

# Government Interventions in a Dynamic Market with Adverse Selection\*

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## Abstract

We study government interventions in a dynamic market with asymmetric information. We show that if the government can only carry out budget-neutral policies, introducing a short tax-exempt trading window followed by short-lived positive taxes creates a Pareto improvement in the market. Under a sufficient condition on the shape of the gains from trade and the distribution of asset values, we show that, even when not requiring budget-neutrality, it is optimal to subsidize trades only at time zero while imposing prohibitively high taxes afterwards. Subsidies can greatly enhance welfare but they can also be detrimental if they are provided with delay.

## 1 Introduction

During times of financial distress, such as those experienced in 2008 after the demise of Lehman Brothers, asset sales are an important source of funds for financial institutions such as banks and insurance companies. Unfortunately, the big gains from trade between those that are liquidity constrained and those that are not may be difficult to realize due to asymmetric information. As in the classic Akerlof (1970) market for lemons, if buyers were to pay the price corresponding to the average quality of the assets in the market, sellers

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\*This paper is closely related to our previous working paper: "Costs and Benefits of Dynamic Trading in a Lemons Market."

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holding the best assets might not wish to trade. Realizing this, buyers would then reduce their offers and end up trading with a small fraction of the sellers or none at all. Absent government intervention, trade either completely stops or slows down, with prices gradually rising and over time better and better assets being traded in the market.

The main questions we seek to answer in this paper are if and how should the government intervene in these situations, even if it has a binding budget constraint. We answer them in a model of a dynamic competitive market in which liquidity-constrained sellers have private information about their assets and homogenous, liquidity-abundant buyers compete to buy those assets.

Several recent papers document how different financial markets had drastic reductions in volume in the aftermath of Lehman Brothers collapse in 2008. Among others, Heider, Hoerova & Holthausen (2009) discuss the collapse of the interbank market, McCabe (2010) the money market funds and Duffie (2009) discusses the OTC and repo markets. These contractions were largely driven by the uncertainty over the counterparty's ability to meet its obligations and the disagreement over the value of securities that could be used as collateral. This was clearly reflected in the OTC market where the types of securities acceptable as collateral significantly changed. Information sensitive securities were largely replaced by cash. Similarly, the assets under management of money market funds saw a big compositional change at the time of Lehman's collapse with a pronounced drop in the amount of asset-backed commercial paper and a large increase of government securities. These recent events motivate our interest in dynamic markets with asymmetric information and the impact of government interventions on such markets.

Our first and main result (Theorem 1) is that, even if the government can use only a budget-neutral tax/subsidy policy, introducing a short tax-exempt trading window followed by short-lived positive taxes creates a Pareto improvement in the market. Absent any intervention, the equilibrium is characterized by a smooth flow of trade where worst assets are sold first and both the quality of traded assets and price gradually increase over time. By taxing future trades, the government creates more incentives to trade in the early tax-exempt period. In particular, holders of higher quality assets that would delay trade absent the government policy now prefer to trade earlier in order to avoid the taxes or excessive delay. As the quality of the pool of assets sold early improves, market price increases as well. Higher prices in turn induce even more trade ultimately doubling the amount of trade and making all sellers better off: those who trade early get better prices, those that trade later save on delay costs (buyers are indifferent since they make zero profit in either scenario).

Our second result (Theorem 2) is about optimal, i.e., total-welfare maximizing, budget-

neutral interventions. Under a sufficient regularity condition on the shape of the gains from trade and the distribution of asset values, we show it is optimal to subsidize trades at time zero while imposing prohibitively high taxes afterwards. Intuitively, the regularity condition implies that the ratio of the marginal gains from trade to the marginal information rents of the seller is decreasing in asset quality. Under this condition, the solution to the optimization problem has a bang-bang property: it is optimal to push as much trade as possible to take place immediately even if it comes at the expense of excluding all higher types from trade altogether.<sup>1</sup> We extend this result in Corollary 1 where we allow the government to carry out non-budget neutral interventions. This is important for both normative and positive considerations. We show that there can be big returns from spending some resources in bailing out these markets: even if raising a dollar in revenues from another market induces some deadweight loss, it may still be optimal to bailout these markets. This is best illustrated by showing that in certain cases even with the best budget-neutral intervention the market completely unravels (see Section 4.3). By providing an initial subsidy the government is able to jump-start the market and greatly increase the overall surplus.

Our third result stresses the importance of acting quickly. Via a series of examples we demonstrate that the timing of the subsidies is crucial. If the government moves slowly and the subsidy is expected to arrive in the future (either deterministically or stochastically) then it can actually have a negative effect on welfare. The intuition is that the expectation of future subsidies delays current trade. So, although there is clearly a benefit from subsidizing trades, it is of the essence that the government acts fast. While the flexibility of the Federal Reserve and its ability to act fast was likely crucial in the recent crisis, some of the uncertainty over future interventions/subsidies may have contributed to the reduction in trade volume in private markets.

Lastly, we note that the exact form of the intervention is not important, as long as the information rents collected by the sellers are unchanged (and transaction costs are the same). A proportional subsidy, as assumed for concreteness in the paper, or a government guarantee on the payoff of the assets, as implemented during the crisis with the Public-Private Investment Program for Legacy Assets, would have equivalent effects. Outright purchases of assets, as implemented with the TARP program, would also be equivalent, but only if the program budgeting accounted for the expected future proceeds from the assets purchased by

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<sup>1</sup>This is related to the result that a durable-good monopolist facing a demand curve with decreasing marginal revenue, would like to commit to set constant price. Such price induces high valuation buyers to buy immediately and the rest of the buyers never to trade, that is, under commitment the seller would not want to use time to screen buyers.

the government. For example, a subsidy program with a budget  $b$  is equivalent to a purchase program with a budget  $b$ , if the budget in the latter case is on net losses from the program and not on total purchases.<sup>2</sup>

## Related Literature

Optimal government interventions in similar models have been studied recently by Philippon and Skreta (2012) and Tirole (2012). In these papers, the government offers financing to firms having an investment opportunity and it is secured by assets that the firms have private information about (these are sellers in our model - using an asset as a collateral or selling it to obtain financing are essentially economically equivalent). That round of government financing is followed by a static competitive market in which firms that did not receive funds from the government can raise funds in a private market. This creates a problem of "mechanism design with a competitive fringe" as named by Philippon and Skreta (2012): the government intervention affects the post-intervention equilibrium and vice versa (a feature shared by our model). In sharp contrast to our results, both papers show that tampering with the private markets does not improve welfare: see Proposition 2 in Tirole (2012) and Theorem 2 in Philippon and Skreta (2012). Since the post-intervention market creates endogenous IR constraints for the agents participating in the government program, making the market less attractive could make it easier for the government to intervene. However, these two papers argue that this is never a good idea.

The key difference between our model and these two papers that leads to these opposing results is that Philippon and Skreta (2012) and Tirole (2012) assume a static model of the private market, while we study a dynamic market. It is best seen in the light of our Theorem 2/Corollary 1: under the regularity condition, it is indeed optimal to have government subsidy at time zero and all trade happening at time zero, with no additional trades in the future. Beyond this crucial difference in results and their practical interpretation, our paper differs from these two papers in terms of the focus on the dynamics of trade and the tradeoffs in dynamic interventions.

The bang-bang property of the optimal intervention in Theorem 2 is mathematically related to the findings of Samuelson (1984). In a static setting he shows that the optimal budget-neutral mechanism divides sellers into at most three groups: a group that trades

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<sup>2</sup>That assumes that the government holding the assets to maturity is as efficient as private buyers holding the asset. If not, then after purchasing the assets the government should pool them into a portfolio and sell shares of the portfolio to buyers with liquidity. Since the government can commit to pool all of the assets, there would be no (additional) adverse selection problem in creation and sales of the portfolio.

with probably one, a group that trades with a common intermediate probability and a group that does not trade. In our dynamic setting this translates respectively to a group that trades immediately, a group that trades with delay and a group that never trades. Our Theorem 2 contributes to his result by establishing a sufficient condition for the optimal mechanism having trade only at  $t = 0$ . Moreover, we show how the optimal direct revelation mechanism can be implemented in a decentralized market with a particular government policy that induces a unique competitive equilibrium. In addition, when the regularity condition is satisfied, we also extend the result by allowing for non budget-neutral interventions.

Two related papers, Heider, Hoerova and Holthausen (2009) and Bolton, Santos and Scheinkman (2011), combine the problem of adverse selection with one of maturity mismatch. Although we do not model the maturity mismatch problem explicitly, we believe it had an important role in the recent crisis and our liquidity- constrained sellers likely are in that situation because of it. That is, we see the maturity mismatch problem as a possible micro-foundation of our model of gains from trade and asymmetric information. Heider, Hoerova & Holthausen (2009) have a 3 period model of the interbank market. Banks in need of liquidity can use the interbank market to borrow from those with excess liquidity. Asymmetric information about the quality of the assets in the borrower's balance sheets makes lenders afraid of lending to a "lemon" leading to a reduction or complete disappearance of credit. They discuss some policy interventions but their focus is positive rather than normative and essentially static.<sup>3</sup>

On the theoretical side, our paper is also related to literature on dynamic markets with adverse selection. The closest paper is Janssen and Roy (2002) who study competitive equilibria in a market that opens at a fixed frequency. They show that in equilibrium prices increase over time and eventually every type trades. They do not ask market design or policy questions as we do in this paper. Yet, we share with their model the observation that dynamic trading leads to more and more types trading over time. Camargo and Lester (2013) find the same equilibrium dynamics in a setting with decentralized search rather than a competitive market (in discrete time, with two types of the seller). While their paper is focused on characterizing the set of equilibria of the game with no government intervention, they also show that sunset provisions for subsidies can increase benefits of government subsidies because, for reasons similar to what we describe in this paper, expectation of future subsidies can slow down trade earlier on. Our paper shows other examples of problems of de-

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<sup>3</sup>Bolton, Santos and Scheinkman (2011) is a bit further from our work since they focus more on the ex-ante asset choices and they assume there is no asymmetric information initially but rather that it grows over time.

layed/prolonged interventions and adds to this analysis by characterizing good and optimal policies. On the more technical side, our competitive-equilibrium setup with a continuum of types and continuous time allows us to show uniqueness of equilibrium, which makes it easier to interpret comparative statics. For other papers on dynamic signaling/screening with a competitive market see Noldeke and van Damme (1990), Swinkels (1999), Kremer and Skrzypacz (2007) and Daley and Green (2012). While we share with these papers an interest in dynamic markets with asymmetric information, none of these papers studies government interventions.

