$, the incumbent is willing to reduce his probability of winning to $\max p_I(q_T) < \max p_I(q_I)$ and the opposition obeys if (1) $q_I$ is not too high and (2) the incumbent is not too extreme. The divergence of beliefs, $q_I - q_T$, has only a small effect.

**Corollary 1** Given that $p_I = \max p_I(q_T)$ and that $dp_I/dm > 0$, there is a maximum degree of manipulation, $m^*$, under which the opposition participates in elections.

**Proof.** Obvious. ■

To study fraud, we can now use the model of Gandhi and Przeworski (2009), in which the degree of manipulation is constrained exogenously but the incumbent optimally chooses the extent of fraud, $f^*$, thought of as the proportion of votes the incumbent falsifies.

Now, even if particular acts of fraud can be detected, the extent of fraud is not observable. Suppose that the opposition believes that, after all the manipulation, some proportion $v_I(m^*)$ is certain to vote for the incumbent and some proportion $v_O(m^*)$ is certain to vote against him. If the probability that the voters whose intentions are not known
vote one way or another is uniformly distributed, the probability that the incumbent wins is \( p_I = \frac{0.5 - F(v_O)}{1 - F(v_O) - F(v_I)} \). Suppose that \( v_I = 0.4 \) and \( v_O = 0.3 \), so that \( p_I = 2/3 \). The results of the election are announced and the incumbent is declared to have won 60 percent of the vote. The opposition cannot tell if this result is accurate or fabricated. It may actually calculate that the expected vote share of the incumbent was 60 percent \((Ev_I = v_I + p_I * (1 - v_I - v_O))\) but it does not know what proportion of the 0.2 share the incumbent claims to have won among the voters whose intentions were unknown were cast as such. Even with the best monitoring technology, this is simply unknowable: think of 2000!

This is not to say that monitoring technology does not matter. Let \( r(f) \) be the probability that fraud is detected, given its magnitude, with \( r'(f) > 0, r''(f) > 0 \). Given that the opposition is indifferent between obeying and rebelling when the extent of manipulation is \( m^* \), if the opposition detects fraud it rebels. Hence, \( r(f) \) is the probability that the opposition rebels if the incumbent declares himself to be the winner.

The results of Gandhi and Przeworski (2009) now hold in the present framework:

**Proposition 6** The extent of fraud increases in the degree of manipulation until the probability of detection becomes dissuasive. Formally, \( \frac{\partial f}{\partial m} \geq 0 \) if \( r(f^*) \leq r^*(q, s, \rho) \), where \( r^* \) is some critical value.

**Proposition 7** Incumbents with higher military power commit more fraud, \( \frac{\partial f^*}{\partial q} > 0 \).

**Proposition 8** The optimal degree of fraud is lower when the coercive forces are less partisan, \( \frac{\partial f^*}{\partial s} < 0 \).

**Proof.** See Gandhi and Przeworski (2009) ■

Note that if they can get away with it, that is, if they do not fear that fraud would be detected, incumbents commit some fraud even if they have a good chance to win without it. Secondly, contrary to Schedler (2002: 46), the capacity to repress is what enables fraud: an incumbent who is less afraid to be overthrown if fraud would be detected, engages in more of it. Finally, if the military are less partisan, that is, if they turn against an incumbent caught at fraud, the incumbent uses less of it.

5 **Elections, violence, and competition**

Countries suffer from civil strife when the state does not have the capacity to deter armed challenges and political forces are polarized in terms
of their policy preferences. When the rulers are militarily strong but moderate, they can sustain order without holding elections. Elections occur when the polity is not highly polarized and neither the current incumbent nor the potential challengers have an overwhelming military advantage. Their results are obeyed independently of what happen to be if the military are non-partisan or if the country has a high income. When beliefs about the power of the incumbent diverge, incumbents may attempt to engage in manipulation which the opposition judges to be excessive or may commit fraud, provoking a rebellion.

Peaceful elections are not necessarily, indeed they are rarely, competitive. Most peaceful elections are neither "fair," nor "genuine," nor "democratic," to use the language of election monitoring agencies. Militarily strong but moderate political parties can maintain themselves in office holding "elections" without any opposition or maintaining an overwhelming chance to win, with a passive consent of the potential challengers. Only when the military power is balanced or the military are constitutionalist or incomes are high and when the competing parties are moderate are elections competitive and peaceful.

