Altruism in Networks

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Abstract: We provide the first analysis of altruism in networks. Agents are connected through a fixed, weighted network and care about the well-being of their network neighbors. Given some initial distribution of incomes, agents may provide financial support to their poorer friends. We characterize the Nash equilibria of this transfer game for general networks and utility functions. Our analysis highlights the importance of indirect gifts, where an agent gives to a friend because his friend himself has a friend in need. We uncover four main features of this interdependence. First despite a potentially large multiplicity in transfers, all Nash equilibria lead to the same consumption levels. Second, there is no waste in transfers in equilibrium and transfers flow through paths of highest altruistic strength. Third, equilibrium consumptions vary monotonically with incomes. And a redistribution that decreases income inequality can increase consumption inequality. Fourth, altruistic networks decrease inequality and the shape of the network has a first-order impact on inequality reduction.

Keywords: private transfers, altruism, social networks, income redistribution, inequality.

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I Introduction

Private transfers between individuals represent a significant share of our economies. For instance in 2003, households in the United States gave an estimated amount of $133 bn, or 1.2% of GDP, to other households.[1] In developing countries, migrant remittances alone constitute a main source of income (Yang 2011). For instance, remittances received in 2009 in the Philippines amounted to 12% of GDP (Worldbank 2011). Gifts in kind and in time are also prevalent (Attias-Donfut, Ogg & Wolff 2005). Identifying precisely why people give to each other is empirically challenging.[2] Still, altruism seems to play a central role: individuals give to others they care about.[3] Moreover and as increasingly recognized by economists, individuals are embedded in different social and familial neighborhoods (Jackson 2008). The collection of altruistic linkages defines a network and *private transfers flow through altruistic networks*.

In this paper, we provide the first analysis of altruism in networks. We assume that agents are connected through a fixed, weighted network and care about the well-being of their network neighbors. Altruistic links may vary in intensity and can represent both strong ties (siblings, spouses, close friends) and weak ties (relatives, colleagues, neighbors). Given some initial distribution of incomes, agents may provide financial support to their poorer friends. Incentives to give are intricately linked to the network structure. Gifts made in one part of the network may depend on gifts received or made in other parts. We characterize the Nash equilibria of this non-cooperative game for general networks and utility functions.[4]

[1]See Lee, Donehower & Miller (2011) and the National Transfer Accounts at http://www.ntaccounts.org/. In the same study, the estimated amount of intrahousehold transfers in 2003 reaches $3,216 bn, or 27.9% of GDP.

[2]A large empirical literature, dating back to Cox (1987), aims at disentangling the motives behind private transfers, see Arrondel & Masson (2006).

[3]The importance of altruism has notably been emphasized in biology (de Waal 2010), psychology (Batson 2011) and philosophy (Held 2005). In evolutionary biology, Hamilton's rule shows that altruism should be induced by blood ties and shared genes (Hamilton 1964a,b), see Bergstrom (1995), Cox & Fafchamps (2008) and Alger & Weibull (2010). Empirical evidence, in economics, on altruism is reviewed below.

[4]Private utility functions may differ between agents. They must be twice continuously differentiable, strictly increasing, strictly concave and satisfy an assumption guaranteeing that an agent never gives to a richer friend, see Section II.

Our analysis highlights the importance of indirect gifts. In equilibrium, an agent may give to a friend because his friend himself has a friend in need. Transfers and consumptions ultimately depend on the whole network structure. We uncover four main features of this interdependence: consumption uniqueness, cost efficiency, income monotonicity, and inequality reduction.

We first show that, despite a potentially large multiplicity in transfers, *all Nash equilibria lead to the same consumption levels*. This extends to weighted networks a result obtained in Arrow (1981) for complete graphs.[5] We also investigate equilibrium multiplicity. We find that cycles in the network are necessary for multiplicity to appear and show that the equilibrium set is, in any case, compact and convex.

Second, our analysis reveals *a principle of economy in transfers* at work in altruistic networks. We show that transfers must flow through paths of highest altruistic strength and that, conditional on consumption levels, equilibrium transfers minimize a weighted sum of transfers. When all links have the same strength, transfers flow through shortest paths of the network and minimize the aggregate transfer needed to reach equilibrium consumptions.

Third, we show that *equilibrium consumptions vary monotonically with incomes*. If an agent suffers a loss in income her consumption decreases strictly and the consumption of every other agent decreases weakly. Moreover, the shock affects socially closer agents first. We also look at income redistributions. We identify cases where a redistribution is neutral and public transfers are undone by private transfers. We show that a redistribution which decreases income inequality can increase consumption inequality. These results may have important implications for the design of public policies in altruistic societies.

Fourth, we show that *altruistic networks decrease inequality*, in the sense of second-order stochastic dominance. We then study how inequality is affected by the network structure. We find that an expansion in the altruistic network can increase consumption variance. In contrast, it always leads to a decrease in a measure of the consumption spread. We also look at homophily with respect to income.[6] We find that networks with higher homophily

[5]We discuss the relationship between Arrow (1981)'s setup and ours in Section III.

[6]Homophily, or the tendency of similar individuals to be connected, is prevalent in social networks

tend to generate more inequality. Moreover, the first few connections across income groups have a disproportionate impact on inequality reduction,

Finally, we obtain further results when preferences satisfy Constant Absolute Risk Aversion (CARA). We uncover the existence of a potential function in that case and provide a characterization of Nash equilibria as solutions of a convex quadratic minimization problem.

Our analysis contributes to three literatures. First, we introduce networks to the economics of altruism. Pioneered by Becker (1974), this literature has become a cornerstone of the economics of the family (Kolm & Mercier Ythier 2006). Most theoretical studies consider pairs of altruistic individuals (Alger & Weibull 2010, Bernheim & Stark 1988, Stark 1995). Arrow (1981) analyzes transfers within groups where agents all care equally for everyone else. However, economists have, so far, failed to recognize that even close family ties give rise to complex networks.[7] We extend Arrow (1981)'s analysis to weighted networks. We find that the network has a first-order impact on outcomes in an altruistic economy.

Second, our analysis contributes to the literature on private transfers. We study the first model able to explain three stylized facts identified in the empirical literature. (1) Risk sharing in rural communities of developing countries tends to be good but imperfect.[8] (2) Private transfers generally flow through social links.[9] (3) An important share of these transfers seems redistributive in nature. For instance in a study of informal insurance in rural Tanzania, De Weerdt & Fafchamps (2011) find that unreciprocated transfers between kin, such as in support to the disabled, play a central role.[10]

To explain imperfect risk sharing, economists have considered mutual insurance ar-

(McPherson, Smith-Lovin & Cook 2001).

[7]Sociologists and social psychologists have started to explore on small samples the network aspects of the family, see Widmer (2006). To our knowledge, large-scale statistical studies of familial networks are still lacking.

[8]Early results of Townsend (1994) have been confirmed, for the most part, by a large follow-up empirical literature. See Mazzocco & Saini (2012) and the references therein.

[9]Detailed studies on gifts and informal loans show the key role played by social networks; people help their family, friends and neighbors in case of need. See De Weerdt & Dercon (2006), Fafchamps & Gubert (2007), Fafchamps & Lund (2003).

[10]Empirical studies finding evidence of altruism include Agarwal & Horowitz (2002), Foster & Rosenzweig (2001), Kazianga (2006), Leider et al. (2009), Ligon & Schechter (2012), Mitrut & Nordblom (2010).

rangements under self-enforcement and informational constraints (Dubois, Jullien & Magnac 2008, Ligon, Thomas & Worrall 2002). Friction in exchange does not explain the role played by social networks, however, or the presence of some significant redistribution. Researchers have, recently, started to study models of informal insurance in networks. The analysis of Ambrus, Mobius & Szeidl (2014), in particular, bears some relation to ours.[11] Authors also consider a fixed, weighted network. They assume that links can be used as social collaterals. Links have monetary values, limiting how much money can flow through them. Authors characterize the Pareto-constrained risk-sharing arrangements. In contrast, no limit is placed on financial flows in our setup. Rather, links describe the structure of preferences and transfers are obtained as Nash equilibria of a non-cooperative game. Interestingly, these two different models generate some common predictions. In both models, a positive shock on one agent affects others positively and first impacts agents who are socially closer. As discussed in Ambrus, Mobius & Szeidl (2014), these two features are consistent with the empirical findings of Angelucci & De Giorgi (2009) and Angelucci, De Giorgi & Rasul (2012). On most dimensions, however, the two models yield different predictions.[12] With social collaterals, small shocks are perfectly insured. In contrast, large shocks saturate the network constraints and the amount of support obtained from the network. The opposite happens in altruistic networks. Small shocks do not elicit network support. As shocks get larger, however, direct and indirect transfers increase. We provide a detailed description of the anatomy of private transfers generated by altruistic networks.

Applied researchers have recently explored yet another motive that could explain private transfers. People may give to others because they conform to social pressure and redistributive social norms (Baland, Guirkinger & Mali 2011). Since Nash equilibria depend on the best-replies, our whole analysis also applies to a society prescribing to act towards relatives and neighbors *as if* truly altruistic.[13] In that interpretation, the network

[11]Other studies analyze network stability in various risk sharing contexts. See Bloch, Genicot & Ray (2008) and Bramoullé & Kranton (2007a,b).

