

# Models-as-Usual for Unusual Risks? On the Value of Catastrophic Climate Change\*

Antoine Bommier<sup>†</sup>

Bruno Lanz<sup>‡</sup>

Stéphane Zuber<sup>§</sup>

October 1, 2013

PRELIMINARY DRAFT

## Abstract

We study the role of alternative intertemporal preferences in a model of economic growth, stock pollutant and endogenous risk of catastrophic collapse. The traditional “discounted utility” model assumes risk neutrality with respect to intertemporal utility. We contrast this representation of preferences with a multiplicative choice model that displays risk aversion in that dimension. First, we find that both representations of preferences can rationalize the same “business as usual” economy with no climate externality. Second, we show that once endogenous climate risks are taken into account, multiplicative preferences recommend a much tighter policy response. Plausible parametrizations of the model indicate that switching to the multiplicative preference representation has a similar effect, in terms of policy recommendations, as increasing the hazard risk of collapse by a factor of 100.

**Keywords:** Environmental policy; Climate change; Catastrophic risks; Risk aversion; Discounting.

**JEL Classification numbers:** D63, D81, D99, Q53, Q54.

---

\*The authors acknowledge financial support from Swiss Re.

<sup>†</sup>Chair for Integrative Risk Management and Economics, ETH Zürich, Switzerland. E-mail: abommier@ethz.ch.

<sup>‡</sup>Center for International Environmental Studies, Graduate Institute Geneva, Switzerland; Chair for Integrative Risk Management and Economics, ETH Zürich, Switzerland. E-mail: bruno.lanz@graduateinstitute.ch.

<sup>§</sup>CERSES, Université Paris Descartes and CNRS, France; Chair for Integrative Risk Management and Economics, ETH Zürich, Switzerland. E-mail: stephane.zuber@parisdescartes.fr.

# 1 Introduction

A substantial part of the economic literature on climate change discusses climate policy as an intertemporal trade-off, where the costs of an early intervention are compared with the costs of later measures. Such an approach is for example the baseline of the prominent contributions by Stern (2007) and Nordhaus (2008). This line of literature typically presumes that anthropogenic climate change is gradual and reversible, and that the world, as we know it, will exist for ever or, at least, that its existence may not be endangered by today's action (or inaction). However, recent literature on climate change shows an increasing concern for the risk of abrupt and irreversible changes in the climate system (e.g. Alley et al., 2003; Kriegler et al., 2009; Lenton and Ciscar, 2012).<sup>1</sup> There is then the possibility that preventive measures that could be taken today may no longer be available in the future, as a "collapse" might occur in between. The relevant trade-off is no longer that of present consumption versus future consumption, but that of consumption versus a risk of catastrophic climate change (Weitzman, 2009).

This paper considers a setting where greenhouse gas emissions increase the hazard risk of a catastrophic collapse. We define a catastrophic collapse as an irreversible regime shift, with post-collapse welfare independent of current actions. Such a setting has been studied in several contributions, starting with Cropper (1976) and including Clarke and Reed (1994), Tsur and Zemel (1996, 1998), Gjerde et al. (1999), and Karp and Tsur (2011) among others. The most pessimistic view would be to interpret such a catastrophic collapse as the end of humankind as we know it. However, it can also consist in falling back in a kind subsistence level. What matters for the analysis is that no investment can improve post-collapse welfare, so that trajectories after the regime shift are exogenous.<sup>2</sup>

Our main objective is to highlight the importance of the intertemporal preference specifica-

---

<sup>1</sup> The study by Lenton and Ciscar (2012) characterizes potentially abrupt and long-lasting changes for some major elements of the climate system, such as the weakening or shut down of the North Atlantic thermohaline circulation, the melting of the Greenland and the West Antarctic ice sheets, the die-back of the Amazon rainforest and the increasing frequency and amplitude of El Niño-Southern Oscillation. Such low-probability high-impact events pose major challenges to decision-makers, as the associated hazard risk, how these relate to human emissions, and their impact on humankind are highly uncertain.

<sup>2</sup> An alternative approach is to consider the risk of "tipping points", where the realization of the risk affects capital stock through a damage function, but where the fundamental structure of the system is not affected and thus continues afterward (see Keller et al., 2004; Lemoine and Traeger, 2013; Cai et al., 2013). In this setting, it is possible to invest towards welfare enjoyed after the risk is realized. In contrast, we consider the case where welfare after the collapse is independent of current actions.

tion when considering the trade-off between consumption and catastrophic risk reduction. In presence of collapse risk, the time at which the regime shifts occurs is a random variable that generates a risk on intertemporal utility. For this kind of problems, relying on the standard discounted utility model is a very peculiar choice, as it involves an assumption of risk neutrality with respect to intertemporal utility (Bommier, 2006). One may however think that for a given a expected time of collapse, a social planner should prefer to avoid the possibility of a catastrophic collapse occurring in the very short term, as it would result in very low level of intertemporal utility. In other words, one may want to account for risk aversion with respect to intertemporal utility, or using the terminology of Keeney (1980) and Fishburn (1984), introduce preferences for catastrophe avoidance. This requires departing from the additive separability framework, a simple possibility being to rely on multiplicative preferences. Such preferences, although much less widespread than additive ones, were used to address a number of issues, including portfolio choices (Pye, 1973), precautionary savings (van der Ploeg, 1993), the equity premium (Ahn, 1989). In a social choice setting, Bommier and Zuber (2008) have provided an axiomatic characterization of a multiplicative model fulfilling the assumption of preference stationarity. Pure time preferences are then ruled out, but time discounting arises from the combination of a risk of collapse and preferences for catastrophe avoidance. This multiplicative model thus reconciles time discounting with an equal treatment of generations.

Although the additive and multiplicative models rely on very different assumptions, we will show that both models may look very similar when considering an exogenous and constant catastrophic risk. They may therefore provide two alternative rationalizations for the same economy when climatic risks are not taken into account, what we will refer to as “business as usual” (BAU). Moreover, in absence of climate risks, the similarity between models is shown to extend, for example, to how the economy would respond to shocks affecting total factor productivity (TFP). This similarity between models in absence of climate risks contrasts with what is obtained when an endogenous collapse risk is introduced. The multiplicative model typically recommends a much more drastic intervention than the additive one. The practical relevance of our results is illustrated through a set of numerical simulations. In a first stage both the additive and multiplicative models are calibrated to approximate the dynamics of the dynamic integrated model of climate and the economy (DICE) model by Nordhaus (2008) in the BAU scenario. We

then introduce an endogenous collapse risk. It is found that the multiplicative model suggests a significantly higher willingness to reduce the risk of collapse, and thus a more rapid introduction of emissions control. Thus, although the choice between models might be considered as purely theoretical with little practical implication in a BAU economy (i.e. when ignoring climate risks), we show that it is a major driver of the policy response to the existence of catastrophic climate risks.

The remainder of the paper is organized as follows. In Section 2 we first present the technological setting. Section 3 is about the social planner's preferences. Section 4 exposes the first order conditions that characterize optimal trajectories when working with these models, and also characterize the steady states. Section 5 reports numerical simulations with a simple climate-economy model, contrasting optimal paths for additive and multiplicative models under specific risk profiles. Section 6 concludes.

## 2 The economy

Time is assumed to be continuous and denoted by the letters  $t$ ,  $\tau$  or  $T$ .<sup>3</sup> To simplify notation, for any function  $x(\cdot)$  that depends on time, we denote by  $x_t$  (and not by  $x(t)$ ) its value at time  $t$ . The steady state value of a variable  $x_t$  is denoted  $x^*$ , and its time profile is  $x$ . Consistent with this notation we will denote by  $G(x)$  by the value of the functional  $G$  applied to the profile  $x$  and by  $g(x_t)$  the value of the function  $g$  taken at the point  $x_t$ . For any function of time  $x_t$ , the notation  $\dot{x}_t$  will refer to the derivative of  $x$  with respect to  $t$ , i.e.  $\dot{x}_t = \frac{d}{dt}x_t$ .

Consider an economy where output is associated with a flow of emissions, which we interpret as greenhouse gas (GHG) emissions. Potential output at time  $t$  is denoted  $y_t$ . It is related to physical capital at time  $k_t$  through  $y_t = f(k_t)$ . The production function  $f$  is assumed to be increasing and concave. Like in Stockey (1998), emissions are caused by production depending of an index of the technology  $z_t$ , with  $0 \leq z_t \leq 1$ . Actual output at time  $t$  is  $\bar{y}_t = z_t f(k_t)$ . The ratio of GHG emission  $e_t$  on potential output is given by the differentiable, increasing, convex

---

<sup>3</sup> Working in continuous time allows us to obtain simpler analytical expressions, and we thus use it for the first sections of the paper. For the numerical part later on, we will formulate the problem in discrete time. Moving from continuous time to discrete time does not affect the qualitative properties of the problem.

function  $\varphi(z_t) = \frac{e_t}{f(k_t)}$ . GHG concentration in the atmosphere  $M_t$  develops as:

$$\dot{M}_t = e_t - \psi(M_t - \bar{M}) = \varphi(z_t)f(k_t) - \psi(M_t - \bar{M}) \quad (1)$$

where  $\psi$  is a positive real number (the natural rate of absorption), and  $\bar{M}$  is an equilibrium value for GHG concentration in the absence of anthropogenic GHG emissions.

We assume the existence of a risk of catastrophic collapse that would move the economy to a state where consumption and production trajectories are exogenous (i.e. independent of pre-collapse actions). Intuitively, the nature of this shift is so unpredictable that it is impossible to readily invest for the post-collapse welfare. Following Clarke and Reed (1994), we assume that the likelihood of such a collapse is a function of the pollution stock. Formally, we write the hazard rate of a regime shift as:

$$\mu_t = \mu(M_t)$$

where  $\mu$  is an increasing function. Furthermore, we will assume that  $\mu(0) = \mu_0 > 0$ , with  $\mu_0$  small, so that the hazard rate of collapse also captures exogenous natural events. Note that we do not consider direct damages associated with increased emissions, either through damages reducing consumption or as an independent element of people welfare. This would solely increase incentives to reduce emissions and distract us from the main point of the paper.

The last aspect of the economy is a simple growth model with only one productive sector. Each period, actual output  $\bar{y}_t$  can be either consumed  $c_t$  or invested to accumulate physical capital. The motion equation for capital is:

$$\dot{k}_t = z_t f(k_t) - \delta k_t - c_t$$

where  $\delta$  is the instantaneous rate of capital depreciation. The planner's problem involves choosing the optimal trajectories for  $c_t$  and  $z_t$ , his objective being discussed in the next section.

### 3 Social preferences

We consider a social planner who deems that welfare at time  $t$  exclusively depends on consumption at time  $t$ . The planner is aware of the possibility of a regime shift, and that he cannot

have any influence on what happens after such a shift. Since the regime shift does not occur deterministically, the planner needs to compare random trajectories. We assume that he is an expected utility maximizer, in the general sense that the utility index is a weighted sum of intertemporal utilities enjoyed under alternative state of the worlds. In our context, the planner weights the intertemporal utility associated with all possible collapse date by their respective probabilities.<sup>4</sup>

Formally, the time at which the world will experience a regime shift is viewed ex-ante as random. Therefore, the planner assesses an infinitely long consumption plan  $c$ , which may be interrupted by a regime shift. We will denote by  $U(c, t)$  the utility the planner associates to the case where the consumption profile  $c$  is interrupted by a regime shift at time  $t$ . The planner will choose  $c$  in order to maximize his expected utility, denoted  $W(c)$ , which is simply the expected value of  $U(c, t)$  given  $g_t$ , the distribution function of the random variable  $t$  :

$$W(c) = \int_0^{+\infty} U(c, t)g_t dt \quad (2)$$

Further denote by  $s_t$  the probability that the world will still be in the usual regime at time  $t$  (the survival probability). We know from survival analysis that  $\mu_t = \frac{g_t}{s_t} = -\frac{\dot{s}_t}{s_t}$ . In consequence,  $s_t = \exp\left(-\int_0^t \mu_\tau d\tau\right)$  and

$$g_t = \mu_t \exp\left(-\int_0^t \mu_\tau d\tau\right) = \mu(M_t) \exp\left(-\int_0^t \mu(M_\tau) d\tau\right). \quad (3)$$

We can thus re-express the social expected utility (2) as:

$$W(c) = \int_0^{+\infty} U(c, t)\mu(M_t) \exp\left(-\int_0^t \mu(M_\tau) d\tau\right) dt. \quad (4)$$

Hence the social welfare function is a function of the consumption path as well as the hazard rate associated with the trajectory of the pollution stock.