The glaring weakness of this model, and of the more general approach it implements, is that the repressive agents are not treated as a full-fledged strategic actor. Obviously, the military intervene in politics not only on behalf or behest of civilians but at times for their own reasons (Acemoglu, Ticchi, and Vindigni 2008). Moreover, while the violence analyzed here was treated as strategic, some of it must be spontaneous, unpremeditated, resulting simply from flaring tempers.

Note that while some dynamic implications of the model are transparent they have not been systematically derived. Finally, while this model is inspired by intuitions gleaned from historical data, it must be subjected to a systematic empirical evaluation.

6 Appendix: Proofs

6.1 Value functions

It is useful to distinguish the contenders separately from their incumbency status. Let the current pair of contenders be \( j \in \{L, R\} \) and their incumbency status \( k \in \{I, O\} \). In principle, values must be defined for each party in each incumbency status given its own actions and the ac-

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*European Union observers want elections to be "open and fair," OSCE wants them to be "genuine," while the Declaration of Principles for International Elections Observation of the Carter Center and NDI wants them to be "genuine and democratic." All of them also want them to be non-violent. See respectively European Commission (2008), OSCE (2007), Carter Center (2005).*
tions of the other party in each incumbency status. These values are thus \( V_j^k(a_I^k, a_O^k; a_R^k, o_R^k) \). For example, one such value is \( V_I^I(o_I^o, r_I^r; o_R^O, r_R^R) \): the value of \( L \) as the incumbent if he obeys as an incumbent and rebels in opposition given that \( R \) rebels as an incumbent and obeys in opposition. There are \( 2 \times 2^4 = 32 \) such values for each party. To ease the notation, \( V_j(a) \) stands for the value of \( j \) if the actions of both parties in both incumbency statuses are \( a \).

For party \( L \) these values are given by

\[
V_I^I(o) = U_L(x_L) + \rho p_L V_I^I(o) + \rho(1 - p_L)V_O^O(o)
\]  
(1)

\[
V_O^O(o) = U_L(x_R) + \rho(1 - p_R)V_I^I(o) + \rho p_R V_O^O(o)
\]  
(2)

or

\[
V_I^I(o) = \frac{U_L(x_L) + \rho(1 - p_L)V_O^O(o)}{1 - \rho p_L}
\]  
(3)

\[
V_O^O(o) = \frac{U_L(x_R) + \rho(1 - p_R)V_I^I(o)}{1 - \rho p_R}
\]  
(4)

Making the substitutions yields

\[
V_I^I(o) = \frac{(1 - \rho p_R)U_L(x_L) + \rho(1 - p_L)U_L(x_R)}{(1 - \rho) [(1 - \rho p_R) + \rho(1 - p_L)]}
\]  
(5)

\[
V_O^O(o) = \frac{\rho(1 - p_R)U_L(x_L) + (1 - \rho p_L)U_L(x_R)}{(1 - \rho) [(1 - \rho p_R) + \rho(1 - p_L)]}
\]  
(6)

Note that if \( x_L = x_R = x \), that is, parties adopt the same platform, the probabilities of winning do not matter and \( V_I^I(o) = V_O^O(o) = U_L(x)/(1 - \rho) \).

To derive the value of rebelling, think as follows. If \( L \) wins the current rebellion, it gets \( U_L(x_L^* \mu) \) during the current period and a discounted continuation value. If it loses, it is out of power and \( R \) is in, so that during this period \( L \) gets \( U_L(x_R^*) \). But \( R \) can be deposed and replaced by someone else. Hence, over the long run, \( L \) can expect to get \( U(\mu - c) \), where \( c \) is the discounted cost of fighting, which he bears only if he is deposed. Thus,

\[
V_L(r) = \frac{q U_L(x_L^*) + (1 - q)U_L(\mu - c)/(1 - \rho)}{1 - \rho q}
\]

But \( U_L(x_L^*) = 0 \). Hence,
\[ \frac{1 - q}{1 - \rho q} \frac{U_L(\mu - c)}{1 - \rho} \]  
(7)

