

Optimal Reserve Accumulation in a Financially Constrained Economy¹

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Abstract

Motivated by the Chinese experience, we analyze a semi-open economy where the central bank has access to international capital markets, but the private sector has not. This enables the central bank to choose an interest rate different from the international rate. We examine the optimal policy of the central bank by modeling it as a Ramsey planner who can choose the level of domestic public debt and of international reserves. The central bank can improve savings opportunities of credit-constrained consumers modeled as in Woodford (1990). We find that in a steady state it is optimal for the central bank to replicated the open economy, i.e., to issue debt financed by the accumulation of reserves so that the domestic interest rate equals the foreign rate. When the economy is in transition, however, we find that a rapidly growing economy has a higher welfare without capital mobility and that the optimal interest rate differs from the international rate. Whether the optimal interest rate is above or below the international rate depends on whether the high growth rate is temporary or more permanent. We also find that capital controls can help reach the first best when the planner has more fiscal instruments.

1 Introduction

China has been a key contributor to global imbalances with both a significant current account surplus and a substantial accumulation of international reserves by the central bank. Figure 1 shows the increase in both variables in recent years. The parallel evolution of these two variables illustrates an interesting feature of the Chinese economy. On the one hand, there are strict restrictions on private capital flows, which characterize a closed economy. On the other hand, there are substantial net capital outflows, through the accumulation of international reserves. This hybrid system differs from the usual open economy or closed economy paradigms and has received little attention in the literature. However, to analyze the macroeconomic behavior of the Chinese economy, it seems fundamental to have a good understanding of this specific structure.

The objective of this paper is to analyze a simple economy where the central bank has access to international capital markets, but the private sector has not. We call this situation a *semi-open* economy.¹ We want to address two main questions in this context. First, what is the optimal policy of the central bank? For this purpose, we model the central bank as a Ramsey planner in a closed economy who has access to the international capital market and we examine optimal policies. The second question is how does the semi-open economy compare to an open economy? This question is interesting because we know that an open economy typically gives a higher welfare than a closed economy, in particular because it allows intertemporal trade. But the semi-open economy also allows intertemporal trade. Moreover, the semi-open economy allows to choose the real interest rate, contrarily to the open economy. Thus, an economy with limited capital mobility may have a higher welfare than an open economy. Our analysis actually shows that, when the central bank behaves optimally, a rapidly growing economy has a higher welfare without capital mobility.

We consider a simple framework where the central bank can improve the saving opportunities of credit-constrained consumers. The model is an extension of the endowment

¹Jeanne (2011) also analyzes a semi-open economy, but in a context that gives no role for optimal policies.

economy presented by Woodford (1990). There are two groups of consumers with endowments fluctuating periodically. In each period, one of the groups has a low endowment and may not be able to smooth consumption due to a credit constraint. This may generate precautionary saving in the period of high endowment. In an open economy this model would generate excess saving based on several mechanisms proposed in the recent literature on global imbalances. First, there is precautionary saving as in Mendoza et al. (2009)² associated with a demand for liquidity in high revenue periods as in Bacchetta and Benhima (2010). Second, when credit constraints are tight, there is lack of domestic assets in the spirit of Caballero et al. (2008).

In a steady state, there are two ways such an economy can reach a first best. First, in a closed economy the government can provide internal liquidity, issuing debt financed by lump-sum taxes (see Woodford, 1990). Second, in an open economy consumers get external liquidity on the international capital market. In a related context, Holmstrom and Tirole (2011) show that the international capital market can provide the desired liquidity, while the first best cannot be reached in a closed economy with distortionary taxes. In our context, the central bank provides internal liquidity and has access to external liquidity. The liquidity provided by a central bank is typically made of commercial banks reserves and of central bank bonds. Figure 2 shows that in the Chinese economy there is a close relationship between central bank liquidity and international reserves.

By extending the above results, we find that in a steady state semi-open economy the first best can be reached when the central bank provides liquidity and finances it by the accumulation of international reserves. Moreover, it is optimal to replicate the open economy, so that the amount of international reserves in a semi-open economy is equal to the amount of foreign assets that would prevail in an open economy. This also implies that the optimal domestic interest rate is equal to the international rate, if we assume that the discount rate and the growth rate are the same as in the rest of the world. The reason the open economy is the first best in a steady state is that consumers are able to avoid the constraint when they approach the steady state. Therefore, as suggested by

²Although there is no uncertainty in the Woodford (1990) model, the heterogeneity across consumers allows to capture a mechanism similar to a more complex Bewley-Aiyagari model.

Song et al. (2010) or Wen (2011) the central bank simply serves as intermediary between the private sector and international capital markets when the economy has limited capital mobility.

However, in the context of the Chinese economy, it seems more useful to consider growing economies that converge to their steady state. In this case, the open economy is usually not the first best. Therefore, the optimal interest rate generally differs from the international interest rate. The reason is that credit constraints are binding on the convergence path as consumers are not able to smooth their consumption. Thus, there is an incentive for the planner to relax the credit constraint. However, whether the domestic interest rate should be higher or lower than the international interest rate is ambiguous. This is due to the heterogeneity of consumers and the fact that there are both lenders and borrowers. On the one hand, there is an incentive to lower the interest rate for low-endowment consumers who borrow. On the other hand, there is an incentive to increase the interest rate for high-endowment consumers who save because they will be constrained in the subsequent period. We find that in economies that experience high transitory growth it is optimal to have a lower interest rate, because of a higher social value to soften the credit constraint of today's borrowers. In contrast, economies with more steady growth prefer a higher interest rate.

We focus on a planner, the central bank, who has only two instruments: the levels of domestic public debt and of international reserves. When more instruments are available, like lump-sum taxes or consumption taxes, we can show that the planner could easily reach a first best by fully relaxing the credit constraint.³ In these cases, however, the potential excess saving by the private sector disappears and there is no role for reserve accumulation. Nevertheless, we still find that limited capital mobility gives a higher welfare, and allows to reach the first best, when the planner can choose consumption taxes. The reason is that modifying the domestic interest rate can offset the distortionary impact of consumption taxes on households intertemporal decisions.

The perspective taken in this paper should be seen as complementary to the litera-

³Benigno et al. (2011) also find that the credit constraint stops binding when the planner has a tax on traded or on non-traded goods consumption.

ture that focuses on the role of international reserves as an insurance against aggregate shocks.⁴ Based on this approach, Jeanne and Rancière (2011) actually find that the precautionary motive against aggregate shocks is not sufficient to explain international reserve accumulation in China. A similar complementarity is found in the literature on optimal government debt in contexts where Ricardian equivalence does not hold. An interesting distinction in that literature is whether shocks are at the aggregate level or at the idiosyncratic level. When shocks are at the aggregate level, it is optimal for a government to accumulate assets (e.g. see Aiyagari et al., 2002). This would correspond to the self-insuring role of international reserves analyzed in the literature. However, when there are idiosyncratic shocks in the private sector, it is optimal for the government to issue debt (see Aiyagari and McGrattan, 1998). This corresponds to the case we consider, since private sector heterogeneity is the basic justification for debt issuance and reserve accumulation.⁵

Our analysis differs from the recent literature showing the optimality of capital controls or more generally on limits to borrowing (e.g., Korinek, 2010, Jeanne and Korinek, 2011, Bianchi, 2011, or Bianchi and Mendoza, 2010). In that literature, the justification for limits to capital mobility comes from pecuniary externalities. Typically, external borrowing affects a relative price, the exchange rate or an asset price, so that the financial constraint becomes tighter. The private sector does not internalize the effect, which gives a role for government intervention. In our case, however, the justification comes simply from the presence of credit constraints in a growing economy. Consumers would be better off in reallocating resources from the future to the present and government intervention can increase welfare.

The rest of the paper is organized as follows. In Section 2, we describe the model and the various equilibrium concepts. In Section 3, we analyze the decentralized closed economy and examine the impact of changing the supply of government bonds. In Section 4,

⁴For recent contributions see, for example, Aizenman (2011), Aizenman and Lee (2007), Barnichon (2009), Durdu et al. (2009), or Obstfeld et al. (2011).

⁵In general, we can conjecture that there would be motives for holding international reserves coming both from aggregate and from idiosyncratic shocks. Shin (2006) introduces both motives in a closed economy. It would be interesting to extend such an analysis to a semi-open economy.

we examine the Ramsey planner problem with foreign reserves. In Section 5 we examine the case where the planner can also choose optimally the consumption tax rate. Section 6 concludes.

2 Model

The economy is inhabited by infinitely-lived households who consume every period, but alternate between low and high endowment periods as in Woodford (1990, section I). This structure implies that households save in their periods of high endowment and would like to borrow in their periods with low endowment.⁶ But the extent of borrowing can be limited by creditors, which leads to a desire for additional precautionary saving. Saving is in the form of bonds that we will call liquid assets. There is a gross interest rate r_t on lending and borrowing.

In addition to households there is a Ramsey planner, that we call a central bank, who can issue liquid assets and hold international reserves. When credit constraints are tight, the demand for funds by cash-poor households is small. In a closed economy, this limits the opportunities to save for cash-rich households. In this case the provision of liquid assets by the central bank may be desirable.

2.1 Households

There are two groups of mass one of households. At time t , a first group of households receives an endowment Y_t , while the second group receives aY_t , with $0 \leq a < 1$. At $t + 1$, the first group receives aY_{t+1} while the second receives Y_{t+1} , and so on. We refer to the group with Y as cash-rich households, or depositors, and the group with aY as cash-poor households, or borrowers. Each household alternates between a cash-rich and a cash-poor state, and each period there is an equally-sized population of rich and poor.

Households maximize:

$$\sum_{s=0}^{\infty} \beta^s u(c_s) \tag{1}$$

⁶There are three basic differences with Woodford (1990): i) there is no capital stock; ii) consumers may be able to borrow; iii) there is a Ramsey planner.

We denote consumption during the cash-rich period as c^D . In this period, households will typically hold deposits D . These deposits are one-period contracts, either as direct loans to borrowing cash-poor households or in public debt holdings.⁷ Consumption during the cash-poor period is denoted c^L . In this period, households borrow L . Consider a household that is cash-rich at time t and cash-poor at date $t + 1$. His budget constraints at t and $t + 1$ are:

$$Y_t - r_t L_t = \tau_t c_t^D + D_{t+1} \quad (2)$$

$$aY_{t+1} + r_{t+1} D_{t+1} = \tau_{t+1} c_{t+1}^L - L_{t+2} \quad (3)$$

The income of the household at date t , which is composed of endowment Y_t minus debt repayments $r_t L_t$, is allocated to deposits D_{t+1} and consumption c_t^D , including a flat-rate consumption tax $\tau_t - 1$ with $\tau_t > 0$. In the following period, at $t + 1$, his income is composed of the return on deposits, $r_{t+1} D_{t+1}$, and of aY_{t+1} . This has to pay for consumption c_{t+1}^L and taxes. Typically the cash-poor household will borrow, so that at the optimum $L_{t+2} \geq 0$.

