When do non-linear contracts deter the expansion of (in)efficient rivals?

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Abstract

We study the anti-competitive effects of non-linear contracts (e.g., two-part tariffs, rebates, quantity discounts) between a buyer and an incumbent, dominant firm in the presence of small rivals wishing to expand. While it is known that in a rent-shifting environment with imperfect information two-part tariffs are not different than the exclusive contracts of Aghion and Bolton, we show that alternative non-linear schemes can be remarkably different. Schemes that do not include unconditional payments and, therefore, work entirely through the spot market (e.g., rebates), prevent the buyer and the incumbent to commit ex-ante to the transfer of rents. The effect is so strong that these contracts are rarely anti-competitive; more so when the incumbent's bargaining power and outside option are large. The reason these contracts exist is because they can still be used to extract rents from inefficient rivals, which ultimately limits the amount of inefficient expansion.

1 Introduction

The potential exclusionary effect of rebates and quantity discounts has received widespread attention from scholars and antitrust authorities in recent years. There is a long list of cases from the well known *British Airways*, *Michelin II* and *Intel*[1] to some very recent ones such as

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[1]Description of these cases can be found in *British Airways v. Commission* (EU Case T-219/99), *Virgin v. British Airways* (2nd Cir. 2001 and S.D.N.Y. 1999), *Michelin II v. Commission* (EU Case T-203/01), *Intel v. Commission* (EU Case T-286/09), *AMD v. Intel* (CA No. 05-441, District of Delaware), and *Attorney General Andrew M. Cuomo v. Intel* (C.A. No. 09-827, District of Delaware).

Unilever v. Chile (FNE 2013). Rebates are believed to be a cheaper and more effective way of exclusion as they "allow firms to use the inelastic portion of demand as leverage to decrease the price in the elastic portion of demand, thereby artificially increasing switching costs for buyers" (Maier-Rigaud, 2005). The granting of rebates would then generate a "lock-in" effect inducing buyers to concentrate their purchases from a single supplier, much as exclusive dealing contracts would do (Faella, 2008). In fact, European Courts have been particularly harsh in some of their rulings towards theses non-linear contracts, especially regarding all-unit retroactive rebates (where the per-unit price falls discontinuously for *all* units purchased after a pre-specified sales threshold is reached).[2]

The problem is that quantity discounts can arise without an exclusionary motive and, more importantly, can be efficient. For example, all-unit discounts may be used in a bilateral monopoly setting to avoid a problem of double marginalization when demand is known, and be used as a screening device when retailers have private information regarding demand (Kolay, Ordover and Shaffer, 2004). Moreover, they can also help solve agency and hold-up problems by aligning the incentives of manufacturers and retailers; an argument analogous to the efficiency defense of exclusive dealing contracts (Marvel, 1982; Motta, 2004; and Whinston, 2006). Rebates may also generate efficiency gains for a dominant firm allowing it to exploit its economies of scale and/or save on transaction costs (Rey et. al. 2005). And they may even increase price competition between downstream firms (Ahlborn and Bailey, 2006). For these reasons both authorities and scholars tend to favor a case-by-case approach when looking at these non-linear schemes in which their potential efficiency gains are balanced against their possible anticompetitive effects (Rey et. al. 2005; Spector 2005; Office of Fair Trading 2005; Motta 2006).

The economic view on the anticompetitive effects of exclusive dealing has also evolved significantly over the last fifty years. Chicago schoolars contend that exclusive deals cannot be used to deter the entry of efficient rivals (e.g., Posner 1976; Bork 1978). A buyer would not sign an exclusive contract that reduces competition unless it is fully compensated, which the incumbent cannot afford when the entrant is more efficient. Aghion and Bolton (1987), henceforth A&B, were the first to challenge this so-called Chicago-School view. They show that exclusive deals can occur in equilibrium if the contract generates an externality into a

[2]For an excellent survey of some European Cases see Gyselen (2006), and for an analysis of comparative law between the US and the EU see Ahlborn and Bailey (2006).

third party (i.e., the potential entrant) that is absent at the contracting stage. There is now an extensive post-Chicago literature that can be organized around three distinct families of models with different assumptions and outcomes, namely, "rent-shifting" models, "naked" exclusion models, and "downward competition" models.

Rent-shifting models (e.g., A&B 1987; Spier and Whinston 1995; Marx and Shaffer, 1999 and 2004) are based in an incumbency game between a dominant supplier, a buyer, and a potential rival (e.g., entrant) in a sequential contracting environment. The incumbent has a first-mover advantage when negotiating with the buyer. Quoting Marx and Shaffer (2004), "...these parties will set the terms of their contract with an eye on extracting rents from the potential rival." In the presence of imperfect information, however, rent extraction will not be complete and exclusion of some efficient rivals will emerge as a side effect. Naked-exclusion models (e.g., Rasmussen et. al. 1991; Segal and Whinston 2000; Fumagalli and Motta 2006; Spector 2011), on the other hand, assume that the potential rival requires of a minimum scale of operation due to either scale or network economies. Then, if there are numerous unorganized buyers, there is scope for entry deterrence as the incumbent needs to compensate and lock-in only a subset of them in order to prevent the rival from achieving this minimum scale.[3] Finally, downstream competition models (Simpson and Wickelgren 2007; Abito and Wright 2009; Asker and Bar-Isaac, 2013) show that when buyers are downstream competitors instead of final buyers, signing an exclusive deal may be profitable even in the absence of coordination problems and economies of scale. By preventing the entry of an upstream supplier, exclusive deals soften competition downstream and avoid giving up rents to final consumers.

The potential exclusionary implications of non-linear pricing have been analyzed in all three families of models. In a naked-exclusion setting, Karlinger and Motta (2012) found that rebates have a higher exclusionary potential reducing the set of achievable socially efficient equilibria. On the other hand, Asker & Bar-Isaac (2013) analyze how rebates and other mechanisms can be used in a downstream competition model by a dominant firm to share its monopoly quasi-rents with its retailers, effectively "bribing" them not to distribute a rival's product. Finally, Choné and Linnemer (2012), henceforth C&L, study general pricing schemes in a rent-shifting model, extending Marx and Shaffer (1999) to imperfect information. They found that under one dimensional uncertainty, two-part tariffs work similar to A&B exclusive contracts by setting a tax or penalty on to the expansion of rivals, although again, at the expense of blocking some

[3] Compensations are not even required when consumers compete for them (Whinston, 2006).

moderately efficient competitors. Because two-part tariffs and other nonlinear schedules are typically seen as equivalent for the purposes of rent extraction, for example in the monopoly pricing literature,[4] it is tempting to extend the equivalence between A&B contracts and two-part tariffs to any non-linear scheme such as rebates (see, e.g., Rey et. al. 2005).

Building upon the rent-shifting model of A&B, we investigate whether this equivalence does indeed hold true more generally. We find that the existence of unconditional payments, or more precisely, of an ex-ante commitment (e.g., liquidated damages) to transfer rents from the retailer to the manufacturer is crucial for the equivalence to hold. Interestingly, such transfers are either rarely observed or have not received much attention by antitrust authorities when analyzing rebates and other forms of quantity discounts. More importantly, we found that non-linear contracts that do not rely on such transfers (e.g., rebates) are highly unlikely to deter efficient rivals from expanding, especially when the incumbent's *bargaining position*, which can be understood as some combination of bargaining power and outside option, is strong.

The intuition is that the first-best contract for the incumbent-buyer coalition is to charge a very low marginal price in order to extract rents from efficient rivals and use high infra-marginal prices to distribute surplus from the buyer to the incumbent. When unconditional transfers are part of the contractual arrangement, like in two-part tariffs, the first-best contract is feasible to implement because part of the transfer is done ex-ante, so a significant amount of those high infra-marginal prices become sunk at the time the buyer negotiates with the expanding rival. However, in the absence of such commitment, any transfer of surplus must be done entirely through the spot market. This implies that these high infra-marginal prices are fully internalized by the buyer at the time of purchase, who may refrain to buy any additional units from the incumbent once he decides to buy from the expanding supplier. By placing a cap on the price that can be charged for the infra-marginal units, the buyer's ex-post participation constraint makes rent distribution within the incumbent-buyer coalition much harder.[5] It forces the coalition to move away from the first-best contract towards contracts with less exclusionary potential, that is, with higher marginal prices.

We also find that the stronger the bargaining position of the incumbent —for example, due to the indispensable nature of its product— the larger the surplus that needs to be transfered

[4] But not necessarily in duopoly pricing as recently shown by Calzolari and Denicolo (2013).

[5] Note also that when unconditional payments are restricted, the standard approach in the literature for finding the coalition optimum –the solution that maximizes overall surplus– is not longer valid. We return to this in Section 3.

to him and, hence, the more difficult is for the coalition to implement a contract with an anticompetitively low marginal price. Thus, a small contestable share makes the implementation of an anticompetitive quantity contract less not more likely, as commonly believed (e.g., Mota, 2006).[6] However, even if they do not prevent efficient rivals from expanding, quantity contracts can still emerge in equilibrium as a way to extract rents from inefficient rivals, which limits the amount of inefficient expansion that would otherwise occur in the absence of contracts.[7]

The contribution of the article is twofold. From a theoretical perspective, we question the common view that in a rent-shifting environment non-linear contracts such as rebates work very much like exclusive contracts in terms of deterring efficient entry/expansion. We go over the underlying mechanism that makes this "apparent" equivalence not only to break down but in most cases to revert. And from a policy perspective, our results have important and clear-cut antitrust implications for cases that are best captured by a rent-shifting environment. The rest of the paper is organized as follows. We start the next section with a brief discusion of *Unilever v Chile*, simply to illustrate what seems to make a good case for a "rent-shifting" analysis. We then present a simple model that follows A&B very closely (the only difference is that instead of considering a large potential entrant, we consider a small rival that is already in the market and at best can expand to serve a larger fraction of the total demand). In this simple model, rebates are never anticompetitive and may arise only as a vehicle to extract rents from inefficient rivals. In Section 3 we work with a more general formulation that considers the full range of possible bargaining powers and outside options (i.e., payoffs in the absence of contracts). This generalization allows us to better understand the forces behind the results of the simple model and appreciate how general they seem to be. In section 4 we show that our results also extend to a downward sloping demand. Finally in section 5 we discuss the antitrust implications and contrast them with the implications of alternative models.

[6] In fact, Motta (2006, p. 372) explains "...a better understanding of how to balance exclusionary and efficiency effects of exclusive contracts is needed but it seems safe to assume that the former might dominate the latter only if the firm using exclusive contracts has a very strong market position." In a rent-shifiting environment, we find the exact opposite.

[7] Absent of contracts, there is no inefficient entry in A&B because either supplier can serve the entire demand. In our setting, the rival can at best serve a fraction of the total demand which gives rise to inefficient mixed-strategy equilibria in the spot; very much like in the slot-competition model of Jeon and Meniccuci (2012). In their paper, inefficiencies dissapear when firms are allowed to sell bundles while here when firms are allowed to use non-linear prices in the spot.

2 A Simple A&B Model

2.1 A motivating case

The rent-shifting model suits well for analysis of antitrust cases in which (i) buyers are relatively big, so an entrant or expanding supplier can achieve a minimum scale of operation by serving just one buyer; (ii) downstream competition is not that intense, so upstream competition does not fully permeate into the downstream market; and (iii) the incumbent and buyer are not entirely certain at the time of "contracting" about the value that a rival's product can add to the market either in terms of lower costs or higher quality (or cannot discriminate among several potential rivals of different characteristics). All three conditions seem to apply to *Unilever v. Chile*.

Unilever has been accused of restricting the expansion of rivals in the wholesale market for laundry detergents by agreeing to "all-unit" retroactive discounts with supermarket and other distributors. Unilever has a market share of over 70% in the supermarket segment (its closer competitor has 13% and there are many other small suppliers, some short-lived). Supermarket sales make up for over 60% of sales, and of those supermarket sales, the vast majority is from the three largest supermarket chains with shares of 35%, 28% and 24%, respectively. Any of these supermarkets qualify as a large buyer (even to approach a small supplier to develop its own private label). As for the intensity of competition at the retail level, we do not have evidence on laundry detergents but we do on other items (e.g. instant coffee) showing the two largest chains pricing well above wholesale prices (Noton and Elberg, 2013).[8]

These retroactive discounts are typically negotiated on a yearly basis without discriminating among potential rivals, in part because the quality and cost of their products are unknown at the time of contracting and likely to vary across suppliers depending on advertising and on innovation efforts. There indeed appears to be some "innovation" in production processes, especially on powder detergents, that save on infraestructure requirements but at the cost of providing a lower quality product (FNE 2013).

2.2 Model assumptions

There are three risk-neutral agents. A single buyer B demands one unit of an infinitely divisible good at reservation value v. This good can be either supplied by a dominant, incumbent firm I

[8] Smith (2004) also documents for the UK supermarket industry high price-cost markups.

or by a (small) expanding rival E, who is already in the market (or alternatively, whose entry cost is 0).[9] I has unlimited capacity to produce at constant marginal cost $c_I \in (0, v)$. E, on the other hand, has constant marginal cost c_E, but can sell at most $\lambda \in (0, 1)$ of the good. The literal interpretation of λ is that it corresponds to E's installed capacity but an alternative and more recent interpretation is that it represents the "contestable share" of the market, that is, the amount for which the customer may prefer and be able to find substitutes (European Commission, 2009).

There are three periods: two contracting stages ($t = 1, 2$) followed by a spot market or transaction stage ($t = 3$). As standard in the rent-shifting models, there is sequential contracting. At $t = 1$, I makes a take-it-or-leave-it offer to B (as we will consider different type of non-linear contracts, the specific form of the contract offered is specified below).[10] At this time c_E is unknown to both I and B but it is common knowledge that it distributes according to the cdf $F(\cdot)$, over the interval $[0, \bar{c}_E]$, where $c < \bar{c}_E \leq v$ and F/f is nondecreasing. It is important to notice that what is relevant here is not the actual source of uncertainty —whether is E's cost and/or the buyer's valuation for its product— but rather that the surplus created by the relationship between E and B is unknown at the time when I and B are contracting. As usual we also assume that contracts are not renegotiable ex-post, that is, after c_E is revealed.

At $t = 2$ and after observing the contract signed by I and B, E is free to make a take-it-or-leave-it offer to B for its λ units. At this moment c_E becomes publicly known (whether the uncertainty regarding c_E is resolved at this time or at the opening of the spot market is less relevant, as it is reasonable to allow the EB coalition to sign contingent schemes on c_E absent information asymmetries; the crucial issue is the absence of information asymmetries at this stage). Finally, at $t = 2$ the transaction stage opens and B decides how much to buy from each supplier. In the absence of contracts, I and E compete in the spot market by simultaneously setting uniform prices,[11] otherwise parties adhere to the price conditions established in the

[9] The unit-demand assumption makes the model easier to handle as it eliminates firms' incentives for using non-linear contracts to avoid allocative inefficiencies. In Section 4 we allow for a downward sloping demand curve.

[10] In the simple model, and as in A&B, suppliers have all the bargaining power in their bilateral relations with B. In section 3 we consider a Nash bargaining model that covers the full range of bargaining powers.

[11] The uniform price assumption is not only in the spot market of A&B but also in wholesale spot markets of more recent papers (e.g., Hendricks and McAfee 2008; Jeon and Menicucci 2012). Nevertheless, in Section 3 we relax this assumption as we allow parties to compete in the spot under different formats, for example, short-term bilateral negotiations that may or not involve two part tariffs. The idea here is that more complex non-linear contracts are usually negotiated on a long term basis (1 year), but otherwise parties can still engage in some negotiation on a monthly/weekly basis, although simpler due to bargaining/transaction costs.

contracts.

Definition 1. *A non-linear contract is said to be "anticompetitive" if it blocks the expansion of an efficient rival, that is, the expansion of a rival with cost $c_E < c_I$.*[12]

While closer to the antitrust practice of what constitutes an anticompetitive contract, this definition makes no precision about the welfare implications of an anticompetitive contract. Such precision is unnecessary when in the absence of a contract entry is always efficient, as in the A&B original model, in which case any anticompetitive contract reduces welfare. There can be instances, however, of too much entry in the absence of a contract, in which case an exclusive deal may indeed increase welfare by limiting inefficient entry (Whinston 2006, p. 188). While it is true that this latter possibility complicates the welfare analysis of some of these contracts, it is less relevant for our paper because our focus is to show when and why many of the non-linear contracts we observe (e.g., rebates, quantity discounts) are hardly ever anticompetitive, a sufficient condition for a contract not to be welfare reducing.

2.3 Agents' outside options

We begin by characterizing I and B's outside options, that is, their payoffs in the absence of contracts. If I and B fail to sign a contract, will E and B sign one? Given its capacity constraint, E cannot use B to shift rents from the incumbent using the quantity discounts or rebates we have in mind.[13] And given our inelastic demand, the best E can do is to offer a linear price to B at this contracting stage. Thus, the problem faced by E at $t = 2$ is to either make a price offer to B for its λ units or compete in the spot tomorrow, at $t = 3$. Suppose the latter (below we show this is what happens in equilibrium).

Lemma 1. *Let $c^* \equiv c_I + (1 - \lambda)(v - c_I)$. The equilibria of the spot-market game in which firms simultaneously set uniform prices can be characterized as follows:*[14]

1. *When $c_E \geq c^*$, there is a pure strategy Nash equilibrium with prices $p_I = p_E = c_E$ and payoffs $\pi_I = c_E - c_I$, $\pi_E = 0$ and $\pi_B = v - c_E < \lambda(v - c_I)$.*

[12] *yeths is portant to make the distinction between a contract that is anticompetitive from a contract. Whinston (2006).*

[13] In principle it could using contracts with breach penalties. This would leave I with no rents, which is unlikely when λ is small. We like to think that the contracts that the expanding rival and the buyer can sign, provided I does not sign any, are non-linear schedules that work entirely through the spot market.

