Firm Heterogeneity under Financial Imperfection: Impacts of Trade and Capital Movement

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Abstract
The paper investigates the role of wealth distributions and financial institutions of an economy on within-industry firm heterogeneity in productivity and on international trade in goods and capital movement. When borrowing is constrained under imperfect financial institution, entrepreneurs with different wealth levels may choose different investment levels, resulting in the firm heterogeneity in productivity. The paper examines the impacts of goods and capital trade between North and South, which differ in their wealth distributions and quality of the financial institution. We find that if the quality of the financial institution is significantly better in North than in South, capital flight from South occurs when only capital is allowed to move internationally. Trade in goods alone will not change the production side of the market structure, such as the number of firms for each productivity. Trade in goods, however, affects the production side of the market structure if capital is also allowed to move internationally. Trade in goods and capital movement are complements in this intriguing sense.

Preliminary and incomplete

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1 Introduction

Recent financial turmoil reminded us of the importance of the high-quality credit market on the economy. The subprime loan problem in the United States seriously hurt the financial systems in the United States and other countries in the world, which led to the global economic downturn as banks and other financial intermediaries became cautious and reluctant to lend money that is necessary for firms to smoothly operate.

The purpose of this paper is to examine how international trade in goods and capital movement across countries affect the economy when financial institutions of trading countries are imperfect. We especially focus on the impacts of trade and capital movement on an industry under monopolistic competition (which can be considered as a manufacture industry). It is important to know how the financial condition of the economy affect the market structures and to know how trade and capital movement affect the market structures under financial imperfection. Since no country has a perfect financial institution and some countries, especially less-developed countries, have rather poor financial institutions, it is necessary to take financial imperfection into explicit consideration in order to derive reliable policy implications from economic models.

Indeed, the quality of financial institution has long been recognized to be critical to the economic prosperity. McKinnon (1973, 1993), for example, emphasizes that less-developed countries and countries in transition from socialism to democracy should develop reliable financial institution in order to achieve economic growth. He argues that countries should first improve their internal financial institutions before opening to trade in goods. He also claims that allowing free international capital mobility should be the last stage of economic liberalization to avoid unwarranted capital flight or an accumulation of foreign debt. There is also a body of research on the effect of financial development on the economic growth. Rajan and Zingales (1998), for example, find empirical evidences that financial development contributes positively to the economic growth.

Recently, Matsuyama (2005), Wynne (2005), Ju and Wei (2008), Antràs and Caballero (2009), and others have explicitly considered financial frictions in their models to exam-
ine the impacts of financial frictions (or financial imperfection) on the models’ trade policy implications. Matsuyama (2005), Wynne (2005), and Ju and Wei (2008) argue that the cross-country differences in the quality of financial institutions significantly affect the structure of countries’ comparative advantage and trade patterns. Antràs and Caballero (2009) theoretically examine the complementarity between international trade in goods and capital movement under financial imperfection. They show among others that trade in goods induces international capital movement, which in turn stimulates international trade in goods. This result is in a stark contrast to a typical result in the traditional literature that trade in goods and international capital movement are substitutes (Mundell, 1957).

This paper examines the impacts of trade and capital movement on an industry in which firms with different productivities compete with one another in the presence of financial imperfection. Our analysis reveals the impacts of the liberalizations on the number of firms, productivity distributions, and average productivity in the industry in each country. Moreover, heterogeneity of firms in the industry is not exogenously given, but caused by the financial imperfection; the degree of firm heterogeneity depends on the quality of the financial institution. Our study, therefore, has also a significant contribution to the recent literature on firm heterogeneity pioneered by Bernard, et al. (2003) and Melitz (2003). Ederington and McCulman (2008, forthcoming) show that heterogeneous timing of technology adoption causes firm heterogeneity. Yeaple (2005) considers worker heterogeneity as a source of firm heterogeneity.\footnote{Bustos (2005), Atkeson and Burnstein (2007), and Constantin and Melitz (2007) allow firms to upgrade their production technologies with a fixed amount of investment, and show that inherently productive firms have more incentives than others to upgrade their technology and engage in the export. Furusawa and Sato (2008) demonstrate that firms with inherently different productivities choose different production technologies with different factor intensities.} We propose another source of firm heterogeneity, which is the financial imperfection. Indeed, we show that firm heterogeneity arises if and only if the financial institution is imperfect. This channel of firm heterogeneity is especially important for less-developed countries, as most of these countries have less-developed financial institutions.

In this paper, we consider the countries in which individuals with different wealth levels choose whether or not they become entrepreneurs, producing varieties of the differentiated...
good. Given that the individuals of the economy are endowed with heterogeneous wealth, entrepreneurs with different wealth levels may choose different investment levels, resulting in the firm heterogeneity in productivity. Then, we examine the impact of trade and capital movement on this industry. We find among others that under financial imperfection, international trade in goods alone will not affect the production side of the market structure (i.e., the number of firms, average productivity, etc.) of any trading countries. The impact of international capital movement differs depending on international wealth distribution and an international difference in the quality of the financial institution. Capital flight from South (the country with a less-developed financial institution) to North, which is known as Lucas Paradox (Lucas, 1990), arises if the difference in the financial institution is significant. Moreover, trade in goods affects the production side of the market structure if and only if capital is mobile across countries.

The last complementarity result may appear similar to that of Antràs and Caballero (2009), but is very different indeed. They show that trade in goods itself will change the production structure of the economy (the capital rental rate, in particular), which induces international capital movement that further stimulates international trade in goods. Whereas we show that trade in goods affects the production structure only when accompanied by capital movement. Moreover, we focus on the effects on an industry, deriving detailed impacts on the industry with heterogeneous firms.

Manova (2008) also develops a model with credit-constrained heterogeneous firms. In her model, firms are faced with credit constraint in financing trade costs. Efficient firms are less financially constrained, so efficient firms in financially developed countries are more likely to engage in the export. Our model is quite different from hers in that financial imperfection leads to firm heterogeneity in not just their attitudes toward exporting but their productivities themselves (which of course affect export activities). We also investigate the impact of international trade of capital and goods on the market structure.
2 Model

There are two countries, which we call North (N) and South (S). In country $k \in \{N, S\}$, there is a mass $m_k$ of individuals, each owning one unit of labor and a wealth of $\omega$ that is uniformly distributed on $[0, \bar{\omega}_k]$; thus the density of individuals whose wealth is $\omega \in [0, \bar{\omega}_k]$ equals $m_k/\bar{\omega}_k$. All individuals share the same utility function over the two goods, a differentiated good $X$ and numeraire good $Y$, characterized by

$$u = \log u_x + y,$$  \hspace{1cm} (1)

where

$$u_x = \left[ \int_{\Omega_k} x(i) \frac{\sigma - 1}{\sigma} di \right]^{\frac{\sigma - 1}{\sigma}}; \quad \sigma > 1$$  \hspace{1cm} (2)

denotes the subutility derived from the consumption of continuum varieties of good $X$, \{x(i)\}_{i \in \Omega_k} (where $\Omega_k$ denotes the set of all varieties available in country $k$), and $y$ denotes the consumption of good $Y$. The numeraire good is competitively produced such that one unit of labor produces one unit of good, so the wage rate equals one.

