Theoretical notes on bubbles and the current crisis

Alberto Martin and Jaume Ventura*

January 25, 2010

Preliminary and Incomplete

Do not cite or circulate without permission

Abstract

We explore a view of the crisis as the result of a shock to investor sentiment that led to the collapse of a bubble or pyramid scheme in asset markets. We embed this view in a standard model of the financial accelerator and explore its empirical and policy implications. In particular, we focus on asset price fluctuations, the international propagation of shocks and the role of fiscal policy.

*We dedicate this research to the memory of Paul Samuelson, the best economist of the twentieth century, and the first one to understand that pyramid schemes are possible and might raise welfare even if we are all rational and well informed. Martin: CREI and Universitat Pompeu Fabra, amartin@crei.cat. Ventura: CREI and Universitat Pompeu Fabra, jventura@crei.cat. CREI, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005-Barcelona, Spain. We acknowledge financial support from the Spanish Ministry of Science and Innovation, the Generalitat de Catalunya, and the Barcelona GSE Research Network.

1 Introduction

History shows that capitalists economies alternate expansions and recessions. Thus, even in the heights of the expansion that went from the mid 1990s to the subprime mortgage crisis in the summer of 2007 it was widely understood that a crisis would someday hit the world economy. But nobody anticipated what has happened since. The depth of the current recession and the blazing speed with which it has propagated across countries far exceeds even the most pessimistic scenarios. In fact, we need to go back to the Great Depression of the 1930s to find a crisis of a similar magnitude and global scope. It is still not clear however that the lessons we learned from that earlier crisis are useful to understand what is going on today.

As everybody else, macroeconomists have been taken by surprise by the unfolding of events. Even worse, providing an accurate diagnosis of the problem that afflicts the world economy and coming up with clearcut policy prescriptions to deal with it is turning out to be a really hard challenge. Part of the reason for this, of course, is that state-of-the-art models are poorly adapted to this task. These models typically emphasize nominal rigidities and labor market frictions, and downplay the role of financial frictions. As a profession, we must go back to the drawing board and reverse these priorities. To understand the current crisis we need models that bring back financial frictions to center stage.

Recent attempts to do this build on the seminal contributions by Bernanke and Gertler (1989) and Kiyotaki and Moore (2007) who developed models of the "financial accelerator" mechanism. These models highlight the role of net worth as a key variable affecting the functioning of financial markets. The intuition is simple: the role of financial markets is to intermediate funds from those that have them (i.e. the savers or creditors) to those who know what to do with them (i.e. the entrepreneurs or borrowers). This intermediation is useful because it raises the average efficiency of the economy and thus the welfare of its inhabitants. To be able to do this intermediation, savers need guarantees from entrepreneurs that the funds they lend them (plus an attractive enough return!) will be paid back once the investments give their fruits. The concept of net worth is akin to those guarantees. The net worth of an entrepreneur is the amount of future funds that he/she can pledge today to its creditors. When net worth is low, entrepreneurs cannot borrow enough and the economy operates at low levels of efficiency. When net worth is high, entrepreneurs can borrow enough and the economy operates at high levels of efficiency.

The financial accelerator models receive this name because they were initially built to show

how financial frictions could amplify the effects of technology shocks on economic activity through expansions and contractions of credit. The idea was that small productivity shocks could have large effects in economic activity through their effects on net worth. The need for such an amplification mechanism arose from the observation that measured productivity shocks were too small for the Real Business Cycle (RBC) model to provide a reasonable description of the data. In the RBC model, the net worth of entrepreneurs is always large enough to implement the optimal degree of intermediation and productivity shocks do not affect credit. From the perspective of the current crisis, the financial accelerator models are thus interesting because of two reasons: (i) the current crisis has clearly been associated with a large contraction of credit; (ii) it is difficult to find a technology shock that has the magnitude required to explain the severity of the recession.