[12]In addition, the two models do not have the same dimensionality. In a network with n agents and L links, Pareto-constrained risk sharing arrangements depend on $L+n-1$ parameters: the link values and the Pareto weights. In contrast, incomes after transfers in altruistic networks only depend on L parameters: the links' altruistic strengths.

[13]Similarly, Alger & Weibull (2008) assume that social norms prescribe to act as if truly altruistic in their study of incentives and family ties.

would describe the structure of the social prescriptions.

Finally, our paper contributes to the literature on games played on fixed networks (Ballester, Calvó-Armengol & Zenou 2006, Galeotti et al. 2010, Bramoullé, Kranton & D'amours 2014). We analyze the first model where agents' strategies are multidimensional and best-replies are non-linear. Our results for CARA utility functions confirm the interest of potential functions to study network games emphasized by Bramoullé, Kranton & D'amours (2014).

The remainder of the paper is organized as follows. We introduce the model and establish existence in Section II. We illustrate the effect of the network in Section III. We analyze key properties of Nash equilibria in Section IV. We study comparative statics with respect to incomes and to the network in Section V and we conclude in Section VI.

II The model

We consider a community of $n \geq 2$ agents. Agents have some initial income and may make private transfers to each other. Formally, agent i has income $y_i^0 \geq 0$ and may give $t_{ij} \geq 0$ to agent j. The collection of transfers defines a n by n matrix $\mathbf{T}$ with non-negative entries. By convention, $t_{ii} = 0$. Income after transfers, or consumption, y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_j t_{ji} \tag{1}$$

where $\sum_j t_{ij}$ represents total gifts made *by* i and $\sum_j t_{ji}$ total gifts made *to* i. Define S as the set of transfer profiles: $S = \{\mathbf{T} \in \mathbb{R}_+^{n^2} : \forall i, t_{ii} = 0\}$. In this setup, aggregate gifts made are equal to aggregate gifts received: $\sum_i y_i = \sum_i y_i^0$. Aggregate income is redistributed through private transfers.

We assume that agents care about each other. Preferences have a private and a social component. Agent i's private, or material, preferences are represented by utility function $u_i : \mathbb{R} \to \mathbb{R}$. We assume that u_i is twice continuously differentiable and satisfies $u_i' > 0$ and $u_i'' < 0$. Agent i's social, or altruistic, preferences are represented by utility function

$v_i : \mathbb{R}^n \to \mathbb{R}$ such that

$$v_i(y_i, \mathbf{y}_{-i}) = u_i(y_i) + \sum_j \alpha_{ij} u_j(y_j) \tag{2}$$

and $\forall i, j, 0 \leq \alpha_{ij} < 1$. When $\alpha_{ij} > 0$, i cares about j's material well-being and the size of the coefficient measures the strength of the altruistic linkage.[14] By convention, $\alpha_{ii} = 0$. The collection of bilateral coefficients α_{ij} defines a *directed and weighted altruistic network* α. We say that this network is *binary* when all links have the same strength and $\alpha_{ij} \in \{0, \alpha\}$. Our analysis applies to heterogenous private utilities and weighted altruistic networks. To gain insight into the effect of the network structure, we will often illustrate our results with common utilities and binary networks.

We make the following joint assumption on private utilities and altruistic coefficients:

$$\forall i, j, \forall y, u_i'(y) > \alpha_{ij} u_j'(y) \tag{3}$$

This provides a counterpart in our context to the assumption of "selfish preferences" in Arrow (1981, p. 203). With common private utilities $\forall i$, $u_i = u$, Assumption (3) simply reduces to: $\forall i, j$, $\alpha_{ij} < 1$. In general, this condition guarantees that an agent never gives to a richer friend. In equilibrium, $t_{ij} > 0 \Rightarrow y_i > y_j$. This implies, in particular, that an agent never gives more than he can afford to: $\forall i, y_i \geq 0$.[15]

The collection of agents, altruistic utilities and transfers define a non-cooperative simultaneous move game. Our main objective is to study the Nash equilibria of this game. Each agent chooses her transfers to maximize her altruistic utility, conditional on transfers received from and given by others. A Nash equilibrium is a matrix of transfers $\mathbf{T} \in S$ such that $\forall i, \forall \mathbf{T}_i' \in \mathbb{R}_+^{n-1}$, $v_i(\mathbf{T}_i, \mathbf{T}_{-i}) \geq v_i(\mathbf{T}_i', \mathbf{T}_{-i})$.

We next present a useful reformulation of equilibrium conditions. Observe that v_i is concave in $\mathbf{T}_i$ for any $\mathbf{T}_{-i}$. This means that the first-order conditions of i's utility

[14] Alternatively, we could assume that agents care about others' social well-being: $v_i = u_i + \sum a_{ij} v_j$. Assume that $\mathbf{I} - \mathbf{A}$ is invertible, where $\mathbf{I}$ is the identity matrix, and let $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$. Then, $\mathbf{v} = \mathbf{Bu}$ and setting $\alpha_{ij} = b_{ij}/b_{ii}$ leads us back to our formulation. Our analysis applies as long as $\forall i \neq j, 0 \leq b_{ij} < b_{ii}$.

[15] We show in Proposition 4 below that the network of transfers is acyclic in equilibrium. If i makes some transfer, it means that there exist j_1,..., j_l such that $t_{ij_1} > 0$, $t_{j_1 j_2} > 0$,..., $t_{j_{l-1} j_l} > 0$ and j_l does not give. Therefore, $y_i > y_{j_1} > ... > y_{j_l} > 0$.

maximization problem are necessary and sufficient. This yields:

Lemma 1 *A matrix of transfers* $\mathbf{T}$ *is a Nash equilibrium of the transfer game iff (1)* $\forall i, j,\ u_i'(y_i) \geq \alpha_{ij} u_j'(y_j)$ *and (2)* $\forall i, j$ *such that* $t_{ij} > 0,\ u_i'(y_i) = \alpha_{ij} u_j'(y_j)$.

In equilibrium an agent cannot be much richer than any of his friends. And a positive transfer must be such that the ratio of the giver's marginal utility on the receiver's is precisely equal to the altruistic coefficient. In particular, agents only give to others they care about: $\alpha_{ij} = 0 \Rightarrow t_{ij} = 0$. Still, as we will see below, agents may end up being affected by friends of friends and by others far away in the network.

To illustrate, consider common private utilities satisfying Constant Absolute Risk Aversion, or CARA: $\forall i, u_i(y) = -e^{-Ay}/A$ with $A > 0$. Lemma 1's conditions become: (1) $\forall i, j,\ y_i \leq y_j - \ln(\alpha_{ij})/A$ and (2) $\forall i, j$ such that $t_{ij} > 0,\ y_i = y_j - \ln(\alpha_{ij})/A$. The difference between two friends' incomes cannot be greater than some threshold value. Alternatively, suppose that preferences satisfy Constant Relative Risk Aversion, or CRRA: $\forall i, u_i(y) = y^{1-\gamma}/(1-\gamma)$ for $\gamma \neq 1$ or $u_i(y) = \ln(y)$ for $\gamma = 1$. The Lemma's conditions are now: (1) $\forall i, j,\ y_i/y_j \leq \alpha_{ij}^{-1/\gamma}$ and (2) $\forall i, j$ such that $t_{ij} > 0,\ y_i/y_j = \alpha_{ij}^{-1/\gamma}$. The ratio of two friends' incomes cannot be greater than some threshold value.

We observe that the transfer game exhibits a complex pattern of strategic interactions and externalities. On strategic interactions, first, note that the optimal t_{ij} is weakly decreasing in t_{kj}. An agent tends to reduce his gift to a friend when this friend receives more gifts from others. In contrast, t_{ij} is weakly increasing in t_{jk}. The agent tends to increase his gift when his friend makes more gifts himself. Therefore, gifts to an agent from different givers are strategic substitutes while gifts to and from an agent are strategic complements. Next, consider externalities. Suppose that i cares about j but does not care about k: $\alpha_{ij} > 0$ and $\alpha_{ik} = 0$. Then, v_i is decreasing in t_{jk} but increasing in t_{kj}. Agent i suffers a loss in utility from j's gifts to others, but benefits from the gifts j receives. Externalities may be positive or negative and the externality pattern is rooted in the network structure.[16]

[16]These externalities imply that Nash equilibria are typically not Pareto-optima. A well-known exception is a situation where one agent is both very rich and very altruistic (Becker 1974, Arrow 1981).

In contrast, this game captures relatively simple rules of behavior. With common private utilities and a binary network, each agent provides financial support to his poorest friends. Depending on his financial ability, he brings them all up to a neighborhood-specific level of minimal consumption. In general, agent i seeks to reduce the maximal level of altruistic marginal utility $\alpha_{ij}u'_j$ in his neighborhood. What happens when everyone is simultaneously behaving in this way, however, is not obvious. Our objective is to analyze the interplay of altruistic behavior on arbitrary networks.

We conclude this section by establishing existence. In this setup, existence follows from results due to Mercier Ythier (1993, 2006). In Appendix, we derive an alternative proof based on the theory of concave games (Rosen 1965).