[14] *Note that in the case of $c_E < c^*$, as $\lambda \to 1$, $c^* \to c_I$ and spot competition becomes efficient; moreover, E's price distribution collapses into a singleton, $p_E^* = c_I$, yielding the standard Bertrand outcome. We do not exactly converge to the result in the A&B model because they assume that the entrant faces a positive, although very small, fixed cost of entry.*

2. *When $c_E < c^*$, there are only mixed strategy equilibria with*

   (a) *both firms randomizing over the support $[c^*, v]$*

   (b) *and payoffs $\pi_I^e(p_I) = (1-\lambda)(v-c_I)$, $\pi_E^e(p_E) = \lambda\,(c^* - c_E)$, and $\pi_B^e < \lambda(v - c_I)$.*

Proof. See the Appendix. □

Now, to understand E's decision as to whether make a price offer at $t = 2$ or compete in the spot at $t = 3$, consider first the case in which $c_E > c^*$. Since the buyer will not accept any price higher than c_E from E, the latter is indifferent between selling at c_E at $t = 2$ or going to the spot market at $t = 3$; in either case he makes zero profits. Both the buyer and the incumbent are also indifferent as to what E does. The second case, $c_E < c^*$, is a bit more involved. If E sets $p_E > c^*$ at $t = 2$, it is optimal for I to undercut that price at $t = 3$ and leave E with zero profits. Alternatively, if E sets $p_E \leq c^*$ at $t = 2$, it is not longer optimal for I to price right below p_E but rather to price at v and get the residual monopoly profit, $(1-\lambda)(v - c_I)$, which is equal to the profit she would get when competing in the spot. So the best E can do is to set $p_E = c^*$, which yields the exact same payoff than any of the mixed-strategy equilibria in the spot. Whether B accepts E's offer $p_E = c^*$ at $t = 2$ will depend on $\pi_B^e \gtreqless \lambda(v - c^*)$; although in any case its surplus will be less than $\lambda(v - c_I)$. Therefore, regardless of whether c_E is lower than, equal to or greater than c^*, E gains nothing by contracting at $t = 2$.

We can summarize this discussion as follows

Lemma 2. *When the incumbent and the buyer fail to sign a contract at $t = 1$, their outside options, denoted, respectively, by $\bar{\pi}_I$ and $\bar{\pi}_B$, satisfy*

$$\begin{aligned} \bar{\pi}_I &= (v - c_I)(1-\lambda) + [1 - F(c^*)]\,\{\mathbb{E}(c_E \,|\, c_E > c^*) - c^*\} \geq (1-\lambda)(v - c_I) \\ \bar{\pi}_B &< \lambda(v - c_I) \end{aligned}$$

Proof. Immediate from the payoffs in Lemma 1.

2.4 A&B exclusive contracts and two-part tariffs

Consider first an A&B exclusive dealing contract (w, l) according to which —if accepted at $t = 1$— the buyer commits to buy exclusively from the incumbent at $t = 3$ at the wholesale price w. In case the buyer decides to buy some units from the expanding rival, he must pay the incumbent a per-unit penalty (or liquidated damages) l.[15] At $t = 1$, the incumbent anticipates that only those expanding rivals with costs $c_E \leq w - l$ can, at $t = 2$, induce the buyer to agree to purchase units from them because no buyer is willing to switch supplier at a unit-price higher than $w - l$. Thus, the probability of expansion at the time of contracting is $F(w - l)$.

The A&B contract the incumbent offers the buyer is then obtained from the following problem

$$\begin{aligned} \max_{w,l} \mathbb{E}\pi_I(w,l) &= [(1-\lambda)(w-c_I)+\lambda l]\,F(w-l)+(w-c_I)\,[1-F(w-l)] \\ \text{s.t.} \quad v - w &\geq \bar{\pi}_B \end{aligned} \tag{1}$$

which has solution

$$\begin{aligned} w^* - l^* &= c_I - \frac{F(w^*-l^*)}{f(w^*-l^*)} \\ w^* &= v - \bar{\pi}_B \end{aligned}$$

It is clear that $w^* - l^* < c_I$ and straightforward to verify that $\mathbb{E}\pi_I(w^*, l^*) > \bar{\pi}_I$ and $w^* > c_I$.

As first shown by A&B, these exclusive deals are not only profitable for both the incumbent and buyer to sign but they have anticompetitive implications in that they block the expansion of some efficient rivals, those with costs $c_E \in [w^* - l^*, c_I)$.[16] The idea of an A&B contract is not to block the expansion of all efficient rivals, but to extract rents from the expansion of the most efficient ones, those with costs $c_E < w^* - l^*$. In expectation, this efficient expansion reports the incumbent profits equal to $F(w^* - l^*)[\lambda l^* - \lambda(w^* - c_I)] = \lambda F(w^* - l^*)(c_I + l^* - w^*)$.

A&B contracts are not the only way to extract rents from efficient rivals. Consider now a two-part tariff contract (p, T) according to which the buyer commits to buy from the incumbent at $t = 3$ at the wholesale price p and to an *unconditional* lump-sum transfer of T, that is, independent of how much B ends up buying from I (possibly T is paid at the signing of the contract). Facing this contract, the only way for E to induce B to buy some units from him is with a price $p_E \leq p$, which implies that the probability of expansion as of period 1 is equal to $F(p)$. Thus, irrespective of whether E expands or not, the buyer gets $v - p - T$ in case of

[15]It makes no difference to consider a lump-sum penalty because the buyer will never breach, if at all, for less than λ units.

[16]Note that if this A&B contract is not allowed there is too much entry, so it is not entirely clear from a welfare perspective the right course of action here. Our analysis contributes to this welfare analysis by identifying contracts for which this ambiguity disappears.

signing the contract.

The two part-tariff (2PT) contract the incumbent offers the buyer is again obtained from the problem

$$\begin{aligned}\max_{p,T} \mathbb{E}\pi_I(p,T) &= T + (1-\lambda)(p-c_I)F(p) + (p-c_I)\left[1-F(p)\right] \\ \text{s.t. } v - p - T &\geq \bar{\pi}_B\end{aligned}$$

Denote by p^* and T^* the solution to the incumbent's 2PT problem. Relabing p as $w - l$ and T as l, it becomes clear that the 2PT problem reduces exactly to the A&B problem and $p^* = w^* - l^* < c_I$ and $T^* = l^*$. The two problems are equivalent in all respects, including their anticompetitive implications, except for the timing of transfers (which is irrelevant under risk-neutrality). In two-part tariffs the surplus transfer from the buyer to the incumbent is done or commited ex-ante, before any expansion, while in A&B contracts is done ex-post, after the expansion has actually ocurred.

The conceptual equivalence between A&B and two-part tariff contracts was first noted by Marx and Shaffer (1999) in a perfect information setting and later extended by Choné and Linnemer (2012) to an imperfect information environment. Intuitively, the optimal two-part tariff is designed to extract rents from the most efficient rivals charging a low marginal price for the contestable units $p^* < c_I$ and to distribute those rents charging high inframarginal prices via the up-front payment T^*. Uncertainty regarding c_E at the time of contracting prevents perfect discrimination which generates the well-known side effect of blocking some efficient rivals.

Curiously, when analyzing the *Standard Fashion* case, Marvel (1982) argues that the emergence of up-front charges with lower marginal prices after the outlaw of exclusivity contracts, was evidence against any "anticompetitive" properties this exclusivity clauses might have had in the first place. The equivalence between A&B contracts and two-part tariffs points otherwise. More importantly, because two-part tariffs and other nonlinear schedules are typically seen as equivalent for the purposes of rent extraction, for example in the literature of monopoly pricing, it is tempting to extend the equivalence between A&B contracts and two-part tariffs to other non-linear contracts (see for example Rey et. al. 2005). We show next however that there is no such equivalence.

2.5 Rebates

Following the contractual arrangements seen in some recent cases (e.g., *Michelin II*, *Unilever*, etc), consider now the all-unit retroactive rebate contract $(r, R, \bar{Q} = 1)$, where r is the listed price, $\bar{Q}$ is a pre-specified sales threshold above which the rebate applies, in this case is equal to 1, and R is a lump-sum rebate.[17] Under this contractual arrangement the buyer commits to a price r when pruchasing $q \in (0, \bar{Q})$ units from the incumbent and to $r - R$ when purchasing above that amount.

If B signs the contract, an expanding rival E can still induce B to buy λ units from him if he is compensated for the forgone rebate. For that to be the case, E's price offer p_E must satisfy

$$v - (1 - \lambda)r - \lambda p_E \geq v - r + R$$

or

$$p_E \leq r - \frac{R}{\lambda}$$

If E decides to expand/enter it will offer exactly $p_E = r - R/\lambda$, which sets the probability of expansion at the time of contracting equal to $F(r - R/\lambda)$.

The cutoff price $r - R/\lambda$ is typically known as the *effective price of the contestable demand* and represents the marginal cost the buyer faces when purchasing from an alternative supplier. This price differs from $r - R$ because the contestable share is smaller than the total demand, which is what allows the incumbent to leverage its position. Indeed, it is relatively easy to conceive a profitable, yet anticompetitive, rebate scheme when λ is particularly small

$$r - \frac{R}{\lambda} < c_I < r - R \tag{2}$$

Since the smaller the contestable demand, the easier is for the incumbent to deter efficient expansions (in the limit as $\lambda \to 0$, it becomes virtually costless), it is not surprising the great deal of attention and controversy around the estimation of the contestable demand that we have seen in some recent cases; notably, *Intel v. Commission*.

[17] In Section 3 we explore the optimality of these two-step quantity discounts. Note also that in this inelastic-demand setting we do not need to make any distinction between own-supply discounts and market-share discounts that are a function of both how much the buyer purchases from the dominant supplier and how much from rival suppliers. In Section 4 we make this distinction when we work with a downward sloping demand.

Because the buyer pays $r - R$ regardless, the incumbent now faces the following program

$$\begin{aligned} \max_{r,R} \mathbb{E}\pi_I(r,R) &= (1-\lambda)(r-c_I)F(r-R/\lambda) + (r-R-c_I)\left[1-F(r-R/\lambda)\right] \\ \text{s.t.} \quad v - r + R &\geq \bar{\pi}_B \end{aligned} \tag{3}$$

The first term is the profit from selling $1-\lambda$ units at price r, which happens with probability $F(r-R/\lambda)$, and the second term is the profit from selling all units at price $r-R$, which happens with probability $1-F(r-R/\lambda)$. Now, relabeling r as $p+T/(1-\lambda)$ and R as $\lambda T/(1-\lambda)$, the rebate program becomes

$$\begin{aligned} \max_{p,T} \mathbb{E}\pi_I(p,T) &= T + (1-\lambda)(p-c_I)F(p) + (p-c_I)\left[1-F(p)\right] \\ \text{s.t.} \quad v - p - T &\geq \bar{\pi}_B \end{aligned} \tag{4}$$

which is the incumbent's 2PT program.

This result explains the apparent equivalence between exclusive deals and two-part tariffs, on the one hand, and between two-part tariffs and quantity discounts, on the other. From (3) and (4), one can see that the rebate scheme works very much like a two-part tariff scheme, that is, charging a very low marginal price for the contestable units and high prices for the infra-marginal units which is what allows the distribution of rents among the members of the incumbent-buyer coalition. One can also see from (3) that the rebate scheme works very much like an A&B contract, that is, imposing a tax or penalty to the buyer when switching supplier as the breaching clause in A&B is.

There is a missing element in all this comparison, however, that makes this apparent equivalence to fall apart. Unlike A&B and 2PT contracts, the rebate contract is not equiped with unconditional payments. The rebate scheme works entirely through the spot market in that both the rent extraction and distribution are done ex-post during the transaction stage. This subtle but crucial difference imposes an additional participation constraint on the buyer side. In period 3, the buyer will not be purchasing units in equilibrium at prices above its reservation price v, from either supplier. This requires that in equilibrium must hold

$$r - R < r \leq v$$

However, it can be established that

Lemma 3. *The solution to the "unrestricted" rebate program (3), which we denote by* (r^u, R^u)*, violates the buyer's ex-post participation constraint, i.e.,* $r^u > v$*.*

Proof. From the relabeling of variables in programs (3) and (4) and using $\bar{\pi}_B = v - p^* - T^*$ yields

$$r^u - v = \frac{T^*}{1-\lambda} + p^* - v = \left(\frac{1}{1-\lambda}\right)[\lambda(v - p^*) - \bar{\pi}_B]$$

Furthermore, from Lemma 2 we know that $\bar{\pi}_B < \lambda(v - c_I)$, therefore

$$r^u - v > \left(\frac{1}{1-\lambda}\right)[\lambda(v - p^*) - \lambda(v - c_I)] = \left(\frac{\lambda}{1-\lambda}\right)(c_I - p^*) > 0$$

which finishes the proof.

The rebate scheme (r^u, R^u) does not allow for both optimal rent extraction and rent distribution because of the ex-post "opportunistic" behavior of the buyer, or which is the same, because of the absence of unconditional payments. Moving away from the first-best contract —either (w^*, l^*) or (p^*, T^*)— brings up three important questions: what is the optimal rebate scheme the incumbent can offer the buyer while satisfying its ex-post participation constraint? is it still anticompetitive? and if not, does it still pay the incumbent to sign it?

To answer these questions, let us add to the incumbent's rebate program (3), the buyer's ex-post constraint $r \leq v$. Let (r^*, R^*) denote the solution to this "updated" program and $c_E^* \equiv r^* - R^*/\lambda$ the critical cost level below which a rival supplier expands.

Proposition 1. *The rebate contract* (r^*, R^*) *is never anticompetitive, i.e.,* $c_E^* \geq c_I$*.*

Proof. For I to offer the rebate contract (r^*, R^*), it must be true that $\mathbb{E}\pi_I(r^*, R^*) \geq \bar{\pi}_I$, that is

$$[(r^* - c_I)(1-\lambda) - \bar{\pi}_I] + \lambda(c_E^* - c_I)[1 - F(c_E^*)] \geq 0 \tag{5}$$

But we know that $r^* \leq v$, so $(r^* - c_I)(1-\lambda) \leq (v - c_I)(1-\lambda) \leq \bar{\pi}_I$. In turn, these inequalities indicate that the first term in (5) is non-positive which requires the second term to be non-negative

$$\lambda(c_E^* - c_I)[1 - F(c_E^*)] \geq 0$$

And since $F(c_E^*) < 1$, we have that $c_E^* \geq c_I$.

Lemma 4. *The optimal rebate is characterized by* $r^* = v$ *and* $c_E^* \in (c_I, \bar{c}_E)$*. Moreover, a sufficient condition for the incumbent to find it profitable offer the contract is* $\bar{c}_E \leq c^*$*.*

Proof. See the Appendix. □

Proposition 1 and Lemma 4 convey two remarkable messages of the simple model: Rebates are never anticompetitive, yet, they can emerge in equilibrium. The two results are intimately connected. To get an intuition for the first result notice that it is the buyer the one that directly appropriates the rents extracted from rivals when facing the low marginal prices set by either I or E. In the absence of unconditional payments, it is ex-post unfeasible for B to transfer a large fraction of those rents to I enough cover his outside payoff. When transfers are so restricted, relative to the incumbent's outside option, the incumbent will never agree on an anticompetitive schedule.

As for the second result, notice that a higher effective price c_E^* makes the transfer restriction less demanding by closing the gap between the rent B captures from rivals and I's outside option. And because there is too much entry in the spot in case I and B do not sign a contract, there might be still some scope for rent-shifting from moderately inefficient rivals. Understanding why $\bar{c}_E \leq c^*$ is a sufficient, but not necesary, condition to ensure that rebates emerge in equilibrium shed further light on the latter. Given that rebates cannot be used to extract rents from efficient rivals (Proposition 1) and that there are no allocative inefficiencies (inelastic demand), the only reason to use rebates is to deter the expansion of some inefficient rivals. But in doing so, the incumbent has actually two choices: (i) two, non-contingent instruments (the two-step rebate contract) and (ii) an imperfect but contingent instrument (an ex-post uniform price). When $\bar{c}_E \leq c^*$, that is, when rivals are relatively efficient, choice (i) is the preferred one by the incumbent.

The simple model has served to advance three of our main results, namely, (i) that unconditional payments are critical for the equivalence between exclusive dealing contracts and non-linear contracts to hold in a rent-shifting environment; (ii) that in the absence unconditional payments contrats such as rebates and quantity discounts are highly unlikely to be anticompetitive; and (iii) that these contracts can still emerge in equilibrium as a mean to extract rents from inefficient rivals. In the next two sections we show how these results stand to different extensions of the simple model including the consideration of more flexible non-linear schedules, different bargaining powers and outside options, and a downward sloping demand.

These extensions help not only appreciate how general the results of the simple model are but also better understand the underlying forces that explain them.

3 General Setting

[TO BE EDITED AND COMPLETED]

We generalize our simple model in several directions except for the inelastic demand assumption that we relax in Section 4. First, we allow the agents to have different bargaining power in their bilateral relations, each modeled as a Nash barganing game (recall that negotiations are sequential). We let $\eta \in [0,1]$ and $\beta \in (0,1]$ be the bargaining power of manufacturers I and E respectively, in their relations with B. Second, we introduce reduced-form approach to characterize the subgame when I and B fail to sign a contract. We let $\bar{\pi}_I$ and $\bar{\pi}_B$ take values within a range of plausible values and where $\bar{\pi}_I + \bar{\pi}_B = \bar{W}$. For example, in the simple model, as in A&B, outside payoffs were the result of a uniform-price setting game: $\bar{\pi}_I \geq (1-\lambda)(v-c_I)$ and $\bar{\pi}_B < \lambda\,(v-c_I)$, but $\bar{W} < v - c_I$.

