

GENDER WAGE GAPS RECONSIDERED: A STRUCTURAL APPROACH USING MATCHED EMPLOYER-EMPLOYEE DATA^{*}

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Abstract

In this paper I propose and estimate an equilibrium search model using matched employer-employee data to study the extent to which wage differentials between men and women can be explained by differences in productivity, disparities in friction patterns or wage discrimination. The availability of matched employer-employee data is essential to empirically disentangle differences in workers productivity across groups from differences in wage policies toward those groups. The model features rent splitting, on-the-job search and two-sided heterogeneity in productivity. It is estimated using German microdata. I find that female workers are less productive and more mobile than males. The total gender wage gap is 34 percent. It turns out that most of the gap is accounted for by differences in productivity and that differences in destruction rates explain 1.4 percent of the total wage-gap. Netting out differences in offer-arrival rates would increase the gap by 2.6 percent. I find no significant evidence of discrimination against women in Germany.

JEL Code: J70, C51, J64

KEYWORDS: Labor market discrimination, search frictions, structural estimation, matched employer-employee data.

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1 Introduction

In this paper I propose and estimate an equilibrium search model using matched employer-employee data to study the extent to which wage differentials between men and women can be explained by differences in ability, disparities in friction patterns or wage discrimination. The model features rent-splitting, on-the-job search and two-sided heterogeneity in productivity. Its estimation involves several steps: firstly, I estimate group-specific productivities from firm-level production functions. Secondly, I compute job-retention and job-finding rates using employee-level data. Finally, I calculate rent-splitting parameters (bargaining power) relying on individual wage data, transition parameters and productivity measures estimated in the previous stages.

There has been a large number of studies trying to estimate how much of the unconditional mean wage differential between groups may be understood as wage discrimination¹. The traditional approach takes the unexplained gap in wage regressions as evidence of discrimination. This method estimates Mincer-type equations for both groups and then it decomposes the difference of mean wages into “explained” and “unexplained” components. The fraction of the gap that cannot be explained by differences in observable characteristics is considered as discrimination. This kind of analysis has been very informative from a descriptive perspective but the causal interpretation and the nature of discrimination are not clear.

Discrimination refers to differences in wages that are caused by the fact of belonging to a given group, therefore causality is an essential issue in this context. Ideally, detecting discrimination would require to test if the group effect is significant once we have controlled for between groups differences in wage determinants.

The availability of matched employer-employee data allowed a new approach pioneered in Hellerstein and Neumark (1999)². Their method uses

¹See Blau and Kahn (2003) and Altonji and Blank (1999) for good surveys.

²The main papers in this branch are: Hellerstein and Neumark, (1999) with Israeli data, Hellerstein, Neumark and Troske, (1999) with U.S. data, Crepon, Deniau and Pérez-Duarte (2003) with french data, Kawaguchi (2007) with Japanese data and Van Biesebroeck,

firm level data to estimate relative marginal products of various worker types, which are then compared with their relative wages. This analysis implies a clear causality from productivity to wages. Whenever perfect competition holds in the labor market, wage is equal to productivity and therefore any difference in wages that is not driven by a difference in productivity may be considered as discrimination.

However, a frictionless scenario has been shown to be not very useful to understand the labor market. In a labor market with frictions the relationship between productivity and salaries is not so clear, and a direct comparison between both is less informative. Moreover, wage differentials across groups are often accompanied by unemployment rate and job duration differentials. There is a vast literature estimating differentials in job-finding and job-retention rates across groups, directly observing duration in the unemployment and employment or with experiments in audit studies. Although there is agreement in predicting an effect of frictions on wages³, there is scarce empirical evidence on how much of the wage gap can be accounted for differences in friction patterns.

Estimated structural models may provide an interpretation of observed wage gaps as a consequence of disparities in group-specific fundamentals of labor market performance like ability, bargaining power and job creation and destruction rates. Nevertheless, progress in this direction has been slow mainly due to the difficulty in separately identifying the impacts of skill differentials and discrimination from worker-level survey data. The main references are Eckstein and Wolpin (1999) and Bowlus and Eckstein (2002). Both papers study racial discrimination in the U.S. and deal with this empirical identification problem through structural assumptions. Eckstein and Wolpin (1999) proposed a method based on a two-sided, search-matching model that formally accounts for unobserved heterogeneity and unobserved offered wages. They argued that differences in the bargaining power parameter (their index of discrimination) are not identified unless some firm side data are available, and so they are forced to simply compute bounds for

(2007) with subsaharian data.

³See van den Berg and van Vuuren (2003) for a good discussion on this issue.

discrimination that end up being not informative on the estimation sample they work with. Bowlus and Eckstein (2002) also proposed a search model with heterogeneity in workers' productivity but including an appearance-based employer desutility factor. As long as there are firms that do not discriminate, they are able to identify between-group differences in the skill distribution and thereafter the discrimination parameter, which in their case, is the proportion of discriminatory employers⁴. Their paper is not focused on estimation, its main objective is to propose an identification strategy⁵.

The first attempt to use an equilibrium search model to study gender discrimination was made by Bowlus (1997). In her paper, Bowlus only focused on the effect of gender differences in friction patterns over wage differentials without distinguishing between differences in productivity and discrimination. In a recent paper Flabbi (2007) uses a similar strategy as Bowlus and Eckstein (2002), but allowing heterogeneity in matches productivity⁶. He estimates the model by Maximum Likelihood to study whether gender labor market differentials are due to labor market discrimination or to unobserved productivity differences.

In this paper I propose and estimate an equilibrium search model with on-the-job search, rent-splitting, and productivity heterogeneity in firms and workers⁷. The availability of matched employer-employee data furthers identification by allowing me to disentangle differences in workers productivity across groups from differences in wage policies toward those groups. I combine productivity measures estimated at the firm level *a la* Hellerstein et al, group specific friction patterns estimated from individual duration data, and individual wages to estimate the wage equation provided by the structural model. This structural wage equation states the precise relationship between

⁴Mondal (2006) also estimates a similar model to study racial wage differentials in the U.S.

⁵The model assumes no firm heterogeneity and generates counterfactual implications on wage distributions. They are only able to match some moments generated by the model with moments estimated using a sample from the NLSY.

⁶In order to have a model that is estimable with employee-level data, Flabbi (2007) only includes heterogeneity at the match level and do not allow for on-the-job search. Although he allows for wage bargaining, the bargaining power is not estimated.

⁷From now on, I will refer to worker productivity as ability.

wages, worker's ability, firm's productivity, friction patterns, and bargaining power.

The wage equation may be understood as the structural counterpart of a standard Mincer-equation but only including *theoretically relevant* wage determinants⁸. It allows me to undertake counterfactual analysis, like comparing wages of two ex-ante identical workers in terms of ability and outside options, who only differ in the rent-splitting parameter corresponding to their gender⁹.

Distinguishing between the part of the wage gap that is driven by differences in ability and the part that is due to differences in outside options and bargaining power is crucial for social policy. The first one may be due to differences in the skills workers bring to the market, and not to discrimination within the labor market and, therefore, it has to be tackled at the skill formation level (Heckman, 1998). But differences in the job offer arrival rate, job duration and bargaining power are inequalities within the labor market and there should be specific policies or regulations to deal with each of them.

I use a 1996-2005 panel of matched employer-employee data provided by the German Labor Agency, called LIAB¹⁰. This dataset is especially useful for this study for two reasons. Firstly, it contains essential individual variables like gender, wages and occupation. Secondly, it is a panel that tracks firms as opposed to individuals, which is important in order to be able to estimate production functions using panel estimation methods. As far as I know, this paper presents the first structural estimation that uses matched employer-employee data to study labor market discrimination.

⁸The structural wage equation may be also understood as an equation that completes the Hellerstein et al (1999) approach, where wages were assumed to be simply equal to productivity. The model provide a close form solution in which wages are found to be a function of productivity but also function of friction patterns and bargaining power.

⁹Note that a difference in the bargaining power between men and women is considered as wage discrimination. This has already been assumed in Eckstein and Wolpin (1999) and it is meaningful in the sense that an inequality in the rent-splitting parameter generates a difference in wages between two workers with the same ability and outside option that are working in similar jobs in terms of sector and qualification and they only differ in terms their gender.

¹⁰This dataset is subject to strict confidentiality restrictions. It is not directly available but only after the IAB has approved the research project, The Research Data Center (FDZ) provides on site use or remote access to external researchers.

The empirical analysis proceeds by first calculating differences in productivity between men and women, following the approach in Hellerstein et al. As in these studies, I find important negative productivity differentials against women. Next, I analyze group-specific dynamics. I find that women have higher job-creation rates than equivalent men, and that females have also higher job-destruction rates than males. Finally, I estimate group-specific bargaining power. In spite of having large wage differentials, women are not found to have significantly lower bargaining power than men.

In terms of wages, the total gender wage gap is 34 percent. It turns out that most of the gap is accounted for by differences in productivity and that differences in destruction rates explain 1.4 percent of the total wage-gap. Netting out differences in offer-arrival rates would increase the gap by 2.6 percent. Differences in the rent-splitting parameter are responsive for a very small fraction of the wage gap. In general, the structural estimation gives no significant evidence of discrimination against women in Germany.

The rest of the paper is organized as follows. In the next section I describe the structural model. In section 3 data are described. In section 4 I estimate the structural model inputs, namely productivity measures and friction parameters, I present and discuss these intermediate results, and finally I estimate the structural wage equation. In section 5 I perform and discuss some counterfactual experiments and I compare my empirical results with those resulting from other strategies for detecting discrimination using the same data. A conclusion is offered in section 6.

2 Structural framework

In this section I describe the behavioral model of labor market search with matching and rent-splitting. The main goal of estimating a structural model is to clearly state a wage setting equation that allows me to measure the effect of each wage determinant. Having this wage equation estimated, it is straightforward to obtain the effect of discriminatory wage policies, comparing a man's wage with the wage that a woman with the same wage determinants would receive.

Previous research has shown the ability of this kind of models in describing the labor market outputs and dynamics. Building on these assessments, in this paper I am interested in using the structural model as a measurement tool that allows me to identify the effect of discrimination on wages. Search-matching models has been used as an instrument to address empirical questions in a variety of papers. Examples are the previously mentioned papers in the discrimination literature, but there are also interesting contributions in measuring returns to education (Eckstein and Wolpin, 1995) or in analyzing the effect of a change in the minimum wage (Flinn, 2006).

2.1 Assumptions

I propose a continuous time, infinite horizon, stationary economy. This economy is populated by infinitely lived firms and workers. All agents are risk neutral and discount future income at rate $\rho > 0$.

Workers: I normalize the measure of workers to one. Workers may belong to one of different groups (k) defined in terms of gender¹¹. Workers have different abilities (ε) measured in terms of efficiency units they provide per unit of time. The distribution of ability in the population of workers is exogenous and specific for each group, with cumulative distribution function $L_k(\varepsilon)$. This source of heterogeneity is perfectly observable by every agent in the economy. Each worker may be either unemployed or employed. The workers from a generic group k that are not actually working receive a flow utility, proportional to their ability, $b_k\varepsilon$.