Proposition 1 *For any altruistic network and any utility functions, a Nash equilibrium of the transfer game exists.*

One difficulty in showing existence is that the set of strategies S is unbounded. To address this issue, we show that transfer networks in equilibrium are acyclic (see Proposition 4 below) and that aggregate transfers in acyclic networks are bounded from above. Once this bound is established, existence follows from classical fixed point arguments.

III Illustrations

In this section, we provide a first analysis of two simple benchmark cases. We look at complete graphs and stars with binary links and common private utilities. This allows us to discuss the results of Arrow (1981) and to illustrate the impact of the network on equilibrium behavior.

Consider complete graphs first. Everyone is equally altruistic towards everyone else: $\forall i \neq j$, $\alpha_{ij} = \alpha$. This case is covered by Arrow (1981)'s analysis.[17] Our discussion follows his and we refer to his paper for detailed explanations. On the complete graph, equilibrium conditions can be expressed through a unique endogenous variable: minimal

[17] Arrow (1981) considers utility functions of the form $v_i(\mathbf{y}) = u_i(y_i) + \sum_{j\neq i} w(y_j)$. The two setups coincide when $\forall i, u_i = u$ and $w = \alpha u$. Our analysis also applies to complete networks with heterogeneous private utilities: $v_i(\mathbf{y}) = u_i(y_i) + \alpha \sum_{j\neq i} u_j(y_j)$, which are not studied in Arrow (1981).

consumption $y_{\min}$. To see why, denote by $\hat{y}_{\max}$ the consumption level such that $u'(\hat{y}_{\max}) = \alpha u'(y_{\min})$. Condition (1) of Lemma 1 implies that $\forall i, u'(y_i) \geq \alpha u'(y_{\min})$ and hence $y_i \leq \hat{y}_{\max}$. Condition (2) means that if $t_{ij} > 0$, then $u'(y_i) = \alpha u'(y_j) \leq \alpha u'(y_{\min}) = u'(\hat{y}_{\max})$ and hence $y_i = \hat{y}_{\max}$ and $y_j = y_{\min}$. There are two cases. If $\forall i, j, u'(y_i^0) \geq \alpha u'(y_j^0)$, there is no transfer in equilibrium. If, on the other hand, $u'(y_i^0) < \alpha u'(y_j^0)$ for some pair i, j, then there is some transfer and society is endogenously partitioned into three groups: the poor, the rich and a middle class. If $y_i^0 < y_{\min}$, then $y_i = y_{\min}$. If $y_i^0 > \hat{y}_{\max}$, then $y_i = \hat{y}_{\max}$. Otherwise, $y_i = y_i^0$. Minimal consumption $y_{\min}$ solves the following accounting equation

$$\sum_{i: y_i^0 > \hat{y}_{\max}} (y_i^0 - \hat{y}_{\max}) = \sum_{i: y_i^0 < y_{\min}} (y_{\min} - y_i^0) \tag{4}$$

This equation says that aggregate gifts made by the rich are equal to aggregate gifts received by the poor. Since the left hand side is decreasing in $y_{\min}$ while the right hand side is increasing in $y_{\min}$, this equation has a unique solution. Therefore, incomes after transfers, y_i, are uniquely determined.

Equilibrium behavior can be described as a system of minimal consumption for the poor, paid for by the rich. Agents act *as if* any income above $\hat{y}_{\max}$ is contributed to a fund which serves to bring the income of the poorest up to $y_{\min}$. Note that an agent never gives and receives at the same time. Also, rankings in the distribution of incomes cannot be reversed: $y_i^0 \geq y_j^0 \Rightarrow y_i \geq y_j$. We provide a numerical illustration in Figure 1. Four agents have common CARA private utility $u_i(y) = -e^{-Ay}/A$ and such that $-\ln(\alpha)/A = 1$. The upper panel depicts initial incomes. The lower panel depicts two equilibria; there are many others. In any equilibrium, a rich agent consumes 4 while a poor one consumes 3.

Consider stars next. Agent 1, the center, cares about all peripheral agents who only care about the center. Formally, $\alpha_{1i} = \alpha_{i1} = \alpha$, $\forall i \neq 1$ and $\alpha_{ij} = 0$ if $i, j \neq 1$. Equilibrium conditions on stars can also be expressed through a unique parameter, the center's consumption y_1. Denote by $\hat{y}_R$ and $\hat{y}_P$ the income levels such that $u'(\hat{y}_R) = \alpha u'(y_1)$ and $u'(y_1) = \alpha u'(\hat{y}_P)$. Consider a peripheral agent $i \neq 1$ and rewrite the equilibrium conditions of Lemma 1. There are three cases. If $\hat{y}_P \leq y_i^0 \leq \hat{y}_R$, then $t_{i1} = t_{1i} = 0$ and $y_i = y_i^0$. If

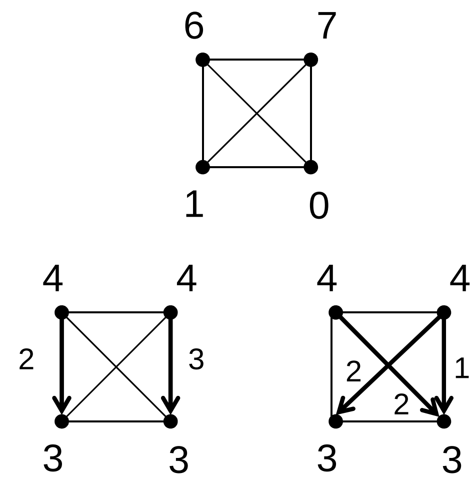

Figure 1: Nash equilibria on a complete graph

$y_i^0 > \hat{y}_R$, then $t_{i1} > 0$ and $y_i = \hat{y}_R$. If $y_i^0 < \hat{y}_P$, then $t_{1i} > 0$ and $y_i = \hat{y}_P$. The center's consumption y_1 solves the following equation

$$y_1 = y_1^0 + \sum_{i:y_i^0 > \hat{y}_R} (y_i^0 - \hat{y}_R) - \sum_{i:y_i^0 < \hat{y}_P} (\hat{y}_P - y_i^0) \qquad (5)$$

This is a reformulation of accounting equality (1) for the center. It says that the center's consumption is equal to his income plus what he receives minus what he gives. Observe that the left hand side is now increasing in y_1 while the right hand side is decreasing in y_1. Therefore, this equation also has a unique solution. This means that on stars, there is a unique equilibrium in transfers.

This characterization allows us to illustrate key properties of altruism in networks. Consider the example depicted in Figure 2 with common CARA private utilities and $-\ln(\alpha)/A = 1$. Initial incomes are represented in the upper panel. Center is richest with an income of 8. Left is the second richest with an income of 7. If society were composed of these two agents only, there would not be any transfer. However, Right is poor with an income of 0. The lower panel depicts equilibrium consumptions and transfers. Left gives 1 to Center, who gives 4 to Right and consumptions are 6, 5 and 4. Here, Center gives and receives at the same time. The transfer from Left to Center is, in a way, indirect.

Left gives to Center because his friend has a friend in need. In addition, Center is richest ex-ante but not ex-post.

Thus, many features uncovered by Arrow (1981) do not extend to altruistic networks. In some structures at least, the equilibrium in transfers is unique. Agents may both give and receive in equilibrium. Income rankings can be reversed and the individual position in the network may be an important determinant of consumption. In the remainder of the paper, our objective is to characterize equilibrium behavior in general networks.

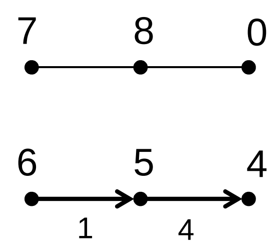

Figure 2: Transfers in an altruistic network

IV Nash equilibria

In this section, we analyze key properties of Nash equilibria, holding the network and incomes constant. We first investigate the uniqueness in transfers and consumptions. We then look at the relationship between the altruistic network and the network of transfers. We finally describe a useful reformulation of equilibrium conditions for CARA utilities.

A Uniqueness

We now establish our first main result: the uniqueness of incomes after transfers.

Theorem 1 *For any altruistic network and any utility functions, all Nash equilibria lead to the same incomes after transfers.*

Proof: Consider two equilibria $\mathbf{T}$ and $\mathbf{T}'$ with income after transfers $\mathbf{y}$ and $\mathbf{y}'$. Assume that $\mathbf{y} \neq \mathbf{y}'$. Without loss of generality, assume that $y'_i > y_i$ for some i. Define $U = \{i : y'_i > y_i\}$. Our proof unfolds in two steps.