While in this paper we study government interventions in terms of taxes and subsidies, there are other ways the government or market designer can affect trade in equilibrium. For example, in our related working paper, Fuchs and Skrzypacz (2013), we allow the market designer to determine the times the market should be open or closed. The market microstructure literature (see Biais, Glosten and Spatt (2005)) has also considered the question of how different trading protocols perform in the presence of adverse selection. That literature has mainly focused on the stock markets where there are potentially many competing sellers, divisible assets and dispersed information. A different design question for dynamic markets with asymmetric information is asked in Hörner and Vieille (2009), Kaya and Liu (2012), Kim (2012) and Fuchs, Öry and Skrzypacz (2012). These papers ask how information about past rejected offers affects efficiency of trade. Moreno and Wooders (2012) ask a yet another design question: they compare decentralized search markets with centralized competitive markets.

## 2 The Model

There is a mass of size one of financially distressed banks (the sellers). Each seller owns one unit of an indivisible asset. When the seller holds the asset, it generates for him a revenue stream with net present value  $c \in [0, 1]$  that is private information of the seller. The seller types,  $c$ , are distributed according to  $F(c)$ , which is common knowledge, atomless and has a continuous, strictly positive density  $f(c)$ . We assume that the private information is never revealed.<sup>4</sup>

There is a competitive market of potential buyers. Each buyer values the asset at  $v(c)$

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<sup>4</sup>Most of our results can be extended to a setting in which at some deterministic or random time the private information becomes public, but the players cannot contract on the realization of this information (see the working paper Fuchs and Skrzypacz 2013). While the cashflows generated by the asset (which are correlated with  $c$ ) are realized by the buyer, we assume that the buyer and the seller cannot contract on their realization, i.e., we assume that the seller cannot offer warranty contracts on the assets he sells.

which is strictly increasing, twice continuously differentiable, and satisfies  $v(c) > c$  for all  $c \in (0, 1)$ ,  $v(0) \geq 0$ , and  $v(1) = 1$  (i.e. no gap on the top).<sup>5</sup>

Time is  $t \in [0, \infty]$  and the market is continuously open. There is also a benevolent government that can intervene by subsidizing or taxing trades proportionally.<sup>6</sup> The government publicly commits to a path of taxes  $\tau_t$  for  $t \in [0, \infty]$  before the market opens at  $t = 0$ . If at time  $t$  buyers pay price  $p_t$ , the sellers receive  $p_t(1 - \tau_t)$ ;  $\tau_t < 0$  represents a subsidy.

All players discount payoffs at a rate  $r$ . If bank with type  $c$  sells at time  $t$  at a price  $p_t$ , its payoff is

$$(1 - e^{-rt})c + e^{-rt}p_t(1 - \tau_t)$$

and the buyer's payoff at the time of purchase is:

$$v(c) - p_t$$

Given a path of prices and taxes, the sellers face an optimal stopping problem. Namely, when to sell and collect  $p_t(1 - \tau_t)$ :

$$\max_x \int_0^x e^{-rt}rcdt + e^{-rx}p_x(1 - \tau_x). \quad (1)$$

Since the stopping problem is supermodular in  $c$  and  $x$ , if seller of type  $c$  has an optimal stopping time  $t$  then all types  $c' < c$  have optimal stopping times  $t' \leq t$  (even if the optimal stopping time for some types is not unique). The intuition is that the lower types get the same payoff from selling as type  $c$ , but forego less of future cashflows. This is known as the “skimming property” and it simplifies equilibrium analysis since in equilibrium the set of seller types remaining in the market at any time is a truncation of the original seller distribution.

Let  $x(c)$  be some selection of the optimal stopping times given the net-price process,  $p_t(1 - \tau_t)$ .<sup>7</sup> Let  $k_t$  denote the lowest quality asset that has not been traded by time  $t$ :

$$k_t = \inf \{c : x(c) \geq t\}$$

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<sup>5</sup>Assuming  $v(1) = 1$  allows us not to worry about out-of-equilibrium beliefs after a history where all sellers were supposed to trade but some did not trade. The equilibria we characterize in this paper continue to exist even if  $v(1) > 1$  but may no longer be unique.

<sup>6</sup>As discussed in Remark 2, the exact form of the subsidy turns out not to matter.

<sup>7</sup>To assure the stopping problem has a solution, we restrict  $\tau_t$  to be such that we can construct equilibrium  $p_t$  so that  $p_t(1 - \tau_t)$  is right-continuous when it is increasing and left-continuous when it is decreasing.

Note that  $k_t$  is left-continuous and it is independent of the selection of the optimal stopping times (since for any  $t$  at most zero measure of types are indifferent between stopping at that time and some other time).

We use  $K_t$  to describe the (set of) types that trade at  $t$ . There are three possibilities: (i) if  $k_t$  is constant to the right of  $t$ , that means there is no trade at  $t$ , and we denote it by  $K_t = \emptyset$ ; (ii) if  $k_t$  increases continuously to the right of  $t$ , it means trade is smooth at  $t$ , and we denote it by  $K_t = k_t$ ; (iii) if  $k_t$  jumps discontinuously from  $k_t$  to  $k_{t+} = \lim_{s \rightarrow t+} k_s$ , it means that there is an atom of trade at  $t$ , and we denote by  $K_t = [k_t, k_{t+}]$ .<sup>8</sup>

With this notation we define a competitive equilibrium:

**Definition 1** *Given a tax schedule  $\{\tau_t\}$ , a **competitive equilibrium** is a pair of functions  $\{p_t, k_t\}$  for  $t \geq 0$  that satisfy:*

(E1) *Zero Profit Condition: if  $K_t \neq \emptyset$ , then  $p_t = E[v(c) | c \in K_t]$*

(E2) *Seller Optimality: Sellers optimally choose their stopping times (i.e.,  $k_t$  is consistent with seller optimization given  $p_t$  and  $\tau_t$ ).*

(E3) *Market Clearing: for all  $t$ ,  $p_t \geq v(k_t)$ .*

Conditions (E1) and (E2) are standard. Condition (E3) deserves a bit of explanation. It is needed because condition (E1) provides no discipline when  $K_t = \emptyset$ . We justify it by a market clearing reasoning, that is, that given the market prices demand equals supply. Suppose at some  $t$  the assets were offered at  $p_t < v(k_t)$ . Then, since all buyers believe that the value of the asset is at least  $v(k_t)$ , they would all demand it. Demand would not equal supply and the market would not clear.<sup>9</sup> This condition removes some trivial multiplicity of equilibria. For example, it removes as a candidate equilibrium the path  $(p_t, k_t) = (0, 0)$  for all periods (i.e. no trade and very low prices) even though this path satisfies the first two conditions.<sup>10</sup>

We assume that all market participants publicly observe all the trades. Hence, once a buyer purchases an asset, if he tries to put it back on the market, the market makes a correct inference about  $c$  based on the history. Since we assume that all buyers have the same value of the asset, there would not be any profitable re-trading of the asset (after the initial seller transacts) and hence we ignore that possibility.

<sup>8</sup>We use the notation  $k_{t+}$ ,  $p_{t+}$ , etc., to denote right-limits of the corresponding functions at  $t$ .

<sup>9</sup>We thank Andrew Postlewaite for pointing this out to us.

<sup>10</sup>Condition (E3) is analogous to the condition (iv) in Janssen and Roy (2002) and is weaker than the No Unrealized Deals condition in Daley and Green (2012) (see Definition 2.1 there; since they study the gap case, they need a stronger condition to account for out-of-equilibrium beliefs).



## 2.1 Laissez-faire Equilibrium

Absent government interventions, i.e., if  $\tau_t = 0$  for all  $t$ , the equilibrium has no atoms of trade and is given by:

**Proposition 1 (laissez-faire)** *If  $\tau_t = 0$  for all  $t$  then there exists a unique competitive equilibrium that is the unique solution to:*

$$\begin{aligned} p_t &= v(k_t) \\ k_0 &= 0 \\ r(v(k_t) - k_t) &= v'(k_t) \dot{k}_t \end{aligned} \tag{2}$$

To see the intuition (proof is in the appendix), note that if an interval of types traded at time  $t$ , Condition (E3) would require that an instant later  $p_{t+} \geq v(k_{t+})$ . However, that would imply a jump in prices at  $t$  since  $p_t = E[v(c) | c \in K_t] < v(k_{t+})$ . But if prices jumped, no type would trade the instant before the jump. Hence,  $k_t$  is a continuous function in equilibrium. Therefore, by Condition (E1),  $p_t = v(k_t)$  when there is trade. The differential equation for  $k_t$  comes from seller optimality: at each point in time, the current cutoff type  $k_t$  must be indifferent between trading or delaying trade for an instant. The gain from waiting for an instant of time is that prices rise over time while the cost of delaying trade is the lost interest on the gains from trade. Together:

$$r(p_t - k_t) = \dot{p}_t$$

Using  $p_t = v(k_t)$ , this tradeoff can be stated as:

$$r(v(k_t) - k_t) = v'(k_t) \dot{k}_t \tag{3}$$

This differential equation, together with the boundary condition  $k_0 = 0$ , pins down the equilibrium path of  $k_t$ .