In order to focus the discussion, we impose further structure on the planner's preferences.

---

<sup>4</sup> It is worth emphasizing that assuming expected utility does not involve making assumptions regarding how consumption is aggregated over time to provide an intertemporal utility. The most common approach consists in assuming that intertemporal utility is the sum of instantaneous utilities, but other aggregation procedures (e.g. a multiplicative aggregation) are of course possible.

First, we assume that preferences are stationary in the sense of Koopmans (1960). This reflects the view that preferences regarding the future should be independent of the past and of the calendar date, and ensures that the ensuing plans are time consistent. Second, we assume that preferences are weakly separable in the sense that their restriction to deterministic consumption paths fulfills the separability assumption described in Gorman (1968). In other words, in absence of uncertainty, the trade off between welfare at two different moments in time is independent from what happens in other periods.

As is shown in Bommier (2012), these standard assumptions imply that the utility index, whose expectation is maximized by the planner, must have either an additive or a multiplicative structure. More specifically, assuming that exogenous post-collapse welfare is equivalent to the welfare associated with consuming  $\underline{c}$  for ever, and normalizing (without loss of generality) the utility function  $u(\cdot)$  so that  $u(\underline{c}) = 0$ , we must have:

$$U(c, t) = \int_0^t e^{-\theta\tau} u(c_\tau) d\tau \quad \text{in the additive case,} \quad (5)$$

$$U(c, t) = \frac{1 - \exp\left\{-\varepsilon \int_0^t u(c_\tau) d\tau\right\}}{\varepsilon} \quad \text{in the multiplicative case,} \quad (6)$$

where  $\theta \geq 0$  is the pure rate of time preference and  $\varepsilon \geq 0$  measures the degree of absolute risk aversion with respect to intertemporal utility (i.e. an increase in  $\varepsilon$  increases risk aversion in the sense of Kihlstrom and Mirman (1974)). In the remainder of the paper, we call  $\varepsilon$  the *temporal risk aversion parameter*.

Normative and ethical aspects of additive and multiplicative preferences have been discussed in Bommier and Zuber (2008). In short, additive preferences assume pure time preference, implying that different generations are given different utility weights as a consequence of their years or birth. This aspect has often been criticized for being contrary to intergenerational equity. Multiplicative preferences rule out this possibility but make it possible for the the planner to exhibit risk aversion with respect to intertemporal utility, or equivalently, preference for catastrophe avoidance. This may lead the social planner to choose policies that do not optimize individual welfare in order to reduce the aggregate risk. There is a debate about such ethical implications (Manski and Tetenov, 2007; Fleurbaey, 2010), and we do not suggest that one model

is more appropriate than the other. Rather, our aim is to highlight implications of alternative preferences towards social risk in the context of climate change policy.

The additive and multiplicative models are both extension of the case where utility would be given by  $U(c, t) = \int_0^t u(c_\tau) d\tau$ . This specification is indeed obtained when  $\theta = 0$  in the additive case and when  $\varepsilon = 0$  in the multiplicative case. In the expected utility framework, it is impossible to combine pure time preference and preferences for catastrophe avoidance without giving up the assumption of preference stationarity. Stationary preferences exhibiting both pure time preferences and catastrophe avoidance may be obtained by leaving the domain of expected utility theory, as in the “risk-sensitive preferences” model introduced by Hansen and Sargent (1995) and axiomatized in Bommier and LeGrand (2013). In fact, additive and multiplicative models are just two polar cases in the class of risk-sensitive preferences. Thus, our results could be extended to a continuum of models in which  $\theta > 0$  and  $\varepsilon > 0$ . Focusing on additive and multiplicative specifications is however sufficient, and the simplest way, to provide insights about the importance of the representation of intertemporal preferences.

One final technical consideration worth mentioning is that the integral in Equation (2) does not necessarily converge. For the problem at hand however, and assuming that  $\theta$  and  $\varepsilon$  are non-negative, this issue can be avoided by making the assumption that consumption is bounded.

For both additive and multiplicative preferences, we can apply integration by part on equation (4) to obtain:

$$W(c) = \left[ -U(c, t) \exp\left(-\int_0^t \mu(M_\tau) d\tau\right) \right]_0^{+\infty} + \int_0^{+\infty} \frac{dU(c, t)}{dt} \exp\left(-\int_0^t \mu(M_\tau) d\tau\right) dt \quad (7)$$

The first term of the right hand side is zero, and the second one can be simplified by introducing a function  $v(\mu_t, c_t)$  from  $\mathbb{R} \times \mathbb{R}$  into  $\mathbb{R}$  given by:

- $v(\mu_t, c_t) = \theta + \mu_t$  in the additive case;
- $v(\mu_t, c_t) = \mu_t + \varepsilon u(c_t)$  in the multiplicative case.

We can now write equation (7) as:

$$W(c) = \int_0^{+\infty} \exp\left(-\int_0^t v(\mu(M_\tau), c_\tau) d\tau\right) u(c_t) dt \quad (8)$$



Using equation (8) is convenient, as the difference between both models is restricted to the function  $v(\mu_t, c_t)$ .

For notational purposes, we write the continuation utility  $\mathcal{U}(c, T)$  as the future remaining expected utility discounted at time  $T$ :

$$\mathcal{U}(c, T) = \int_T^{+\infty} \exp\left(-\int_T^t v(\mu(M_\tau), c_\tau) d\tau\right) u(c_t) dt. \quad (9)$$

We shall now define two important concepts.

**Definition 1** *The social discount rate is:*

$$\rho(c, T) = -\frac{d}{dT} \left( \log \frac{\partial W(c)}{\partial c_T} \right) \Big|_{\dot{c}_T=0} \quad (10)$$

The discount rate measures how rapidly the marginal utility of consumption is declining with time, controlling for variations in consumption.<sup>5</sup> A similar definition can for example be found in Epstein (1987).

The second concept quantifies the willingness to sacrifice current consumption to lower the hazard risk of a regime shift.

**Definition 2** *The social value of catastrophic risk reduction is:*

$$\mathcal{V}(c, T) = - \left( \frac{\partial W(c)}{\partial \mu_T} \right) / \left( \frac{\partial W(c)}{\partial c_T} \right)$$

This social value of catastrophic reduction is therefore simply the opposite of the marginal rate of substitution between consumption at time  $t$  and hazard risk of regime shift at time  $t$ . In the parallel literature on individual choice with endogenous mortality (death being a particular regime shift), this is called the value of a statistical life.

The following proposition provides expressions for the social discount rate and for the social value of catastrophic risk reduction for the additive and multiplicative cases:

**Proposition 1**

**(a)** *In the additive case,  $\rho(c, T) = \mu(M_T) + \theta$  and  $\mathcal{V}(c, T) = \frac{\mathcal{U}(c, T)}{u'(c_T)}$ .*

---

<sup>5</sup> As we work in continuous time, the derivatives  $\frac{\partial W(c)}{\partial c_T}$  is to be understood as a Volterra derivative.

(b) In the multiplicative case,  $\rho(c, T) = \frac{\mu(M_T)}{1 - \varepsilon \mathcal{U}(c, T)}$  and  $\mathcal{V}(c, T) = \frac{\mathcal{U}(c, T)}{u'(c_t)(1 - \varepsilon \mathcal{U}(c, T))}$ .

**Proof.** See Appendix A. ■

In the additive model, the discount rate is just the sum of the rate of pure time preferences and of the hazard risk of regime shift, as is well known since Yaari (1965). In particular the rate of time discounting is independent of consumption. In the multiplicative model, the discount rate involves the continuation utility, which depends on future consumption and future hazard risk. This implies that changing the parameters of the multiplicative model has a non-trivial impact on the discount rate as the continuation utility function may depend on these parameters. Nevertheless, one may show that for a given risk of regime shift a higher  $\varepsilon$  would lead to a larger discount rate (Bommier and Zuber, 2008). Indeed if a regime shift occurs, later generations will not be able to benefit from foregone consumption by early generations, which leads a risk averse planner to put even more weight on the welfare of early generations.

The social value of catastrophic risk reduction also has very different expressions in both models. In both cases this value depends positively on the continuation utility (thus on future consumption), as it increases the cost of a regime shift, and thus the willingness to pay to lower its risk. In the additive case, continuation utility is a decreasing function of the rate of time preference. Thus a higher preference for the present imply a lower willingness to pay for risk reduction. In the multiplicative case the role of risk aversion is less obvious, as it combines two effects. On the one hand, a greater  $\varepsilon$  implies a higher aversion to the risk of regime shift, thus increasing the willingness to pay for risk reduction. On the other hand, in presence of a collapse risk, a greater  $\varepsilon$  increases the value of immediate consumption through the discount rate, which lowers the willingness to pay for risk reduction. The overall effect is ambiguous, which is in line with findings on the role of risk aversion on preventive measures (Jullien et al., 1999).

In order to get some intuition about the implications of additive and multiplicative preferences in the presence of an endogenous collapse risk, one can observe that in the additive case:

$$\mathcal{V}(c, T) = \frac{\rho(c, T)\mathcal{U}(c, T)}{u'(c_T)} \frac{1}{\mu_t + \theta}$$

while in the multiplicative case:

$$\mathcal{V}(c, T) = \frac{\rho(c, T)\mathcal{U}(c, T)}{u'(c_T)} \frac{1}{\mu_t}.$$

If consumption and the hazard rate of regime shift were to remain constant over the time, then in both models, one would have  $\rho(c^*, T)\mathcal{U}(c^*, T) = u(c^*)$ . In that case, the value of catastrophic risk reduction would be  $(1 + \frac{\theta}{\mu})$  larger in the multiplicative model than in the additive one. Predictions of both models would then strongly diverge when the pure rate of time preference in the additive model is large compared to the hazard risk.

## 4 Optimal paths and steady states

### 4.1 Business as usual

We first study the problem of choosing an optimal consumption plan when the hazard risk of collapse is constant and exogenous. We label this problem the BAU problem because the planner does not account for the effect of emission on the risk of a catastrophic event. Our objective is to highlight that the choice between the two social objectives does not matter so much in problems that focus on intertemporal consumption trade-offs. This contrasts with a setting in which the planner accounts for the effects of current decisions on the hazard of a catastrophic collapse, which we discuss next (Section 4.2).

Consider a social planner who faces a hazard risk  $\mu_t = \mu_0$  in all periods. His objective, which defines the BAU problem, is:

$$\begin{aligned} \max_{(c, z)} \quad & W(c) = \int_0^{+\infty} \exp\left(-\int_0^t v(\mu(M_\tau), c_\tau) d\tau\right) u(c_t) dt \\ \text{s.t.} \quad & \dot{k}_t = z_t f(k_t) - \delta k_t - c_t \\ & \mu(M) = \mu_0, \quad \forall M \in \mathbb{R} \end{aligned} \quad (11)$$

with  $v(\mu_t, c_t) = \theta + \mu_0$  in the additive case and  $v(\mu_t, c_t) = \mu_0 + \varepsilon u(c_t)$  in the multiplicative case.