We show, first, that if $i \in U$ and $t_{ij} > 0$ or $t'_{ji} > 0$ then $j \in U$. Suppose that $y'_i > y_i$ and $t_{ij} > 0$. By Lemma 1, $u'_i(y_i) = \alpha_{ij} u'_j(y_j)$ and $u'_i(y'_i) \geq \alpha_{ij} u'_j(y'_j)$. Since u'_i is strictly decreasing, $u'_i(y_i) > u'_i(y'_i)$ and hence $\alpha_{ij} u'_j(y_j) > \alpha_{ij} u'_j(y'_j)$, which means that $y'_j > y_j$. Suppose next that $t'_{ji} > 0$. Then, $u'_j(y'_j) = \alpha_{ji} u'_i(y'_i)$ and $u'_j(y_j) \geq \alpha_{ji} u'_i(y_i)$. Since $\alpha_{ji} > 0$, $\alpha_{ji} u'_i(y_i) > \alpha_{ji} u'_i(y'_i)$ and hence $u'_j(y_j) > u'_j(y'_j)$, and $y'_j > \hat{y}_j$.

Next, given two sets U and V, define $t_{U,V} = \sum_{i \in U, j \in V} t_{ij}$ as aggregate transfers from agents in U to agents in V. Let N denote the set of all agents. Observe that $\sum_{i \in U} y_i = \sum_{i \in U}(y_i^0 - t_{i,U} - t_{i,N-U} + t_{U,i} + t_{N-U,i})$. Since $\sum_{i \in U}(t_{U,i} - t_{i,U}) = 0$, this yields

$$\sum_{i \in U} y_i = \sum_{i \in U} y_i^0 - t_{U,N-U} + t_{N-U,U}$$

This accounting equation says that aggregate consumption within is equal to aggregate income within minus gifts given outside plus gifts received within.

Finally, observe that statements in our first step mean that $t_{U,N-U} = 0$ while $t'_{N-U,U} = 0$. This implies that $\sum_{i \in U} y_i = \sum_{i \in U} y_i^0 + t_{N-U,U} \geq \sum_{i \in U} y_i^0$ while $\sum_{i \in U} y'_i = \sum_{i \in U} y_i^0 - t'_{U,N-U} \leq \sum_{i \in U} y_i^0$. Therefore, $\sum_{i \in U} y'_i \leq \sum_{i \in U} y_i$. But $i \in U \Leftrightarrow y'_i > y_i$ which means that $\sum_{i \in U} y'_i > \sum_{i \in U} y_i$, which establishes a contradiction. $\square$

To show this result, we assume that two different consumption profiles exist. We combine Lemma 1 with elementary flow techniques to generate a contradiction. Theorem 1 extends Arrow (1981)'s result for binary complete graphs to arbitrary weighted networks. Our proof is fundamentally distinct, however, from Arrow's reasoning. While he characterizes the equilibria explicitly, we prove uniqueness by contradiction. We note that his reasoning does not extend to networks because equilibrium conditions can generally not be expressed through a single parameter.[18] We also note that classical properties, such as contraction or Rosen (1965)'s conditions, do not apply here due to equilibrium multiplicity. We provide a constructive argument for CARA private utilities in Section IV.C below. Whether we can find a constructive proof valid for any network and any utility functions remains an open question.

[18] Stars and complete graphs seem to be the only two structures for which this property holds.

We next turn to uniqueness in transfers. Multiplicity on the complete graph is noted by Arrow (1981, p. 221): "The actual flows, who gives to whom, are not unique (...), but this nonuniqueness is not very interesting". However, the stars' example shows that actual flows may be unique on incomplete networks. In addition, multiplicity does not mean that anything goes and understanding the structure of the equilibrium set is of some interest.

We show, first, that uniqueness holds on *trees*. An altruistic network is a tree if the undirected graph where i and j are linked when $\alpha_{ij} > 0$ or $\alpha_{ji} > 0$ has no cycles.

Proposition 2 *The transfer game has a unique Nash equilibrium if the altruistic network is a tree.*

Proof: Proceed by induction on the number of agents. Define H_n as the proposition's statement when the number of agents is lower than or equal to n. Direct computations show that H_2 is true. Suppose that H_{n-1} is true and consider a tree with n nodes. There are two cases. Either the tree has an isolated node, and H_n is true. Or it has a node who has a unique neighbor. In this case, let i have j as his only neighbor. Let $\mathbf{T}$ be an equilibrium and $\mathbf{T}_{-i}$ the submatrix obtained by removing the i^{th} row and column. By Theorem 1, equilibrium consumptions $\mathbf{y}$ is unique. By Lemma 1, there are three cases. If $y_i = y_j$, then $t_{ij} = t_{ji} = 0$. If $y_i > y_j$, then $t_{ji} = 0$ and $t_{ij} = y_i^0 - y_i \geq 0$. If $y_i < y_j$, then $t_{ij} = 0$ and $t_{ji} = y_i - y_i^0 \geq 0$. In all cases, t_{ij} and t_{ji} are uniquely determined. Then, note that $\mathbf{T}_{-i}$ is an equilibrium for incomes $\hat{\mathbf{y}}^0$ and network α_{-i} where $\hat{y}_j^0 = y_j^0 - t_{ji} + t_{ij}$ and $\hat{y}_k^0 = y_k^0, \forall k \neq j$. By H_{n-1}, $\mathbf{T}_{-i}$ is uniquely determined and hence H_n is true. $\square$

To prove this result, we combine Theorem 1 with the fact that connected trees always have a node with a unique neighbor. Proposition 2 means that multiplicity can only emerge in the presence of cycles in the altruistic network. And indeed, in a cycle money can flow in two different ways from one agent to another. Our next result characterizes the mathematical structure of the equilibrium set for general networks.

Proposition 3 *For any altruistic network and any utility functions, the set of Nash equilibria is compact and convex .*

Proof: Consider a sequence of equilibria $\mathbf{T}_n$ converging towards $\mathbf{T}$. By condition (1) of Lemma 1, $\forall i, j$, $u_i'(y_i^n) \geq \alpha_{ij} u_j'(y_j^n)$. Taking the limit and by continuity of u', $u_i'(y_i) \geq \alpha_{ij} u_j'(y_j)$. Next consider i, j such that $t_{ij} > 0$. There exists N such that $n > N \Rightarrow t_{ij}^n > 0$. By condition (2), $u_i'(y_i^n) = \alpha_{ij} u_j'(y_j^n)$ and hence $u_i'(y_i) = \alpha_{ij} u_j'(y_j)$. Thus, $\mathbf{T}$ is an equilibrium and the set of Nash equilibria is closed. By the proof of Proposition 1, it is also bounded and hence compact.

Consider $\mathbf{T}$ and $\mathbf{T}'$ two equilibria with consumptions $\mathbf{y}$ and $\mathbf{y}'$, and $\lambda \in [0, 1]$. Let $\mathbf{T}_\lambda = \lambda \mathbf{T} + (1 - \lambda)\mathbf{T}'$, which yields consumption $\mathbf{y}_\lambda$. Note that $\mathbf{y} = \mathbf{y}^0 - \mathbf{T1} + \mathbf{T}^T\mathbf{1}$. Therefore, $\mathbf{y}_\lambda = \mathbf{y}^0 - (\lambda\mathbf{T} + (\mathbf{1} - \lambda)\mathbf{T}')\mathbf{1} + (\lambda\mathbf{T} + (1 - \lambda)\mathbf{T}')^T\mathbf{1} = \lambda\mathbf{y} + (1 - \lambda)\mathbf{y}'$. By Theorem 1, $\mathbf{y} = \mathbf{y}'$ and hence $\mathbf{y}_\lambda = \mathbf{y}$. This means that condition (1) of Lemma 1 is satisfied. As for condition (2), suppose that $\lambda t_{ij} + (1 - \lambda)t_{ij}' > 0$. This implies that $t_{ij} > 0$ or $t_{ij}' > 0$. In either case, it means that $u_i'(y_i) = \alpha_{ij} u_j'(y_j)$ and condition (2) is satisfied. □

An implication of Proposition 3 is that when uniqueness fails to hold, there is a continuum of equilibria.

B The network of transfers

In this section, we look at the pattern of transfers. Observe that any Nash equilibrium $\mathbf{T}$ defines a weighted, directed network where i is linked to j if $t_{ij} > 0$. We already mentioned that agents only give to others they care about. Our first result uncovers a tight geometric relationship between the altruistic network and the network of transfers.

Consider a chain C of agents of length l: i_1,i_2,...,i_l. Define the *altruistic strength* α_C of chain C as the product of the bilateral altruistic coefficients: $\alpha_C = \alpha_{i_1 i_2}\alpha_{i_2 i_3}...\alpha_{i_{l-1} i_l}$. This strength is positive if and only if the chain is a path of the altruistic network and any agent in the chain cares about his successor.

