Trends and Cycles
An Historical Review of the Euro Area[§]

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Abstract

We analyze the euro area business cycle in a medium scale DSGE model where we assume two stochastic trends: one on total factor productivity and one on the inflation target of the central bank. To justify our choice of integrated trends, we test alternative specifications for both of them. We do so, estimating trends together with the model's structural parameters, to prevent estimation biases.

In our estimates, business cycle fluctuations are dominated by investment specific shocks and preference shocks of households. Our results cast doubts on the view that cost push shocks dominate economic fluctuations in DSGE models and show that productivity shocks drive fluctuations on a longer term.

As a conclusion, we present our estimation's historical reading of the business cycle in the euro area. This estimation gives credible explanations of major economic events since 1985.

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Introduction

DSGE models provide a mapping between observable variables and the structural shocks on the business cycle. Usually, this literature attributes a linchpin role to price and wage mark-up shocks in cyclical fluctuations. This decomposition is however highly sensitive to the treatment of the observables. We add two unit roots to the Smets and Wouters (2003) model and do not use employment level as a proxy for hours worked. Doing so, we improve the fit to the data, we significantly increase internal persistence of the model while shocks exhibit low persistence, and we find a convincing identification of shocks replicating major economic historical episodes of the euro area.

Indeed, while the most widely estimation approach was to de-trend variables before the estimation of the model's parameters, Gorodnichenko an Ng (2009) shows that a potential mispecification of the trend can imply sizeable estimation biases. Besides, Ferroni (2008) underlines that a one step estimation of both the trend and the cycle, provides a better fit to the data and avoids estimations biases. This paper therefore undertakes a *one step* estimation of the trend and the structural parameters of the Smets-Wouters model. This unification of trend and cycle inside a same framework allows for the reconstruction of non stationary variables using DSGE techniques. We use a model of closed economy for the euro area, following Christiano et al. (2005) and to Smets and Wouters (2003). We assume stochastic trends on Total Factor Productivity (TFP) and on inflation target. The TFP is modeled with an integrated process with drift while the inflation trend is modeled by a random walk on the central banker inflation target (Ireland (2008)). Once linearized, this model is estimated using GDP, private consumption, private investment, wages, inflation, and interest rate times series from 1985 to 2008 for the euro area. We adopt a *one-step* approach and simultaneously extract the trend and estimate the model. To carry out the estimation, we use a standard approach of partial calibration and partial Bayesian estimation.

This estimation approach yields three results. First, the trends on real variables and inflation are better modeled with first order integrated processes than with autoregressive processes. Second, the contribution to the cycle of the shocks generating the trends is weak. Indeed, the productivity shocks have two effects in our framework. They modify the contemporaneous value of the trend, through the integrated component of TFP. They can also influence the stationary variables, i.e. drive the business cycle. The unit root makes the real trend slowly fluctuate around its deterministic trajectory, but at business cycle frequencies, the impact on the business cycle is clearly dominated by other shocks. Hence, our results contrast those obtained by standard RBC results (e.g. King and Rebelo, 1999). Business cycles appear dominated by preference shocks and investment specific shocks. Particularly, we side with Greenwood et al. (2000), Fisher (2006) and Justiniano et al. (2008) in showing that investment specific shocks plays a crucial role in the business cycle. It is therefore likely that the importance of cost push shocks, identified as a DSGE weakness by Chari et al. (2009) could be due to estimation biases. Third, we estimate much lower persistence of the shocks, i.e. of the exogenous persistence of the model. We therefore address one of the most frequent criticisms of estimated DSGE models which have insufficient "internal propagation to replicate the dynamics of the data" (Canova, 2007).

Other authors have introduced real or nominal trend in their models. Smets and Wouters (2005, 2007) use linear trend on TFP. In Smets and Wouters (2003), while real variables are filtered with an HP-filter, they introduced an AR(1) inflation target. Ireland (2008) has introduced unit roots on the inflation target and TFP for a US model without capital, and Fève, Matheron and Sahuc (2008) did the same for the euro area. Yet, none of them compare the results under alternative specifications. Ferroni (2008) did so on US data, but only for the TFP trend. He uses the Smets and Wouters model but he "considers off model trends", i.e. agents are making decision with regard to the deviation from the trend either it is deterministic or stochastic. Here, we are able to test the two alternatives of integrated or autoregressive trends on both the TFP and the inflation target. Moreover we do not make an "off model trends" assumption, allowing for a stochastic trend to be taken into account in the agents decisions.

The remainder of this paper is organized as follows: section 1 briefly exposes the set up of our DSGE model, section 2 presents the estimation method and data, section 3 details the relationship between trend and cycle, section 4 presents shocks driving the cycle while section 5 tests our shocks identification is consistent with stylized facts by conducting an historical review of the Euro Area.

1 A DSGE model with two unit roots

We consider a closed economy with a continuum of infinitely-lived households who maximize their utility under a set of constraints. They provide differentiated labor skills, which are aggregated by a labor agency as in Erceg et al (2000). As in Christiano et al. (2005), households own capital which they decide to rent to firms and we impose a rigidity on capital adjustment and on the capital utilization rate. We distinguish an intermediate sector that operates under imperfect competition à la Dixit-Stiglitz (1977) from the final sector producing a good used by private and public agents to consume or invest. We add nominal rigidities on prices and wages à la Calvo (1983) as in Smets and Wouters (2003). The departure from the baseline model is the addition of two stochastic trends following Fève et al. (2008) who introduce the same trends in a model with no capital.

We add a TFP trend, modeled as an integrated process with a drift, to account for economic growth. For monetary policy matters, we add an integrated inflation target to account for the change in monetary policy directed toward the convergence to low inflation levels up to the mid 1990s and a constant inflation target afterwards. Hence our model is compatible with long term growth and inflation, in other words with real and nominal trends. Moreover it takes into account the effect of these two trends on the cycle. While productivity shocks make a contribution to the cycle, the inflation target of the central banker is introduced as a monetary policy tool used by other agents in the indexation of prices.

In contrast to Smets and Wouters (2003 and 2007), we do not include two shocks: a shock on labor desutility, which can not be differentiated from the wage mark-up in the linearized model (see Chari et al. (2009)) and a not microfounded shock on the risk premium. These two shocks only account for a negligible part of economic fluctuations in their estimation. We also eliminate the fixed cost in the intermediary sector. In the remainder of this section, we briefly recall the main features of the model.

Households

Households, indexed by τ, maximize their utility defined as:

$$E_0 \sum_{0}^{\infty} \beta^t \varepsilon_t^B \left(\log(C_t^\tau - hC_{t-1}) - \frac{{l_t^\tau}^{1+\frac{1}{\sigma_l}}}{1+\frac{1}{\sigma_l}} + \Upsilon(m_t^\tau) \right) \tag{1}$$

where E_0 is the expectations operator at time zero; C_t, l_t, m_t are respectively, private consumption, hours worked and real balances; β is the discount factor, ε^B is a shock on preference, Υ is a function including money in the utility function and σ_l is the Frish elasticity.

The intertemporal elasticity of substitution is equal to one (log utility) for the model to be consistent with long term growth, see King, Plosser, and Rebelo (1988).

Households face three constraints: the income constraint, the budget constraint and the capital accumulation equation. The first constraint corresponds to the decomposition of the total revenue of households. Total revenue Y_t^τ includes labor and capital revenues. Labor revenue includes an insurance UI_t thanks to which, *ex post*, the agents are identical concerning employment. Capital revenue is diminished by a function of the utilization rate of capital which stands for an adjustment cost in the capital utilization.

$$Y_t^\tau = (w_t^\tau l_t^\tau + UI_t) + (r_t^k z_t^\tau K_{t-1}^\tau - \psi(z_t^\tau)K_t^\tau)$$

where w_t^τ, r^k, K^τ and z^τ are the wages, the renting cost of capital, the capital and the capital utilization rate, respectively. $\Psi(.)$ is the cost related to capital utilization. This function, as in Christiano et al. (2005), is equal to zero at the steady state and convex. The budget constraint of the household is as follows:

$$\frac{M_t^\tau}{P_t} + b_t \frac{B_t^\tau}{P_t} = \frac{M_{t-1}^\tau}{P_t} + \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau$$

where M^τ, P_t, B^τ, b_t and I^τ are the money, the price level, the savings (bonds), the saving return rate and the investment, respectively. Let's turn to the capital dynamic equation. Capital is depreciated with rate δ. Moreover, a function $S(\frac{I_t}{I_{t-1}})$ stands for a cost of investment or the technology of converting investment into capital and ϵ_t^I is a shock on the investment cost or technology, the investment specific shock.

$$K_t = (1-\delta)K_{t-1} + \epsilon_t^I \left(1 - S(\frac{I_t}{I_{t-1}})\right) I_t$$

On the labor market, households are wage setters and know the labor demand function of firms. Labor of each household l_t^τ is aggregated with Dixit-Stiglitz method into total labor L_t, as in Erceg et al. (2000).

$$L_t = \left(\int (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}}\right)^{1+\lambda_{w,t}}$$

where $\lambda_{w,t}$ is the wage mark-up, equal to a steady state value plus a wage mark-up shock $\varepsilon_W(t)$. Wages are set through a Calvo process. If not re-optimized, which happens with probability ξ_w, wages are indexed on productivity growth, $\frac{A_t}{A_{t-1}}$, past inflation, π_{t-1} and the central banker inflation target, π_t^* (with relative weight γ_W).

On the capital market, households are capital owners and rent it to intermediate producers. The model introduces different frictions on this market, an investment cost, $S(\frac{I_t}{I_{t-1}})$, a capital utilization cost, $\psi(z_t^\tau)K_t^\tau$ and a lag on capital utilization (K_t is used for production at date $t+1$).

Intermediate firms and final good producers

Intermediate firms (indexed by j) produce differentiated products with a Cobb-Douglas production function.

$$Y_t^j = \tilde{K}_{t,j}^\alpha (A_t L_{t,j})^{1-\alpha} \tag{2}$$

where $\tilde{K}_{t,j} = z_t K_{t-1,j}$ (respectively $L_{t,j}$) is the capital (resp. labor) used by firm j for production at date t. $K_{t,j}$ and $L_{t,j}$ are undifferentiated fractions of K_t and L_t respectively.

A_t is the total factor productivity. It is modeled as an integrated process with a drift, to account for economic growth.

$$A_t = A_{t-1} e^{a+\varepsilon_A(t)} \tag{3}$$

with a the average GDP growth and $\varepsilon_A(t)$ the productivity shock.

Intermediate firms are also price setters. Their prices follow a Calvo process similar to wages with parameters ξ_p and γ_p without indexation on productivity growth.

Regarding the final good sector, firms produce an undifferentiated good, Y_t with input Y_t^j with the technology:

$$Y_t = \left(\int (Y_t^j)^{\frac{1}{1+\lambda_{p,t}}} \right)^{1+\lambda_{p,t}} \tag{4}$$

where $\lambda_{p,t}$ is the price mark-up, equal to a steady state value plus a price mark-up shock $\varepsilon_P(t)$.

Market clearing condition

This model is a closed economy model without fiscal policy. Hence, government expenditure, together with trade balance, are aggregated into an exogenous expenditure shock $\varepsilon_g(t) = G_t$ in equation (5). National accounting gives the global demand of final goods:

$$Y_t = C_t + I_t + G_t + \Psi(z_t) K_{t-1} \tag{5}$$

Monetary authority

Our sample includes a common monetary policy for the whole euro area also prior to the foundation of the European Central Bank. The central banker sets the nominal interest rate following a Taylor rule where the interest rate is a weighted average of national interest rates before 1999 and the ECB interest rate afterwards. Gerlach and Schnabel (2000) have shown that the European Monetary Union policy prior to the foundation of the ECB can be described by a Taylor rule.

$$R_t = R_{t-1}^{\rho} \left(R_t^* \left(\frac{\pi_t}{\pi_t^*} \right)^{r_\pi} \left(\frac{Y_t A_{t-1}}{A_t Y_{t-1}} \right)^{r_y} \right)^{1-\rho} \epsilon_t^R \tag{6}$$

where π_t^* is the inflation target and R_t^*, the targeted nominal interest rate ($R_t^* = \bar{R}R \pi_t^*$ with $\bar{R}R$ the constant real interest rate targeted by the central banker). We model the inflation target as an integrated process:

$$\pi_t^* = \varepsilon_\pi(t) \pi_{t-1}^* \tag{7}$$

The shock $\varepsilon_\pi(t)$ enables to model possible structural break of inflation target. This feature departs from the original time varying inflation target in Smets and Wouters (2003) which was AR(1), and follows Ireland (2008) in the US and Fève et al. (2008) in the euro area. In particular, it allows for a declining inflation target up to the mid 1990s, when central banks where converging toward lower inflation levels and since the European Central Bank (ECB) foundation, a constant inflation target, consistent with its objective. Contrary to Ireland (2008), we choose not to include correlation between the innovation of inflation target and other structural shocks, because in the Euro Area, the inflation target has to be related to exogenous political decisions.