Because the stock of pollution does not have a direct effect on welfare and the impact of pollution on the regime shift is overlooked by the planner, there will be no emission control. Hence at the optimum we have  $z_t = 1$  for all  $t \in \mathbb{R}$ , and we have:

**Proposition 2**

(a) *If a steady state exists for an interior solution of Problem (11), then it is characterized by the following equations:*

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)} (f'(k_t) - \delta - \rho(c, t)) \quad (12)$$

$$\dot{k}_t = f(k_t) - c_t \quad (13)$$

(b) *If a steady state exists for an interior solution of Problem (11), then:*

(i) *In the additive case, it is characterized by the following equations:*

$$\mu_0 + \theta = f'(k^*) - \delta \quad (14)$$

$$f(k^*) = c^* \quad (15)$$

(ii) *In the multiplicative case, it is characterized by the following equations:*

$$\mu_0 + \varepsilon u(c^*) = f'(k^*) - \delta \quad (16)$$

$$f(k^*) = c^* \quad (17)$$

**Proof.** See Appendix B. ■

The optimal dynamics described in part (a) of Proposition 2 is the familiar optimal growth trajectory in a one-sector model. Equation (12) is the Euler equation describing the evolution of consumption as a function of the difference between the interest rate on capital investments and the social discount rate. Equation (12) holds for both the additive and multiplicative models, but the expression of the discount rate  $\rho(c, t)$  depends on the specification (Proposition 1). Equation (13) is the standard motion equation for the state variable.

A noteworthy feature of Proposition 2 is the similarity of the additive and multiplicative steady state equilibria in the BAU problem, displayed in part (b) of the Proposition. For any  $\theta$  it is possible to find an  $\varepsilon$  such that the additive and multiplicative model yield the same interest rate in the long run, and hence the same equilibrium consumption and physical capital. In particular, it suffices to define  $\varepsilon = \theta/u(c^*)$  for the steady state consumption and physical capital

to be the same in the additive and multiplicative cases.

While the parameters of the models can be calibrated to produce the same BAU steady state equilibrium, short-term dynamics will differ. For such a calibration, our numerical simulations will however suggest that optimal BAU paths for the additive and multiplicative models are very similar (see Section 5.2 below).

## 4.2 Endogenous catastrophic collapse

We now come to the specific problem studied in this paper, namely the case where the social planner anticipates the effect of his actions on the probability of a regime shift. In that case, the social planner seeks to solve the following dynamic optimization problem:

$$\begin{aligned} \max_{(c,z)} \quad & W(c) = \int_0^{+\infty} \exp\left(-\int_0^t v(\mu(M_\tau), c_\tau) d\tau\right) u(c_t) dt \\ \text{s.t.} \quad & \dot{k}_t = z_t f(k_t) - \delta k_t - c_t \\ & \dot{M}_t = \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \end{aligned} \quad (18)$$

with  $v(\mu_t, c_t) = \theta + \mu_t$  in the additive case and  $v(\mu_t, c_t) = \mu_t + \varepsilon u(c_t)$  in the multiplicative case.

Denote  $\pi(z) = z - \frac{\varphi(z)}{\varphi'(z)}$ . The optimal path for an interior solution of Problem (18) is characterized as follows:

### Proposition 3

(a) *If an interior solution of Problem (18) exists, it is characterized by the following equations:*

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)} (\pi(z_t) f'(k_t) - \delta - \rho(c_t, t)) \quad (19)$$

$$\dot{z}_t = \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \varphi'(z_t) \mu'(M_t) \mathcal{V}(c_t, t) + \delta - \left\{ \psi + \pi(z_t) f'(k_t) \right\} \right) \quad (20)$$

$$\dot{k}_t = z_t f(k_t) - c_t \quad (21)$$

$$\dot{M}_t = \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \quad (22)$$

(b) *If a steady state exists for an interior solution of Problem (18), then:*

(i) In the additive case, it is characterized by the following equations:

$$\mu(M^*) + \theta = \pi(z^*)f'(k^*) - \delta \quad (23)$$

$$\pi(z^*)f'(k^*) - \delta + \psi = \frac{u(c^*)}{u'(c^*)} \frac{\mu'(M^*)}{\mu(M^*) + \theta} \varphi'(z^*) \quad (24)$$

$$z^* f(k^*) = c^* \quad (25)$$

$$\varphi(z^*)f(k^*) = \psi(M^* - \bar{M}) \quad (26)$$

(ii) In the multiplicative case, it is characterized by the following equations:

$$\mu(M^*) + \varepsilon u(c^*) = \pi(z^*)f'(k^*) - \delta \quad (27)$$

$$\pi(z^*)f'(k^*) - \delta + \psi = \frac{u(c^*)}{u'(c^*)} \frac{\mu'(M^*)}{\mu(M^*)} \varphi'(z^*) \quad (28)$$

$$z^* f(k^*) = c^* \quad (29)$$

$$\varphi(z^*)f(k^*) = \psi(M^* - \bar{M}) \quad (30)$$

**Proof.** See Appendix C and Appendix D for the additive and multiplicative cases respectively. ■

To understand the optimal dynamics described in part (a) of Proposition 3, first remark that Equation (19) is, like before, the Euler equation. It is adapted to the present setting where net interest rate  $r_t = \pi(z_t)f'(k_t) - \delta$  is adjusted to take into account the pollution externality.

Equation (20) is an arbitrage condition on emission control. Assume that you divert money from investments in physical capital to increase emission control (i.e. decrease  $z$ ). The cost is given by the net interest rate  $r_t$ . The benefits is to decrease emissions (in proportion  $\varphi'(z)$ ), so that the risk decreases through  $\mu(M_t)$ . The value of this risk reduction is given by  $\mathcal{V}(c)$ . Hence the net benefit is  $\varphi'(z)\mu'(M)\mathcal{V}(c) - \psi$ , where  $\psi$  accounts for the reduction in risk that occurs through natural absorption. The arbitrage between costs and benefits governs the evolution of emission control.

A key difference with the BAU problem is that the steady states displayed in part (b) of Proposition 3 are different in the additive and the multiplicative cases when they are calibrated to the same BAU interest rate ( $\varepsilon = \theta/u(c^*)$ ). The key difference is in the right-hand side of Eq. (24) and (28). The difference tends to vanish whenever  $\theta$  is close to 0 (so that the associated  $\varepsilon$  is close to 0). Hence, for very low values of the rate of pure time preference (and of the

temporal risk aversion parameter), the two models behave in a similar way in the long run. This is consistent with the fact that when  $\theta = \varepsilon = 0$  the additive and multiplicative preference models are formally identical. However, as soon as we take some distance with this particular case, the models provide very distinct predictions, as pure time preferences and risk aversion with respect to the date of a catastrophic shift have different behavioral implications.

In the additive model, increasing pure time preference has two direct effects. First, it increases the social discount rate (Equation (23)). Second, it decreases the social value of catastrophic risk reduction, which is  $\frac{u(c^*)}{u'(c^*)(\mu(M^*)+\theta)}$  in steady state (Equation (24)). From the effect on the social discount rate, the future matter less for higher values of pure time preference, inducing less capital accumulation that can result in less consumption and less pollution. On the contrary, higher values of pure time preference will induce a lower value of catastrophic risk reduction, and hence less tight environmental policies that can result in more consumption and more pollution.

In the multiplicative model, increasing temporal risk aversion has only one direct effect, which is to increase the social discount rate (Equation (27)). A higher value of the temporal risk aversion parameter will therefore tend to decrease consumption but it has no direct impact on the steady state value of social value of catastrophic risk reduction risk, which is  $\frac{u(c^*)}{u'(c^*)\mu(M^*)}$ . It may still have an indirect impact through its effect on consumption and GHG concentration. It is worth noting that for the same value of  $c^*$  and  $M^*$  the value of social value of catastrophic risk reduction risk is always higher in the multiplicative model than in the additive one. This suggests that the multiplicative model may be more prone to avoid catastrophic risks.

To study whether this is actually the case, we now turn to a calibrated version of the model. It will allow us to compare not only the steady state equilibria, but also the optimal dynamics of the two models. We will also investigate the role of two key parameters: the rate of pure time preference  $\theta$  (or the associated temporal risk aversion parameter  $\varepsilon$ ) and the consumption level  $\underline{c}$  equivalent to the post regime shift welfare.

Table 1: Parametric specifications for numerical simulations

| Parameter   | Value  | Source          |
|---|--------|-----------------|
| Production technology: $y_t = Ak_t^\alpha$  |        |                 |
| $A$ Total factor productivity   | 8.5    | Calibrated      |
| $\alpha$ Share of capital   | 0.3    | Nordhaus (2008) |
| $k_0$ Initial capital stock   | 0.99   | Calibrated      |
| $\delta$ Rate of depreciation   | 0.1    | Nordhaus (2008) |
| Carbon cycle: $\Delta M_t = \kappa z_t^\beta - \psi(M_t - \bar{M})$                               |        |                 |
| $\kappa$ Emissions intensity  | 0.29   | Calibrated      |
| $\beta$ Abatement cost parameter  | 2.7    | Nordhaus (2008) |
| $\psi$ Exogenous emissions' decay   | 0.005  | Calibrated      |
| $\bar{M}$ Pre-industrial stock of emissions   | 596.4  | Nordhaus (2008) |
| $M_0$ Initial pollution stock   | 808.9  | Nordhaus (2008) |
| Instantaneous utility function: $u(c) = \frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}$ |        |                 |
| $\gamma$ Marginal utility parameter   | 2      | Calibrated      |
| $\underline{c}$ Post-collapse consumption   | 1      | –               |
| $\theta$ Pure rate of time preference (additive model)  | 0.015  | Nordhaus (2008) |
| $\varepsilon$ Temporal risk aversion (multiplicative model)                                       | 0.0014 | Calibrated      |

## 5 A numerical illustration

### 5.1 Calibration and solution method

To quantify the implications of alternative preference representations on optimal paths, we formulate a numerical version of the model presented above. Most of the functional forms and parameter values are taken directly from the Dynamic Integrated model of Climate and the Economy (DICE) model by Nordhaus (2008), or calibrated to reproduce BAU trajectories in the DICE model (i.e. no climate externality). These are summarized in Table 1.

Potential output is represented by the function  $y_t = Ak_t^\alpha$ , with  $\alpha = 0.3$ . Given our assumptions about social preferences and a depreciation rate of 10% taken from Nordhaus (2008), we calibrate the TFP parameter ( $A$ ) such that our BAU steady state output equals per capita output in 2100 in the DICE model. The initial capital stock ( $k_0$ ) is then chosen such that  $y_0$  equals DICE's per capita output in the initial year (2005).

The DICE model features a detailed representation of the carbon cycle, whereas we only have one equation describing the dynamics of the pollution stock (Equation (1)). Nevertheless,



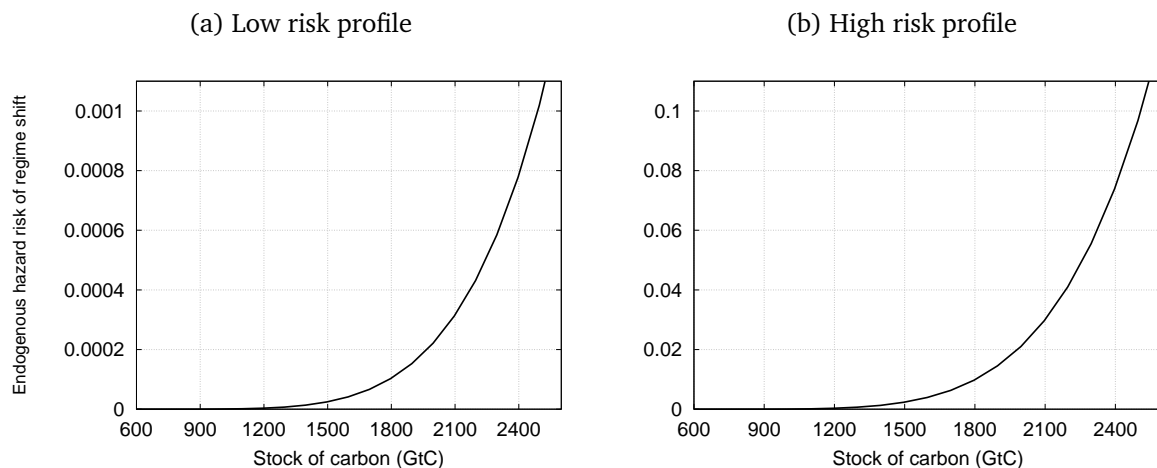
we find that we can approximate GHG dynamics quite well. First, the initial pollution stock  $M_0 = 808.9$  GtC and the pre-industrial GHG concentration  $\bar{M} = 596.4$  GtC are taken directly from DICE. Second, the natural decay of the stock, measured by  $\psi$  in our model, is determined by switching off emissions the DICE model from 2005 onwards, and simulating the evolution of DICE's GHG stock. We then choose the parameter  $\psi$  such that the trajectory of the pollution stock in our model (with no additional emissions) matches that of the DICE model. Third, the function describing emissions per unit of output is specified as  $\varphi(z_t) = \kappa z_t^\beta$ , with  $\beta = 2.7$  as in DICE. The parameter  $\kappa$ , measuring uncontrolled emissions intensity per unit of output, is chosen to closely approximate DICE's BAU pollution trajectory, given BAU values for  $y$ ,  $k$  and natural decay  $\psi$ .