Building largely on the financial accelerator mechanism, the logic of the current research strategy seems to go in two directions. First, there are those who interpret these models in a classic way. According to their views the current crisis is unprecedented because the financial accelerator mechanism has become so powerful that even hard-to-detect productivity shocks can unleash a massive contraction of credit and a deep recession. This is, for instance, the view that lies behind the recent work of Gertler and Kiyotaki (2009), two of the original developers of this class of models. Second, there are those who hold the view that the world economy has not suffered a small shock to productivity, but instead a large shock to net worth. The problem in articulating this view is that there are, in our view, no good models of shocks to net worth. In existing financial-accelerator models, this variable depends on the market's expectation of the future profits of entrepreneurs and the productivity of the financial sector at intermediating. Once we rule out shocks to the future productivity to entrepreneurs, it seems that we are only left with the alternative of having to propose yet another type of productivity shock. Namely, a large shock to the productivity of the financial sector itself!

Here is where we come in, relaxing this choice and proposing another alternative. We welcome the focus on financial accelerator models as we also think they constitute the best available tool to study the macroeconomic implications of current events. We also welcome the notion that the current crisis has been caused by a large shock to net worth. We do not think however that this shock is due to a reduction in the productivity of the financial sector (although we comment on this possibility later on). We argue instead that the shock was a decline in investor confidence that led to the collapse of a number of pyramid schemes or bubbles. These bubbles were useful because they increased net worth and intermediation, allowing the world economy to function at a high level of efficiency. The bursting of these bubbles drastically reduced net worth, leading to a massive credit contraction and a deep recession.

This shift in perspective requires some theoretical work, and we provide it here. One contribution of this paper is to show that the financial accelerator model can accommodate shocks to investor sentiment and the presence of random asset bubbles with minimal modifications. This widens the modelling choices available to macroeconomists, allowing us in particular to explore a whole new view of the crisis, its origins and its possible remedies.

This shift in perspective is more than academic exercise, and we also show this here. Another contribution of this paper is to derive some basic empirical and policy implications of this view:

1. On the empirical side, modelling the crisis as a collapse of a bubble allows us to provide answers to two burning questions for current macroeconomics: (i) Why do asset (stock, housing, ...) prices fluctuate so much and in ways that seem so unrelated to fundamentals? and (ii) How is it that the current crisis has propagated so quickly and so strongly across countries?

2. On the policy side, modelling the crisis as the collapse of a bubble radically affects the role of fiscal policy as an stabilization tool. We show here that the case for a fiscal stimulus package and its optimal design depend crucially on whether the shock that led to the crisis is a small (but very amplified) technology shock or a change in investor sentiment that led to the collapse of the bubble.

This paper is related to two important literatures. On the one hand, we have already stressed its relationship to the literature on the financial accelerator that originated with the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). This literature has highlighted the role of the financial system as an amplifier and a propagator of shocks. In a recent paper, Gertler and Kiyotaki (2009) draw on the insights derived from this body of work in order to interpret the recent crisis. On the other hand, our paper is also closely related to the traditional literature on rational bubbles that goes back to Samuelson (1958). Tirole (1985) analyzed the conditions for the existence of such bubbles in the context of a production economy. Our paper is closely related to the latter with the difference that, in our setup, the presence of financial frictions implies that bubbles can be expansionary and lead to increases in the capital stock. This feature of our model is reminiscent of recent results by Kraay and Ventura (2007), Caballero and Krishnamurthy (2006), and Farhi and Tirole (2009). Our framework differs from these last papers in two important respects, though.

The first is that we derive all of our results in the context of a standard production economy. The second is that, as in Martin and Ventura (2010), bubbles in our setting can arise even if all investments are dynamically efficient in the economy's fundamental equilibrium.

The paper is organized as follows: section **??** presents a stylized version of the financial-accelerator model while section **??** uses it to explain current events. Sections three, four and five introduce bubbles and shocks to investor sentiment into the model and use these to re-interpret current events. Sections six, seven and eight compare the empirical and policy implications of the two views. Section nine provides some final remarks.