Finally, all shocks follow first order autoregressive processes, except for the productivity shock (ε_A) which is a white noise.

2 Bayesian inference

In this section we briefly detail and comment the data and the methodology used to estimate deep parameters of the model presented above.

Data

We use the Area Wide Model data base (see Fagan et al. (2005)), complemented with data from Eurostat, the OECD and the monthly bulletin of the European Central Bank. We use eight series of Euro Area variables : real GDP, real Private Consumption, real Gross Fixed Capital Formation, Total Compensation of Employees, Total Employment, Total Labor Force, Price Inflation calculated on the basis of the GDP Deflator and the short term interest rate in the Euro area (Euribor 3 months). We further develop the construction of our database in the Appendix.

Our model implies that real variables should share the same trend: the TFP trend. We assume that TFP is a first order integrated process with drift. This drift should be the average growth rate of the real variables. Nevertheless, we empirically find differences in the average growth rate of real GDP and wages (see first graph of figure 1). Indeed, the ratio of wages to GDP has been slowly decreasing since 1992, which is incompatible with our model. There is no such difference in the growth rate of wages and GDP in the US. This fact stems from a specific trajectory of all countries before 1998. Yet, there has not been, as far as we know, theoretical works which could reproduce this phenomenon with micro-foundations in the framework. Hence, we choose to add some *ad hoc* trend-correction on the growth rate of real wages.

Other authors have introduced the use of total employment as a proxy of hours worked and an *ad hoc* function of transfer from employment to hours. We do not use this method and do not use total employment as an observable, since the use of hours worked is incompatible with standard DSGE model and needs further theoretical developments[1].

We want to avoid any filtering of the data prior to the estimation since potential problems for business cycle analysis arising for this approach have been exposed by many authors (Cogley Nason (1995), Canova(1998,1999)). To extract as much information as possible from the data, we use raw data as observable variables. Because real variables in levels are not stationary in our model, we use the growth rates of GDP (dY), GFCF (dI), private consumption (dC) and real wages (dW) as observable variables. The same argument with the nominal trend (inflation target) instead of the real trend (TFP), justifies the use of inflation growth rate ($d\pi$) as an observable variable. We also use the real interest rate ($RR_t = r_t - \pi_t$) which is stationary in our set-up.
As a result, observables are: $[dY, dC, dI, d\Pi, dW, RR]$.

[1] Adding employment data into the model with a non-microfounded transfer function from employment to hours worked does not seem adequate in our approach, it would only cast doubts over our results. We leave the development of the model to the labor market for further research.

The following equations link input variables with stationary variables[2]:

$$dY_t = e^a(\hat{y}_t - \hat{y}_{t-1} + \varepsilon_t^A) + a \tag{8}$$
$$dC_t = e^a(\hat{c}_t - \hat{c}_{t-1} + \varepsilon_t^A) + a \tag{9}$$
$$dI_t = e^a(\hat{i}_t - \hat{i}_{t-1} + \varepsilon_t^A) + a \tag{10}$$
$$dW_t = e^{a+errw}(\hat{w}_t - \hat{w}_{t-1} + \varepsilon_t^A) + a + err_w \tag{11}$$
$$RR_t = \hat{r}_t - \hat{\pi}_t - \bar{RR} \tag{12}$$
$$d\Pi_t = \hat{\pi}_t - \hat{\pi}_{t-1} + \epsilon_\pi(t) \tag{13}$$

where a is the TFP drift, err_w is the trend-correction on wages, $\bar{RR}$ is the steady state value of real interest rate, and $\hat{\pi}$ is the ratio of inflation to the inflation target.

Priors and calibrations

Some parameters are calibrated to replicate standard stylized facts and ratios in the raw data, which correspond to "the parameters that determine the steady state" of Del Negro and Schorfheide (2008). Some other parameters are calibrated as in Smets and Wouters (2003) because they are weakly identified and we prefer using common values rather than introducing noise in the estimation. Calibrations are detailed in table 1. The other groups of parameters mentioned by Del Negro and Schorfheide (2008), corresponding to "taste technology and policy parameters" on the one hand and "parameters describing the propagation mechanism" on the other hand, are estimated through a Bayesian approach. We set a prior for each structural parameter before the estimation[3]. Priors are detailed in table 2.

We follow Smets and Wouters (2003) for most priors, except for prior densities of standard deviations and target inflation parameters. The usual prior density of standard deviation is an inverse gamma; we choose Gaussian distributions to let the Markov Chain cover a larger range of value[4][5]. Regarding the standard deviation of shocks, we set the prior densities equal to likely values according to volatility of observable data. For example, the prior's mean for the investment specific shock's standard deviation is set equal to 10%, comparable to the price of investment volatility[6]. For the monetary policy shock standard deviation prior, we use the deviation from a simple Taylor rule estimated outside the model. We set the mean of the inflation target shock standard deviation prior equal to 0.01%, which corresponds to the decrease of the HP-filtered inflation from the mid 1980s to the mid 1990s. The standard deviation of this prior is set to 0.01 to let the possibility of a constant inflation target.

3 Trends

In this section, we detail the rationale for our trends specification. While Smets and Wouters (2003) use an HP-filter to extract the cycle *ex ante*, Smets and Wouters (2005) and Sahuc and Smets (2008) use a "deterministic growth rate driven by labor-augmenting technological

[2]These equations differ from usual ones. We show in appendix on linearization, how using log linear approximation implies a mixed first and zero-order approximation with respect to the average growth rate a. Approximation we don't want to make since a is a parameter.

[3]We use Dynare v4.0 and a Random Walk Metropolis Hastings with 600 000 draws to obtain the posterior density.

[4]When we estimate our model with uniform or inverse gamma priors, it hardly modifies the point estimates and our findings remain unchanged.

[5]Gaussian densities allow for negative values, this is why some posterior densities, are bimodal on plus and minus the standard deviation value, see for instance σ_W in figure 3

[6]See section 4 and Justiniano et al. (2008) for more details on the link between the investment specific shock and the price of investment.

progress" to detrend real variables. However, none of these papers include a nominal trend simultaneously with a TFP trend. This feature of the New Keynesian Phillips Curve has been investigated by Cogley and Sbordonne (2008). Recent papers do such inclusion in a DSGE framework, as Ireland (2008) or Fève et al. (2008) but their models do not include capital. Yet we expose here that investment dynamic is key in explaining cyclical fluctuations in the economy.

Rationale for an integrated productivity process

We have earlier introduced the TFP as a stochastic integrated process:

$$A_t = A_{t-1}e^{a+\varepsilon_{A1}(t)}$$
$$A_t = A_0 e^{at+\sum_{i=0}^{t}\varepsilon_{A1}(i)}$$

with $\varepsilon_{A1}(t)$ a white noise shock.

Even though we are skeptical about modeling technological innovation as transitory, we test the alternative specification used by Smets and Wouters (2005,2007), a linear TFP with autoregressive technology shock:

$$A_t = A_0 e^{at+\varepsilon_{A2}(t)}$$

with $\varepsilon_{A2}(t)$ an AR(1) shock. In order to test which assumption best fits the data we introduce both shocks transitory and integrated in the TFP:

$$A_t = A_0 e^{at+\varepsilon_{A2}(t)+\sum_{i=0}^{t}\varepsilon_{A1}(i)}$$

By eliminating the transitory shock (resp. the permanent shock) we can test the fit of each set-up with the data.

A set-up with both shocks slightly deteriorates the marginal density (-301 against -300 for our integrated specification). Besides, the marginal density of the model with a linear trend and a non integrated process is lower (-311), which implies that if I am agnostic over the choice of model*ex ante*, *ex post* I will find the integrated specification 6 10^4 times more likely than the autoregressive specification. Only if our prior allows for very high persistence of the productivity shock, the marginal density compares to our specification (-302), in this case we find the persistence of this shock equal to 0.97, and the integrated specification is still 7.4 times more likely than the autoregressive specification. Thus Bayesian analysis argues in favor of an integrated process.

In figure 9, we have a closer look at productivity shocks. The graph shows both productivity shocks, AR(1) ($\varepsilon_{A2}(t)$) and I(1) ($\sum_{i=0}^{t}\varepsilon_{A1}(i)$). It also shows the HP-trend of output cleared from the linear trend of TFP. As a matter of fact, when using HP-filters of the data, this HP-trend is actually the equivalent of the productivity in our model. First we see that the AR(1) and I(1) TFP are roughly consistent with raw data and a pure statistical filter. In fact, AR(1) and I(1) estimated TFP are very similar. However, compared to HP-filters, other approaches reveal more information in sharper peaks of productivity. Moreover, there is hardly one cycle of TFP over our sample, which corresponds to a persistence close to one in the AR(1) specification. It calls the AR(1) specification into question.

Because of a better fit to the data and because the AR(1) hypothesis leads to a highly autocorrelated I(1)-shaped process, using an I(1) process to model TFP seems to be the best approach. This result is consistent with Ferroni (2008) estimation of the Smets and Wouters model for the US.

Rationale for an integrated Inflation Target in the euro area

As far as the nominal trend is concerned, the political decision to tackle inflation in the 80s as well as the well-known ECB's objective to maintain inflation "below but close to 2%" economically justify our design of a moving inflation target: an integrated process for inflation target allows it to decrease sharply in the 80s and be stable since 1999.

However, a simply I(1) inflation target has a drawback in a rational expectations framework: agents do not anticipate after the Maastricht treaty the convergence toward lower inflation levels which made the creation of the euro area possible. If one believes that the ECB objective of inflation was anticipated at the beginning of the sample, one should prefer an AR(1) inflation target as in Smets and Wouters (2003). On the other hand, it is hard to know whether the success of the convergence process and after of the ECB in maintaining a low inflation has been anticipated or not, even though the decision to lower inflation was made.

Thus, there are three alternatives left. A pure I(1) inflation target (Ireland (2008)), agents cannot anticipate future levels of inflation target. An AR(1) inflation target (Smets and Wouters (2003)). Or an integrated target where the innovation follows an AR(1) process (Fève et al.(2008)). The third specification allows for long term fluctuation of inflation with partial anticipation of future changes. As for the real trend, we test all three specifications. First, in the integrated inflation target set-up, we find an autocorrelation coefficient for the inflation target innovation equal to 0.71, which rules out the purely I(1) specification. Then, marginal density decreases from -300 with an integrated inflation target to -310 with an autoregressive inflation target, which implies that if I am agnostic over the choice of model*ex ante*, *ex post* I will find the integrated specification $2\ 10^4$ times more likely than the autoregressive specification.

Using only autoregressive processes for both the inflation target and productivity deteriorates the marginal density to -316, the odd ratio is then $9\ 10^6$ in favor of the integrated specifications.

In figure 11, the first graph shows the inflation target derived from our model (in red) compared to the inflation (black) and the Euribor 3 months (dotted black). The inflation target effectively follows the inflation, this finding is consistent with Ireland (2008) for the US and Fève et al. (2008) for the euro area. Our inflation target captures the convergence toward lower inflation levels through a sharp decrease of inflation target from 1992 to 1999. Then, inflation target volatility is significantly smaller which is consistent with the ECB mandate for price stability.

Actually, re-estimating our model on 1999-2008 sub-sample, divides by almost five the standard deviation of the inflation target innovation (from 0.027 to 0.0064). This small standard deviation implies a constant inflation target profile. On this sub-sample, the fit to the data (marginal density) is exactly the same wether we calibrate the inflation target shock constant or not. Hence, our model is able to confirm a true structural break in inflation target strategy through this integrated process[7].

To conclude, the assumption of an integrated inflation target seems to be the most consistent hypothesis in terms of economic and monetary history, as well as of fit to the data.

[7]Estimating on sub-samples implies no major change for deep parameters estimation. Hence we can estimate our model on the whole period without fearing structural break of deep parameters and estimation biases

Defining the business cycle

3.0.1 Three levels of real variables dynamic introduced by the integrated TFP

When TFP is modeled by an integrated process with drift, it introduces three levels of dynamics, which are exemplified in figure 10.

The first graph illustrates the general shape of real GDP and our identification of its trend: the red line is the random walk described by TFP with its drift, while the black line is real GDP. The gap between output and the productivity appears cyclical and relatively small compared to historical range of GDP changes (less than 4% deviation). This first graph shows the ability of our model to replicate trended real variables.

The second figure shows the deviation of output and TFP from their deterministic trend. The red line is the random walk described by TFP without its drift and the black line is the output without its linear trend. According to our model, a shock on productivity has a positive impact on real variables of 100% magnitude in the long term and we can see that the accumulation of these shocks describes a long term cycle. Up to the mid 1990s, the productivity shocks are strictly and strongly positive, indicating a true upward trend of potential production (4% above the linear trend). After 1994, they are negative, except around 2000 (the internet boom).

The third graph shows output over productivity extracted from our model. This component of output is the business cycle. It is stationary. We can see that this variable is cyclical and has a similar range of change to the long term fluctuations of output induced by random walk productivity (4%).