Note that if  $v(0) = 0$  (i.e. no strict gains from trade in the bottom of the distribution), there is no trade in equilibrium.<sup>11</sup> Moreover, the dynamics of  $(p_t, k_t)$  in the laissez-faire equilibrium do not depend on the shape of the distribution  $F(c)$  but only on its support. The shape of  $F(c)$  will play a role once we introduce government interventions that generate

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<sup>11</sup>This can be seen from equation (3). Continuous trading is not necessary for this result to hold. If trading is at discrete intervals of time  $\Delta$  this result would continue to hold for small  $\Delta$  if  $f(0) > 0$ . Even for  $\Delta = \infty$  the result may continue to hold, as we show in Section 4.3.

atoms of trade. Finally, total surplus/gains from trade in the laissez-faire equilibrium are:

$$S_{LF} = \int_0^\infty e^{-rt} (v(k_t) - k_t) \dot{k}_t f(k_t) dt = \int_0^1 e^{-r\tilde{t}(c)} (v(c) - c) f(c) dc$$

where  $\tilde{t}(c)$  is the inverse of  $k_t$ . Condition (3) implies  $\tilde{t}'(c) = \frac{v'(c)}{r(v(c)-c)}$  and hence  $S_{LF}$  is independent of the discount factor (since  $c$  and  $v(c)$  are present values, the first-best surplus is independent of  $r$  as well). The intuition is that since in equilibrium all types eventually trade, the deadweight loss is due to the delay of trade. While a smaller discount implies that any fixed delay is less costly, by condition (3) a smaller  $r$  implies more delay. Since the speed of trading,  $\dot{k}_t$ , is proportional to  $r$ , these two forces cancel each other out.

### 3 Government Interventions: Motivating Examples

We start with the following benchmark example to illustrate the benefits of imposing future taxes to increase early trading. Assume  $c$  is distributed uniformly over  $[0, 1]$  and  $v(c) = \frac{1+c}{2}$ , as illustrated in Figure 1.

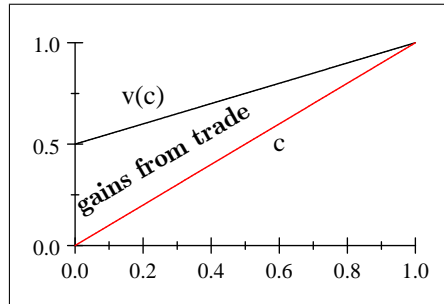


Figure 1: Example of gains from trade.

#### Laissez-faire Economy

Absent any government interventions in this example the equilibrium cutoffs are:

$$k_t^{LF} = 1 - e^{-rt}.$$

The total surplus in the laissez-faire economy is:

$$S_{LF} = \int_0^1 e^{-r\tilde{t}(c)} (v(c) - c) f(c) dc = \frac{1}{6}.$$

Note that even though asymptotically all types trade in equilibrium, the equilibrium is inefficient due to delay. The first-best has all types trading immediately (i.e.  $\tilde{t}(c) = 0$ ) and surplus is  $\int_0^1 (v(c) - c) f(c) dc = \frac{1}{4} > S_{LF}$ .

### Initial Subsidy Followed by Constant Permanent Tax

Now consider the following government intervention. The government provides an initial subsidy  $s = -\tau_0 \geq 0$  per unit traded at time 0 and then finances this subsidy with a constant tax rate  $\tau_t = \tau$  for  $t > 0$ , to (dynamically) achieve budget balance. One interpretation of this intervention is that the government levies a constant tax rate on the market to finance a subsidized auction at time 0.

To construct the equilibrium we solve the following fixed point problem. We first solve for the equilibrium for any  $s$  and  $\tau$ . Then, we look for pairs of  $(s, \tau)$  that are budget neutral. The amount of initial trade depends on the initial subsidy; how much of a subsidy can be provided depends on the amount of trade after  $t > 0$  which, in turn, is a function of which types trade at  $t = 0$ .

The one-time subsidy at time 0 induces an atom of trade at time 0 and the constant tax later implies either no trade (if the tax is high enough) or smooth trade. The equilibrium conditions are as follows. First, cutoff type at time 0,  $\kappa \equiv k_{0+}$ , must be indifferent between pooling with lower types and getting the initial subsidized price  $p_0$ , or waiting an instant to separate from them and getting a higher price but being taxed:

$$p_0(1 + s) = \max \left\{ \underbrace{v(\kappa)}_{=p_{0+}}(1 - \tau), \kappa \right\}. \quad (4)$$

where the maximization captures the two possibilities: there will either be trade after  $t = 0$  (with buyers paying  $p_{0+} = v(\kappa)$ ) or no trade at all.

Second, the buyer zero profit condition at time  $t = 0$  is:

$$p_0 = E[v(c) | c \in [0, \kappa]]. \quad (5)$$

The unique solution to (4) and (5) pins down the equilibrium  $\kappa$  and price  $p_0$  given  $(s, \tau)$ .

Third, if there is any more trade after time  $t = 0$  (i.e. if  $\tau$  is not too high) it must be smooth. The same reasoning as in the laissez-faire equilibrium holds after time zero.

Equilibrium is then pinned down by the sellers' indifference condition:

$$r(p_t(1 - \tau) - k_t) = (1 - \tau)\dot{p}_t$$

and the zero-profit condition  $p_t = v(k_t)$ . Using the assumed form of  $v(c)$  and given a boundary condition  $\kappa$ , the unique solution of this differential equation is:

$$k_t = \frac{1 - \tau}{1 + \tau} - \left( \frac{1 - \tau}{1 + \tau} - \kappa \right) e^{-r\frac{1+\tau}{1-\tau}t}.$$

Inverting it, we get the following expression for the time at which each cutoff type  $k$  trades:

$$\tilde{t}(k) = \frac{1 - \tau}{r(1 + \tau)} \left( \ln \frac{k(1 + \tau) + \tau - 1}{\kappa(1 + \tau) + \tau - 1} \right) \text{ for } k \in \left[ \kappa, \frac{1 - \tau}{1 + \tau} \right].$$

Note that because of the tax, types such that  $(1 - \tau)v(c) \leq c$  do not trade in equilibrium. That completes the characterization of the equilibrium for any  $(s, \tau)$ .

Finally, to verify that in equilibrium the intervention is budget-neutral, we require that for any fixed  $\tau$  the subsidy  $s$  satisfies:

$$sp_0\kappa = \tau \int_{\kappa}^1 e^{-r\tilde{t}(k)} \frac{1 + k}{2} dk \tag{6}$$

The unique positive solution  $(s, \kappa, p_0)$  to (4), (5), (6) pins down the unique competitive equilibrium in this example. With it we can then calculate the total surplus associated with a given tax rate  $\tau$ :

$$S(\tau) = \int_0^{\kappa} (v(c) - c) dc + \int_{\kappa}^1 e^{-r\tilde{t}(k)} (v(k_t) - k_t) dk.$$

*How does the equilibrium depend on  $\tau$ ?*

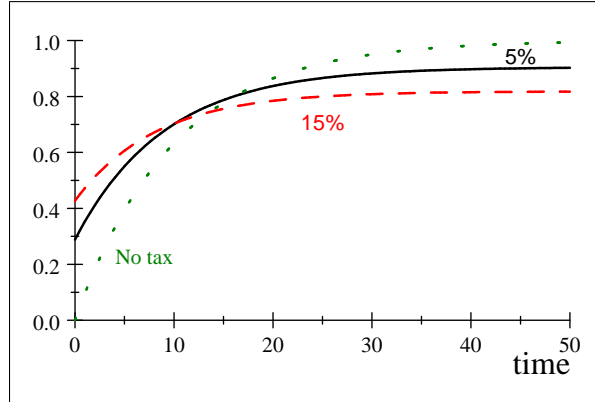


Figure 2: Taxes and Cutoff Dynamics

In Figure 2 we plot equilibrium cutoffs,  $k_t$ , for different tax rates: the dashed line has  $\tau = 15\%$ , the solid line  $\tau = 5\%$  and the dotted line has  $\tau = 0\%$ . As shown, a higher tax rate leads to a higher initial cutoff but slower trade thereafter. In other words, as taxes increase, there is a tradeoff between trading faster with the lower types at the expense of slower trade with higher types.

*How does the total surplus change with  $\tau$ ?*

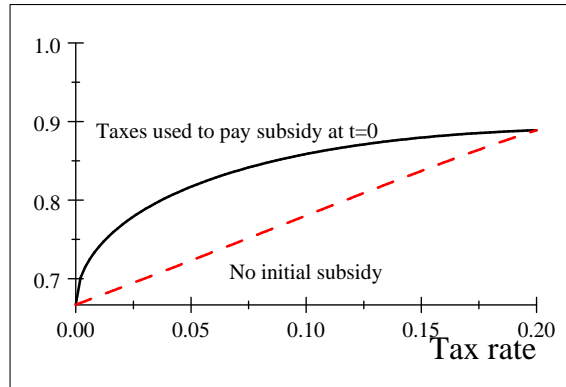


Figure 3: Surplus relative to first best with a tax-exempt auction followed by a constant tax.

Figure 3 shows that in this example surplus (the solid line, represented as a fraction of first-best surplus) monotonically increases in the level of taxes. For sufficiently high taxes ( $\tau \geq 20\%$ ), there is no trade after  $t = 0$  and the surplus is only 11% below first best, With no intervention it is 33% below.

Somewhat surprisingly, even if the taxes are levied but not used for the initial subsidy (i.e. if  $s = 0$  but  $\tau > 0$ ), the surplus is also increasing in  $\tau$ , as shown by the dashed line in Figure 3. Comparing the two curves, for small tax rates, the initial subsidy has a large contribution to the welfare gains; yet, for large tax rates the difference is small. The reason is that for high tax rates the Laffer Curve implies that total tax revenues decrease in the tax rate and hence the subsidy becomes again small. The main effect of taxes is then that they push more sellers to participate in the initial tax-exempt auction, and that improves efficiency.

## 4 Optimal Government Interventions

In this section we return to our general model and provide two results. First, we show that for general  $F(c)$  and  $v(c)$  there is always room for the government to improve over laissez-faire and even more, that a tax-exempt auction followed by a short-lived tax achieves a Pareto improvement. This improvement is possible even if the government can only use budget-neutral policies. Then, under a regularity condition on  $F(c)$  and  $v(c)$ , we characterize the optimal interventions for a zero (and positive) budget constraint.