The cash-poor household might face a credit constraint when borrowing at date $t + 1$. Due to standard moral hazard arguments, a fraction $0 \leq \phi < 1$ of the endowment is used as collateral for bond repayments:

$$r_{t+2} L_{t+2} \leq \phi Y_{t+2} \quad (4)$$

The multiplier associated with this constraint is denoted $u'(c_{t+2}^D) \lambda_{t+2}$.

Cash-rich households at time t satisfy the following Euler equation:

$$u'(c_t^D) = \beta r_{t+1} \frac{\tau_t}{\tau_{t+1}} u'(c_{t+1}^L) \quad (5)$$

Similarly, poor households at date t satisfy the following Euler equation:

$$u'(c_t^L) = \beta r_{t+1} \frac{\tau_t}{\tau_{t+1}} u'(c_{t+1}^D) (1 + \lambda_{t+1}) \quad (6)$$

⁷Alternatively, we could introduce a banking sector that allocates deposits between loans and public debt.

The intertemporal choice of a cash-poor household is distorted when the credit constraint is binding, because $\lambda_{t+1} > 0$. The following slackness condition has also to be satisfied:

$$(\phi Y_{t+1} - r_{t+1} L_{t+1}) \lambda_{t+1} = 0 \quad (7)$$

2.2 Central Bank Policy

The central bank issues domestic liquidity B_{t+1} that pays an interest rate r_{t+1} and has access to foreign reserves B_{t+1}^* that yield the world interest rate r^* . We assume that the world interest rate is $r^* = 1/\beta$. Private agents cannot buy external bonds directly, so the domestic interest rate is determined in the domestic bond market. Equilibrium in this market is:

$$B_{t+1} = D_{t+1} - L_{t+1} \quad (8)$$

In the presence of capital controls, only the central bank has access to external liquidity, so it has a monopoly over the supply of liquidity to domestic agents. It can therefore manipulate the domestic interest rate r_t by appropriately setting the supply of liquidity B . The possibility of accumulating reserves B^* enables the central bank to change the domestic liquidity by simply expanding its balance sheet. The central bank can then match the desired domestic liquidity by accumulating reserves.⁸

When the central bank policy creates a wedge between r_t and r^* , this generates revenues or losses that have to be financed. In practice, central banks transfer their revenues and losses to the government. Therefore, there is a fiscal side to central bank policy. We assume that the changes in central bank revenues affect the consumption tax $\tau_t - 1$. Absent government consumption, the aggregate budget constraint of the central bank and the government is then:

$$r^* B_t^* + B_{t+1} + (\tau_t - 1)(c_t^D + c_t^L) = B_{t+1}^* + r_t B_t. \quad (9)$$

⁸Notice that this institutional framework is well illustrated by Figure 2, where the sum of the central bank bonds and reserve deposits of commercial banks (B) closely matches the difference between deposits and credits of commercial banks ($D - L$) and moves together with international reserves (B^*).

In the following, we will consider two cases. First, for the sake of realism, Section 4 assumes that the central bank cannot change the tax rate $\tau_t - 1$ which is then constant. This is justified by the fact that, in practice, monetary policy is more flexible than fiscal policy. The sequence of budget constraints (9), together with the usual no-ponzi condition, implies that the constant level of the tax rate is then simply set to balance the net present value of revenues and losses by the central bank:

$$(\tau - 1) \sum_{t \geq 0} \frac{c_t^D + c_t^L}{(r^*)^t} + r^*(B_0^* - B_0) = \sum_{t \geq 0} \frac{(r_t - r^*)B_t}{(r^*)^t}.$$

Then, Section 5 examines the case where τ_t is an additional instrument for the Ramsey planner which can be freely chosen every period. In practice, this would mean that fiscal policy is highly flexible and that the central bank can coordinate with the fiscal authority to maximize welfare. In this second case, the planner can supply liquidity without accumulating reserves if it chooses to finance the bonds B with the consumption tax.

Notice that the wedge between r and r^* is akin to a subvention (tax) on deposits (loans).⁹ Setting a higher domestic interest rate is therefore equivalent to subsidizing deposits and taxing loans, while setting a lower interest rate is equivalent to taxing deposits and subsidizing loans. When setting the domestic interest rate, the central bank faces a trade-off between depositors and borrowers.

When the central bank sets $r = r^*$ it can replicate the open economy. In general, we consider three policy regimes, which correspond to different constraints imposed on the set of policy instruments:

Definition 1 (Policy regimes) *We define the following policy regimes:*

- *The closed economy, where $B^* = 0$ and $r \in \mathfrak{R}^+$;*
- *The open economy, where $B^* \in \mathfrak{R}$ and $r = r^*$;*

⁹Indeed, the net asset returns $r^*B^* - rB$ of the central bank can be rewritten as $r^*(B^* - B) - (r - r^*)B = r^*(B^* - B) - (r - r^*)D + (r - r^*)L$, which is the net return on central bank assets at the world interest rate, minus subsidies on deposits, plus taxes on loans, if $r > r^*$.

- *The semi-open economy, where $B^* \in \mathfrak{R}$ and $r \in \mathfrak{R}^+$*

In the semi-open economy regime, the central bank uses its exclusive access to foreign bonds to set the domestic supply of liquidity and manipulate the domestic interest rate as it wishes. Since this is the more general case, we will consider optimal policy within the semi-open economy regime. Indeed, both the open economy and the closed economy are nested in the semi-open economy. In the closed economy, the central bank's access to reserve accumulation is restricted, so neither the central bank nor the private agents can trade foreign bonds. In the open economy regime, we assume that the central bank provides the desired supply B_t at interest rate r^* to the private sector. This is equivalent to let the private sector buy directly foreign assets as B_t and B_t^* are perfect substitutes in this case. This implies that, if the optimal interest rate is equal to the world interest rate r^* , then capital controls are unnecessary. But as long as they differ, capital controls are welfare-improving.

As a Ramsey planner, the central bank will choose a decentralized equilibrium to maximize its social objective:

$$\sum_{s=0}^{\infty} \beta^s [u(c_s^D) + u(c_s^L)] \quad (10)$$

In the next section, we describe the decentralized equilibrium and in Section 4 we analyze optimal policies.

3 Decentralized equilibrium

In this section, we examine the properties of a decentralized equilibrium for a given policy. First, we describe how the credit constraint affects consumption behavior and leads to additional saving. Then, we analyze the steady state and determine the conditions under which the economy is constrained. At the end of the section, as a benchmark case, we analyze a growing open economy without any policy intervention. In order to get analytical results, this section considers the case of a logarithmic utility function $u(c_t) = \log(c_t)$.

We define a decentralized equilibrium as follows:

Definition 2 (Decentralized equilibrium) *Given initial conditions r_0L_0 , r_0D_0 , and B_0^* , a decentralized equilibrium under a given policy regime is a sequence of prices $\{r_t\}_{t \geq 0}$, allocations $\{D_{t+1}, L_{t+1}, c_t^D, c_t^L\}_{t \geq 0}$ and central bank policy $\{B_{t+1}, B_{t+1}^*, \tau_t\}_{t \geq 0}$ such that: (i) given the price system and the central bank policy, the allocation solves the households' problems (equations (2)–(7) are satisfied); (ii) given the allocation and the price system, the central bank policy satisfies the sequence of central bank budget constraints (9) and a given policy regime; (iii) equilibrium in the domestic bond market (8) is satisfied; (iv) $B_t^* > -A$ for some large positive A .*

The last condition rules out ponzi schemes by the central bank. As explained in Section 2, the sequence of taxes τ_t can be constant or variable across time depending on whether the central bank have access to the fiscal instrument.

3.1 Precautionary Saving

In the absence of credit constraints and with log-utility, households would consume a fixed fraction of their intertemporal wealth:

$$c_t^{D*} = \frac{1 - \beta}{\tau_t} \left(\sum_{k=0}^{\infty} \frac{Y_{t+2k}}{\prod_{i=1}^{2k} r_{t+i}} + \sum_{k=0}^{\infty} \frac{aY_{t+2k+1}}{\prod_{i=1}^{2k+1} r_{t+i}} - r_t L_t \right), \quad (11)$$

$$c_t^{L*} = \frac{1 - \beta}{\tau_t} \left(\sum_{k=0}^{\infty} \frac{aY_{t+2k}}{\prod_{i=1}^{2k} r_{t+i}} + \sum_{k=0}^{\infty} \frac{Y_{t+2k+1}}{\prod_{i=1}^{2k+1} r_{t+i}} + r_t D_t \right). \quad (12)$$

Then, the unconstrained (or notional) demands for deposits and loans D_{t+1}^* and L_{t+1}^* are given by replacing c_t^{D*} and c_t^{L*} in the budget constraints (2) and (3).

To understand what happens when cash-rich households are constrained in $t + 1$, we use the Euler equation to substitute for consumptions in the budget constraints and derive the demand for deposits:

$$D_{t+1} = \frac{1}{1 + \beta} \left(\beta(Y_t - r_t L_t) - \frac{aY_{t+1}}{r_{t+1}} - \frac{L_{t+2}}{r_{t+1}} \right) \quad (13)$$

Since deposits are used to smooth consumption between the cash-rich period and the cash-poor period, they depend negatively on future borrowings L_{t+2} . As the credit constraint imposes that $L_{t+2} \leq L_{t+2}^*$, we have then $D_{t+1} \geq D_{t+1}^*$. This means that when households anticipate that their borrowing capacity will be limited in the future, they save more. More specifically, the level of loans contracted by cash-poor households that are constrained in t is given by:

$$L_{t+1} = \frac{\phi Y_{t+1}}{r_{t+1}} \quad (14)$$

The interest rate r_{t+1} that clears the market for domestic liquidity must be such that total deposits D_{t+1} equal outside liquidity B_{t+1} and inside liquidity L_{t+1} , as stated by equation (8), for a given level of B_{t+1} .

3.2 Liquidity Supply in the Steady State

Whether the economy is constrained or not depends on the adequation of supply and demand for liquidity. An economy with a tight constraint needs a larger supply of liquidity B . This can be analyzed precisely in a steady state. For a given constant level of Y , a steady state is a decentralized equilibrium with constant liquidity provision B , reserve holdings B^* and taxes τ satisfying the central bank budget constraint (9).