To sum up, we obtain a decomposition of fluctuations into three parts depending on the horizon: a prominent long term linear trend, a long term trend induced by random walk productivity and the business cycle (the resultant).

3.0.2 Two levels of dynamic for inflation and interest rate

Symmetrically, the integrated inflation target, since it has no drift, introduces two levels of dynamic for the inflation and the interest rate, which are exemplified in figure 11.

The first graph shows the real values of inflation and interest rate and the estimated inflation target. A shock on the inflation target has a permanent impact on both inflation and interest rate, in particular, we can interpret a positive inflation target shock as a permanent accommodative monetary policy shock, it positively impacts GDP.

The second and third graph show the business cycle of the inflation and the interest rate, respectively. Inflation target shocks also have a transitory effect on the business cycles of both the inflation and the interest rate. This effect is positive for inflation and negative for interest rate, like a negative monetary shock.

4 Results implications for the business cycle

Estimation results in our most probable specification of the trends are presented in table 2, while figures 3 and 4 depict the prior and posterior densities of the estimated parameters and show the quality of the estimation. In these graphs, the prior density is represented in grey, the posterior density in black and the posterior mode in green. One can check that the mode corresponds to the posterior mode and that the posterior distribution has a lower variance than the prior except for the Frish elasticity, σ_l and the weight of inflation in the Taylor rule, r_π which are often weakly identified. Regarding the structural parameters, our results are generally similar to the ones found in literature. We find a smaller indexation of prices and wages on past inflation than Smets and Wouters (2003), but our results are very similar to those of Feve et al. (2008), who use the same indexation on both past inflation and current inflation target and also close to Sahuc and Smets (2008) and Smets and Wouters (2007). The Calvo parameter on prices is larger than the Calvo parameter on wages which is also found by Smets and Wouters (2003), Fève et al. (2008) and Sahuc and Smets (2008).

While our estimates are in line with the literature, we identify two main differences in the economic transmission mechanisms in our model. First, a larger internal persistence than what is usually found, second a different set of shocks driving the short term economic fluctuations.

Internal persistence

As in the data, we find strong persistence of the endogenous variables: 0.96 for GDP, investment and consumption, 0.99 for capital, 0.94 for labor, 0.92 for the interest rate, and 0.67 for inflation. This persistence is induced by the economic model rather than by the shocks. Indeed, on the investment specific shock and the preference shock we have much smaller persistence (0.17, 0.38, respectively) than Smets and Wouters (2003, 2005), Sahuc and Smets (2008) who find them around 0.9. Also we find no need to use ARMA processes to avoid unit roots on mark-up shocks as in Smets and Wouters (2007) and Sahuc and Smets (2008). The residual demand shock has the highest persistence (0.94), this value is logically close to the persistence of endogenous variables mentioned above since this shock embodies the rest of the world and this is consistent with the literature. In comparison with the literature, we find a much higher habit formation parameter, which partly accounts for the high persistence of the model anyhow.

Sources of fluctuation at business cycle frequencies

Table 3 documents the decomposition of each endogenous variables' variance in terms of shocks and enables to understand what are the main sources of the fluctuations.

Nominal fluctuations stem from price and wage mark-up shocks; however, contrary to Smets and Wouters (2003 and 2005), we find no role for price mark-up shocks and a small role for wage mark-up shocks in real variables' business cycle (see columns 5 and 8 in table 3). However, contrary to Smets and Wouters (2003 and 2005), we find no role for price mark-up shocks and a small role for wage mark-up shocks in real variables' business cycle (see columns 5 and 8 in table 3). For instance, wage mark-up shock, respectively price mark-up shock, only explains 11%, respectively 0% of GDP.

Turning to the productivity shock (first column, table 3), contrary to RBC supporters, we find a little role for productivity in explaining both the real and nominal variables' business cycle. For instance, GDP fluctuations are driven for only 5% by productivity shock, for inflation and interest rate, these figures are 0% and 2%, respectively. Overall, only a few percentage points of variance are due to one of these shocks and none of them is the main source of variance for any variable[8].

The inflation target shock (sixth column) also have a negligible impact on both the real and nominal variables' cycle. It accounts for 2% of the GDP business cycle and for the inflation and the interest rate, 0% and 2%, respectively.

[8]The fact that productivity shocks account for 46% of volatility in wage growth variance reflects our assumption of perfect indexation of real wages on productivity.

Actually, we find a linchpin role for the investment shock in cyclical fluctuations. It accounts for 22% of GDP, 84% of capital. This finding is consistent with the recent findings of Justiniano et al. (2008), but also Greenwood et al.(1997) or Fischer (2002), who find that investment specific shocks explain a large part of GDP fluctuations in the United-States. Ferroni (2008) find similar results: a little role for productivity shocks, central investment specific shocks in a *one-step* approach. He also find that mark-up shocks can be important sources of fluctuation, but under the less likely specifications of the trend.
In addition to the investment specific shock, we find that 40% of consumption's business cycle is driven by the preference shock. This shock is a wedge in the Euler equation on consumption. Canzoneri et al. (2007) have estimated on US data the interest rate from this equation without wedge. They find that the interest rate which should explain fluctuations in private consumption is negatively correlated to the monetary policy instrument. Hence, consumption being driven by preference shocks is not surprising and argues in favor of "animal spirit" as an important source of the business cycle. The prevalence of investment specific shocks and preference shock in explaining the business cycle also translates in the historical decomposition of endogenous variables. Figure 13 exemplifies the domination of preference shocks over private consumption's business cycle (second graph), while the investment specific shock explain the investment business cycle (third graph). The two combined play a major role for the GDP (first graph).

On the shocks driving the cycle and their structural characteristics

As the preference shock and the investment specific shock are central in understanding the cycle, we broaden the study by testing the ability of these shocks to match their definition.

First the preference shock affects the subjective discount rate of households and can be interpreted as their confidence in the future. To illustrate its identification, we compare it with the confidence indicator of the households in the euro area published by the European Commission. The correlation with the preference shock is equal to 0.26 which is non-negligible. Because the estimated preference shocks are more volatile than the confidence indicators, we provide moving average of this time series on 4 quarters. The correlation of the synthetical confidence indicator with the smoothed preference shock is equal to 0.29. A regression of the smoothed preference shock on the different items of the confidence indicator (General economic situation over last 12 months, General economic situation over next 12 months, Price trends over next 12 months, Unemployment expectations over next 12 months, Statement on financial situation of household) explains 69% of the smoothed preference shock variance. Figure 14 shows these time series. Hence, the preference shock we estimate cannot be said to be orthogonal to the measurement of households' confidence.

Regarding the investment specific shock, it describes the conversion of one unit of investment into capital. One may interpret it as the combined effects of a shock affecting the transformation of consumption into investment goods (the relative price of investment) and a shock describing the difficulty of firms to finance their investment. A key question consists in disentangling the two components of this shock[9]
First, we follow Greenwood et al. (2000), Fisher (2006), Justiniano et al. (2008) and compare the investment specific shock with the inflation of investment relative to GDP inflation using the time series of the AWM database. Figure 15 shows this comparison. Both time series exhibit high volatility and their correlation is equal to 0.20, which is again non-negligible.
We also compare the investment specific shock with the spread BBB-OAT since 2000 for the non-financial corporate rate published by Merrill Lynch. We find a clear negative correlation

[9]Justiniano et al. (2009) have investigated this shock for the US. They show that the investment specific shock is mainly explained by its financial part.

of the investment specific shock with the BBB corporate rate equal to -0.30.
We then have evidence showing that the investment specific shock embodies market conditions of investment: relative inflation of investment goods, risk premium for external financing and entrepreneurs confidence.

As a conclusion, the comparisons between these two shocks and some related time series show that one can not reject the hypothesis that these shocks satisfactorily replicate structural shocks in the economy.

5 Booms and busts under the scope of historical decomposition

In this section, we provide an illustration of the credibility of our model and our identification of shocks. To this aim, we turn to the historical decomposition of variables in terms of shocks (figure 12 for GDP growth, inflation and interest rate, figure 13 for GDP, consumption and investment) and the interpretation it gives for the economic history of the euro area since 1985.

Up to the 1993 recession and the Maastricht treaty

In the beginning of the sample we estimate negative and persistent contribution of the residual demand shock (in pale blue). This contribution can be interpreted as the combination of 2 facts. First in 1985, the exchange rate between US dollar and the Deutsche Mark was quite high. Hence, the DM and other European currencies pegged on it were relatively depreciated with respect ti the US dollar and the trade balance between the euro area and its first commercial partner was larger. As the currencies appreciated in the late 1980s and early 1990s, the trade balance effect became smaller. Second, European government started coordinated policies of reducing government expenditures in order to create the European Union. The Maastricht treaty was written in 1992, it was adopted in 1994. The two combined have a depressionary impact on the euro area which translates in our model to a negative contribution to GDP of the residual demand shock. Figure 16 shows the negativity of the trade balance and figure 16 illustrate the decline of the government expenditures between 1980 and 1990.

The German reunification, and the exchange rate crisis

From 1991 to 1993, monetary policy is very restrictive (deviation from the average Taylor rule is in orange). Indeed, the German reunification induced a huge inflow of liquidity in the German economy in 1990. To counter the inflationist risks, the Bundesbank implemented a restrictive monetary policy starting the second half of 1991. Other European countries, for fear of having their currency depreciated against the Deutsche Mark, tightened their monetary policy as well. On top of that, protection against speculative attacks forced some countries (for instance UK in July 1992, Italy in September 1992, Spain and Portugal in November 1992, Ireland in February 1993 ...) caused these countries to temporarily increase their rates even more. Thus, this huge positive deviation from the Taylor rule have a clear depressionary impact on GDP according to our estimates. Up to 1993, it is somewhat counterbalanced by preference shocks, investment specific shocks and in a smaller extent wage mark up shock.
But in 1993, preference shocks and investment specific shocks were such that their positive effect on GDP was reduced to almost zero while at the same time, productivity started declining. Simultaneously the effect of mark up shock on GDP (in red) inverted and became

negative amplifying the crisis. Yet, monetary policy stayed restrictive for another three years even though its contribution to growth became positive during 1992-1993. Monetary policy went closer to the Taylor rule but the discretionary impact of monetary policy never became really accommodative.
The difficult situation for monetary policy coordination in the euro area may explain the slow ease of monetary policy in front of the 1993 crisis. Gerlach and Schnabel (2000) have shown for instance that, between 1992 and 1993, the interest rate of the European Monetary Union (EMU) has significatively departed from its usual Taylor rule. In the early nineties, the European Monetary System (EMS) faced numerous devaluation and revaluation of its currencies. All currencies have suffered from a reconsideration of their parity with the DM except the florin, which took advantage of a perfectly aligned macroeconomic policy with Germany. The pound sterling and the Italian lire even left the EMS in 1992. In this context, the EMS was weakened and its fluctuation margin was dramatically increased to $\pm 15\%$. In addition, the policies directed toward the convergence to low inflation levels added to the restrictiveness of monetary policy from 1992 to 1999, which deepened the recessionary effect of monetary policy (inflation target shock in yellow). Quantitatively, the inflation target during 1993 had a negative impact on GDP growth (-0.30% in annual growth). Even if this quantitative impact is lower than the investment specific shock (-1.24%) or the preference shock (-0.72%) for the same period, it remains comparable to GDP growth (-0.41%).

Climax in 2000 and collapse

This period was characterized by a succession of events.

First the Asian crisis in the late nineties had by contagion a recessionary impact in Europe. Its effect translates to the residual demand shock which includes the trade balance. According to our identification of shocks, the small contraction from 1998-Q1 to 1999-Q1 preceding the dot-com bubble is widely due to this contractionary effect of the trade balance.

Second, during the dot-com bubble, we identify the GDP growth as the combination of the investment specific shock and an increase in productivity with an almost neutral monetary policy. When burst of the dot-com bubble in 2001 we find a slow down in productivity growth (see second graph of figure 10) and a shift in the investment specific shock which became recessionary. As a consequence of this crisis the governments' deficits in the euro area increased dramatically. This automatic stabilizer can explain the temporary positive effect of residual demand shock, see figure 16 and is reinforced by an improvement of the trade balance as shown in figure 16.

Third, the 9-11 attacks negatively impacted households confidence tremendously and might be responsible for the further decline in consumption growth in the third and fourth quarter of 2001 and the first quarter of 2002. The historical decomposition of output shows that households' confidence had a positive impact on GDP as the dot-com bubble grew, but this effect shifted in the fourth quarter of 2001 just after the terrorist attacks in USA.

Because the crash was global, the contribution of residual demand shock is negative which is consistent with a potential negative impact of the trade balance, see figure 16. Meanwhile monetary policy shock was expansionary and significantly contributed to reduce the contraction.

The subprime crisis

In the summer 2007, the subprime bubble burst in USA affecting all the other economies mainly by the end of 2008. In the euro area, GDP growth is at its minimum over our sample in the fourth quarter 2008. According to our estimates, a strong recessionary impact of both the preference shock and the investment specific shock explain this large decrease in GDP growth. As we have shown above, we can interpret the negative preference shock as the collapse of the households' confidence, while the investment specific shock embodies financial market situation, among others, a channel which, with no doubt, faced a major negative shock during the subprime crisis.