The hazard risk of catastrophic collapse is described by the function  $\mu(M) = \mu_0 + \mu_1(M - \bar{M})^\sigma$ . The exogenous hazard parameter is set to  $\mu_0 = 10^{-5}$ , which implies a survival probability of 90% after 10,000 years and 40% after 100,000 years. As the GHG stock increases above  $\bar{M}$ , the hazard risk increases with parameters  $\mu_1$  and  $\sigma$ . We consider two alternative parameterizations for the hazard function, illustrated in Figure 1. Under the “low” risk profile, the endogenous part of the hazard risk,  $\mu_1(M - \bar{M})^\sigma$ , remains below  $\mu_0$  when the stock doubles relative to  $\bar{M}$ , but it increases to 0.01% when the stock triples. This implies that a tripling the GHG stock would reduce the survival probability by 2.5 percentage points after 1,000 years. Under the “high” risk profile, the endogenous part of the hazard rate is 100 times larger for the same pollution stock. The risk of collapse under BAU reaches 1% by 2100, which is in line with Weitzman (2009). Under this profile, a doubling of the stock reduces the survival probability by 3 percentage points after 100 years.

For both the additive and the multiplicative models, the instantaneous utility function is given by  $u(c_t) = \frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}$ . We calibrate  $\gamma$  such that the initial saving rate in the BAU is 22% as in the Dice model. The post-collapse consumption level  $\underline{c}$  is set to 1, which corresponds to an 85% reduction of the initial consumption level (around \$6,500). We stress that our aim is not to portray a specific catastrophe linked with climate change. Rather we see this numerical exercise as a way to illustrate the magnitude of the modeling choices. We will examine the importance of  $\underline{c}$  for our results in Section 5.4.

In the additive model, the pure rate of time preference  $\theta$  is 1.5% per year, as in the DICE

Figure 1: Pollution stock and hazard risk of collapse



model. As discussed in Section 4.1, we calibrate the temporal risk aversion parameter in the multiplicative model such that both models predict the same BAU steady state interest rate. This is achieved by setting  $\varepsilon = \theta/u(c^*)$ , where  $c^*$  is the BAU steady consumption in the additive model (determined by solving the additive model with  $\mu_1 = 0$ ).

We solve the numerical versions of problems (11) and (18) as discrete time non-linear programs. Indeed the uncertainty about collapse time  $T$  is summarized in the function  $v(\mu_t, c_t)$ , so that tools from mathematical programming can be applied. In an optimization framework however it is not possible to formulate an infinite horizon program, since it would require maximizing a sum with an infinite number of terms subject to an infinite number of constraints. We thus truncate the horizon of the problem to a finite number of periods, and approximate the solution to the infinite horizon problem with a state variable targeting approach (Lau et al., 2002).<sup>6</sup> This approach involves formulating the problem in the mixed complementarity format, which exploits the equilibrium conditions relating the constraints of the primal problem and their associated multipliers, as defined by the Karush-Kuhn-Tucker conditions. We formulate the

<sup>6</sup> If the time horizon of the problem is finite, the optimal shadow values of the stock variables are zero in finite time. Hence typically, the solution of the finite horizon problem differs from its infinite horizon counterpart. These terminal effects can be avoided by adding equilibrium conditions targeting the post-terminal evolution of the shadow values to be consistent with the infinite horizon solution. Following Lau et al. (2002), we use steady-state conditions relating the rate of change of investment to that of output to target the post-terminal value of the capital stock. For the pollution stock we relate the rate of change of the technology index to that of output to target the (negative of the) post-terminal value of the emissions stock.

system of complementarity relationships in GAMS and solve it with the PATH solver (Dirkse and Ferris, 1995).

## 5.2 Business as usual and TFP shock

We start by comparing the BAU paths implied by the additive and multiplicative models. Given our calibration procedure both models admit the same steady state interest rate, and by Proposition 2 the steady state capital stock and consumption level are identical. In the additive case, it is possible to derive closed-form solutions for the steady state values of the capital stock and consumption level, and this can be used to validate both the calibration procedure and the accuracy of the solution method.<sup>7</sup>

Figure 2 reports the optimal paths for  $c_t$  and  $k_t$ , as well as the value of their respective analytical steady state. All paths converge to their theoretical values, which demonstrates that both models can rationalize the same BAU economy, but also that we numerically approximate the true steady state with a high precision. Moreover, while the transition paths can potentially differ between additive and multiplicative specifications, optimal trajectories are almost identical. We note that the multiplicative model suggests a slightly more rapid convergence towards the steady state. This is because the discount rate is endogenous according to the multiplicative model, and thus changes during the transition.

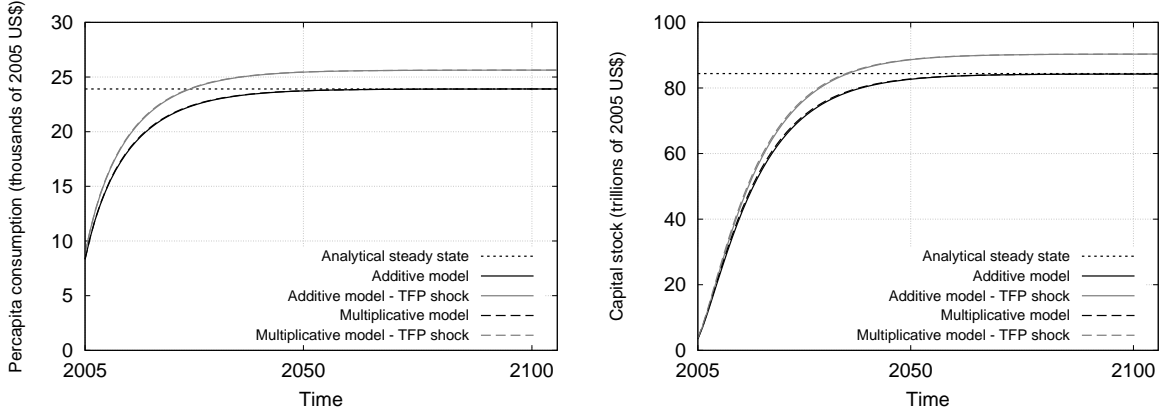
Further evidence about the similarity between the two choice models in the absence of an endogenous catastrophic risk is provided by looking at the response to a TFP shock. Figure 2 displays the optimal transition paths under a 5% increase in the TFP parameter ( $A$ ) from  $t = 0$  onwards (curve labeled “TFP shock”). We find that both the steady state values and the transition paths remain almost identical in both models.

---

<sup>7</sup> The analytical steady state in the additive model with  $\mu_1 = 0$  and  $z_t = 1$  is given by:

$$k^* = \left( \frac{A\alpha}{\delta + \theta} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad c^* = A(k^*)^\alpha - \delta k^*.$$

Figure 2: Business as usual calibration and TFP shock



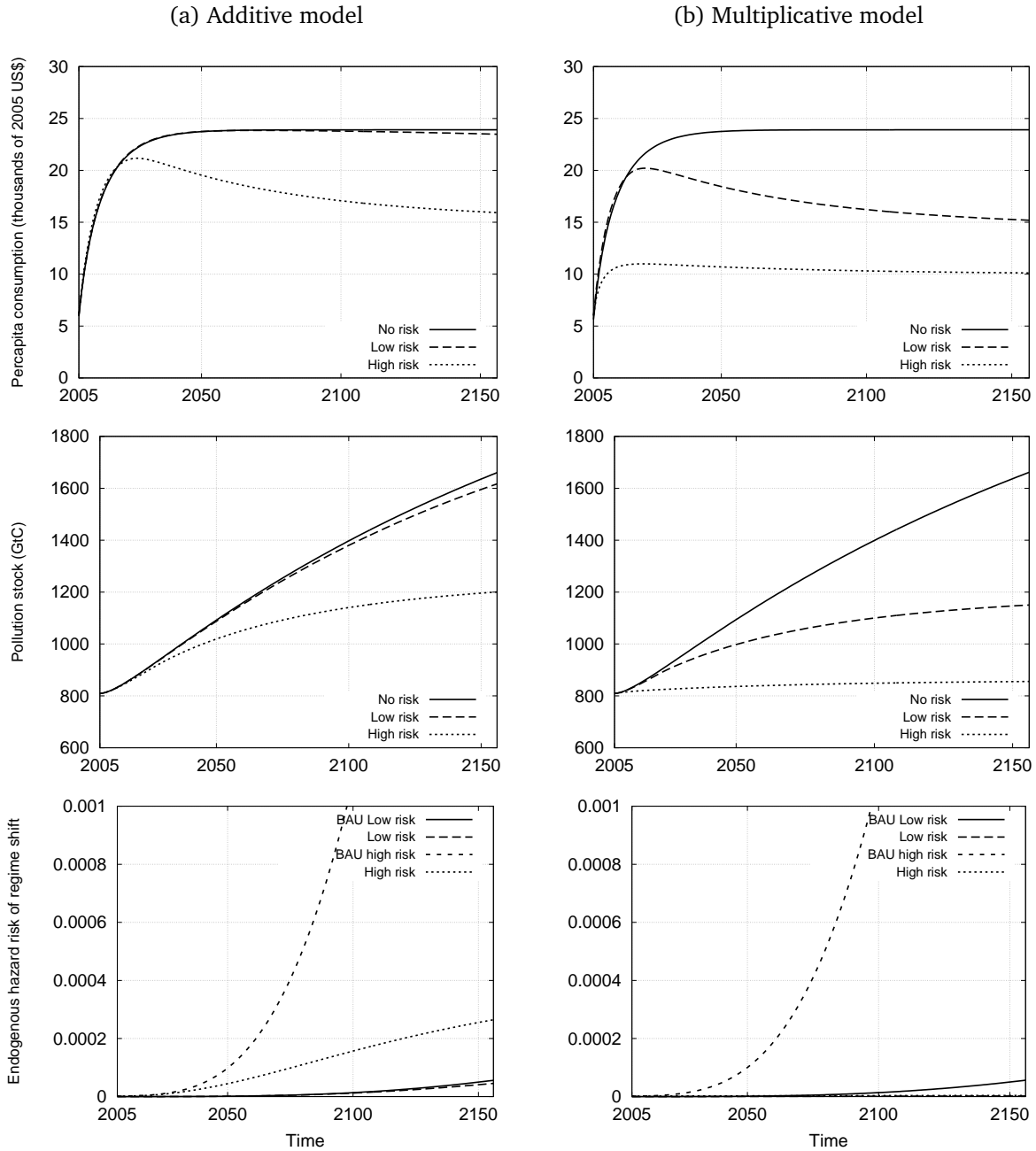
### 5.3 Optimal climate policy under endogenous catastrophic collapse

We now consider the case where the planner takes into account the relationship between the stock of pollution and the hazard risk of catastrophic collapse. Optimal paths for consumption  $c_t$ , pollution stock  $M_t$  and the hazard risk  $\mu_t$  under the two alternative specifications of the risk profiles and social preferences are reported in Figure 3. We also report BAU trajectories as a reference, which are labeled as “no risk”.