Firms: Every firm is characterized by its productivity (p). I Assume that there are only frictions in the labor market. Firms can adjust capital instantaneously in every period without adjustment costs. I assume that p is distributed across firms according to a given cumulative distribution function $H(p)$, which is continuously differentiable with support $[p_{\min}, p_{\max}]$. This source of heterogeneity is perfectly observable by every agent in the economy. The opportunity cost of recruiting a worker is zero.

¹¹The structural model abstract many dimensions that may be relevant in the wage setting. In order to compare jobs as similar as possible, the empirical analysis is clustered at sector and occupation level. See section 4 for details.

Each firm contacts a worker of a given group k at the same constant rate, regardless of the firm's bargained wage, its productivity or how many filled job it has. Unemployed workers receive job offers at a Poisson rate $\lambda_{0k} > 0$. Employed workers may also search for a better job while employed and they receive job offers at a Poisson rate $\lambda_{1k} > 0$. I treat λ_{0k} and λ_{1k} as exogenous parameters specific for each group k . Searching while unemployed as searching while employed has no cost. Employment relationships are exogenously destroyed at a constant rate $\delta_k > 0$, leaving the worker unemployed and the firm with nothing. The marginal product of a match between a worker with ability ε and a firm with productivity p is εp .

Whenever an employed worker meets a new firm, the worker must choose an employer and then, if she switches employers, she bargains with the new employer with no possibility of recalling her old job. If she stays at her old job, nothing happens. Consequently when a worker negotiates with a firm, her alternative option is always the unemployment. The surplus generated by the match is split in proportions β_k and $(1 - \beta_k)$, for the worker and the firm respectively, where $\beta_k \in (0, 1)$ is exogenously given and specific to each group k . I will refer to β_k as the rent-splitting parameter. As in Wolpin and Eckstein (1999), I interpret $\beta_{male} - \beta_{female}$ as an index of the level of discrimination in the labor market. A difference in β in the same kind of job and sector, reveals differential payments unrelated to productivity and outside options, which are only driven by belonging to a given group.

Since the worker does not have the option of recalling the old employer, there is no possibility of Bertrand competition between firms as in Cahuc, Postel-Vinay and Robin (2006). Whether to allow firm competition a la Bertrand or not is controversial. While the Cahuc et al bargaining scenario may be conceptually more appealing and may help to avoid the Shimer critique, it is not clear how realistic this assumption is. Mortensen (2003) argues that counteroffers are uncommon empirically, and Moscarini (2008) shows that, in a model with search effort, firms may credibly commit to ignore outside offers to their employees, letting them go without a counteroffer, and suffer the loss, in order to keep in line the other employees' incentives to not search on the job.

In an environment where contracts cannot be written and wages are continuously negotiated, the alternative option of the job is always unemployment. In this context, if a worker receives an offer from a firm with higher productivity, she must switch. She cannot use this offer to renegotiate with her actual firm, because she knows that tomorrow this offer will not be available and then her future option will be the unemployment again¹². This possibility is also discussed in Flinn and Mabli (2008).

It is not clear whether β can be interpreted as a Nash bargaining power. Shimer (2006) argues that in a simple search-matching model with on-the-job search, the standard axiomatic Nash bargaining solution is inapplicable, because the set of feasible payoffs is not convex. This non-convexity arises because an increase in the wage has a direct negative effect over the firm's rents but an indirect positive effect raising the duration of the job. This critique will hold out depending on the shape of the productivity distribution. Whether β can be understood as a Nash Bargaining Power, is not essential for this study. If the critique holds, I interpret β as a rent-splitting parameter that simply states how much of the surplus goes to the worker. A difference in this parameter remains informative about discrimination.

This model is similar to the model presented in independent work by Flinn and Mabli (2008). The main difference is in the distributions of productivity. In order to have a model that is estimable with employee-level data only, they assume that there is a technologically-determined discrete distribution of worker-firm productivity. In other words, they assume discrete heterogeneity at the match level while here I assume two-side continuous heterogeneity. The model presented here also have the convenient property of producing a closed form solution for the wage setting equation.

¹²If wages are continuously negotiated, firms could increase the wage of the worker at the moment of the on-the-job offer to try to avoid the worker quitting. If the alternative employer is more productive it can force the transition by also paying a premium. This auction for the worker finishes when the actual firm cannot pay more than the full productivity and transition holds as in a Bertrand competition. This premium may be considered as a hiring cost for the firm. Modelling this possibility is outside the scope of this paper.

2.2 Value Functions

The expected value of income for a worker with ability ε , who belongs to group k , currently employed at wage $w(p, \varepsilon, k)$ is denoted by $E(w(p, \varepsilon, k), \varepsilon, k)$ and it satisfies:

$$\begin{aligned} & \rho E(w(p, \varepsilon, k), \varepsilon, k) \\ = & w(p, \varepsilon, k) + \delta_k (U(\varepsilon, k) - E(w(p, \varepsilon, k), \varepsilon, k)) + \\ & \lambda_{1k} \int_{w(p_{\min, \varepsilon, k})}^{w(p_{\max, \varepsilon, k})} [E(\tilde{w}(p, \varepsilon, k), \varepsilon, k) - E(w(p, \varepsilon, k), \varepsilon, k)] dF(\tilde{w}(p, \varepsilon, k)) \end{aligned} \quad (1)$$

The expected value of being unemployed for a worker with ability ε , who belongs to group k is given by:

$$\begin{aligned} & \rho U(\varepsilon, k) \\ = & b_k \varepsilon + \lambda_{0k} \int_{w(p_{\min, \varepsilon, k})}^{w(p_{\max, \varepsilon, k})} [E(\tilde{w}(p, \varepsilon, k), \varepsilon, k) - U(\varepsilon, k)] dF(\tilde{w}(p, \varepsilon, k)) \end{aligned}$$

Finally, the value of the match with productivity $p\varepsilon$ for the firm when paying a wage $w(p, \varepsilon, k)$ to a worker of group k is given by:

$$\begin{aligned} & \rho J_p(w(p, \varepsilon, k), p\varepsilon, k) \\ = & p\varepsilon - w(p, \varepsilon, k) - (\delta_k + \lambda_{1k} \bar{F}(w(p, \varepsilon, k)|\varepsilon)) J_p(w(p, \varepsilon, k), p\varepsilon, k) \end{aligned} \quad (2)$$

where $\bar{F}(w(p, \varepsilon, k)|\varepsilon) = 1 - F(w(p, \varepsilon, k)|\varepsilon)$ and $F(w(p, \varepsilon, k)|\varepsilon)$ is the equilibrium cumulative distribution function of wages received by workers with ability ε who belongs to group k in firms with productivity p . Note that every parameter is group-specific. As the alternative value of the match for the firm is always zero, this value does not depend on alternative matches and therefore it is independent on parameters of the other groups of workers. Although every group is sharing the same labor market, all the value functions may be considered group by group as if they were in independent markets. For notation simplicity I then omit the k -index.

These expressions are equivalent to the value functions of the model with heterogenous firms in Shimer (2006) including heterogeneity in workers ability. But here, wages are determined by the following surplus splitting rule:

$$(1 - \beta) [E(w(p, \varepsilon), \varepsilon) - U(\varepsilon)] = \beta J_p(w(p, \varepsilon), \varepsilon) \quad (3)$$

after some algebra (see the appendix for the whole proof), it can be shown that:

$$\begin{aligned} w(p, \varepsilon) &= p\varepsilon - (\rho + \delta + \lambda_1 \bar{F}(w(p, \varepsilon)|\varepsilon)) \\ &\quad \times \frac{(1 - \beta)}{\beta} \int_{w(\varepsilon)_{\min}}^{w(p, \varepsilon)} \frac{1}{(\rho + \delta + \lambda \bar{F}(\tilde{w}(p, \varepsilon)|\varepsilon))} d(\tilde{w}(p, \varepsilon)) \end{aligned}$$

Noting that $\bar{F}(w(p, \varepsilon)|\varepsilon) = \bar{H}(p)$ and changing the variable within the integral, I obtain a first-order differential equation,

$$w(p, \varepsilon) = p\varepsilon - (\rho + \delta + \lambda_1 \bar{H}(p)) \frac{(1 - \beta)}{\beta} \int_{p_{\min}}^p \frac{1}{(\rho + \delta + \lambda \bar{H}(p'))} \frac{d(w(p, \varepsilon))}{dp'} dp'$$

Solving the differential equation, after some algebra the wage equation takes the following form:

$$w(p, \varepsilon) = \varepsilon p - \varepsilon(1 - \beta)(\rho + \delta + \lambda_1 \bar{H}(p))^\beta \int_{p_{\min}}^p (\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} dp' \quad (4)$$

This expression states a clear relationship between wages $w(p, \varepsilon)$, workers' ability (ε), firm productivity (p), friction patterns (λ_1, δ) and the rent-splitting parameter (β). This wage equation is relatively similar to the one proposed by Cahuc, Postel-Vinay and Robin (2006) when the wage is bargained between a firm with productivity p and an unemployed worker with ability ε ¹³.

As expected the model predicts that the mean equilibrium wage increases in β and that the mean wage paid by a firm with productivity p increases in p . Note in (4) that, if $\beta = 1 \Rightarrow w(p, \varepsilon) = p\varepsilon$, the maximum wage that a firm with productivity p can pay to a worker with ability ε is the full productivity.

If $\beta = 0 \Rightarrow w(p, \varepsilon) = p_{\min}\varepsilon$, that is the minimum wage that a worker would accept to leave unemployment, see Figure 1¹⁴.

As it can be seen in Figure 1, the mean equilibrium wage increases when λ_1 increases and when δ decreases. Many models in the literature predict that the mean equilibrium wage decreases in the amount of frictions (see for example the models in Burdett and Mortensen, 1998, Bontemps, Robin and Van den Berg, 2000, Postel-Vinay and Robin, 2002 and Cahuc, Postel-Vinay and Robin 2006). The intuition behind this fact is clearly explained in van den Berg and van Vuuren (2003). They argue that all of these models are asymmetric in workers and employers. This asymmetry is due to the fact that workers correspond to a relatively long-lived unit whereas firms can expand and contract and can be created and destroyed relatively quickly. When frictions decrease, the value of creating a vacancy increases, and this may prompt an instantaneous inflow of new firms. The latter mitigates the effect of the reduction in frictions on the firms whereas it increases the effect on the workers, and hence the wage increases.

I have assume that the economy is in steady state. The stationary equilibrium conditions that I will exploit are the standard ones. The inflow must balance the outflow for every stock of workers, defined in terms of individual ability, employment status and, for those workers that are employed, firm's productivity.

- The inflow to the unemployment must be equal to its outflow, $\lambda_0\mu = \delta(1 - \mu)$, where μ is the unemployment rate given by:

$$\mu = \frac{\delta}{\delta + \lambda_0} \tag{5}$$

- The inflow to jobs in firms with productivity p or lower than p must be equal to its outflow:

$$\lambda_0 H(p)\mu = (\lambda_1 \bar{H}(p) + \delta) G(p)(1 - \mu),$$

¹⁴These simulations are calibrated using the estimated parameters of male skilled workers in the manufacturing sector, see Section 4. Those parameter are: $\beta = 0.292$, $\lambda_1 = 0.217$ and $\delta = 0.034$.