The monetary policy has become accommodative only in the fourth quarter 2008 corresponding to the fact that the ECB decided to decrease its main refinancing interest rate only in October 2008. This delay may have resulted from an upward risk on inflation as well as uncertainty during this period. Indeed, the euro area has experienced inflationary shocks interpreted by our model as wage and price mark-up shocks instead of shocks on energy prices and commodity prices since 2007-Q1 as figure 12 shows.

Finally, our estimates passes the economic test of historical credibility as our shocks identification gives credible causes of economic fluctuations for the euro area.

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Linearized Model

Linearized observation equations

Let X_t be a real trended variable of the economy (GDP, investment, consumption or wages). Note $\tilde{X}_t = X_t/A_t$ the corresponding stationary variable.
$\bar{X}$ is the variable's steady steady and $\hat{X}_t$ is the rate of deviation of $\tilde{X}_t$ from its steady state value.
$\hat{X}_t = (\tilde{X}_t - \bar{X})/\bar{X}$ or $log(\tilde{X}_t) - log(\bar{X})$ with a first order approximation.
We recall that productivity verifies $A_t = A_{t-1}e^{a+\varepsilon_A(t)}$ and productivity shocks ε_A is AR(1).

The growth rate of X_t is our observable. The following equations link it to the stationary variables taken in deviation from their steady state values which are the variables used for computations.

$$
\begin{aligned}
dX_t &= \frac{X_t - X_{t-1}}{X_{t-1}} \\
dX_t &= \frac{\tilde{X}_t * A_t - \tilde{X}_{t-1} * A_{t-1}}{\tilde{X}_{t-1} * A_{t-1}} \\
dX_t &= \frac{e^{a+\varepsilon_a(t)}(1+\hat{X}_t) - (1+\hat{X}_{t-1})}{1+\hat{X}_{t-1}} \\
dX_t &= \frac{e^{a}(1+\varepsilon_a(t))(1+\hat{X}_t) - (1+\hat{X}_{t-1})}{1+\hat{X}_{t-1}} + o(\varepsilon_a^2) \\
dX_t &= (e^a - 1) + e^a * (\hat{X}_t - \hat{X}_{t-1} + \varepsilon_a(t)) + o(\hat{X}^2, \varepsilon_a^2)
\end{aligned}
$$

As a consequence, the following equation, which is used by other authors,

$$dX_t = \hat{X}_t - \hat{X}_{t-1} + \varepsilon_a(t) + a$$

is an approximation of the equation above with a, the average growth rate, close to zero. It is false for two reasons: it is an approximation with respect to a parameter in addition to variables, it is a mixed first-order/zero-order approximation.

Yet a being very small, such a mistake does not cast much doubts on the results found with this method.

Others observables are the growth rate of inflation, and the real interest rate.
Before linearization, the model uses the variables $R_t = 1 + r_t$ and $\Pi_t = 1 + \pi_t$ where r_t and π_t are interest rate and inflation rate.
R_t/Π_t^* and Π_t/Π_t^* are the stationary variables. We write $\hat{r}_t$ and $\hat{\pi}_t$ their deviation rate from steady state value.
Thus, the following equations link the real interest rate and the growth rate of inflation to the stationary variables.

$$
\begin{aligned}
RR_t &= r_t - \pi_t = \hat{r}_t - \hat{\pi}_t - \bar{RR} \\
d\pi_t &= \hat{\pi}_t - \hat{\pi}_{t-1} + \epsilon_\pi(t)
\end{aligned}
$$

where $\bar{RR}$ is the steady state value of the real interest rate and ϵ_π the inflation target shock.

Steady State equations

Output in national accounting

$$\bar{y} = \bar{c} + \bar{i} + \bar{g} \tag{14}$$

Output as final production

$$\bar{y} = \bar{z}^{\alpha}\bar{k}^{\alpha}\bar{L}^{1-\alpha}e^{-\alpha a} - \Phi \tag{15}$$

Marginal utility of households

$$\beta\frac{\bar{R}}{\bar{\bar{\Pi}}} = e^{a} \tag{16}$$

Capital dynamic

$$\frac{\bar{i}}{\bar{\bar{k}}} = 1 - e^{-a}(1-\delta) \tag{17}$$

Tobin-Q and investment maximization program

$$1 = \bar{Q}(1 + S(e^{a}) - e^{a}S'(e^{a})) + \beta e^{-a}\bar{Q}e^{2a}S'(e^{a}) \tag{18}$$

$$with\ S(e^{a}) = 0 = S'(e^{a}) \tag{19}$$

$$gives\ \bar{Q} = 1 \tag{20}$$

$$\bar{Q} = \beta e^{-a}(\bar{Q}(1-\delta) + \bar{r}^{k}\bar{z}) \tag{21}$$

$$knowing\ \bar{Q} = 1 = \bar{z} \tag{22}$$

$$gives\ \bar{r}^{k} = \frac{e^{a}}{\beta} + \delta - 1 \tag{23}$$

Capital utilization rate and rental cost

$$\bar{r^{k}} = \psi'(\bar{z}) \tag{24}$$

$$with\ \bar{z} = 1 \tag{25}$$

$$czcap = \frac{\bar{\psi}''}{\bar{\psi}'} \tag{26}$$

Marginal cost of production

$$M\bar{C} = \bar{w}^{1-\alpha}(\bar{r}^{k})^{\alpha}(\alpha^{-\alpha}(1-\alpha)^{\alpha-1}) \tag{27}$$

Constant ratio of factors remuneration

$$\bar{w}\bar{L} = \frac{1-\alpha}{\alpha}\bar{r}^{k}\bar{z}\bar{k}e^{-a} \tag{28}$$

Price setting

$$1 = \xi_{p}\bar{\pi}^{\frac{1-\gamma_{p}}{\bar{\lambda}_{p}}} + (1-\xi_{p})\bar{\tilde{\Pi}}^{\frac{1}{\bar{\lambda}_{p}}} \tag{29}$$

$$(1+\bar{\lambda}_{p})M\bar{C} = \bar{\tilde{\Pi}}\delta\bar{\Pi}^{(\gamma_{p}-1)(T-1-t)} \tag{30}$$

knowing $\bar{\Pi^{*}} = 1$. we find $\delta\bar{\Pi} = 1$

Wage setting

$$\bar{w}^{\frac{1}{\bar{\lambda}_{w}}} = \xi_{w}\bar{w}^{\frac{1}{\bar{\lambda}_{w}}} + (1-\xi_{w})\bar{\tilde{w}}^{\frac{1}{\bar{\lambda}_{w}}} \tag{31}$$

hence $\bar{w} = \bar{\tilde{w}}$

Dynamic equations

Output in national accounting

$$\hat{y}_t = ccons\ \hat{c}_t + cinv\ \hat{i}_t + \varepsilon_g(t) + \frac{cinv\ e^{-a}}{1-e^{-a}(1-\delta)}(\frac{e^a}{\beta}+\delta-1)\ \hat{z}_t \tag{32}$$

Output as final production

$$\hat{y}_t = (1+\phi_y)(\alpha\hat{z}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{L}_t - \alpha\ \varepsilon_a(t)) \tag{33}$$

Households' maximization program

$$\hat{c}_t = \frac{h}{e^a+h}\hat{c}_{t-1} + \frac{e^a}{e^a+h}\hat{c}_{t+1} - \frac{h}{e^a+h}\varepsilon_a(t) + \frac{e^a}{e^a+h}\varepsilon_a(t+1) + \frac{e^a-h}{e^a+h}(\varepsilon_b(t)-\varepsilon_b(t+1)-(\hat{r}_t-\hat{\pi}_{t+1}-\varepsilon_{\pi^*}(t)) \tag{34}$$

Tobin-Q

$$\hat{Q}_t = -\hat{r}^k_t + \hat{\pi}_{t+1} + \hat{\pi}^*_{t+1} + (1-\delta)\beta e^{-a}\hat{Q}_{t+1} + (1-(1-\delta)\beta e^{-a})\hat{r}^k_{t+1} \tag{35}$$

Investment maximisation program

$$\hat{i}_t = \frac{1}{1+\beta}\left(\hat{i}_{t-1} + \beta\hat{i}_{t+1} + \frac{e^{2a}}{S''(e^a)}(\hat{Q}_t + \hat{\varepsilon}^I_t) - \varepsilon_a(t) + \beta\varepsilon_a(t+1)\right) \tag{36}$$

Capital utilization rate and rental cost

$$\hat{r}^k_t = czcap\ \hat{z}_t \tag{37}$$

Marginal cost of production

$$\hat{MC}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}^k_t \tag{38}$$

Constant ratio of factors remuneration

$$\hat{w}_t + \hat{L}_t = \hat{r}^k_t(1+\frac{1}{czcap}) + \hat{k}_{t-1} - \varepsilon_a(t) \tag{39}$$

Phillips curve

$$\begin{aligned}\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} + \gamma_p\varepsilon_{\Pi^*}(t) &= \beta e^a(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t + \gamma_p\varepsilon_{\Pi^*}(t+1)) \\ &\quad + \frac{(1-\beta\xi_p e^a)(1-\xi_p)}{\xi_p}[\hat{MC}_t + \frac{\bar{\lambda}_p\hat{\lambda}_{p,t}}{1+\bar{\lambda}_p}]\end{aligned} \tag{40}$$

Wage Phillips curve

$$\begin{aligned}\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \gamma_w\hat{\pi}_{t-1} + \gamma_w\varepsilon_{\Pi^*}(t) &= \beta(\hat{w}_{t+1} - \hat{w}_t + \hat{\pi}_{t+1} - \gamma_w\hat{\pi}_t + \gamma_w\varepsilon_{\Pi^*}(t+1)) \\ + \frac{(1-\xi_w)(1-\xi_w\beta)}{\xi_w(1+\sigma_l\frac{1+\bar{\lambda}_w}{\bar{\lambda}_w})}&(-\hat{w}_t + \sigma_l\hat{l}_t + \frac{\bar{\lambda}_w\hat{\lambda}_{w,t}}{1+\bar{\lambda}_w} - \varepsilon_b(t) + \frac{\hat{c}_t - he^{-a}(\hat{c}_{t-1}-\varepsilon_a(t))}{1-he^{-a}})\end{aligned} \tag{41}$$

Data

The updating of the AWM database was achieved as follows:
Real GDP, private consumption and GFCF were extrapolated using the growth of the corresponding Eurostat series.
Inflation was simply completed with Eurostat data.
Total compensation of employees and total employment were extrapolated using the growth of the corresponding series published in the monthly bulletin of the European Central Bank.
Total labor force was completed using the OECD series of unemployment rate and the extrapolated series of Total employment.

Our data cover the 1985 Q1 to 2008 Q4 period for Euro Area (16 countries).

The model evades the labor market, it is then based on the modeling of the labor force to explain the economy. In others words, the question of participation to the labor market is voluntarily left aside and any consumer or household is a worker. In order to model a representative household in this framework, we divided the extensive data (real GDP, private consumption and GFCF) by the total labor force. As a consequence, these per capita variables must be handled with care while commenting the results since they overestimate the real value which would be divided by the total population.

Regarding the labor market variables, the correspondence between data and the model is more complicated.

In the model, households are wage setters and firms adjust their labor demand to this wage level. Hence, the best definition of wage would be the wage per hour worked. Not having at our disposal the series of total hours worked, we use the series of total employment as its proxy and calculate the wage per employment.