The following proposition characterizes the steady states of the model depending on the amount of liquidity B . For the sake of simplicity, the analysis is restricted to symmetric equilibria where both types of households have the same (possibly state-dependent) level of consumptions and wealths.¹⁰

Proposition 1 *If $\frac{B}{Y} < \beta \left(\frac{1-a}{1+\beta} - 2\phi \right)$, there is a unique steady state with a binding credit constraint where the interest rate $r < 1/\beta$ increases with $\frac{B}{Y}$. The ratio of relative consumption depends on r : $\frac{c^L}{c^D} = \beta r < 1$.*

If $\frac{B}{Y} \geq \beta \left(\frac{1-a}{1+\beta} - 2\phi \right)$, the credit constraint does not bind and symmetric steady states are characterized by $\beta r = 1$.

¹⁰Equilibria where the borrowing constraint binds for both groups of households when cash-poor are always symmetric as wealth is pinned down by the borrowing constraint. However, when the constraint does not bind for at least one group of households when cash-poor, there is a continuum of equilibria corresponding to different levels of wealth.

Proof. See Appendix A. ■

Proposition 1 implies that a binding credit constraint leads to a lower interest rate. This is the case when ϕ is low and B/Y is small. When the constraint binds, cash-poor households are not able to supply enough liquid assets to cash-rich households because of their limited collateral. As a result, liquidity is overpriced compared to the first best, which corresponds to a depressed interest rate. It also prevents cash-poor households from transferring some of their consumption from the next to the current period, so that consumption is lower in the L-state than in the D-state.

The provision of liquidity by the central bank affects the decentralized equilibrium in the following way. A larger amount of bonds B provides more saving instruments to cash-rich households, alleviating the limited supply of bonds by cash-poor households and decreasing the price of liquidity. As shown in Proposition 1, this results in a higher interest rate. As the steady-state supply of public bonds increases, the interest rate increases until it reaches $r = 1/\beta$. At that point, the constraint stops binding and the supply of public bonds has no more effect on the interest rate and the allocation of resources.

Based on the Proposition, we define liquidity-scarce and liquidity-abundant countries as follows:

Definition 3 *A liquidity-scarce country is one where $\phi < \frac{1-a}{2(1+\beta)}$. A liquidity-abundant country is one where $\phi \geq \frac{1-a}{2(1+\beta)}$.*

In a liquidity-scarce country, agents need a positive supply of bonds by the central bank to achieve a first-best steady state, because private agents supply insufficient liquidity. In contrast, in a liquidity-abundant country there is no need for public supply of liquidity since private agents issue more bonds.

Corollary 1 (Closed economy) *Consider a closed economy with no taxes ($\tau = 1$) and therefore $B = 0$. If the economy is liquidity-abundant, the credit constraint does not bind in the steady state and $\beta r = 1$. If it is liquidity-scarce, the constraint binds in the steady state and $c^L/c^D = \beta r < 1$.*

Corollary 2 (Open economy) *Consider an open economy with $\beta r^* = 1$ and no taxes ($\tau = 1$). The constraint does not bind in the steady state and $\frac{B^*}{Y} = \frac{B}{Y} \geq \beta \left(\frac{1-a}{1+\beta} - 2\phi \right)$. If the economy is liquidity-scarce, then $\frac{B^*}{Y} \geq 0$.*

Corollary 1 shows that, if the country is liquidity-scarce and in the absence of taxation, there is an insufficient domestic supply of liquidity in the closed economy, so households are credit constrained in the long run. In contrast, Corollary 2 shows that reserve accumulation by the central bank in the open economy allows to eliminate the credit constraints. These results will be further discussed in the next section.

While the steady state provides tractable solutions and interesting insights, it seems more relevant to analyze growing economies. In this case, even the open economy might face binding borrowing constraints on the convergence path before reaching the steady state. We examine the optimal policy in the next section, but as a benchmark it is useful to examine the dynamics of the open economy.

3.3 The Convergence of Open Economies

Consider a growing open economy without planner's intervention, that is $\tau = 1$ and $r = r^* = 1/\beta$. The endowment is growing at rate g_t , i.e., $Y_{t+1} = (1 + g_{t+1})Y_t$. We consider an economy where g_t is driven by the following process:

$$g_{t+1} = \mu g_t, \tag{15}$$

where $0 \leq \mu < 1$. This is for example the case of an economy that catches up toward the world's productivity frontier. We examine in particular whether the economy is constrained on the convergence path and whether it is asymptotically constrained or unconstrained.

In the open economy case, each household faces the world interest rate, so it behaves independently from the others. It is therefore sufficient to examine the behavior of a given household. The open economy is then only the aggregation over the two groups of households.

Proposition 2 *Consider an open economy with $\beta r^* = 1$ and no taxes ($\tau = 1$). If cash-poor households are constrained in t , then they are constrained in all their subsequent cash-poor periods and unconstrained in all their subsequent cash-rich periods. Moreover, $\lambda_{t+2k+1} \simeq \mu^{2k} (1 + \mu) \frac{1-\phi+\mu\beta a+\mu^2\beta^2\phi}{1-\phi+\beta a+\beta^2\phi} g_t$, $k \geq 0$.*

Proof. See the Appendix. ■

The Proposition gives conditions for the credit constraints to stay binding, as well as an approximation for λ . Notice that when μ goes to one, which corresponds to permanent growth, the value of λ goes to $2g$. More importantly, the economy stays constrained on the convergence path but is asymptotically unconstrained ($\lim_{t \rightarrow \infty} \lambda_{t+1} = 0$).

Notice that the growing economy does not need to be liquidity-scarce in the sense of Definition 3 to be constrained. Definition 3 corresponds to liquidity needs in a closed-economy steady state with no growth. In an open economy, liquidity is available in infinite amounts on international markets. However, a growing economy can be constrained for any level of $\phi < 1$ because growth generates a strong need for borrowing to smooth consumption.

If agents are initially unconstrained, they liquidate their assets progressively as g goes to zero. If initial wealth is low, if g converges to zero slowly and if credit constraints are stringent (ϕ is small), then they eventually end up with binding constraints.

4 Optimal policy

The optimal policy crucially depends on the set of instruments available to the planner. As explained in Section 2, we first consider the case where the planner can change the supply of liquidity B_t and holdings of international reserves B_t^* but not the level of the distortive tax, which is fixed once and for all to τ . The consumption tax τ is then used to balance the intertemporal budget constraint of the central bank, but not to affect the intertemporal choices of the agents. In the next section, we consider the case where the planner can freely choose the level of the consumption tax $\tau_t - 1$ every period.

We consider the optimal policy both in the steady state and in an economy converging

to its steady state. In both cases, we find that it is optimal to accumulate reserves. Moreover, when the economy is away from its steady state, it is optimal to have r_t diverge from r^* .

4.1 The Ramsey Problem

To analyze optimal policy we consider the Ramsey planner under the semi-open economy regime. The planner maximizes its objective (10) subject to the household budget constraints (2) and (3), their first order conditions (5) and (6), the credit constraint (4) and the aggregate resource constraint. The latter is found using the equilibrium on the bond market (8) and the agents' budget constraints. Maximization is carried out with respect to $\{L_{t+1}, D_{t+1}, B_{t+1}^*, c_t^D, c_t^L, r_{t+1}\}_{t \geq 0}$. The Lagrangian of the Ramsey's problem in the semi-open economy is then defined as follows:

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t^D) + u(c_t^L) \right. \\ & + \gamma_t^D [Y_t - \tau c_t^D - D_{t+1} - r_t L_t] \\ & + \gamma_t^L [aY_t + r_t D_t + L_{t+1} - \tau c_t^L] \\ & + \gamma_t^G [r^* B_t^* - B_{t+1}^* + (1+a)Y_t - c_t^D - c_t^L] \\ & + \kappa_t^D [u'(c_t^D) - \beta r_{t+1} u'(c_{t+1}^L)] \\ & + \kappa_t^L [u'(c_t^L) - \beta r_{t+1} u'(c_{t+1}^D)(1 + \lambda_{t+1})] \\ & \left. + \Lambda_t [\phi Y_t - r_t L_t] \right\} \end{aligned}$$

To determine whether the planner wants to deviate from the open economy regime with $r = r^*$, we focus on the first order condition with respect to r_{t+1} :

$$\gamma_{t+1}^L D_{t+1} - (\gamma_{t+1}^D + \Lambda_{t+1}) L_{t+1} - \kappa_t^D u'(c_{t+1}^L) - \kappa_t^L u'(c_{t+1}^D)(1 + \lambda_{t+1}) = 0 \quad (16)$$

The first two terms reflect the direct distributive effects between depositors and borrowers (or cash-rich and cash-poor households). The last two terms reflect the effect of the interest rate on the intertemporal choices of households. These terms reflect the potential

need for pigovian taxation.

Optimal policy obeys complex motives. In order to understand what the trade-offs of the central bank are, we proceed in two steps. First, we evaluate the above first-order condition in an open economy with no central bank intervention in a constrained transition path as the one studied in the previous section. We then simulate the optimal semi-open regime, i.e. the one that satisfies the FOCs of the above Ramsey problem.

4.2 Is the Open Economy Optimal?

We define by I_{t+1}^* the left-hand side of (16) evaluated at $r_{t+1} = r^*$. To determine whether r_{t+1} should be lower or higher than r^* , we look at the sign of I_{t+1}^* . If I_{t+1}^* is positive, then the central bank can increase social welfare by raising the interest rate. If I_{t+1}^* is negative, then it can increase social welfare by lowering the interest rate. In general, any deviation of I_{t+1}^* from zero means that the open economy is suboptimal.

First, we can show that, if the economy is constrained along the convergence path, $\kappa_t^D = \kappa_{t+1}^L = 0$, for all $t \geq 0$. This means that there is no Pigovian tax after the first period. This is because distorting the households' intertemporal choices is ineffective when credit constraints are binding. As a result, I_{t+1}^* is simply equal to the direct redistributive effect of the interest rate. Using the other FOCs of the planner's program, it can be written as follows for all $t \geq 1$ (see the Appendix for more details):

$$I_{t+1}^* = \left(\sum_{i=1}^{\infty} \Lambda_{t+2i} \right) D_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+1+2i} \right) L_{t+1} \quad (17)$$

The first term corresponds to the net effect of the interest rate on depositors and is obviously positive. The second term corresponds to the net effect of the interest rate on loan issuers. This term is negative because a high interest rate hurts the households who issued loans in the previous period (the current cash-rich households) through higher interest payment, and makes the credit constraint more stringent for the cash-poor households.