Analogously for \( R \), skipping some steps,

\[ \frac{U_R(x_R) + \rho(1 - p_R)V_R^O(o)}{1 - \rho p_R} \]  
(8)

\[ \frac{U_R(x_L) + \rho(1 - p_L)V_R^I(o)}{1 - \rho p_L} \]  
(9)

\[ \frac{q}{1 - \rho(1 - q)} \frac{U_R(\mu - c)}{1 - \rho} \]  
(10)

### 6.2 Proposition 1

#### 6.2.1 Assumptions

The following assumptions simplify the algebra and reduce the proliferation of cases:

**Assumptions:**

(i) Without a loss of generality, assume that the distribution \( F(x_R^+) \) is symmetric, so that \( \mu = 0 \), and with some loss of generality that (ii) \( D = \bar{x} - \underline{x} > 1 \) and that (iii) all \( L \)'s are drawn from \([\underline{x}, 0)\) and all \( R \)'s from \((0, \bar{x})\).

#### 6.2.2 Lemma 1

Substituting \( V_R(r) \) into \( V_R^I(o) \) shows that \( R \) obeys as an incumbent only if

\[ \frac{U_R(x_R) + \rho(1 - p_R)V_R(r)}{1 - \rho p_R} > V_R(r) \]

or

\[ U_R(x_R) > \frac{q}{1 - \rho(1 - q)} U_R(\mu - c), \]

which defines \( q^{**} \). After substituting the utilities into this expression, we learn that

**Condition 1** \( R \) obeys as the incumbent if \( q > \frac{(D-1)(1-\rho)}{1+(D-1)(1-\rho)+c/x_R} \equiv q^{**} \).

Note that because \( x_R > 0 \), \( \partial q^{**}/\partial x_R > 0 \), and since \( x_R = x_R^*/D, \partial q^{**}/\partial x_R^* > 0 \). Moreover, as \( x_R^* \to 0 \), so does \( q^{**} \), meaning that if \( R \) is very moderate he never rebels. The intuitive reason is that as \( x_R^* \to \mu \), there is little to gain by rebelling.

In turn, \( L \) obeys as the incumbent if
\[
U_L(x_L) > \frac{1 - q}{1 - \rho q} U_L(\mu - c),
\]

which yields

**Condition 2** \(L\) obeys as the incumbent if \(q < \frac{1-c/x_L}{1+(D-1)(1-q)-c/x_L} \equiv q^*.\)

Note again that \(\partial q^*/\partial x_L^* > 0\), and that \(q^* \to 1\) as \(x_L^* \to 0\), so that an \(L\) close to the mean never rebels.

While it was easier to keep track of who is who by labelling parties, the results for \(L\) apply to any current incumbent and for \(R\) to any current opposition. If \(R\) is the incumbent, then these conditions apply by placing \(c=x_L\) for \(c=x_R\) in Condition 1 and vice versa in Condition 2. Note only that if we took \(R\) to be the incumbent, then \(\partial q^*/\partial x_L^* < 0\) and \(\partial q^*/\partial x_R^* < 0\).

### 6.2.3 Lemma 2

Comparing these conditions shows that \(q^{**} < q^*\) if

\[
(D - 1)^2(1 - \rho)^2 < 1 + cD\frac{x_L^* - x_R^* - cD}{x_L^* x_R^*}.
\]

Consider first the case when \(c = 0\) and rewrite this condition as \(D < 1 + \frac{1}{1-\rho} \sqrt{1},\) yielding \(D(c = 0) < \frac{2-\rho}{1-\rho}\). Now, let \(c > 0\) and rewrite it as \(D < 1 + \frac{1}{1-\rho} \sqrt{1 + g(D)}\). Because \(g(D) > 0, D(c > 0) > D(c = 0)\). Hence, if fighting is costly to the loser, the range of \(q\) in which both parties can obey is larger.