When eluding the question of the labor market, we make an even stronger assumption: by hypothesis, there is no unemployment in the model. The difficulty to overcome then, is the correct definition of labor supply and demand at equilibrium. To model the labor market, Smets & Wouters (following Christiano, Eichenbaum and Evans 2005) stated the existence of a perfect insurance against unemployment and labor income variation using state-contingent securities which ex-post guarantees that the labor income of each household matches the aggregate labor income. As a consequence, all members of the labor force can be treated equally.
Usually, labor in this model is total hours worked. From this definition of employment, we first followed Smets & Wouters (2003) to derive the total hours worked through a "Calvo" process. Yet, the series of total employment introduced to much non stationarity and we finally abandoned the employment as an observed variable.

parameter	value	comments
Cobb-Douglas param. α	0.34	corresponds to yields of capital to output ratio
households discount factorβ	0.9926	compatible with steady state equation
capital depreciation rate δ	0.025	as in Smets & Wouters (2003)
SS cons. share in GDP c_{cons}	0.57	equal to average ratio in the data
SS invest. share in GDP c_{inv}	0.21	equal to average ratio in the data
SS wage mark-up λ_w	0.1	weakly identified, set as Feve et al. (2008)
SS price mark-up λ_p	0.2	unidentified, set as Feve et al. (2008)
SS real interest rate $\bar{R}R$	0.4762	equal to average value

Table 1: Parameters calibration

	Prior			Posterior			
Estimated parameters	distribution	mean	stdev	mode	mean	conf.	interval
GDP growth	norm	0.004	0.0005	0.0033	0.0033	0.0029	0.0038
Trend correction on wages	norm	-0.002	0.0002	-0.0014	-0.0015	-0.0017	-0.0012
habit formation	beta	0.700	0.1500	0.9592	0.9484	0.9223	0.9746
invest. adj. cost	beta	0.100	0.1000	0.0711	0.0628	0.0370	0.0880
σ labour	norm	2.000	0.5000	2.0529	2.0848	1.2959	2.8659
calvo wages	beta	0.750	0.0500	0.7251	0.7310	0.6615	0.8046
calvo prices	beta	0.750	0.0500	0.7349	0.7283	0.6594	0.7991
wage indexation	beta	0.500	0.1500	0.3631	0.3772	0.1673	0.5837
price indexation	beta	0.500	0.1500	0.1319	0.1598	0.0514	0.2641
capital util. adj. cost	norm	0.200	0.1000	0.4140	0.4218	0.2778	0.5623
polmone inflation	norm	1.700	0.1500	1.6038	1.6078	1.3544	1.8456
polmone smoothing	beta	0.750	0.1000	0.8614	0.8577	0.8211	0.8962
polmone output	norm	0.125	0.0500	0.1552	0.1535	0.0725	0.2303
Shocks persistence	distribution	mean	stdev	mode	mean	conf.	interval
preference shock	beta	0.250	0.1500	0.1065	0.1631	0.0200	0.2919
residual demand shock	beta	0.750	0.1500	0.9494	0.9365	0.8994	0.9746
investment specific shock	beta	0.500	0.1500	0.3804	0.3729	0.2259	0.5256
wage mark-up shock	beta	0.500	0.1500	0.4019	0.4194	0.2229	0.6204
monetary policy shock	beta	0.500	0.1500	0.5329	0.5325	0.4041	0.6601
price mark-up shock	beta	0.500	0.1500	0.2258	0.2477	0.0904	0.4036
inflation target shock	beta	0.500	0.1500	0.7087	0.6630	0.4575	0.8779
Standard deviation of shocks	distribution	mean	stdev	mode	mean	conf.	interval
productivity shock	norm	0.300	0.3000	0.2959	0.3093	0.2317	0.3826
preference shock	norm	5.000	5.0000	12.8731	11.6560	6.2559	16.7775
residual demand shock	norm	0.300	0.3000	0.9617	0.9329	0.6506	1.2030
investment specific shock	norm	10.000	10.0000	13.9785	17.2123	10.7292	23.6268
wage mark-up shock	norm	0.200	0.2000	0.1221	0.1265	0.1039	0.1480
monetary policy shock	norm	0.200	0.2000	0.1744	0.1804	0.1463	0.2143
price mark-up shock	norm	0.010	0.0100	0.0249	0.0269	0.0167	0.0371
inflation target shock	norm	0.200	0.2000	0.2306	0.1686	-0.1882	0.3288

Table 2: Point estimates of our baseline model on the 1985-2008 period

	Shocks contribution to each variable cyclical dynamic							
	prod.	pref.	res. demand	invest.	price m-u	infl. targ.	mon. pol.	wage m-u
GDP	5	20	14	22	0	2	26	11
Cons	14	40	6	25	0	1	10	4
Invest.	1	9	5	61	0	1	13	8
Capital	4	6	3	78	0	0	4	4
Cap. rent. cost	3	11	7	72	0	0	5	3
Euribor	0	7	5	6	4	2	62	14
Labour	3	25	17	16	0	2	25	12
Wage	2	5	3	51	7	0	4	28
Tobin-Q	0	1	1	5	2	5	75	9
Inflation	0	5	3	2	33	2	31	24
Marg. Cost	0	3	2	2	20	1	12	61
GDP growth	2	32	29	28	0	1	6	1
cons. growth	1	96	0	0	0	0	2	0
invest. growth	0	1	1	86	0	1	8	3
wage growth	49	0	0	1	17	0	0	33
real interest rate	0	1	0	1	11	3	78	6
inflation growth	0	1	1	0	76	2	6	13

Table 3: Variance decomposition of our baseline model estimated on the 1985-2008 period

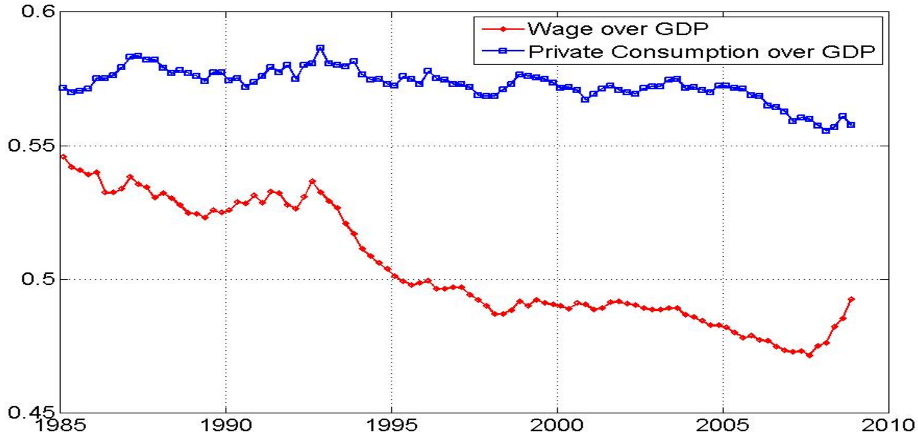
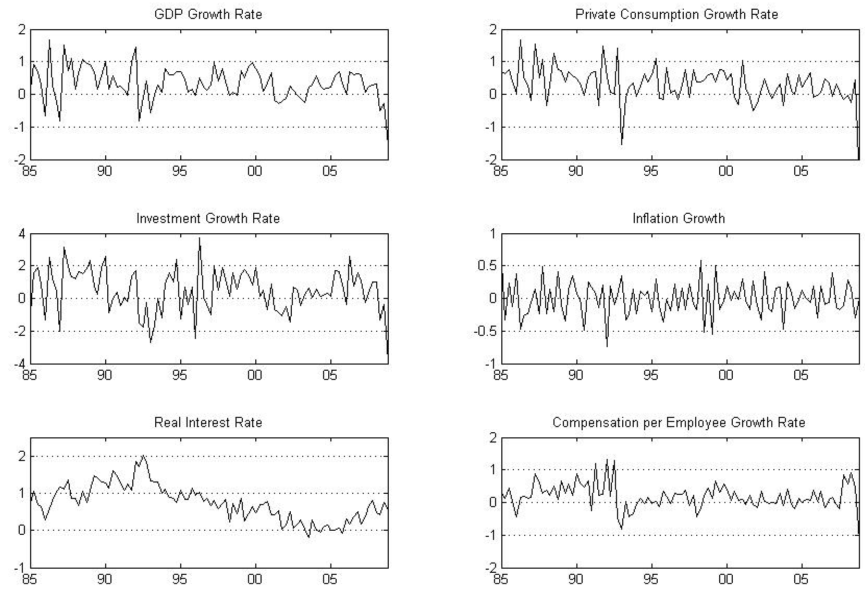


Figure 1: Long term evolution of wages over GDP and employment rate

Observable variables



Estimated innovations

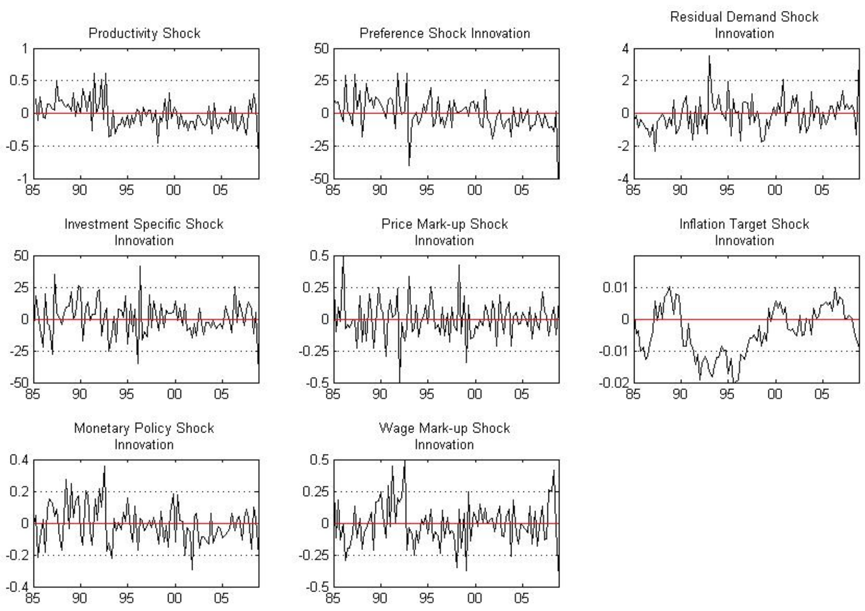


Figure 2: Input (observable variables) and output (shock innovations)

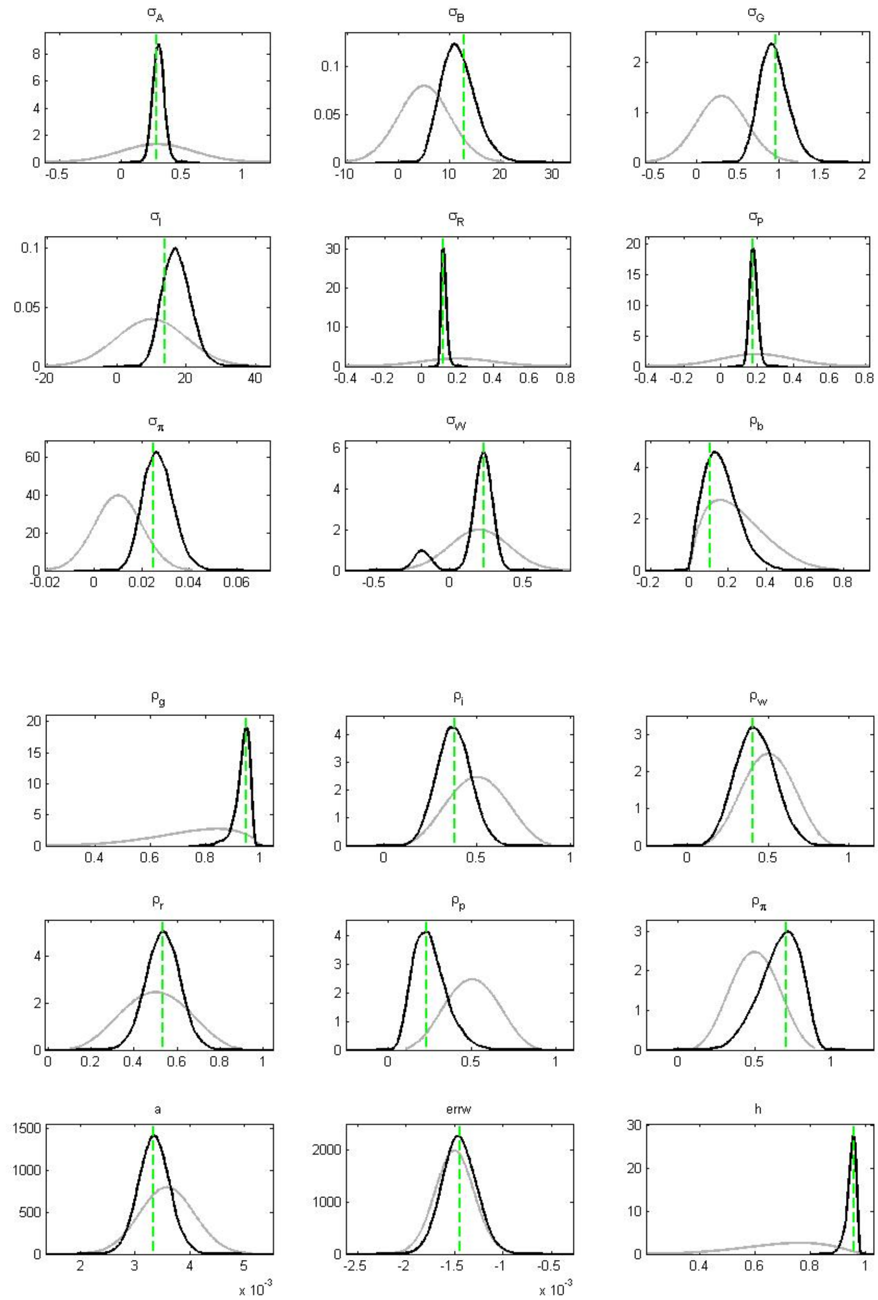


Figure 3: Priors and posteriors of our baseline model estimation on 1985 2008 period -MH: 1 million iterations and 4 chains (1/2)