Proposition 4 *There are no cycles in transfers and transfers flow through paths of highest altruistic strength.*

Proof: Suppose that i and j are connected through a path P in the transfer network: $t_{ii_2} > 0$, $t_{i_2 i_3} > 0$, ..., $t_{i_{l-1} j} > 0$. We have: $y_i > y_{i_2}$, ...,$y_{i_{l-1}} > y_j$ and hence $y_i > y_j$ and there is no cycle. Condition (2) of Lemma 1 means that $u_i'(y_i) = \alpha_{ii_2}u_{i_2}'(y_{i_2})$, $u_{i_2}'(y_{i_2}) = \alpha_{i_2i_3}u_{i_3}'(y_{i_3})$, ..., $u_{i_{l-1}}'(y_{i_{l-1}}) = \alpha_{i_{l-1}j}u_j'(y_j)$. Successive substitutions yield $u_i'(y_i) = \alpha_{ii_2}\alpha_{i_2i_3}...\alpha_{i_{l-1}j}u_j'(y_j)$ and hence $u_i'(y_i)/u_j'(y_j) = \alpha_P$. Next, consider an arbitrary chain C connecting i and j: $i_1 = i$, i_2, ..., i_{l-1}, $i_l = j$. Condition (1) of Lemma 1 means that $u_i'(y_i) \geq \alpha_{ii_2}u_{i_2}'(y_{i_2})$, ..., $u_{i_{l-1}}'(y_{i_{l-1}}) \geq \alpha_{i_{l-1}j}u_j'(y_j)$. Successive substitutions yield: $u_i'(y_i) \geq \alpha_{ii_2}\alpha_{i_2i_3}...\alpha_{i_{l-1}j}u_j'(y_j)$ and hence $\alpha_P \geq \alpha_C$. Therefore, P is a chain of highest altruistic strength. □

To prove Proposition 4, we simply put together the bilateral conditions of Lemma 1 for all links in a path or a chain. This result shows that the underlying altruistic network imposes strong restrictions on the network of transfers. While many paths may connect two agents, very few of these paths have highest strength. For instance if all links have strength α, then $\alpha_P = \alpha^l$ for all path P of length l in the altruistic network. In this case, the altruistic strength is stronger when the length of the path is shorter. Proposition 4 yields:

Corollary 1 *When all links have the same strength, transfers flow through shortest paths of the altruistic network.*

Money flows in this context share some characteristics of water flows across a rugged landscape (Viessman & Lewis 2002). Water always follows paths of least resistance. Similarly, gifts always follow paths of highest altruism. Chains of transfers never take roundabout ways to connect two agents.

Proposition 4 reveals a geometric relationship between the altruistic network and the network of transfers. We next obtain a related quantitative result. Suppose momentarily that equilibrium consumptions $\mathbf{y}$ are known. Observe that any transfer profile $\mathbf{T}$ such that $\mathbf{T1} - \mathbf{T}^t\mathbf{1} = \mathbf{y}^0 - \mathbf{y}$ leads to consumptions $\mathbf{y}$ from incomes $\mathbf{y}^0$. This is a large set, which contains many profiles that are not Nash equilibria. The following result clarifies the properties satisfied by equilibrium transfers within this set.

Proposition 5 *Take consumptions* $\mathbf{y}$ *as given. Then, equilibrium transfers* $\mathbf{T}$ *minimize* $\sum_{i,j:\alpha_{ij}>0} -\ln(\alpha_{ij})t_{ij}$ *subject to:* $\forall i, j, t_{ij} \geq 0$, $\forall i, j$*:* $\alpha_{ij} = 0$, $t_{ij} = 0$ *and* $\forall i, \sum_j t_{ij} - \sum_j t_{ji} = y_i^0 - y_i$.

Proof: The objective function and the constrained set are convex, therefore first-order conditions are necessary and sufficient. The lagrangian of this problem is $L = \sum_{i,j:\alpha_{ij}>0} -\ln(\alpha_{ij})t_{ij} + \sum_i \lambda_i(\sum_j t_{ij} - \sum_j t_{ji} - y_i^0 + y_i)$ where λ_i is the lagrange multiplier associated with i's accounting equality. Kuhn-Tucker first-order conditions are: $\forall i,j$: $\alpha_{ij} > 0, \lambda_i - \lambda_j \geq \ln(\alpha_{ij})$ and $t_{ij} > 0 \Rightarrow \lambda_i - \lambda_j = \ln(\alpha_{ij})$. Next, take the logarithm in the conditions of Lemma 1. A transfer profile $\mathbf{T}$ is a Nash equilibrium iff $\forall i,j$: $\alpha_{ij} = 0$, $t_{ij} = 0$ and $\forall i,j$: $\alpha_{ij} > 0$, (1) $\ln(u_i'(y_i)) - \ln(u_j'(y_j)) \geq \ln(\alpha_{ij})$ and (2) $t_{ij} > 0 \Rightarrow \ln(u_i'(y_i)) - \ln(u_j'(y_j)) = \ln(\alpha_{ij})$. Setting $\lambda_i = \ln(u_i'(y_i))$ yields the correspondence. □

To prove this result, we show that equilibrium conditions coincide with the first-order conditions of the minimization problem for judicious choices of the lagrange multipliers. Thus, among all the transfer profiles leading to equilibrium consumptions, equilibrium transfers must minimize a weighted sum of transfers.[19] Stronger links have lower weights in the sum. When all links have the same strength, equilibrium transfers simply minimize the aggregate transfer needed to reach equilibrium consumptions under the constraint that money flows through links.

Taken together, Propositions 4 and 5 illustrate a principle of *economy in transfers* at work in altruistic networks. Even though agents maximize their utility functions independently, the interplay of their decentralized altruistic decisions eliminates waste in transfers.

C Constant Absolute Risk Aversion

In this section, we obtain further results for CARA private utilities. We show that the transfer game in this case has a potential. This allows us to characterize Nash equilibria as the solutions of a concave quadratic maximization problem. We then derive some implications of this reformulation.

Consider CARA utilities, $u_i(y) = -e^{-A_i y}/A_i$, $\forall y$ for some $A_i > 0$. Assume that agents can only give to others they care about.[20] Lemma 1 becomes: (1) $\forall i, j$:$\alpha_{ij} > 0$, $A_i y_i \leq$

[19]A potentially useful implication of Proposition 5 is that if $\mathbf{T}$ and $\mathbf{T}'$ are two equilibria, then $\sum_{i,j:\alpha_{ij}>0} \ln(\alpha_{ij})t_{ij} = \sum_{i,j:\alpha_{ij}>0} \ln(\alpha_{ij})t'_{ij}$. The weighted sum of transfers is the same for all Nash equilibria.

[20]Since in equilibrium agents only give to others they care about, this restriction is without loss of

$A_jy_j - \ln(\alpha_{ij})$ and (2) $\forall i, j{:}\alpha_{ij} > 0$ and $t_{ij} > 0$, $A_iy_i = A_jy_j - \ln(\alpha_{ij})$. Following Monderer & Shapley (1996) and Voorneveld (2000), a function φ defined over transfer profiles is a *best-response potential* of the transfer game if $\arg\max_{\mathbf{T}_i} v_i(\mathbf{T}_i, \mathbf{T}_{-i}) = \arg\max_{\mathbf{T}_i} \varphi(\mathbf{T}_i, \mathbf{T}_{-i})$, $\forall i, \mathbf{T}_{-i}$.

Lemma 2 *Suppose that* $u_i(y) = -e^{-A_iy}/A_i$, $\forall i, \forall y$ *with* $A_i > 0$. *The function* $\varphi(\mathbf{T}) = \sum_{i,j:\alpha_{ij}>0} \ln(\alpha_{ij})t_{ij} - \frac{1}{2}\sum_i A_iy_i^2$ *is a concave best-response potential of the transfer game.*

Proof: Since $\mathbf{y} \mapsto \sum_i A_iy_i^2$ is convex in $\mathbf{y}$ and $\mathbf{y}$ is linear in $\mathbf{T}$, φ is concave in $\mathbf{T}$. Consider i, j such that $\alpha_{ij} > 0$. First, compute $\partial\varphi/\partial t_{ij}$. Note that $\partial y_i/\partial t_{ij} = -1$, $\partial y_j/\partial t_{ij} = +1$ and $\partial y_k/\partial t_{ij} = 0$, for $k \neq i, j$. Therefore, $\partial\varphi/\partial t_{ij} = A_iy_i - A_jy_j + \ln(\alpha_{ij})$. Next, we have: $\partial v_i/\partial t_{ij} = -e^{-A_iy_i} + \alpha_{ij}e^{-A_jy_j} = e^{-A_iy_i}[e^{A_iy_i - A_jy_j + \ln(\alpha_{ij})} - 1]$. This shows that $\partial v_i/\partial t_{ij} > 0 \Leftrightarrow \partial\varphi/\partial t_{ij} > 0$ and $\partial v_i/\partial t_{ij} = 0 \Leftrightarrow \partial\varphi/\partial t_{ij} = 0$. The problems of maximizing v_i and φ over $\mathbf{T}_i$ have the same necessary and sufficient first-order conditions. $\square$

Since the potential is concave, Lemma 2 yields the following reformulation of equilibrium conditions.

Proposition 6 *Suppose that* $u_i(y) = -e^{-A_iy}/A_i$, $\forall i, \forall y$ *with* $A_i > 0$. *A matrix* $\mathbf{T}$ *is a Nash equilibrium of the transfer game if and only if it maximizes* $\varphi(\mathbf{T})$ *subject to:* $\forall i, j, t_{ij} \geq 0$ *and* $\forall i, j{:}\ \alpha_{ij} = 0,\ t_{ij} = 0$.