Each individual chooses a consumption profile of good $X$ to maximize $u_x$ subject to $\int_{\Omega_k} p(i) x(i) di \leq E$, where $p(i)$ and $E$ denote the price for variety $i$ and the total expenditure on all varieties of good $X$, respectively. It is immediate to obtain $x(i) = p(i)^{-\sigma} E / P_k^{1-\sigma}$, where $P_k = \left[ \int_{\Omega_k} p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$ denotes the price index of good $X$. We substitute this result into (2) to obtain $u_x = E / P_k$. Therefore, an individual’s utility function can be written as $u = \log E - \log P_k + y$. Maximizing this with the constraint $E + y \leq I$, where $I$ denote the individual’s income (which is the sum of her labor income and the investment return from her wealth), we obtain $E = 1$. That is, each individual spends $E = 1$ on good $X$, so the country $k$’s aggregate expenditure on good $X$ is $m_k$.

The differentiated-good industry is characterized by the monopolistic competition with free-entry and free-exit. When a firm enters, however, it incurs an R&D (or setup) cost. There are two types of production technology (or facility). The higher the investment, the lower is the marginal cost of production. More specifically, if a firm invests $g_h$ ($g_l$) units of the numeraire good, its marginal cost becomes $1/\varphi_h$ ($1/\varphi_l$). We assume that $g_l < g_h < \tilde{\omega}_k$. 


\( \varphi_l \equiv \varphi \), and \( \varphi_l < \varphi_h \equiv \beta \varphi \), where \( \beta > 1 \) represents the productivity gap. To obtain the profits for firm \( i \) in country \( k \) (in autarky), we define the competition index

\[
\tilde{\varphi}_k \equiv \left[ \int_{i \in \Omega_k} \varphi(i)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}. \quad (3)
\]

Since there is a continuum of varieties, each firm naturally ignores the impact of its pricing on the price index, so that firms select prices that are \( \sigma/(\sigma - 1) \) times their individual marginal costs. It is easy to see that the profits for firm \( i \) in country \( k \) equal

\[
\pi_k(\varphi(i), \tilde{\varphi}_k) = \frac{m_k}{\sigma} \left( \frac{\varphi(i)}{\tilde{\varphi}_k} \right)^{\sigma-1}. \quad (4)
\]

Individuals in country \( k \) decide whether or not they become entrepreneurs who can borrow money at a gross interest rate of \( R_k \) to finance their investments if necessary. If an individual decides to become an entrepreneur, she will choose the high-productivity technology or the low-productivity technology with which her firm operates. If she decides not to be an entrepreneur or if part of her wealth is left after the investment for her firm, she will lend out her (remaining) wealth.

The critical feature of the model is that entrepreneurs are faced with a financial constraint: entrepreneur \( i \) can borrow only up to \( \theta_k \pi_k(\varphi(i), \tilde{\varphi}_k) \), the fraction \( \theta_k \in (0, 1] \) of the profits that her firm will earn. The fraction \( \theta_k \) represents the quality of the financial institution of the country. A financial institution is perfect if \( \theta_k = 1 \); any entrepreneur with any amount of wealth can finance the investment for either high-productivity technology or low-productivity technology, effectively without any constraint. A financial institution is imperfect if \( \theta_k < 1 \); individuals with small amounts of wealth may not be able to finance the investment costs in this case.

We can list several reasons why \( \theta \) is smaller than 1.\(^2\) One natural explanation is that the imperfection of legal enforcement generates financial imperfection.\(^3\) In the traditional literature, legal enforcement is assumed to be perfect. When a borrower has been contracted to pay the amount \( \pi \), which indeed is realized, a court can force the borrower to pay the

\(^2\)Matsuyama (2005, footnote 1) lists various possible causes for financial imperfection of this kind.

\(^3\)See for example Hart (1995).
contracted amount $\pi$. Empirical evidences show, however, the actual enforcement power is not perfect (La Porta, et al., 1998). If the legal enforcement power is not so strong, a court may be able to force a borrower to pay only the amount $\theta \pi$, where $\theta < 1$, even though realized profit is $\pi$. Hence, as long as the non-pecuniary penalty on the default is not so high, the borrower is likely to refuse to pay more than $\theta \pi$ even though the promised payment is $\pi$. This behavior is called the “strategic default.” Even if the borrower defaults on the loan, the court seizes only $\theta \pi$ and the borrower can still get $(1 - \theta) \pi$. Hence, the borrower will not pay more than $\theta \pi$, so the lender has no incentive to lend more than the amount such that the return from the lending equals $\theta \pi$. A contract cannot be a perfect commitment device if the legal enforcement is imperfect; it is difficult for a lender to expect that a borrower will sincerely make the promised payment. Another cause of financial imperfection is the agency problems of the lender-borrower relationship, which is explained briefly with a simple model in the Appendix.

In the economy that we consider, there are two types of the constraints, the profitability constraints and borrowing constraints, which must be satisfied. The profitability constraints

$$\pi_k(\varphi_h) - R_k g_h \geq 0,$$
$$\pi_k(\varphi_l) - R_k g_l \geq 0,$$

for the high-productivity firm and the low-productivity firm, respectively, simply mean that the net profits must be non-negative if firms of the respective type operate at all. The borrowing constraints, on the other hand, can be written as

$$\theta_k \pi_k(\varphi_h) \geq R_k (g_h - \omega),$$
$$\theta_k \pi_k(\varphi_l) \geq R_k (g_l - \omega),$$

which mean that entrepreneurs can borrow money only up to the amount such that the payment does not exceed the fraction $\theta_k$ of the profits. It is easy to see that for each type of firm, the profitability constraint is tighter than the borrowing constraint if $\theta_k$ is large, whereas the borrowing constraint is tighter than the profitability constraint if $\theta_k$ is small.
We investigate the effects of international capital movement and trade in goods on the economy, mainly on the market structure of the differentiated-good industry, under the imperfect financial institution. But before that, we analyze the benchmark case in which the financial institution is perfect. For the rest of our analysis, we assume that North is capital-abundant (i.e., $\bar{\omega}_N > \bar{\omega}_S$) and North has a better financial institution than South (i.e., $\theta_N > \theta_S$).

3 Equilibrium under perfect financial institution

This section shows that if there is no financial constraint, all entrepreneurs choose the same production technology and hence all firms in the differentiated good sector become homogeneous. Moreover, international capital movement and trade in goods are shown to be substitutes in a sense that is made clear later.

Consider a decision made by an individual with the wealth $\omega$. If she invests $g_h$ on the high-productivity technology, she would obtain $\pi_k(\varphi_h, \hat{\varphi}_k) - R_k(g_h - \omega)$. If $\omega < g_h$, she borrows $g_h - \omega$ to earn $\pi_k(\varphi_h, \hat{\varphi}_k)$ and pay $R_k(g_h - \omega)$ back to the lenders. If $\omega \geq g_h$, on the other hand, she obtains $\pi_k(\varphi_h, \hat{\varphi}_k)$ from the production of good $X$ (from the investment of $g_h$) and $-R_k(g_h - \omega)$ from lending out. Similarly, if she invests $g_l$, she would obtain $\pi_k(\varphi_l) - R(g_l - \omega)$. Finally, if she lends out the entire wealth of hers, she would get $R_k\omega$.