Alternatively, one could see still see manufacturers and buyers interacting on a short-term basis using two-part tariffs (p_i, S_i). As it is shown in our online appendix, this would imply that the incumbent and the buyer's outside option would then be given by

$$\begin{aligned}\bar{\pi}_I &= (1-\lambda)(v-c_I) + \lambda[1-F(c_I)]\left\{\mathbb{E}(c_E \mid c_E > c_I) - c_I\right\} \\ \bar{\pi}_B &= \lambda\,(v-c_I) - \lambda[1-F(c_I)]\left\{\mathbb{E}(c_E \mid c_E > c_I) - c_I\right\}\end{aligned} \tag{6}$$

and $\bar{W} = v - c_I$. Or maybe neither of those options is satisfactory, since even in the transaction stage there might still be scope to negotiate some sort of simple consensual deal, which could result in completely different values for $\bar{\pi}_I$ and $\bar{\pi}_B$.

The advantage of this reduced-form approach is that is allow us to omit how exactly outside options are determined, and to focus instead on the mechanisms underlying rent-shifting when non-linear contracts are used. It must be noted however, that the basic idea underlying this approach is the same as previously discussed: if the negotiations of complex non-linear contracts fail, the parties involved still have incentives to engage afterwards in some form of relationship, although of simpler nature. Let then $\mathbb{E}\pi_I$ and $\mathbb{E}\pi_B$ be I's and B's expected payoffs if they agree on a "complex" schedule at $t=1$, and denote $\mathbb{E}W_{IB} \equiv \mathbb{E}\pi_I + \mathbb{E}\pi_B$. The above discussion then translates on requiring at a minimum, that the maximum value $\mathbb{E}W_{IB}$ can achieve is greater than $\bar{W}$. In other words, that the best non-linear contract possible at $t = 1$, does generates positive surplus for the IB coalition in aggregate terms. This for example immediately rules the case where $\beta = 0$, as it would imply that I and B jointly could achieve perfect rent extraction or discrimination agains rivals, so obviously, there is no space for signing an imperfect contract "ex-ante".

Finally, we do not assume an a priori shape for the contract being negotiated between I and B in the first period.[18] Throughout the remaining sections of the article both market-share and own-supplied contracts, $T(q_I, q_E)$ and $T(q_I)$, will be analyzed. However, as it will be clear shortly, in the inelastic demand model both types are equivalent. The distinction between market-share and own-supplied discounts, will only be relevant once we introduce a downward slopping demand in section ?.

3.1 A General (Non-)Equivalence Result

We begin by deriving the optimal non-linear contract for the $I - B$ coalition, using the "coalitional" approach outlined by Marx & Shaffer (1999), and Choné & Linnemer (2012). This approach, which omits any transfers problems, has been at the center of the arguments claiming the conceptual equivalence between exclusive dealing contracts and non-linear contracts.

The equilibrium is found by backward induction: first we found the optimal quantities purchased by B to each supplier at the transaction stage conditional on $T(q_I, q_E)$ or $T(q_I)$ depending on the case, and on $p_E q_E$. Then, we characterize the optimal schedule negotiated at $t = 2$ that maximizes the joint surplus of the $E - B$ relationship. Finally, we found the non-linear contract $T(q_I, q_E)$ or $T(q_I)$ that maximizes the (expected) surplus of the IB coalition at $t = 1$.

As our simple example already hinted, a crucial issue in the negotiation between I and B is whether the non-linear contract has an unconditional transfer or not. We already showed that when unconditional payments are allowed, then the equivalence between exclusive dealing contracts and non-linear contract does hold, and therefore this latter were indeed "anticompetitive" in the sense that they blocked more efficient rivals. In this section then, we will focus exclusively on the opposite limiting case where all transfers must be done entirely through the spot market/transaction stage.

[18] As we already discussed, given the inelastic demand assumption, simple linear prices are optimal in the $E - B$ relationship.

Now, because only the marginal penalties $T_{q_E}(q_I, q_E)$, and prices $T'(q_I)$, and not the total amounts affect the extraction of rents, then both, market share contracts and own-supplied discount, must satisfy the equilibrium constraint $T(0, q_E) = 0$ or $T(0) = 0$. If that were not the case, then B would always refrain from buying additional units to the incumbent after E made a sufficiently attractive offer. Hence, from the $I - B$ coalition's perspective, all schedules that do not satisfy this condition are weakly dominated by ones that do, as they do not distort rent extraction from rivals and give the coalition the option of selling more units[19].

Moreover, notice that since both c_I and c_E are less than v, then the marginal price of both I's and E's schedules must be below v. That is, $T_{q_I}(q_I, q_E) \leq v$ or $T'(q_I) \leq v$ for all $q_I \in [0, 1]$ and that $p_E \leq v$[20]. This is because units that are offered at marginal prices higher than v are never purchased in equilibrium, so any schedule violating the aforementioned condition, will be weakly dominated by one that does not[21].

This two conditions combined imply that the equilibrium schedule agreed on by I and B must satisfy that $T(q_I, q_E) \leq vq_I$ or that $T(q_I) \leq vq_I$ for all $q_I \in [0, 1]$, and any q_E. Moreover, summed with our inelastic demand assumption they also translate onto the fact $q_I^* + q_E^* = 1$, where q_I^* and q_E^* are the equilibrium quantities bought by the consumer from I and E at the transaction state respectively. Hence, when the incumbent and the buyer agree on an (equilibrium) own-supplied contract to implement q_I^* they are indirectly choosing q_E^* at the same time, and therefore this is equivalent as if they had the power to select both. Consequently, under the inelastic demand assumption, both market share discount and own-supplied contracts in this setting are equivalent[22]. Indeed, in the case of own-supplied contract the problem faced by B at $t = 2$ is

$$\max_{\substack{q_E, q_I \\ q_I + q_E \leq 1}} \pi_B = vq_I + (v - p_E)\min(\lambda, q_E) - T(q_I) \tag{7}$$

Hence, conditional on p_E (and equilibrium condition $T(0) = 0$), the marginal price of the $I - B$ schedule $T'(\cdot)$ determines both q_I^* and q_E^*.

Suppose then, without loss of generality, that I and B agree on the market-share discount

[19]Notice that the recent literature studying own-supplied and market-share contract usually assume that this schemes do not involve unconditional payment. For example, the equilibrium conditions $T(0, q_E) = 0$ and $T(0) = 0$ are also imposed in Calzolari & Denicolo (2013)

[20]Remember that, given our inelastic demand assumption, without loss of generality we could work assuming that contract between E and B consisted on a linear price

[21]This Proposition 2, from Choné & Linnemer (2012)

[22]Disposable costs do not play a role, in contrast to for example Choné & Linnemer (2012) as we do not have, using their terminology, "super-efficient" rivals in this model.

$T(q_I, q_E)$ that satisfies all the equilibrium conditions previously discussed. What is the optimal contract negotiated between E and B? Notice that when this bargaining takes place, there is no space for posterior opportunistic behavior from the consumer's side, as both parties are bargaining under full information. Then this coalition selects the optimal $q_E(c_E)$ conditional on the schedule $T(\cdot)$ that maximizes the sum of the surpluses to be obtained by each party at the transaction stage $W_{EB} = \pi_B + \Pi_E$, with $\Pi_E \equiv q_E(p_E - c_E)$ and $\pi_B \equiv v - p_E q_E - T(q_I)$

$$\max_{q_E \leq \lambda} W_{EB} = v - c_E q_E - T(q_I, q_E) \tag{8}$$

The price $p_E(c_E)$ on the other hand is chosen such that the transaction stage outcome gives each party the corresponding rents as dictated by the Nash Bargaining solution we assumed, with β as E's relative bargaining power against B. Disagreement payoff on the other hand are given by $v - T(1,0)$ (the surplus B obtains if he purchase the entire production to I) and 0 for B and E respectively. Hence if we denote $\Delta W_{EB} \equiv W_{EB} - v + T(1,0)$ as the net surplus generated by this relationship, the optimal $p_E^*(c_E)$ is given by either of the following conditions

$$\begin{aligned} \Pi_E^*(c_E) &= \beta[T(1,0) - c_E q_E^*(c_E) - T(q_I, q_E^*(c_E))] = \beta \Delta W_{EB}^*(c_E) \\ \pi_B^*(c_E) &= (1-\beta)\Delta W_{EB}^*(c_E) + v - T(1,0) \end{aligned} \tag{9}$$

Finally, given the optimal schedule negotiated by the EB coalition, I and B decide theirs. However, at $t = 0$ the exact value of c_E is not known, and therefore the particular contract that E and B will end up signing cannot be determined a priori[23], E will therefore end up with some ex-ante informational rents. More formally, the IB coalition chooses the optimal schedule $T(\cdot)$ in order to maximize the joint expected surplus they will obtain from the transaction stage $\mathbb{E}W_{IB} = \mathbb{E}\pi_I + \mathbb{E}\pi_B^*$, where

$$\begin{aligned} \mathbb{E}\pi_I &= \int_0^{\bar{c}_E} \{T(q_I(c_E), q_E(c_E)) - c_I q_I(c_E)\} f(c_E) \mathrm{d}c_E \\ \mathbb{E}\pi_B^* &= \mathbb{E}[(1-\beta)\Delta W_{EB}^*(c_E)] + v - T(1,0) \end{aligned} \tag{10}$$

It is possible to prove (see lemma also 5), that, given the absence of bilateral inefficiencies $\mathbb{E}W_{IB}$

[23] In some sense, this is hidden information problem

can be rewritten as

$$\mathbb{E}W_{IB} = \mathbb{E}[(v - c_I)q_I(c_E) + (v - c_E)q_E(c_E)] - \mathbb{E}\Pi_E(c_E) \tag{11}$$

In other words, total rents being appropriated by the IB coalition are equal to the total expected surplus less the share the rival, by inducing the opportunistic behavior of B once c_E is known, is obtaining.

However, using the envelope theorem we have $\partial\Pi_E/\partial c_E = -\beta q_E$, so

$$\mathbb{E}\Pi_E^* = \Pi_E(\bar{c}_E) + \int_0^{\bar{c}_E} \beta q_E F(c_E) \mathrm{d}c_E \tag{12}$$

And moreover, given $q_I(q_E) = 1 - q_E^*(c_E)$, the problem faced by the IB coalition can be rewritten as the following lemma states:

Lemma 5. *The "coalitional" problem faced by the IB coalition can be written as:*

$$\max_{q_E(c_E)} \mathbb{E}W_{IB} = v - c_I - \int_0^{\bar{c}_E} \left[(c_E - c_I) + \frac{\beta F(c_E)}{f(c_E)}\right] q_E(c_E)\ f(c_E)\mathrm{d}c_E - \Pi_E(\bar{c}_E) \tag{13}$$

Subject to (1) both participation constraints, and that (2) $q_E'(c_E) \leq 0$

Proof. See Appendix ☐

The surplus created in this transaction is $\Delta W_{IB}^* = \mathbb{E}W_{IB}^* - \bar{\pi}_B - \bar{\pi}_I = \mathbb{E}W_{IB}^* - \bar{W}$, and the corresponding payoffs to B and I, $\Pi_B^* = (1-\eta)\Delta W_{IB}^* + \bar{\pi}_B$, $\Pi_I^* = \eta\Delta W_{IB}^* + \bar{\pi}_I$. Hence, $T(\cdot)$ must not only implement the optimal allocation rule $q_E^*(c_E)$ and $q_I^*(c_E) = 1 - q_E^*(c_E)$, $T(\cdot)$ but must also ensure that $\mathbb{E}\pi_B^* = \Pi_B^*$, and $\mathbb{E}\pi_I = \Pi_I^*$.

Now, before characterizing the optimal non-linear pricing scheme chosen by the IB coalition, we introduce the notion of "simple" allocation rules that will play an important role in our analysis.

Definition 2. *An allocation* $q_E(\cdot) : [0, \bar{c}_E] \to [0, \lambda]$ *is said to be "simple at* c_E^**" if*

$$q_E^*(c_E) = \begin{cases} \lambda & if\ c_E \in [0, c_E^*] \\ 0 & otherwise \end{cases} \tag{14}$$

For these allocation rules, only two amounts become relevant for the incumbent: the one when the realization of c_E is relatively low and it receives $T(1-\lambda, \lambda)$, as there is entry/expansion;

and the amount $T(1,0)$ it gets when c_E turns out to be high and B purchases his entire demand from him[24]. Moreover, all the relevant price schedules that implement this sort of allocations share the following properties:

Lemma 6. *Consider any market-share discount $T(q_I, q_E)$ that satisfies $T_{q_I}(q_I, q_E) \leq v$ and $T(q_I, q_E) \leq vq_I$ for all $q_I \in [0,1]$ and any q_E, and define $p^e(q_E)$ as*

$$p^e(q_E) \equiv \frac{T(1,0) - T(1-q_E, q_E)}{q_E} \tag{15}$$

Then,

1. *If $T(\cdot)$ induces a simple allocation at c_E^* then $p^e(\lambda) = c_E^*$.*
2. *Moreover, if*

   (a) *$p^e(q_E)$ is non-decreasing in q_E for all $q_E \in [0, \lambda]$*

   And,

   (b) *$p^e(\lambda) = c_E^*$.*

   Then $T(q_I, q_E)$ implements a simple allocation at c_E^*

Proof. For own-supplied contracts the lemma is analogous. Just replace $T(1,0)$ for $T(1)$, and $T(1-q_E, q_E)$ for $T(1-q_E)$. For the proof, see Appendix. □

$p^e(\lambda)$ represent the *"effective price"* the consumer internalizes when buying q_E to the rival. Hence, all pricing schedules that implement simple allocations at c_E^*, have $p^e(\lambda)$ equal to that same value. We now complete the characterization of the optimal non-linear contract the IB coalition agree upon.

Proposition 2. *The optimal market-share discount (without unconditional payments) that maximizes the surplus created by the IB coalition, ΔW_{IB}, is characterized by:*

1. *$T_{q_I}(q_I, q_E) \leq v$, for all $q_I \in [0,1]$ and any q_E*
2. *$T(0, q_E) = 0$ for all q_E*

[24] $T(1-\lambda)$ and $T(1)$ in own-supplied discounts

3. $T(q_I, q_E)$ induces a simple allocation at $\tilde{c}_E$, where $\tilde{c}_E$ satisfies:

$$f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0 \tag{16}$$

Proof. For own-supplied contracts the lemma is analogous. For the proof, see the Appendix. □

As expected, and due to the existence of imperfect information at the time when I and B are contracting, the optimal schedule sets to B an average penalty of $c_I - p^e(\lambda)$ for each unit he purchases from the E, and hence acts as a tax over entry in the same way as a breaching clause in an exclusive dealing contract. As in A&B model, this distort the entry decision of moderately more efficient rivals, generating the well-known side effect of inefficient foreclosure. Hence the optimal schedule in this setting is "anticompetitive".

Given that the optimum is a simple allocation, then the (private) efficiency of the coalition can be entirely summarizes by the "cut-off" type $\hat{c}_E = \tilde{c}_E$. Hence, the maximum surplus generated by the transaction can be denoted $\Delta W^*_{IB} \equiv \Delta W_{IB}(\tilde{c}_E)$. Notice moreover, that setting $\beta = 1$ above, we obtain the same cutoff $\tilde{c}_E$ as in our simple example.

Moreover, since market-share are equivalent to own-supplied contract in this setting, and the optimal schedule only needs to induce a simple allocation, there are apparently several schemes that can achieve the coalition's optimum. Indeed, using the second part of Lemma 6, examples of own-supplied contracts that would in principle work include:

1. "Unconditional" Two-Part Tariffs ($T = T^*, p^* = \tilde{c}_E$), where T^* is a lump-sum payment made immediately, and p^* the marginal price charged:

$$p^e_{2PT}(q_E) = \frac{T^* + \tilde{c}_E - T^* - (1 - q_E)\tilde{c}_E}{q_E} = \tilde{c}_E \qquad \forall\ q_E \in [0, \lambda] \tag{17}$$

2. Incremental Discounts ($P = P^* \leq v, P^*_d = \tilde{c}_E$), where P^* is the price per-unit of the first $1 - \lambda$ units, and $P^*_d = \tilde{c}_E$ the price of each additional unit purchased:

$$p^e_{Inc.D}(q_E) = \frac{(1-\lambda)P^* + \lambda\tilde{c}_E - (1-\lambda)P^* - (\lambda - q_E)\tilde{c}_E}{q_E} = \tilde{c}_E \qquad \forall\ q_E \in [0, \lambda] \tag{18}$$

3. Conditional, "all-unit" retroactive rebates ($P_r = P^*_r \in [\tilde{c}_E, v], R^* = \lambda(P^*_r - \tilde{c}_E), \bar{Q}^* = 1$), where P_r is the "price list", R the (monetary amount of the) discount, and $\bar{Q}$ the pre-

specified threshold that triggers the rebate:

$$p^e_{Ret.D}(q_E) = \frac{P_r^* - \lambda(P_r^* - \tilde{c}_E) - (1-q_E)P_r^*}{q_E} = \frac{\lambda\tilde{c}_E - P_r^*(\lambda - q_E)}{q_E} \tag{19}$$

So

$$\begin{aligned} &\frac{\partial p^e_{Ret.D}}{\partial q_E} = \frac{\lambda(P_r - \tilde{c}_E)}{q_E} \geq 0 \qquad \forall\ q_E \in [0,\lambda] \\ &p^e_{Ret.D}(\lambda) = \tilde{c}_E \end{aligned} \tag{20}$$

However in this setting this characterization is incomplete as it crucially omits the issue of rent distribution.