2 A canonical model of financial frictions and business cycles

In a useful recent paper, Gertler and Kiyotaki (2009) take stock of the state of the art in this class of models and develop a "canonical framework to help organize thinking about credit market frictions and aggregate economic activity in the context of the current crisis" (p.1). This framework is built around an agency cost that limits the use of future profits of firms as collateral for obtaining financing today. This friction generates a credit constraint that fundamentally affects the way the economy works. The goal of this section is to develop a stripped-down model that captures this view and also allows us to take it in the direction we want later.

The simplest model we could come up with builds on Samuelson's overlapping-generations structure. The world economy contains an infinite sequence of generations, indexed by $t \in (-\infty, +\infty)$. Each generation lives two periods, young and old; and it contains a continuum of individuals of size one, indexed by $i \in I_t$. Let c_{it+1} be the old-age consumption of individual i of generation t. Individuals maximize the expected value of this consumption, i.e. $U_{it} = E_t \{c_{it+1}\}$; where U_{it} is its utility function when young. To finance their consumption, individuals supply one unit of labor when young and receive a wage equal to w_t. Since individuals only care about old age consumption, they save the entire wage. Therefore, w_t is also their savings or wealth. Since individuals are risk-neutral, they always invest their wealth so as to maximize its expected return.

Each individual manages one firm, building capital during youth and producing output during old age. Output is produced with labor and capital. To do this, all individuals/firms have access to the same Cobb-Douglas technology, i.e.

$$F(l_{it}, k_{it}) = l_{it}^{1-\alpha} \cdot k_{it}^{\alpha} \tag{1}$$

where l_{it+1} and k_{it+1} are the labor and capital used by the firm managed by individual i. Capital in period $t+1$ is produced with output in period t. In particular, we assume that each unit of output in period t produces A_{it+1} units of capital next period:[1]

$$k_{it+1} = A_{it+1} \cdot I_{it} \tag{2}$$

where I_{it} is investment. Gertler and Kiyotaki refer to A_{it} as the "quality of capital" and it plays a central role in their analysis. Firms (or, more precisely, the individuals that manage them) differ in this variable. To keep things simple, we consider the case in which there are two types, efficient and inefficient. With these assumptions at hand, we can write:

$$A_{it+1} = \begin{cases} \pi_t & \text{if } i \in I_t^E \\ 1 & \text{if } i \notin I_t^E \end{cases} \tag{3}$$

We assume that π_t is known as of time t, although it might fluctuate randomly over time. Fluctuations in π_t affect both the potential efficiency at which the economy can operate and also the efficiency differential between firms. Since $\pi_t > 1$ in all dates and states of nature, it makes sense to call I_t^E the subset of efficient individuals in generation t. A fraction $\varepsilon \in [0,1]$ of each generation belongs to this set. Since all individuals have the same wealth, this parameter measures the share of the economy's savings or wealth that is in the hands of efficient firms and need not be intermediated.

Investment is financed with internal and external funds:

$$I_{it} = w_t + f_{it} \tag{4}$$

The internal funds are the wage or wealth of the individual, while the external funds are the financing obtained during youth, i.e. $f_{it} \in [-w_t, \infty)$. Efficient individuals/firms would like to borrow from inefficient ones. We introduce however an agency cost that limits the ability of firms to obtain financing. In particular, we assume that firms can commit or pledge only a fraction $\phi \in [0,1]$ of future revenues to their creditors:

$$R_{t+1}^f \cdot f_{it} \le \phi \cdot \left[F\left(l_{it+1}, k_{it+1}\right) - w_{t+1} \cdot l_{it+1}\right] \tag{5}$$

[1] For simplicity, we have assumed that capital fully depreciates in production.

where R^f_{t+1} is the cost (per unit) of this financing. This cost can be made contingent on any variable which is known in period $t+1$. In fact, in this world of risk-neutral individuals the optimal financial contract will ensure that, if the credit constraint is binding in one state of nature, it is also binding in all the rest. Equation (5) is central in the analysis, as its right-hand side captures the concept of net worth of individuals/firms.[2] As discussed in the introduction, in the canonical model net worth depends on future profits and the financial friction.