### 4.1 Laissez Faire is Never Optimal

Our main result follows. Consider the following tax policy:

$$\tau^\Delta \equiv \tau_t = \begin{cases} 0 & \text{for } t \in \{0\} \cup [\Delta, \infty) \\ \tau > 0 & \text{otherwise} \end{cases}$$

That is, there is tax-exempt trading at  $t = 0$ , followed by a (short) time interval  $\Delta$  in which transactions are taxed at  $\tau$ , after which taxes are reduced back to zero.

We show that for small  $\Delta > 0$  all sellers prefer  $\tau^\Delta$  over the laissez-faire equilibrium (i.e.  $\tau_t = 0$ ):

**Theorem 1** *Suppose  $v(0) > 0$ . For every  $r$ ,  $\tau > 0$ ,  $F(c)$ , and  $v(c)$ , there exists  $\Delta > 0$  such that an equilibrium with government policy  $\tau^\Delta$  yields strictly higher gains from trade than the laissez-faire equilibrium. Moreover, it is preferred by all seller types (Pareto improvement).*

To establish that the  $\tau^\Delta$  intervention increases overall efficiency we show in the proof (see appendix for details) an even stronger result: for small  $\Delta$ , under  $\tau^\Delta$  every type trades sooner

than under laissez-faire. We start with characterizing the equilibrium with  $\tau^\Delta$  for small  $\Delta$ . First, since after the initial period taxes increase discontinuously, it will attract an atom of types to trade at  $t = 0$ . Let  $\kappa_\Delta$  be the highest type that trades at  $t = 0$  with  $\tau^\Delta$ . Moreover, the fact that taxes drop discontinuously at  $t = \Delta$  implies that for small  $\Delta$  there will be no trade in the time interval  $[0, \Delta]$  (roughly, waiting increases prices from  $(1 - \tau)v(k_\Delta)$  to  $v(k_\Delta)$  and for small  $\Delta$  that is a much larger benefit than the discounting cost).

Let  $k_\Delta^{LF}$  denote the equilibrium cutoff at time  $t = \Delta$  with no taxes (laissez-faire). As we show in the proof, for small  $\Delta$ ,  $k_\Delta^{LF} < \kappa_\Delta$ . Since with  $\tau^\Delta$  after time  $\Delta$  the equilibrium is the same as in case of the laissez-faire economy but with a different boundary condition, every type trades sooner under  $\tau^\Delta$  and the claim follows.

The key step of the proof is to show that:

$$\lim_{\Delta \rightarrow 0} \frac{\partial \kappa_\Delta}{\partial \Delta} = 2 \lim_{\Delta \rightarrow 0} \frac{\partial k_\Delta^{LF}}{\partial \Delta}.$$

Since as  $\Delta \rightarrow 0$  both  $\kappa_\Delta$  and  $k_\Delta^{LF}$  converge to 0, this means that for small  $\Delta$  approximately twice as many types trade before  $\Delta$  if the government intervenes in  $(0, \Delta)$ . The intuition is as follows. As we announce the tax plan  $\tau^\Delta$ , some types that were planning to trade in  $(0, \Delta)$  now would prefer to trade at 0 *even if the price at 0 did not change*. The reason is that not taking the price  $p_0$  implies a fixed delay cost. It turns out that the set of types that decide to take that fixed  $p_0$  grows in  $\Delta$  approximately as fast as does  $k_\Delta^{LF}$ .

The doubling of early trade is then achieved because pooling of trade at time 0 reduces adverse selection faced by buyers and hence *price  $p_0$  increases*. For small  $\Delta$  the price is approximately half way between  $v(0)$  and  $v(\kappa_\Delta)$ . As the price goes up, even more types prefer to trade at 0 and the adverse selection problem is reduced even further, making  $p_0$  even higher, and so on. Because prices grow at half the speed of  $v(k_\Delta^{LF})$ , the resulting cutoff,  $\kappa_\Delta$ , is twice as high as  $k_\Delta^{LF}$ .

To see the improvement is Pareto, note that type  $\kappa_\Delta$  could choose to trade at  $t = \Delta$  for  $p = v(\kappa_\Delta)$  which is strictly better than what he would get in the laissez-faire economy since it would take the economy longer to reach that price absent taxes (and in equilibrium he trades at that price). Types  $c > \kappa_\Delta$  are also better off because they trade at the same price but earlier than they would absent the intervention. To see that types  $c < \kappa_\Delta$  are better off, note that given that they all trade immediately their payoff ends up being the same as that for  $c = \kappa_\Delta$  whereas in the laissez-faire economy it would be strictly lower.

The same logic can be used to show that even when initial trades are subsidized (i.e.  $\tau_0 < 0$  even if this subsidy is not financed by subsequent taxes), it is still optimal to introduce

positive taxes into the market for some small time  $\Delta$ .

We required for the result that  $v(0) > 0$ . If  $v(0) = 0$  whether the tax policy  $\tau^\Delta$  leads to a strict increase of surplus or no change depends on whether it induces any trade. That, in turn, depends on the shape of  $F(c)$  and  $v(c)$ . Both cases are possible: in the example in Section 4.3  $\tau^\Delta$  does not induce trade for any  $\Delta$ , while in case  $F(c) = c$  and  $v(c) = \sqrt{c}$ , if  $e^{-r\Delta} < \frac{2}{3}$ , then  $\tau^\Delta$  does induce trade and thus generates a Pareto improvement.

## 4.2 When Extreme Policy is Optimal

We showed so far that a short-lived budget-neutral intervention improves over the laissez-faire equilibrium and that in fact we can get a Pareto improvement. Although the proof of Theorem 1 uses  $\Delta \rightarrow 0$  it is important to note that the optimal policy would generally call for  $\Delta \gg 0$ . Furthermore, the intervention can also continue to be a Pareto improvement outside of the limit. If a small intervention generates an improvement, what about a larger one? For our example economy, with  $c$  distributed uniformly over  $[0, 1]$ ,  $v(c) = \frac{1+c}{2}$ , and  $r = 10\%$ , we can see in Figure 4A that total welfare is actually increasing in  $\Delta$ . In Figure 4B we show the surplus gain/loss of the cutoff type,  $\kappa_\Delta$ , over what his surplus would be without any intervention.<sup>12</sup>

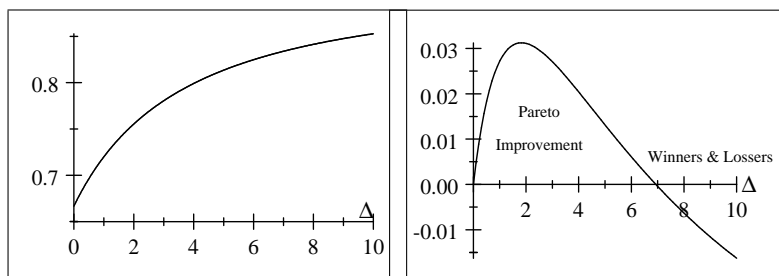


Figure 4A: Total surplus relative to first best. 4B: Gain/loss of cutoff type relative to no intervention.

Is this true in general? To answer this question we introduce the following condition:

**Definition 2** We say that the environment is (strictly) regular if  $\frac{f(c)}{F(c)} (v(c) - c)$  is (strictly) decreasing.

<sup>12</sup>The intervention is a Pareto improvement if the cutoff type is weakly better off.



As we explain below, the ratio  $\frac{f(c)}{F(c)}(v(c) - c)$  represents the relative marginal effect of speeding up trade of type  $c$  on the social surplus and the information rent of the seller.<sup>13</sup> Under this condition we can show that the optimal policy induces trade that is extremely concentrated in time:

**Theorem 2** *If the environment is regular, a competitive equilibrium for a tax policy*

$$\tau_t = \begin{cases} 0 & \text{for } t = 0 \\ \tau \geq H & \text{otherwise} \end{cases}$$

where  $H$  is high enough so that there is no trade after  $t = 0$ , maximizes total surplus over all possible budget-neutral policies and all corresponding equilibria.

Moreover, if the environment is strictly regular, the competitive equilibrium for this tax policy is unique.

The benchmark example in Section 3 ( $v(c) = \frac{1+c}{2}$  and  $F(c) = c$ ) satisfies the regularity condition and hence we have described there the optimal budget-neutral intervention: tax-exempt initial trade followed by prohibitively high taxes for  $t > 0$ . Note that it is welfare-maximizing, but compared to the laissez-faire equilibrium there are winners and losers from this intervention: for example, type  $3/4$  eventually trades in the laissez-faire economy, but does not trade under this optimal intervention, so that type prefers the former. Therefore, it would be harder to build consensus to implement these type of policies ex-post, once agents know their types.

Our proof of Theorem 2 considers a mechanism design problem with a market designer who maximizes expected gains from trade. The designer is allowed to cross-subsidize sellers trading in different periods but has to break even on average.

Letting  $G_t(c)$  denote, for a given type, the (cumulative) distribution over times of trade, the expected discounted time to trade for this type is:

$$x(c) = \int_0^\infty e^{-rt} dG_t(c),$$

and since all the traders are risk-neutral, their expected payoffs depend on  $G_t(c)$  only via  $x(c)$ .

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<sup>13</sup>This regularity condition is also similar to the standard condition in price theory that the marginal revenue is monotone. In particular, think about a static problem of a monopsonist buyer choosing volume of trade,  $F(c)$ , by making a take-it-or-leave-it offer equal to  $P(c) = c$ . The FOC of this problem is:  $f(c)(v(c) - c) - F(c) = 0$ . A decreasing ratio  $\frac{f(c)}{F(c)}(v(c) - c)$  guarantees that the marginal profit (the left-hand-side of the FOC) crosses zero exactly once.