Proposition 3. *When $T(0,q_E) = 0$, there exists a unique $\bar{\pi}_I^*(\eta) \in (0,(1-\lambda)(v-c_I))$ defined as:*

$$\bar{\pi}_I^*(\eta) = (v-c_I)(1-\lambda) + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] - \eta\Delta W_{IB}(\tilde{c}_E) \tag{21}$$

Which is decreasing for all $\eta \in [0,1]$, such that the schedule $T(q_I,q_E)$ satisfying

1. *The coalition optimality conditions*

2. *And simultaneously that:*

$$\mathbb{E}\pi_I = \int_0^{\bar{c}_E} \{T(q_I^*(c_E), q_E^*(c_E)) - c_I q_I^*(c_E)\} f(c_E)\mathrm{d}c_E = \Pi_I^* \tag{22}$$

Exists if and only if $\bar{\pi}_I \leq \bar{\pi}_I^*(\eta)$

Proof. Obviously $\bar{\pi}_I^*(\eta)$ is decreasing in all η, since $\Delta W_{IB}(\tilde{c}_E) > 0$ by assumption. Moreover, notice that $\bar{\pi}_I^*(\eta)$ can be written as

$$\bar{\pi}_I^*(\eta) = \bar{\pi}_I^*(\eta = 0) - \eta\Delta W_{IB}(\tilde{c}_E) \tag{23}$$

For the rest see the Appendix. □

Proposition 3 states that when unconditional payments are not allowed and I's outside option is sufficiently large, the coalition is unable to find an schedule that achieves both the (unconstrained) maximization of the surplus created by the coalition, and that simultaneously transfers the corresponding payment to the incumbent through the spot market. This in marked contrast to exclusive dealing contracts and "unconditional"-tranfer schedules.

Intuitively, non-linear pricing schemes/exclusive dealing contracts in this model are designed to charge a very low marginal price in order to extract efficiency rents from E, while using very high infra-marginal prices to distribute its share of the surplus to the incumbent. Hence, the greater its outside option $\bar{\pi}_I$, higher the infra-marginal prices charged. When unconditional payments are allowed (e.g. in "unconditional" two-part tariffs), the scheme works smoothly as the transference is done at any event, or "ex-ante", implying that a significant amount of those high infra-marginal prices, which turn marginal when E expands, are sunk, so the behavior of the consumer conditional on E expanding, remains unaltered. However, when unconditional payments are not allowed/available, all the surplus transference must be executed through the transaction stage. This in turn implies that the totality of the high infra-marginal price is internalized by the consumer, who opportunistically refrains from buying any additional units from I once buying from E, making harder the rent distribution among the members of the $I - B$ coalition. This also implies that optimal non-linear schedule in this restricted scenario cannot solved using the coalition approach consisting on maximizing the sum of the parties' profits. Indeed this approach is valid only when transfers are not bounded, and/or when transfer restriction is not binding.

The best way to see it, and that will actually turn to be useful once we begin characterizing the optimal schedule taking into account transfer restrictions, is by returning to the primitive problem underlying Nash's bargaining solution. That is selecting a $T(q_I, q_E)$ to maximize the Nash product

$$(\mathbb{E}\pi_I - \bar{\pi})^{\eta}(\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \tag{24}$$

Subject to $\mathbb{E}\pi_I \geq \bar{\pi}_I$, $\mathbb{E}\pi_B \geq \bar{\pi}_B$, $T(q_I, q_E) \leq vq_I$, for all $q_I \in [0, 1]$, and any q_E; $q_E(c_E) \in [0, \lambda]$ and $q'_E(c_E) \leq 0$ for all $c_E \in [0, \bar{c}_E]$. It is possible to prove then, that if we omit the transfer restriction the solution to the problem is equivalent to maximize the sum of the surpluses. This is intuitive: when transfers are not restricted, the coalition first select the schedule that maximizes the overall surplus and only afterwards deals with the distribution of rents.

The problem arises once the transfer restriction begins to be active, as some sort of distortion over the optimal schedule must be executed in order to ensure the correct distribution of rents. This not only may modify the nature of the schedule in hand, but may also destroy enough coalitional rents so that signing an "ex-ante" agreement may not be optimal for both parties.

This insights will play a key role in the next section when we discuss the nature of the optimal agreement when transfer are indeed restricted.

The conclusion is that the apparent equivalence between exclusive dealing in rent shifting models and "ex-post" quantity discounts is much more subtle than originally thought. The crucial issue is the existence of unconditional transfers, and therefore they cannot be analyzed/treated lightly as similar practices before verifying their existence and other features of the contracting setting (i.e. bargaining powers and disagreement payoffs).

3.2 Optimal Contract without Unconditional Payments

We now characterize the optimal non-linear contract when unconditional payments are unfeasible. The problem faced by the IB coalition is then to choose an allocation rule $q_E(c_E)$ that maximizes the Nash product:

$$\max_{q_E(c_E)} (\mathbb{E}\pi_I - \bar{\pi})^{\eta}(\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \tag{25}$$

Subject to $\mathbb{E}\pi_I \geq \bar{\pi}_I$, $\mathbb{E}\pi_B \geq \bar{\pi}_B$, $T(q_I, q_E) \leq vq_I$, for all $q_I \in [0,1]$, and any q_E; $q_E(c_E) \in [0, \lambda]$ and $q_E'(c_E) \leq 0$ for all $c_E \in [0, \bar{c}_E]$. Where (see proof lemma 5):

$$\begin{aligned}
\mathbb{E}\pi_I &= v - c_I - W_{EB}^*(\bar{c}_E) - \int_0^{\bar{c}_E} \left\{ c_E - c_I + \frac{F(c_E)}{f(c_E)} \right\} q_E^*(c_E) f(c_E) \mathrm{d}c_E \\
\mathbb{E}\pi_B &= \int_0^{\bar{c}_E} (1-\beta) q_E(c_E) F(c_E) \mathrm{d}c_E + (1-\beta) W_{EB}^*(\bar{c}_E) + \beta[v - T(1,0)]
\end{aligned} \tag{26}$$

As we mentioned in the previous section, is relatively straightforward to show that if the transfer restriction is not binding, that is if $\bar{\pi}_I \leq \bar{\pi}_I^*(\eta)$, the solution coincides with A&B. We therefore restrict our attention to the region where $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$. Now, even if the transfer constraint is active, the optimal schedule necessarily satisfies:

Lemma 7. *There exists a threshold α such that the optimal schedule $q_E^*(c_E) = 0$ for all $c_E \geq \alpha$.*

Proof. See Appendix □

The previous characterization allows us to write the problem faced by the IB coalition in a more tractable way. In particular, collapsing the constraint $T(q_I, q_E) \leq vq_I$, for all $q_I \in [0,1]$ in a restriction over a single constant.

Lemma 8. *The problem faced by the IB coalition can be rewritten as:*

$$\max_{q_E(c_E)} (\mathbb{E}\pi_I - \bar{\pi})^{\eta}(\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \tag{27}$$

Subject to $\mathbb{E}\pi_I \geq \bar{\pi}_I$, $\mathbb{E}\pi_B \geq \bar{\pi}_B$, $T(1-q_E(0), q_E(0)) \leq v\,(1-q_E(0))$ *and any* $q_E(0) \in [0,\lambda]$*;* $q_E(c_E) \in [0,\lambda]$ *and* $q_E'(c_E) \leq 0$ *for all* $c_E \in [0, \dot{c}_E]$*. Where:*

$$\begin{aligned} \mathbb{E}\pi_B &= v - T(1-q_E(0), q_E(0)) - \int_0^{\dot{c}_E} \frac{\{1-(1-\beta)[1-F(c_E)]\}}{f(c_E)} q_E^*(c_E) f(c_E)\mathrm{d}c_E \\ \mathbb{E}\pi_I &= T(1-q_E(0), q_E(0)) - c_I - \int_0^{\dot{c}_E} \left\{ c_E - c_I - \frac{[1-F(c_E)]}{f(c_E)} \right\} q_E^*(c_E) f(c_E)\mathrm{d}c_E \end{aligned} \tag{28}$$

Proof. See Appendix. □

We now try to shed some light about the solution to this problem. Let's start considering the unrestricted solution to the maximization problem when we only take into account the buyer. That is, when we maximize $\mathbb{E}\pi_B - \bar{\pi}_B$. The solution to this relaxed program is $q_E(c_E) \equiv 0$, which may initially seems counterintuitive since it completely avoids entry. However, the expansion of E is blocked not because B wants to restrict competition between manufacturers, but because the schedule implementing such allocation must involve a very low marginal price, which increases the buyer's surplus. If we consider the unrestricted maximization of the seller's surplus, $\mathbb{E}\pi_I - \bar{\pi}$, we find $q_E(c_E) = \lambda$ for $c_E \leq \dot{c}_E$ and $q_E(c_E) = 0$ for $c_E > \dot{c}_E$. Since blocking expansion is costly (the incumbent must charge a low marginal price) and there is no recoupment, I allows inefficient entry (since $\dot{c}_E > c_I$). Define then $\bar{\pi}_I^{***} \equiv \mathbb{E}\pi_I(\dot{c}_E)$ as the profits the incumbent gets with an a schedule that induces a simple allocation at $\dot{c}_E$. Given that $\bar{\pi}_I^{***}$ is the maximum value $\mathbb{E}\pi_I$ can attain, if I's outside option is greater than such value, the incumbent would never agree on an ex-ante schedule. We further restrict attention to all $\bar{\pi}_I \in [\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***}]$.

In both cases, the solution to the relaxed problem satisfies the implementability condition (q_E is weakly decreasing). These solutions, however, may fail to satisfy the participation constraints. We now tackle the characterization of the optimal solution in these two extreme cases.

Lemma 9. *Consider a pair of outside options* $(\bar{\pi}_I, \bar{\pi}_B)$*.*

1. *Suppose that* $\eta = 1$ *and consider* $\hat{c}_E = \min\{\dot{c}_E, \delta\}$*, where* δ *is the solution to*

$$\lambda(v-\delta) + \lambda(1-\beta)F(\delta)[\delta - \mathbb{E}(c_E \mid c_E \leq \delta)] = \bar{\pi}_B$$

If a solution exists, then it is given by $T^*(1-\lambda,\lambda) = v(1-\lambda)$ *and* $q_E^*(c_E) = \lambda$ *if* $c_E \le \hat{c}_E$, *and 0 otherwise.*

2. *Suppose that* $\eta = 0$ *and consider* $\check{c}_E$ *the solution to*

$$(1-\lambda)(v - c_I) + \lambda\,(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I$$

If a solution exists, then it is given by $T^*(1-\lambda,\lambda) = v(1-\lambda)$ *and* $q_E^*(c_E) = \lambda$ *if* $c_E \le \check{c}_E$, *and 0 otherwise.*

Proof. See Appendix. □

As we mentioned before, the preferred solution of the incumbent (without taking into account participation constraints) involves inefficient entry, while the preferred solution of the buyer is one without entry. In order to satisfy participation constraints, both solutions allow for some entry. What is natural, though, is that the solution with $\eta = 0$ involves more entry than the solution with $\eta = 1$. In fact, such a condition guarantees the existence of a solution in both extreme cases. [25]

Lemma 10. *If* $\check{c}_E \le \hat{c}_E$ *then the problem has a solution for* $\eta = 0$ *and* $\eta = 1$.

It is important to remark that in both extreme cases, the solution is a "simple" allocation rule. It involves either full entry or no entry at all. We now show that this result extends to a general bargaining position η. Moreover, we show that the solution involves a level of entry that is in between the levels for $\eta = 0$ and $\eta = 1$.

Proposition 4. *For a given* $\eta \in [0,1]$, *and a given pair* $(\bar{\pi}_I, \bar{\pi}_B)$ *such that* $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$, *a solution to the general problem exists if and only if* $\check{c}_E \le \hat{c}_E$. *Moreover the optimal schedule satisfies* $T(1-\lambda,\lambda) = v(1-\lambda)$, *and there exists* $c_E^*(\eta,\bar{\pi}_I) \in [\check{c}_E, \hat{c}_E]$, *such that the optimal allocation induced by such schedule, is given by*

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \in [0, c_E^*(\eta,\bar{\pi}_I)] \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

Proof. See Appendix □

[25] In fact, it is possible to show that the condition $\check{c}_E \le \hat{c}_E$ is equivalent to the gains from trade being big enough.

Given the simple nature of the solution, then the "anticompetitive" nature of a non-linear contract in this setting is entirely summarized on the optimal cutoff $c_E^*(\eta, \bar{\pi}_I)$. This simple solution allow us to characterize very neatly sufficient conditions to ensure when contract are anticompetitive or not. In particular, notice the optimality conditions imply the optimal payoff to each party are given by

$$\begin{aligned}\mathbb{E}\pi_B(c_E^*) &= \lambda\,(v - c_E^*) + \lambda(1-\beta)F(c_E^*)[c_E^* - \mathbb{E}(c_E \mid c_E \leq c_E^*)] \\ \mathbb{E}\pi_I(c_E^*) &= (1-\lambda)(v - c_I) + \lambda\,(c_E^* - c_I)[1 - F(c_E^*)]\end{aligned} \tag{30}$$

Then if $\bar{\pi}_I \geq (1-\lambda)(v - c_I) \equiv \bar{\pi}_I^{**} \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***})$, which is equivalent to $\check{c}_E = c_I$, then "anticompetitive" contracts cannot arise in equilibrium, as they do not satisfy I's participation. Hence, $\bar{\pi}_I \geq \bar{\pi}_I^{**}$ is sufficient to ensure that anticompetitive contracts are not signed. Contracts with $c_E^* \geq c_I$ however, can emerge in this setting and be optimal if $\check{c}_E = c_I < \hat{c}_E$.

The above discussion help us partition the set of potential $\bar{\pi}_I$ in different regions, which share different properties. In particular if $\bar{\pi}_I$ is between $(0, \bar{\pi}_I^*(\eta))$, even without unconditional payments, the optimal schedule is equivalent to the A&B exclusive dealing contract. On the other hand, when $\bar{\pi}_I \in [\bar{\pi}_I^*(\eta), \bar{\pi}_I^{**}]$ the sufficient conditions already derived do not apply, so we need a finer characterization. If $\bar{\pi}_I \in [\bar{\pi}_I^{**}, \bar{\pi}_I^{***}]$ then non-linear contracts cannot be anticompetitive, but may still emerge if $\check{c}_E \leq \hat{c}_E$. And finally if $\bar{\pi}_I > \bar{\pi}_I^{***}$ then with certainty no contract is signed at $t = 0$. This is summarized in figure 1.

[FIGURE 1]

In our simple example, in that case we had

$$\bar{\pi}_I^L = (v - c_I)(1-\lambda) + [1 - F(c^*)]\left\{\mathbb{E}(c_E \,|\, c_E > c^*) - c^*\right\} \geq (1-\lambda)(v - c_I) = \bar{\pi}_I^{**} \tag{31}$$

Hence when transfer were restricted, anticompetitive contract could emerge in equilibrium. Moreover, it is straightforward to prove that $\bar{c}_E \leq c^*$ is sufficient to ensure that $\check{c}_E = c_I < \hat{c}_E$ when $\bar{\pi}_I = \bar{\pi}_I^L$, and hence in that scenario contracts where used only to extract rents from more inefficient rivals. Since in the simple example we were assuming $\eta = 1$, then the optimal schedule will induce a simple allocation at $\hat{c}_E$.

On the other hand, if firms can use two-part tariffs to compete at the transaction stage without contracts, then $\bar{\pi}_I^{2PT} = (1-\lambda)(v - c_I) + \lambda[1 - F(c_I)]\left\{\mathbb{E}(c_E \| \; c_E > c_I) - c_I\right\}$, however it is easily proven that such expression is greater than $\bar{\pi}_I^{***}$. Hence in this setting, if unconditional payments are unfeasible no contract will ever be signed in $t = 0$ between I and B, i.e. $\check{c}_E > \hat{c}_E$ for $\bar{\pi}_I = \bar{\pi}_I^{2PT}$, and any $\bar{U}$, since $\check{c}_E$ in this case is greater than $\dot{c}_E$, the upper bound of $\hat{c}_E$. Intuitively, when I can use two-part tariffs in the spot market, it allows it to appropriate an important fraction of the total surplus generated as this 2PT are contingent on c_E (since c_E is known already at this stage) while giving enough pricing flexibility due to its two instruments. Hence, if transfers are restricted, the distortion over the ex-ante schedule needed to transfer that amount of rents is so important, than the total surplus generated by the ex-ante scheme is not sufficient to satisfy both participation constraints, i.e. "gains from trade" are entirely depleted.

Returning to the general formulation, we still need to characterize the optimal non-linear contract when $\bar{\pi}_I \in [\bar{\pi}_I^*(\eta), \bar{\pi}_I^{**}]$. While it is not possible to determine exactly whether a contract in this region is anticompetitive for a given pair of outside options, without further specifying the distribution of E's costs, we can determine when is more likely for a contract in this region the be anticompetitive. For this we use the following lemma:

Lemma 11. *$c_E^*(\eta, \bar{\pi}_I)$ is increasing in η and $\bar{\pi}_I$*

Proof. See Appendix □

Hence if we define a *bargaining position* as a combination $(\eta, \bar{\pi}_I)$, the main implication of the model is that without unconditional payments, the *stronger* the bargaining position of the incumbent against the buyer, the *less likely* the resulting contract is anticompetitive. This result goes against the common belief that the stronger the incumbent, the more likely is the foreclosure of efficient rivals. Indeed, if I's good is a "must-stock" item (and transfers are restricted) this makes exclusion *harder* by improving I's bargaining position, not easier as in other models. The intuition behind this result should already be clear by now.