In the presence of a collapse risk, consumption in every period is lower than under BAU. However, the magnitude of the decline varies greatly across risk profiles and preferences representations. In the additive model, the consumption trajectory under the low risk profile does not differ significantly from BAU. Similarly, the stock of GHG remains very close to BAU, and the reduction in the hazard risk is negligible. By 2150, the risk of collapse related to the stock of pollution is around 0.01% per year.

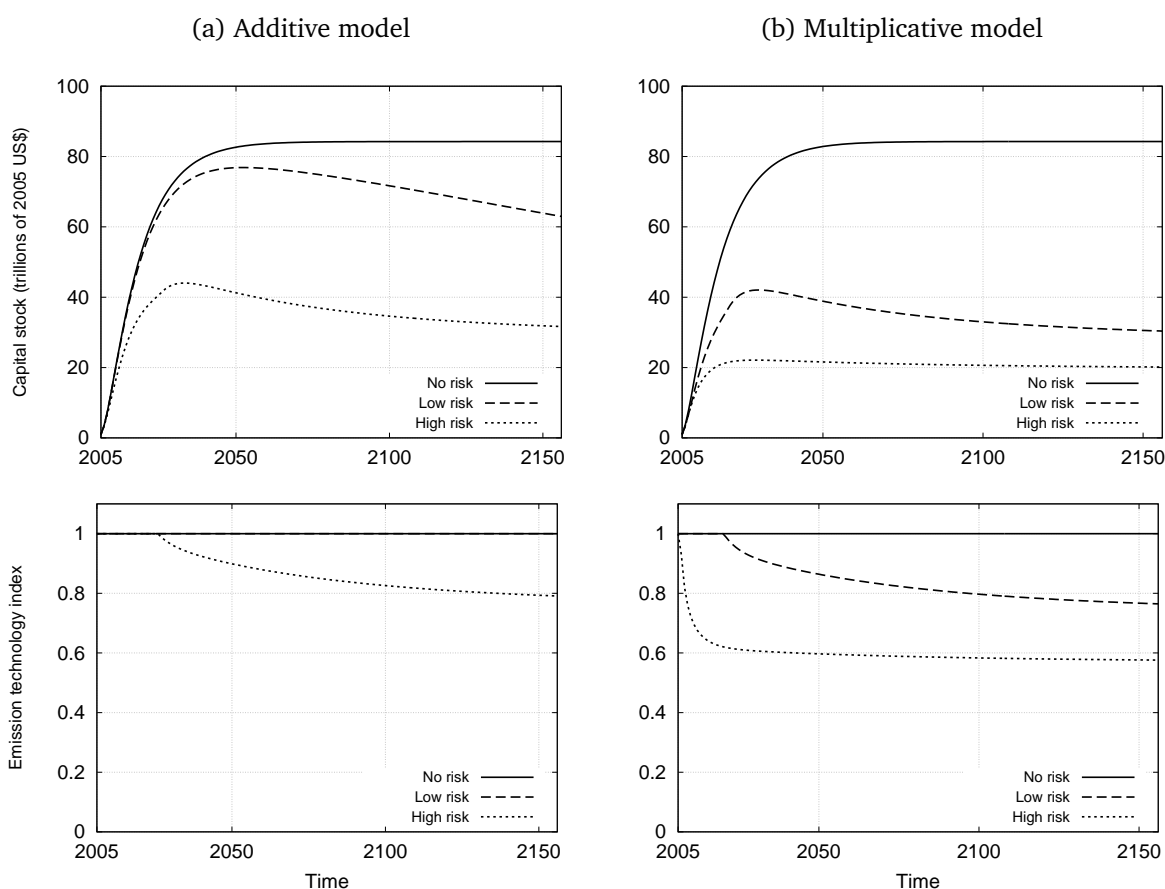
In our model, the reduction of emissions can be achieved either by using a cleaner but costly technology (i.e. choosing  $z_t < 1$ ), or slowing down the accumulation of capital by increasing consumption. As shown in Figure 4, the optimal solution under additive preferences and a low risk profile only uses the latter option. Since the economy produces with the dirty but efficient technology ( $z_t = 1$ ), slowing capital accumulation allows the consumption trajectory to remain close to BAU. In the steady state, however, both capital and consumption are lower than their BAU counterpart.

Figure 3: Optimal consumption and pollution under endogenous catastrophic risk



In contrast to the additive model, under multiplicative preferences emissions reductions are drastic even when the planner faces a low risk profile. The dirty technology is used for the first 15 periods, but then  $z$  falls below one and the consumption trajectory significantly diverges from BAU. This keeps the steady state stock of emissions below 1200 GtC. This threshold, which cor-

Figure 4: Optimal abatement under endogenous catastrophic risk



responds to a doubling of GHG concentration compared to the pre-industrial level, is therefore never reached with multiplicative preferences.

When we consider the high risk profile, the multiplicative model suggests an immediate change in the emission technology, with around 40% of output being diverted to lower emissions through  $z_t$ . Given this, both the consumption level and the stock of emissions do not grow beyond their 2010 levels. With the additive model there is still no change in technology in the first years, and a much smaller adjustment in the long term. In the multiplicative model the endogenous part of the risk is reduced below the level of the exogenous risk ( $\mu_0 = 0.001\%$ ), whereas the risk tolerated by the planner under additive preferences is around 0.02% by 2100, which would reduce the survival probability below 80% after 1,000 years.

One may notice that the trajectory obtained with the additive model and high risk is quite close to the one derived with the multiplicative model and low risk. Therefore, switching from

the additive to the multiplicative models has about the same impact as a 100-fold increase in the hazard risk profile. This suggests that the attention given to the form of social preferences matters as much as the quantification of risks associated with GHG emissions. In particular, in the presence of an unusual catastrophic risk such as climate change, where there is little observations to inform the representation of social preferences, it should at least be recognized that the assumptions underlying the traditional additive model have tremendous consequences for policy recommendations.

#### 5.4 On the role of post-collapse welfare and time preferences

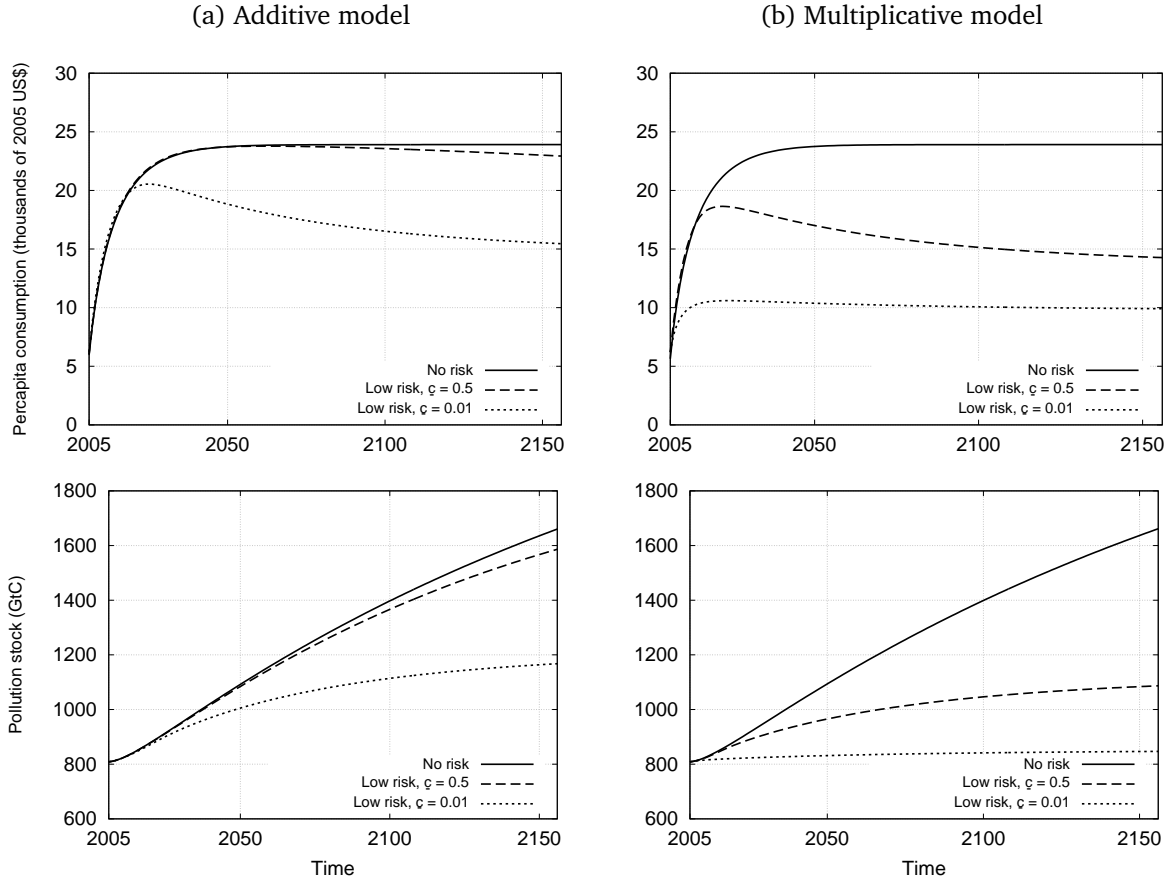
Let us now turn to the role of main parameters determining magnitude of the catastrophe, namely the level of post-collapse consumption  $\underline{c}$ , and the choice of the discount rate in the additive model.

The choice of  $\underline{c}$  does not influence the qualitative nature of our results, but Figure 5 shows that it greatly matters for the magnitude of the policy response. The value for  $\underline{c}$  depends on the kind of collapse risk one may think of, and is thus debatable. Our aim here is only to illustrate how this parameter influences our results. The value used so far ( $\underline{c} = 1$ ) corresponds to an 85 % decrease of consumption. We now consider two additional post-collapse consumption levels: (i)  $\underline{c} = 0.5$  (more severe poverty); (ii)  $\underline{c} = 0.01$ , a prospect that could be associated with human extinction.

Reducing post-collapse consumption from  $\underline{c} = 1$  to  $\underline{c} = 0.5$  has a relatively small impact on optimal paths (compare Figure 3 and Figure 5). Moreover, the change in optimal trajectories is similar under additive and multiplicative preferences, although the differences between models remain (i.e. the stock of emissions is significantly lower with the multiplicative model). Similarly, moving to  $\underline{c} = 0.01$  involves much more drastic intervention but the qualitative differences between models are preserved.