An entrepreneur chooses the high-productivity technology rather than the low-productivity technology if

$$\pi_k(\varphi_h, \hat{\varphi}_k) - R_k(g_h - \omega) > \pi_k(\varphi_l, \hat{\varphi}_k) - R_k(g_l - \omega),$$

which can be written as

$$\pi_k(\varphi_h, \hat{\varphi}_k)(1 - \beta^{1-\sigma}) > R_k(g_h - g_l).$$

(9) Note that this inequality does not depend on $\omega$, so all entrepreneurs choose the same technology.

Whether or not the inequality (9) holds depends on the productivity and investment-cost parameters. In this paper, we focus on the natural case in which entrepreneurs choose the
high-productivity technology if they are not financially constrained, so the inequality (9) holds. In equilibrium, some individuals become entrepreneurs while some others must be lending money to entrepreneurs, and hence the net benefit of being an entrepreneur and that of lending money must be the same. That is,

$$\pi_k(\varphi, \tilde{\varphi}_k) - R_k(gh - \omega) = R_k \omega,$$

which is reduced to

$$\pi_k(\varphi_h, \tilde{\varphi}_k) = R_k gh.$$  \hspace{1cm} (10)

Note that this equality simply shows that profits for high-tech firms are zero: running a business does not yield extra-benefits to individuals. Now, substituting this equality into (9) and rearranging terms, we obtain $\beta^{\sigma-1} > gh/g_l$, which we assume for the rest of our analysis.

**Assumption 1**

$$\beta^{\sigma-1} > gh/g_l.$$  

This assumption indicates that the productivity gap is so large that the more-costly high-productivity technology is effectively more economical than the low-productivity technology. Consequently, all entrepreneurs choose the high-productivity technology while some individuals lend their wealth to those entrepreneurs. Moreover, it is easy to check that under this assumption, there does not exist equilibrium in which entrepreneurs choose the low-productivity technology.

**Proposition 1** Under perfect financial institution, all entrepreneurs in the differentiated-good sector choose the same production technology upon entry, and hence firms are homogeneous within the sector.

### 3.1 Autarkic Equilibrium

In this subsection, we derive the details of the autarkic equilibrium. To this end, we first investigate the credit market. Let $n_k$ denote the mass of firms (or equivalently the mass of
entrepreneurs) in country \( k \) in equilibrium. Then, the total investment demands equal \( n_k g_h \), while the total loan supply equals

\[
\frac{m_k}{\bar{\omega}_k} \int_{0}^{\bar{\omega}_k} \omega d\omega = \frac{m_k \bar{\omega}_k}{2}.
\]

By equating the asset demands and supplies, we find that the mass of firms is given by

\[
n_k = \frac{m_k \bar{\omega}_k}{2 g_h}.
\] (11)

We need the following assumptin to ensure that \( n_k < m_k \).

**Assumption 2**

\[
\bar{\omega}_k < 2 g_h.
\]

Recall that the decision as to whether or not an individual becomes an entrepreneur does not depend on her wealth. This means that despite that the number of entrepreneurs is unambiguously determined, who become entrepreneurs is indeterminate. But if we suppose that only the wealthiest individuals become entrepreneurs, the wealth level of the poorest entrepreneur \( \omega^*_{h,k} \) must satisfy

\[
\frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega^*_{h,k}) = \frac{m_k \bar{\omega}_k}{2 g_h},
\]

which gives us

\[
\omega^*_{h,k} = \bar{\omega}_k - \frac{\bar{\omega}_k^2}{2 g_h}.
\] (12)

In this case, individuals become entrepreneurs if and only if their wealth levels lie in the interval \([\omega^*_{h,k}, \bar{\omega}_k]\).

To obtain the equilibrium profits and interest rate, we calculate the autarkic competition index \( \bar{\varphi}^A_k \) from (3) using (11) to obtain

\[
\bar{\varphi}^A_k = \beta \varphi \left( \frac{m_k \bar{\omega}_k}{2 g_h} \right)^{\frac{1}{\sigma}}.
\] (13)

Substituting this into (4), we have

\[
\pi_k(\varphi_h, \bar{\varphi}^A_k) = \frac{m_k}{\sigma n_k} = \frac{2 g_h}{\sigma \bar{\omega}_k}.
\] (14)
In autarky, we have $\pi_N(\varphi_h, \tilde{\varphi}_N^A) \geq \pi_S(\varphi_h, \tilde{\varphi}_S^A)$; profits in capital-abundant North are lower than those in South, since there are more firms per capita in North than in South. As for the interest rate, it follows from (10) that

$$R_k = \frac{2}{\sigma \bar{\omega}_k}.$$  

In autarky, we have $R_N \leq R_S$ because $\bar{\omega}_N \geq \bar{\omega}_S$; the interest rate in the capital-abundant North is lower than that in South.

### 3.2 Equilibrium with Free Trade in Goods

Let us investigate the effect of opening to trade on the economy. In free trade, firms in each country sell their products abroad without incurring any trade costs as well as in their individual home countries. Opening to trade does not affect the credit market in each country, so the number of firms in country $k$ is still given by $n_k = m_k \bar{\omega}_k/(2g_h)$ as shown in (11). Consequently, the competition index in country $k$ can be written as

$$\tilde{\varphi}^T = \beta \varphi(n_N + n_S) \frac{1}{\sigma} = \beta \varphi \left(\frac{m_N \bar{\omega}_N + m_S \bar{\omega}_S}{2g_h}\right)^{\frac{1}{\sigma - 1}}.$$  

The competition index increases as a result of trade as the comparison between (13) and (16) reveals.

Although the competition in each country becomes tougher, firms are now able to sell their products in both countries. The worldwide profits for any firm of any country are

$$\pi_W(\varphi_h, \tilde{\varphi}^T) = \frac{m_N + m_S}{\sigma} \left(\frac{2g_h}{m_N \bar{\omega}_N + m_S \bar{\omega}_S}\right) = \frac{2g_h (m_N + m_S)}{\sigma (m_N \bar{\omega}_N + m_S \bar{\omega}_S)}.$$  

Comparison between (15) and (17) reveals that opening to trade will increase the profits for Northern firms and decrease the profits for Southern firms. Similar observation is made as for the equilibrium interest rates: the interest rate increases in North and decreases in South to

$$R^T_N = R^T_S = \frac{2(m_N + m_S)}{\sigma (m_N \bar{\omega}_N + m_S \bar{\omega}_S)}.$$  

Trade in goods will equate the interest rates between the two countries, eliminating individuals’ incentive to invest abroad even when capital is allowed to move internationally.
3.3 Equilibrium with International Capital Movement

Since the interest rate is lower in North than in South in autarky, capital moves from North to South when capital is allowed to move internationally. The interest rates will be equated between the two countries in equilibrium. Then, it follows from the profitability constraint (5) that profits will also be the same, i.e., \( m_N/(\sigma n_N) = m_S/(\sigma n_S) \) (see equation (14), which can be solved for \( n_S \) as \( n_S = m_S n_N/m_N \).