[FIGURE 2]

That the bargaining position of the incumbent relative to the buyer affects the anticompetitive potential of the non-linear contract is a novel, although expected, feature of the model, especially considering that results in this section are driven by the difficulty of transferring rents between the members of the IB coalition.

4 Downward Sloping Demand

In this section we revisit our non-equivalence result when unconditional payments are not allowed in the case of a downward sloping demand $D(p)$ with inverse demand function $P(q)$. We keep the perfect substitutability between both goods though, while outside options are again given by $\bar{\pi}_I$ and $\bar{\pi}_B$ with $\bar{\pi}_I + \bar{\pi}_B = \bar{W}$.

We moreover made that rather standard assumptions that $D(p)$ is such, that $\pi_I^M \equiv (p_I - c_I)D(p_I)$ is an strictly concave function of p_I that achieves a unique global optimum p_I^M, the monopoly price. Finally, we denote $S(q)$ as

$$S(q) \equiv \int_0^q P(s)\mathrm{d}s \tag{32}$$

Revisiting the (Non-)Equivalence

Suppose the incumbent offers a market share discount $T(q_I, q_E)$ to B. We will solve the problem faced by I and B at $t = 1$ using, as before, the coalitional approach and then show that when unconditional payments are not allowed, the solution is only valid when $\bar{\pi}$ is in a neighbourhood near 0.

Given $T(q_I, q_E)$ the EB coalition solves:

$$\max_{\substack{q_E \in [0,\lambda] \\ q_I \geq 0}} W_{EB}(q_I, q_E) = S(q_I + q_E) - c_E q_E - T(q_I, q_E) \tag{33}$$

So again by envelope theorem we have:

$$\mathbb{E}\Pi_E^* = \Pi_E(\bar{c}_E) + \int_0^{\bar{c}_E} \beta q_E F(c_E)\mathrm{d}c_E \tag{34}$$

Therefore, the IB coalition problem is

$$\max_{\substack{q_E(c_E) \in [0,\lambda] \\ q_I(c_E) \geq 0}} \mathbb{E}W_{IB} = \int_0^{\bar{c}_E} \left[S(q_I(c_E) + q_E(c_E)) - c_I q_I(c_E) - c_E q_E(c_E) - \beta q_E(c_E) \frac{F(c_E)}{f(c_E)} \right] f(c_E)\mathrm{d}c_E - \Pi_E(\bar{c}_E) \tag{35}$$

The surplus created in this relationship is $\Delta W_{IB}^* = \mathbb{E}W_{IB}^* - \bar{\pi}_B - \bar{\pi}_I = \mathbb{E}W_{IB}^* - \bar{W}$, so the corresponding payoffs to each party involved $\Pi_B^* = (1-\eta)\Delta W_{IB}^* + \bar{\pi}_B$ and $\Pi_I^* = \eta\Delta W_{IB}^* + \bar{\pi}_I$. The solution to problem (35), is clearly "bang-bang" as the following lemma proposition states:

Proposition 5. *The optimal allocation rule in the downward sloping demand case, is given by*

$$(q_I^*(c_E), q_E^*(c_E)) = \begin{cases} (D(c_I)-\lambda, \lambda) & if\ c_E \leq \tilde{c}_E \\ (D(c_I), 0) & otherwise \end{cases} \tag{36}$$

Where $\tilde{c}_E$ is uniquely and implicitly defined by

$$f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0 \tag{37}$$

Proof. See Appendix. □

As it is shown in the Appendix, one way to implement the above outcome is with a three-part exclusive contract (p_i, t_i, K) exclusionary contract, where the marginal price for I's units p_i is set at c_I, t_i the per-unit liquidated damage clause is set at $c_I - \tilde{c}_E$, and K is an unconditional sum paid immediately once the contract is signed.

Then, the EB coalition finds optimal to purchases the entire capacity λ to E, and the remaining units $D(c_I) - \lambda$ to the incumbent whenever $c_E \leq p_i - t_i = \tilde{c}_E$, or to purchase $D(c_I)$ units exclusively to the incumbent when $c_E > \tilde{c}_E$. That is (K^*, p_i^*, t_i^*) implements the optimum prescribed by the coalition approach.

Regarding non-linear contracts, notice that with a downward-sloping demand there is a need for additional instruments. Indeed, a single marginal price (as for example in the case of own-supplied discounts), is not sufficient to implement the coalitional optimum, as it cannot extract rents (i.e. marginal price below c_I) without sacrificing own-supplied efficiency (i.e. marginal price equal to c_I). Therefore, own-supplied are no longer equivalent as market-share contracts. Only this latter can, omitting rent transfer issues, implement the optimum using its two-instrument to separate the rent-extraction from the own-supply efficiency motive. Indeed, any market share contract that implements the coalition optimum, must satisfy the following properties

Lemma 12. *If the market share discount $T(q_I, q_E)$ implements the coalition optimum then*

1. *$T(q_I, q_E)$ is separable in q_I and q_E, and the marginal price for I's unit is c_I: $T(q_I, q_E) = c_I q_I + \xi(q_E)$*

2. *And* $b^e(\lambda) \equiv \frac{\xi(\lambda)-\xi(0)}{\lambda}$ *is equal to* $c_I - \tilde{c}_E$

Proof. See Appendix. □

Regarding their equivalence with three-part generalized A&B contracts (p_i, t_i, K), again the crucial issue with market-share contracts is whether there is an unconditional payment or not. When this are not allowed we then need to check whether the consumer still buys additional units from I once he decides to purchase also from E. This puts an upper bound over the total penalty imposed from buying units elsewhere.

Indeed, if the consumer only buys from E he receives $\int_0^{\lambda} P(q)\mathrm{d}q - P_E(\lambda)$, where $P_E(\lambda)$ depends on β the relative bargaining power on the EB relationship. While if he buys additional units from I he gets $\int_0^{D(c_E)} P(q)\mathrm{d}q - P_E(\lambda) - c_I[D(c_I) - \lambda] - \xi(\lambda)$. Hence $\xi(\lambda)$ is bounded by:

$$\int_0^{\lambda} P(q)\mathrm{d}q - P_E(\lambda) \leq \int_0^{D(c_I)} P(q)\mathrm{d}q - P_E(\lambda) - c_I[D(c_I) - \lambda] - \xi(\lambda)$$
$$\Longrightarrow \xi(\lambda) \leq \int_{\lambda}^{D(c_I)} P(q)\mathrm{d}q - c_I[D(c_I) - \lambda] \tag{38}$$

We then have the following result:

Proposition 6. *When the penalty* $\xi(q_E)$ *is paid only conditional on dealing with* I*, there exists a unique* $\bar{\pi}_I^{D*}(\eta)$ *strictly between* 0 *and* $\int_{\lambda}^{D(c_I)} P(q)\mathrm{d}q - c_I[D(c_I) - \lambda]$ *defined as:*

$$\bar{\pi}_I^*(\eta) = \int_{\lambda}^{D(c_I)} P(q)\mathrm{d}q - c_I[D(c_I) - \lambda] + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] - \eta\Delta W_{IB}(\tilde{c}_E) \tag{39}$$

Which is decreasing for all $\eta \in [0, 1]$*, such that a market share discount* $T(q_I, q_E)$ *satisfying*

1. *The coalition optimality conditions*

2. *And simultaneously:*

$$\mathbb{E}\pi_I = \int_0^{\bar{c}_E} \{T(q_I^*(c_E), q_E^*(c_E)) - c_I q_I^*(c_E)\} f(c_E)\mathrm{d}c_E = \Pi_I^* \tag{40}$$

Exists if and only if $\bar{\pi}_I \leq \bar{\pi}_I^{D*}(\eta)$

Proof. See the Appendix. □

Hence as it is easily seen, results are easily generalized in this direction. The only difference, is the need of more complex non-linear contracts, as own-supply efficiency is not immediately guaranteed as in the inelastic demand model.

5 Discussion

[TO BE COMPLETED]

The literature has usually relied on exclusive dealing models to analyze the anticompetitive effect of rebates and other non-linear contracts, as the underlying mechanisms are usually regarded as conceptually the same. They have been treated almost as equal from an antitrust perspective (cite examples). Our results show that there is no such equivalence, however. In particular, at least in the rent-shifting family of models there is one critical assumption explaining why the equivalence falls apart but that has remained unnoticed: the existence of unconditional payments, or more generally, of an ex-ante credible commitment to transfer rents.

This is interesting since in practice rebates and other types of non-linear contracts do not usually involve such kind of binding commitments, or at least they have not been documented in the most prominent cases. Two complementary hypothesis can explain this: either unconditional payments have not received the attention they deserve, and/or such rent transfer is not nakedly executed but masked under alternative principal-agent rationales. For example, instead of using "up-front" lump-sum payments or signing liquidated damage clauses, which could raise antitrust suspicious, rent transfers may be achieved more subtly using payments to supposedly boost advertising efforts. Alternatively, the relationship between the dominant manufacturer and the buyer/downstream retailer may be of a long-term and repeated nature, which could partially discourage the opportunistic behavior from this latter, making the transfer restriction less severe despite the absence of such explicit payments.

APPENDIX

Proof of Lemma 1

Part 1.

Claim 1. *If $c_E \geq c_I + (1-\lambda)(v - c_I) = c^*$, then $p_I = c_E$, $p_E = c_E$ is a pure-strategy equilibrium.*

Proof. Suppose first that $p_I = v$ and $p_E = v$. Obviously then, I has incentives to deviate to $p_I' = v - \varepsilon$ and sell both units because as it obtains $\pi_I' = v - c_I - \varepsilon > (1-\lambda)(v - c_I) = \pi_I'$. But if $p_I' = v - \varepsilon$, then E has incentives to undercut its price to $p_E' = v - 2\varepsilon$ and obtain strictly positive profits. This undercutting dynamic continues until $p_E = c_E$ and $p_I = c_E - \varepsilon \approx c_E$, where $\pi_I = c_E - c_I$. But when $p_E = c_E$, I also has the alternative to act as a residual monopoly, whose optimal price comes from

$$\pi_B(\textit{Split Purchases}) = \lambda(v - c_E) + (1-\lambda)(v - p_I) = \lambda(v - c_E) = \pi_B(\textit{Only to E}) \tag{41}$$

That is charging a price of $p_I^{RM} = v$ to obtain profits of $\pi_I^{RM} = (1-\lambda)(v - c_I)$. Hence, $p_I = c_E$, $p_E = c_E$ is a pure-strategy equilibrium only when $c_E - c_I \geq (1-\lambda)(v - c_I)$, that is when $c_I + (1-\lambda)(v - c_I) = c^*$ □

Part 2. The second part is more involved.

Claim 2. *If $c_E < c^* = c_I + (1-\lambda)(v - c_I)$, then no pure strategy equilibrium exists.*

Proof. If $p_E < p_I$, firm E has incentives to rise its price; if $c_E < p_I < p_E$, firm E has the incentive to undercut I's price; if $p_I \leq p_E = c_E$, the incumbent prefers to act as a residual monopoly given that $c_E \in [c_I, c^*)$; and if $p_I = p_E = v$ the incumbent has incentives to undercut E's price. Hence all possible candidates for pure-strategy equilibrium are discarded. □

For the following claims, let $\{r, R\}$ and $\{z, Z\}$ the boundaries of the supports of the distributions of E and I respectively.

Claim 3. *If $c_E < c^*$, in any mixed strategy equilibrium $\pi_I^e(p_I) = (1-\lambda)(v - c_I) = \pi_I^{RM}$*

Proof. Realizing that $\pi_I^e(p_I) \geq \pi_I^{RM}$ is straightforward, as the incumbent can always secures for itself at least the residual monopoly profits by charging the residual monopoly price, this also implies that $p_I \geq c^*$ so $z \geq c^*$. Proving however that in any equilibrium $\pi_I^e(p_I) \leq \pi_I^{RM}$, is far more involved.

Now, it is easy to realize that in any mixed strategy equilibrium both firms must be randomizing because otherwise, the rival has incentives to slightly undercut with probability one. Now notice that

$$\begin{aligned} \pi_I^e(p_I) &= \mathbb{P}(p_I \leq p_E)(p_I - c_I) + \mathbb{P}(p_I > p_E)(1-\lambda)(p_I - c_I) \\ &= (1-\lambda)(p_I - c_I) + \lambda(p_I - c_I)\mathbb{P}(p_I \leq p_E) \end{aligned} \tag{42}$$

Suppose then that $\pi_I^e(p_I) > \pi_I^{RM}$, this implies

$$[(1-\lambda)(p_I - c_I) - (1-\lambda)(v - c_I)] + \lambda(p_I - c_I)\mathbb{P}(p_I \leq p_E) > 0 \tag{43}$$

So $\mathbb{P}(p_I \leq p_E) > 0$ for all p_I admissible, since $(1-\lambda)(p_I - c_I) \leq (1-\lambda)(v - c_I) = \pi_I^{RM}$. In particular, this must hold for $p_I = Z$, which means that $\pi_E^e(R) = 0$ (either $Z < R$, or $Z = R$ and R has an atom).

Hence, defining $S \subseteq [Z, R]$ the support of E's distribution must be either S or $\{c_E\} \cup S$, with an atom θ at c_E. However the support S cannot be equilibrium as I has the incentives to play $p_I = Z$ with probability one, a contradiction. On the other hand, $\{c_E\} \cup S$, cannot be equilibrium either because

$$\pi_I^e(p_I) = \theta(p_I - c_I) + (1-\theta)(1-\lambda)(p_I - c_I) \tag{44}$$

So $\pi_I^e(z) = \pi_I^e(Z) \iff z = Z$, again implying the I plays a singleton with prob. 1.

Hence, assuming $\pi_I^e(p_I) > \pi_I^{RM}$ always leads to a contradiction, so $\pi_I^e(p_I) \leq \pi_I^{RM}$ also, implying that $\pi_I^e(p_I) = \pi_I^{RM} = (1-\lambda)(v - c_I)$. □

Claim 4. *If $c_E < c^*$, in any mixed strategy equilibrium $r = z \leq v$*

Proof. If $z > v$, then I would get null profits which contradicts the previous claim. Now, because $z \leq v$, then $r \geq z$ because otherwise, $\pi_E(r) = \lambda(r - c_E) < \pi_E(r + \varepsilon) = \lambda(r + \varepsilon - c_E)$ for an $\varepsilon > 0$ but arbitrarily small, so E would be better off by deviating a playing $r + \varepsilon$ with prob. 1.

However, if $z < r$ then $\pi_I(z) = z - c_I < \pi_I(z + \varepsilon) = z + \varepsilon - c_I$ for a $\varepsilon > 0$ but arbitrarily small, which again is a contradiction. Hence $r = z < v$. $\square$

Claim 5. *If $c_E < c^*$, in any mixed strategy equilibrium $\pi^e_E(p_E) = \lambda\,(c^* - c_E)$*

Proof. That $\pi^e_E(p_E) \geq \lambda\,(c^* - c_E)$ follows from the fact that E can always secure such payoff by playing $p_E = c^* - \varepsilon \equiv c^*$ without risking being undercut by I.

However, yet again, showing that $\pi^e_E(p_E) \leq \lambda\,(c^* - c_E)$ is not that straightforward. Suppose on the contrary that $\pi^e_E(p_E) > \lambda\,(c^* - c_E)$, that is

$$[1 - \mathbb{P}(p_I \leq p_E)]\lambda(p_E - c_E) > \lambda(c^* - c_E) \Longrightarrow p_E > c^*,\ \forall\, p_E \tag{45}$$

Hence $z = r > c^*$. However, $z > c^*$ can only be equilibrium, if the distribution over which E randomizes is atomless at r, otherwise, I would be strictly better off by charging a price $c^* < p_I = r - \varepsilon < r$, with probability one. But if $z = r > c^*$, and r is atomless we would have that $\pi^e_I(z = r) = z - c_I > c^* - c_I = (1 - \lambda)(v - c_I)$, contradicting claim 3. Hence $r > c^*$ leads to a contradiction, implying that $\pi^e_E(p_E) \leq \lambda\,(c^* - c_E)$. Therefore $\pi^e_E(p_E) = \lambda\,(c^* - c_E)$. $\square$

Claim 6. *If $c_E < c^*$, there exists a mixed strategy equilibrium.*

Proof. We will prove existence by constructing a mixed strategy equilibrium. Suppose E randomizes over the connected interval $[c^*, v]$ using the pdf.

$$g(x) = \frac{c^* - c_I}{\lambda\,(x - c_I)^2} = \frac{(1 - \lambda)\,(v - c_I)}{\lambda\,(x - c_I)^2} \tag{46}$$

Now,

$$\pi^e_I(p_I) = [1 - G(p_I)](p_I - c_I) + G(p_I)(1 - \lambda)(p_I - c_I) \tag{47}$$

So using $g(x)$ we have,

1. If $p_I \in [c^*, v] \Longrightarrow \pi^e_I(p_I) = c^* - c_I = (1 - \lambda)(v - c_I) = \pi^{RM}_I$, and invariant in p_I.