According to Proposition 2, an open economy that is constrained on its transition path is unconstrained in the long run as long as its growth is asymptotically zero. This

implies that I_{t+1}^* converges to zero, as Λ goes to zero. The central bank has no incentives to redistribute wealth by distorting the interest rate in the steady state as the utilities of the two groups of agents converge. This means that an open economy in the steady state is at the Ramsey optimum. This is because, as described in Corollary 2, this economy has accumulated a sufficient amount of reserves.

As argued below, in the open economy, it does not matter whether the accumulation of foreign assets takes place through reserve accumulation or private capital flows. In the long run, the open economy would still be optimal if private agents had the right to trade foreign bonds. However, in the transition, since $\Lambda > 0$, I_{t+1}^* can be either positive or negative, which means that the central bank has incentives to manipulate the domestic interest rate by setting capital controls and taking charge of the foreign trade in bonds. In that case, the accumulation of foreign bonds necessarily takes place through the central bank, even if capital controls are irrelevant in the long run.

The sign of I_{t+1}^* depends on the relative size of D_t and L_t as well as on the evolution of the shadow cost of credit constraints, Λ_t . We can derive the following Lemma:

Lemma 1 *If $\Lambda_{t+3} = \xi^2 \Lambda_{t+1}$ for all $t \geq 1$, with $0 \leq \xi < 1$, then I_{t+1}^* is of the same sign as $D_{t+1}/L_{t+1} - \Lambda_{t+1}/\Lambda_{t+2}$ for all $t \geq 1$.*

Proof. This Lemma derives directly from the definition of I_{t+1}^* . Indeed, $I_{t+1}^* = (\Lambda_{t+2}D_{t+1} - \Lambda_{t+1}L_{t+1})/(1 - \xi^2)$. ■

The interpretation is as follows. The incentives for the central bank to raise the interest rate are strong if the amount of deposits is large as compared to the amount of loans, since $(r - r^*)(D - L)$ represent the net quantity of resources that will be distributed to households. On the other hand, I_{t+1}^* is decreasing in the ratio between the shadow costs of the current and future credit constraints. Indeed, if the constraints faced by today's borrowers are more stringent than those faced by future borrowers (today's depositors), then it is better to transfer resources to the former by decreasing debt repayments.

When we consider the specific process for converging endowments described in (15), we can derive the following proposition.

Proposition 3 *Consider an economy with binding credit constraints. If $g_{t+1} = \mu g_t$, $0 \leq \mu < 1$, $g_t > 0$, then I_{t+1}^* is of the same sign as $\frac{1-a}{(1+1/\mu)(1+\beta)} - \phi$.*

As mentioned, the sign of I_{t+1}^* is ambiguous and depends both on the size of loans L as compared to the size of deposits D and on the size of Λ_{t+1} as compared to Λ_{t+2} . The size of Λ_{t+1} as compared to Λ_{t+2} is governed by μ . When μ is small, the convergence speed of g is high and future constraints are less stringent than the current ones. In that case, it is optimal for the central bank to decrease the domestic interest rate. However, this also depends on D/L , which is governed by ϕ and a . If ϕ and a are small enough, the liquidity needs of cash-poor households are strong and the central bank can still have an incentive to increase the interest rate. In general, whether I_{t+1}^* is positive or negative, the open economy regime is suboptimal on a constrained transition path.

4.3 The Ramsey Policy

The basic conclusion is that it is generally optimal to set $r_t \neq r^*$, i.e., to deviate from the open economy regime. The actual optimal policy, however, cannot be derived analytically so that we simulate the semi-open economy under the Ramsey policy, with different parameter specifications. The resulting optimal interest rates are shown in Figure 3. In the baseline, we set $\mu = 0.4$, $a = 0$ and $\phi = 0.1$. At $t = 0$, the economy is supposed to be in a steady state where $\frac{B}{Y} = \beta \left(\frac{1-a}{1+\beta} - 2\phi \right)$ so that the borrowing constraint is (marginally) non-binding. At $t = 1$, the economy start growing at an initial rate of 10%. This case generates a higher interest rate in the transition to the steady state, as the convergence speed is low and liquidity needs are strong. We then lower μ to 0.1. The figure shows that, as predicted by Proposition 3, the central bank has then an incentive to lower the interest rate. The picture is the same when we raise ϕ to 0.2 and a to 0.5.

Figure 4 gives the dynamics of the semi-open economy in the baseline case where growth is sustained and liquidity needs are strong and compares it with the open economy. In this case, borrowers at the beginning of the growth episode suffer because the high interest rate makes their constraint more stringent and increases their debt repay-

ments (c^L is below its open-economy value). Consistently, they borrow less than in the open economy. But later, borrowers benefit from the larger return of their liquid assets. Depositors consume slightly less than in the open economy because they have to spend more on interest payments. This policy is welfare-improving because future constraints are stringent and the demand for deposits is large as compared to the demand for loans: increasing the return on liquid assets alleviates future constraints without making the current ones excessively stringent. To increase the domestic interest rate, the Central Bank increases the supply of public bonds B . In the short run, reserve accumulation is also faster. But in the long run, because this policy is costly (more interest on public bonds have to be paid), the level of reserves is lower.

Figure 5 gives the dynamics with a more short-lived growth and stronger liquidity needs (we set $\mu = 0.3$ and $\phi = 0.2$). In this case, borrowers benefit from the lower interest rate at the beginning of the growth episode as they can borrow more. But later, they suffer from the lower return on their liquid assets. Depositors consume more because they pay less interests on their debt. This policy is welfare-improving because credit constraints are front-loaded, so it is more efficient to alleviate the current constraints by lowering the interest rate. Besides, the demand for loans is large as compared to the demand for liquidity. The central bank can therefore transfer more resources to the agents by decreasing the interest rate. To decrease the domestic interest rate, the Central Bank decreases the supply of public bonds. This translates into a slower reserve accumulation in the short run. But in the long run, since this policy creates additional resources for the central bank (the central bank pays a lower interest rate on its debt), the country accumulates more external reserves.

5 Optimal Policy with Flexible Tax Rates

Giving more instruments to the planner may drastically change the optimal policy. In this section, we consider the case where the planner can choose the optimal tax rate of consumption $\tau_t - 1$ at each period. On a transition path, it is still optimal to have

$r_t \neq r^*$, but it is no longer optimal to accumulate reserves.

When the planner has the full use of the consumption tax, it can actually overcome the borrowing constraint and transfer consumptions both across periods and across households. It can thus achieve a first best allocation where borrowing constraints never bind and the marginal utility of consumption is constant both across periods and households. With a growing endowment as in (15), the planner adjusts the tax rate (potentially negative) so that households consume part of their future income and have a constant consumption. In this process, the planner borrows abroad (or depletes foreign reserves). This case is described in the following proposition.

Proposition 4 *Consider an endowment stream $\{Y_t\}_{t \geq 0}$, initial conditions $r_0 D_0$, $r_0 L_0$, B_0^* , with $r_0 L_0 < Y_0$, and a real number $\rho > 0$. There exists a sequence of policy instruments $\{B_{t+1}, B_{t+1}^*, \tau_t\}_{t \geq 0}$ such that in the corresponding decentralized equilibrium borrowing constraints never bind, each household has a constant consumption, and the ratio of consumptions for both types of households is equal to $c_0^L / c_0^D = \rho$.*

Proof. See Appendix E. ■

As the Proposition shows, the planner can achieve any allocation such that the borrowing constraints do not bind and the ratio of consumptions for both types of households stays constant. In particular, it can choose the same level of consumption for both types of households by setting $\rho = 1$: this allocation maximizes the welfare function (10). Although this does not necessarily maximize welfare, the planner can also replicate the open economy with no borrowing constraints by setting an appropriate ratio ρ . This implies that the planner can always Pareto-improve on the open economy with (potentially) binding constraints.

The proof of the Proposition consists in explicitly constructing the family of allocations with constant consumptions and lax constraints. Intuitively, the planner can tax or subsidize consumption to transfer wealth across agents and periods. Subsidies are financed by borrowing abroad or depleting reserves and the product of taxes can be saved by buying foreign reserves. However, as can be seen from equations (5) and (6), changes in the tax rate distort the Euler equation of households. To make sure that agents choose

a constant consumption stream, the planner has to move the domestic real interest rate away from the world interest rate so that the tax-adjusted interest rate $r_{t+1}\tau_t/\tau_{t+1}$ stays equal to the world rate. Thus, in general, the optimal policy requires the economy to deviate from the open economy, as shown in the following Corollary.

Corollary 3 *Consider an endowment stream $\{Y_t\}_{t \geq 0}$, initial repayments on deposits and loans r^*D_0 and r^*L_0 , and initial foreign reserves $B_0^* = D_0 - L_0$ such that the borrowing constraint binds at some date $t \geq 0$ in the open economy with zero taxes (i.e. with $\tau_t = 1, \forall t \geq 0$). Then, along the optimal policy, r_t deviates from r^* for some date t .*

Proof. We know from Proposition 4 that optimal policy requires lax constraints and constant consumption. Then, compared with the open economy with no taxes, optimal policy requires either a change in the tax rate or in the domestic interest rate. Also, from the first order conditions (5) and (6), we must have $r_{t+1}\tau_t = r^*\tau_{t+1}$. Suppose the domestic interest rate stays equal to r^* . Then the tax rate can only change once and for all at $t = 0$. From the government budget constraint (9), this implies that B_t^* will diverge either to $+\infty$ or $-\infty$. The former is not optimal. The latter violate the upper limit on net foreign debt. We conclude that the domestic interest rate has to diverge from the world interest rate. ■

This case provides another justification for imposing capital controls during the convergence to the steady state. In the previous section, it was the binding constraint that justified a different interest rate. In this section, the justification comes from distortionary taxes. The planner needs the domestic interest rate to deviate from the world interest rate to offset the distortionary effect of the tax. Notice that both reasons disappear once we reach a steady state (assuming $\beta r^* = 1$).

Figure 6 shows an example of optimal policy when taxes are flexible and compares it to the case where they are not in an economy on the convergence path described by (15). The parameters are set to their baseline values ($\mu = 0.4, a = 0, \phi = 0.1$). When taxes are flexible, the planner sets a large subsidy in the first period so that consumption jumps to its constant first best level, and taxes consumption later on. It also issues bonds, allowing agents to save part of their subsidy to overcome the borrowing constraints in

the subsequent periods. This raises the domestic interest rate and exactly offset the future increase in tax rates in households' Euler equations. As the economy, in the aggregate, consumes the permanent value of its growing endowment from the start, this policy requires running a current account deficit during the transition: foreign reserves decrease.