To obtain the number of firms and profits in each country, we write the worldwide credit market clearing condition as

\[
(n_N + n_S)g_h = \frac{m_N \bar{\omega}_N}{2} + \frac{m_S \bar{\omega}_S}{2}.
\]

Substituting \( n_S = m_S n_N/m_N \) into this equation gives us

\[
n_N = \frac{m_N(m_N \bar{\omega}_N + m_S \bar{\omega}_S)}{2g_h(m_N + m_S)}
\]

and

\[
n_S = \frac{m_S(m_N \bar{\omega}_N + m_S \bar{\omega}_S)}{2g_h(m_N + m_S)}.
\]

It is readily verified that the number of firms decreases in North and increases in South, just as expected because capital moves from North to South. Given these numbers of firms, it is also easy to show that profits are the same between the two countries:

\[
\pi_N(\varphi_h, \hat{\varphi}_N) = \pi_S(\varphi_h, \hat{\varphi}_S) = \frac{2g_h(m_N + m_S)}{\sigma(m_N \bar{\omega}_N + m_S \bar{\omega}_S)}.
\]

Note that the equilibrium profits under capital movement are the same as those under free trade in goods. Trade in goods effectively expands the market for Northern firms and shrinks the market for Southern firms, which increases Northern firms’ profits and decreases Southern firms’. Capital movement, on the other hand, decreases the number of Northern firms while increases the number of Southern firms, and thereby changes their profits accordingly.

In addition, it follows again from the profitability constraint that the equilibrium interest rates are the same between the two countries, and it is the same as the equilibrium interest rate under free trade in goods.
Proposition 2  Under the perfect financial institution, trade in goods and international capital movement are perfect substitutes in the sense that (i) the profits are equated between the two countries at a common level in either of the two cases and (ii) the interest rates are also equated between the two countries at a common level in either of the two cases.

Although some important economic variables, such as the profits and interest rates, are the same between the two cases, consumers can enjoy more varieties in the case of free trade in goods than in the case of free capital mobility, as trade allows consumers in either country to consume varieties produced in the foreign country as well as those produced in the home country. Trade in goods and capital movement are substitutes on the production side of the economy. But trade in goods is a better alternative than capital movement when we consider the consumption side.

3.4 Equilibrium with Capital Movement and Trade in Goods

As expected from the analysis in the last two subsection, equilibrium profits and interest rates are the same as those in the last two cases. The total number of firms in the world is determined from the credit market as

\[ n_N + n_S = \frac{m_N \bar{\omega}_N + m_S \bar{\omega}_S}{2gh}, \]

while the number of firms in each country is indeterminate in this case. Given that goods are freely traded between the two countries, capital movement simply changes the distribution of firms across the countries, without affecting profits and interest rates. Letting \( K \) denote the amount of capital that moves from North to South, the numbers of firm in the respective countries are given by

\[ n_N = \frac{m_N \bar{\omega}_N}{2gh}, \quad n_S = \frac{m_S \bar{\omega}_S}{2gh}, \]

where \( K \) can take any value as long as \( n_N \geq 0 \) and \( n_S \geq 0 \) (and as long as labor is still allocated to sector \( Y \)).
4 Financial Imperfection and Firm Heterogeneity

We have shown that under perfect financial institution, an individual’s wealth is irrelevant in her decision as to whether or not she becomes an entrepreneur. As expected, individual’s wealth will be an important factor under financial imperfection. Due to a financial constraint, only wealthy individuals can borrow money so that they become entrepreneurs. Moreover, since individuals are heterogeneous in their wealth, their choice of technology may be heterogeneous leading to the firm heterogeneity in productivity.

In the rest of the analysis, we focus on the case in which \( \theta \) is so small that the borrowing constraints, (7) and (8), hold with equality while profitability constraints, (5) and (6), hold with strict inequalities. Thus, the relevant constraints are the borrowing constraints, (7) and (8). If \( \theta_k \) is small enough that the borrowing constraint for either type is binding, wealthiest individuals become entrepreneurs with the high-productivity technology, those who own intermediate levels of wealth become entrepreneurs with the low-productivity technology, and the poorest individuals lend out their wealth.

We define critical levels of wealth, \( \omega_{h,k} \) and \( \omega_{l,k} \), such that all individuals with \( \omega \in [\omega_{h,k}, \bar{\omega}_k] \) become entrepreneurs choosing the high-productivity technology while all individuals with \( \omega \in [\omega_{l,k}, \omega_{h,k}] \) become entrepreneurs choosing the low-productivity technology.

The condition that \( \omega_{h,k} \) and \( \omega_{l,k} \) must satisfy is the credit-market clearing condition. In autarky, it is written as

\[
\frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega_{h,k}) g_h + \frac{m_k}{\bar{\omega}_k} (\omega_{h,k} - \omega_{l,k}) g_l = \frac{m_k \bar{\omega}_k}{2},
\]

which can be solved for \( \omega_{l,k} \) to define the function \( \hat{\omega}_{l,k} \):

\[
\hat{\omega}_{l,k}(\omega_{h,k}) = \frac{2g_h \bar{\omega}_k - \bar{\omega}_k^2}{2g_l} - \frac{g_h - g_l}{g_l} \omega_{h,k}.
\]

This function represents the relation between \( \omega_{l,k} \) and \( \omega_{h,k} \) under the credit-market clearing condition. We can easily see that \( \hat{\omega}_{l,k} \) is decreasing and that \( \omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}) \) increases with \( \omega_{h,k} \). An increase in \( \omega_{h,k} \) releases part of capital used for the high-tech firms, which is absorbed by the low-tech entrants whose mass exceeds that of the exiting high-tech firms.
4.1 Autarkic Equilibrium

We use the relation (20) to write profits for firms as functions of \( \omega_{h,k} \). In this case, the competition index defined by (3) can be written as

\[
\bar{\phi}_k(\omega_{h,k}) = \left\{ \left( \frac{\beta \varphi}{\sigma} - 1 \right) \frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega_{h,k}) + \varphi \left( \frac{m_k}{\bar{\omega}_k} \omega_{h,k} - \bar{\omega}_l, \omega_{h,k} \right) \right\}^{\frac{1}{\sigma - 1}},
\]

(21)

where

\[
\phi_k(\omega_{h,k}) = \beta \sigma - 1 \frac{\bar{\omega}_k - \omega_{h,k}}{\bar{\omega}_k} + \frac{\omega_{h,k} - \bar{\omega}_l, \omega_{h,k}}{\bar{\omega}_k}.
\]

The competition index \( \bar{\phi}_k(\omega_{h,k}) \) is decreasing in \( \omega_{h,k} \) as the derivative of the normalized average productivity \( \phi_k(\omega_{h,k}) \) with respect to \( \omega_h \) equals \( \left( \frac{g_h}{g_l} - \beta \sigma^{-1} \right) \bar{\omega}_k \), which is negative under Assumption 1; the effect of the contraction of the high-tech group outweighs the effect of the expansion of the entire mass of firms. The profits for the firms can be written as

\[
\pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) = \frac{m_k}{\sigma} \left( \frac{\beta \varphi}{\bar{\phi}_k(\omega_{h,k})} \right)^{\sigma - 1} = \frac{\beta \sigma - 1}{\sigma \phi_k(\omega_{h,k})},
\]

(22)

\[
\pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) = \frac{1}{\sigma \phi_k(\omega_{h,k})},
\]

(23)

for the high-tech and low-tech firms, respectively. Since \( \phi_k(\omega_{h,k}) \) decreases with \( \omega_{h,k} \), both \( \pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) \) and \( \pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) \) increase with \( \varphi_h \).