2. If $p_I < c^* \Longrightarrow \pi^e_I(p_I) = p_I - c_I < c^* - c_I = \pi^{RM}_I$

3. If $p_I > 0 \Longrightarrow \pi^e_I(p_I) = 0 < \pi^{RM}_I$

Hence, given the way E randomizes, any distribution with a support being a subset of $[c^*, v]$ is a best reply from I's perspective. Suppose for example that I also randomizes over the interval $[c^*, v]$, but using the pdf.

$$h(x) = \begin{cases} \frac{c^*-c_E}{v-c_E} & if\ x = v \\ \frac{c^*-c_E}{2\,(x-c_E)^2} = \frac{c_I+(1-\lambda)\,(v-c_I)-c_E}{(x-c_E)^2} & if\ x \in [c^*, v] \end{cases} \tag{48}$$

Which indeed satisfies the conditions for being a best-reply from I's perspective. Then,

$$\pi_E^e(p_E) = \lambda(p_E - c_E)\,[\phi + (1-\phi)\,\{1 - H(p_E)\}] \tag{49}$$

So given $h(x)$ we have:

1. If $p_E \in [c^*, v] \Longrightarrow \pi_E^e(p_E) = \lambda(c^* - c_E)$, and invariant in p_E.
2. If $p_E < c^* \Longrightarrow \pi_E^e(p_E) = \lambda(p_E - c_E) < \lambda(c^* - c_E)$
3. If $p_E > p_I^{RM} \Longrightarrow \pi_E^e(p_E) = 0 < \lambda(c^* - c_E)$

Hence any distribution over $[c^*, v]$ is a best reply function from E's perspective, in particular

$$g(x) = \frac{c^* - c_I}{\lambda\,(x - c_I)^2} = \frac{(1-\lambda)\,(v - c_I)}{\lambda\,(x - c_I)^2} \tag{50}$$

Consequently, the following represents a mixed strategy Nash equilibrium of the spot competition subgame:

1. Firm E randomizes over $[c^*, v]$, using the atomless pdf.:

$$g(x) = \frac{c^* - c_I}{\lambda\,(x - c_I)^2} = \frac{(1-\lambda)\,(v - c_I)}{\lambda\,(x - c_I)^2} \tag{51}$$

2. Firm I randomizes over $[c^*, v]$, with atom at v according to the pdf:

$$h(x) = \begin{cases} \frac{c^*-c_E}{v-c_E} & if\ x = v \\ \frac{c^*-c_E}{2\,(x-c_E)^2} = \frac{c_I+(1-\lambda)\,(v-c_I)-c_E}{(x-c_E)^2} & if\ x \in [c^*, v] \end{cases} \tag{52}$$

□

Claim 7. *If $c_E < c^*$ any mixed strategy equilibrium is (weakly) welfare inefficient for all c_E, while strictly inefficient for all $c_E \neq c_I$.*

Proof. First, notice that because $Z \leq v$, in all mixed strategy equilibria the consumer buys 1 unit with prob. 1.

Now, suppose $c_E < c_I$, then the maximum welfare achievable is $W^* = v - \lambda c_E - (1-\lambda)c_I$. However, total (expected) welfare (across realization) in any mixed strategy equilibrium would be

$$W^e = \mathbb{P}(p_I \leq p_E)(v - c_I) + [1 - \mathbb{P}(p_I \leq p_E)]\{v - \lambda c_E - (1-\lambda)c_I\} \tag{53}$$

Then, given that $v - c_I < v - \lambda c_E - (1-\lambda)c_I$ and $\mathbb{P}(p_I \leq p_E) \in (0,1)$ (since both firms must be randomizing), we get $W^e < W^*$.

On the other hand, if $c_E \in (c_I, c^*)$ maximum welfare achievable is $W^* = v - c_I$. While total expected welfare would again be W^e. Hence, given that $\mathbb{P}(p_I \leq p_E) \in (0,1)$ and $v - \lambda c_E - (1-\lambda)c_I < v - c_I$, again $W^e < W^*$.

Finally, if $c_I = c_E$ it is straightforward to see that $W^e = W^*$. $\square$

The only thing left to prove is that $\pi_B^e < \lambda\,(v - c_I)$.

Claim 8. *If $c_E < c^*$, in any mixed strategy equilibrium $\pi_B^e < \lambda\,(v - c_I)$.*

Proof. Suppose $c_E \leq c_I$. Then $W^* = v - \lambda c_E - (1-\lambda)c_I$, and

$$\begin{aligned} W^e &= \pi_I^e + \pi_E^e + \pi_B^e \\ &= (1-\lambda)(v - c_I) + \lambda(c^* - c_E) - \pi_B^e \end{aligned} \tag{54}$$

But we know $W^e \leq W^*$, which implies

$$\pi_B^e \leq \lambda\,(v - c^*) = \lambda^2(v - c_I) < \lambda(v - c_I) \tag{55}$$

On the other hand, suppose $c_E \in (c_I, c^*)$. Then $W^* = v - c_I$, while W^e is the same as above. But again $W^e \leq W^*$, which implies

$$\pi_B^e \leq \lambda(v - c_I) - \lambda(v - c^*) < \lambda(v - c_I) \tag{56}$$

Hence, in any case $\pi_B^e < \lambda(v - c_I)$. $\square$

Proof of Lemma 4

The problem at hand is

$$\max_{r^*, c_E^*} \mathbb{E}\pi_I^R = (r^* - c_I)(1-\lambda) + \lambda(c_E^* - c_I)[1 - F(c_E^*)] \tag{57}$$

Subject to $v - r^* + \lambda(r^* - c_E^*) \geq \bar{\pi}_B$, and $r^* \leq v$. Obviously, at least one of the restriction must be binding, since otherwise the unconstrained optimization of the incumbent expected profits leads to a violation of the consumer participation. On the other hand, this latter constraint cannot be the only one active, because it would imply a $r^* > v$ as we already showed. Hence in any case $r^* = v$.

This implies that the incumbent's expected payoff $\mathbb{E}\pi_I^R(c_E^*) = (v - c_I)(1-\lambda) + \lambda(c_E^* - c_I)[1 - F(c_E^*)]$ and that B's participation constraint can be rewritten as $\lambda(v - c_E^*) \geq \bar{\pi}_B$.

On the other hand, we already saw "anticompetitive" rebates cannot emerge in equilibrium as they violate the incumbent's participation constraint. It remains to see whether non-anticompetitive contracts can emerge in equilibrium. That is, if $\exists\ c_E^* > c_I$, such that both participation constraints are met simultaneously. To tackle this we use the following claim.

Claim 9. *If $\bar{c}_E \leq c^*$, then $\mathbb{E}\pi_I^R(c_E^*) \geq \bar{\pi}_I$ and simultaneously $\lambda(v - c_E^*) \geq \bar{\pi}_B$, only if c_E^* belongs to the non-empty interval $[a, b] \subset (c_I, v)$*

Proof. Using the fact that $r^* = v$, and the expression for $\bar{\pi}_I$ given by lemma 1, I's participation constraint can be rewritten as

$$\left[\frac{1 - F(c^*)}{1 - F(c_E^*)}\right] \{\mathbb{E}(c_E \mid c_E > c^*) - c^*\} \leq \lambda(c_E^* - c_I) \tag{58}$$

On the other hand, the consumer's participation constraint can be rearranged as $\lambda(c_E^* - c_I) \leq \lambda(v - c_I) - \bar{\pi}_B$. Combining both inequalities we have that necessary condition for the emergence of non-anticompetitive contracts is that

$$\left[\frac{1 - F(c^*)}{1 - F(c_E^*)}\right] \{\mathbb{E}(c_E \mid c_E > c^*) - c^*\} \leq \lambda(v - c_I) - \bar{\pi}_B \tag{59}$$

Intuitively the fulfillment of the above condition is equivalent to asking that the contract setting c_E^* generates positive surplus for the IB coalition i.e. there is scope for "trade". The condition then is immediately satisfied when $\bar{c}_E \leq c^*$, since the left side is 0, while the right hand strictly positive as $\lambda(v - c_I) > \bar{\pi}_B$.

However, we still have to check each condition individually. The incumbents participation requires that

$$\left[\frac{1-F(c^*)}{1-F(c_E^*)}\right]\{\mathbb{E}(c_E \mid c_E > c^*) - c^*\} = 0 \leq \lambda(c_E^* - c_I) \tag{60}$$

While the consumer's condition asks for $\lambda(c_E^* - c_I) \leq \lambda(v - c_I) - \bar{\pi}_B$. Therefore, there $\exists\ c_E^*$ greater, but sufficiently close to c_I, that satisfies both conditions. So define then the non-empty interval $[a, b]$ as all such values. It is easily seen then $[a, b] \subset (c_I, v)$. Hence, both conditions are simultaneously satisfied for all $c_E^* \in [a, b]$. □

Finally, in order to show that $c_E^* < \bar{c}_E$ (so the upper bound is not really v as the above claim stated), notice that the consumer participation constraint is less active the lower c_E^*. On the other hand, differentiating $\mathbb{E}\pi_I^{Rb}(c_E^*)$ with respect to c_E^* we get:

$$\frac{\partial \mathbb{E}\pi_I^R}{\partial c_E^*} = \lambda[1 - F(c_E^*) - f(c_E^*)(c_E^* - c_I)] \tag{61}$$

Hence, $\mathbb{E}\pi_I^{Rb}$ attains its unconstrained maximum at a $c_E^* < \bar{c}_E$. Therefore, the incumbent would never choose a $c_E^*\bar{c}_E$, since it decreases its payoff and makes the consumer participation harder to satisfy.

Therefore, using all the results above we have that the optimal rebates without unconditional payments in this case is given by $r^* = v$, $c_E^* \in (c_I, \bar{c}_E)$, and that a sufficient condition for its emerge in equilibrium is that $\bar{c}_E \leq c^*$.

Proof of Lemma 5

The expected profit and total surplus I and B obtain from dealing through the spot market are given by:

$$\begin{aligned} \mathbb{E}\pi_I &= \int_0^{\bar{c}_E} \{T(q_I(c_E), q_E(c_E)) - c_I q_I(c_E)\} f(c_E) \mathrm{d}c_E \\ \mathbb{E}\pi_B &= \mathbb{E}[(1-\beta)\Delta W_{EB}^* + v - T(1,0)] \end{aligned} \tag{62}$$

Moreover, we have that

$$W_{EB}^*(c_E) = \max_{q_E \leq \lambda} W_{EB} = v - c_E q_E - T(q_I, q_E) \tag{63}$$

Se we have the usual implementation condition $q'(c_E) \leq 0$ given by the second order condition of the above problem. Using then the envelope theorem:

$$\mathbb{E}W^*_{EB} = W^*_{EB}(\bar{c}_E) + \int_0^{\bar{c}_E} q_E(c_E)F(c_E)\mathrm{d}c_E \tag{64}$$

So

$$\begin{aligned}\mathbb{E}\pi_B &= \mathbb{E}[(1-\beta)\Delta W^*_{EB} + v - T(1,0)] \\ &= \int_0^{\bar{c}_E} (1-\beta)q_E(c_E)F(c_E)\mathrm{d}c_E + (1-\beta)W^*_{EB}(\bar{c}_E) + \beta[v - T(1,0)]\end{aligned} \tag{65}$$

Moreover, we know that

$$\mathbb{E}W^*_{EB} = \int_0^{\bar{c}_E} \{v - c_E\ q^*_E(c_E) - T(q_I(c_E), q^*_E(c_E))\}\, f(c_E)\mathrm{d}c_E \tag{66}$$

Therefore, using (64) and (66) we get can obtain an expression for $\int_0^{\bar{c}_E} T(q_I(c_E), q^*_E(c_E))f(c_E)\mathrm{d}c_E$ as function of the allocation rule $[q_I(c_E), q^*_E(c_E)]$. Replacing then in $\mathbb{E}\pi_I$, and using the fact that $q_I(c_E) = 1 - q^*_E(c_E)$ we get:

$$\mathbb{E}\pi_I = v - c_I - W^*_{EB}(\bar{c}_E) - \int_0^{\bar{c}_E} \left\{c_E - c_I + \frac{F(c_E)}{f(c_E)}\right\} q^*_E(c_E)f(c_E)\mathrm{d}c_E \tag{67}$$

Using then (65) and (67), and realizing that $\Pi^*_E(\bar{c}_E) = \beta[W^*_{EB}(\bar{c}_E) - v + T(1,0)]$, we get that

$$\mathbb{E}W_{IB} = v - c_I - \int_0^{\bar{c}_E} \left[(c_E - c_I) + \frac{\beta F(c_E)}{f(c_E)}\right] q_E(c_E)\ f(c_E)\mathrm{d}c_E - \Pi_E(\bar{c}_E) \tag{68}$$

Hence the problem faced by the IB coalition, is to choose a suitable allocation rule $q^*_E(c_E)$ that maximizes (68) subject to both participation constraints, and that $q^{*'}_E(c_E) \leq 0$ for all $q_E \in [0, \lambda]$.

Proof of Lemma 6

Since $T(q_I, q_E)$ satisfies $T_{q_I}(q_I, q_E) \leq v$ and $T(q_I, q_E) \leq vq_I$ for all $q_I \in [0,1]$ and any q_E, we know that $q_I = 1 - q_E$. Therefore, the $E - B$ coalition solves:

$$\max_{q_E \leq \lambda} W_{EB} = v - c_E q_E - T(1 - q_E, q_E) \tag{69}$$

And the surplus created by it is

$$\Delta W_{EB}^{*}(c_E) = W_{EB}^{*}(c_E) - v + T(1,0) = T(1,0) - c_E q_E^{*}(c_E) - T(1-q_E^{*}(c_E), q_E^{*}(c_E)) \tag{70}$$

The EB coalition will not be active in equilibrium ($q_E^{*}(c_E) = 0$) if and only if $\Delta W_{EB}(c_E) < 0$, for all $q_E \in [0, \lambda]$.

Part 1. Now, suppose $T(q_I, q_E)$ implements a simple allocation at c_E^{*}, but $(1/\lambda)[T(1,0) - T(1-\lambda, \lambda)] > c_E^{*}$, and consider $c_E = c_E^{*} + \varepsilon$ Notice that if $q_E(c_E) = \lambda$, then

$$\Delta W_{EB}(c_E) = T(1,0) - (c_E^{*} + \varepsilon)\lambda - T(1-\lambda, \lambda) > 0 \Longleftrightarrow \frac{T(1,0) - T(1-\lambda,\lambda)}{\lambda} - c_E^{*} > \varepsilon \tag{71}$$

Which is true for a sufficiently small ε. Hence a if the entrant has marginal cost $c_E \in [c_E^{*}, c_E^{*}+\varepsilon]$, he will also sell an strictly positive amount in equilibrium. The argument is analogous for $(1/\lambda)[T(1,0) - T(1-\lambda,\lambda)] < c_E^{*}$,

Part 2. Take any $T(q_I, q_E)$ such that $p^e(q_E)$ is non-decreasing in q_E for all $q_E \in [0, \lambda]$ and $p^e(\lambda) = c_E^{*}$, and rewrite the surplus as $\Delta W_{EB} = q_E[p^e(q_E) - c_E]$. Therefore, the $E-B$ coalition problem is

$$\max_{q_E \leq \lambda} \Delta W_{EB} = q_E[p^e(q_E) - c_E] \tag{72}$$

Noticing that $p^e(q_E)$ is maximized at $q_E = \lambda$ we get that if $c_E \leq c_E^{*} = p^e(\lambda)$, ΔW_{EB} is maximized at λ, and moreover that surplus is non-negative $\Delta W_{EB}* = \lambda[c_E^{*} - c_E] \geq 0$, so all types $c_E \leq c_E^{*}$ will choose $q_E^{*}(c_E) = \lambda$. On the other hand, if $c_E > c_E^{*}$, then $p^e(q_E) \leq c_E^{*} < c_E$ for all $q_E \in [0, \lambda]$, so ΔW_{EB} is strictly decreasing in q_E, implying that all types $c_E > c_E^{*}$ will chose $q_E^{*}(c_E) = 0$. Hence, the allocation induced by this price schedule is simple at c_E^{*}.

The proof for own-supplied contracts, is equivalent.

Proof of Proposition 2

Conditions 1 and 2 have already been derived, so we focus here on 3. The problem faced by the coalition, ignoring for the moment any constraints, is

$$\max_{q_E} \mathbb{E}W_{IB} = v - c_I - \int_0^{\bar{c}_E} \left[(c_E - c_I) + \frac{\beta F(c_E)}{f(c_E)}\right] q_E^{*}(c_E)\ f(c_E)\mathrm{d}c_E - \Pi_E(\bar{c}_E) \tag{73}$$

Therefore doing point-wise optimization, and given the monotonic hazard rate assumption, there exists a unique $\tilde{c}_E \in [0, \bar{c}_E]$ given by $f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0$, which fully characterizes the optimal allocation:

$$q_E^*(c_E) = \begin{cases} \lambda & if\ c_E \in [0, \tilde{c}_E] \\ 0 & otherwise \end{cases} \tag{74}$$

This allocation indeed satisfies the constraint $q_E'(c_E) \leq 0$. Hence the optimal schedule chosen by the $I - B$ coalition induces a simple allocation rule at $\tilde{c}_E$. Given lemma 6, this implies that $p^e(\lambda) = \tilde{c}_E$, leaving entrants with type $c_E \in [\tilde{c}_E, \bar{c}_E]$ with zero surplus. Hence $\Pi_E(\bar{c}_E) = 0$.

Finally, both participation constraints are obviously met given out Nash-Bargaining solution, and the fact that $\mathbb{E}W_{IB}^* > \bar{\pi}_B + \bar{\pi}_I = \bar{W}$ by assumption.

Proof of Proposition 3

Suppose $T^*(q_I, q_E)$ satisfies the optimality conditions. We already saw, that conditions 1. and 2. implied $T^*(q_I, q_E) \leq vq_I$ for all $q_I \in [0, 1]$ and any q_E. Assume also, on the contrary, that $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$.