6 Conclusion

This paper has analyzed optimal policy in a semi-open economy, where the Ramsey planner is a central bank that has access to international capital markets. We found that the accumulation of reserves combined with capital controls gives a higher welfare than full capital mobility in a rapidly growing economy. This characterization clearly corresponds to the Chinese economy. However, our model is too stylized to determine quantitatively whether the accumulation of reserves in China is optimal.

The accumulation of assets by a Ramsey planner when consumers are credit constrained may be surprising. Indeed it would seem more intuitive if the planner would borrow in the international market as an intermediary for the consumers. This would actually correspond to the first best allocation in a converging phase, as shown in Section 5. But with a limited set of instruments, the central bank is not able to relax the credit constraint and the optimal policy is to lend to, rather than borrow from, the rest of the world.

Changes in foreign exchange reserves are naturally associated with exchange rate policy. In our context, the exchange rate does not play a role as the analysis is conducted in a one-good economy. A natural extension is to add traded and non-traded goods. In such an extension, international reserve accumulation obviously goes in parallel with a real depreciation (see Jeanne, 2011). Consequently, a real depreciation would be optimal if reserve accumulation is. However, this would only be the case in a transition phase. As the economy approaches its steady state, a higher level of reserves should lead to an appreciated currency.

Another interesting extension would be to model explicitly the domestic financial sector. This would give more instruments to the central bank. For example, central bank liabilities are made of commercial banks' reserves and of central bank bonds. The central bank can typically adjust the interest rate on reserves as well as the reserve requirements. However, reserve requirements do not seem useful in our context as it is optimal to have high interest rate on deposits.

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A Proof of Proposition 1

Assume first that the borrowing constraint is binding. Then, using the expression for deposits (13) and loans (14) in the steady state, the bond market equilibrium (8) can be written $P(1/r) = 0$, with

$$P(X) = \phi X^2 + [a + \phi(1 + \beta)]X - [\beta(1 - \phi) - (1 + \beta)\frac{B}{Y}].$$

It is easy to see that $P(\beta) < 0$ if and only if $\frac{B}{Y} < \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$. In that case, there is a unique positive equilibrium interest rate $r < 1/\beta$. This equilibrium interest rate is an increasing function of $\frac{Y}{B}$. The ratio of relative consumption is given by the first order condition (5). The other first order condition, equation (6), implies that the constraint must be binding when $r < \beta$.

Assume now that the borrowing constraint does not bind. From the first order conditions (5) and (6), it is clear that we must have $\beta r = 1$ so that consumption will be constant for both types of households. In a symmetric steady state, it follows that $c^L = c^D$. Then, taking the difference of budget constraints (2) and (3) in the steady state, together with the bond market equilibrium (8), we get $2L = \frac{\beta}{1+\beta}(1-a)Y - B$. The condition such that the constraint does not bind is $L \leq \beta\phi Y$, which becomes $\frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$.

B Proof of Proposition 2

We denote by $\tilde{X} = X/Y$ the normalized variables. Take an agent who is constrained in $t-2$, her cash-poor period. We look for conditions under which the agent is constrained for all $t+2k$ and unconstrained for all $t+2k+1$, $k \geq 0$. We assume that such conditions exist and derive an expression for λ_{t+2k+1} . We then derive conditions under which $\lambda_{t+2k+1} > 0$ for all $k \geq 0$ and $\tilde{D}_{t+2k+1} \geq 0$.

If the agent is constrained for all $t+2k$ and unconstrained for all $t+2k+1$, $k \geq 0$, then:

$$\tilde{c}_{t+2k}^L = \frac{1}{1+\beta} \left[\frac{1-\phi}{1+g_{t+2k}} + \beta a + \beta^2 \phi (1+g_{t+2k+1}) \right] \quad (18)$$

$$\tilde{c}_{t+2k+1}^D = \frac{1}{1+\beta} \left[(1-\phi) + \beta a(1+g_{t+2k+2}) + \beta^2 \phi(1+g_{t+2k+2})(1+g_{t+2k+3}) \right] \quad (19)$$

Equations (18) and (19) are found by combining the budget constraint (2) and (3) with (13) and (14).

We then use the Euler equation (6) and $\beta r^* = 1$ to derive the multiplier of the credit constraint:

$$\begin{aligned} 1 + \lambda_{t+2k+1} &= (1 + g_{t+2k+1}) \frac{\tilde{c}_{t+2k+1}^D}{\tilde{c}_{t+2k}^L} \\ &= (1 + g_{t+2k})(1 + g_{t+2k+1}) \frac{1 - \phi + (1 + g_{t+2k+2})\beta a + (1 + g_{t+2k+2})(1 + g_{t+2k+3})\beta^2 \phi}{1 - \phi + (1 + g_{t+2k})\beta a + (1 + g_{t+2k})(1 + g_{t+2k+1})\beta^2 \phi} \end{aligned}$$

Using the fact that $g \simeq 0$, we approximate λ :

$$\lambda_{t+2k+1} \simeq g_{t+2k}(1 + \mu) \frac{1 - \phi + \mu\beta a + \mu^2\beta^2\phi}{1 - \phi + \beta a + \beta^2\phi}$$

As long as $\phi < 1$, $\lambda_{t+2k+1} > 0$ for all $k \geq 0$. This implies that agents are constrained in all their cash-poor periods.

Finally, we check whether the agents are unconstrained in all their cash-rich periods.

We have:

$$\tilde{D}_t = \frac{1}{1+\beta} \left[\beta \frac{1-\phi}{1+g_t} - \beta a - \beta^2 \phi(1+g_{t+1}) \right]$$

For the agents not to be constrained in their cash-rich periods, we must have $\tilde{D}_t \geq -\beta\phi$.

For g small, this amounts to $a \leq 1$.

C Derivation of equation 17

For $t \geq 0$, by taking the derivative of the Lagrangian \mathcal{L} with respect to λ_{t+1} , we can show that $\kappa_t^L = 0$. Using the central bank FOCs, we now show that $\kappa_{t+1}^L = 0$ implies $\kappa_t^D = 0$. Differentiating with respect to reserves gives, with $\beta r^* = 1$:

$$\gamma_t^G = \gamma_{t+1}^G \quad (20)$$

The first-order conditions with respect to D and L are, with $r = r^*$:

$$\gamma_t^D = \gamma_{t+1}^L \quad (21)$$

$$\gamma_t^L = \gamma_{t+1}^D + \Lambda_{t+1} \quad (22)$$

The planner FOC with respect to c_t^D and c_{t+1}^L are the following:

$$u'(c_t^D) - \gamma_t^D + \kappa_t^D u''(c_t^D) - \gamma_t^G = 0 \quad (23)$$

$$u'(c_{t+1}^L) - \gamma_{t+1}^L + \kappa_{t+1}^L u''(c_{t+1}^L) - \gamma_{t+1}^G = 0 \quad (24)$$

From equation (5), we know that $u'(c_t^D) = u'(c_{t+1}^L)$. Combining with the above equations, we obtain:

$$\gamma_{t+1}^L + \gamma_{t+1}^G - \kappa_{t+1}^L u''(c_{t+1}^L) = \gamma_t^D + \gamma_t^G - \kappa_t^D u''(c_t^D)$$

Using the planner FOCs with respect to D_{t+1} and B_{t+1}^* , we get:

$$\kappa_{t+1}^L u''(c_{t+1}^L) = \kappa_t^D u''(c_t^D)$$

Since $\kappa_{t+1}^L = 0$ and $u'' < 0$, then $\kappa_t^D = 0$ for all $t \geq 0$. This yields, for $t \geq 1$:

$$I_{t+1}^* = \gamma_{t+1}^L D_{t+1} - (\gamma_{t+1}^D + \Lambda_{t+1}) L_{t+1}$$

Using the planner FOCs with respect to D and L , we solve this equation forward and obtain:

$$I_{t+1}^* = \left(\gamma_\infty^L + \sum_{i=1}^{\infty} \Lambda_{t+2i} \right) D_{t+1} - \left(\gamma_\infty^D + \sum_{i=0}^{\infty} \Lambda_{t+1+2i} \right) L_{t+1}$$

Since the economy is asymptotically unconstrained according to Proposition 2, we have $\gamma_\infty^L = \gamma_\infty^D = \gamma_\infty$, as implied by equations (21) and (22). Combining with equations (23) and (24) in the steady state, we derive: $u'(c_\infty^D) = u'(c_\infty^L)$. This allocation is the steady-state solution to the maximization of (10) under the aggregate budget constraint alone. Therefore, the social cost of the individual budget constraints is zero in the long

run, that is $\gamma_\infty = 0$. Hence equation (17).

D Proof of Proposition 3

According to Proposition 2, an economy with binding credit constraint and $g_{t+1} = \mu g_t$ for all $t \geq 0$ satisfies $\lambda_{t+2k+1} \simeq \mu^{2k}(1 + \mu) \frac{1-\phi+\mu\beta a+\mu^2\beta^2\phi}{1-\phi+\beta a+\beta^2\phi} g_t$ for all $k \geq 1$. To derive Lemma 1, we need to relate Λ to λ . We use equations (20)-(24) and the households' Euler equations to achieve that.

Using equation (23) expressed in period $t + 1$ and equation (24) expressed in period t , $t \geq 1$, and using $\kappa_t^D = \kappa_t^L = 0$, we derive:

$$u'(c_t^L) - u'(c_{t+1}^D) - \gamma_t^L + \gamma_{t+1}^D - \gamma_t^G + \gamma_{t+1}^G = 0$$

Using equations (20)-(22) and the Euler equation for the cash-poor agent, we obtain:

$$u'(c_{t+1}^D)\lambda_{t+1} = \Lambda_{t+1}$$

With log-utility, we have:

$$\frac{\Lambda_{t+1}}{\Lambda_{t+2}} = \frac{\lambda_{t+1}}{\lambda_{t+2}} \frac{(1 + g_{t+2})\tilde{c}_{t+2}^D}{\tilde{c}_{t+1}^D}$$

When g is small, this can be approximated as follows:

$$\frac{\Lambda_{t+1}}{\Lambda_{t+2}} \simeq \frac{1}{\mu}$$

This means that $\Lambda_{t+3} = \mu^2 \Lambda_{t+1}$ for all $t \geq 1$, which, according to Lemma 1, implies that I_{t+1}^* is of the same sign as $D_{t+1}/L_{t+1} - \Lambda_{t+1}/\Lambda_{t+2} \simeq D_{t+1}/L_{t+1} - 1/\mu$

Finally, we can approximate:

$$\frac{D_{t+1}}{L_{t+1}} = \frac{\frac{1}{1+\beta} \left[\beta \frac{1-\phi}{1+g_{t+1}} - \beta a - \beta^2 \phi (1 + g_{t+2}) \right]}{\beta \phi} \simeq \frac{1-a}{(1+\beta)\phi} - 1$$

This implies that I_{t+1}^* is of the same sign as $D_{t+1}/L_{t+1} - 1/\mu \simeq (1-a)/(1+\beta)\phi - 1 - 1/\mu$ for all $t \geq 1$.