We are now ready for determining equilibrium levels of \( \omega_{h,k}, \omega_{l,k}, \) and \( R_k \). The binding borrowing constraint for the high-tech firms, \( \theta_k \pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) = R_k(g_h - \omega_{h,k}) \), can be written as

\[
R_k = \frac{\theta_k \beta \sigma - 1}{\sigma \phi_k(\omega_{h,k})(g_h - \omega_{h,k})},
\]

(24)

while the one for the low-tech firms, \( \theta_k \pi_k(\varphi, \bar{\phi}_k(\omega_{h,k})) = R_k(g_l - \bar{\omega}_l, \omega_{h,k}) \), can be written as

\[
R_k = \frac{\theta_k}{\sigma \phi_k(\omega_{h,k})(g_l - \bar{\omega}_l, \omega_{h,k})}.
\]

(25)

It immediately follows from (24) and (25) that \( \omega_{h,k}^A \) is given by

\[
\beta \sigma - 1 = \frac{g_h - \omega_{h,k}^A}{g_l - \bar{\omega}_l, \omega_{h,k}^A}.
\]

(26)
For the solution of (26) to make sense, \( \hat{\omega}_{l,k}(\omega^A_{h,k}) < \omega^A_{h,k} \) must hold. We substitute (20) into this inequality to find that \( \hat{\omega}_{l,k}(\omega^A_{h,k}) < \omega^A_{h,k} \) is equivalent to \( \omega_{h,k} > \omega^*_h,k \), where \( \omega^*_h,k \) is given by (12). Recalling that \( \hat{\omega}_{l,k} \) is a decreasing function, therefore, we need the following assumption to ensure \( \hat{\omega}_{l,k}(\omega^A_{h,k}) < \omega^A_{h,k} \).

**Assumption 3**

\[
\beta^{\sigma-1} < \frac{g_h - \omega^*_h,k}{g_l - \hat{\omega}_{l,k}(\omega^*_h,k)}.
\]

With this assumption, we can assert the following proposition.

**Proposition 3** Firm heterogeneity within the differentiated good sector arises under a poor financial institution.

Note that equation (26) does not involve \( \theta_k \). As long as \( \theta_k \) is small so that the borrowing constraints are binding for both high-tech and low-tech firms, the production side of the market structure (i.e., the masses of the high-tech and low-tech firms) is not affected by a change in the quality of the financial institution; the market structure is solely determined by the credit-market clearing condition.

With the market structure given by (26), the interest rate \( R^A_k \) is determined by the borrowing constraint (for high-tech firms, for example):

\[
R^A_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_k(\omega^A_{h,k})(g_h - \omega^A_{h,k})}.
\]  

(27)

As (27) indicates, any change in \( \theta_k \) will induce an offsetting change in \( R_k \). In partial equilibrium analyses, the development of financial institution generally increases the number of firms because it becomes easier for entrepreneurs to finance the investment costs. But this seemingly obvious causality breaks down in this general equilibrium model. The production side of the market structure hinges critically on the total credit supply that is fixed in the autarkic economy. That is why the financial development, for example, will increase the interest rate to offset an induced increase in credit demands. In addition, it can be verified from (20) that an increase in \( \bar{\omega}_k \) leads to decreases in \( \omega^A_{h,k}/\bar{\omega}_k \) and \( \hat{\omega}_{l,k}(\omega^A_{h,k})/\bar{\omega}_k \), which in turn increases \( \phi_k(\omega^A_{h,k}) \), and hence increases the competition index \( \tilde{\varphi}_k \). The industry is more competitive in the capital-abundant country than the other.
Lemma 1 If the financial institution is so poor that the borrowing constraints are binding for both high-productivity and low-productivity firms, (i) an increase in $\bar{\omega}_k$ reduces $\omega_{A,h,k}/\bar{\omega}_k$ and $\hat{\omega}_{l,k}(\omega_{A,h,k})/\bar{\omega}_k$, and hence raises the normalized average productivity $\phi_k(\omega_{A,h,k})$, and (ii) an improvement of financial institution would only increase the interest rate, leaving the production side of the market structure unchanged.

It immediately follows from $\bar{\omega}_N \geq \bar{\omega}_S$ and Lemma 1 that $g_h - \omega_{h,N} > g_h - \omega_{h,S}$ and $\phi_N(\omega_{A,h,N}^A) \geq \phi_S(\omega_{A,h,S}^A)$. Therefore, we see from (27) that (i) $R_{N}^A < R_{S}^A$ if $\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$, and (ii) $R_{N}^A > R_{S}^A$ if $\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$. In a general case where $\theta_N \geq \theta_S$ and $\omega_N \geq \omega_S$, which of $R_{N}^A$ and $R_{S}^A$ is greater than the other depends on whether or not the difference in the quality of the financial institutions between the two countries is more significant than the difference in the wealth levels.

Proposition 4 In autarky, the interest rate is higher in North than in South if the difference in the quality of the financial institution between North and South is more significant than the difference in the wealth levels.

4.2 Equilibrium with Free Trade in Goods

In this subsection, we show that trade in goods alone will not change the production side of the market structure while it will generally narrow the gap between the countries’ interest rates.

To obtain the profits, we calculate the competition index, defined by (3), as

$$\hat{\phi}_W = \left\{ (\beta \varphi)^{\sigma - 1} \sum_{k=N,S} \frac{m_k}{\bar{\omega}_k} (\bar{\omega}_k - \omega_{h,k}) + \varphi^{\sigma - 1} \sum_{k=N,S} \frac{m_k}{\bar{\omega}_k} \left[ \omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k}) \right] \right\}^{\frac{1}{\sigma - 1}}$$

$$= \varphi (m_N + m_S) \frac{1}{\sigma - 1} \phi_W(\omega_{h,N},\omega_{h,S})$$

where

$$\phi_W(\omega_{h,N},\omega_{h,S}) \equiv \beta^{\sigma - 1} \sum_{k=N,S} \frac{m_k}{m_N + m_S} \frac{\omega_{h,k} - \omega_{h,N}}{\bar{\omega}_k} + \sum_{k=N,S} \frac{m_k}{m_N + m_S} \frac{\omega_{h,k} - \hat{\omega}_{l,k}(\omega_{h,k})}{\bar{\omega}_k}$$

$$= \sum_{k=N,S} \frac{m_k}{m_N + m_S} \phi_k(\omega_{h,k})$$

(30)
Then the profits for high-tech firms are given by

\[
\pi_W(\varphi, \tilde{\varphi}^T) = \frac{m_N + m_S}{\sigma} \left( \frac{\beta \varphi}{(m_N + m_S)^{\frac{1}{\sigma-1}}} \phi_W(\omega_{h,N}, \omega_{h,S})^{\frac{1}{\sigma-1}} \right)^{\sigma-1}
\]

(31)

while the profits for low-tech firms are given by

\[
\pi_W(\varphi_l, \tilde{\varphi}^T) = \frac{1}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S})}.
\]

The borrowing constraints for high-tech and low-tech firms can be written respectively as

\[
R_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S})(g_h - \omega_{h,k})},
\]

(33)

\[
R_k = \frac{\theta_k}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S})}[g_l - \hat{\omega}_{l,k}(\omega_{h,k})].
\]

(34)

The equilibrium values of \(\omega_{h,k}\) and \(R_k\), which we call as \(\omega^T_{h,k}\) and \(R^T_k\), satisfy the two equations for each \(k = N, S\):

\[
\beta^{\sigma-1} = \frac{g_h - \omega^T_{h,k}}{g_l - \hat{\omega}_{l,k}(\omega^T_{h,k})},
\]

(35)

\[
R^T_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_W(\omega^T_{h,N}, \omega^T_{h,S})(g_h - \omega^T_{h,k})},
\]

(36)

which are directly derived from (33) and (34).