### 6.2.4 Lemma 3

The \(p_L\) that satisfies \(V_R^O(o) \geq V_R(r)\) is

\[
p_L \leq 1 + \frac{1}{\rho - p_R} \frac{(1 - \rho(1-q))U_R(x_L) - qU_R(\mu - c)}{(1 - \rho(1-q))U_R(x_R) - qU_R(\mu - c)} \equiv \max p_L \quad (11)
\]

We already know that if \(q > q^{**}\), the denominator of the fraction is positive. Hence, \(\max p_L < 1\) if the numerator is negative. Substituting the utilities shows that the value of the numerator is 0 if \(U_R(\mu - c) = -x_R^*-c = U_R(x_L) = -x_R^*+x_L\), or \(x_L = -c\). If \(x_L = -c\), the numerator is negative for all \(q \leq 1\); otherwise it becomes positive at some \(q^{**} < q < 1\).

In turn, the value \(p_R\) which satisfies \(V_L^O(o) \geq V_L(r)\) is

\[
p_R \leq 1 + \frac{1}{\rho - p_L} \frac{(1 - \rho q)U_L(x_R) - (1-q)U_L(\mu - c)}{(1 - \rho q)U_L(x_L) - (1-q)U_L(\mu - c)} \equiv \max p_R \quad (12)
\]

and the analysis is analogous.
6.2.5 Proposition 1
Lemma 1, 2, and 3 imply Proposition 1. Note only that Lemma 1 holds if the current opposition is indifferent between obeying and rebelling, which means that it applies only if the current incumbent obeys or if the current incumbent rebels and the current opposition revolts in response, but not if the current incumbent rebels and the current opposition obeys.
Note as well that the best response of incumbent $L$ to a rebellion by $R$ cannot be to choose a lower $p_L$. No promise by $R$ to lower $\max p_R$ would be credible, so that if $L$ induces $R$ to obey in opposition it has to anticipate that $R$ will operate at $\max p_R$ if it becomes the incumbent. Because $\partial V_L^I/\partial p_L > 0$, reducing $p_L$ would make $L$ even worse off.

6.3 Proposition 2
6.3.1 Lemma 4
If $q < q^{**}$, $R$ rebels as the incumbent. If $L$ obeys in opposition in response to $R$ rebelling as the incumbent, $L$ can expect to get $U_L(x^*_R)$ for ever. Hence, $L$ obeys if

$$U_L(x^*_R) > \frac{1-q}{1-\rho q} U_L(\mu - c).$$

Substituting utilities yields

$$x^*_L - x^*_R > \frac{1-q}{1-\rho q} (x^*_L - c).$$

This inequality defines $q' < q^*$ such that if $q < q'$, $L$ obeys in opposition when $R$ rebels as the incumbent. Note that if $x^*_R > c$ this inequality is never satisfied.

If $q > q^*$, $L$ rebels as the incumbent. $R$ obeys if

$$x^*_L - x^*_R > \frac{q}{1-\rho(1-q)} (x^*_R - c),$$

which defines $q'' > q^{**}$ such that if $q > q''$, $R$ obeys in opposition when $L$ rebels as the incumbent. This inequality can be satisfied only if $-x^*_L < c$.

6.3.2 Lemma 5
Suppose now that $L$ is the incumbent and $L$ knows it would obey if $R$ was to rebel as the incumbent. The $L$ incumbent obeys if

$$\frac{U_L(x_L) + \rho (1-p_L) U_L(x^*_R) / (1-\rho)}{1-\rho p_L} > V_L(r).$$
Now, if $L$ obeys as incumbent when in opposition it is indifferent between obeying and rebelling and if $U_L(x_R^*)/(1 - \rho) > V_L(r)$, that is, $q < q'$, then it obeys as incumbent knowing it would rebels in opposition in response to a rebellion by the $R$ incumbent as long as $q < q'$. If $q > q'$ then it rebels in opposition facing a rebellion by the $R$ incumbent, which by Lemma 1 implies that $L$ obeys as the incumbent as long as $q < q^*$. Analogous reasoning leads to the conclusion that $R$ obeys as the incumbent knowing that it would obey in opposition if $L$ rebels as the incumbent if $q > q^*$.\[\text{6.3.3 Proposition 2}\]

Follows directly from the two Lemmas. There are several cases:

(1) If $q < q^{**} < q' < q^*$, the equilibrium is \{L obeys as the incumbent, R obeys in opposition; L obeys in opposition, R rebels as the incumbent\},

(2) If $q^{**} < q'' < q^* < q$, \{L rebels as the incumbent, R obeys in opposition; L obeys in opposition, R obeys as the incumbent\}.