As for alternative hazard risk schedule, we observe that the path derived with additive preferences and a low value of post-collapse consumption,  $\underline{c} = 0.01$ , is close to the one derived with multiplicative preferences and  $\underline{c} = 1$ . This suggests a correspondence between the magnitude of the catastrophic collapse and the choice of the social preference representation. Moreover, our simulations show that, for both models,  $\underline{c} = 0.01$  with a low risk profile implies roughly the

Figure 5: The role of post-collapse consumption level ( $\underline{c}$ ) under endogenous hazard risk



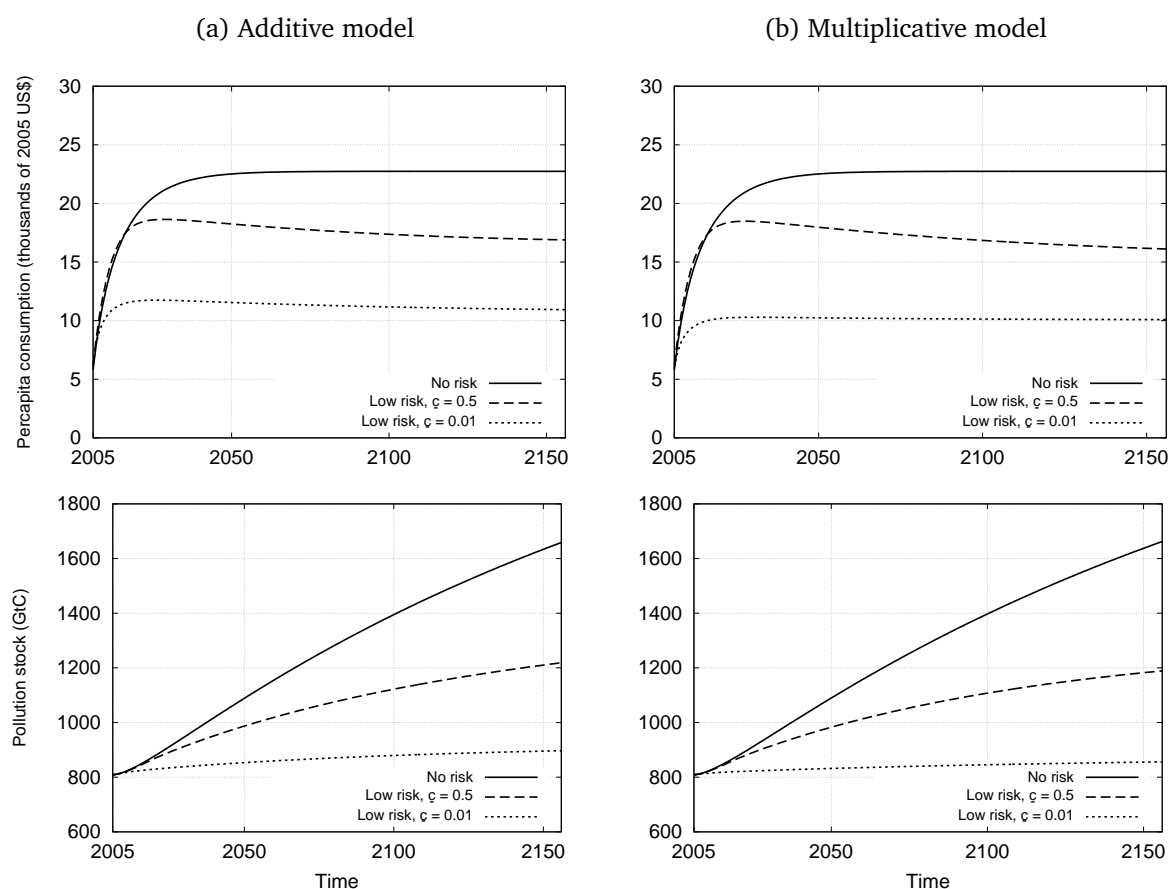
same equilibrium as  $\underline{c} = 1$  with a high risk profile.

The second parameter we consider, the pure rate of time preferences in the additive model ( $\theta$ ), is a highly contentious parameter in the economic literature on climate change. As we have shown earlier, it partly determines the difference in how the multiplicative and the additive model respond to an endogenous hazard risks (see Section 4.2). The value used by Nordhaus (2008) of 1.5% has been criticized by Stern (2007) among others as being unethically high. In the following, we thus use a discount rate of 0.1% as in Stern (2007).

We calibrate both choice models so that they yield the same BAU steady state interest rate. Therefore, the choice of  $\theta$  also determines the temporal risk aversion parameter in the multiplicative model  $\varepsilon$ . Moreover, changing the discount rate also modifies incentives to accumulate capital in the BAU, and we re-calibrate the TFP parameter to remain on the same benchmark



Figure 6: Comparing additive and multiplicative models under low time preferences ( $\theta = 0.1\%$ )



path for output and emissions.<sup>8</sup> Figure 6 compares the additive model with a discount rate of 0.1%, with a multiplicative model re-calibrated to produce the same BAU steady state interest rate. As above, we contrast optimal paths under the low risk profile and two alternative post-collapse consumption levels, namely  $\underline{c} = 0.5$  and  $\underline{c} = 0.01$ . We stress that our objective remains to compare how additive and multiplicative models that rationalize the same BAU economy respond to information about a hazard risk of catastrophic collapse.

Under a low value for the pure rate of time preference, here  $\theta = 0.1\%$ , optimal trajectories for additive and multiplicative preferences are very close. This was expected from Proposition 3: the equations defining the steady state become very similar when the pure rate of time preference approach zero. The simulations further show that, for the particular parametrization we

<sup>8</sup> Since our calibration procedure targets the path for output, the BAU consumption path slightly differs from that reported in Figure 3. The BAU emissions path and associated hazard risk are however exactly identical.

consider, reducing  $\theta$  in the additive model significantly increases the abatement efforts, whereas trajectories for the multiplicative model remain close to those reported in Figure 5. This is so because reducing  $\theta$  in the additive model increases the value of catastrophic risk reduction, and in the present configuration this effect dominates the discounting effect.

We thus conclude that differences between both models are not irreconcilable, and hinge upon the view of the appropriate pure rate of time preference prevailing in the additive model. Nevertheless, our simulations indicate that it is the paths of the additive model that converge towards those of the multiplicative models rather than the opposite. This therefore reinforces our message that basing policy recommendations solely upon the additive model in conjunction with a more conventional value for the pure rate of time preference ( $\theta > 1\%$ ) has very strong implications when considering the trade-off between consumption and hazard risk reduction.

## 6 Concluding remarks

Economic studies about individual portfolio choice have long discussed the role of risk aversion. It is now well accepted that individuals may dislike taking risk, and that risk aversion is an element that has to be integrated in studies about the optimal degree of risk taking. Very similarly, one might think that discussions on optimal catastrophic risk prevention should account for risk aversion with respect to intertemporal utility, or equivalently preferences for catastrophe avoidance. Unfortunately, the literature on climate change has not done so, thus far. The reason is that most studies on climate change rely on models assuming additively separable preferences, which are risk neutral with respect to intertemporal utility.

In the current paper, we have considered multiplicative preferences displaying such risk aversion. In the case where catastrophic risks are exogenous and constant, the multiplicative model behaves almost like the standard additive model. However, as soon as we consider endogenous catastrophic risks, the two models radically differ, the multiplicative model advocating a much tighter policy response.

Doing so, we highlight that including preferences for catastrophe avoidance in the analysis could radically change the conclusion regarding the appropriate level of action. Loosely speaking, the choice of the preference model induces changes in the optimal plans whose magnitude

is similar to large changes in the catastrophic risk faced by the society. Considering alternative choice models may therefore seem relevant for the kind of risks we consider.

Ethical questions surrounding preference for catastrophe avoidance have been discussed elsewhere. Some, like Fleurbaey (2010) would actually argue in favor of risk equity, that is the opposite of preferences for catastrophe avoidance, because the social planner may dislike ex-post inequalities. Others, like Keeney (1980), Fishburn (1984), Manski and Tetenov (2007) or Bommier and Zuber (2008) argue that positive preferences for catastrophe avoidance is the most natural assumption. Divergence in opinion is possible but so far the debate was limited to theoretical contributions in social choice theory. Potential implications for concrete problems, as those related to climate change, have mostly been ignored because of the popularity of the additive model. The current paper shows that far from being a theoretical curiosity, preferences for catastrophe avoidance is a key element that cannot reasonably be ignored when considering endogenous catastrophic risks and, a fortiori, climate change.

## Appendix A Proof of Proposition 1

### Proof of (a).

Using Eq. (8) in the additive case, we obtain:

$$\frac{\partial W(c)}{\partial c_T} = \exp\left(-\int_0^T (\theta + \mu_t) dt\right) u'(c_T)$$

and

$$\frac{\partial W(c)}{\partial \mu_T} = -\int_T^{+\infty} \exp\left(-\int_0^t (\theta + \mu_\tau) d\tau\right) u(c_t) dt$$

Hence,

$$-\frac{d}{dT} \left( \log \frac{\partial W(c)}{\partial c_T} \right) \Big|_{\dot{c}_T=0} = -\frac{-(\theta + \mu_T) \exp\left(-\int_0^T (\theta + \mu_t) dt\right) u'(c_T)}{\exp\left(-\int_0^T (\theta + \mu_t) dt\right) u'(c_T)} = \theta + \mu_T$$

and

$$-\left(\frac{\partial W(c)}{\partial \mu_T}\right) \Big/ \left(\frac{\partial W(c)}{\partial c_T}\right) = \frac{\int_T^{+\infty} \exp\left(-\int_0^t (\theta + \mu_\tau) d\tau\right) u(c_t) dt}{\exp\left(-\int_0^T (\theta + \mu_t) dt\right) u'(c_T)} = \frac{\int_T^{+\infty} \exp\left(-\int_T^t (\theta + \mu_\tau) d\tau\right) u(c_t) dt}{u'(c_T)}.$$

### Proof of (b).

Using Eq. (8) in the multiplicative case, we obtain:

$$\begin{aligned} \frac{\partial W(c)}{\partial c_T} &= \exp\left(-\int_0^T (\varepsilon u(c_t) + \mu_t) dt\right) u'(c_T) - \varepsilon u'(c_T) \int_T^{+\infty} \exp\left(-\int_0^t (\varepsilon u(c_\tau) + \mu_\tau) d\tau\right) u(c_t) dt \\ &= \exp\left(-\int_0^T (\varepsilon u(c_t) + \mu_t) dt\right) u'(c_T) (1 - \varepsilon \mathcal{U}(c, T)) \end{aligned}$$

and

$$\frac{\partial W(c)}{\partial \mu_T} = -\int_T^{+\infty} \exp\left(-\int_0^t (\varepsilon u(c_\tau) + \mu_\tau) d\tau\right) u(c_t) dt = -\exp\left(-\int_0^T (\varepsilon u(c_t) + \mu_t) dt\right) \mathcal{U}(c, T)$$

Hence,

$$\begin{aligned}
& - \frac{d}{dT} \left( \log \frac{\partial W(c)}{\partial c_T} \right) \Big|_{\dot{c}_T=0} = \\
& - \frac{-(\varepsilon u(c_T) + \mu_T) \exp \left( - \int_0^T (\varepsilon u(c_t) + \mu_t) dt \right) u'(c_T) + \varepsilon u'(c_T) \exp \left( - \int_0^T (\varepsilon u(c_t) + \mu_t) dt \right) u(c_T)}{\exp \left( - \int_0^T (\varepsilon u(c_t) + \mu_t) dt \right) u'(c_T) (1 - \varepsilon \mathcal{U}(c, T))} \\
& = \frac{\mu_T}{1 - \varepsilon \mathcal{U}(c, T)}
\end{aligned}$$

and

$$- \left( \frac{\partial W(c)}{\partial \mu_T} \right) / \left( \frac{\partial W(c)}{\partial c_T} \right) = \frac{\exp \left( - \int_0^T (\varepsilon u(c_t) + \mu_t) dt \right) \mathcal{U}(c, T)}{\exp \left( - \int_0^T (\varepsilon u(c_t) + \mu_t) dt \right) u'(c_T) (1 - \varepsilon \mathcal{U}(c, T))} = \frac{\mathcal{U}(c, T)}{u'(c_T) (1 - \varepsilon \mathcal{U}(c, T))}.$$

## Appendix B Optimal dynamics and steady state: BAU problem

In the case of the additive model, the BAU problem is just the standard Ramsey optimal growth problem (with  $\mu_0 + \theta$  replacing  $\theta$ ). So the optimal dynamics and the steady state are well-known.