Since equation (35) is identical to the one in (26), we find that the critical levels of wealth are the same between the two cases, i.e., \(\omega^T_{h,N} = \omega^A_{h,N}\) and \(\omega^T_{h,S} = \omega^A_{h,S}\).

Proposition 5 *International trade in goods between two countries under financial imperfection will not affect the production side of the market structure in either country.*

International trade in goods, however, affects the interest rates in general through its effects on firms’ profits. We investigate the impacts of trade on profits and interest rates separately in the case where the countries are different in a traditional sense (\(\theta_N = \theta_S\) and \(\bar{\omega}_N > \bar{\omega}_S\)) and in the case where they are different in the sense that we want to emphasize (\(\theta_N > \theta_S\) and \(\bar{\omega}_N = \bar{\omega}_S\)).
4.2.1 Interest rates when \( \theta_N = \theta_S \) and \( \bar{\omega}_N > \bar{\omega}_S \)

In this case, the Northern market is more competitive than the Southern market in autarky, so opening to trade will increase the Northern firms’ profits and decrease the Southern firms’. Consequently, the interest rate increases in North and decreases in South.

To see this effect of trade on the interest rates, we note that \( \omega_{h,N}^T = \omega_{h,N}^A \), \( \omega_{h,S}^T = \omega_{h,S}^A \) and (30) imply that

\[
\phi_N(\omega_{h,N}^A) > \phi_W(\omega_{h,N}^T, \omega_{h,S}^T) > \phi_S(\omega_{h,S}^A),
\]

and hence \( \pi_N(\varphi_h, \tilde{\varphi}_N^A) < \pi_W(\varphi_h, \tilde{\varphi}_W^T) < \pi_S(\varphi_h, \tilde{\varphi}_S^A) \) as (22), (23), (31), and (32) indicate. That is, the Northern firms’ profits increase while the Southern firms’ profits decrease as a result of opening to trade. These effects on profits imply that the interest rate rises in North and drops in South, i.e., \( R_N^T > R_N^A \) and \( R_S^T < R_S^A \) as the comparison between (27) and (36) also reveals.

We also find that the interest rate is still lower in North than in South, i.e., \( R_N^T < R_S^T \). It follows from \( \bar{\omega}_N > \bar{\omega}_S \) and \( \omega_{h,k}^T = \omega_{h,k}^A \) that \( g_h - \omega_{h,N}^T > g_h - \omega_{h,S}^T \). Then, since \( \theta_N = \theta_S \), (36) implies that \( R_N^T < R_S^T \).

The interest rate is lower in North than in South in autarky, reflecting the difference in their wealth levels. The gap between them narrows, although not completely, as a result of trade.

4.2.2 Interest rates when \( \theta_N > \theta_S \) and \( \bar{\omega}_N = \bar{\omega}_S \)

In this case, the normalized average productivities are the same between the two countries, and so are the profits for the firms of each type, in autarky. The autarkic interests are higher in North than in South, reflecting the difference in the quality of the financial institutions. Since the firms’ profits are the same between the two countries in autarky, trade will not change their profits; the market expansion effect of trade liberalization completely offsets the competition enhancement effect. Consequently, the individual interest rates of the two countries will not change as a result of trade.

Similarly to the case where \( \theta_N = \theta_S \) and \( \bar{\omega}_N > \bar{\omega}_S \), we compare the normalized average
productivities before and after trade liberalization to find that $\phi_N(\omega_{h,N}^A) = \phi_W(\omega_{h,N}^T, \omega_{h,S}^T) = \phi_S(\omega_{h,S}^A)$ and that $\pi_N(\varphi_h, \tilde{\varphi}_N^A) = \pi_W(\varphi_h, \tilde{\varphi}_W^T) = \pi_S(\varphi_h, \tilde{\varphi}_S^T)$. Then, it follows from (27), (36), and $\theta_N > \theta_S$ that $R_N^T = R_N^A > R_S^A = R_S^T$.

### 4.3 Equilibrium with International Capital Movement

As Proposition 4 indicates, whether or not North has a higher interest rate than South depends on whether or not the difference in the quality of the financial institution between North and South is more significant than the difference in the wealth levels. If capital is allowed to move internationally, it moves from the country with a lower interest rate to the country with a higher interest rate, shrinking the industry in the former country and expanding the industry in the latter.

To see the impacts of capital movement more closely, we first derive the equilibrium competition index for each country. Let us define the amount of capital that moves from North to South by $K$ (which takes a negative value when capital moves from South to North). Then, in North, for a given $\omega_{h,N}$, the smaller threshold wealth $\omega_{l,N}$ changes from $\tilde{\omega}_{l,N}(\omega_{h,N})$ to $\tilde{\omega}_{l,N}(\omega_{h,N}) + (K/g_l)/(m_N/\bar{\omega}_N)$ as a result of the capital movement. Thus, we can write Northern competition index as

$$\hat{\varphi}_N(\omega_{h,N}, K) = \left\{ \left( \beta \varphi \right)^{\sigma - 1} \left( \frac{m_N}{\bar{\omega}_N} \right) (\bar{\omega}_N - \omega_{h,N}) + \varphi^\sigma \left( \frac{m_N}{\bar{\omega}_N} \right) \left[ \omega_{h,N} - \tilde{\omega}_{l,N}(\omega_{h,N}) - \left( \frac{\omega_N}{m_N} \right) \frac{K}{g_l} \right] \right\}^{\frac{1}{\sigma - 1}},$$

where

$$\phi_N(\omega_{h,N}, K) \equiv \beta^{\sigma - 1} \frac{\bar{\omega}_N - \omega_{h,N}}{\bar{\omega}_N} + \frac{\omega_{h,N} - \tilde{\omega}_{l,N}(\omega_{h,N})}{\bar{\omega}_N} - \frac{K}{m_N g_l}.$$  

For a given $\omega_{h,N}$, Northern competition index falls if capital flows out of North because the number of low-tech firms decreases while the number of high-tech firms remain the same. Similarly, Southern competition index is derived as

$$\hat{\varphi}_S(\omega_{h,S}, K) = \varphi m_S^{-\frac{1}{\sigma - 1}} \phi_S(\omega_{h,S}, K)^{\frac{1}{\sigma - 1}},$$

where

$$\phi_S(\omega_{h,S}, K) \equiv \beta^{\sigma - 1} \frac{\bar{\omega}_S - \omega_{h,S}}{\bar{\omega}_S} + \frac{\omega_{h,S} - \tilde{\omega}_{l,S}(\omega_{h,S})}{\bar{\omega}_S} + \frac{K}{m_S g_l}.$$
It follows from (38) and (39) that profits for high-tech firms, for example, in North and South can be written as

\[
\pi_N(\varphi_h, \check{\varphi}_N) = \frac{m_N}{\sigma} \left( \frac{\beta \varphi}{\varphi m_N^\frac{1}{\sigma} \phi_N(\omega_{h,N}, K)^\frac{1}{\sigma-1}} \right)^{\sigma-1} = \frac{\beta}{\sigma \phi_N(\omega_{h,N}, K)},
\]

\[
\pi_S(\varphi_h, \check{\varphi}_S) = \frac{\beta}{\sigma \phi_S(\omega_{h,S}, K)}.
\]