In the direct revelation mechanism the designer chooses  $x(c)$  and a net transfer to type  $c$ ,  $P(c)$ , to maximize:

$$\max_{x(c), P(c)} \int_0^1 x(c) (v(c) - c) f(c) dc \quad (7)$$

subject to budget-neutrality:

$$\int_0^1 [x(c) v(c) - P(c)] f(c) dc \geq 0,$$

and truth-telling constraint:

$$c \in \arg \max_{\tilde{c}} (1 - x(\tilde{c})) c + P(\tilde{c})$$

and individual rationality for the seller. Truth-telling implies that the equilibrium payoff of type  $c$ ,  $U(c)$ , has to satisfy  $U'(c) = (1 - x(c))$  almost everywhere. We use this to express the budget constraint in terms of the allocation only,  $x(c)$ :

$$\int_0^1 (x(c) (v(c) - c)) f(c) dc - \int_0^1 x(c) F(c) dc \geq 0 \quad (8)$$

The first term is the amount of money the mechanism designer can collect from the buyers and the second term is the information rent he has to pay the sellers so that they report their types truthfully.

The ratio of derivatives of the constraint (8) and the objective function (7) with respect to  $x(c)$  is  $\left(1 - \frac{F(c)}{(v(c)-c)f(c)}\right)$  which is decreasing under our regularity condition. That is a sufficient condition for the optimal solution to have a bang-bang property: types below a threshold trade immediately and types above the threshold never trade. That solution can be implemented by a competitive equilibrium imposing sufficiently high taxes after the initial trade. Given these taxes, if the regularity condition holds strictly, the equilibrium is unique. Hence this extreme intervention is the most efficient.<sup>14</sup>

**Remark 1** *Without the regularity condition, it can be shown that the optimal policy (with zero or positive-balance interventions) induces trade in at most two periods, with  $t = 0$  being*

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<sup>14</sup>Details of the proof are in the appendix. The proof uses standard mechanism design tools, similar to Samuelson (1984) in a static environment. The contribution of our proof is to apply these methods to characterize optimal intervention in a dynamic market. On the technical side, we contribute by establishing a sufficient condition for the optimal mechanism having trade only at  $t = 0$ , and showing that the described policy induces a unique competitive equilibrium that implements the outcome of the optimal direct revelation mechanism.

*one of them. Thus, except at these two times, the government sets sufficiently high taxes so that no trade would take place.*

Our two main results are in stark contrast to recent results in the literature. Optimal government interventions in similar models (although, admittedly richer) have been studied recently by Philippon and Skreta (2012) and Tirole (2012). In these papers, the government offers financing to firms having an investment opportunity and it is secured by assets that the firms have private information about. That intervention is followed by a *static* competitive market in which firms that did not receive funds from the government can trade in a private market to raise funds. This creates a problem of "mechanism design with a competitive fringe" as named by Philippon and Skreta (2012): the government intervention affects the post-intervention equilibrium and vice versa. This effect is shared by our model: for example, under the policy in Theorem 1, the amount of trade at  $t = 0$  under  $\tau^\Delta$  depends on the price after the taxes are removed and that in turn depends on which types trade at time 0.

Both papers obtain a result that shutting down the private market does not improve welfare: see Proposition 2 in Tirole (2012) and Theorem 2 in Philippon and Skreta (2012). Since the post-intervention market creates endogenous IR constraints for the agents participating in the government program, making the market less attractive could make it easier for the government to intervene. However, these two papers argue that this is never a good idea.

As hinted by our benchmark example and generalized in Theorems 1 and 2, taking into account the dynamic nature of the market changes this conclusion. The key difference in the models that leads to these opposing results is that both Philippon and Skreta (2012) and Tirole (2012) assume a static model of the private market, while we study a dynamic market. In our example setting tax rate prohibitively high so that all trade takes place at  $t = 0$  turns out to be equivalent to assuming that the private market is opened only once after the government intervention, as in their papers. Theorem 1 showed that some shut-down is always optimal and Theorem 2 showed that under our regularity condition a complete shutdown is optimal within our model.

We don't think one should take this extreme policy as a literal recommendation of how governments should intervene since this extreme policy is unlikely to be optimal in a richer environment where additional shocks or gains from trade continue to arise over time. An additional consideration with closing the market forever is that it might require a lot of commitment power from the government - a shorter intervention would suffer less from such a concern. Therefore, we think that Theorem 1 is a more realistic policy prescription.

### 4.3 Non Budget-Neutral Interventions: Jump Starting the Market

We now show via an example that if the adverse selection problem is sufficiently severe, even the best budget-neutral intervention is incapable of generating any trade. Consider the following example:  $F(c)$  is uniform in  $[0, 1]$  and

$$v(c) = \begin{cases} 1.5c & \text{if } c < \frac{1}{2} \\ \frac{1+c}{2} & \text{if } c \geq \frac{1}{2} \end{cases}$$

This example satisfies the regularity condition. Hence, Theorem 2 implies that the optimal budget-neutral intervention would induce trade only at  $t = 0$ . Unfortunately, as in Akerlof's original example, even if there is only one opportunity to trade, we get complete unraveling.<sup>15</sup> This follows since the equilibrium cutoff type  $\kappa$  must satisfy:

$$\kappa = p_0$$

but the zero-profit condition is

$$p_0 = E[v(c) | c < \kappa].$$

Since in this example  $E[v(c) | c < \kappa] \leq \frac{3}{4}\kappa$ , the only solution is  $\kappa = 0$ .

Now suppose the government allocates a budget  $b \geq 0$  to bailout this market. Suppose it uses it to subsidize the initial trades with a proportional subsidy  $s$ . When  $b < \frac{1}{16}$  the optimal intervention has  $s = \frac{1}{3}$  (and prohibitively high taxes after  $t = 0$ ), types below  $\kappa_0 = 2\sqrt{b}$  trade at  $t = 0$  and buyers pay  $p_0 = \frac{3}{2}\sqrt{b}$  per unit. Total gains from trade (net of transfers) are:

$$S(b) = \int_0^{2\sqrt{b}} (0.5c) dc = b$$

Since the buyers still break-even, every dollar the government spends increases the welfare of the sellers by 2 dollars, one from the direct transfer and one from the improvement in the efficiency of the market (if  $b > \frac{1}{16}$ , the marginal effect is even higher). Thus, if the deadweight loss associated with raising taxes from other markets is not too large, bailouts are welfare-improving.

A natural question to ask is if  $b > 0$ , and so the government can restart the market at

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<sup>15</sup>The classic example from Akerlof has  $v(c) = 1.5c$  and no trade in (static) equilibrium. We keep close to his example, but modify  $v(c)$  slightly to have  $v(1) = 1$ . Note that it implies even less gains from trade than when  $v(c) = 1.5c$ .

$t = 0$ , would it be even better to have low taxes after  $t > 0$  so that there would be additional trade (and government could use the revenues to finance an even larger subsidy at  $t = 0$ )? Theorem 2 describes the optimal zero-budget policy, but the proof can be easily modified to allow the government a total budget  $b$  to be spent over time (including interest on savings) by putting  $b$  on the right-hand side of (8). This leads to:

**Corollary 1** *Suppose the government has a budget  $b \geq 0$  and the environment is regular. Then, a welfare-maximizing intervention has  $\tau_0 = s \leq 0$  and  $\tau_t \geq H$  for  $t > 0$ , where  $H$  is high enough that there is no trade after  $t = 0$ .*

In other words, to maximize welfare given a budget  $b$ , all the subsidy should be used at time zero and it would not be worth trying to raise additional revenue by taxing future trade - instead, the future taxes should be high enough to induce as many types as possible to trade at time 0.<sup>16</sup>

**Remark 2** *It is important to note that the exact form of the intervention is not important, as long as the information rents collected by the sellers are unchanged. A proportional subsidy, as assumed for concreteness in the paper, or a government guarantee on the payoff of the assets, as implemented during the crisis with the Public-Private Investment Program for Legacy Assets would have equivalent effects. The outright purchase of assets done with the TARP program is also equivalent as long as we take into account the proceeds in the budgeting for the program, that is, consider just the expected net cost of the program as the government's budget.<sup>17</sup>*

## 5 The Cost of Delaying Interventions

In practice it might take time for the government to act upon a crisis. In this section we show that speed is often of the essence. Not only is it usually optimal to act immediately, as established by Theorem 2, but delayed interventions can actually *decrease surplus compared to the laissez-faire equilibrium*. In particular, we show that if the government is expected to provide a bailout at some future time, it slows down (or even shuts down) trade before. Even though the bailout has a positive direct effect on efficiency, it creates also this negative

<sup>16</sup>If  $b$  is large enough,  $b \geq 1 - E[v(c)]$ , first-best is achievable.

<sup>17</sup>As we discussed in the Introduction, the government proceeds can either come from holding the assets to maturity or alternatively from creating a portfolio and selling its shares.

endogenous/equilibrium effect of delay due to anticipation. The net effect can be negative: the equilibrium welfare with a delayed subsidy can be lower than absent any intervention.

We show this claim via two examples using the benchmark example from Section 3:  $v(c) = \frac{1+c}{2}$  and  $F(c) = c$ . We consider first the case of a deterministic date for the intervention and then the case when the timing of the intervention is uncertain.

## 5.1 Delayed Interventions at Announced Date

Consider the following policy:

$$\tau^T = \begin{cases} -s & \text{for } T > 0 \\ 0 & \text{otherwise} \end{cases}$$

The government has a dynamic budget constraint  $e^{-rT}sp_T|K_T| \leq b$ , where  $|K_T|$  is the measure of types that trade at  $T$  and  $p_T$  is the market price buyers pay at  $T$ , so that the left hand side is the time-zero present value of the total subsidy at  $T$ .

For any  $b > 0$  there exists a  $\bar{T}$  such that if  $T \leq \bar{T}$  there is no trade in equilibrium until  $T$  since all sellers prefer to wait for the subsidy than to trade immediately. In this range of  $T$  the competitive equilibrium has atom of trade at  $T$  followed by smooth trading as in the laissez-faire equilibrium. The following conditions pin down the equilibrium for  $T \leq \bar{T}$ .<sup>18</sup>

First, denote by  $\kappa$  the highest type trading at  $T$ . This type has to be indifferent between trading at the subsidized price and trading after the subsidy is removed:

$$p_T(1+s) = p_{T+} = v(\kappa),$$

where the second equality follows because after  $T$  the equilibrium coincides with the laissez-faire equilibrium with a boundary condition  $k_T = \kappa$  (and hence the zero-profit condition with smooth trading implies  $p_t = v(k_t)$ ). The zero-profit condition for prices at  $T$  is:

$$p_T = E[v(c) | c \leq \kappa].$$

Finally, the budget constraint is:

$$e^{-rT}p_Ts\kappa = b.$$

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<sup>18</sup>When  $T$  is large, the analysis is more complex. The equilibrium then has continuous trading until some time  $T^*$ ; from  $T^*$  to  $T$ , the market shuts down waiting for the subsidy, an atom of sellers trade at  $T$ , and smooth trading follows from then on.