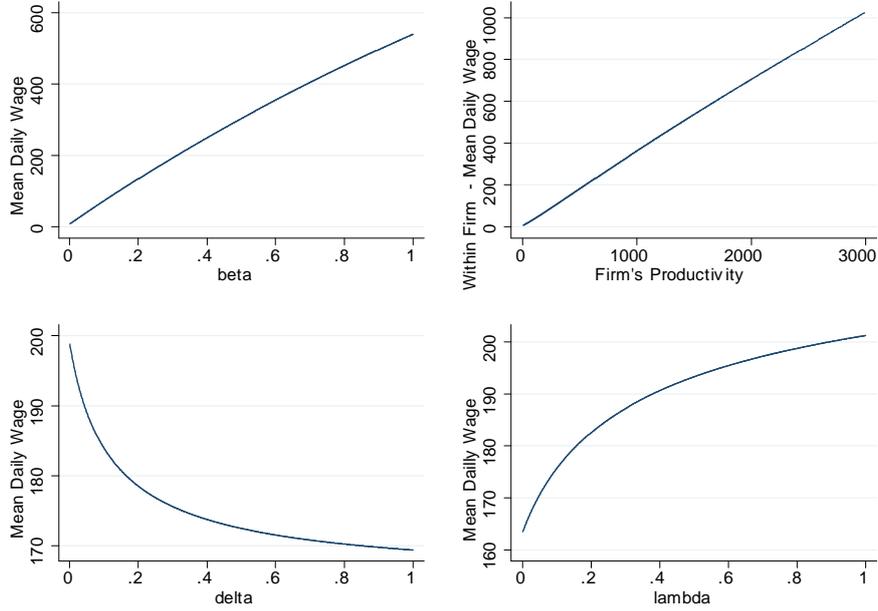


Figure 1: Wage Setting Equation

where $G(p)$ is the fraction of workers employed at a firm with productivity p or lower than p . Then using condition (5) and rearranging:

$$G(p) = \frac{H(p)}{1 + \kappa_1 \bar{H}(p)} \quad (6)$$

where κ_1 is $\frac{\lambda_1}{\delta}$. This stationarity condition, (or its counterpart in terms of wages) is quite common and has been broadly used after Burdett and Mortensen (1998) to infer the primitive distribution of productivity (or the primitive distribution of wages) when only the distribution of productivity (or distribution of wages) within employed workers is observable. Since here I use matched employer-employee data I can directly observe the empirical distribution of productivity at firm level. I only use this stationarity condition in order to construct the likelihood for the duration analysis in section 4.

- The fraction of employed workers with ability ε or lower than ε that are working in firms with productivity p or lower than p are $(1 - \mu)\tilde{F}(\varepsilon, p)$,

where $\tilde{F}(\varepsilon, p)$ is the joint cdf of ε and p . These workers leave this group due to a better offer or because they become unemployed, such event occurs with probability $(\delta + \lambda_1 \bar{H}(p))$. The inflow to this group is given by the unemployed workers with ability ε or lower than ε , (ie: $L(\varepsilon)\mu$) who receive an offer from a firm with productivity p or lower than p . This last event occurs with probability $\lambda_0 H(p)$. Then I have the following condition:

$$(1 - \mu)(\delta + \lambda_1 \bar{H}(p))F(\varepsilon, p) = \lambda_0 H(p)L(\varepsilon)\mu$$

Next using conditions (5) and (6), and rearranging:

$$F(\varepsilon, p) = \frac{H(p)}{(1 + \kappa_1 \bar{H}(p))}L(\varepsilon) = G(p)L(\varepsilon) \quad (7)$$

This expression says that *there is no sorting between firm's productivity and worker's ability*¹⁵.

This statement is controversial, and there is an active debate in the assortative matching literature about it. Becker (1973) showed that in a model without search frictions but with transferable utility, if there are supermodular production functions, any competitive equilibrium exhibits positive assortative matching. In more recent work, Shimer and Smith (2000) and Atakan (2006) show that in search models, complementarities in production function are not sufficient to ensure assortative matching. Assuming different cost functions the first one predicts a negative correlation while the second the opposite.

After Abowd, Kramarz, and Margolis (1999), the empirical literature has mainly focused on estimating the correlation between worker's and firm's fixed effects using matched employer-employee data. However, there are still no definitive results. Abowd et al found a negative and small correlation between firms and workers fixed effects for France, and zero correlation for the U.S. while Lindeboom, Mendez and van den Berg (2008), using a Portuguese

¹⁵To show that there is no sorting, condition (7) is necessary but not sufficient. We also need that the p_{\min} , the minimum productivity, is independent of the worker ability. This condition also holds in this model (the proof is in the appendix).

matched employer-employee dataset, find that there is positive assortative matching.

3 Data

Linked Employer-Employee Data from the German Federal Employment Agency or LIAB.

I use the linked employer-employee dataset of the IAB (denoted LIAB) covering the period 1996-2005. LIAB was created by matching the data of the IAB establishment panel and the process-produced data of the Federal Employment Services (Social security records). The distinctive feature of this data is the combination of information about individuals with details concerning the firms in which these people work. The workers source contains valuable data on age, sex, nationality, daily wage (censored at the upper earnings limit for social security contributions), schooling/training, the establishment number and occupation based on a 3-digit code that in this paper is collapsed into two categories: skilled and unskilled jobs.¹⁶

The firm's data give details on total sales, value added, investment, depreciation¹⁷, number of workers and sector¹⁸. In particular, only firms with more than 10 workers, positive output and positive depreciated capital have been included in my subsample. Since firms of different sectors do not share the same market I construct separate samples for each sector. LIAB has a very detailed industry classification. I focus on four main industries: Manufacturing, Construction, Trade, and Services¹⁹. Participation of establishments is voluntary, but the response rates are high, exceeding 70 per cent. Moreover, the response rate in some key-variables for my purpose is lower. Among survey respondent, only 60% of firms in the previous four industries provide

¹⁶I have assigned the following groups to the unskilled category: Agrarian occupations, manual occupations, services and simple commercial or administrative occupations. While I have classified as skilled jobs: Engineers, professional or semi-professional occupations, qualified commercial or administrative occupations, and managerial occupations.

¹⁷The survey gives information about investment made to replace depreciated capital.

¹⁸For a more detailed description of this dataset, see Alda et al (2005)

¹⁹The service sector includes three kind of services defined in the survey: industrial services, transport and communication, and other services.

valid responses for output. To estimate productivity I need data on output and number of workers in each category. I only consider observations from the old Federal Republic of Germany (West-Germany). Finally, firms with strictly less than 10 employees were removed. The final number of observations in my sample of firms is 15,174. Table 1 provides descriptive statistics of the final sample of firms.

Table 1: Firms - Descriptive Statistics

	n° of firms	Output (mean)*	n° of workers	Women (%)		Men (%)	
				Unsk.	Skill	Unsk.	Skill
MANUFACT.	7,354	151.0	4,297,762	11.9	8.1	55.3	24.7
CONSTRUCT.	1,491	30.2	170,786	12.9	11.5	58.1	17.6
TRADE	2,078	67.4	247,884	30.6	17.5	34.4	17.6
SERVICES	4,251	30.4	1,043,678	21.1	21.0	38.9	20.0
TOTAL	15,174	92.8	5,760,110	14.2	11.0	51.5	23.4

* Per annum total output in millions of euros

One of the main advantages of this data-set is that it has information on all the employees subject to social security in each firm²⁰. The employee data are matched to firms for which I have valid estimates of productivity through a unique firm identifier. The raw data contains 21,246,022 observations between 1996 and 2004, but after this final trimming I have a 9-year unbalanced panel, including a total of 5,760,110 workers' observations distributed into 15,174 firms' observations.

Women are, on average, younger than men, they have less tenure and less experience. Women tend to have high-skill occupations with higher frequency than men. The proportion of immigrants is higher within the men's group. See Table 2 for details of the workers sample.

²⁰Employees subject to social security are workers, other employees and trainees who are liable to health, pension and/or unemployment insurance or whose contributions to pension insurance is partly paid by the employer. The following forms of employment are not considered liable to social security: civil servants, self-employed persons, unpaid family workers and so-called "marginal" part-time workers (A "marginal" part-time worker is a person who is either: employed only short-term or paid a maximum wage of €400 per

Table 2: Workers - Descriptive Statistics

	WOMEN	MEN
IMMIGRANT (%)	8.4	10.4
AGE (YEARS)	39.2	40.7
TENURE (YEARS)	10.1	12.0
EXPERIENCE (YEARS)	15.3	17.1
SKILLED (%)	46.4	31.9
OBSERVATIONS	1,290,156	4,130,453

The main goal of this study is to understand the gender wage gap. The difference in conditional means is 21 percent. This is the result from a Oaxaca-Blinder decomposition calculated from worker-level wage equation estimates (see Tables 11 and 12 in the appendix). This would mean that women, on average, have salaries 21 percent lower than men with the same observable characteristics. The unconditional wage differential averages 33 percent, but it is not stable across sectors and occupations (see Table 3). Mean-wages estimated across industries and occupations show that the gap ranges between 22 percent and 51 percent. Wage gaps are significantly different from zero in every sector and in every group, and they are always larger for skilled workers.

German Socio-Economic Panel

Our LIAB dataset is a panel of firms complemented with workers data. As it does not track workers, it is not possible to distinguish between attrition²¹ and job-termination²². For that reason I use GSOEP (German Socio-Economic Panel) to estimate group-specific transition parameters²³.

month).

²¹There is no attrition in a establishment, which is the unit of observation in the sample. I lack individuals that may have changed their identifier or that have changed establishment without changing firm.

²²Unless the worker leaves the establishment and moves to another establishment within the panel.

²³Cahuc, Postel-Vinay and Robin (2006) follow the same strategy for estimating transition parameters with the French Labor Force Survey.

Table 3: Gender Wage Gap

		MEAN DAILY-WAGE		W-GAP
		WOMEN	MEN	(%)
MANUFACT.	UNSKILLED	75.07 (0.08)	96.33 (0.03)	22.07% (0.08%)
	SKILLED	103.61 (0.12)	188.65 (0.14)	45.08% (0.08%)
CONSTRUCT.	UNSKILLED	54.68 (0.29)	85.59 (0.15)	36.12% (0.36%)
	SKILLED	74.78 (0.40)	153.30 (0.77)	51.22% (0.36%)
TRADE	UNSKILLED	56.65 (0.19)	89.71 (0.27)	36.85% (0.28%)
	SKILLED	76.93 (0.27)	135.90 (0.59)	43.39% (0.32%)
SERVICES	UNSKILLED	50.72 (0.14)	92.53 (0.12)	45.18% (0.17%)
	SKILLED	81.85 (0.14)	152.97 (0.29)	46.49% (0.14%)
WEIGHTED AVERAGE		78.01	121.87	33.29%

Note: Standard errors are given in parentheses. Means of log-wage are estimated using worker-level data maximizing saturated normal-likelihoods. Means of wages are calculated by the moment generating function. Standard errors are obtained by Delta-Method.

The German Socio-economic panel is a representative repeated survey of households in Germany. This survey has been carried out annually with the same people and families in Germany since 1984 (but I only use 1996-2005)²⁴. This dataset is the German equivalent to the PSID in the US.