Each individual's consumption equals the value of the firm he/she manages, i.e. $c_{it+1} = V_{it+1}$; where V_{it+1} is the value of that firm. Maximization therefore implies that:

$$E_t V_{it+1} = \max_{\langle l_{it+1}, k_{it+1}, f_{it} \rangle} E_t \left\{ F\left(l_{it+1}, k_{it+1}\right) - w_{t+1} \cdot l_{it+1} - R^f_{t+1} \cdot f_{it} \right\} \tag{6}$$

$$s.t. \text{ Equations (1), (2), (3), (4) and (5)}$$

Since the capital fully depreciates in production, Equation (6) says that the value of the firm tomorrow will equal its profits, i.e. revenue net of labor and financing costs. Profit maximization is subject to both technological and financial constraints.

In the canonical model there are two key markets, the labor and financial markets, that allocate workers and funds to firms. [3] We first turn to the labor market and impose the full-employment or market-clearing condition, i.e. $l_t \equiv \int_{i \in I} l_{it} = 1$. This means that:[4]

$$w_t = (1 - \alpha) \cdot k_t^{\alpha} \tag{7}$$

where $k_t \equiv \int_{i \in I} k_{it}$ is the aggregate capital stock. Since all firms use the same capital-labor ratio, this must be the aggregate one. Equation (7) says that the wage equals the marginal product of labor evaluated at the aggregate capital-labor ratio. It makes sense that wages, and therefore the wealth of each generation, be determined by the capital stock that the economy has accumulated over time.

We turn next to the financial market and how it allocates funds to firms. There are two key

[2] As in Bernanke and Gertler (1989), our focus is on collateralizable net worth.

[3] A lot of useful recent research has focused on how frictions in the labor market affect unemployment and its cyclical behavior. Since our focus is on financial frictions, we ignore this literature here and assume that the economy always operates at full employment. See Galí (2009) for a summary of recent efforts to introduce labor market frictions in standard business cycle models, with a focus on monetary economies.

[4] Profit maximization implies that firms hire up to the point in which their marginal product equals the wage: $l_{it+1} = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} \cdot k_{it+1}$. Imposing the full-employment condition leads to Equation (7).

parameters that determine the outcome in this market: (i) the smaller is ε, the smaller is the wealth of efficient investors and the larger is the volume of trade that is required to reach efficiency; and (ii) the smaller is ϕ, the larger is the financial friction and the more difficult it is to implement this trade. We shall assume throughout that:

Assumption 1 (binding financial friction): $\phi \cdot \pi_t < 1 - \varepsilon$ *in all dates and states of nature.*

This restriction ensures that the credit constraint is binding and efficient firms do not have enough net worth to absorb all the savings of the economy. Imposing the market-clearing condition: $\int_{i \in I} f_{it} = 0$, we find that:

$$R_{t+1} = R^f_{t+1} = \alpha \cdot k^{\alpha-1}_{t+1} \tag{8}$$

$$m_t = \frac{\phi \cdot \pi_t}{1 - \phi \cdot \pi_t} \cdot \varepsilon \cdot (1 - \alpha) \cdot k^{\alpha}_t \tag{9}$$

where R_{t+1} is the interest rate and m_t is the volume of intermediation, i.e. $m_t \equiv 0.5 \cdot \int_{i \in I} |f_{it}|$. Equation (8) says that the interest rate equals the return to investing in those firms. Moreover, risk-neutrality implies that the interest rate must also equal the expected cost of financing a firm. Since the net worth is predictable as of time t, the cost of financing need not be contingent on any variable. Equation (9) shows the amount of intermediation that the economy does. Despite the optimal design of financial contracts, there is a loss of efficiency associated to the presence of credit market frictions since some of the investment is done using second-best technology.