These two equilibria are peaceful but not electoral.

(3) If $q' < q < q^{**} < q^*$, \{L obeys as the incumbent, R obeys in opposition; L rebels in opposition, R rebels as the incumbent\},

(4) If $q^{**} < q^* < q < q''$, \{L rebels as the incumbent, R rebels in opposition; L obeys in opposition, R obeys as the incumbent\}.

These equilibria are violent, respectively when $L$ or $R$ are the incumbents.

\[\text{6.4 Proposition 3}\]

The proof proceeds in two steps: I first analyze the case when $c = 0$, so the types do not matter, and then modify the result to incorporate different types.

**Step 1: $c = 0$** By convention, the first incumbent who holds an election without a revolt is $L$. Suppose that $L$ loses an election. Now he believes that his power is $q_T - s$ and that the power of the $R$ incumbent is $1 - (q_T - s)$. Hence, $L$ obeys in opposition if $1 - (q_T - s) > q^{**}$ or if $s > q_T + q^{**} - 1$. In turn, the $R$ incumbent believed that the power of $L$ was $q_T$ and its own power was $1 - q_T$, so that $R$ incumbent obeys if $1 - q_T + s < q^*$ or $s < q^* + q_T - 1$. But $R$ knows that if the military is perfectly neutral, his power as an incumbent cannot be higher than that of the incumbent just deposed, which $R$ believed to be $q_T$. Hence $R$ cannot believe that $1 - q_T + s > q_T$ or that $s > 2q_T - 1$. And because $q^* + q_T - 1 > 2q_T - 1$, if $L$ obeys when he loses then $R$ always obeys when he wins.

Hence, both contenders obey when $L$ loses the first election if $s >$
Now suppose that the incumbent $R$ lost a subsequent election and $s = s^*$. Moving to the opposition, $R$ believes that the power of the $L$ incumbent is $(q_T - s) + s = q_T > q^{**}$, while the $L$ incumbent believes that this power is $1 - (1 + s - q_T) + s = q_T < q^*$. Hence, both obey again.

Consider now what happens in the first election lost by $L$ if $s = s^* - \varepsilon < s^*, \varepsilon \approx 0$. $L$ knows that the power of the $R$ incumbent is $1 - (q_T - (s^* - \varepsilon)) = 1 - q_T + (q^{**} + q_T - 1) - \varepsilon = q^{**} - \varepsilon < q^{**}$. Hence, $L$ does not accept the outcome of the election.

**Step 2:** $c > 0$ When $c > 0$, the thresholds $q^{**}$ and $q^*$ depend on the types of contenders who are, respectively, in opposition and in office. Hence, we need to think in terms of $q^{**}(x^*_O)$ and $q^*(x^*_I)$, $\{O, I\} \in \{L, R\}$. Assume that the current pair of contenders is such that $q^{**}(x^*_R) < q_T < q^*(x^*_L)$. Again, $L$ loses an election and obeys if in opposition if $s > q_T + q^{**}(x^*_L) - 1$. Now, we know that when $R$ is the incumbent $\partial q^{**}/\partial x^*_L < 0$. Hence, $\partial s^*/\partial x^*_L < 0$: if the first defeated incumbent is more moderate (closer to the other party), $s$ can be lower for the alternation to be peaceful, meaning that the military can be more partisan.

### 6.5 Proposition 4

If $U_j(x^*_j, x; y, c) = -d(x^*_j, x)/y - \delta c$, the incumbent $L$ obeys if $U_L(x^*_L, x_L; y) \geq \frac{1-q}{1-pq} U_L(\mu, y, c)$ or $-d_L(x^*_L, x_L)/y \geq \frac{1-q}{1-pq} (-d_L(x^*_L, \mu)/y - c)$ or if

$$q < \frac{1 + cy/x_L}{1 + (D - 1)(1 - \rho) + cy/x_L} \equiv q^*(y),$$

and $\partial q^*(y)/\partial y > 0$.