4 Empirical Strategy and Results

The discrete nature of annual data implies a complicated censoring of the continuous-time trajectories generated by the theoretical model. Because of

²⁴See Wagner, Burkhauser, and Behringer (1993) for further details on the GSOEP.

these complications a potentially efficient, full information maximum likelihood is not considered as a candidate for the estimation. Instead, I perform a multi-step estimation procedure²⁵.

Even though it may be theoretically inefficient, I prefer a step-by-step method. One reason is that the efficiency of full information maximum likelihood is only guaranteed in the case of correct specification. However I am interested in having productivity differences and transition parameter estimates that are robust to misspecification in other parts of the model. Another reason is that transition parameters are better estimated using a standard labor force survey such as SGOEP.

A multi-step estimation procedure allows me to have control of the source of variation that is effectively identifying each parameter. The empirical identification of productivity differences with firm level data is weak and imprecise. Full-information maximum likelihood may have helped empirically because I would use data on wages to improve on the productivity estimates, but on the other hand I would not be able to guarantee that such estimates are solely revealing productivity differences as opposed to wage setting inequalities. If the model were the true data generating process this caveat would not be necessary, because the model does not imply any reverse causality from wages to productivity, and the noise in productivity estimates would be only due to the contemporary productivity shock uncorrelated with wages. However, even in an informal way, it seems prudent to use estimators that are as robust to misspecification as possible.

The structural model abstract many dimensions that may be relevant in the wage setting, for example amenities or union pressure. These omitted dimensions may be mainly associated with different types of jobs. As it can be seen in Tables 1 and 2 there are important differences between men and women in terms of occupation and sector. In order to compare jobs which are as similar as possible, the empirical analysis is clustered at the sector and occupation level. The model is estimated independently for each

²⁵Multi-step estimation has been done in many papers. Good examples are Bontemps, Robin, and Van den Berg (2000), Postel-Vinay and Robin (2002), and Cahuc, Postel-Vinay and Robin (2006).

of the four sectors. In order to control for occupation; transition parameters and the rent-splitting parameter are also estimated independently for both types of jobs, in each sector and gender group. I only control for occupation parametrically when I estimate productivity, because I need to consider the full workforce in each firm.

4.1 Productivity

The production function specification chosen in the empirical section, is a standard Cobb-Douglas function with constant returns to scale and quality adjusted labor input. This function has already been used in the discrimination literature to estimate between-group productivity differences and it is also consistent with the theory proposed in the previous section. The value added, Y_{jt} , produced by firm j in period t , is given by:

$$Y_{jt} = A_j K_{jt}^{(1-\alpha)} Q_{jt}^\alpha e^{u_{jt}}$$

Where K_{jt} is the total capital, A_j is a firm specific productivity parameter, u_{jt} is a zero mean stationary productivity shock and Q_{jt} is the total amount of labor in efficiency units given by:

$$Q_{jt} = \sum_K \tilde{\gamma}_k L_{jt}^k$$

As it was mentioned above I have four types of workers depending on gender (men and women) and occupation (skilled and unskilled). I normalize $\gamma_{ms} = 1$ considering male skilled workers as the reference group²⁶. Now $\gamma_k = \tilde{\gamma}_k / \tilde{\gamma}_{ms}$ is the proportional productivity of group k relative to the productivity of male skilled workers. Imposing constant returns to scale and assuming that firms can adjust capital instantaneously makes this specification totally consistent with the theory, where I have assumed that the productivity of a match is $p\varepsilon$. Section A.3 in the appendix provides more details and robustness checks on this assumption.

²⁶Due to this normalization, the firm specific productivity \tilde{A}_j is redefined as $A_j \gamma_{ms}^{\alpha}$.

Using the panel with firm level data on value-added²⁷ (Y_{jt}), depreciated capital²⁸ (K_{jt}^d) and number of workers in each category, I estimate the production function in logs forcing constant return to scale and constant proportionality between occupation across gender ($\gamma_{wu} = \gamma_w \times \gamma_u$).

$$\begin{aligned} \log(Y_{jt}) = & \log(A_j) + (1 - \alpha_l) \log(K_{jt}^d) + \\ & \alpha_l \log(L_{jt}^{ms} + \gamma_w L_{jt}^{ws} + \gamma_u L_{jt}^{mu} + \gamma_w \gamma_u L_{jt}^{wu}) + u_{jt} \end{aligned} \quad (8)$$

where L_{jt}^{ws} and L_{jt}^{ms} are, respectively, the number of women and men in skilled occupations in firm j at time t while L_{jt}^{wu} and L_{jt}^{mu} are, respectively, the number of women and men in unskilled occupations in firm j at time t .

The model predicts that more productive firms are able to attract more workers of every type. As a result the total labor input would be correlated with the firm fixed effect. Therefore, I estimate (8) by Within-Groups Non Linear Least Squares to remove the firm fixed effects.

The Non-Linear Within Groups results are shown in Table 4. Women's productivity is lower than men's productivity in similar jobs. This difference ranges between 20 percent and 41 percent. Unskilled workers are also found to be between 51 percent and 70 percent less productive than skilled workers. As in most production function estimations, α_l is found to be very near one and hence, α_k is very small but statistically different from zero. One interpretation of this result is that K_{jt}^d only captures variable capital whereas fixed capital is subsumed in the firm effect. But if so, the constant returns restriction is dubious.

Differences in productivity across gender are well documented in the literature, following Hellerstein and Neumark (1999) and Hellerstein, Neumark

²⁷I only use value-added in manufacturing. Output measures are used in construction, trade and services due to lack of convergence of estimates based on value-added. Assuming that a constant fraction of output is spent in materials, both types of estimates are consistent for the same parameters, the difference going to the constant term. In order to use firm productivity measures in the structural wage equation both measures are not equivalent because the constant term matters, and hence, value-added is used in every sector.

²⁸Assuming that a constant fraction (d) of capital depreciates by unit of time: $K_{jt}^d = d \times K_{jt} \Rightarrow \log(K_{jt}^d) = \log(d) + \log(K_{jt})$. Therefore $\alpha_k \log(d)$ goes to the constant term.

Table 4: Production Function Estimates

WG-NLLS ESTIMATES OF (8)			
	α_l	γ_w	γ_u
MANUFACTURING	0.963 (0.005)	0.672 (0.062)	0.484 (0.042)
CONSTRUCTION	0.961 (0.006)	0.701 (0.052)	0.444 (0.039)
TRADE	0.971 (0.009)	0.804 (0.092)	0.487 (0.056)
SERVICES	0.945 (0.007)	0.588 (0.068)	0.298 (0.030)
WEIGHTED AVERAGE	0.96	0.67	0.43

Note: Time dummies included. Robust Standard errors are given in parentheses. Weighted averages take into account the number of firms in each sector.

and Troske (1999). The first paper finds, with Israeli firm-level data, a productivity gap of 17 percent while the second, using a U.S. sample of manufacturing plants reports a productivity gap of 15 percent. These studies have been criticized mainly due to the potential endogeneity of the proportion of female workers in the firm²⁹. In this paper, I treat the number of workers of each group as potentially correlated with the firm fixed effect³⁰. Estimating (8) by Within-Groups Non Linear Least-Squares the firm fixed effect is completely removed, hence my estimates are robust to any correlation of the labor input level and the labor input composition with the firm fixed effect.

In this dataset there is strong evidence of correlation between the firm's fixed effect and the firm's labor input. Estimating (8) by NLLS without fixed effects, γ 's estimates are significantly lower, the average of γ_w^{NLLS} across sectors is 0.380 and the average of γ_u^{NLLS} across sectors is 0.256. See Table 8 in the appendix.

Estimating (8) by non-linear within-groups the firm fixed effect is re-

²⁹See Altonji & Blank (1999).

³⁰Indeed, the model predicts that more productive firms are able to attract more workers of every type. Therefore, the total labor input will be correlated with the firm fixed effect but not the labor input composition.

moved, but the simultaneity problem is not totally solved. One alternative would be to treat Ql_{jt} and k_{jt} as predetermined variables and estimate the production function by Non Linear GMM. I tried this possibility but I have a severe problems of lack of precision of the GMM estimates of the γ parameters.

The precision in the non-linear GMM estimates of γ 's is low in every sector and with different sets of instruments included. I have test using: Only lagged levels for the equation in differences as in Arellano and Bond (1991); lagged levels for the equation in differences and lagged differences to instrument the equation in levels as in Arellano and Bover (1995) and only lagged differences to instrument the equation in levels as in Cahuc et al. (2006). I also tried these three alternative sets of moment conditions treating the proportion of each kind of worker as exogenous, but the estimated γ 's remained imprecise.

I obtained extremum estimators that minimized the two-stage robust GMM2 objective function and iterated-GMM but also Chernozhukov and Hong (2003) MCMC type of estimators for Continuously-updated GMM.

Non-Linear System GMM and NLLS estimates of the production function are reported in section A.3 in the appendix.

Apparently, this problem of precision is pervasive in this kind of production function specification. Cahuc et al have decided to estimate the productivity parameters and the wage equation parameter simultaneously by an iterated non-linear least squares procedure without removing the firm's fixed effect³¹.

4.2 Labor Market Dynamics

Given that job termination occurs due to job-to-job transitions and exogenous job destruction and that both processes are Poisson, the model defines the precise distribution of job durations t conditional on the firm productivity p :

³¹MATA codes for computing the non-linear estimators previously described are available from the author upon request.

$$\mathcal{L}(t|p) = [\delta + \lambda_1 \bar{H}(p)] e^{-[\delta + \lambda_1 \bar{H}(p)]t} \quad (9)$$

As I use GSOEP to estimate transition parameters and it does not have productivity measures, λ_1 and δ are estimated treating p as an unobservable. Therefore, I maximize the unconditional likelihood $\mathcal{L}(t) = \int \mathcal{L}(t|p)g(p)dp$, where $g(p)$ is the probability density function of firm's productivity among employed workers.

Taking derivatives with respect to p in equation (6), I get the density of firm's productivity in the population of workers:

$$g(p) = \frac{(1 + \kappa_1)h(p)}{1 + \kappa_1 \bar{H}(p)} \quad (10)$$

In the appendix I show the individual contribution to the unconditional likelihood becomes simple enough to be estimated and it is given by:

$$\mathcal{L}(t) = \frac{\delta(1 + \kappa_1)}{\kappa_1} \left[\int_{\delta t}^{(1+\kappa_1)\delta t} \frac{e^{-x}}{x} dx \right]$$

Integrating unobserved productivity out of the conditional likelihood removes p and all reference to the sampling distribution $H(p)$ (Cahuc et al, 2006). This method is robust to any misspecification in the wage bargaining. The only property of the structural model that is required, is that there exist a scalar firm index, in this case p , which monotonously defines transitions.

In the appendix, I show how to obtain the exact form of the likelihood that takes into account that some duration are right-censored while some others started before the survey's beginning. Finally, an individual contribution to the log-likelihood is:

$$l(t_i) = (1 - c_i) \log \left(\frac{\int_{\delta t}^{(1+\kappa_1)\delta t} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right) +$$

$$c_i \log \left(\frac{\frac{e^{-\delta t_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)t_i}}{\delta(1+\kappa_1)} - t_i \int_{\delta t_i}^{(1+\kappa_1)\delta t_i} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right)$$

Where c_i is a right-censored spell indicator and H_i is the time period elapsed before the sample started³².