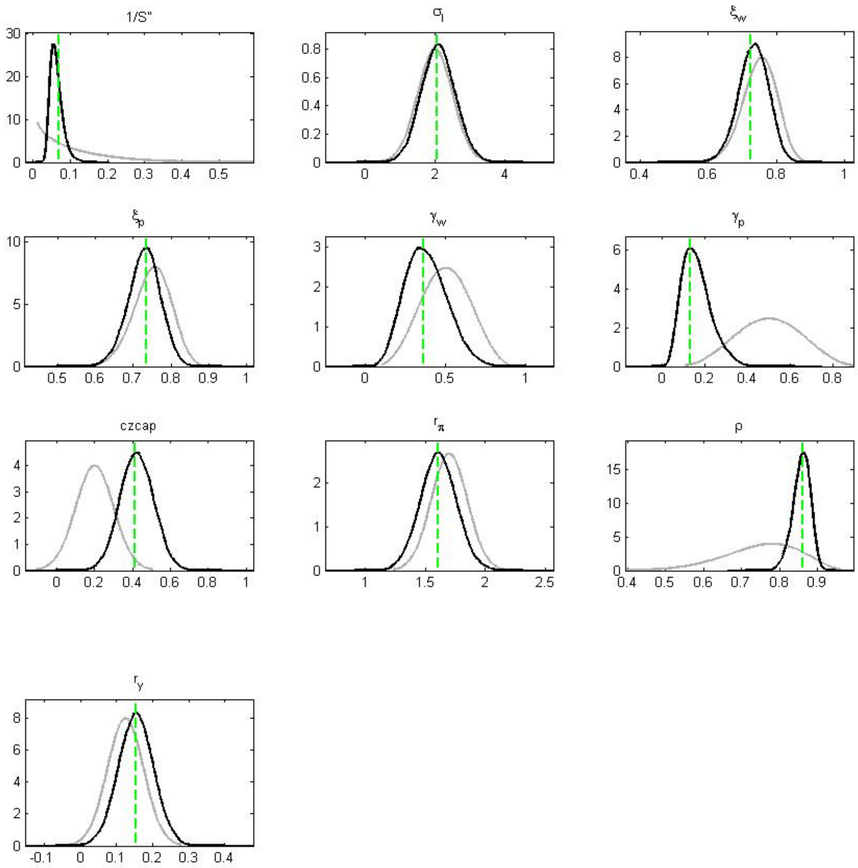
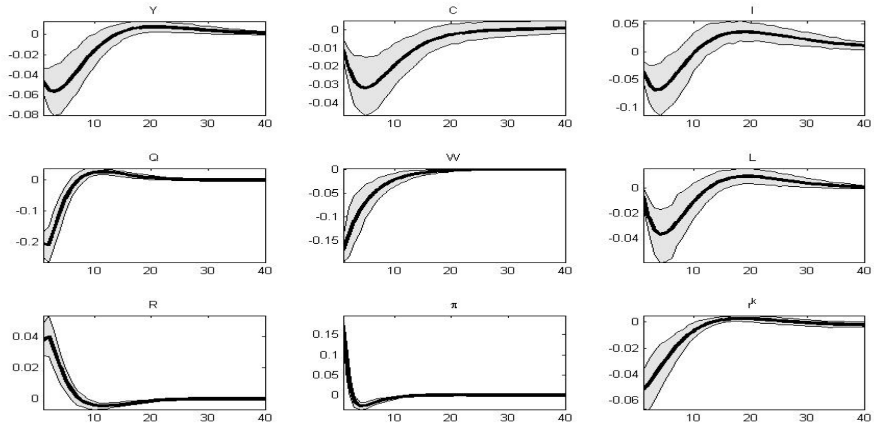
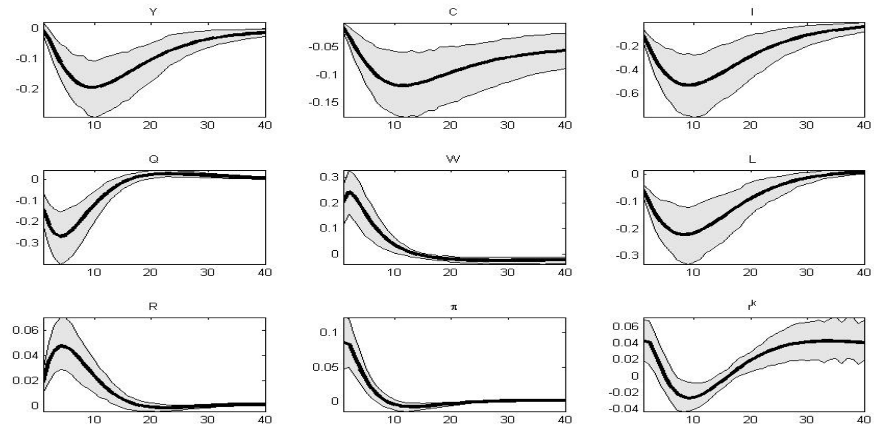


Figure 4: Priors and posteriors of our baseline model estimation on 1985 2008 period -MH: 1 million iterations and 4 chains (2/2)

Price mark-up shock



Wage mark-up shock



Investment specific shock

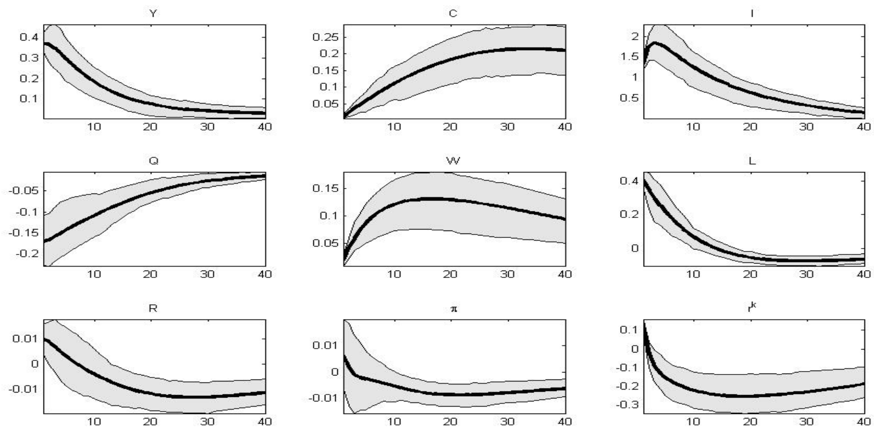
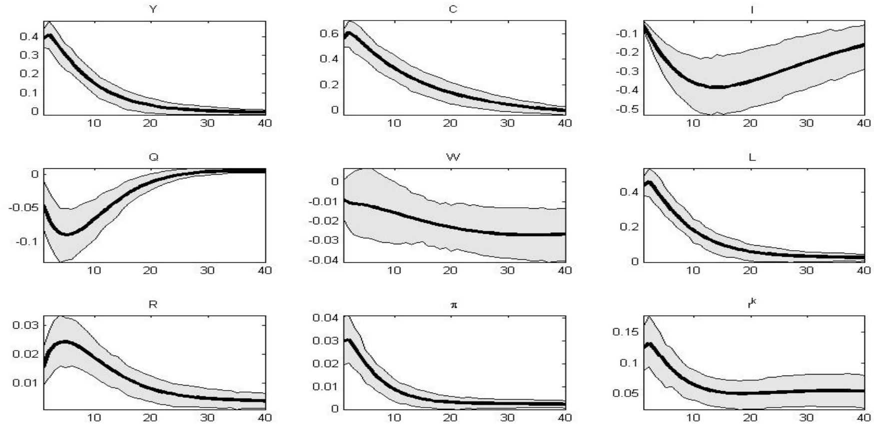


Figure 5: IRF of our baseline model to the offer shocks: price mark-up shock, wage mark-up shock and investment cost shock

Households' preference shock



Government spending shock

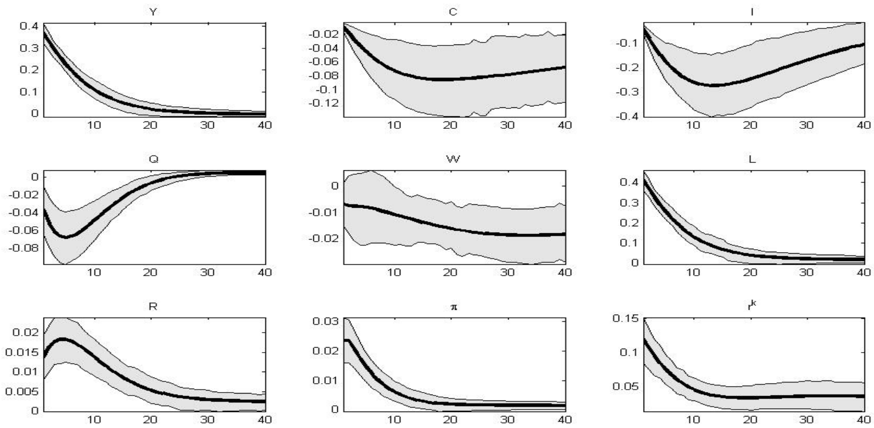


Figure 6: IRF of our baseline model to the demand shocks: households' preference shock and government spending shock

Productivity shock

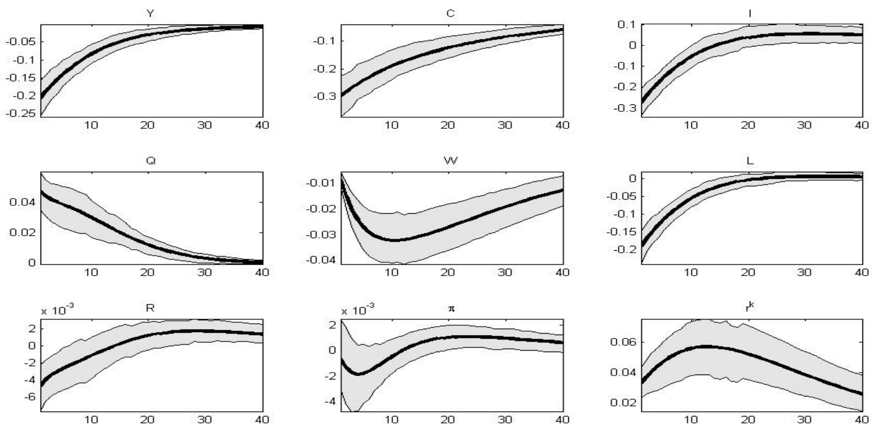
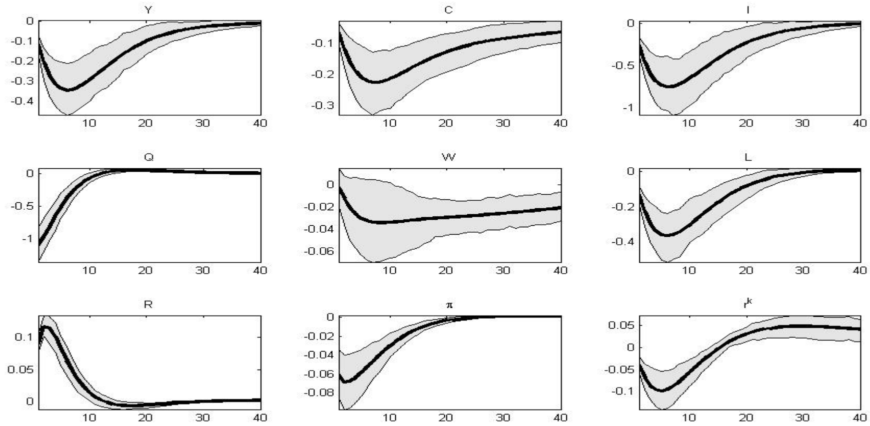


Figure 7: IRF of our baseline model to the productivity shock

Monetary policy shock



Inflation target shock

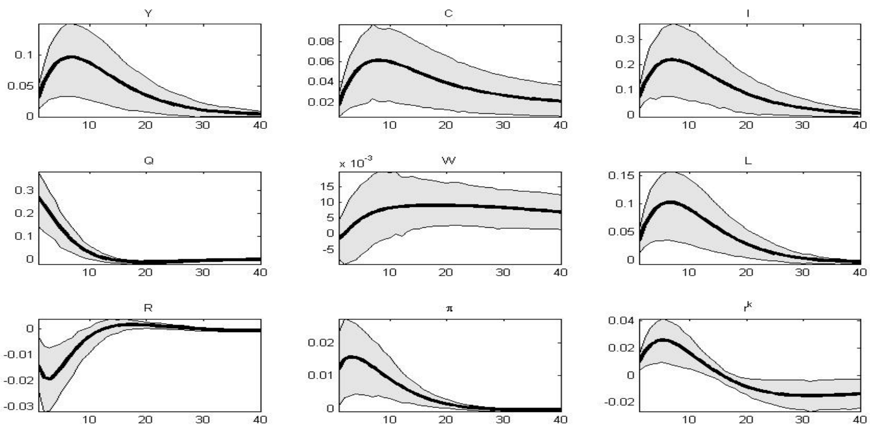


Figure 8: IRF of our baseline model to the monetary shocks: monetary policy shock and inflation target shock