From these equations (using  $v(c)$  and  $F(c)$  from the benchmark example) we obtain:

$$\kappa = 2\sqrt{be^{Tr}}.$$

Assuming that  $b$  is small enough that  $\kappa < 1$ , after  $T$  trade is smooth and we can use the characterization of the laissez-faire equilibrium in Section 3 to compute:

$$k_t = 1 - (1 - \kappa) e^{-r(t-T)}.$$

Inverting it yields the time at which types  $k > \kappa$  trade:

$$\tilde{t}(k) = -\frac{1}{r} \left( \ln \frac{1-k}{1-\kappa} \right) + T.$$

That completes the characterization of the equilibrium.<sup>19</sup>

Present value of the gains from trade in equilibrium is:

$$S(b, T < \bar{T}) = e^{-rT} \left( \int_0^\kappa \left( \frac{1-c}{2} \right) dc + \int_\kappa^1 \left( \frac{1-c}{1-\kappa} \right) \left( \frac{1-c}{2} \right) dc \right)$$

Since  $b = 0$  corresponds to no intervention, we have  $S(0, T) = S_{LF} = \frac{1}{6}$ .

How does delay,  $T$ , affect gains from trade?<sup>20</sup> There are two opposing effects. On one hand, later subsidy implies that there is more delay until the market starts trading at  $T$ . On the other hand, since the unused budget earns interest, it allows for more efficient trade at  $T$  and afterwards. It turns out that in our benchmark case the first force is stronger. In particular, for  $b > 0$  and  $T \leq \bar{T}$  and  $2\sqrt{be^{Tr}} < 1$  we get the following result:

**Proposition 2** *Assume  $v(c) = \frac{1+c}{2}$  and  $F(c) = c$ .*

1) *Despite the government being able to save at rate  $r$ , the equilibrium welfare is decreasing in  $T$ ,  $\frac{\partial S(b, T)}{\partial T} < 0$ .*

2) *Moreover, there exists  $T^* < \bar{T}$  such that the subsidy delayed by more than  $T^*$  destroys surplus, that is  $S(b, T) < S_{LF}$  for  $T \in (T^*, \bar{T})$ .*

In words, delay is costly despite the budget growing with delay. Even more surprisingly, the second part of the proposition states that the decrease in the surplus can be so large as

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<sup>19</sup>It implies  $\bar{T}$  is the solution to  $e^{-r\bar{T}}(1+s)p_T = v(0)$  which simplifies to a solution of  $b = \frac{1(1-z)^2}{4z}$  where  $z = e^{-r\bar{T}}$

<sup>20</sup>The effect of  $b$  is obvious: if  $b$  is small enough so that not all types trade at  $T$ ,  $S(b, T)$  is increasing in  $b$  for all  $T$ , because higher  $b$  uniformly speeds up trade.

to drive the total surplus below the surplus with no intervention. For example with  $b = \frac{1}{10}$  and  $r = 1$ , if  $T > T^* = 0.387$  then  $S(b, T) < \frac{1}{6} = S_{LF}$  (in this case  $\bar{T} = 0.622$ ). This is illustrated in Figure 5.

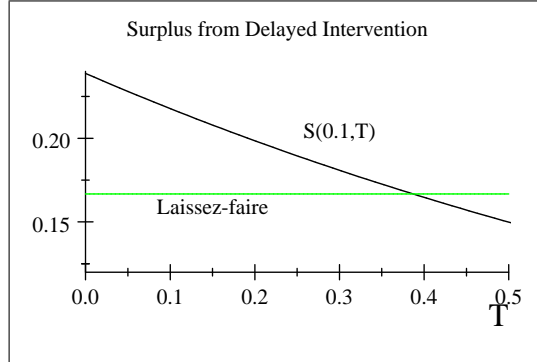


Figure 5: The Cost of Delayed Interventions.

## 5.2 Delayed Interventions with Uncertain Timing

In practice the market might expect the possibility of a government subsidy but be uncertain about its timing. We argue that this creates incentives to wait for the arrival of the intervention and may be detrimental to welfare even taking into account the benefits of the subsidy if it materializes.

To illustrate this problem, we analyze a model in which the government intervention arrives at a random time as a Poisson process with intensity  $\lambda$ . Suppose that when it finally intervenes, the government subsidizes trade sufficiently that all types trade (by offering sufficient subsidy for  $p_t(1 + s) = 1$ ).

The equilibrium dynamics depend crucially on the level of  $\lambda$ . If  $\lambda$  is small, there will be trade even before the government subsidy arrives. If it is high, the market will shut down completely until the arrival of the subsidy.

We start with the first, more interesting case. If  $\lambda < r$ , then trade is smooth until arrival of the subsidy. Equilibrium cutoffs  $k_t$  are characterized by the following differential equation (with a boundary condition  $k_0 = 0$ ):

$$r(v(k_t) - k_t) = v'(k_t)\dot{k}_t + \lambda(1 - v(k_t)) \quad (9)$$

The left-hand side is the familiar discounting cost of not taking the price today. The first term on the right-hand side is the familiar increase in price in case the intervention does not



arrive (as before, both use the zero-profit condition  $p_t = v(k_t)$ ). The new term is the last term on the right-hand side: by delaying trade, the current cutoff type can hope to receive the subsidized price 1 instead of the non-subsidized price  $v(k_t)$ .

A higher  $\lambda$  has two opposing welfare effects. The direct effect is that it speeds up the arrival of the subsidy, which increases welfare. The indirect, equilibrium effect, is that it slows down trade before arrival and it decreases welfare (the indirect effect can be seen from (9) since  $\dot{k}_t$  that solves it is decreasing in  $\lambda$ ).

Connecting this observation to real-life events, market participants' beliefs that Federal Reserve and/or the US Treasury would intervene in some financial markets, might have contributed to the reduction of trade volume in some of the those markets after the collapse of Lehman Brothers. While asymmetric information was likely the primary culprit, the expectations of future actions could make it worse.

Using  $v(c)$  and  $F(c)$  from the benchmark example, the differential equation (9) simplifies to:

$$(r - \lambda)(1 - k_t) = \dot{k}_t.$$

After solving it and computing total welfare, we can show that the negative, indirect effect always dominates (unless  $\lambda$  is so high that the market shuts down completely):

**Proposition 3** *Assume  $v(c) = \frac{1+c}{2}$  and  $F(c) = c$ . Suppose the government subsidy that induces first-best trade arrives at a Poisson rate  $\lambda$ .*

- 1) *If  $\lambda < r$ , there is trade in equilibrium even before the subsidy arrives and the equilibrium welfare is strictly decreasing in  $\lambda$ .*
- 2) *If  $\lambda > r$  then the market shuts down in the anticipation of the subsidy and equilibrium welfare is increasing in  $\lambda$  (as  $\lambda \rightarrow \infty$  the surplus converges to first-best). In that range, the surplus is higher than the laissez-faire equilibrium surplus if and only if  $\lambda$  is sufficiently higher than  $r$ .*

In words, over a large range of arrival rates, the expectation of the possibility of arrival reduces welfare in our benchmark example. In fact, for the delayed subsidy to have a hope at improving welfare, the arrival rate has to be sufficiently high so that the market closes down completely.

Figure 6 illustrates the results for  $r = 1$ .

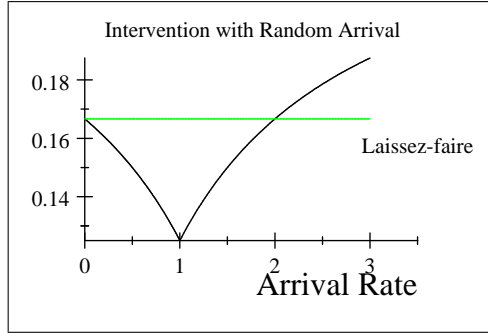


Figure 6: Cost of Random Arrival of Subsidy.

The intuition behind the second part of the above proposition is straightforward. For  $\lambda$  sufficiently large there will not be any trade until the government intervenes. Hence, there is only the first, direct effect. Since equilibrium surplus is continuous in  $\lambda$  and it decreases from  $S_{LF}$  as  $\lambda$  increases from 0 to  $r$ , it has to increase sufficiently more to recover back to  $S_{LF}$ .

## 6 Conclusions

In this paper we have analyzed government interventions in a dynamic market with asymmetric information. Our main result is that even without the use of subsidies, efficiency can be improved over the laissez-faire equilibrium. Setting high taxes for a short period, after an initial tax-exempt auction or trading window, induces more sellers to trade early and enhances efficiency. Remarkably, under a fairly commonly used regularity condition, the optimal government policy is to set high taxes for  $t > 0$ , effectively shutting down private markets. Of course these results have to be interpreted with caution since they rely on the assumption that the liquidity shock arrives once at the same time for all sellers. In reality, since additional shocks may arrive over time, the optimal policy has to balance the flexibility to respond to new shocks and the benefits of pooling trade early. Finding an optimal policy in such an environment would be additionally complicated because, for the same reasons as we discuss in Section 5, if current market participants expected future shocks and corresponding future interventions, it would create additional negative effects of slowing down trade.

Freeing the government from the requirement that its intervention must be budget-neutral,

i.e. allowing for bailouts, can be very valuable.<sup>21</sup> Moreover, although the particular form in which the subsidy is provided is not important, its timing is. Subsidies can greatly enhance welfare when provided immediately or quickly after the shock, but they can even destroy surplus if they are delayed.