Table 5: Transition Parameters - Maximum Likelihood Estimates

	UNSKILLED					
	WOMEN			MEN		
	λ_1	δ	κ_1	λ_1	δ	κ_1
MANUFACT.	0.406	0.044	9.202	0.314	0.031	10.095
	(0.041)	(0.004)	(0.127)	(0.021)	(0.002)	(0.176)
CONSTRUCT.	0.601	0.098	6.085	0.437	0.105	4.162
	(0.188)	(0.031)	(0.150)	(0.025)	(0.006)	(0.118)
TRADE	0.5257	0.094	5.613	0.432	0.074	5.478
	(0.050)	(0.009)	(0.085)	(0.042)	(0.008)	(0.090)
SERVICES	0.559	0.095	5.866	0.458	0.086	5.313
	(0.051)	(0.009)	(0.073)	(0.037)	(0.007)	(0.049)
	SKILLED					
	WOMEN			MEN		
	λ_1	δ	κ_1	λ_1	δ	κ_1
MANUFACT.	0.308	0.044	7.025	0.217	0.034	6.373
	(0.031)	(0.004)	(0.316)	(0.015)	(0.002)	(0.103)
CONSTRUCT.	0.510	0.090	5.691	0.255	0.071	3.620
	(0.095)	(0.017)	(0.107)	(0.025)	(0.006)	(0.048)
TRADE	0.327	0.060	5.459	0.353	0.050	7.093
	(0.020)	(0.004)	(0.078)	(0.032)	(0.004)	(0.219)
SERVICES	0.393	0.073	5.363	0.227	0.050	4.547
	(0.024)	(0.004)	(0.062)	(0.015)	(0.003)	(0.051)

Note: Per annum estimates. Standard errors are given in parentheses.

Maximum likelihood estimates are reported in Table 5. The average duration of an employment spell, $1/\delta$ (possibly changing employer) is between 10 and 32 years, but the mean-duration across sectors is 16.8 years. The average time between two outside offers, $1/\lambda_1$, ranges from 1.7 to 4.6 years. These results seem to be fairly large but they are compatible with others in the literature. van den Berg and Ridder (2003) using a similar specification

³²The MATA code for computing the exponential integral and the MATA code to maximize this likelihood are available from the author upon request.

but with German aggregated data, find δ equal to 0.060 and κ_1 equal to 6.5³³, while here a weighted average of δ across sectors and groups is 0.0592 and the weighted average of κ_1 is 6.275³⁴.

Skilled workers have in general lower transition rates to unemployment and lower on-the-job offer arrival rates. Women are more mobile than men in terms of job-to-job transitions and, in general, they also have higher job-destruction rates. Considering κ_i as an index of frictions it is noteworthy that in general, women and unskilled workers suffer higher labor market frictions than men and skilled workers respectively.

4.3 The Wage Equation: Closing the Model

The structural wage equation (4) can be written as:

$$w_{j,t,i} = \varepsilon_i w_{j,t,k(i)}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)})$$

where $w_{j,t,i}$ is the daily wage of a worker i , who belongs to a group $k(i)$, in a firm j with productivity p_j at time t , and:

$$\begin{aligned} & w_{j,t,k(i)}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)}) \\ = & p_{j,t} - (1 - \beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p_{j,t}))^{\beta_{k(i)}} \\ & * \int_{p_{\min}}^{p_{j,t}} (\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p'))^{-\beta_{k(i)}} dp' \end{aligned}$$

As shown in equation (7) ε is statistically independent of p thus,

$$\begin{aligned} E(w_{j,t,i}) &= E(\varepsilon_i w_{j,t,k(i)}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)})) \\ &= E_k(\varepsilon) E(w_{j,t,k}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)})) \end{aligned}$$

$E_k(\varepsilon) = \gamma_k$ is the mean efficiency units of workers in group k in that market relative to the male skilled group. Therefore the predicted mean wage for workers of group k working in firms with productivity $p_{j,t}$, at time t is:

³³van den Berg and Ridder (2003, p.237) report monthly rates for $\lambda_1 = 0.028$ and $\kappa_1 = \frac{\lambda_1}{\delta} = 6.5$.

³⁴Groups are weighted by their size in the sample reported in Table 1. The unweighted averages are 0.0687 for δ and 6.061 for κ_1 .

$$E(w_{jtk}) = \gamma_k w_{jtk}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)}) \quad (11)$$

The group chosen for normalization is unimportant. Changing this group to a generic group k , we would change our measure of productivity. Instead of p_j , that is, the productivity measured in terms of efficiency units of skilled males, we would have $p_j^k = \gamma_k p_j$, that is, the productivity measured in term of efficiency units of group k . In fact, to define (11) in terms of the productivity of group k , we only need to put γ_k inside the expectation operator:

$$\begin{aligned} E(w_{j,t,i}) &= E(\gamma_k p_{j,t} - \\ &\quad (1 - \beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p_{j,t}))^{\beta_{k(i)}} \\ &\quad * \int_{p_{\min}}^{p_{j,t}} (\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p'))^{-\beta_{k(i)}} \gamma_k dp') \end{aligned}$$

Noting that $\frac{dp}{dp^k} = \frac{1}{\gamma_k}$, and also that $\bar{H}(p_{j,t}) = \bar{H}(p_{j,t}^k)$, and changing the variable within the integral, we have

$$\begin{aligned} E(w_{j,t,i}) &= E(p_{j,t}^k - \\ &\quad (1 - \beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p_{j,t}^k))^{\beta_{k(i)}} \\ &\quad * \int_{p_{\min}^k}^{p_{j,t}^k} (\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p^{k'}))^{-\beta_{k(i)}} dp^{k'}) \quad \blacksquare \end{aligned}$$

For each firm in the sample I estimate the average daily wage \bar{w}_{jtk} paid to workers of group k at time t . Since wages are top-coded, I estimate the firm mean-wage for each worker group (*ie* : \bar{w}_{jtk}) by maximum likelihood at the firm level assuming that wages are log-normal. Under the steady state assumption and according to the theory presented in section 2, \bar{w}_{jtk} exhibits stationary fluctuation around the steady state mean wage $E(w_{jtk})$ paid by firm j with productivity p_j .

I estimate equation (11) in logs with firm-level data:

$$\begin{aligned}
\log \bar{w}_{jtk} &= \ln(\gamma_k) + & (12) \\
&\ln \left(p_{j,t} - (1 - \beta_{k(i)}) (\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p_{j,t})) \right)^{\beta_{k(i)}} \\
&* \int_{p_{\min}}^{p_{j,t}} (\rho + \delta_{k(i)} + \lambda_{1,k(i)} \bar{H}(p'))^{-\beta_{k(i)}} dp' \\
&+ v_{jtk},
\end{aligned}$$

by weighted non-linear least squares at the firm level, where γ_k , δ_k and λ_k are parameters estimated in previous stages, $p_{j,t}$ is the productivity³⁵ of firm j in time t and v_{jtk} is a transitory shock with unrestricted variance. As usual, the discount factor has been set to an annual rate of 5% (daily rate of 0.0134%).

Standard errors have to take into account that γ , λ_1 and δ are estimated in previous stages. To solve this problem I combine bootstrap for γ with an analytical solution for λ_1 and δ . Hence, I obtain standard errors replicating the productivity estimation and the bargaining power estimation in 200 resamples of the LIAB original sample, with replacement, but taking the transition parameters as the population ones. To correct these preliminary standard errors I add to them the analytical term corresponding to the standard errors of λ_1 and δ reported in Table 5. Finding the analytical solution is not difficult in this case because estimators come from different samples so that I can omit the term corresponding to the outer product of scores in the first and second stages.

Consistent standard errors are given by:

$$\begin{aligned}
\widehat{Var}(\hat{\beta}) &= Var(\hat{\beta} | \hat{\lambda}_1, \hat{\delta})_{bootstrap} + \\
&\frac{\hat{H}_{\beta\lambda_1} \widehat{Var}(\hat{\lambda}_1) \hat{H}_{\beta\lambda_1} + \hat{H}_{\beta\delta} \widehat{Var}(\hat{\delta}) \hat{H}_{\beta\delta}}{\hat{H}_{\beta\beta} * \hat{H}_{\beta\beta}}
\end{aligned}$$

where H is the objective function in the optimization, which in this case is the weighted sum of squares and $H_{\xi\phi} = \frac{\partial(\frac{\partial H}{\partial \xi})}{\partial \phi}$. Second derivatives of H are

³⁵See section A.4 in the appendix for details about how I recover p_{jt} using parameters estimated in previous stages. In section A.4 I also present robustness checks on different assumptions regarding the construction of p_{jt} .

obtained numerically³⁶.

The results are presented in Table 6. Women are found to have lower rent-splitting parameters than men in construction and trade for both skilled and unskilled occupations, and in manufacturing skilled occupations. Female workers receive larger portion of the surplus than males in services and in manufacturing unskilled occupations but these differences are not significant.

Table 6: Rent-Splitting Parameter Estimates

		WNLLS ESTIMATES OF (12)	
		WOMEN	MEN
		β	β
MANUFACTURING	UNSKILLED	0.419 (0.129)	0.398 (0.096)
	SKILLED	0.226 (0.088)	0.292 (0.036)
CONSTRUCTION	UNSKILLED	0.214 (0.090)	0.408 (0.045)
	SKILLED	0.113 (0.078)	0.186 (0.104)
TRADE	UNSKILLED	0.339 (0.173)	0.382 (0.106)
	SKILLED	0.152 (0.066)	0.222 (0.064)
SERVICES	UNSKILLED	0.849 (0.125)	0.757 (0.125)
	SKILLED	0.413 (0.104)	0.324 (0.073)

Note: Corrected Bootstrap Standard errors are given in parentheses.

There is a clear pattern in terms of skilled and unskilled occupations. Unskilled workers receive larger shares of the surplus in every sector for female and male workers³⁷. These findings are not consistent with the results

³⁶The analytical correction $\frac{\hat{H}_{\beta\lambda_1} \widehat{Var}(\hat{\lambda}_1) \hat{H}_{\beta\lambda_1} + \hat{H}_{\beta\delta} \widehat{Var}(\hat{\delta}) \hat{H}_{\beta\delta}}{\hat{H}_{\beta\beta} * \hat{H}_{\beta\beta}}$ is not significant in any industry either for skilled or like the unskilled workers.

³⁷These differences, as differences between industry may be understood as consequences

found in Cahuc, Postel-Vinay and Robin (2006), where they report a positive association between bargaining power and job qualification.

Estimates of the rent-splitting parameter are considerably higher than those of Cahuc et al. This is probably due to differences in our definition of match rent. In a similar model estimated with US employee-level data by Flinn and Mably (2008), the overall bargaining power found is 0.45 while here the weighted average across cells is remarkably similar, 0.404.

Differences in rent-splitting parameters are not significant in every sector. I only find that male workers receive larger shares of the surplus than female ones in construction, bootstrap p-values are 95.5% for unskilled workers and 90.4% for skilled ones.