E Proof of Proposition 4

The proof of Proposition 4 proceeds by construction and consists in three steps.

1. Lemma 2 establishes that an equilibrium allocation associated to an endowment stream and a certain class of policy instruments is also the equilibrium allocation of the open economy with zero tax associated to another stream of endowments.
2. We show how the planner can choose a sequence of policy instruments to get a stream of endowments in the modified open economy problem such that the borrowing constraint never binds and the ratio of consumption between both types of agents equals ρ .
3. From Lemma 2 this allocation also obtains in the semi-open economy with the chosen set of policy instruments.

Lemma 2 *Consider an endowment stream $\{Y_t\}_{t \geq 0}$, and initial conditions $r_0 D_0$, $r_0 L_0$, B_0^* . Let $\{\bar{c}_t^D, \bar{c}_t^L\}_{t \geq 0}$ denote the corresponding equilibrium allocation. For any set of policy instruments $\{\bar{B}_t, \bar{B}_{t+1}^*, \bar{\tau}_t\}_{t \geq 0}$ that satisfy the central bank's budget constraint (9), and such that*

$$\sum_{t \geq 0} \frac{1}{(r^*)^t} \left[(1+a) \frac{Y_t}{\bar{\tau}_t} \right] + r_0 \frac{D_0 - L_0}{\bar{\tau}_0} = \sum_{t \geq 0} \frac{1}{(r^*)^t} [(1+a)Y_t] + r^* B_0^* \quad (25)$$

and that $r_{t+1} = r^ \frac{\bar{\tau}_{t+1}}{\bar{\tau}_t}$, the allocation $\{\bar{c}_t^D, \bar{c}_t^L\}_{t \geq 0}$ is the decentralized equilibrium associated to the open economy regime, the sequence of taxes $\tau_t = 1, \forall t \geq 0$, the endowment stream $\{\frac{Y_t}{\bar{\tau}_t}\}_{t \geq 0}$, and initial deposits and loans $r^* \frac{D_0}{\bar{\tau}_0}$ and $r^* \frac{L_0}{\bar{\tau}_0}$.*

Proof. First note that the central bank can always set $\{\bar{B}_t, \bar{B}_{t+1}^*\}_{t \geq 0}$ to target any sequence of domestic interest rates $\{r_{t+1}\}_{t \geq 0}$: it just needs to provide loans and deposits at those rates and let agents choose their desired levels of loans and deposits. The

corresponding sequence $\{\bar{B}_t, \bar{B}_{t+1}^*\}_{t \geq 0}$ then follows from equilibrium in the bond market and the central bank's budget constraints.

Denote p_t the date-0 price of a unit of consumption good at date t for households. We normalize $p_0 = 1$. We have $p_{t+1} = \frac{p_t}{r_t}$.

The equilibrium allocation $\{\bar{c}_t^D, \bar{c}_t^L\}_{t \geq 0}$ is the solution to the following set of equations,

$$\sum_{t \geq 0} p_{2t}(Y_{2t} - \bar{\tau}_{2t}c_{2t}^D) + p_{2t+1}(aY_{2t+1} - \bar{\tau}_{2t+1}c_{2t+1}^L) - r_0L_0 = 0, \quad (26)$$

$$\sum_{t \geq 0} p_{2t}(aY_{2t} - \bar{\tau}_{2t}c_{2t}^L) + p_{2t+1}(Y_{2t+1} - \bar{\tau}_{2t+1}c_{2t+1}^D) + r_0D_0 = 0, \quad (27)$$

$$\sum_{t \geq 0} \frac{1}{(r^*)^t} [(1+a)Y_t] + r^*B_0^* = \sum_{t \geq 0} \frac{1}{(r^*)^t} (\bar{c}_t^D + \bar{c}_t^L), \quad (28)$$

$$\sum_{s=0}^t p_{2s}(Y_{2s} - \bar{\tau}_{2s}c_{2s}^D) + p_{2s+1}(aY_{2s+1} - \bar{\tau}_{2s+1}c_{2s+1}^L) - r_0L_0 + p_{2t+2}\phi Y_{2t+2} \geq 0, \quad (29)$$

$$\sum_{s=0}^t p_{2s}(aY_{2s} - \bar{\tau}_{2s}c_{2s}^L) + \sum_{s=0}^{t-1} p_{2s+1}(Y_{2s+1} - \bar{\tau}_{2s+1}c_{2s+1}^D) + r_0D_0 + p_{2t+1}\phi Y_{2t+1} \geq 0, \quad (30)$$

$$u'(c_t^D) = \beta r_{t+1} \frac{\bar{\tau}_t}{\bar{\tau}_{t+1}} u'(c_{t+1}^L), \quad (31)$$

$$u'(c_t^L) = \beta r_{t+1} \frac{\bar{\tau}_t}{\bar{\tau}_{t+1}} (1 + \lambda_{t+1}) u'(c_{t+1}^D), \quad (32)$$

and the associated slackness conditions. Equations (26) and (27) are the intertemporal budget constraints of both agents, equation (28) the intertemporal resource constraint, and equations (29) and (30) are the borrowing constraints written as of date 0.

The condition on domestic interest rates $r_{t+1} = r^* \frac{\bar{r}_{t+1}}{\bar{r}_t}$ implies that $p_t = \frac{\bar{r}_0}{\bar{r}_t} \frac{1}{(r^*)^t}$. The equations defining the equilibrium allocation can then be rewritten

$$\begin{aligned} \sum_{t \geq 0} \frac{1}{(r^*)^{2t}} \left(\frac{Y_{2t}}{\bar{r}_{2t}} - c_{2t}^D \right) + \frac{1}{(r^*)^{2t+1}} \left(a \frac{Y_{2t+1}}{\bar{r}_{2t+1}} - c_{2t+1}^L \right) - r_0 \frac{L_0}{\bar{r}_0} &= 0, \\ \sum_{t \geq 0} \frac{1}{(r^*)^{2t}} \left(a \frac{Y_{2t}}{\bar{r}_{2t}} - c_{2t}^L \right) + \frac{1}{(r^*)^{2t+1}} \left(\frac{Y_{2t+1}}{\bar{r}_{2t+1}} - c_{2t+1}^D \right) + r_0 \frac{D_0}{\bar{r}_0} &= 0, \\ \sum_{t \geq 0} \frac{1}{(r^*)^t} [(1+a)Y_t] + r^* B_0^* &= \sum_{t \geq 0} \frac{1}{(r^*)^t} (\bar{c}_t^D + \bar{c}_t^L), \\ \sum_{s=0}^t \frac{1}{(r^*)^{2s}} \left(\frac{Y_{2s}}{\bar{r}_{2s}} - c_{2s}^D \right) + \frac{1}{(r^*)^{2s+1}} \left(a \frac{Y_{2s+1}}{\bar{r}_{2s+1}} - c_{2s+1}^L \right) - r_0 \frac{L_0}{\bar{r}_0} + \frac{1}{(r^*)^{2t+1}} \phi \frac{Y_{2t+2}}{\bar{r}_{2t+2}} &\geq 0, \\ \sum_{s=0}^t \frac{1}{(r^*)^{2s}} \left(a \frac{Y_{2s}}{\bar{r}_{2s}} - c_{2s}^L \right) + \sum_{s=0}^{t-1} \frac{1}{(r^*)^{2s+1}} \left(\frac{Y_{2s+1}}{\bar{r}_{2s+1}} - c_{2s+1}^D \right) + r_0 \frac{D_0}{\bar{r}_0} + \frac{1}{(r^*)^{2t+1}} \phi \frac{Y_{2t+1}}{\bar{r}_{2t+1}} &\geq 0, \\ u'(c_t^D) &= \beta r^* u'(c_{t+1}^L), \\ u'(c_t^L) &= \beta r^* (1 + \lambda_{t+1}) u'(c_{t+1}^D). \end{aligned}$$

Given (25), it is straightforward to see that the allocation $\{\bar{c}_t^D, \bar{c}_t^L\}_{t \geq 0}$ is the decentralized equilibrium associated to the open economy regime, zero taxes, the endowment stream $\{\frac{Y_t}{\bar{r}_t}\}_{t \geq 0}$, and initial repayments on deposits and loans $r_0 \frac{D_0}{\bar{r}_0}$ and $r_0 \frac{L_0}{\bar{r}_0}$. ■

We can now prove Proposition 4.

Proof. Denote $\{\bar{c}_t^D, \bar{c}_t^L\}$ an allocation that satisfies the intertemporal resource constraint, such that consumption is constant for both types of households and $c_0^L = \rho c_0^D$.

Denote $\bar{y} = \frac{\sum_{t \geq 0} y_t / (r^*)^t}{\sum_{t \geq 0} 1 / (r^*)^t}$ the permanent value of the endowment stream, and set $\bar{r}_t = Y_t / \bar{y}$ for $t \geq 3$.

Choose \bar{r}_0 sufficiently low so that the following conditions hold:

$$a \frac{Y_0}{\bar{r}_0} - \bar{c}_0^L + r_0 \frac{D_0}{\bar{r}_0} \geq 0 \quad (33a)$$

$$\frac{Y_0}{\bar{r}_0} - \bar{c}_0^D - \frac{\bar{c}_1^L}{r^*} - r_0 \frac{L_0}{\bar{r}_0} \geq 0 \quad (33b)$$

$$a \frac{Y_0}{\bar{r}_0} - \bar{c}_0^L - \frac{\bar{c}_1^D}{r^*} - \frac{\bar{c}_2^D}{(r^*)^2} + r_0 \frac{D_0}{\bar{r}_0} + \frac{1}{(r^*)^3} \phi \frac{Y_3}{\bar{r}_3} \geq 0 \quad (33c)$$

$$\frac{Y_0}{\bar{r}_0} + a \frac{1}{(r^*)^3} \frac{Y_3}{\bar{r}_3} - \bar{c}_0^D - \frac{\bar{c}_1^L}{r^*} - \frac{\bar{c}_2^D}{(r^*)^2} - \frac{\bar{c}_3^L}{(r^*)^3} - r_0 \frac{L_0}{\bar{r}_0} + \frac{1}{(r^*)^4} \phi \frac{Y_4}{\bar{r}_4} \geq 0. \quad (33d)$$

This is possible from the assumption that $r_0 L_0 < Y_0$. Remark that τ_0 does not depend on τ_1 and τ_2 which have not been set, yet.