In the multiplicative case, the BAU problem can be rewritten:

$$\begin{aligned}
& \max_c \int_0^{+\infty} e^{-\Omega_t} u(c_t) dt \\
& s.t. \quad \dot{k}_t = f(k_t) - \delta k_t - c_t \\
& \quad \quad \dot{\Omega}_t = \mu_0 + \varepsilon u(c_t)
\end{aligned}$$

The Hamiltonian of this problem is:

$$\begin{aligned}
H(t, c_t, z_t, k_t, M_t, \Omega_t) &= e^{-\Omega_t} u(c_t) + \lambda_t \left\{ f(k_t) - \delta k_t - c_t \right\} \\
& \quad + \eta_t \left\{ \mu_0 + \varepsilon u(c_t) \right\}.
\end{aligned}$$

The first order necessary conditions of are:

$$\frac{\partial H}{\partial c} = e^{-\Omega_t} u'(c_t) - \lambda_t + \eta_t \varepsilon u'(c_t) = 0 \tag{31}$$

$$\dot{\lambda}_t = - \frac{\partial H}{\partial k} = -\lambda_t (f'(k_t) - \delta) \tag{32}$$

$$\dot{\eta}_t = - \frac{\partial H}{\partial \Omega} = e^{-\Omega_t} u(c_t) \tag{33}$$

Equation (33) implies that  $\eta_t = \eta(0) + \int_0^t e^{-\Omega\tau} u(c_\tau) d\tau$ . The transversality condition  $\lim_{t \rightarrow +\infty} \eta_t = 0$  yields  $\eta(0) = - \int_0^{+\infty} e^{-\Omega\tau} u(c_\tau) d\tau$ . Hence  $\eta_t = - \int_t^{+\infty} e^{-\Omega\tau} u(c_\tau) d\tau$ . Denote  $\mathcal{U}_t = \int_t^{+\infty} e^{-(\Omega\tau - \Omega t)} u(c_\tau) d\tau$  the future expected utility discounted at period  $t$ . We obtain

$$\eta_t = -e^{-\Omega t} \mathcal{U}_t, \quad (34)$$

so that, using (33),

$$\dot{\mathcal{U}}_t = \dot{\Omega} \mathcal{U}_t - u(c_t). \quad (35)$$

Using Equation (34), Equation (31) can be rewritten:

$$e^{-\Omega t} \{1 - \varepsilon \mathcal{U}_t\} u'(c_t) = \lambda_t$$

Differentiating each term in the above equation with respect to time and dividing by the corresponding term in the equation, we obtain:

$$-\dot{\Omega}_t - \frac{\varepsilon \dot{\mathcal{U}}_t}{1 - \varepsilon \mathcal{U}_t} + \dot{c}_t \frac{u''(c_t)}{u'(c_t)} = \frac{\dot{\lambda}_t}{\lambda_t}$$

Given Equation (35), we obtain  $\dot{\Omega}_t + \frac{\varepsilon \dot{\mathcal{U}}_t}{1 - \varepsilon \mathcal{U}_t} = \frac{(1 - \varepsilon \mathcal{U}_t) \dot{\Omega}_t + \varepsilon (\dot{\Omega} \mathcal{U}_t - u(c_t))}{1 - \varepsilon \mathcal{U}_t} = \frac{\dot{\Omega}_t - \varepsilon u(c_t)}{1 - \varepsilon \mathcal{U}_t} = \frac{\mu_0}{1 - \varepsilon \mathcal{U}_t}$ .

Combining this equality, Equation (32) and the previous equation, we have:

$$\begin{aligned} \dot{c}_t &= \frac{u'(c_t)}{u''(c_t)} \left( \frac{\mu_0}{1 - \varepsilon \mathcal{U}_t} + \frac{\dot{\lambda}_t}{\lambda_t} \right) \\ &= - \frac{u'(c_t)}{u''(c_t)} \left( f'(k_t) - \delta - \frac{\mu_0}{1 - \varepsilon \mathcal{U}_t} \right) \end{aligned} \quad (36)$$

In the end, the optimal dynamics for the BAU problem with multiplicative preferences are given by the dynamic system:

$$\dot{c}_t = - \frac{u'(c_t)}{u''(c_t)} \left( f'(k_t) - \delta - \frac{\mu_0}{1 - \varepsilon \mathcal{U}_t} \right) \quad (37)$$

$$\dot{k}_t = f(k_t) - c_t \quad (38)$$

$$\dot{\mathcal{U}}_t = \dot{\Omega} \mathcal{U}_t - u(c_t) = (\mu_0 + \varepsilon u(c_t)) \mathcal{U}_t - u(c_t) \quad (39)$$

In a steady state,  $\dot{\mathcal{U}}_t = 0$  implies that  $\mathcal{U}^* = \frac{u(c^*)}{\mu_0 + \varepsilon u(c^*)}$ , so that  $\frac{\mu_0}{1 - \varepsilon \mathcal{U}^*} = \mu_0 + \varepsilon u(c^*)$ . Conse-

quently, in steady state, Equations (37)-(38) yield the Equations (16)-(17), which characterize a steady state for an interior solution of Problem (11) in the multiplicative case.

## Appendix C Optimal dynamics and steady state: Additive model

Denote  $\Omega_t = \int_0^t \{\mu(\tau) + \theta\} d\tau$  and notice that  $\dot{\Omega}_t = \mu_t + \theta = \mu(M_t) + \theta$ . Problem (18) can be rewritten:

$$\begin{aligned} \max_{c,z} \quad & \int_0^{+\infty} e^{-\Omega_t} u(c_t) dt \\ \text{s.t.} \quad & \dot{k}_t = z_t f(k_t) - \delta k_t - c_t \\ & \dot{M}_t = \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \\ & \dot{\Omega}_t = \mu(M_t) + \theta \end{aligned}$$

The Hamiltonian of this problem is

$$\begin{aligned} H(t, c_t, z_t, k_t, M_t, \Omega_t) = & e^{-\Omega_t} u(c_t) + \lambda_t \{z_t f(k_t) - \delta k_t - c_t\} \\ & + \kappa_t \{\varphi(z_t) f(k_t) - \psi(M_t - \bar{M})\} \\ & + \eta_t \{\mu(M_t) + \theta\}. \end{aligned}$$

The first order necessary conditions of are:

$$\frac{\partial H}{\partial c} = e^{-\Omega_t} u'(c_t) - \lambda_t = 0 \quad (40)$$

$$\frac{\partial H}{\partial z} = \lambda_t f(k_t) + \kappa_t \varphi'(z_t) f(k_t) = 0 \quad (41)$$

$$\dot{\lambda}_t = -\frac{\partial H}{\partial k} = -\lambda_t z_t f'(k_t) + \lambda_t \delta - \kappa_t \varphi(z_t) f'(k_t) \quad (42)$$

$$\dot{\kappa}_t = -\frac{\partial H}{\partial M} = -\mu'(M_t) \eta_t + \kappa_t \psi \quad (43)$$

$$\dot{\eta}_t = e^{-\Omega_t} u(c_t) \quad (44)$$

Equation (41) can be rewritten:

$$\kappa_t = -\frac{\lambda_t}{\varphi'(z_t)} \quad (45)$$

Equation (42) hence becomes:

$$\dot{\lambda}_t = - \left( \left\{ z_t - \frac{\varphi(z_t)}{\varphi'(z_t)} \right\} f'(k_t) - \delta \right) \lambda_t \quad (46)$$

We denote  $\pi(z_t) = z_t - \frac{\varphi(z_t)}{\varphi'(z_t)}$ . Since  $\varphi$  is convex,  $\pi$  is increasing.

Equation (40) yields  $e^{-\Omega t} u'(c_t) = \lambda_t$ . Differentiating both terms of the equation with respect to time and dividing by the corresponding term of the original equation, we obtain:

$$\frac{u''(c_t)}{u'(c_t)} \dot{c}_t - \dot{\Omega}_t = \frac{u''(c_t)}{u'(c_t)} \dot{c}_t - \theta - \mu(M_t) = \frac{\dot{\lambda}_t}{\lambda_t}$$

Using Equation (46), we finally get:

$$\dot{c}_t = - \frac{u'(c_t)}{u''(c_t)} \left( \pi(z_t) f'(k_t) - \delta - \theta - \mu(M_t) \right)$$

Equation (44) implies that  $\eta_t = \eta(0) + \int_0^t e^{-\Omega \tau} u(c_\tau) d\tau$ . The transversality condition  $\lim_{t \rightarrow +\infty} \eta_t = 0$  yields  $\eta(0) = - \int_0^{+\infty} e^{-\Omega \tau} u(c_\tau) d\tau$ . Hence  $\eta_t = - \int_t^{+\infty} e^{-\Omega \tau} u(c_\tau) d\tau$ , so that:

$$\eta_t = -e^{-\Omega t} \mathcal{U}_t, \quad (47)$$

and, by (40),

$$\eta_t = - \frac{\mathcal{U}_t}{u'(c_t)} \lambda_t. \quad (48)$$

Using (45) and (48), Equation (43) becomes:

$$\dot{\kappa}_t = \left( \psi - \mu'(M_t) \varphi'(z_t) \frac{\mathcal{U}_t}{u'(c_t)} \right) \kappa_t$$

Differentiating each term in Equation (45) with respect to time and dividing by the corresponding term in (45), we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = - \dot{z}_t \frac{\varphi''(z_t)}{\varphi'(z_t)} + \frac{\dot{\lambda}_t}{\lambda_t}$$



Hence:

$$\begin{aligned}\dot{z}_t &= \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{\kappa}_t}{\kappa_t} \right) \\ &= \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \mu'(M_t) \varphi'(z_t) \frac{u_t}{u'(c_t)} + \delta - \left\{ \psi + \pi(z_t) f'(k_t) \right\} \right)\end{aligned}$$

Gathering all the results and the equations of motion for  $k$  and  $M$  yields the dynamic system:

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)} (\pi(z_t) f'(k_t) - \delta - \rho(c_t, t)) \quad (49)$$

$$\dot{z}_t = \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \mu'(M_t) \varphi'(z_t) \mathcal{V}(c_t, t) + \delta - \left\{ \psi + \pi(z_t) f'(k_t) \right\} \right) \quad (50)$$

$$\dot{k}_t = z_t f(k_t) - c_t \quad (51)$$

$$\dot{M}_t = \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \quad (52)$$

In a steady state,  $\mathcal{U}^* = \frac{u(c^*)}{\mu(M^*) + \theta}$ . Hence Equations (49)-(52) yield Equations (49)-(52), which characterize a steady state for an interior solution of Problem (18) in the additive case.

## Appendix D Optimal dynamics and steady state: Multiplicative model

Problem (18) can be rewritten:

$$\begin{aligned}\max_{c, z} \quad & \int_0^{+\infty} e^{-\Omega t} u(c_t) dt \\ \text{s.t.} \quad & \dot{k}_t = z_t f(k_t) - \delta k_t - c_t \\ & \dot{M}_t = \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \\ & \dot{\Omega}_t = \mu(M_t) + \varepsilon u(c_t)\end{aligned}$$

The Hamiltonian of this problem is

$$\begin{aligned}H(t, c_t, z_t, k_t, M_t, \Omega_t) &= e^{-\Omega t} u(c_t) + \lambda_t \left\{ z_t f(k_t) - \delta k_t - c_t \right\} \\ &\quad + \kappa_t \left\{ \varphi(z_t) f(k_t) - \psi(M_t - \bar{M}) \right\} \\ &\quad + \eta_t \left\{ \mu(M_t) + \varepsilon u(c_t) \right\}.\end{aligned}$$

The first order necessary conditions of are:

$$\frac{\partial H}{\partial c} = e^{-\Omega t} u'(c_t) - \lambda_t + \eta_t \varepsilon u'(c_t) = 0 \quad (53)$$

$$\frac{\partial H}{\partial z} = \lambda_t f(k_t) + \kappa_t \varphi'(z_t) f(k_t) = 0 \quad (54)$$

$$\dot{\lambda}_t = -\frac{\partial H}{\partial k} = -\lambda_t z_t f'(k_t) + \lambda_t \delta - \kappa_t \varphi(z_t) f'(k_t) \quad (55)$$

$$\dot{\kappa}_t = -\frac{\partial H}{\partial M} = \kappa_t \psi - \eta_t \mu'(M_t) \quad (56)$$

$$\dot{\eta}_t = -\frac{\partial H}{\partial \Omega} = e^{-\Omega t} u(c_t) \quad (57)$$

Equation (54) can be rewritten:

$$\kappa_t = -\frac{\lambda_t}{\varphi'(z_t)} \quad (58)$$

Equation (55) hence becomes:

$$\dot{\lambda}_t = -\left(\pi(z_t) f'(k_t) - \delta\right) \lambda_t \quad (59)$$

where  $\pi$  is defined like in Appendix C.