Now, the borrowing constraints for high-tech firms in country \( k \) can be written as

\[
R_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_k(\omega_{h,k}, K)(g_h - \omega_{h,k})},
\]

whereas those for low-tech firms in North and South are respectively written as

\[
R_N = \frac{\theta_N}{\sigma \phi_N(\omega_{h,N}, K) \left[ g_l - \check{\omega}_{l,N}(\omega_{h,N}) - \left( \frac{\omega_N}{m_N} \right) \frac{K^K}{g_l} \right]};
\]

\[
R_S = \frac{\theta_S}{\sigma \phi_S(\omega_{h,S}, K) \left[ g_l - \check{\omega}_{l,S}(\omega_{h,s}) + \left( \frac{\omega_S}{m_S} \right) \frac{K^K}{g_l} \right]}.
\]

In equilibrium with international capital movement, interest rates \( R_N \) and \( R_S \) are equated at a level, which we call \( R_W \). The equilibrium values of \( \omega_{h,k} \) and \( R_W \), which we call as \( \omega^K_{h,k} \) and \( R^K_W \), satisfy the following three equations:

\[
\beta^{\sigma-1} = \frac{g_h - \omega^K_{h,N}}{g_l - \check{\omega}_{l,N}(\omega^K_{h,N}) - \left( \frac{\omega_N}{m_N} \right) \frac{K^K}{g_l}},
\]

\[
\beta^{\sigma-1} = \frac{g_h - \omega^K_{h,S}}{g_l - \check{\omega}_{l,S}(\omega^K_{h,S}) + \left( \frac{\omega_S}{m_S} \right) \frac{K^K}{g_l}},
\]

\[
\frac{\theta_N \beta^{\sigma-1}}{\sigma \phi_N(\omega^K_{h,N}, K^K)(g_h - \omega^K_{h,N})} = \frac{\theta_S \beta^{\sigma-1}}{\sigma \phi_S(\omega^K_{h,S}, K^K)(g_h - \omega^K_{h,S})}.
\]

The effects of capital movement on the industry are very different from those of trade in goods. Capital movement, induced by the difference in the financial development and the difference in wealth levels, will change the production side of the market structure. To see these effects, we examine the two cases separately again.

**4.3.1 Effects of capital movement when \( \theta_N = \theta_S \) and \( \check{\omega}_N > \check{\omega}_S \)**

As we have seen in the above, in autarky, the normalized average productivity is higher in North than in South, i.e., \( \phi_N(\omega^A_{h,N}) > \phi_S(\omega^A_{h,S}) \), so firms’ profits are higher in lower in North.
Consequently, the interest rate is smaller in North than in South. If capital is allowed to move internationally, therefore, capital flows out of North to South, which shrinks Northern industry (an increase in $\omega_{h,N}$) and expand Southern industry (a decrease in $\omega_{h,S}$). As a result, $\phi_N(\omega^A_{h,N})$ goes down and $\phi_S(\omega^A_{h,S})$ to equilibrium values.

We know from (45) that

$$\phi_N(\omega^K_{h,N}, K^K)(g_h - \omega^K_{h,N}) = \phi_S(\omega^K_{h,S}, K^K)(g_h - \omega^K_{h,S})$$

must be satisfied in equilibrium. Here, we show that (46) implies that $\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K)$ and $g_h - \omega_{h,N} < g_h - \omega_{h,S}$. Suppose on the contrary that $g_h - \omega_{h,N} \geq g_h - \omega_{h,S}$. Then, we have $\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K)$ because $\bar{\omega}_N > \bar{\omega}_S$ and $\omega_{h,N} \leq \omega_{h,S}$. But then (46) is violated, so we must have $g_h - \omega_{h,N} < g_h - \omega_{h,S}$ and hence $\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K)$ again from (46).

Capital movement from the capital-abundant North shrinks Northern industry and expands Southern industry. This change can be considered as large because Northern threshold wealth $\omega_{h,N}$ is smaller than Southern counterpart $\omega_{h,S}$ in autarky, but is greater now in the case where capital is allowed to move internationally; the poorest entrepreneurs who adopt the high-productivity technology must be richer now in North than in South. Note, however, that $\phi_N(\omega^K_{h,N}, K^K) > \phi_S(\omega^K_{h,S}, K^K)$ so that Northern firms still earn less profits than Southern firms in equilibrium. As Northern firms’ profits increase while Southern firms’ profits decrease, the interest rate increases in North and decreases in South to the common rate $R^K_W$.

**Proposition 6** When the two countries differ in their wealth levels, capital moves from the capital-abundant North to South, shrinking the industry in North and expanding the industry in South.

**4.3.2 Effects of capital movement when $\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$**

The normalized average productivities are the same between the two countries in autarky due to $\bar{\omega}_N = \bar{\omega}_S$, so the fact that $\theta_N > \theta_S$ leads to $R^A_N > R^A_S$. Consequently, capital flight from South arises if capital is allowed to move.
It follows from (45) that

\[
\frac{\theta_N}{\theta_S} = \frac{\phi_N(\omega_{h,N}, K)}{\phi_S(\omega_{h,S}, K)} g_h - \omega_h N
\]

must be satisfied in equilibrium. Since capital moves from South to North, equilibrium \(\omega_{h,N}\) and \(\omega_{h,S}\) that satisfy this condition are such that \(\phi_N(\omega_{h,N}^K, K^K) > \phi_S(\omega_{h,S}^K, K^K)\) and \(\omega_{h,N} < \omega_{h,S}\). Note that the normalized average productivity is higher in North than in South also in this case.

**Proposition 7** When the two countries differ in the quality of the financial institution, capital moves from South with the relatively poor financial institution to North, shrinking the industry in South and expanding the industry in North.

### 4.4 Equilibrium with Capital Movement and Trade in Goods

We have seen that the normalized average productivity is higher in North than in South even in the equilibrium with international capital movement. Thus, if trade is allowed (in addition to capital movement) so that all firms compete in a level field, Northern firms’ profits rise while Southern firms’ profits fall, which will induce further capital movement from South to North.