In turn, the opposition $R$ would obey as an incumbent if

$$q > \frac{(D - 1)(1 - \rho)}{1 + (D - 1)(1 - \rho) + cy/x_R} \equiv q^{**}(y),$$

and $\partial q^{**}(y)/\partial y < 0$.

Note that contenders with extreme ideal points may revolt given an income level under which moderate contenders would not.

### 6.6 Proposition 5

The incumbent $L$ believes that his strength is $q_I$ and he is told by the opposition that his strength is $q_T < q_I$ and that it will rebel if the incumbent runs the election with $\max p_L(q_I)$. Now, expecting that if the opposition wins it will operate elections with $\max p_R(q_T)$, $L$ asks himself what is the maximum $p_L$ that the opposition would accept. The answer is the $p_L$ that solves
\[
V_R^Q(o; \max p_R(q_T), p_L) = V_R(r; q_T)
\] (13)

which is given by (13) with \( q = q_T \) and \( p_R = \max p_R(q_T) \).

Now, given that the opposition rebels unless \( p_L \leq \max p_L(q_T) < \max p_L(q_I) \), the incumbent asks himself if he is willing to accept \( \max p_L(q_T) \). He is willing to do so if \( p_L \geq \min p_L(q_I) \) given by \( V_L'(o; p_L, \max p_R(q_T)) \geq V_L(r; q_T) \). The solution is

\[
p_L \geq 1 + \left( \frac{1}{\rho} - \max p_R(q_T) \right) \frac{(1 - \rho q_U L(x_L) - (1 - q)U_L(\mu - c)}{(1 - \rho q_U L(x_R) - (1 - q)U_L(\mu - c)} \equiv \min p_L(q_I)
\]

Hence, a compromise is possible if \( \min p_L(q_I) \leq \max p_L(q_T) \). Tedious algebra shows that

**Condition 3** \( \min p_L(q_I) \leq \max p_L(q_T) \) if

\[
\frac{1 - q_I}{1 - \rho q_I} - \frac{q_T}{(1 - \rho(1-q_T))} > 0
\]

Now, \( \frac{1 - q_I}{1 - \rho q_I} - \frac{q_T}{(1 - \rho(1-q_T))} > 0 \) if \( q_T + q_I < 1 \) or \( q_I < 1 - q_T \). In turn, the right-hand side is positive if \( x_R^* < -x_L^* \), meaning that the incumbent \( L \) is more extreme (further from the mean) than the opposition \( R \). Hence, we must consider the particular cases:

1. If \( x_R^* > -x_L^* \), meaning that \( R \) is more extreme, and \( q_I < 1 - q_T \), this condition is always satisfied. The incumbent does not feel strong, the opposition believes him to be weak and the opposition cares less about avoiding a fight

2. If \( x_R^* > -x_L^* \), \( R \) is more extreme, and \( q_I > 1 - q_T \), this condition is satisfied only if

\[
\left( \frac{1 - \rho(1-q_T-q_I)}{(1 - \rho q_I)(1 - \rho(1-q_T))} \right) > \frac{(x_L^* - x_R^*)^2}{x_L^* - c(D-1)(x_L^* + x_R^*)}
\]

3. If \( x_R^* < -x_L^* \), \( L \) is more extreme, and \( q_I < 1 - q_T \), it is satisfied only if

\[
\left( \frac{1 - \rho(1-q_T-q_I)}{(1 - \rho q_I)(1 - \rho(1-q_T))} \right) > \frac{(x_L^* - x_R^*)^2}{x_L^* - c(D-1)(x_L^* + x_R^*)}
\]

4. If \( x_R^* < -x_L^* \), \( L \) is more extreme, and \( q_I > 1 - q_T \), the condition cannot be satisfied. In this case, the incumbent believes he is strong and he risks a lot by letting \( R \) win elections.

Although the left-hand side of Condition (3), \( LHS = \frac{(1 - \rho)(1-q_T-q_I)}{(1 - \rho q_I)(1 - \rho(1-q_T))} \), increases in the distance in the beliefs, let it be \( k = q_I - q_T \), this distance matters little. Basically, a compromise is possible if the incumbent does not believe that he is very strong and if he is not too extreme in his preferences. The following figure illustrates the condition (3) for different values of \( k \).
7 References


Alonso


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