5 Discussion: Counterfactual analysis

The structural wage setting equation provides us with a direct way of isolating the effect of each wage determinant, and hence we are able to calculate which fraction of the wage gap is due to differences in bargaining power, productivity or frictions patterns.

Using the structural wage equation (4):

$$w_{j,t,i} = \varepsilon_i w_{j,t,k(i)}^p(p_{j,t}, \beta_{k(i)}, \lambda_{1k(i)}, \delta_{k(i)})$$

For each sector and each worker group it is possible to measure the wage differential caused by differences in each wage determinant. For example I can estimate the wage gap accounted by δ as the relative difference between the mean-wage that women actually receive and the mean wage that women would receive if they had the male destruction rate.

$$1 - \frac{E \left[\varepsilon_i w_{j,t,k(i)}^p(p_{j,t}, \beta_{women}, \lambda_{1women}, \delta_{women}) \right]}{E \left[\varepsilon_i w_{j,t,k(i)}^p(p_{j,t}, \beta_{women}, \lambda_{1women}, \delta_{men}) \right]} \quad (13)$$

As shown in equation (7) ε is independent of p , therefore I can estimate equation (13) at the firm level:

of compensating differentials or differences in union pressure. They cannot be understood as discrimination because we are comparing occupations and not workers.

$$1 - \frac{\gamma_{women} \sum_j N_j \times w_{j,t,k(i)}^p(p_{j,t}, \beta_{women}, \lambda_{1women}, \delta_{women})}{\gamma_{women} \sum_j N_j \times w_{j,t,k(i)}^p(p_{j,t}, \beta_{women}, \lambda_{1women}, \delta_{men})}$$

Where N_j is the number of female workers in each firm. In order to have a complete decomposition of the wage gap, I replace sequentially each female parameter for a male parameter until I have the male predicted mean wage³⁸.

The wage setting equation implies that the higher the offer-arrival rate, the higher the wage, and that, the higher the job destruction rate the lower the wage. As women have higher offer arrival rates but also higher job-destruction rates, the effect of differences in friction patterns as a whole is undetermined. Differences in δ explain, on average, 1.4 percent of the total wage gap. Netting out λ_1 would increase the wage-gap in 2.6 percent. The size of these effects is surprisingly small, specially if we take into account Bowlus (1997), where using samples of high school and college graduates from the National Longitudinal Survey of Youth (NLSY), these behavioral patterns were found to be significantly different across the genders and account for 20% - 30% of the wage differentials.

The proportion of the wage gap that is due to differences in productivity connects directly with a branch of the literature initiated with Hellerstein et al (1999). In this line of work, they assumed equality between wages and productivity, and therefore any inequality in wages that is not driven by differences in productivity may be considered as discrimination. Here wages and productivity are connected in a more sophisticated manner, in fact this relationship has been shown to be not an equality, not even for the non-discriminated group³⁹. On average, 99.7 percent of the total wage gap is accounted for by differences in productivity. Differences in productivity, and hence their effect over the wage-gaps, are very heterogenous across groups. In services for low and high qualification occupations as in manufacturing for low qualification occupations, differences in productivity imply larger

³⁸ ie: $\gamma_{men} \times E \left[w_{j,t,k(i)}^p(p_{j,t}, \beta_{men}, \lambda_{1men}, \delta_{men}) \right]$

³⁹ $w_{ijt} = \varepsilon_i p_{jt} \Leftrightarrow \beta = 1$, but β is statistically different from one in every sector and in every worker group.

differences in wages than the ones observed in the data.

On average, 1.5 percent of the total wage gap is accounted for by differences in the rent-splitting parameter. The structural estimation provides no significant evidence of discrimination against women in Germany. This finding is different to what I obtained using the traditional approach based on Mincer-equations, where female workers are found to receive wages that are 14.5 percent lower than those of equivalent male workers (see Table 12 in the appendix). As it can be seen in Table 6, there are only significant differences in β between gender in construction. In that industry, differences in β account for 62 percent of the total wage gap in unskilled occupations and 69 percent of the total wage gap in the skilled ones.

6 Concluding Remarks

This paper is the first attempt to estimate an equilibrium search model using matched employer-employee data to study wage discrimination. This kind of data is essential for analyzing discrimination because it allows us to have a clear distinction between differences in workers productivity across groups and differences in wage policies toward those groups.

The structural estimation involved several steps: Firstly, I estimated group-specific productivity relying on production functions estimation at the firm-level using LIAB. Secondly, I computed job-retention and job-finding rates using GSOEP employee-level data. Finally, I estimated the wage-setting parameters (bargaining power) using individual wage records in LIAB, and transition parameters and productivity measures specific for each firm estimated in previous steps.

When analyzing productivity, I observe that women are 16 percent less productive than men in similar jobs. The main findings in terms of friction patterns are that women are in general more mobile than men in terms of job-to-job transitions and they have also higher job-destruction rates. In spite of having large wage differentials, women are not found to have significantly lower bargaining power than men.

In terms of wages, I find that the total gender wage gap is 34 percent. It

turns out that most of the gap is accounted for by differences in productivity and that differences in destruction rates explain 1.4 percent of the total wage-gap. Netting out differences in offer-arrival rates would increase the gap by 2.6 percent. Differences in the rent-splitting parameter are responsible for a very small fraction of the wage gap. In general, my structural estimation gives no significant evidence of discrimination against women in Germany.

There are two desirable extensions that I would like to perform. Firstly, when estimating the model I have fixed the discount factor. However it would be potentially informative, to estimate this parameter, and measure the effect of different discount rates over the wage gap. Secondly, firm's productivity is estimated in a large N , but small T panel, hence there is an issue of a non-vanishing small- T measurement error in estimated firm's fixed effects. It would be interesting to obtain β estimates that take this problem into account.

A Appendix

A.1 Model Equations: Proofs

Here I derive analytically the close form of the equilibrium wage equation. The first step is to find the partial derivative with respect to the wage of the value of a job in a firm with productivity p for a worker with ability ε .

Applying the Leibniz integral rule in (1).

$$\frac{\partial [E(w(p, \varepsilon))]}{\partial w(p, \varepsilon)} = \frac{1}{(r + \delta + \lambda_1 \bar{F}(w(p, \varepsilon)|\varepsilon))} \quad (14)$$

Integrating (14) between $w(\varepsilon)_{\min}$ and $w(p, \varepsilon)$.

$$\int_{w(\varepsilon)_{\min}}^{w(p, \varepsilon)} \frac{1}{(r + \delta + \lambda_1 \bar{F}(\tilde{w}(p, \varepsilon)|\varepsilon))} d(\tilde{w}(p, \varepsilon)) = \int_{w(\varepsilon)_{\min}}^{w(p, \varepsilon)} \frac{\partial [E(\tilde{w}(p, \varepsilon), \varepsilon)]}{\partial \tilde{w}(p, \varepsilon)} d(\tilde{w}(p, \varepsilon))$$

$$E(w(p, \varepsilon), \varepsilon) - E(w(\varepsilon)_{\min}, \varepsilon) = E(w(p, \varepsilon), \varepsilon) - U(\varepsilon)$$

Using the surplus splitting rule (3), the value of the job for the worker (1), the value of the job for the firm (2) and rearranging:

$$w(p, \varepsilon) = p\varepsilon - \quad (15)$$

$$\begin{aligned} & (\rho + \delta + \lambda_1 \bar{F}(w(p, \varepsilon)|\varepsilon)) \frac{(1 - \beta)}{\beta} \\ & * \int_{w(\varepsilon)_{\min}}^{w(p, \varepsilon)} \frac{1}{(\rho + \delta + \lambda \bar{F}(\tilde{w}(p, \varepsilon)|\varepsilon))} d(\tilde{w}(p, \varepsilon)) \end{aligned} \quad (16)$$

noting that

$$\begin{aligned} & \int_{w(\varepsilon)_{\min}}^{w(p, \varepsilon)} \frac{1}{(\rho + \delta + \lambda \bar{F}(\tilde{w}(p, \varepsilon)|\varepsilon))} d(\tilde{w}(p, \varepsilon)) \\ & = \int_{p_{\min}}^p \frac{1}{(\rho + \delta + \lambda \bar{H}(p'))} \frac{d(w(p, \varepsilon))}{dp'} dp' \end{aligned}$$

and taking derivatives with respect to p .

$$\begin{aligned} \frac{d(w(p, \varepsilon))}{dp'} & = \varepsilon - \frac{(1 - \beta)}{\beta} \frac{d(w(p, \varepsilon))}{dp'} \\ & + \lambda_1 h(p) \frac{(1 - \beta)}{\beta} \int_{p_{\min}}^p \frac{1}{(\rho + \delta + \lambda \bar{H}(p'))} \frac{d(w(p, \varepsilon))}{dp'} dp' \end{aligned}$$

Then, plugging equation (15):

$$\begin{aligned} \frac{d(w(p, \varepsilon))}{dp'} & = \varepsilon + \\ & \lambda_1 h(p) \frac{w(p, \varepsilon) - p\varepsilon}{(\rho + \delta + \lambda \bar{H}(p'))} - \frac{(1 - \beta)}{\beta} \frac{d(w(p, \varepsilon))}{dp'} \end{aligned}$$

Rearranging, I have a first order differential equation,

$$\frac{d(w(p, \varepsilon))}{dp'} + \frac{\beta \lambda_1 h(p)}{\rho + \delta + \lambda_1 \bar{H}(p)} w(p, \varepsilon) = \varepsilon \beta \left[\frac{\rho + \delta + \lambda_1 \bar{H}(p) + \lambda_1 h(p)p}{\rho + \delta + \lambda_1 \bar{H}(p)} \right] \quad (17)$$

To solve this differential equation, note that:

$$\frac{d(\rho + \delta + \lambda_1 \bar{H}(p))^{-\beta}}{dp} = (\rho + \delta + \lambda_1 \bar{H}(p))^{-\beta} \frac{\beta \lambda_1 h(p)}{\rho + \delta + \lambda_1 \bar{H}(p)}$$

Then, multiplying both sides of equation (17) by $(\rho + \delta + \lambda_1 \bar{H}(p))^{-\beta}$ and rearranging

$$\frac{d [w(p, \varepsilon)(\rho + \delta + \lambda_1 \bar{H}(p))^{-\beta}]}{dp} = \varepsilon \beta \left[\frac{\rho + \delta + \lambda_1 \bar{H}(p) + \lambda_1 h(p)p}{(\rho + \delta + \lambda_1 \bar{H}(p))^{1+\beta}} \right] \quad (18)$$