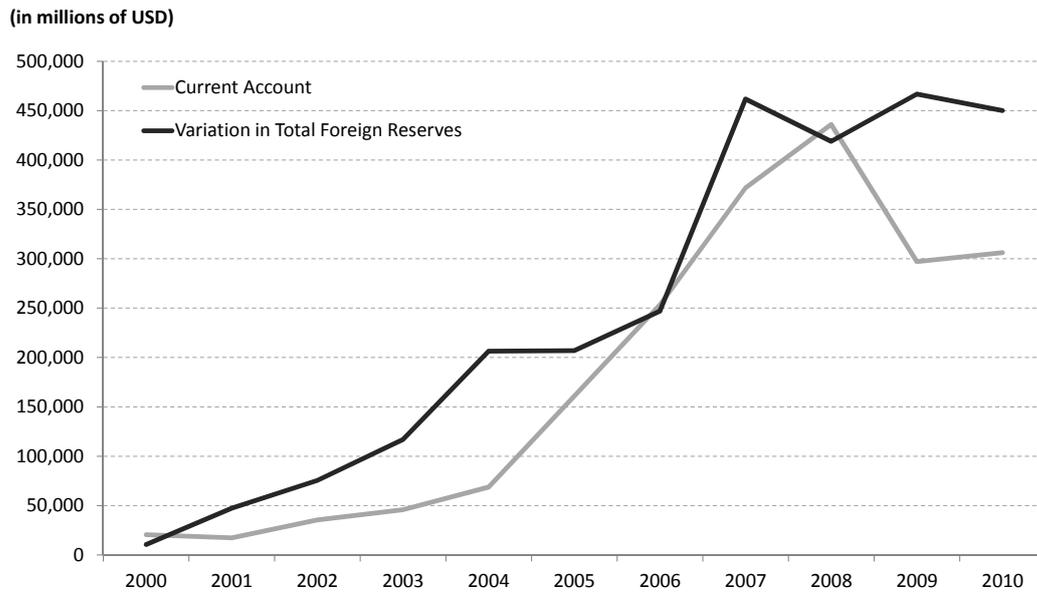
Set $\bar{\tau}_1$ and $\bar{\tau}_2$ to satisfy equation (25) and

$$\sum_{t \geq 0} \frac{1}{(r^*)^{2t}} \frac{Y_{2t}}{\bar{\tau}_{2t}} + \frac{1}{(r^*)^{2t+1}} a \frac{Y_{2t+1}}{\bar{\tau}_{2t+1}} - r_0 \frac{L_0}{\bar{\tau}_0} = \frac{1}{1 + \rho} \left[\sum_{t \geq 0} \frac{1}{(r^*)^t} \left[(1 + a) \frac{Y_t}{\bar{\tau}_t} \right] + r_0 \frac{D_0 - L_0}{\bar{\tau}_0} \right]$$

Finally, suppose the central bank sets $\{\bar{B}_{t+1}, \bar{B}_{t+1}^*\}_{t \geq 0}$ to target the interest rates $r_{t+1} = r^* \frac{\bar{\tau}_{t+1}}{\bar{\tau}_t}$.

The sequence $\{\bar{\tau}_t\}_{t \geq 0}$ satisfies the assumptions of Lemma 2. We can therefore look for the equilibrium allocation of the associated open economy problem with zero tax. In this modified problem, the endowments are constant and equal to \bar{y} for $t \geq 3$, so the borrowing constraints do not bind for $t \geq 4$: $\lambda_{t+1} = 0, \forall t \geq 4$.

Consider the candidate allocation $\{\bar{c}_t^D, \bar{c}_t^L\}$. It is the equilibrium allocation if the constraint does not bind either for $t \in [0, 3]$, which is ensured by conditions (33). Therefore, $\{\bar{c}_t^D, \bar{c}_t^L\}$ is an equilibrium allocation of the original problem. It is such that consumptions are constant for each type of households and the borrowing constraints never bind. From (33), it is such that the ratio of consumptions is equal $c_0^L/c_0^D = \rho$. ■



Source: IMF IFS, WEO (2011)

Figure 1: Evolution of Chinese Current Account and Foreign Reserves Accumulation

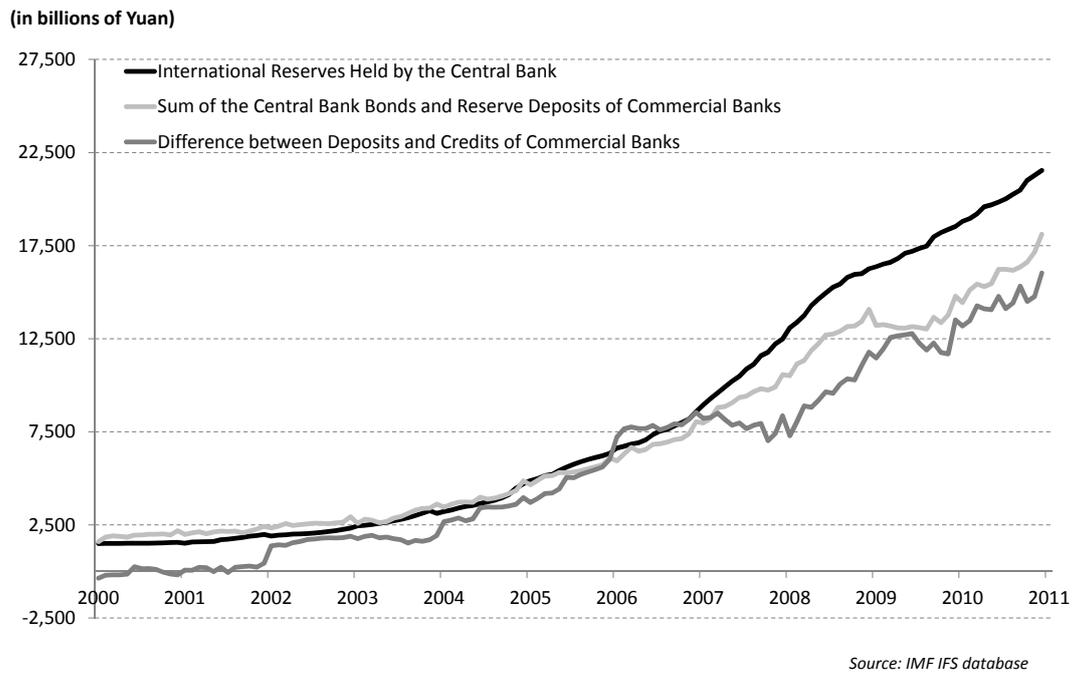


Figure 2: Saving Instruments of the Central Bank and the Excess of Deposits over Lending

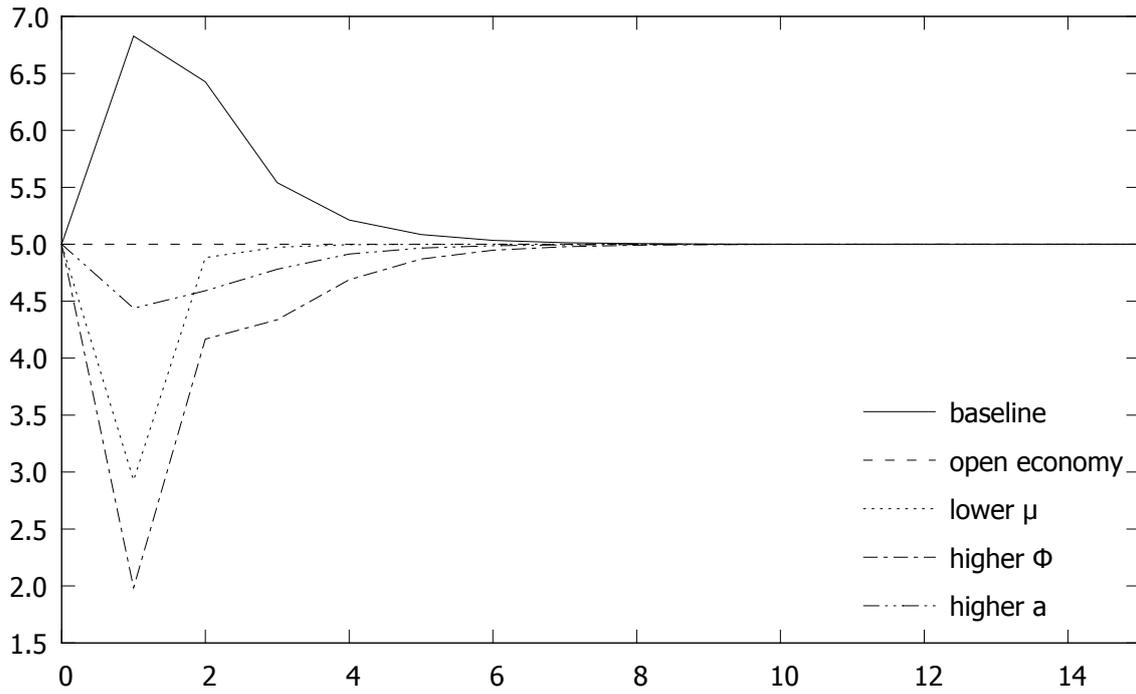


Figure 3: Optimal interest rate $r - 1$, in percentage points.