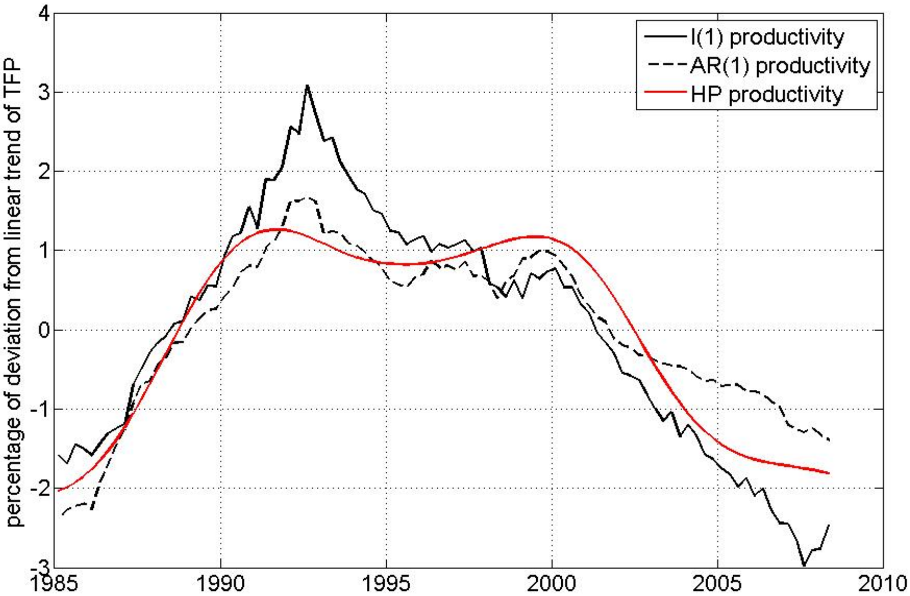


Figure 9: Comparison of integrated processes estimated by the model and corresponding variables

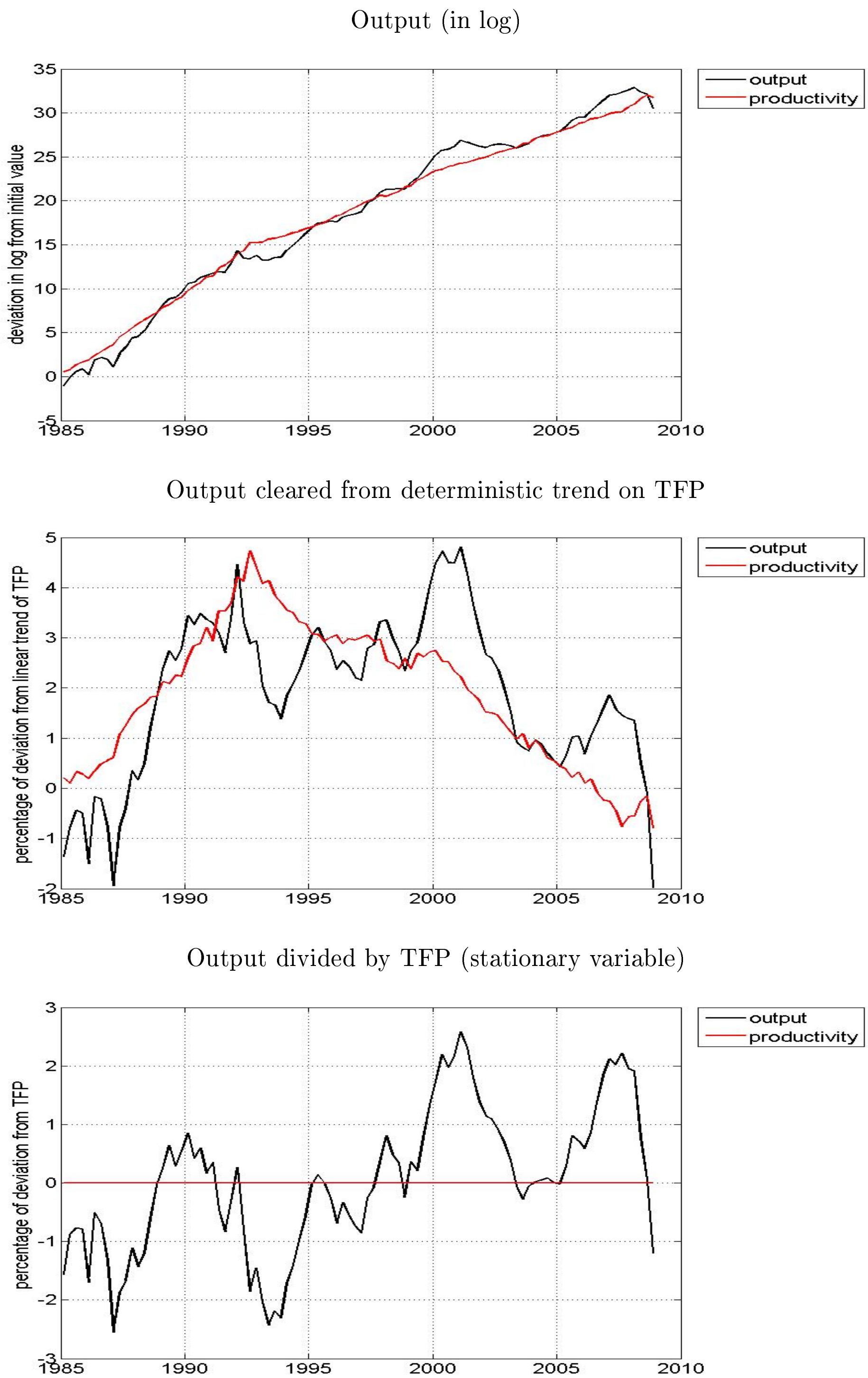


Figure 10: Three levels of dynamic on real variables

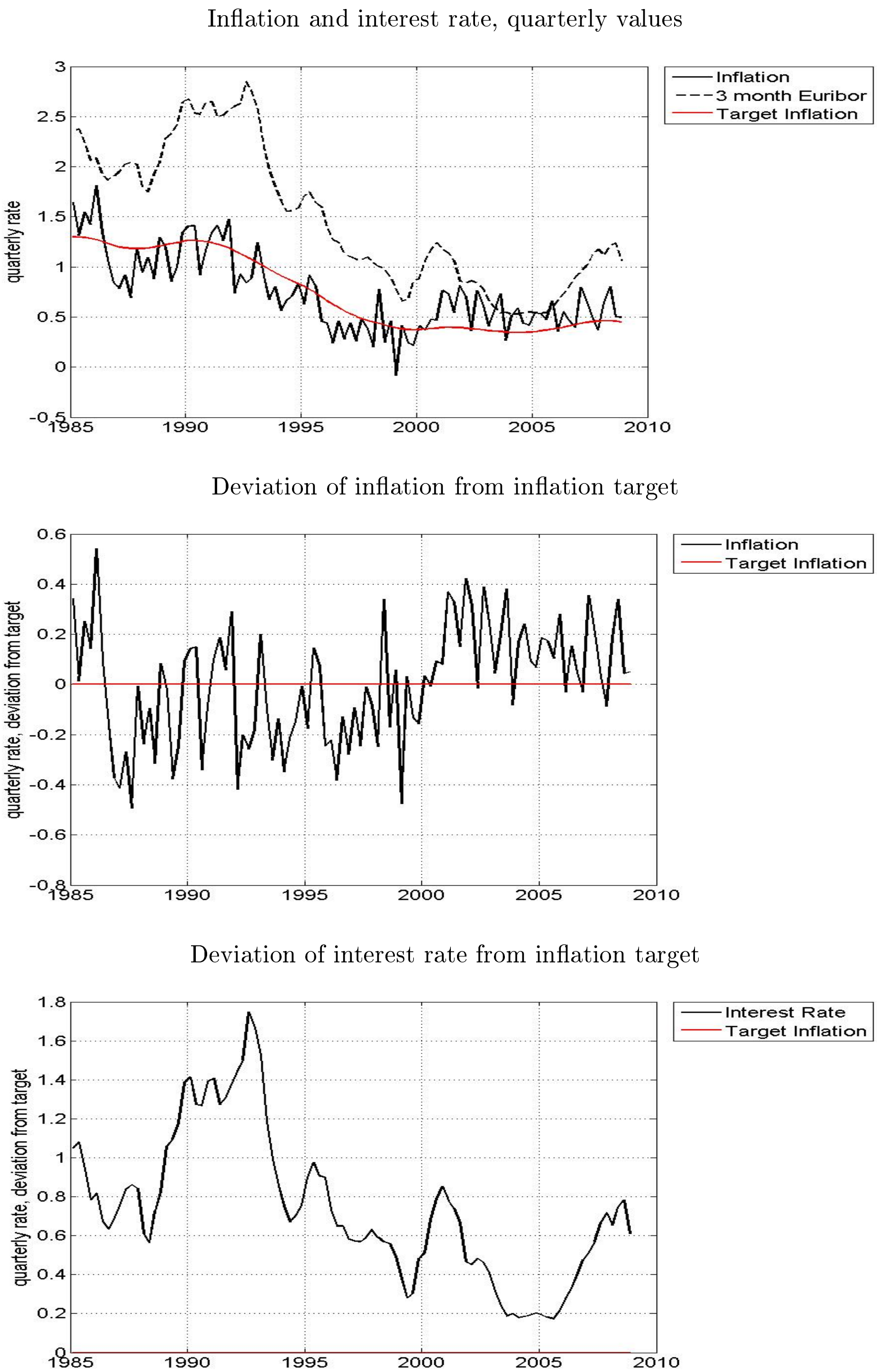


Figure 11: Two levels of dynamic on inflation and interest rate

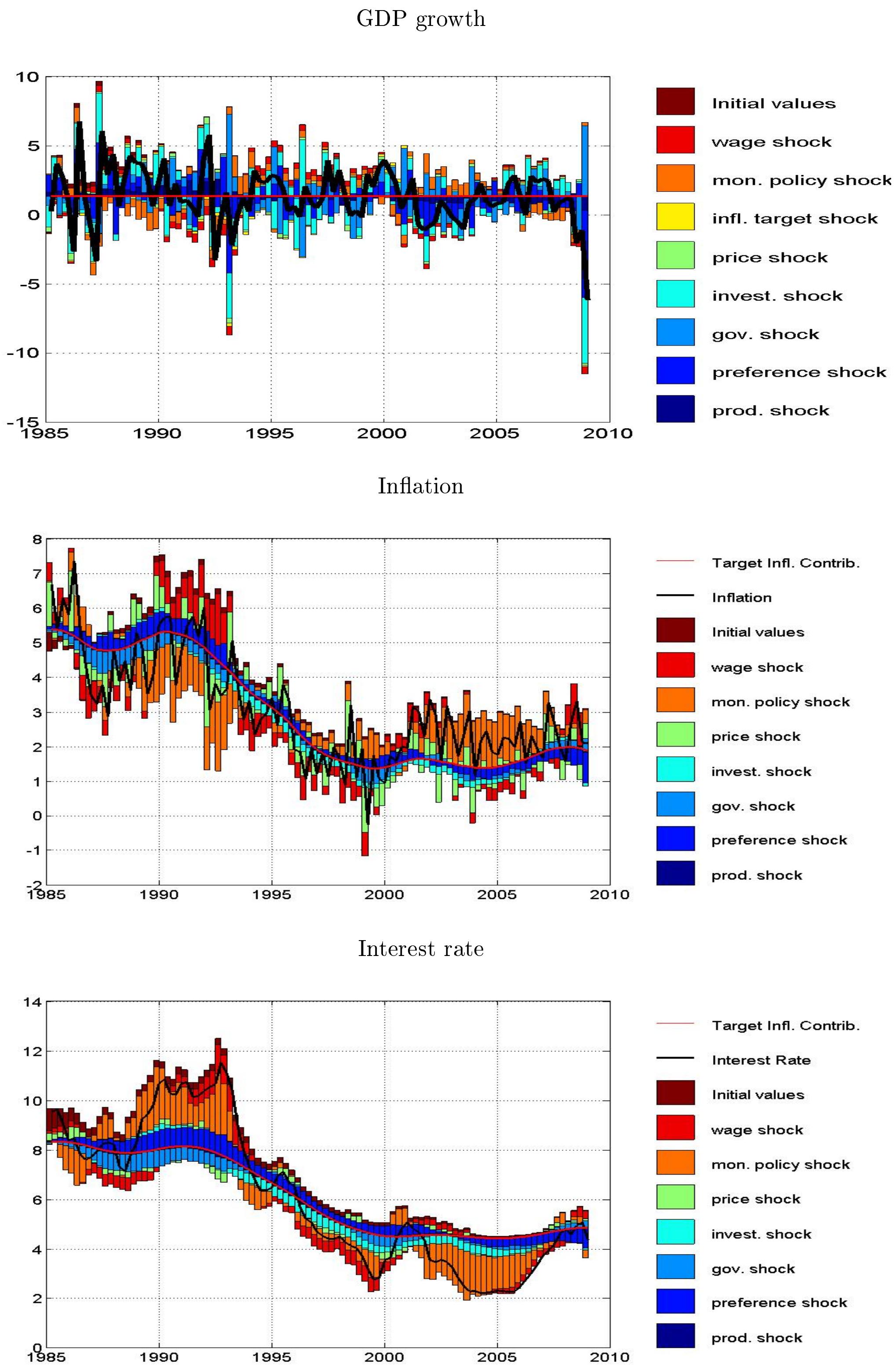


Figure 12: Historical decomposition of macroeconomic time series : output growth, inflation and interest rate