Integrating (18) between p_{\min} and p , and noting that the lowest productivity firm will produce no surplus $\Leftrightarrow w(p_{\min}, \varepsilon) = p_{\min} \varepsilon$, straightforward algebra shows that:

$$\begin{aligned} & w(p, \varepsilon)(\rho + \delta + \lambda_1 \bar{H}(p))^{-\beta} \\ = & (\rho + \delta + \lambda_1)^{-\beta} p_{\min} \varepsilon + \varepsilon \beta \int_{p_{\min}}^p \left[\frac{\rho + \delta + \lambda_1 \bar{H}(p') + \lambda_1 h(p')p'}{(\rho + \delta + \lambda_1 \bar{H}(p'))^{1+\beta}} \right] dp' \end{aligned}$$

separating the integral in a convenient way and noting that:

$$\frac{\partial \left((\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} p' \right)}{\partial p'} = (\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} + \frac{\beta \lambda_1 h(p') p'}{(\rho + \delta + \lambda_1 \bar{H}(p'))^{1+\beta}} dp'$$

it solves as:

$$\begin{aligned} w(p, \varepsilon) &= \frac{(\rho + \delta + \lambda_1 \bar{H}(p))^\beta}{(\rho + \delta + \lambda_1)^\beta} p_{\min} \varepsilon - \\ & \varepsilon (1 - \beta) (\rho + \delta + \lambda_1 \bar{H}(p))^\beta \int_{p_{\min}}^p (\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} dp' + \\ & \varepsilon (\rho + \delta + \lambda_1 \bar{H}(p))^\beta \int_{p_{\min}}^p \frac{\partial \left((\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} p' \right)}{\partial p'} dp' \end{aligned}$$

rearranging I get the wage equation as a function of individual skill (ε), friction patterns (δ and λ_1) and firm's productivity (p).

$$w(p, \varepsilon) = \varepsilon p - \varepsilon (1 - \beta) (\rho + \delta + \lambda_1 \bar{H}(p))^\beta \int_{p_{\min}}^p (\rho + \delta + \lambda_1 \bar{H}(p'))^{-\beta} dp' \quad \blacksquare$$

Now I show that p_{min} is independent of ε . p_{min} is the minimum observed productivity level. Firms with productivity p_{min} make zero profit, and therefore the whole productivity goes to the worker, who receive εp_{min} this wage exactly compensate the worker to leave the unemployment, Therefore:

$$E(p_{min}\varepsilon, \varepsilon) = U(\varepsilon)$$

$$\begin{aligned} & p_{min}\varepsilon + \lambda_1 \int_{w(p_{min}, \varepsilon)}^{w(p_{max}, \varepsilon)} [E(w(p'), \varepsilon), \varepsilon] - U(\varepsilon)] dF(W(p', \varepsilon)) \\ = & b\varepsilon + \lambda_0 \int_{w(p_{min}, \varepsilon)}^{w(p_{max}, \varepsilon)} [E(w(p'), \varepsilon), \varepsilon] - U(\varepsilon)] dF(W(p', \varepsilon)) \end{aligned}$$

$$p_{min}\varepsilon = b\varepsilon + (\lambda_0 - \lambda_1) \int_{w(p_{min}, \varepsilon)}^{w(p_{max}, \varepsilon)} [E(w(p'), \varepsilon), \varepsilon] - U(\varepsilon)] dF(W(p', \varepsilon))$$

Using the surplus splitting rule (3):

$$p_{min}\varepsilon = b\varepsilon + (\lambda_0 - \lambda_1) \frac{\beta}{1 - \beta} \int_{w(p_{min}, \varepsilon)}^{w(p_{max}, \varepsilon)} \frac{p'\varepsilon - w(p', \varepsilon)}{(\rho + \delta + \lambda \bar{F}(w(p', \varepsilon)|\varepsilon))} dF(W(p', \varepsilon))$$

This is the value function for a worker of a given ε , therefore we can rearrange everything in terms of p .

$$p_{min}\varepsilon = b\varepsilon + (\lambda_0 - \lambda_1) \frac{\beta}{1 - \beta} \int_{p_{min}}^{p_{max}} \frac{p'\varepsilon - w(p', \varepsilon)}{(\rho + \delta + \lambda \bar{H}(p'))} dH(p')$$

using equation (4) and rearranging:

$$\begin{aligned} p_{min}\varepsilon &= b\varepsilon + \varepsilon(\lambda_0 - \lambda_1) \frac{\beta}{1 - \beta} \\ &\times \int_{p_{min}}^{p_{max}} \frac{(1 - \beta) \int_{p_{min}}^{p'} (\rho + \delta + \lambda_1 \bar{H}(\tilde{p}))^{-\beta} d\tilde{p}}{(\rho + \delta + \lambda \bar{H}(p'))^{(1-\beta)}} dH(p') \end{aligned}$$

Solving for p_{min} :

$$p_{min} = b + (\lambda_0 - \lambda_1) \beta \int_{p_{min}}^{p_{max}} \frac{\int_{p_{min}}^{p'} (\rho + \delta + \lambda_1 \bar{H}(\tilde{p}))^{-\beta} d\tilde{p}}{(\rho + \delta + \lambda \bar{H}(p'))^{(1-\beta)}} dH(p') \quad \blacksquare$$

Note that p_{\min} is a function of the distribution of p and the parameters of the model. The intuition, in discrete time, is clear because the value of being employed and the value of being unemployed are infinite additions of flows linear on ε ($w(\varepsilon, p)$ and $b\varepsilon$). Each flow is multiplied by the discount rate and the probability of being in each state, that do not depend on ε . Hence the value of being employed and the value of being unemployed are both linear in ε . This condition must hold in order to avoid sorting between p and ε .

A.2 Duration model - Maximum Likelihood Specification

The unconditional likelihood of job spell durations is:

$$\mathcal{L}(t) = \int \mathcal{L}(t|p)g(p)dp.$$

$$\mathcal{L}(t) = \int_{p_{\min}}^{p_{\max}} \frac{(1 + \kappa_1)h(p)}{1 + \kappa_1\bar{H}(p)} [\delta + \lambda_1 H(p)] e^{-[\delta + \lambda_1 H(p)]t} dp$$

Rearranging and recalling that $\kappa_1 = \lambda_1/\delta$.

$$\mathcal{L}(t) = \frac{(1 + \kappa_1)\delta}{\kappa_1} \int_{p_{\min}}^{p_{\max}} \frac{1}{\delta + \lambda_1\bar{H}(p)} e^{-[\delta + \lambda_1 H(p)]t} \lambda_1 h(p) dp$$

Changing the variable within the integral, $x = [\delta + \lambda_1\bar{H}(p)] t$. After straightforward algebra I get:

$$\mathcal{L}(t) = \frac{(1 + \kappa_1)\delta}{\kappa_1} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)]$$

where $E_1(t) = \int_t^\infty \frac{e^{-x}}{x} dx$ is the exponential integral function ■.

My sample covers a fixed number of periods, so that some job durations are right censored, and other job spells started before the panel's beginning. Then, the exact likelihood function that takes into account these events is:

$$l(t_i) = (1 - c_i) \log \left(\frac{\mathcal{L}(t_i)}{\int_{H_i}^\infty \mathcal{L}(t) dt} \right) + c_i \log \left(\frac{\int_{t_i}^\infty \mathcal{L}(t) dt}{\int_{H_i}^\infty \mathcal{L}(t) dt} \right)$$

where c_i is a truncated spell indicator and H_i is the time period elapsed before the sample.

$$\begin{aligned}
l(t_i) &= (1 - c_i) \log \left(\frac{[E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)]}{\int_{H_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt} \right) \\
&\quad + c_i \log \left(\frac{\int_{t_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt}{\int_{H_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt} \right)
\end{aligned}$$

Using the fact that $\int E_1(at)dt = -\int E_i(-at)dt = -\left(tE_i(-at) + \frac{e^{-at}}{a}\right)$ (see Abramowitz and Stegun, 1972), and noting that $E_1(-\infty) = 0$

$$\begin{aligned}
\int_{t_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt &= \int_{t_i}^{\infty} E_1(\delta t) dt - \int_{t_i}^{\infty} E_1(\delta(1 + \kappa_1)t) dt \\
&= -tE_i(-\delta t) + \frac{e^{-\delta t}}{\delta} \Big|_{t_i}^{\infty} + \\
&\quad tE_i(-(1 + \kappa_1)\delta t) + \frac{e^{-\delta t}}{(1 + \kappa_1)\delta} \Big|_{t_i}^{\infty}
\end{aligned}$$

$$\begin{aligned}
&\int_{t_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt \\
&= t_i E_i(-\delta t_i) + \frac{e^{-\delta t_i}}{\delta} - t_i E_i(-(1 + \kappa_1)\delta t) - \frac{e^{-\delta t_i}}{(1 + \kappa_1)\delta}
\end{aligned}$$

since $E_i(-at) = -E_1(at) = -\int_{at_i}^{\infty} \frac{e^{-x}}{x} dx$.

$$\begin{aligned}
&\int_{t_i}^{\infty} [E_1(\delta t) - E_1(\delta(1 + \kappa_1)t)] dt \\
&= \frac{e^{-\delta t_i}}{\delta} - t_i \int_{\delta t_i}^{\delta(1 + \kappa_1)t} \frac{e^{-x}}{x} dx - \frac{e^{-\delta t_i}}{(1 + \kappa_1)\delta}
\end{aligned}$$

The same is true for $\int_{H_i}^{\infty} \mathcal{L}(t) dt$. Then the likelihood takes the following form:

$$\begin{aligned}
l(t_i) &= (1 - c_i) \log \left(\frac{\int_{\delta t}^{(1 + \kappa_1)\delta t} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1 + \kappa_1)H_i}}{\delta(1 + \kappa_1)} - H_i \int_{\delta H_i}^{(1 + \kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right) + \\
&\quad c_i \log \left(\frac{\frac{e^{-\delta t_i}}{\delta} - \frac{e^{-\delta(1 + \kappa_1)t_i}}{\delta(1 + \kappa_1)} - t_i \int_{\delta t_i}^{(1 + \kappa_1)\delta t_i} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1 + \kappa_1)H_i}}{\delta(1 + \kappa_1)} - H_i \int_{\delta H_i}^{(1 + \kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right) \quad \blacksquare
\end{aligned}$$

A.3 Robustness Check: Production Function

Credible productivity estimates are crucial to be able to reach reliable conclusions about wage discrimination. As robustness a check, I present results of the productivity estimates under different sets of assumptions.

No Frictions in the Capital Market: As it has been assumed in the theory, the labor input is given for the firm because it has not control over job-creation and job-destruction Poisson processes but capital is chosen to maximize profit. Since there are no frictions or adjustment cost in the capital market, when a firm knows the total labor QL_{jt} it will have in the present period, it solves the following problem:

$$\max_{k_{jt}} (A_j K_{jt}^{(1-\alpha)} Q L_{jt}^\alpha - r_t K_{jt})$$

Substituting the first order condition into the production function and rearranging, I have that:

$$Y_{jt} = \left[\left(A_j^{\frac{1}{1-\alpha}} \frac{1-\alpha}{r_t} \right)^{\frac{1-\alpha}{\alpha}} e^{u_{jt}} \right] Q L_{jt} = p_{jt} Q L_{jt}$$

where r_t is the cost of capital. Note that this production function is equivalent to $p\varepsilon$, the production function assumed in the theory, where p is time and firm specific: $p_{jt} = A_j^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{r_t} \right)^{\frac{1-\alpha}{\alpha}} e^{u_{jt}}$.

$$\log(Y_{jt}) = a_j + b_t + \log(L_{jt}^{ms} + \gamma_w L_{jt}^{ws} + \gamma_u L_{jt}^{mu} + \gamma_w \gamma_u L_{jt}^{wu}) + u_{jt} \quad (19)$$

In Table 7 I report within-groups non linear least squares estimates of (8) and (19). Relative productivity estimates (ie: γ_w and γ_u) are very similar in both estimations, differences between parameters are always smaller than a standard deviation. This robustness check is essential because in this kind of model, for simplicity it is usually assumed that capital adjusts instantaneously to match the number of workers in each period. Although this assumption may seem controversial, it turns out that using observed capital or using the theoretical optimal choice of capital does not change the relative productivity estimates.