Given that the optimal scheme induces a simple allocation at $\tilde{c}_E$, the profits the incumbent gets from the spot market transactions are

$$\begin{aligned} \mathbb{E}\pi_I &= \int_0^{\bar{c}_E} \{T^*(1 - q_E^*(c_E), q_E^*(c_E)) - c_I(1 - q_E^*(c_E)) - \} f(c_E) \mathrm{d}c_E \\ &= \{T^*(1-\lambda, \lambda) - (1-\lambda)c_I\} F(\tilde{c}_E) + \{T^*(1, 0) - c_I\} [1 - F(\tilde{c}_E)] \end{aligned} \tag{75}$$

But $T^*(\cdot)$ must necessarily fulfill

$$p^e(\lambda) \equiv \frac{T^*(1,0) - T^*(1-\lambda, \lambda)}{\lambda} = \tilde{c}_E \Longrightarrow T^*(1,0) = T^*(1-\lambda, \lambda) + \lambda \tilde{c}_E \tag{76}$$

So replacing in $\mathbb{E}\pi_I$:

$$\mathbb{E}\pi_I = T^*(1-\lambda, \lambda) - (1-\lambda)c_I + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] \tag{77}$$

But because $T(\cdot)$ must also appropriately distribute rents, we need $\mathbb{E}\pi_I = \Pi_I^*$. Therefore

$$\begin{aligned} & T^*(1-\lambda,\lambda) - (1-\lambda)c_I + \lambda(\tilde{c}_E - c_I)[1-F(\tilde{c}_E)] = \eta\Delta W_{IB}(\tilde{c}_E) + \bar{\pi}_I \\ \Longrightarrow\ & T^*(1-\lambda,\lambda) = \bar{\pi}_I + \eta\Delta W_{IB}(\tilde{c}_E) + (1-\lambda)c_I - \lambda(\tilde{c}_E - c_I)[1-F(\tilde{c}_E)] \end{aligned} \tag{78}$$

But then,

$$\begin{aligned} T^*(1-\lambda,\lambda) - v(1-\lambda) &= \bar{\pi}_I + \eta\Delta W_{IB}(\tilde{c}_E) - (1-\lambda)(v-c_I) - \lambda(\tilde{c}_E - c_I)[1-F(\tilde{c}_E)] \\ &= \bar{\pi}_I - \bar{\pi}_I^*(\eta) > 0 \end{aligned} \tag{79}$$

By assumption. Hence $T^*(1-\lambda,\lambda) > v(1-\lambda)$, but this condition contradicts $T^*(q_I,q_E) \le vq_I$ for all $q_I \in [0,1]$ and any q_E. The converse is analogous.

Proof of Lemma 7

We have

$$\begin{aligned} \mathbb{E}\pi_I &= v - c_I - W_{EB}^*(\bar{c}_E) - \int_0^{\bar{c}_E} \left\{ c_E - c_I + \frac{F(c_E)}{f(c_E)} \right\} q_E^*(c_E) f(c_E) \mathrm{d}c_E \\ \mathbb{E}\pi_B &= \int_0^{\bar{c}_E} (1-\beta) q_E(c_E) F(c_E) \mathrm{d}c_E + (1-\beta) W_{EB}^*(\bar{c}_E) + \beta[v - T(1,0)] \end{aligned} \tag{80}$$

However, $\partial W_{EB}^*(c_E)/\partial c_E = -q_E(c_E)$ so

$$W_{EB}^*(\bar{c}_E) = W_{EB}^*(0) - \int_0^{\bar{c}_E} q_E(c_E) \mathrm{d}c_E \tag{81}$$

But $W_{EB}^*(0) = v - T(1-q_E(0), q_E(0))$, hence

$$W_{EB}^*(\bar{c}_E) = v - T(1-q_E(0), q_E(0)) - \int_0^{\bar{c}_E} q_E(c_E) \mathrm{d}c_E \tag{82}$$

So replacing in (65) and (67) we get

$$\begin{aligned} \mathbb{E}\pi_B &= v - T(1,0) + (1-\beta)[T(1,0) - T(1-q_E(0), q_E(0))] - \int_0^{\bar{c}_E} (1-\beta)[1-F(c_E)] q_E(c_E) \mathrm{d}c_E \\ \mathbb{E}\pi_I &= T(1-q_E(0), q_E(0)) - c_I - \int_0^{\bar{c}_E} \left\{ c_E - c_I - \frac{[1-F(c_E)]}{f(c_E)} \right\} q_E^*(c_E) f(c_E) \mathrm{d}c_E \end{aligned} \tag{83}$$

Now, it is easy to see $\mathbb{E}\pi_B$ is maximized by $q_E(c_E) = 0$ for all $c_E \in [0, \bar{c}_E]$. On the other hand, $\mathbb{E}\pi_I$ is maximized by $q_E(c_E) = \lambda$ for all $c_E \leq \dot{c}_E$, and $q_E(c_E) = 0$ for all $c_E > \dot{c}_E$, where this latter is defined implicitly by $f(\dot{c}_E)(\dot{c}_E - c_I) - [1 - F(\cdot c_E)] = 0$, and therefore is strictly between c_I and $\bar{c}_E$. Hence neither the incumbent or the consumer will push for $q_E(c_E) > 0$ for $c_E \in (\dot{c}_E, \bar{c}_E]$, and therefore the optimal schedule necessarily satisfies that $q_E(c_E) = 0$ for all $c_E \geq \alpha = \dot{c}_E$.

Proof of Lemma 8

A crucial issue in this transfer restriction setting, is that the only way $q_E(c_E) = 0$ for all $c_E > \dot{c}_E$ is for $\Pi^*_E(c_E) = \beta[W^*_{EB}(c_E) - v + T(1,0)] \leq 0$ for all c_E's in the relevant region. By monotonicity of $\Pi^*_E(c_E)$, this is equivalent as asking $\Pi^*_E(\dot{c}_E) = 0$. But since $W^*_{EB}(\dot{c}_E) = v - T(1 - q_E(0), q_E(0)) - \int_0^{\dot{c}_E} q_E(c_E)\mathrm{d}c_E$, this implies that

$$T(1,0) - T(1 - q_E(0), q_E(0)) = \int_0^{\dot{c}_E} q_E(c_E)\mathrm{d}c_E \tag{84}$$

So replacing this latter condition in $\mathbb{E}\pi_B$ and $\mathbb{E}\pi_I$, and using the fact that $q_E(c_E) = 0$ for all $c_E > \dot{c}_E$ is optimal, we get:

$$\begin{aligned}
\mathbb{E}\pi_B &= v - T(1 - q_E(0), q_E(0)) - \int_0^{\dot{c}_E} \frac{\{1 - (1-\beta)[1 - F(c_E)]\}}{f(c_E)} q^*_E(c_E) f(c_E)\mathrm{d}c_E \\
\mathbb{E}\pi_I &= T(1 - q_E(0), q_E(0)) - c_I - \int_0^{\dot{c}_E} \left\{c_E - c_I - \frac{[1 - F(c_E)]}{f(c_E)}\right\} q^*_E(c_E) f(c_E)\mathrm{d}c_E
\end{aligned} \tag{85}$$

And the contraint $T(q_I, q_E) \leq v q_I$, for all $q_I \in [0,1]$ can be simplified to $T(1 - q_E(0), q_E(0)) \leq v(1 - q_E(0))$.

Proof of Lemma 9

[To be completed]

Proof of Lemma 10

When $\eta = 0$, the candidate given by lemma 9 states $T(1-\lambda, \lambda) = v(1-\lambda)$ and that $q_E(c_E)$ is simple at $\check{c}_E$, where this latter satisfied

$$(1-\lambda)(v - c_I) + \lambda\,(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I \tag{86}$$

This candidates satisfies the implementability condition, the transfer restriction and the incumbent's participation constraint (which is binding in this case). Therefore, the only condition missing is whether it also satisfies B's participation. In other words, this would be a solution if

$$\lambda(v - \check{c}_E) + \lambda(1-\beta)F(\check{c}_E)[\check{c}_E - \mathbb{E}(c_E \mid c_E \leq \check{c}_E)] \geq \bar{\pi}_B \tag{87}$$

Now $\hat{c}_E$ is defined by $\hat{c}_E = \min\{\dot{c}_E, \delta\}$, where δ is the solution to

$$\lambda(v - \delta) + \lambda(1-\beta)F(\delta)[\delta - \mathbb{E}(c_E \mid c_E \leq \delta)] = \bar{\pi}_B$$

Suppose then $\delta \leq \dot{c}_E$, so $\hat{c}_E = \delta$, and that $\check{c}_E \leq \hat{c}_E = \delta$. Then, B's participation constraint would be

$$\lambda(v-\check{c}_E)+\lambda(1-\beta)F(\check{c}_E)[\check{c}_E-\mathbb{E}(c_E \mid c_E \leq \check{c}_E)] \geq \lambda(v-\delta)+\lambda(1-\beta)F(\delta)[\delta-\mathbb{E}(c_E \mid c_E \leq \delta)] \tag{88}$$

Which will obviously be fulfilled as $\check{c}_E \leq \delta$, and $f(x) = \lambda(v-x)+\lambda(1-\beta)F(x)[x-\mathbb{E}(c_E \mid c_E \leq x)]$ is decreasing in x.

On the other hand, if $\dot{c}_E < \delta$ so $\hat{c}_E = \dot{c}_E$ this implies that

$$\lambda(v - \dot{c}_E) + \lambda(1-\beta)F(\dot{c}_E)[\dot{c}_E - \mathbb{E}(c_E \mid c_E \leq \dot{c}_E)] > \bar{\pi}_B \tag{89}$$

And therefore, if $\check{c}_E \leq \hat{c}_E = \dot{c}_E$, then

$$\begin{aligned}\lambda(v - \check{c}_E) &+ \lambda(1-\beta)F(\check{c}_E)[\check{c}_E - \mathbb{E}(c_E \mid c_E \leq \check{c}_E)] \\ &\geq \lambda(v - \dot{c}_E) + \lambda(1-\beta)F(\dot{c}_E)[\dot{c}_E - \mathbb{E}(c_E \mid c_E \leq \dot{c}_E)] > \bar{\pi}_B\end{aligned} \tag{90}$$

Therefore, if $\check{c}_E \leq \hat{c}_E$, the problem with $\eta = 0$ has a solution. An analogous proof can be used for the case of $\eta = 1$.

Proof of Proposition 4

Part 1: Existence. Fix and $\eta \in [0,1]$, and a combination of outside option $(\bar{\pi}_I, \bar{\pi}_B)$, and remember that $\bar{W} = \bar{\pi}_I + \bar{\pi}_B$.

Suppose $\check{c}_E \leq \hat{c}_E$. Then, any schedule $T(\cdot)$ with $T(1-\lambda, \lambda) = v(1-\lambda)$ that induces a bang-bang allocation at $c^*_E \in [\check{c}_E, \hat{c}_E]$ satisfies both participation constraints, and the implementation condition $q'(c_E) \leq 0$ for all c_E. Hence a solution exists.

For the converse, suppose $q_E^*(c_E) : [0, \bar{c}_E] \to [0, \lambda]$ solves the general problem (27) for a given $(\eta, \bar{\pi}_I, \bar{\pi}_B)$, and define as in the text $\mathbb{E}\pi_I^* \equiv \mathbb{E}\pi_I(q_E(c_E; \eta = 1))$. Now, because $q_E^*(c_E)$ solves the general problem, we have $\mathbb{E}\pi_I^* \geq \mathbb{E}\pi_I(q_E^*(c_E)) \geq \bar{\pi}_I$, and that $\mathbb{E}\pi_B(q_E^*(c_E)) \geq \bar{\pi}_B$. Hence $\mathbb{E}\pi_I^* + \bar{\pi}_B \geq \bar{\pi}_I + \bar{\pi}_B = \bar{W}$.

Now, if $\hat{c}_E = \dot{c}_E$ the results follows immediately, since $\check{c}_E$ is defined by

$$(1-\lambda)(v - c_I) + \lambda\,(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I \tag{91}$$

Where $f(x) = (1-\lambda)(v - c_I) + \lambda\,(x - c_I)[1 - F(x)]$ is increasing in x for all $x \leq \dot{c}_E$, and given that $\bar{\pi}_I \leq \bar{\pi}_I^{***}$. Consequently $\check{c}_E \leq \dot{c}_E = \hat{c}_E$.

We focus then on the case where $\hat{c}_E = \gamma$. We therefore have:

$$\begin{aligned} &\check{c}_E: \ (1-\lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I \\ &\hat{c}_E: \ \lambda(v - \hat{c}_E) + \lambda(1-\beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] = \bar{W} - \bar{\pi}_I \end{aligned} \tag{92}$$

Consequently

$$\bar{W} - \lambda(v - \hat{c}_E) - \lambda(1-\beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] = (1-\lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] \tag{93}$$

But using the fact that $\mathbb{E}\pi_I^* = (1-\lambda)(v - c_I) + \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)]$, and that $\lambda(v - \hat{c}_E) + \lambda(1-\beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] = \bar{\pi}_B$, we have

$$\bar{W} - [\mathbb{E}\pi_I^* + \bar{\pi}_B] = \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] - \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)] \tag{94}$$

But, $\bar{W} \leq \mathbb{E}\pi_I^* + \bar{\pi}_B$ so $\lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] \leq \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)]$. Finally, since $\lambda(r - c_I)[1 - F(r)]$ is increasing in r for all $r \leq \dot{c}_E$, we conclude $\hat{c}_E \geq \check{c}_E$.

Part 2: Characterization. Because the extreme cases with $\eta = 0$ and $\eta = 1$ have already been characterized, we focus on $\eta \in (0, 1)$. Now, fix $\eta \in (0, 1)$, and a pair of outside options $(\bar{\pi}, \bar{\pi}_B)$ such that $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$, and that $\check{c}_E < \hat{c}_E$ (the case with $\check{c}_E = \hat{c}_E$ is trivial). First notice that neither participation constraints can be binding as the objective functional goes to zero, and therefore can be easily dominated with, for example, a bang-bang allocation rule with $c_E^* \in (\check{c}_E, \hat{c}_E)$. Moreover, the constraint $T(1 - q_E(0)) \leq v(1 - q_E(0))$ since otherwise we would arrive at the A&B "coalitional" solution. But since $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$, this violates the aforementioned constraint.

Now, using calculus of variation over the unconstrained function (25), the Euler-Lagrange equation for the general problem involving a composition of functionals (see also Castillo et. al. 2008) is

$$\frac{-\eta\,\Gamma}{(\mathbb{E}\pi_I-\bar{\pi})}\left\{c_E-c_I-\frac{[1-F(c_E)]}{f(c_E)}\right\}-\frac{(1-\eta)\,\Gamma}{(\mathbb{E}\pi_B-\bar{\pi}_B)}\left[\frac{1-(1-\beta)F(c_E)}{f(c_E)}\right]\qquad \forall c_E\in[0,\dot{c}_E]\quad (95)$$

Where, $\Gamma\equiv(\mathbb{E}\pi_I-\bar{\pi})^{\eta}(\mathbb{E}\pi_B-\bar{\pi}_B)^{1-\eta}$. This condition can be rewritten as $\theta H(c_E)$, with

$$\begin{aligned}
\theta &\equiv \frac{\Gamma[\eta\;(\mathbb{E}\pi_B-\bar{\pi}_B)+(1-\eta)(\mathbb{E}\pi_I-\bar{\pi})]}{(\mathbb{E}\pi_I-\bar{\pi})(\mathbb{E}\pi_B-\bar{\pi}_B)}>0\\
H(c_E) &\equiv -\left\{(1-\gamma)(c_E-c_I)-2\frac{[1-F(c_E)]}{f(c_E)}\left(\frac{1}{2}-\gamma\right)+\frac{\gamma\;\beta F(c_E)}{f(c_E)}\right\}
\end{aligned}\qquad (96)$$

Where

$$\gamma\equiv\frac{(1-\eta)\;(\mathbb{E}\pi_I-\bar{\pi})}{\eta\;(\mathbb{E}\pi_B-\bar{\pi}_B)+(1-\eta)(\mathbb{E}\pi_I-\bar{\pi})}\in(0,1)\qquad (97)$$

And where the sign of the point-wise derivative is given by $H(c_E)$. Notice that the allocation rule $q(\cdot):[0,\dot{c}_E]\rightarrow[0,\lambda]$ enters only as a constant in $\mathbb{E}\pi_I$ and $\mathbb{E}\pi_B$ through γ. Hence given $q(\cdot)$, the Euler-Lagrange equation is independent of $q_E(c_E)$.

Now,

$$\begin{aligned}
H(0) &= -\left[-(1-\gamma)c_I-\frac{2}{f(0)}\left\{\frac{1}{2}-\gamma\right\}\right]\\
H(\dot{c}_E) &= -\frac{\gamma[1-(1-\beta)F(\dot{c}_E)]}{f(\dot{c}_E)}\\
H'(c_E) &= -\left\{(1-\gamma)-2\left(\frac{1-F(c_E)}{f(c_E)}\right)'\left(\frac{1}{2}-\gamma\right)+\gamma\;\beta\left(\frac{F(c_E)}{f(c_E)}\right)'\right\}
\end{aligned}\qquad (98)$$

Therefore, when η is near 1, γ is near 0 implying that $H(0)>0$, $H(\dot{c}_E)<0$ and $H'(c_E)\leq 0$, and therefore the optimal allocation is given

$$q_E^*(c_E)=\begin{cases}\lambda & \text{if } c_E\in[0,x^*]\\ 0 & \text{otherwise}\end{cases}\qquad (99)$$

Which is indeed implementable, and where x^* is implicitly defined by

$$(1-\gamma(x^*))(x^*-c_I)-2\frac{[1-F(x^*)]}{f(x^*)}\left(\frac{1}{2}-\gamma(x^*)\right)+\frac{\gamma(x^*)\;\beta F(x^*)}{f(x^*)}=0\qquad (100)$$

With

$$\gamma(x^*) = \frac{(1-\eta)\ (\mathbb{E}\pi_I(x^*) - \bar{\pi})}{\eta\ (\mathbb{E}\pi_B(x^*) - \bar{\pi}_B) + (1-\eta)(\mathbb{E}\pi_I(x^*) - \bar{\pi})} \tag{101}$$

And, where, because $q_E^*(c_E)$ is bang-bang and $T^*(1-\lambda) = v(1-\lambda)$, $\mathbb{E}\pi_B(x^*)$ and $\mathbb{E}\pi_I(x^*)$ are

$$\begin{aligned} \mathbb{E}\pi_B(x^*) &= \lambda\,(v - x^*) + \lambda(1-\beta)F(x^*)[x^* - \mathbb{E}(c_E \mid c_E \leq x^*)] \\ \mathbb{E}\pi_I(x^*) &= (1-\lambda)(v - c_I) + \lambda\,(x^* - c_I)[1 - F(x^*)] \end{aligned} \tag{102}$$

The more troublesome case is when η is in a neighbourhood of 0, and therefore γ may be higher that 1/2 and therefore $H(c_E)$ may not be monotonous (the "irregular" case). We therefore rely on Myerson's ironing to show the the optimal allocation rule must still be bang-bang with a single cutoff.