To see this more formally, we first note that the formula for the competition index is the same as in the case where only trade is allowed, i.e., the formula given in (29), since capital movement simply reallocate the firms from one country to the other without affecting the total number of firms of each type for given \(\omega_{h,N}\) and \(\omega_{h,S}\). Thus, the borrowing constraints for high-tech firms in country \(k\) can be written as

\[
R_k = \frac{\theta_k \beta^{\sigma-1}}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S})(g_h - \omega_{h,k})},
\]

whereas those for low-tech firms in North and South are respectively written as

\[
R_N = \frac{\theta_N}{\sigma \phi_W(\omega_{h,N}, \omega_{h,s})} \left[ g_l - \hat{\omega}_{l,N}(\omega_{h,N}) - \left( \frac{\sigma \omega_{m,N}}{g_l} \right) K \right],
\]

\[
R_S = \frac{\theta_S}{\sigma \phi_W(\omega_{h,N}, \omega_{h,S})} \left[ g_l - \hat{\omega}_{l,S}(\omega_{h,S}) + \left( \frac{\sigma \omega_{m,S}}{g_l} \right) K \right].
\]

22
When trade in goods is allowed in addition to capital movement, the normalized average productivity that faces Northern firms decrease from $\phi_N(\omega^K_{h,N}, K^K)$ and the one that faces Souther firms increase from $\phi_S(\omega^K_{h,S}, K^K)$ to $\phi_W(\omega_{h,N}, \omega_{h,S})$. Induced changes in profits will tend to increase the interest rate in North and decrease the interest rate in South. Capital moves from South to North to counter these movement in order to keep the interest parity between the two countries.

**Proposition 8** Trade in goods induces further capital movement when capital has been allowed to move between the two countries, and thereby changing the production side of the market structures. Trade tends to induce capital movement from South to North, expanding the industry in North and shrinking the industry in South.

**4.4.1 Effects of trade and capital movement when $\theta_N = \theta_S$ and $\bar{\omega}_N > \bar{\omega}_S$**

It follows from (48) and $R_N = R_S$ that $\omega^{KT}_{h,N} = \omega^{KT}_{h,S}$ and consequently $\omega^{KT}_{l,N} = \omega^{KT}_{l,S}$. Note that although the threshold wealth levels are the same between the two countries, the ratio of the mass of high-tech firms to the mass of low-tech firms is higher in North than in South due to $\bar{\omega}_N > \bar{\omega}_S$. Moreover, despite that trade induces further capital movement from South to North, capital moves out of capital-abundant North to South relative to the autarky. Figure 1(a) shows the equilibrium thresholds for the choice of productivity, i.e., $\omega_{h,k}$, in the four scenarios that we have considered. The arrows there indicate the movement of threshold when we move from autarky (or equilibrium with trade) to equilibrium with capital movement and from equilibrium with capital movement to equilibrium with trade and capital movement. Note again that trade will change the threshold only when accompanied by capital movement. The change in the threshold for being an entrepreneur, i.e., $\omega_{l,k}$, is similar, so it is not depicted in the figure for clarity.

**4.4.2 Effects of trade and capital movement when $\theta_N > \theta_S$ and $\bar{\omega}_N = \bar{\omega}_S$**

It follows from (48) and $R_N = R_S$ that the equilibrium $\omega_{h,N}$ and $\omega_{h,S}$ satisfy

$$\frac{\theta_N}{\theta_S} = \frac{g_h - \omega^{KT}_{h,N}}{g_h - \omega^{KT}_{h,S}}.$$

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Comparing this condition to the one in the case where only capital is allowed to move, i.e., condition shown in (47), we immediately find (also from $\phi_N(\omega_{h,N}, K) > \phi_S(\omega_{h,S}, K)$) that $\omega_{h,N}^{KT} < \omega_{h,N}^K$ and $\omega_{h,S}^{KT} > \omega_{h,S}^K$. This, of course, is consistent with our observation that $\phi_N(\omega_{h,N}, K) > \phi_S(\omega_{h,S}, K)$ trade (in addition to capital movement) induces further capital movement from South to North. Figure 1(b) shows the thresholds for the choice of technology. Again, it shows that trade changes the production side of the market structure only when accompanied with capital movement.

5 Concluding Remarks

In the model where entrepreneurs with different wealth levels choose technology levels when they enter a differentiated good sector, we have shown that the firm heterogeneity in productivity arises only if there exists financial imperfection. We have also examined the impact of international trade in goods and capital movement between two countries. We have found among others that (i) trade in goods alone will not affect the production side of the market structure, (ii) capital tends to move from a wealthy country to the other and from a country with a poorer financial institution to the other, shrinking the industry in the source country and expanding the industry in the host country, (iii) trade in goods affects the market structures only when it is accompanied by capital movement, and (iv) when capital is also allowed to move, trade in goods itself induces capital movement from South (with less wealth and/or with less-developed financial institution) to North.

These findings regarding the impacts of financial imperfection on the differentiated good industry (which can be thought of as a manufacture industry) are in general quite different from the conventional wisdom in international trade theory without any consideration of financial imperfection. Since no country has the perfect financial institution in practice, it is important to know how the traditional theories should be modified when we incorporate financial imperfection into the models.

This paper is one of the first attempts to investigate interactions between financial development and international trade, so there are many related topics to be explored. It would be
interesting, for example, to endogenize financial development by incorporating political and legal systems explicitly into the model. It would also be interesting to extend the model to a dynamic one so that the wealth distribution, which has been shown to play an important role in the analysis of this paper, is endogenously determined.
Appendix

In this Appendix, we present a simple model to justify an imperfect financial institution. This model setting is a simplified version of Tirole’s (2006).

Let us consider the situation in which an agent tries to borrow \( g \) from a lender to implement a profitable project. This project potentially generates profits of \( \pi(> Rg) \) where \( R \) is the exogenous (gross) interest rate. In order to complete the project successfully with a high probability, however, the agent must exert effort, which is unobservable to the lender.

If the agent exerts effort, the project generates \( \pi \) with the probability 1; if the agent shirks, on the other hand, the project generates \( \pi \) with the probability \( p^L(< 1) \) and it generates 0 with probability \( 1 - p^L \). By shirking, however, the agent can get non-pecuniary benefits \( b\pi \), where \( 0 < b < 1 \).

The agent unambiguously shirks if the entire \( \pi \) goes to the lender. In order to induce the agent to exert effort, therefore, the lender must abandon some of \( \pi \), giving a contingent reward \( w \) to the agent; the reward is given to the agent if and only if the project has successfully generated \( \pi \). The reward \( w \) should satisfy the incentive condition, \( w \geq p^L w + b\pi \), where the left-hand side is the agent’s payoff when she exerts effort, while the right-hand side is her expected payoff when she shirks. We assume that negative rewards (i.e., penalties) are not allowed perhaps because the asset held by the agent is limited. This incentive condition can be written as

\[
w \geq \frac{b}{1 - p^L} \pi.
\]

The lender expects to obtain at most \( [1 - (b/(1 - p^L))]\pi \) if he induces the agent to exert effort. Alternatively, he may set \( w = 0 \) so that he obtains the expected payoff of \( \pi^L \pi \). Consequently, the lender obtains the returns at most \( \theta \pi \), where

\[
\theta = \max \left\{ 1 - \frac{b}{1 - p^L}, p^L \right\}.
\]

Obviously, the lender will not lend \( g \) if \( Rg \) exceeds \( \theta \pi \). Note that if \( p^L \) is small enough, \( \theta \) is equal to \( 1 - (b/(1 - p^L)) \). Under a developed financial institution with a solid legal system, non-pecuniary benefits tend to be small. The parameter \( \theta \) can be considered to represent
the quality of a financial institution because $\theta$ increases as $b$ diminishes.
References


Figure 1. Thresholds for the choice of high-productivity technology

(a) Case where $\theta_N = \theta_S$, $\bar{\omega}_N > \bar{\omega}_S$

(b) Case where $\theta_N > \theta_S$, $\bar{\omega}_N = \bar{\omega}_S$