Table 7: Production Function: Optimal Capital Input

	WG-NLLS OF (8)		WG-NLLS OF (19)	
	γ_w	γ_u	γ_w	γ_u
MANUFACTURING	0.672 (0.062)	0.484 (0.042)	0.642 (0.132)	0.478 (0.070)
CONSTRUCTION	0.701 (0.052)	0.444 (0.039)	0.839 (0.141)	0.485 (0.833)
TRADE	0.804 (0.092)	0.487 (0.056)	0.745 (0.176)	0.541 (0.982)
SERVICES	0.588 (0.068)	0.298 (0.030)	0.595 (0.115)	0.301 (0.040)

Note: Time dummies included. Robust to heteroscedasticity Standard errors are given in parentheses.

Worker Composition Endogeneity: One of the main criticisms to the productivity estimates reported in Hellerstein and Neumark (1999) and in Hellerstein, Neumark, and Troske (1999) was that the proportion of women in the firm is likely to be correlated with the firm's technology⁴⁰.

$$\begin{aligned} \log(Y_{jt}) = & \log(\check{A}_j) + \alpha_k \log(K_{jt}^d) + \\ & \alpha_l \log(L_{jt}^{ms} + \gamma_w L_{jt}^{ws} + \gamma_u L_{jt}^{mu} + \gamma_w \gamma_u L_{jt}^{wu}) + u_{jt} \end{aligned} \quad (20)$$

Note that (20) is the original Cobb-Douglas production function in logs without imposing constant returns to scale and including the depreciated capital instead of the optimal capital input. In this estimation I am assuming that the depreciation rate is constant, and hence depreciated capital is a constant fraction of the total capital.

The results are presented in Table 8. Female productivity estimates are significantly smaller. This finding is true for all sectors. Comparing the results in this Table with those in Table (4), where firm's fixed effects were removed confirm the Altonji & Blank (1999) suspicion about correlation between the women proportion and the firm fixed effect. Hausman tests reject

⁴⁰See Altonji and Blank (1999).

Table 8: Production Function: Non Linear Least Squares in Levels

NLLS OF (20)				
	α_k	α_l	γ_w	γ_u
MANUFACTURING	0.153 (0.007)	0.905 (0.018)	0.352 (0.032)	0.300 (0.016)
CONSTRUCTION	0.084 (0.017)	1.034 (0.028)	0.451 (0.082)	0.382 (0.043)
TRADE	0.110 (0.018)	0.963 (0.029)	0.562 (0.068)	0.365 (0.039)
SERVICES	0.180 (0.098)	0.839 (0.015)	0.356 (0.029)	0.292 (0.017)

Note: Time dummies included. Standard errors are given in parentheses.

equality of γ_w in every sector. In terms of γ_u , the results are not so different and I only reject equality for manufacturing.

Predetermined inputs:

Estimating (8) the firm fixed effect is completely removed, but the simultaneity problem is not totally solved. One alternative would be to treat Ql_{jt} and k_{jt} as predetermined variables. In Table 9 I report System-GMM estimates of (20). However the precision in the γ 's GMM estimates is poor. Capital coefficients are not significantly different from zero and constant returns to scale are not rejected in any sector. Sargan tests do not reject compatibility of instruments in any sector. Any test of equality between γ 's reported in Table 9 and those in Table 4 do not reject equality, but this fact may be mainly due to the large standard errors of the System-GMM estimated γ 's⁴¹.

⁴¹The Hausman test has to be calculated with these standard errors because System-GMM is robust to predeterminedness of inputs, and I have not imposed constant returns to scale.

Table 9: Production Function: Non Linear SYSTEM-GMM

SYSTEM-GMM OF (20)					
	α_k	α_l	γ_w	γ_u	SARGAN P-V
MANUFACTURING	0.060 (0.046)	0.938 (0.011)	0.70 (0.454)	0.15 (0.069)	58%
CONSTRUCTION	0.016 (0.043)	1.164 (0.160)	0.442 (0.267)	0.188 (0.083)	98%
TRADE	0.053 (0.050)	1.003 (0.170)	0.370 (0.290)	0.322 (0.215)	97%
SERVICES	0.117 (0.071)	0.786 (0.146)	0.434 (0.158)	0.204 (0.090)	93%

Note: Time dummies included. Standard errors are given in parentheses.

A.4 Robustness Check: Constructing firm's productivity

The productivity of firm j in time t , is constructed with the parameters obtained in Section 4.1. Although the production function was estimated using output, productivity is constructed using value-added which is conceptually more accurate.

There are two possibilities to construct marginal productivity measures:

- To include the residual term in the production function:

$$p_{jt} = \alpha_l A_j K_{j,t}^{(1-\alpha_l)} (Ql_{jt})^{(\alpha_l-1)} e^{u_{jt}} = \alpha_l \frac{Y_{jt}}{Ql_{jt}}$$

- To neglect this residual term:

$$\tilde{p}_{jt} = \alpha_l A_j K_{j,t}^{(1-\alpha_l)} (Ql_{jt})^{(\alpha_l-1)}$$

For simplicity and following Cahuc et al (2006), I choose the first option. In this case the productivity is simply $p_{jt} = \alpha_l Y_{jt}/Ql_{jt}$. Whether firms insure temporary shocks to workers, is an open debate in the literature (see Guiso,

Table 10: Robustness Check: Productivity

		WOMEN		MEN	
		WITH $e^{u_{jt}}$	WITHOUT $e^{u_{jt}}$	WITH $e^{u_{jt}}$	WITHOUT $e^{u_{jt}}$
		β	β	β	β
M	UNSK.	0.419	0.438	0.398	0.551
		(0.129)		(0.096)	
	SK.	0.226	0.134	0.292	0.245
		(0.088)		(0.046)	
C	UNSK.	0.214	0.195	0.408	0.407
		(0.090)		(0.045)	
	SK.	0.113	0.109	0.186	0.286
		(0.078)		(0.104)	
T	UNSK.	0.339	0.326	0.382	0.435
		(0.173)		(0.106)	
	SK.	0.152	0.135	0.222	0.194
		(0.066)		(0.064)	
S	UNSK.	0.849	0.794	0.757	0.686
		(0.125)		(0.125)	
	SK.	0.413	0.423	0.324	0.402
		(0.104)		(0.073)	

Note: Bootstrap Standard errors in parentheses for the model with $e^{u_{jt}}$

Pistaferri and Schivardi, 2005). If firms totally insure temporary shocks to workers, and wages are determined as a function of the expected marginal productivity, \tilde{p}_{jt} would be the relevant measure. If not, \check{p}_{jt} would be the correct one. In Table 10 I show that estimated β with or without including the residual term are very similar. Results with $e^{u_{jt}}$ (*ie.* using \check{p}_{jt}) are the same estimates reported in Table 6

A.5 Detecting Discrimination - Traditional Approach

In order to compare different strategies to detect wage discrimination. I perform the traditional approach using Mincer-type wage equations. As it can be seen in Table 11, women have positive negative differentials. Controlling for observable characteristics, they receive wages, on average, 21 percent

lower than men. This difference is significant and consistent with what has been found in previous research: Blau and Kahn (2000), with OECD data reports a difference of 25.5 percent between male and female mean wages, while Fitzenberger and Wunderlich (2002) with the same data as in this paper, but using quantile regression, the estimated German gender wage gap ranges between 16 percent and 25 percent depending on job's qualification.

Oaxaca-Blinder Decomposition

Using the results presented in Table 11, I perform a Oaxaca-Blinder decomposition, which is to simply decompose the wage-gap between differences in observable and unobservable characteristics.

The Oaxaca-Blinder decomposition results are presented in Table 12. The counterfactual female mean-wage has to be interpreted as the mean-wage that women would have if they had the men's distribution of observable characteristics. Therefore, the difference between the counterfactual female mean-wage and the observed women mean-wage is the portion of the gap that is due to differences in observable characteristics.

The portion of the unconditional wage-gap that is not accounted for observable characteristics has usually been interpreted as wage discrimination. In this case, I would conclude that women are being discriminated. They are receiving wages almost 10 percent lower than similar men.

These results are fairly different to those obtained in this paper and a reasonable hypotheses is that this difference is due to the fact that the traditional approach is not able to control for non-observable differences in productivity between groups.

Table 11: Mincer Wage Equations - Censored-Normal Regression. Maximum Likelihood Estimates

Y=LOG(WAGE)	ALL	MEN	WOMEN
WOMEN	-0.211 (0.0004)	- -	- -
IMMIGRANT	0.073 (0.0006)	0.061 (0.0016)	0.076 (0.0006)
SKILLED	0.255 (0.0004)	0.178 (0.0010)	0.276 (0.0005)
AGE	0.056 (0.0002)	0.068 (0.0004)	0.054 (0.0002)
AGE ²	-0.001 (0.0000)	-0.001 (0.0000)	-0.001 (0.0000)
PRIMARY EDUCATION	0.236 (0.0004)	0.257 (0.0011)	0.234 (0.0005)
COLLEGE (INCOMPLETE)	-0.127 (0.0014)	-0.082 (0.0028)	-0.162 (0.0015)
TECHNICAL COLLEGE (COMPLETED)	0.386 (0.0010)	0.436 (0.0021)	0.354 (0.0012)
COLLEGE	0.609 (0.0011)	0.616 (0.0033)	0.566 (0.0011)
UNIVERSITY DEGREE	0.757 (0.0011)	0.819 (0.0027)	0.700 (0.0012)
TENURE	0.017 (0.0001)	0.025 (0.0002)	0.015 (0.0001)
EXPERIENCE	0.033 (0.0001)	0.021 (0.0003)	0.036 (0.0001)
PART-TIME	-0.638 (0.0007)	-0.651 (0.0010)	-0.608 (0.0011)
MANUFACTURING	0.178 (0.0008)	0.175 (0.0016)	0.103 (0.0010)
CONSTRUCTION	0.063 (0.0013)	0.026 (0.0029)	-0.081 (0.0014)
SERVICES	0.037 (0.0009)	0.025 (0.0017)	-0.023 (0.0011)
CONSTANT	2.500 (0.0029)	2.189 (0.0066)	2.599 (0.0031)
PSEUDO R ²	47.23	30.92	52.53
SIGMA	0.38	0.48	0.34

Note: Std. errors are given in parentheses. Time Dummies included.

Table 12: Oaxaca-Blinder Decomposition

(A) OBSERVED MEN MEAN DAILY WAGE	(B) OBSERVED WOMEN MEAN DAILY WAGE	(C) COUNTERFACTUAL WOMEN MEAN DAILY WAGE
107.12 €	73.51 €	91.54 €
TOTAL W-GAP $((B)-(A))/(A)$	EXPLAINED W-GAP= $((B)-(C))/(A)$	UNEXPLAINED W-GAP= $((A)-(C))/(A)$
-31.4%	-16.8%	-14.5%

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