Under CARA, agents act *as if* they are all trying to maximize the potential function under the constraint that money flows through links. Consider a binary network and common utilities: $\forall i, A_i = A$. Denote by $Var(\mathbf{x})$ the variance of profile $\mathbf{x}$ and observe that $\sum_i A_iy_i^2 = An[Var(\mathbf{y}) + (\bar{\mathbf{y}}^0)^2]$. Equilibrium behavior can be viewed as trading off a reduction in variance against a decrease in aggregate transfer. At one extreme, consumption variance is minimized by the equal distribution: $y_i = \bar{y}^0$, $\forall i$. However, reaching this distribution on an arbitrary network typically requires a lot of indirect transfers. At the other extreme, aggregate transfer is minimized when there is no transfer and incomes are unchanged. Overall, the distribution of equilibrium consumptions lies inbetween the initial and the equal income distributions.

generality. It is needed, however, for the application of potential techniques in our results below.

The existence of a best-response potential has useful implications. For instance, we can derive from Proposition 6 alternative proofs of Proposition 3 and Theorem 1. Convexity of the equilibrium set follows from the fact that the set of maximizers of a concave function over a convex set is convex. Then, convexity of the equilibrium set can be shown to imply uniqueness of incomes after transfers. In Section V.B below, we derive further comparative statics results

V Comparative statics

In this section, we study comparative statics with respect to incomes and to the altruistic network.

A Changes in incomes

We first look at the impact of changes in incomes. We consider two kinds of changes: increases and redistributions. We find, first, that an agent's consumption is strictly increasing in her income and weakly increasing in the income of every other agent. We illustrate how an income shock propagates throughout the network. We then study income redistributions. We identify specific situations under which redistributions are neutral and have no impact on consumptions. In general, however, neutrality does not hold and we show that redistributions that decrease income inequality can increase consumption inequality.

We first show that consumption in altruistic networks varies monotonically with incomes.

Theorem 2 *For any utility functions and any altruistic network, y_i is strictly increasing in y_i^0 and weakly increasing in y_j^0, $\forall j \neq i$.*

Proof: Consider two income distributions $\mathbf{y^0}$ and $\mathbf{y^{0\prime}}$ such that $y_i^0 > y_i^{0\prime}$ and $y_j^0 = y_j^{0\prime}$ $\forall j \neq i$. Let $\mathbf{T}$ and $\mathbf{T}'$ be the corresponding equilibria and $\mathbf{y}$ and $\mathbf{y}'$ the corresponding consumptions. Define $U = \{j : y_j' > y_j\}$ and suppose that $U \neq \emptyset$. We adapt arguments from the proof of Theorem 1. If $j \in U$ and $t_{jk} > 0$ or $t_{kj}' > 0$ then $k \in U$. This implies that $\sum_{j \in U} y_i = \sum_{j \in U} y_i^0 + t_{N-U,U}$ and $\sum_{j \in U} y_i' = \sum_{j \in U} y_i^{0\prime} - t'_{U,N-U}$. Therefore, $\sum_{j \in U} y_j \geq \sum_{j \in U} y_j^0 \geq \sum_{j \in U} y_j^{0\prime} \geq \sum_{j \in U} y_j'$, which establishes a contradiction. This implies that $U = \emptyset$, and hence $\forall j \in N$, $y_j \geq y_j'$. Next, define $V = \{j : y_j' \geq y_j\}$. Suppose that $i \in V$. By definition of $\mathbf{y^0}$ and $\mathbf{y^{0\prime}}$, $\sum_{j \in V} y_j^{0\prime} < \sum_{j \in V} y_j^0$. And through a similar reasoning, we can show that $\sum_{j \in V} y_j \geq \sum_{j \in V} y_j^0 \geq \sum_{j \in V} y_j^{0\prime} \geq \sum_{j \in V} y_j'$ which establishes a contradiction. Therefore, $i \notin V$ and $y_i' < y_i$. $\square$

As it turns out, we can adapt the arguments developed to prove Theorem 1 to establish monotonicity. Note that $\mathbf{y}$ is continuous but, in general, not differentiable in $\mathbf{y}^0$. More precisely, $\mathbf{y}$ is a piecewise differentiable function of $\mathbf{y}^0$ with a potentially large number of pieces. Thus, explicit expressions giving $\mathbf{y}$ as a function $\mathbf{y}^0$ become quickly unwieldy as the number of agents grows. We sidestep this difficulty by reasoning by contradiction.

Theorem 2 says that when an agent suffers a negative income shock, her consumption drops and the consumption of every other agent in the altruistic network either decreases or is unaffected. Who ends up being affected? To gain some insight in how a shock propagates throughout the network, consider the following simple example. Agents have a common private utility; links have the same strength; all agents have initial income y^0 except for agent i for whom $y_i^0 = y^0 - L$. Consumptions in equilibrium are characterized by the following properties. There exist threshold levels l_k, $k = 1, 2, ...$ If $L < l_1$, there is no transfer and incomes are unchanged. If $l_1 < L < l_2$, i is supported by his direct friends, who end up with consumption y_j such that $u'(y_j) = \alpha u'(y_i)$, and agents at distance 2 or more from i do not give or receive any money. If $l_2 < L < l_3$, i is supported by his direct friends who are themselves supported by agents at distance 2 and agents at distance 3 or more do not give any money. If j is a friend of i and k is at distance 2, then $u'(y_k) = \alpha u'(y_j) = \alpha^2 u'(y_i)$. And so on. Consumption is weakly decreasing in distance from i and all agents at the same distance end up with the same consumption level.[21] Thus, a shock on one agent first affects other agents who are socially closer.

We next consider income redistributions. We compare $\mathbf{y^0}$ and $\mathbf{y^{0\prime}}$ such that $\sum_i y_i^0 = \sum_i y_i^{0\prime}$. Following a large literature on the neutrality of public policies, we first identify

[21] Moreover, consumption and threshold levels depend on the numbers of agents at finite distances from i, but do not depend on the link patterns over and above this number distribution.

special circumstances under which the redistribution leaves consumptions unaffected. Say that a matrix of transfers $\mathbf{S} \in S$ implements the redistribution $\mathbf{y^{0\prime}}$ if $\mathbf{y^{0\prime}} = \mathbf{y^0} - \mathbf{S1} + \mathbf{S}^T\mathbf{1}$. In that case, $\mathbf{y^{0\prime}}$ can be obtained from $\mathbf{y^0}$ by applying the bilateral transfers $\mathbf{S}$.

Proposition 7 *Suppose that there is an equilibrium* $\mathbf{T}$ *and transfers* $\mathbf{S}$ *implementing* $\mathbf{y^{0\prime}}$ *such that* $t_{ij} = 0 \Rightarrow s_{ij} = 0$ *and* $t_{ij} > 0 \Rightarrow s_{ij} < t_{ij}$. *Then* $\mathbf{y} = \mathbf{y}'$.

Proof: Introduce $\mathbf{T}' = \mathbf{T} - \mathbf{S}$ and show that $\mathbf{T}'$ is an equilibrium for incomes $\mathbf{y}^{0\prime}$. Note that $\mathbf{T}' \in S$. In addition, $\mathbf{T}'$ yields consumptions $\mathbf{z}$ such that $\mathbf{z} = \mathbf{y^{0\prime}} - \mathbf{T}'\mathbf{1} + (\mathbf{T}')^T\mathbf{1} = \mathbf{y}^0 - \mathbf{T1} + \mathbf{T}^T\mathbf{1} = \mathbf{y}$. Since $\mathbf{T}$ is an equilibrium, $\forall i, j$, $u_i'(y_i) \geq \alpha_{ij}u_j'(y_j)$ and the first condition of Lemma 1 is satisfied for $\mathbf{S}$. Next, $t_{ij}' > 0 \Rightarrow t_{ij} > s_{ij} \geq 0$. Then, $u_i'(y_i) = \alpha_{ij}u_j'(y_j)$ and the second condition is also satisfied. By Theorem 1, $\mathbf{z} = \mathbf{y} = \mathbf{y}'$. □

Thus, redistributions obtained from truncated equilibrium transfers are neutral. We illustrate this result in Figure 3. Three agents are placed on a binary line and have CARA private utilities, $u_i(y) = -e^{-Ay}/A$ with $-\ln(\alpha)/A = 2$. The upper panel depicts the initial incomes of 4, 10 and 0. In equilibrium, Center transfers 4 to Right. In the lower left panel, two units of income are redistributed, initially, from Center to Right. Proposition 7 applies and hence consumptions are unchanged. Here, there is perfect crowding out between public and private transfers. Center reduces his transfers to Right by precisely the amount that was initially redistributed. A redistribution that decreases income inequality leaves consumptions unchanged.

We emphasize that the conditions of application of Proposition 7 are quite specific, however. They will typically not hold for arbitrary redistributions on complex networks. In particular, we show next that a redistribution that decreases income inequality can increase consumption inequality. Consider the lower right panel in Figure 3. Two units of income are redistributed from Center to Left. In the new equilibrium, Center gives 3 to Right and consumptions are 6, 5 and 3. We see that consumption of the poorest agent is lower. Moreover, consumptions after the redistribution are more unequal than the original consumptions in the sense of second order stochastic dominance. Here, Center plays an important role of private support in his neighborhood. Public policies that decrease his resources may lead to a deterioration of his neighbors' situations.