It is natural to ask how these insights extend to normal times when we might still think there is some amount of adverse selection in the market. Several interesting complications can arise. For example, as opposed to the whole market being hit by a liquidity shock, as in the recent financial crisis which gives us a clear notion of time zero, in many markets the time the game actually starts is ill-defined and/or sellers arrive to the market at different times. Janssen and Karamychev (2002) show that equilibria in dynamic markets with dynamic entry can be qualitatively different from markets with one-time entry if the "time on the market" is not observed by the market (see also Hendel, Lizzeri and Siniscalchi 2005 and Kim 2012 about the role of observability of past transaction/time on the market). As pointed out recently by Roy (2012), a dynamic market can suffer from an additional inefficiency if buyers are heterogeneous, because the high valuation buyers are more eager to trade sooner and it may be that they are the efficient buyers of the high quality goods. Incorporating these considerations into our design questions would introduce new tradeoffs which are interesting avenues for future research.

## 7 Appendix

**Proof of Proposition 1.** First note that our requirement  $p_t \geq v(k_t)$  implies that there cannot be any atoms of trade, i.e. that  $k_t$  has to be continuous. Suppose not, that at time  $s$  types  $[k_s, k_{s+}]$  trade with  $k_s < k_{s+}$ . Then at time  $s + \varepsilon$  the price would be at least  $v(k_s)$  while at  $s$  the price would be strictly smaller to satisfy the zero-profit condition (1E). If so, then for small  $\varepsilon$  all types in  $[k_s, k_{s+}]$  would be better off not trading at  $s$ , a contradiction. Therefore we are left with processes such that  $k_t$  is continuous and  $p_t = v(k_t)$  at any time such that  $\dot{k}_{t+} > 0$ . If  $k_t$  is strictly increasing over time, we need that  $r(p_t - k_t) = \dot{p}_t$ : if price was rising faster, current cutoffs would like to wait, a contradiction. If prices were rising slower, over any time interval starting at  $s$ , there would be an atom of types trading at  $s$ , another contradiction. So the only remaining possibility is that  $k_t$  is constant over some interval  $[s_1, s_2]$ . Since the price at  $s_1$  is  $v(k_{s_1})$  and the price at  $s_2$  is  $v(k_{s_2})$ , if there is indeed no trade in that time interval, then  $p_{s_1} = p_{s_2}$ . But then there exist a positive measure of

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<sup>21</sup>Note that these bailouts do not affect the ex-ante incentives of the banks to screen the quality of their assets but they do affect their choices of liquidity. Potentially inducing them to take illiquid positions.

types  $k > k_{s_1}$  such that

$$v(k_{s_1}) > (1 - e^{r(s_2-s_1)})k + e^{r(s_2-s_1)}v(k_{s_1})$$

Since after  $s_2$  there are no atoms of trade, the equilibrium continuation payoff of types  $k > k_{s_1}$  is smaller than  $(1 - e^{r(s_2-s_1)})k + e^{r(s_2-s_1)}v(k)$  since these types trade at price  $v(k)$  but later than  $t = s_2$ . Since  $v(k)$  is continuous, there exists an  $\varepsilon$  such that types  $k \in [k_{s_1}, k_{s_1} + \varepsilon]$  would strictly prefer to trade at  $t = s_1$  than to follow the postulated equilibrium. That leads to the final contradiction. ■

**Proof of Theorem 1.** To establish that the market with  $\tau^\Delta$  is more efficient than under laissez-faire, we show an even stronger result: that for small  $\Delta$  there is more trade at  $t = 0$  with  $\tau^\Delta$  than with  $\tau_t = 0$  over the time interval  $[0, \Delta]$ . Since under  $\tau^\Delta$  the equilibrium after  $\Delta$  is characterized by the same differential equation as under laissez-faire, if the boundary condition at  $\Delta$  is higher, all types trade faster in equilibrium under  $\tau^\Delta$ .

Let  $\kappa_\Delta$  be the highest type that trades at  $t = 0$  under  $\tau^\Delta$  (i.e.  $\kappa_\Delta = k_{0+}$ ). Let  $k_\Delta^{LF}$  the equilibrium cutoff at time  $\Delta$  under laissez-faire. Since  $\lim_{\Delta \rightarrow 0} \kappa_\Delta = \lim_{\Delta \rightarrow 0} k_\Delta^{LF} = 0$  (for  $\kappa_\Delta$  see discussion in Step 1 below), to establish that  $\kappa_\Delta > k_\Delta^{LF}$  for small  $\Delta$ , it is sufficient to show:

$$\lim_{\Delta \rightarrow 0} \frac{\partial \kappa_\Delta}{\partial \Delta} > \lim_{\Delta \rightarrow 0} \frac{\partial k_\Delta^{LF}}{\partial \Delta}$$

**Step 1:** Characterizing  $\lim_{\Delta \rightarrow 0} \frac{\partial \kappa_\Delta}{\partial \Delta}$ .

Consider policy  $\tau^\Delta$ .

First notice that since  $\tau_t = \tau > 0$  for  $t \in (0, \Delta]$  and  $\tau_t = 0$  for  $t > \Delta$ , for small  $\Delta$  there cannot be any trade in  $t \in (0, \Delta]$ . Suppose not. Let  $k^*$  be the supremum over types that trade in that time interval. All types that trade in that interval get a payoff no higher than  $(1 - \tau)v(k^*)$  (buyers would lose money if they paid more than  $p = v(k^*)$ , the government takes  $\tau$  of that price, and the best case scenario is that they trade with no delay). For small  $\Delta$ , that payoff is smaller than  $(1 - e^{-r\Delta})k^* + e^{-r\Delta}v(k^*)$ . Since  $p_{\Delta+} \geq v(k^*)$ , all types that trade in  $(0, \Delta]$  would be strictly better off waiting for the tax to be removed, a contradiction.

When the taxes are removed after  $t = \Delta$ , the continuation equilibrium is unique and is characterized in Proposition 1 albeit with a different starting lowest type. Namely, for  $t > \Delta$ :

$$\begin{aligned} p_t &= v(k_t) \\ r(v(k_t) - k_t) &= v'(k_t) \dot{k}_t \end{aligned}$$

with a boundary condition:

$$k_\Delta = \kappa_\Delta.$$

The break even condition for buyers (1E) at  $t = 0$  implies:

$$p_0 = E[v(c) | c \in [0, \kappa_\Delta]].$$

Seller optimality (2E) implies that type  $\kappa_\Delta$  must be indifferent between trading at this price at  $t = 0$  and selling for  $p_{\Delta+} = v(\kappa_\Delta)$  at  $t = \Delta$ :

$$v(\kappa_\Delta) - p_0 = (1 - e^{-r\Delta})(v(\kappa_\Delta) - \kappa_\Delta)$$

Combining these two conditions we get that  $\kappa_\Delta$  is a solution to:

$$v(\kappa_\Delta) - E[v(c) | c \in [0, \kappa_\Delta]] = (1 - e^{-r\Delta})(v(\kappa_\Delta) - \kappa_\Delta) \quad (10)$$

For small  $\Delta$  this equation has a unique solution. Using implicit function theorem we can show that:

$$\lim_{\Delta \rightarrow 0} \frac{\partial \kappa_\Delta}{\partial \Delta} = \frac{2rv(0)}{v'(0)}$$

Intuitively, for small  $\Delta$ ,  $E[v(c) | c \leq c \in [0, \kappa_\Delta]] \approx \frac{v(0)+v(\kappa_\Delta)}{2}$  (because we have assumed that  $f(c)$  and  $v(c)$  are positive and continuous). So the benefit of waiting, the left-hand side of (10), is approximately  $\frac{v(\kappa_\Delta)-v(0)}{2}$ ; while the cost of waiting, the right-hand side of (10), is approximately  $r\Delta v(0)$ . So for small  $\Delta$ ,  $\kappa_\Delta$  solves approximately

$$\frac{v(\kappa_\Delta) - v(0)}{2} = r\Delta v(0)$$

which yields  $\frac{\partial \kappa_\Delta}{\partial \Delta} = \frac{2rv(0)}{v'(0)}$  as  $\Delta \rightarrow 0$ .

**Step 2:** Characterizing  $\lim_{\Delta \rightarrow 0} \frac{\partial k_\Delta^{LF}}{\partial \Delta}$ .

Consider the laissez-faire economy. Since  $k_t$  is defined by the differential equation

$$r(v(k_t) - k_t) = v'(k_t) \dot{k}_t,$$

for small  $\Delta$ :

$$k_\Delta^{LF} \approx r\Delta \frac{v(0)}{v'(0)},$$

and more precisely:

$$\lim_{\Delta \rightarrow 0} \frac{\partial k_{\Delta}^{LF}}{\partial \Delta} = \frac{rv(0)}{v'(0)}.$$

Summing up steps 1 and 2, we have:

$$\lim_{\Delta \rightarrow 0} \frac{\partial \kappa_{\Delta}}{\partial \Delta} = 2 \lim_{\Delta \rightarrow 0} \frac{\partial k_{\Delta}^{LF}}{\partial \Delta}$$

which implies the claim.

### Step 3: Pareto Improvement

Take any  $\Delta$  such that there is no trade in  $t \in (0, \Delta]$  under  $\tau^{\Delta}$  and that  $k_{\Delta}^{LF} < \kappa_{\Delta}$ . Since all types  $c > \kappa_{\Delta}$  trade at the same price but sooner in the market with  $\tau^{\Delta}$  than in the laissez-faire economy, it is immediate that they all prefer the former.

Type  $c = \kappa_{\Delta}$  also strictly prefers  $\tau^{\Delta}$ : while he is trading at  $t = 0$ , he has the option to trade at  $p = v(\kappa_{\Delta})$  at  $t = \Delta$  while in the laissez-faire economy he trades at the same price but later. By revealed preference he is strictly better off under  $\tau^{\Delta}$ .