Suppose then that instead of being bang-bang, $q_E^*(c_E)$ has a continuously decreasing section between k and K. If $H(c_E) \geq 0$ for all $c_E \in [k, K]$ then the above schedule is dominated by one that induces $q_E(c_E) = \lambda$ in the whole interval. On the other hand, if $H(c_E) \leq 0$ for all $c_E \in [k, K]$, then $q_E^*(c_E)$ is dominated by one that makes $q_E(c_E) = 0$ for all $c_E \in [k, K]$.

Finally, suppose that $H(c_E)$ changes sign one or many times in $[k, K]$. First notice that if $H(c_E) \geq 0$ for all $c_E \in [k, k_1]$, for a k_2 sufficiently close but greater than k, then $q_E^*(c_E)$ allocation is dominated by $q_E(c_E) = \lambda$ for all $c_E \in [k, k_1]$. On the other hand, if $H(c_E) \leq 0$ for all $c_E \in [k_2, K]$, for a k_2 sufficiently close, but less than K, then the $q_E^*(c_E)$ allocation is dominated by $q_E(c_E) = 0$ for all $c_E \in [k_2, K]$. The interesting case then is when $H(c_E)$ is negative in $[k, k_3]$ and positive in $[k_5, K]$ for some $k_3 > k$ and $k_5 < K$. Now a continuously decreasing $q(\cdot)$ cannot be solution to the above case, since it is choosing higher $q_E(c_E)$ in $[k, k_3]$ than in any subsequent subinterval where $H(c_E)$ is positive (for example in $[k_5, K]$). Hence the solution in this case is either bang-bang or a decreasingly step function. But since $H(c_E)$ does not depend on c_E, if the area... [To be completed]

Consequently all the above candidates are dominated by a bang-bang schedule with a cutoff y^* being one of roots of $H(c_E) = 0$ in which the function c_E intersects the x-axis from above. Therefore, in both the regular (η near 1), and irregular (η near 0) cases, the solution to the problem is bang-bang.

Proof of Lemma 11

We will only analyze the effect of η, since the proof for $\bar{\pi}$ is completely analogous.

By proposition 4 , we know that the cutoff $c_E^*(\eta, \bar{\pi}_I)$ is given by one of the roots of

$$H(c_E^*) = (1-\gamma(c_E^*))(c_E^* - c_I) - 2\frac{[1-F(c_E^*)]}{f(c_E^*)}\left(\frac{1}{2} - \gamma(c_E^*)\right) + \frac{\gamma(c_E^*)\ \beta F(c_E^*)}{f(c_E^*)} = 0 \tag{103}$$

And that, at such c_E^*, $H'(c_E^*) \leq 0$. Where

$$\gamma(c_E^*) = \frac{(1-\eta)\ (\mathbb{E}\pi_I(c_E^*) - \bar{\pi})}{\eta\ (\mathbb{E}\pi_B(c_E^*) - \bar{\pi}_B) + (1-\eta)(\mathbb{E}\pi_I(c_E^*) - \bar{\pi})} \tag{104}$$

And

$$\begin{aligned} \mathbb{E}\pi_B(c_E^*) &= \lambda\,(v - c_E^*) + \lambda(1-\beta)F(c_E^*)[c_E^* - \mathbb{E}(c_E \mid c_E \leq c_E^*)] \\ \mathbb{E}\pi_I(c_E^*) &= (1-\lambda)(v - c_I) + \lambda\,(c_E^* - c_I)[1 - F(c_E^*)] \end{aligned} \tag{105}$$

It is straightforward to prove that $\gamma(c_E^*)$ is increasing in c_E^*, and that it is decreasing in the direct effect of η. Hence, by the implicit function theorem we have

$$\frac{\partial c_E^*}{\partial \eta} = \frac{(\partial\gamma/\partial\eta)A(c_E^*)}{-H'(c_E^*) - (\partial\gamma/\partial c_E^*)A(c_E^*)} \tag{106}$$

Where

$$A(c_E^*) \equiv c_E^* - c_I - 2\left(\frac{1-F(c_E^*)}{f(c_E^*)}\right) - \frac{\beta F(c_E^*)}{f(c_E^*)} \tag{107}$$

Now, notice that the condition (103) can be rewritten as

$$\left[c_E^* - c_I - \left(\frac{1-F(c_E^*)}{f(c_E^*)}\right)\right] - \gamma A(c_E^*) = 0 \tag{108}$$

Moreover $\left[c_E^* - c_I - \left(\frac{1-F(c_E^*)}{f(c_E^*)}\right)\right] < 0$ since $c_E^* \leq \dot{c}_E$, so $-\gamma A(c_E^*) > 0$, implying that $A(c_E^*) < 0$.

Consequently, given that $A(c_E^*) < 0$, $-H'(c_E^*) \geq 0$, $(\partial\gamma/\partial\eta) < 0$ and $(\partial\gamma/\partial c_E^*) > 0$, then $(\partial c_E^*/\partial\eta) > 0$.

Proof of Proposition 5

By point-wise optimization we have:

$$\begin{aligned} \frac{\partial}{\partial q_I(c_E)} &= P(q_I(c_E) + q_E(c_E)) - c_I \\ \frac{\partial}{\partial q_E(c_E)} &= P(q_I(c_E) + q_E(c_E)) - c_E - \beta\frac{F(c_E)}{f(c_E)} \end{aligned} \tag{109}$$

Therefore:

1. $q_E(c_E) = \lambda$, and $P(q_I(c_E) + \lambda) - c_I = 0 \Longrightarrow q_I(c_E) = D(c_I) - \lambda$, if

$$f(c_E)(c_I - c_E) - \beta F(c_E) \geq 0 \tag{110}$$

2. And $q_E(c_E) = 0$, and $P(q_I(c_E)) - c_I = 0 \Longrightarrow q_I(c_E) = D(c_I)$, if

$$f(c_E)(c_I - c_E) - \beta F(c_E) < 0 \tag{111}$$

Optimal three-part contract with fixed breaching clause

Suppose instead the incumbent offered a contract (K, p_i, T), where T is a fixed sum paid if B decides to buy at least some unit elsewhere. To keep things simple, lets take A&B assumption of $\eta = \beta = 1$, though it will be clear that the argument is easily generalized to arbitrary bargaining powers.

In this setting, E expands and sells all its units whenever $c_E \leq p_i - \frac{T}{\lambda}$. The consumer surplus on the other hand is in any event $S(D(p_i)) - p_i D(p_i) - K$, and therefore the program that determines the optimal contract is:

$$\max_{(K,p_i,T)} \mathbb{E}\pi_I = F\left(p_i - \frac{T}{\lambda}\right)\{\lambda t_i + (p_i - c_I)[D(p_i) - \lambda]\} + \left[1 - F\left(p_i - \frac{T}{\lambda}\right)\right](p_i - c_I)D(p_i) + K \tag{112}$$

Subject to $S(D(p_i)) - p_i D(p_i) - K \geq \bar{\pi}_B$. Which has solution $p_i^* = c_I$, $T^* = \lambda(c_I - \tilde{c}_E)$ and $K = S(D(p_i)) - p_i D(p_i) - \bar{\pi}_B$. Hence, it is easily verified that the optimal (K, p_i, T) contract implements the coalition optimum.

Proof of Lemma 12

Part 1. Suppose $T(q_I, q_E)$ implements the IB coalition optimum. Given the market share discount, lets analyze the EB coalition problem:

$$\max_{\substack{q_E \in [0,\lambda] \\ q_I \geq 0}} W_{EB}(q_I, q_E) = S(q_I + q_E) - c_E q_E - T(q_I, q_E) \tag{113}$$

And first derivatives

$$\begin{aligned}\frac{\partial}{\partial q_E} &= P(q_I + q_E) - c_E q_E - T_{q_E}(q_I, q_E) \\ \frac{\partial}{\partial q_I} &= P(q_I + q_E) - T_{q_I}(q_I, q_E)\end{aligned} \tag{114}$$

Now, the IB optimum has two distinct properties: $q_I^*(c_E)$ is interior, and $q_I^*(c_E) + q_E^*(c_E) = D(c_I)$. Hence using (114) we need

$$c_I = T_{q_I}(q_I, q_E) \tag{115}$$

But integrating across q_I we have

$$c_I q_I = T(q_I, q_E) - T(0, q_E) \tag{116}$$

And relabeling $T(0, q_E)$ as $\xi(q_E)$, we get that $T(q_I, q_E) = c_I q_I + \xi(q_E)$ necessarily.

Part 2. Suppose $T(q_I, q_E) = c_I q_I + \xi(q_E)$ implements the coalition optimum, but that $(1/\lambda)[\xi(\lambda) - \xi(0)] < c_I - \tilde{c}_E$, and consider a type $c_E = \tilde{c}_E + \varepsilon$. Notice then that if $q_E(\tilde{c}_E + \varepsilon) = \lambda$, then

$$\Delta W_{EB}(\tilde{c}_E + \varepsilon) = \lambda \left[(c_I - \tilde{c}_E) - \varepsilon - \left\{ \frac{\xi(\lambda) - \xi(0)}{\lambda} \right\} \right] > 0 \Longleftrightarrow (c_I - \tilde{c}_E) - \left\{ \frac{\xi(\lambda) - \xi(0)}{\lambda} \right\} > \varepsilon \tag{117}$$

Which is true for a sufficiently small ε. Hence all entrants with marginal cost $c_E \in [\tilde{c}_E, \tilde{c}_E + \varepsilon]$, will also sell an strictly positive amount in equilibrium, and hence $T(q_I, q_E) = c_I q_I + \xi(q_E)$ does not implement the coalition's optimum. An analogous argument can be made to show that $(1/\lambda)[\xi(\lambda) - \xi(0)] > c_I - \tilde{c}_E$, again leads to a contradiction.

Proof of Proposition 6

Let $T(q_I, q_E) = c_I q_I + \xi(q_E)$ be a market share discount that satisfies IB optimality conditions and distributes rent accordingly, but suppose on the contrary that $\bar{\pi}_I > \bar{\pi}_I^{D*}$.

We have that

$$\begin{aligned}\mathbb{E}\pi_I &= \int_0^{\bar{c}_E} \left\{ T(q_I^*(c_E), q_E^*(c_E)) - c_I q_I^*(c_E) \right\} f(c_E) \mathrm{d}c_E \\ &= \xi(\lambda) F(\tilde{c}_E) + \xi(0)[1 - F(\tilde{c}_E)](c_I - \tilde{c}_E)\end{aligned} \tag{118}$$

However, given that $T(q_I, q_E)$ implements the IB optimum, then

$$\frac{\xi(\lambda) - \xi(0)}{\lambda} = \tilde{c}_E \Longrightarrow \xi(0) = \xi(\lambda) - \lambda(c_I - \tilde{c}_E) \tag{119}$$

Hence, $\mathbb{E}\pi_I = \xi(\lambda) - \lambda[1 - F(\tilde{c}_E)](c_I - \tilde{c}_E)$.

Moreover, given that $T(q_I, q_E)$ also distributes rent accordingly we have:

$$\begin{aligned} \mathbb{E}\pi_I = \xi(\lambda) - \lambda[1 - F(\tilde{c}_E)](c_I - \tilde{c}_E) = \Pi_I^* = \eta\Delta W_{IB}^* + \bar{\pi}_I \\ \Longrightarrow \xi(\lambda) = \eta\Delta W_{IB}^* + \bar{\pi}_I + \lambda[1 - F(\tilde{c}_E)](c_I - \tilde{c}_E) \end{aligned} \tag{120}$$

But,

$$\begin{aligned} &\xi(\lambda) - \int_{\lambda}^{D(c_I)} P(q)\mathrm{d}q + c_I[D(c_I) - \lambda] \\ &= \eta\Delta W_{IB}^* + \lambda[1 - F(\tilde{c}_E)](c_I - \tilde{c}_E) + \bar{\pi}_I - \int_{\lambda}^{D(c_I)} P(q)\mathrm{d}q + c_I[D(c_I) - \lambda] = \bar{\pi}_I - \bar{\pi}_I^{D*}(\eta) > 0 \end{aligned} \tag{121}$$

So in case $c_E < \tilde{c}_E$, the buyer ends up purchasing zero units to I, and therefore $T(q_I, q_E)$ does not satisfies the IB optimality conditions, a contradiction.

Figure 1: Implicit Regions

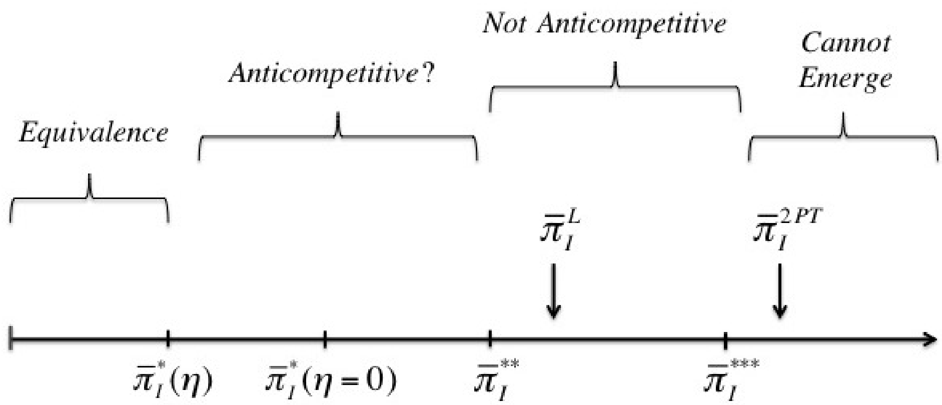
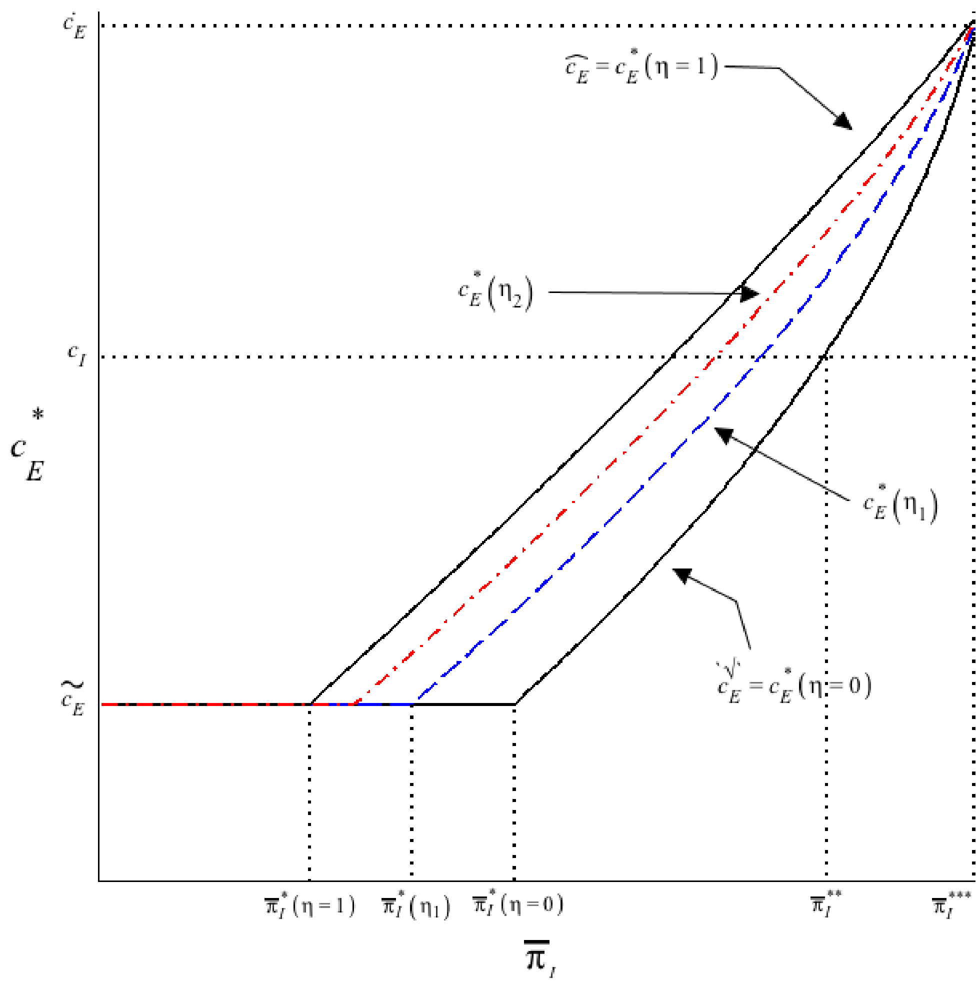


Figure 2: Optimal $c_E^*(\eta)$ - $\eta_2 > \eta_1$



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