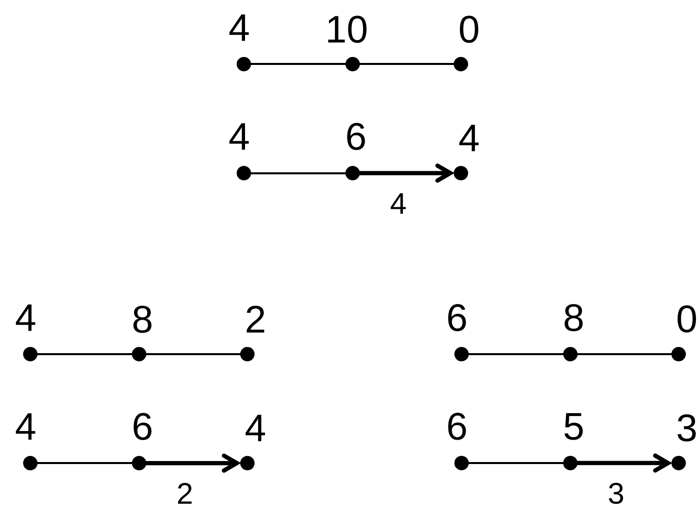

Figure 3: Redistributions that decrease income inequality can be neutral (on the Left) or can increase consumption inequality (on the Right)

B Changes in the altruistic network

In this section, we study the impact of the altruistic network. We first show that consumption inequality is lower than income inequality on any network. We then look at expansions of the altruistic network. We find that more altruism can increase consumption inequality. This effect does not hold for all measures of inequality, however. While consumption variance can increase, a measure of the maximal consumption spread always decreases. In addition with CARA private utilities a linear combination of variance and transfers decreases weakly. Finally, we look at the relationship between income homophily and consumption inequality.

Let us establish, first, that altruistic networks indeed reduce inequality.

Theorem 3 *On any altruistic network and for any utility functions, the distribution of incomes after transfers in equilibrium second-order stochastically dominates the distribution of incomes before transfers.*

Proof: We show that the distribution of incomes after transfers can be obtained from the initial distribution through a series of Pigou-Dalton transfers from richer to poorer agents. Consider an equilibrium $\mathbf{T}$. Order transfers t_{ij} as follows. By Proposition 4, the transfer network is acyclic. Thus, there is an agent i who does not receive. From the initial distribution, apply i's transfers first, in any order. Then remove i and repeat. Pick a second agent who does not receive from others in the remaining network. Apply this second agent's transfers, in any order. Repeat til no more agent is left. This procedure leads to an ordering of all pairwise transfers. Therefore, final incomes are indeed equal to equilibrium incomes. This ordering also guarantees that a transfer always takes place from a richer to a poorer agent. $\square$

Theorem 3 shows that consumption inequality is lower on any network than on the empty network. This raises the more general question of how the shape of the altruistic network affects society's move towards equality.

We first consider expansions in the altruistic network. We ask whether Theorem 3 extends. If new links are added to an altruistic network, or if existing links become stronger, does this necessarily reduce inequality? The answer turns out to be negative. Consider the numerical example presented in Figure 4 with CRRA private utilities, $\forall i, u_i(y) = \ln(y)$, and links with strength $\alpha = 0.5$. The Nash conditions of Lemma 1 become: $\alpha_{ij} > 0 \Rightarrow y_i \leq 2y_j$ and $t_{ij} > 0 \Rightarrow y_i = 2y_j$. There are four agents with incomes 27, 6, 2 and 16. Number agents from left to right. In the initial network, on the left panels, 2 is connected with 1 and 3, and 4 is isolated. Equilibrium consumptions, in the lower left panel, are 20, 10, 5 and 16 with $t_{12} = 7$ and $t_{23} = 3$ and $Var(\mathbf{y}) \approx 32.7$. Next, add a connection between 3 and 4 as depicted on the right panel. Consumptions on the lower right panel become 22, 11, 6 and 12 with $t_{12} = 5$, $t_{23} = 0$ and $t_{43} = 4$ and $Var(\mathbf{y}) \approx 33.7$. The new link connects the poorest agent, 3, with relatively wealthy agent 4. Thanks to his new connection, 3 does not need support from 2 any more and this cuts down indirect gifts from 1. Situation of the richest agent improves and overall variance increases. Thus, adding links to an altruistic network can lead to an increase in consumption variance.

Even when consumption variance increases, however, other measures of consumption inequality may decrease. For instance in the previous example, the ratio of top to bottom consumption drops from 4 to 3.7 when the network expands. This illustrates the following general property. Consider common private utilities $\forall i, u_i = u$ and focus on the lowest

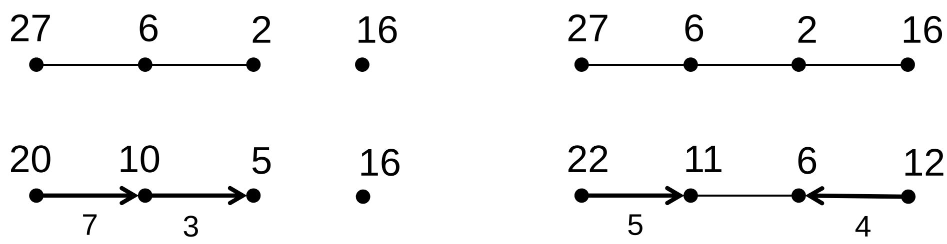

Figure 4: Adding an altruistic link can increase inequality

and highest consumptions $y_{\min}$ and $y_{\max}$. Note that $u'(y_{\max})/u'(y_{\min})$ is lower than 1 and tends to be lower when the consumption spread $y_{\max} - y_{\min}$ is higher. For any pair i, j, define $\hat{\alpha}_{ij}$ as the highest altruistic strength among chains connecting i and j. And let $\hat{\alpha}_{\min} = \min_{i,j} \hat{\alpha}_{ij}$ be the lowest of these pairwise coefficients. Thus, $\hat{\alpha}_{\min}$ captures the altruistic strength of the *weakest indirect link* in the network.

Proposition 8 *On any altruistic network and for any common private utility function u such that $u'(\infty) < \hat{\alpha}_{\min} u'(0)$,*

$$\min_{\mathbf{y}^0} \frac{u'(y_{\max})}{u'(y_{\min})} = \hat{\alpha}_{\min}$$

Proof: Consider an equilibrium and i and j such that $y_i = y_{\max}$ and $y_j = y_{\min}$. From the proof of Proposition 4, we know that $u'(y_{\max})/u'(y_{\min}) \geq \hat{\alpha}_{ij} \geq \hat{\alpha}_{\min}$. Next, let i and j be such that $\hat{\alpha}_{ij} = \hat{\alpha}_{\min}$. Consider the following distribution: $y_i^0 = Y$; $y_k^0 = 0$, $\forall k \neq i$. Note that $y_i = y_{\max}$ and hence $y_i \geq \bar{y}^0 = Y/n$. As Y tends to $+\infty$, y_i tends to $+\infty$. Since $u'(y_i) \geq \hat{\alpha}_{ij} u'(y_j)$, $u'(\infty) \geq \hat{\alpha}_{ij} u'(y_j)$. By assumption, $\hat{\alpha}_{ij} u'(0) > u'(\infty)$ and hence $y_j > 0$. Thus, if Y is high enough, j is a net receiver. Since i is the only individual with positive initial income, money must flow somehow from i to j and hence $u'(y_i)/u'(y_j) = \hat{\alpha}_{ij}$. $\square$

This result says that the relationship between top and bottom consumption is controlled by the strength of the weakest indirect link in the altruistic network. For instance when all links have strength α, $\hat{\alpha}_{\min} = \alpha^d$ where d is the network's *diameter*, i.e., the largest distance between any two agents. With CARA utilities, $u(y) = -e^{-Ay}/A$, and Proposition 8 reduces to: $\max_{\mathbf{y}^0}(y_{\max} - y_{\min}) = d(-\ln(\alpha))/A$. The highest consumption difference sustainable in equilibrium is proportional to the network's diameter. With CRRA preferences, $u(y) = y^{1-\gamma}/(1-\gamma)$ for $\gamma \neq 1$ and $u(y) = \ln(y)$ for $\gamma = 1$, and Proposition 8 becomes $\max_{\mathbf{y}^0} \ln(y_{\max}) - \ln(y_{\min}) = d(-\ln(\alpha))/\gamma$. The highest consumption ratio is now proportional to the diameter.

A direct implication of Proposition 8 is that $\min_{\mathbf{y}^0} u'(y_{\max})/u'(y_{\min})$ is weakly increasing in α. When the altruistic network expands, the weakest indirect link can only become stronger. Relatedly, diameter drops. This necessarily reduces the largest level of inequality sustainable in equilibrium, as captured by $u'(y_{\max})/u'(y_{\min})$.

Next, we consider CARA private utilities. We build on our potential characterization to obtain a stronger comparative statics result.