Equation (57) implies that  $\eta_t = \eta(0) + \int_0^t e^{-\Omega \tau} u(c_\tau) d\tau$ . The transversality condition  $\lim_{t \rightarrow +\infty} \eta_t = 0$  yields  $\eta(0) = -\int_0^{+\infty} e^{-\Omega \tau} u(c_\tau) d\tau$ . Hence  $\eta_t = -\int_t^{+\infty} e^{-\Omega \tau} u(c_\tau) d\tau$ . Denote  $\mathcal{U}_t = \int_t^{+\infty} e^{-(\Omega \tau - \Omega t)} u(c_\tau) d\tau$  the future expected utility discounted at period  $t$ . We obtain

$$\eta_t = -e^{-\Omega t} \mathcal{U}_t, \quad (60)$$

so that, using (57),

$$\dot{\mathcal{U}}_t = \dot{\Omega} \mathcal{U}_t - u(c_t). \quad (61)$$

Using Equation (60), Equation (53) can be rewritten:

$$e^{-\Omega t} \left\{ 1 - \varepsilon \mathcal{U}_t \right\} u'(c_t) = \lambda_t$$

Differentiating each term in the above equation with respect to time and dividing by the

corresponding term in the equation, we obtain:

$$-\dot{\Omega}_t - \frac{\varepsilon \dot{\mathcal{U}}_t}{1 - \varepsilon \mathcal{U}_t} + \dot{c}_t \frac{u''(c_t)}{u'(c_t)} = \frac{\dot{\lambda}_t}{\lambda_t}$$

Given Equation (61), we obtain  $\dot{\Omega}_t + \frac{\varepsilon \dot{\mathcal{U}}_t}{1 - \varepsilon \mathcal{U}_t} = \frac{(1 - \varepsilon \mathcal{U}_t) \dot{\Omega}_t + \varepsilon (\dot{\Omega}_t \mathcal{U}_t - u(c_t))}{1 - \varepsilon \mathcal{U}_t} = \frac{\dot{\Omega}_t - \varepsilon u(c_t)}{1 - \varepsilon \mathcal{U}_t} = \frac{\mu(M_t)}{1 - \varepsilon \mathcal{U}_t}$ .

Combining this equality, Equation (59) and the previous equation, we have:

$$\begin{aligned} \dot{c}_t &= \frac{u'(c_t)}{u''(c_t)} \left( \frac{\mu(M_t)}{1 - \varepsilon \mathcal{U}_t} + \frac{\dot{\lambda}_t}{\lambda_t} \right) \\ &= -\frac{u'(c_t)}{u''(c_t)} \left( \pi(z_t) f'(k_t) - \delta - \frac{\mu(M_t)}{1 - \varepsilon \mathcal{U}_t} \right) \end{aligned} \quad (62)$$

Equations (53) and (60) also imply  $\frac{\lambda_t}{\eta_t} = \left( \frac{e^{-\Omega_t + \varepsilon \eta_t}}{\eta_t} \right) u'(c_t) = -\frac{1 - \varepsilon \mathcal{U}_t}{\mathcal{U}_t} u'(c_t)$ , so that  $\frac{\eta_t}{\kappa_t} = \frac{\mathcal{U}_t}{1 - \varepsilon \mathcal{U}_t} \frac{\varphi'(z_t)}{u'(c_t)}$ . Hence Equation (56) can be rewritten:

$$\dot{\kappa}_t = \left\{ \psi - \frac{\mathcal{U}_t}{u'(c_t) - \varepsilon u'(c_t) \mathcal{U}_t} \mu'(M_t) \varphi'(z_t) \right\} \kappa_t \quad (63)$$

Differentiating each term in Equation (54) with respect to time and dividing by the corresponding term in (54), we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = -\dot{z}_t \frac{\varphi''(z_t)}{\varphi'(z_t)} + \frac{\dot{\lambda}_t}{\lambda_t}$$

Hence, by Equations (59) and (63):

$$\begin{aligned} \dot{z}_t &= \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{\kappa}_t}{\kappa_t} \right) \\ &= \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \frac{\mathcal{U}_t}{u'(c_t) - \varepsilon u'(c_t) \mathcal{U}_t} \mu'(M_t) \varphi'(z_t) + \delta - \left\{ \psi + \pi(z_t) f'(k_t) \right\} \right) \end{aligned} \quad (64)$$

Gathering Equations (61), (62), (64), and the equations of motion for  $k$  and  $M$  yields the

dynamic system:

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)} (\pi(z_t)f'(k_t) - \delta - \rho(c_t, t)) \quad (65)$$

$$\dot{z}_t = \frac{\varphi'(z_t)}{\varphi''(z_t)} \left( \mathcal{V}(c_t, t)\mu'(M_t)\varphi'(z_t) + \delta - \left\{ \psi + \pi(z_t)f'(k_t) \right\} \right) \quad (66)$$

$$\dot{k}_t = z_t f(k_t) - c_t \quad (67)$$

$$\dot{M}_t = \varphi(z_t)f(k_t) - \psi(M_t - \bar{M}) \quad (68)$$

$$\dot{\mathcal{U}}_t = \dot{\Omega}\mathcal{U}_t - u(c_t) = \left( \mu(M_t) + \varepsilon u(c_t) \right) \mathcal{U}_t - u(c_t) \quad (69)$$

In a steady state,  $\dot{\mathcal{U}}_t = 0$  implies that  $\mathcal{U}^* = \frac{u(c^*)}{\mu(M^*) + \varepsilon u(c^*)}$ , so that  $\rho = \frac{\mu(M^*)}{1 - \varepsilon \mathcal{U}^*} = \mu(M^*) + \varepsilon u(c^*)$  and  $\mathcal{V}^* = \frac{\mathcal{U}^*}{u'(c^*) - \varepsilon u'(c^*) \mathcal{U}^*} = \frac{u(c^*)}{u'(c^*) \mu(M^*)}$ . Consequently, in steady state, Equations (65)-(68) yield the Equations (27)-(30), which characterize a steady state for an interior solution of Problem (18) in the multiplicative case.

## References

- Ahn, C. M. (1989) “The effect of temporal risk aversion on optimal consumption, the equity premium, and the equilibrium interest rate,” *Journal of Finance*, 44 (5), pp. 1411 – 1420.
- Alley, R. B., J. Marotzke, W. D. Nordhaus, J. T. Overpeck, D. M. Peteet, R. A. Pielke, R. T. Pierrehumbert, P. B. Rhines, T. F. Stocker, L. D. Talley, and J. M. Wallace (2003) “Abrupt climate change,” *Science*, 299 (5615), pp. 2005 – 2010.
- Bommier, A. (2006) “Uncertain lifetime and intertemporal choice: risk aversion as a rationale for time discounting,” *International Economic Review*, 47 (4), pp. 1223 – 1246.
- (2012) “Life-cycle preferences revisited,” *Journal of the European Economic Association*, forthcoming.
- Bommier, A. and F. LeGrand (2013) “A robust approach to risk aversion.” CER-ETH Working Paper 13/172.
- Bommier, A. and S. Zuber (2008) “Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity?” *Social Choice and Welfare*, 31, pp. 415 – 434.
- Cai, Y., K. L. Judd, and T. S. Lontzek (2013) “The social cost of stochastic and irreversible climate change.” NBER Working Paper 18704.
- Clarke, H. and W. Reed (1994) “Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse,” *Journal of Economic Dynamics and Control*, 18, pp. 991 – 1010.
- Cropper, M. (1976) “Regulating activities with catastrophic environmental effects,” *Journal of Environmental Economics and Management*, 3, pp. 1 – 15.
- Dirkse, S. P. and M. C. Ferris (1995) “The PATH solver: a non-monotone stabilization scheme for mixed complementarity problems,” *Optimization Methods and Software*, 5, pp. 123–156.
- Epstein, L. G. (1987) “A simple dynamic general equilibrium model,” *Journal of Economic Theory*, 41, pp. 68–95.
- Fishburn, P. (1984) “Equity axioms for public risks,” *Operation Research*, 32, pp. 901 – 908.

- Fleurbaey, M. (2010) "Assessing risky social situations," *Journal of Political Economy*, 118 (4), pp. 649 – 680.
- Gjerde, J., S. Grepperud, and S. Kverndokk (1999) "Optimal climate policy under the possibility of a catastrophe," *Resource and Energy Economics*, 21 (3-4), pp. 289 – 317.
- Gorman, W. G. (1968) "The structure of utility functions," *The Review of Economic Studies*, 35, pp. 367–390.
- Hansen, L. P. and T. J. Sargent (1995) "Discounted linear exponential quadratic gaussian control," *IEEE Transactions on Automatic Control*, 40, 5, pp. 968–971.
- Jullien, B., B. Salanié, and F. Salanié (1999) "Should more risk-averse agents exert more effort?" *Geneva Papers on Risk and Insurance Theory*, 24, pp. 19–28.
- Karp, L. and Y. Tsur (2011) "Time perspective and climate change policy," *Journal of Environmental Economics and Management*, 62, pp. 1 – 14.
- Keeney, R. (1980) "Equity and public risk," *Operation Research*, 28, pp. 527 – 534.
- Keller, K., B. Bolker, and D. Bradford (2004) "Uncertain climate thresholds and optimal economic growth," *Journal of Environmental Economics and Management*, 48, pp. 723 – 741.
- Kihlstrom, R. E. and L. J. Mirman (1974) "Risk aversion with many commodities," *Journal of Economic Theory*, 8 (3), pp. 361 – 388.
- Koopmans, T. C. (1960) "Stationary ordinal utility and impatience," *Econometrica*, 28, 2, pp. 287–309.
- Kriegler, E., J. W. Hall, H. Held, R. Dawson, and H. J. Schellnhuber (2009) "Imprecise probability assessment of tipping points in the climate system," *Proceedings of the National Academy of Sciences*, 106, pp. 5041 – 5046.
- Lau, M., A. Pahlke, and T. Rutherford (2002) "Approximating infinite-horizon models in a complementarity format: a primer in dynamic general equilibrium analysis," *Journal of Economic Dynamics and Control*, 26, pp. 577 – 609.

- Lemoine, D. and C. Traeger (2013) “Watch your step: Optimal policy in a tipping climate,” *American Economic Journal: Economic Policy*, forthcoming.
- Lenton, T. and J.-C. Ciscar (2012) “Integrating tipping points into climate impact assessments,” *Climatic Change*, forthcoming.
- Manski, C. F. and A. Tetenov (2007) “Admissible treatment rules for a risk-averse planner with experimental data on an innovation,” *Journal of Statistical Planning and Inference*, 137, pp. 1998–2010.
- Nordhaus, W. D. (2008) *A question of Balance: Weighting the options on global warming policies*: Yale University Press.
- Pye, G. (1973) “Lifetime portfolio selection in continuous time for a multiplicative class of utility functions,” *American Economic Review*, 63, pp. 1013 – 1016.
- Stern, N. (2007) *The Economics of Climate Change: The Stern Review*: Cambridge (UK): Cambridge University Press.
- Stockey, N. (1998) “Are there limits to growth?” *International Economic Review*, 39, pp. 1 – 31.
- Tsur, Y. and A. Zemel (1996) “Accounting for global warming risks: Resource management under event uncertainty,” *Journal of Economic Dynamics and Control*, 20, pp. 1289 – 1305.
- (1998) “Pollution control in an uncertain environment,” *Journal of Economic Dynamics and Control*, 22, pp. 967 – 975.
- van der Ploeg, F. (1993) “A closed-form solution for a model of precautionary saving,” *Review of Economic Studies*, 60 (2), pp. 385 – 395.
- Weitzman, M. (2009) “On modeling and interpreting the economics of catastrophic climate change,” *The Review of Economics and Statistics*, 91 (1), pp. 1 – 19.
- Yaari, M. E. (1965) “Uncertain lifetime, life insurance, and the theory of the consumer,” *The Review of Economic Studies*, 32, 2, pp. 137–150.