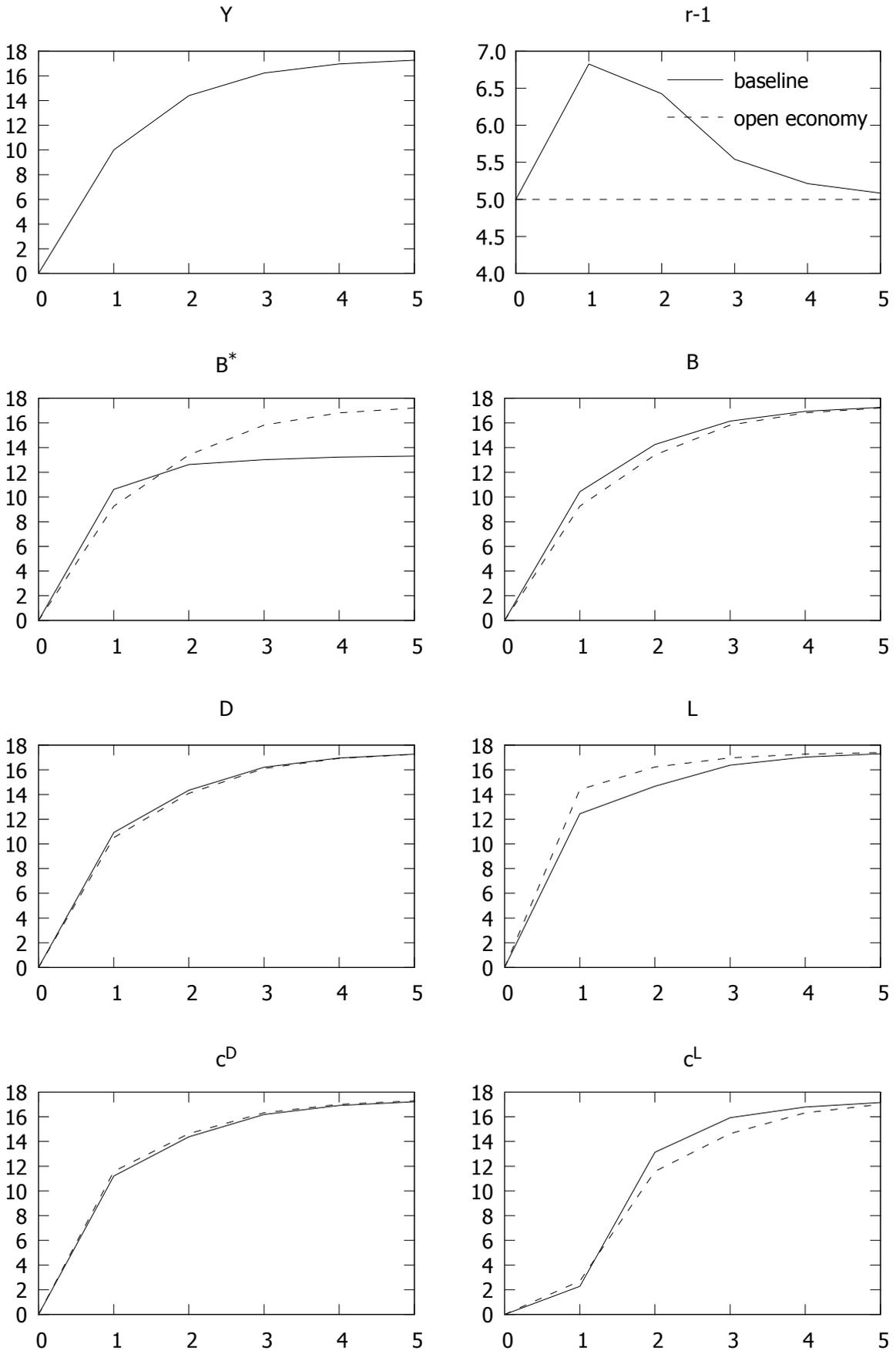


Figure 4: High optimal domestic interest rate. All variables are displayed as percentage changes from their initial value. The interest rate $r - 1$ is in percentage points.

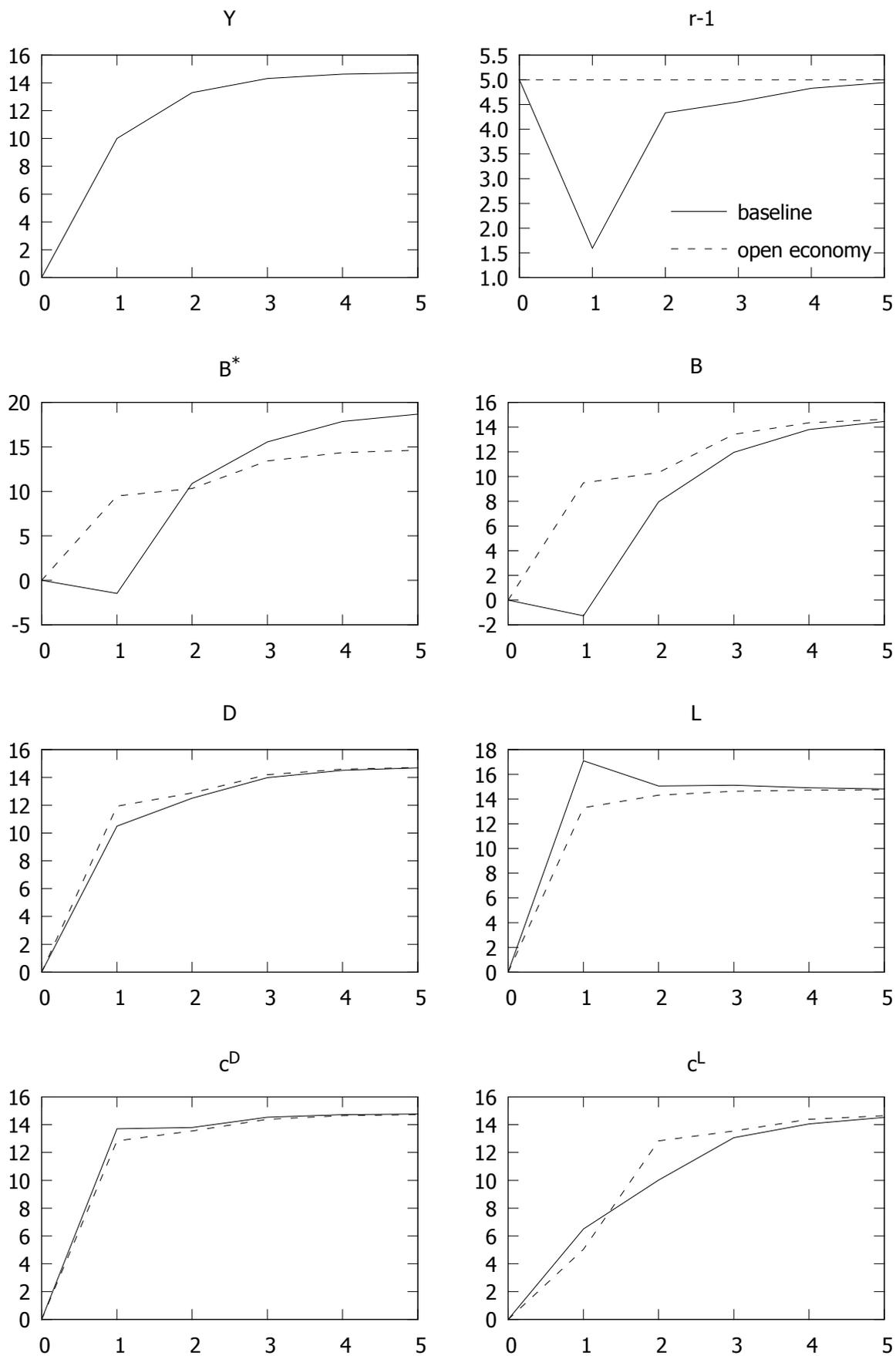


Figure 5: Low optimal domestic interest rate. All variables are displayed as percentage changes from their initial value. The interest rate $r - 1$ is in percentage points.

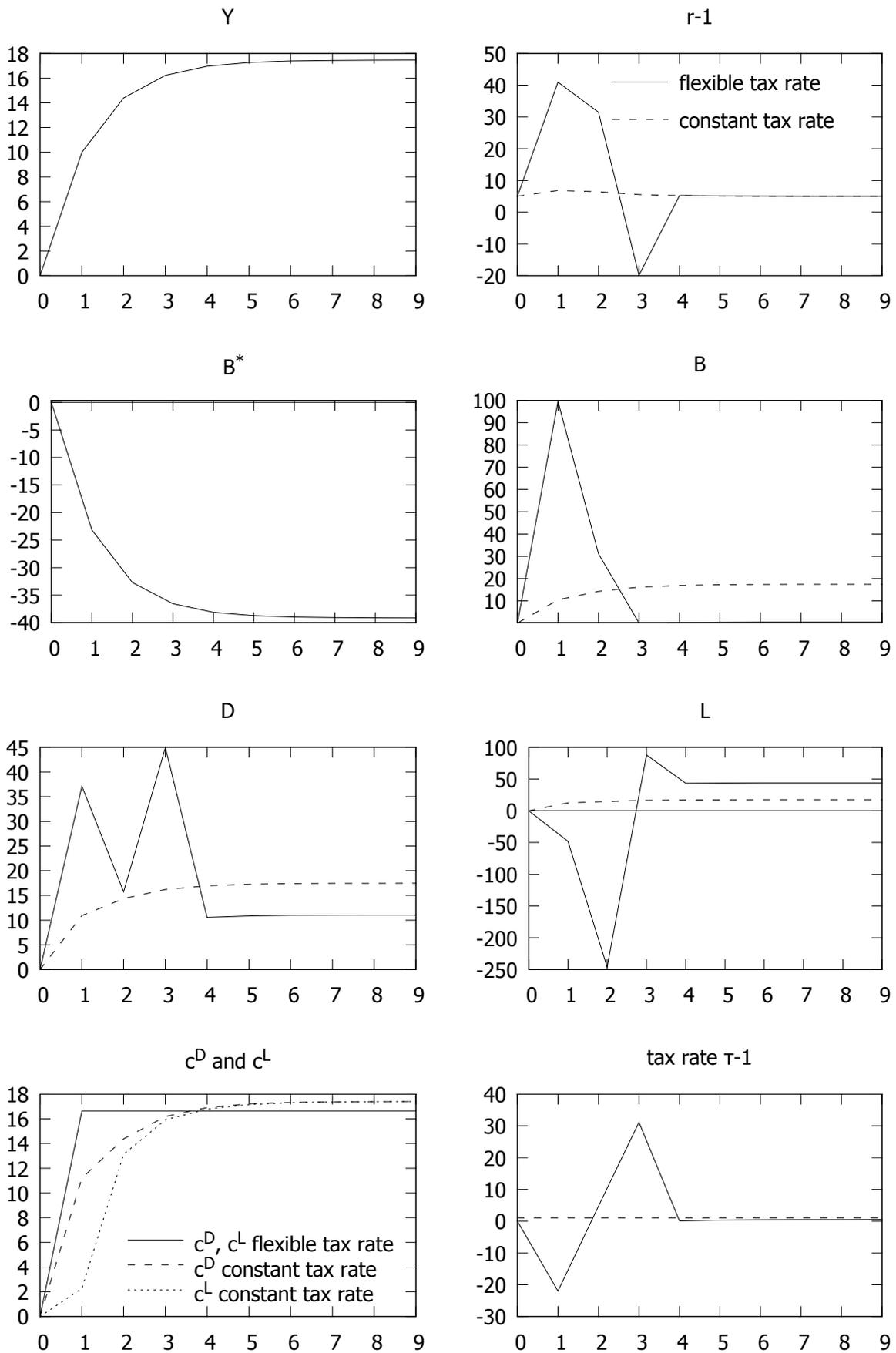


Figure 6: Optimal policy with flexible tax rates. All variables are displayed as percentage changes from their initial value. The interest rate $r - 1$ and the tax rate $\tau - 1$ are in percentage points.