Proposition 9 *Suppose that $u_i(y) = -e^{-A_i y}/A_i$, $\forall i$, $\forall y$ with $A_i > 0$. Consider an equilibrium $\mathbf{T}$ with consumption $\mathbf{y}$ for the network α and an equilibrium $\mathbf{T}'$ with consumption $\mathbf{y}'$ for the network $\alpha' \geq \alpha$. Then*

$$\sum_{i,j} \ln(\alpha_{ij}) t_{ij} - \frac{1}{2} \sum_i A_i y_i^2 \leq \sum_{i,j} \ln(\alpha'_{ij}) t'_{ij} - \frac{1}{2} \sum_i A_i y_i'^2$$

Proof: As α increases, the objective function increases weakly and the constrained set expands in the optimization program of Proposition 6. Therefore, the value of the objective function at the maximum increases weakly. □

Proposition 9 says that the value of the potential at equilibrium increases weakly following an expansion in the network. With common utilities and binary networks, a linear combination of consumption variance and aggregate transfer must decrease when altruism expands. This means, in particular, that increases in variance are necessarily associated with relatively strong decreases in aggregate transfer.

Finally, we provide a first look at the relationship between income homophily and consumption inequality. We consider the following example. There are 20 agents divided in two groups. Ten agents are poor with $y_i^0 = 0$ and the other ten are rich with $y_i^0 = 10$. All links have strength α and agents have CARA utilities with $-\ln(\alpha)/A = 2$. The network is built as follows. Start from the network where agents are fully connected within income groups and there is no link between. This is a situation of maximum homophily. Then, remove l links at random within each group and add $2l$ links at random between the poor and the rich. The overall number of links stays constant. As l increases, the relative proportion of links between increases and hence income homophily decreases. We pick 1000 network realizations for each value of l and for each network we compute equilibrium consumptions.[22] We depict in Figure 5 consumption variance as a function of l. The three curves depict the 5th percentile, the median, and the 95th percentile of the distribution of variances over network realizations.

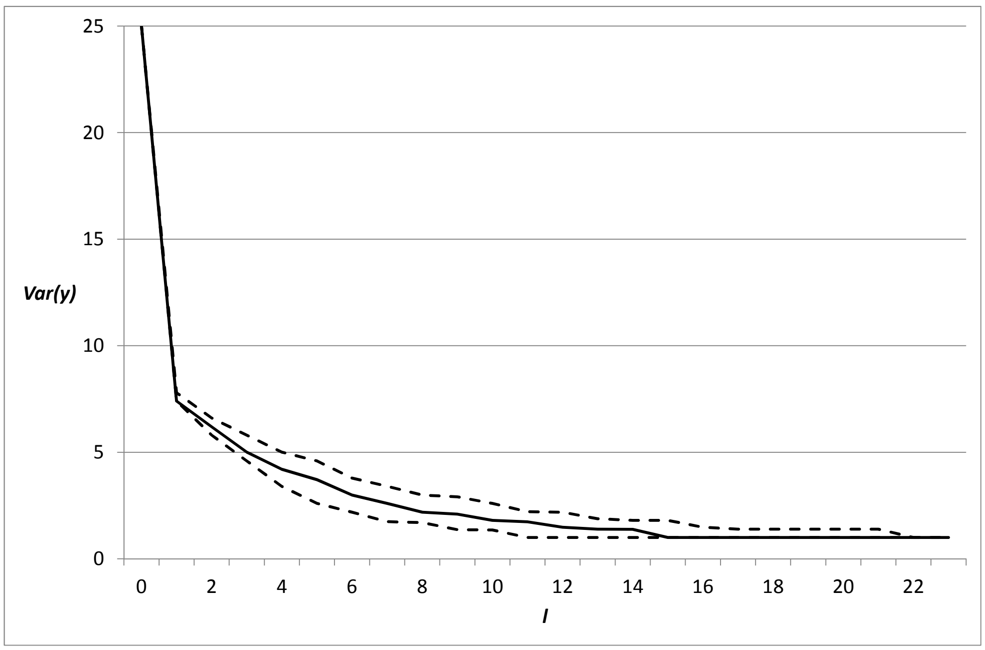

Figure 5: Homophily and Inequality

In this setup, altruistic networks with more homophily clearly tend to generate more inequality.[23] This is not surprising as more links between the rich and the poor represent more opportunities for private transfers. This tendency is not absolute, however, and we

[22] We compute equilibrium consumptions as follows: (0) Start from the profile of zero transfers. (1) Order agents in any way. (2) Have each agent in turn plays a best-reply. Repeat (1) and (2) til convergence, which is guaranteed by the existence of a concave best-response potential.

[23] More precisely, the extent of inequality reduction induced by private transfers tends to be lower on altruistic networks with more homophily.

see that networks with more links between can generate more consumption inequality.[24] Network structure thus matters over and above the homophily level and the overall number of links. In addition, we observe the emergence of *small-world effects.* The first altruistic links between the poor and the rich lead to a strong reduction in variance. Then, additional links have a much smaller and decreasing impact. This is a direct consequence of equilibrium behavior and indirect gifts. Even a single link between communities may end up affecting every agent, as poor agents receiving financial support from rich friends help out other poor agents lacking in such connections.

VI Conclusion

To conclude, we provide the first analysis of altruism in networks. Agents are connected through an arbitrary weighted network and care for the well-being of their network neighbors. We analyze the resulting transfer game: Who gives to whom and how much? How do transfers and consumptions depend on incomes and on the network? We uncover four main features of equilibrium behavior in altruistic networks: consumption uniqueness, cost efficiency, income monotonicity, and inequality reduction. Overall, we provide a detailed description of the anatomy of private transfers generated by altruistic networks.

A number of potentially fruitful issues could be explored in future research.

First, *the risk sharing properties of altruistic networks.* Suppose that incomes are stochastic and that transfers operate, ex-post, as described in our analysis. How would expected utilities depend on the network structure? Would friends' friends help by providing a source of support for direct friends, or would they reduce payoffs by acting as competitors for direct friends' gifts?

Second, *the effect of altruistic networks on incentives.* Alger & Weibull (2010) show that altruism may help to address moral hazard in a model with two agents. Altruistic individuals may exert effort to reduce risk, in order to be in a better position to help. How would their analysis extend to more general social structures? What kind of networks

[24] For instance, consumption variance for networks in the 95^{th} percentile with $l = 10$ is greater than for networks in the 5^{th} percentile with $l = 6$.

would be better able to alleviate moral hazard?

Third, *the design of public policies in altruistic societies*. How to optimize the targets of cash transfers programs in altruistic networks? How to avoid the aggravation of inequality highlighted in Section V? How to introduce formal insurance in altruistic communities? What would be the optimal fiscal policy with respect to gifts?

Fourth, *the empirical identification of real motives behind private transfers*. Many of our results have identifying power. With detailed data on transfers and incomes, these results could potentially be tested or be used in structural estimations.

Fifth, *the interaction between altruism and other motives on networks*. In reality, altruism, exchange and social pressure likely all play a role in explaining private transfers. Few studies have explored the interaction between motives. Foster & Rosenzweig (2001) looks at the impact of altruism on mutual insurance arrangements under limited commitment. Alger & Weibull (2008) study the combined effect of altruism and social pressure on incentives. These two studies consider individuals interacting in pairs. It would be interesting to look at the interaction between altruism, exchange and social pressure in social networks.

APPENDIX

Proof of Proposition 1.

Define $S_M = \{\mathbf{T} \in S : \sum_{i,j} t_{ij} \leq M\}$ for any $M > 0$. Then, S_M is compact and convex. Agent i's utility v_i is concave in $\mathbf{T}_i$ and continuous in $\mathbf{T}$. By Theorem 1 in Rosen (1965), an equilibrium exists on S_M. In this equilibrium, the added constraint does not change the property that $t_{ij} > 0 \Rightarrow y_i > y_j$. This property implies that the transfer network is acyclic. Indeed, suppose that $t_{ii_2} > 0$, $t_{i_2 i_3} > 0$,..., $t_{i_{l-1}j} > 0$. Then: $y_i > y_{i_2}$,...,$y_{i_{l-1}} > y_j$ and hence $y_i > y_j$ and there is no cycle.

Next, let us show by induction that aggregate transfers in acyclic networks are bounded from above. The induction hypothesis, H_n, is as follows: In an acyclic transfer network with n agents, $\sum_{i,j} t_{ij} \leq (n-1)\sum_i y_i^0$. Suppose first that $n = 2$. Then $t_{12}t_{21} = 0$. If $t_{12} > 0$, then $t_{12} \leq y_1^0$. If $t_{21} > 0$, then $t_{21} \leq y_2^0$. In any case, H_2 holds. Suppose, next, that H_{n-1} is true. Consider an acyclic transfer network with n agents. Without loss of generality, suppose that agent 1 does not receive. This means that $\sum_j t_{1j} \leq y_1^0$. Remove 1 and apply H_{n-1} to the resulting network: $\sum_{i,j=2}^n t_{ij} \leq (n-2)\sum_{i=2}^n (y_i^0 + t_{1i})$. Therefore, $\sum_{i,j} t_{ij} = \sum_j t_{1j} + \sum_{i,j=2}^n t_{ij} \leq (n-1)y_1^0 + (n-2)\sum_{i=2}^n y_i^0 \leq (n-1)\sum_i y_i^0$ and H_n holds.

Finally, choose $M > (n-1)\sum_i y_i^0$. An equilibrium on S_M is an equilibrium on S. $\square$

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