Finally consider any type  $c < \kappa_{\Delta}$ . All these types get the same payoff under  $\tau^{\Delta}$ :

$$U_{\tau}(c) = U_{\tau}(\kappa_{\Delta}) = E[v(c) | c \in [0, \kappa_{\Delta}]] \text{ for all } c \leq \kappa_{\Delta}.$$

On the other hand, it is immediate that in the laissez-faire economy the equilibrium payoff is weakly increasing in type (by revealed preference, type  $c'$  can trade at the same time and price as any type  $c < c'$  and since  $c'$  gets a higher payoff flow from holding his asset, his payoff from this strategy is at least as high as type's  $c$ ). Combining these observations yields:

$$U_{LF}(c) \leq U_{LF}(\kappa_{\Delta}) < U_{\tau}(\kappa_{\Delta}) = U_{\tau}(c) \text{ for all } c \leq \kappa_{\Delta}$$

and that finishes the proof. ■

**Proof of Theorem 2.** We use mechanism design to establish the result. The mechanism designer chooses a direct revelation mechanism that maps reports of the sellers to a probability distribution over times they trade and to transfers from the buyers to the mechanism designer and from the designer to the sellers. The constraints on the mechanism are: incentive compatibility for the sellers (to report truthfully); individual rationality for the sellers and buyers (sellers prefer to participate in the mechanism rather than hold the asset forever and the buyers do not lose money on average); and that the mechanism designer does not lose money on average.

Using the regularity condition, we characterize a direct mechanism that maximizes dis-

counted gains from trade. We then show that if the environment is regular, the postulated policy has a corresponding equilibrium that implements the outcome of this best mechanism.

An optimal mechanism leaves the buyers with no surplus (since he could reduce the payment to the buyers and use the savings to increase efficiency of trade). Hence, we can focus on general direct revelation mechanisms described by 2 functions,  $x(c)$  and  $P(c)$ , where  $x(c)$  is the discounted probability of trade over all possible trading times and  $P(c)$  is the transfer received by the seller. Letting  $G_t(c)$  denote for a given type the distribution function over the times of trade:

$$x(c) = \int_0^T e^{-rt} dG_t(c).$$

Since all players are risk neutral, the mechanism depends on  $G_t$  only via  $x(c)$ .

The objective function of the mechanism designer is to maximize

$$\max_{x(c), P(c)} \int x(c) (v(c) - c) f(c) dc. \quad (11)$$

We now describe the constraints.

The seller's value function in the mechanism is:

$$U(c) = P(c) + (1 - x(c))c \quad (12)$$

$$= \max_{c'} P(c') + (1 - x(c'))c \quad (13)$$

Using the envelope theorem:

$$U(c) = U(1) - \int_c^1 (1 - x(c)) dc \quad (14)$$

Seller *IR* constraint is  $U(c) \geq c$  and in the optimal mechanism it binds at  $c = 1$ .<sup>22</sup> Incentive compatibility for the sellers requires that the envelope formula (14) holds and that  $x(c)$  is weakly decreasing.

Since the buyers are willing to pay at most  $\int_0^1 x(c) v(c) f(c) dc$ , the budget constraint of the seller is:

$$\int_0^1 (x(c) v(c) - P(c)) f(c) dc \geq 0$$

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<sup>22</sup>Since  $U'(c) = 1 - x(c) \leq 1$ , if the IR constraint is satisfied at  $c = 1$ , it is satisfied for all types. IR binds at  $c = 1$  since otherwise the mechanism designer could reduce  $P(c)$  by a constant and still satisfy all constraints.

From (12), we can write  $P(c)$  as:

$$P(c) = U(c) - (1 - x(c))c$$

Substituting this to the left-hand-side of the budget constraint we get express it as a function of the allocation alone:

$$\int_0^1 (x(c)(v(c) - c)) f(c) dc - \int_0^1 (U(c) - c) f(c) dc \geq 0$$

We can use (14) and integration by parts to write the last term as

$$\begin{aligned} & \int_0^1 (U(c) - c) f(c) dc \\ &= (U(c) - c) F(c) \Big|_{c=0}^{c=1} - \int_0^1 (U'(c) - 1) F(c) dc \\ &= \int_0^1 x(c) F(c) dc \end{aligned}$$

to obtain the final form of the budget constraint:

$$\int_0^1 (x(c)(v(c) - c)) f(c) dc - \int_0^1 x(c) F(c) dc \geq 0 \quad (15)$$

The first term of the constraint is the revenue the designer can obtain from the buyers and the second term is the information rent he has to pay the sellers to participate in the mechanism.

We now optimize (11) subject to (15), ignoring necessary monotonicity of  $x(c)$  that assures that reporting  $c$  truthfully is incentive compatible (we check later that it is satisfied in the solution).

The derivative of the Lagrangian with respect to  $x(c)$  is:

$$L(c) = (v(c) - c) f(c) \left( 1 + \Lambda \left( 1 - \frac{F(c)}{(v(c) - c) f(c)} \right) \right)$$

where  $\Lambda > 0$  is the Lagrange multiplier.<sup>23</sup>

Note that  $L(c)$  is (weakly) positive for  $c = 0$  (strictly if  $v(0) > 0$ ). Suppose  $\frac{f(c)}{F(c)}(v(c) - c)$  is decreasing (which is our regularity assumption). Then  $L(c)$  crosses zero only once because

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<sup>23</sup> $\Lambda$  is strictly positive in the solution since otherwise the budget constraint would not be binding and we would get  $x(c) = 1$  for all  $c$  (first-best), but that would violate 15.



$\left(1 + \Lambda \left(1 - \frac{F(c)}{(v(c)-c)f(c)}\right)\right)$  is decreasing and  $(v(c) - c) f(c)$  is positive. Let  $c^*$  be the largest solution to

$$1 + \Lambda \left(1 - \frac{F(c)}{(v(c) - c) f(c)}\right) = 0.$$

An optimal  $x(c)$  is then:

$$x(c) = \begin{cases} 1 & \text{if } c \leq c^* \\ 0 & \text{if } c > c^* \end{cases}$$

Since  $x(c)$  is monotone, a mechanism with this allocation (and appropriate  $P(c)$ ) is incentive compatible.

That describes the optimal allocation in the relaxed problem: there exists a  $c^*$  such that types below  $c^*$  trade immediately and types above it never trade. The higher the  $c^*$ , the higher the gains from trade.

The largest  $c^*$  that satisfies the budget constraint (15) is the largest solution of:

$$E[v(c) | c \leq c^*] = c^* \tag{16}$$

since the LHS is the IR constraint of the buyers and the RHS is the IR constraint of the  $c^*$  seller.

A tax policy  $\tau_0 = 0$  and  $\tau_t = H$  for  $t > 0$  clearly induces an equilibrium such that there is trade only at time zero and that equilibrium satisfies (16) for some  $c^*$ . To finish the proof, we need to show that the solution to (16) exists and if the environment is strictly regular, the solution is unique.

1) **Existence.** To see that there exists at least one solution to (16) note that

$$E[v(c) | c \leq k] - k \tag{17}$$

is continuous in  $k$ , positive at  $k = 0$  and negative at  $k = 1$ . So there exists at least one solution.

2) **Uniqueness.** To see that there is a unique solution under the regularity assumption, note that the derivative of (17) at any  $k$  is

$$\frac{f(k)}{F(k)} (v(k) - E[v(c) | c \leq k]) - 1$$

When we evaluate it at points where (16) holds, the derivative is

$$\frac{f(k)}{F(k)} (v(k) - k) - 1$$

and that is by assumption decreasing in  $k$ .

Suppose that there are at least two solutions and select two: the lowest  $k_L$  and second-lowest  $k_H$ . Since  $k_L$  is the lowest solution, at that point the curve (17) must have a weakly negative slope (since the curve crosses zero from above). However, our assumption implies that curve has even strictly more negative slope at  $k_H$ . That leads to a contradiction since by assumption between  $[k_L, k_H]$  expression (17) is negative, so with this ranking of derivatives, it cannot become 0 at  $k_H$ . ■

**Proof of Proposition 2.** Denote  $z = e^{-rT}$ . Then the surplus can be written as:

$$S(z) = z \left( \int_0^{\sqrt{\frac{4b}{z}}} \left( \frac{1-c}{2} \right) dc + \int_{\sqrt{\frac{4b}{z}}}^1 \left( \frac{1-c}{1 - \sqrt{\frac{4b}{z}}} \right) \left( \frac{1-c}{2} \right) dc \right) = \frac{1}{6}z - \frac{1}{3}b + \frac{1}{3}z\sqrt{\frac{b}{z}}$$

so the surplus is increasing in  $e^{-rT}$ .

Evaluating the surplus at  $T = \bar{T}$  so that  $b = \frac{1}{4} \frac{(1-z)^2}{z}$ , for  $\bar{T}$  small enough that  $\kappa < 1$  (so that  $e^{-rT} \in (\frac{1}{2}, 1)$ ), we get

$$S(b, \bar{T}) = \frac{1}{12z} (4z - z^2 - 1)$$

which is less than  $\frac{1}{6}$  for all  $e^{-rT} < 1$  (for  $z \leq 1$  it is an increasing function and at  $z = 1$  it is  $\frac{1}{6}$ ). ■

**Proof of Proposition 3.** As we explained in the text, in case  $r > \lambda$  the equilibrium is described by the differential equation

$$(r - \rho)(1 - k_t) = \dot{k}_t.$$

With a boundary condition  $k_0 = 0$  it has a unique solution:

$$k_t = 1 - e^{-(r-\rho)t}.$$

If the subsidy arrives at time  $t$ , total surplus is:

$$\begin{aligned} S(t|\lambda) &= \int_0^t e^{-rs} (v(k_s) - k_s) \dot{k}_s ds + e^{-rt} \int_{k_t}^1 (v(c) - c) dc \\ &= \frac{1}{4} \frac{2(r - \lambda) + re^{-t(3r-2\lambda)}}{3r - 2\lambda} \end{aligned}$$

Taking expectation over the arrival time:

$$S(\lambda) = \int_0^\infty S(t|\lambda) (\lambda e^{-\lambda t}) dt = \frac{1}{4} \frac{2r - \lambda}{3r - \lambda}$$

which is decreasing in  $\lambda$ .

For the second part, when  $\lambda > r$ , total surplus is

$$S(\lambda) = \frac{1}{4} \frac{\lambda}{r + \lambda}$$

since upon arrival the government induces the first-best surplus which is  $\frac{1}{4}$  in our benchmark example. ■

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