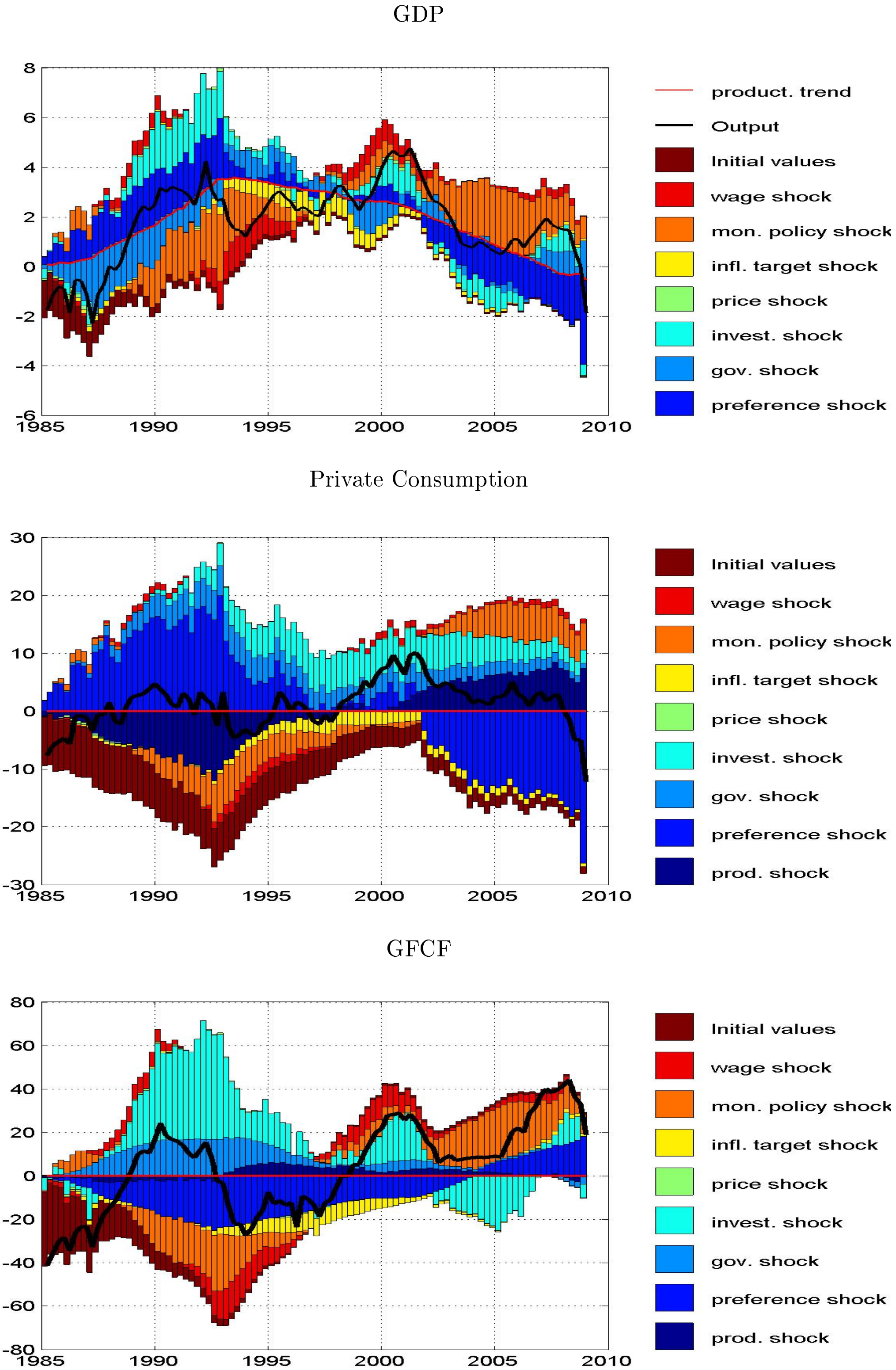


Figure 13: Historical decomposition of macroeconomic time series: GDP, consumption and investment

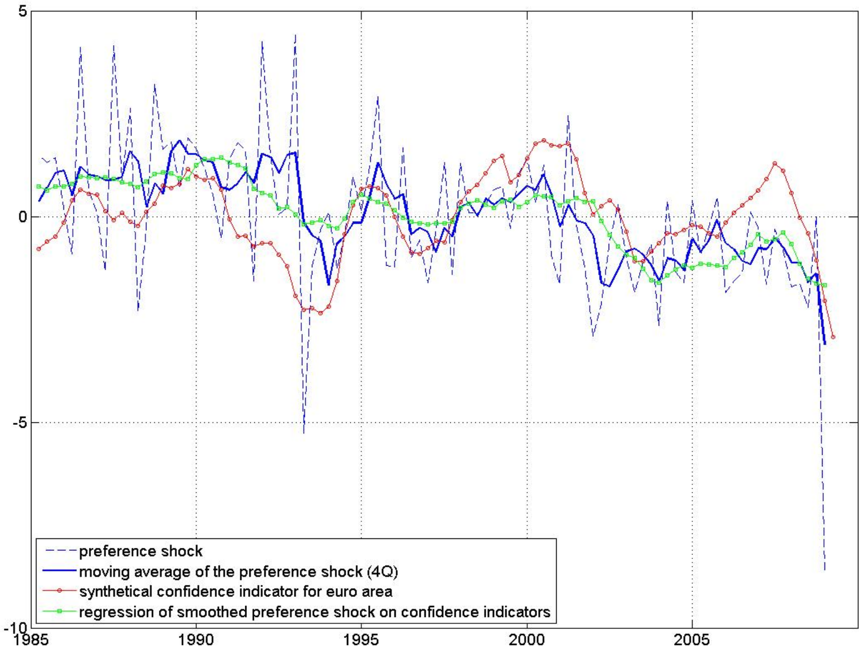
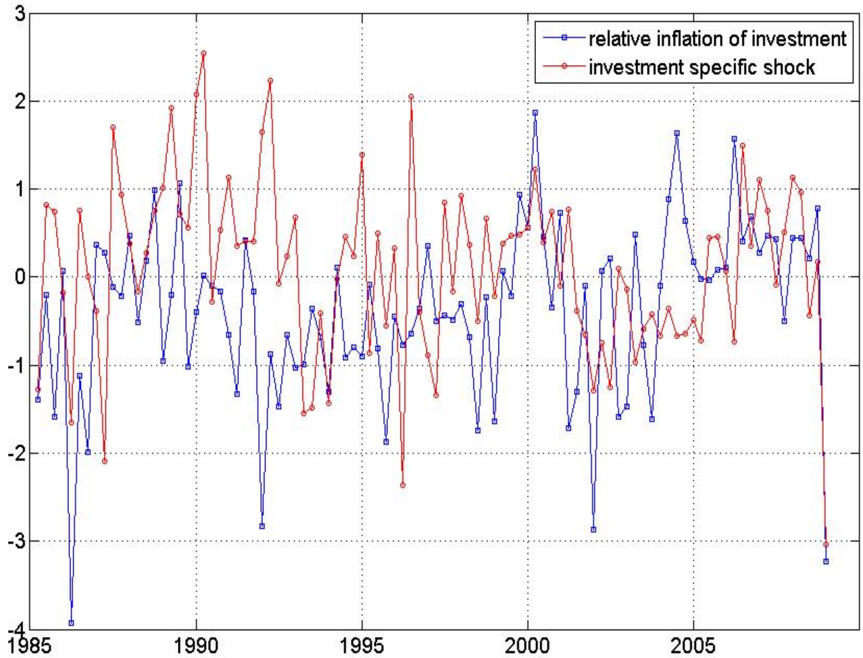


Figure 14: Comparison of the preference shock with the confidence indicator of households

Comparison of the investment specific shock with the relative inflation of investment



Comparison of the investment specific shock with the confidence indicator of entrepreneurs

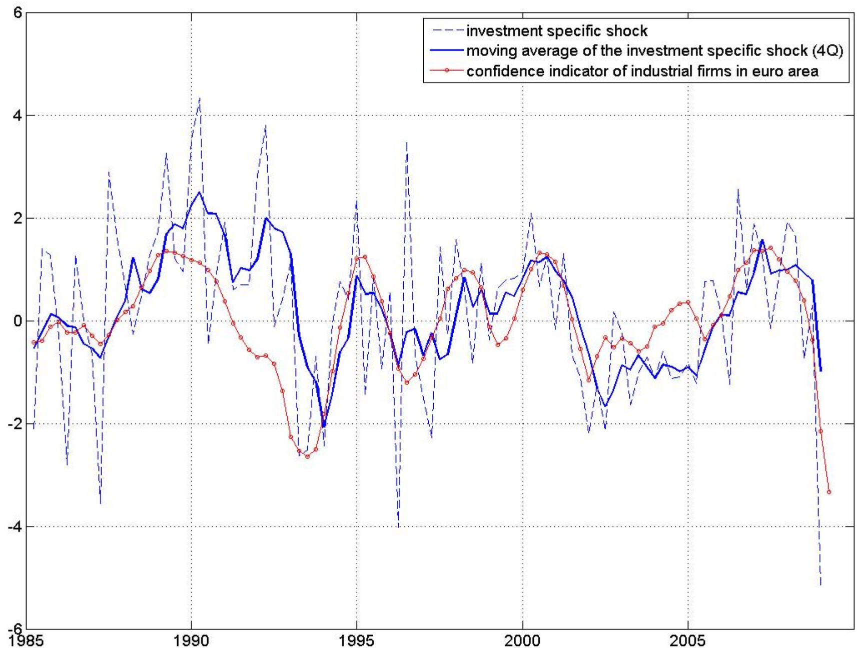
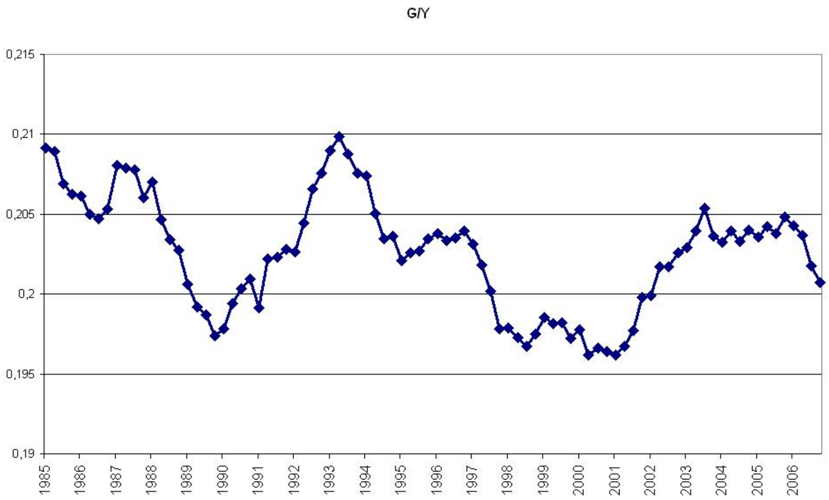


Figure 15: Comparison of the investment specific shocks with related indicators



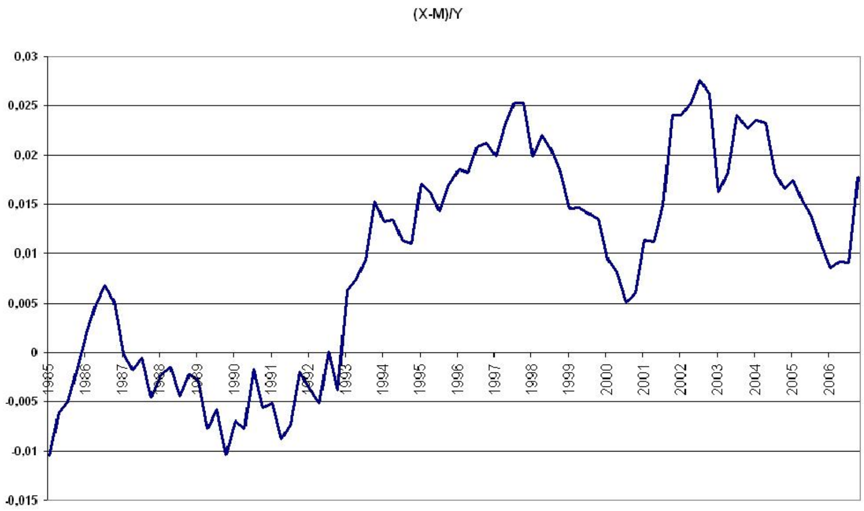


Figure 16: Government expenditures and trade balance for the EA