In this economy, there are two state variables, π_t and k_t. The former is exogenous, i.e. the "shock"; the latter is determined endogenously. Combining Equations (3), (4), (7) and (9), we find the law of motion for the aggregate capital stock:

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_t}\right] \cdot (1 - \alpha) \cdot k^{\alpha}_t \tag{10}$$

Equation (10) says that the capital stock in period $t+1$ equals the amount or quantity of investment, i.e. $(1 - \alpha) \cdot k^{\alpha}_t$, times its average efficiency or quality, i.e. $1 + \dfrac{(\pi_t - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_t}$. We discuss each of these pieces in turn.

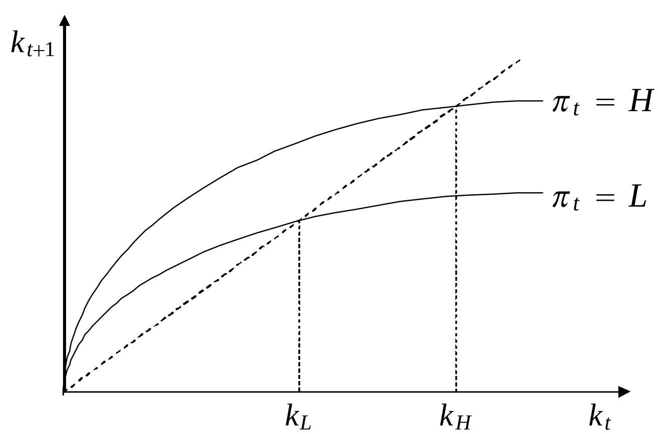
The amount of investment results from two crude assumptions: (i) since capital fully depreciates in production, the investment of this economy equals its wealth; and (ii) since the young save all their income and the old dissave all of theirs, the wealth of this economy equals its labor income. Therefore, it follows from Equation (7) that investment equals $(1 - \alpha) \cdot k^{\alpha}_t$.

The average efficiency of investment depends on the available technology and also the ability of financial markets to allocate funds to efficient firms. An improvement in technology affects the efficiency of investment through two channels. The direct channel is that, for a given allocation of investment, an increase in π_t raises efficiency. The indirect channel is through its expected value since the higher is π_t the higher are future expected profits of efficient firms relative to those of inefficient ones, relaxing the credit constraint and raising intermediation. This is the financial-accelerator mechanism.

We are finally ready to use this model in the way that Gertler and Kiyotaki suggest, namely, as a framework to help organize our thinking about the current crisis.

3 Thinking about the current crisis with the canonical model

Gertler and Kiyotaki view the current crisis as the result of a negative shock to the quality of capital or investment efficiency. To explore this view, we shall be concrete and assume that this variable can take two values: $\pi_t \in \{\pi_H, \pi_L\}$, with $\pi_H > \pi_L$. Figure 1 describes the dynamics of the model, under this assumption. There are two laws of motion, labeled "Expansions" and "Recessions" that correspond to $\pi_t = H$ and $\pi_t = L$, respectively. From any initial condition, the capital stock converges to the interval $[k_L, k_H]$.[5] Once this interval is reached the economy fluctuates forever within it. During expansions, investment efficiency is high and the economy experiences positive growth as it converges towards k_H. During recessions, investment efficiency is low and the economy experiences negative growth as it converges towards k_L.



[5]These are defined as follows: $k_j = \left[(1-\alpha)\cdot\left(1+\frac{(\pi_j - 1)\cdot\varepsilon}{1-\phi\cdot\pi_j}\right)\right]^{\frac{1}{1-\alpha}}$ for $j = L, H$.

A model-based narrative of the crisis would start as follows: sometime in the mid 1990s the world economy entered into a expansionary phase. Firms suddenly faced unusually good investment opportunities (high π) and this created a virtuous investment cycle. Their net worth was high and the best ones could borrow against future profits. Intermediation grew and this further raised the efficiency at which the world economy was operating. As the good investment opportunities were realized, the world economy was able to afford higher consumption and simultaneously increase investment further, starting the cycle again. During this virtuous cycle, the economy rapidly moved towards a steady state characterized by a high capital stock and consumption.

This model-based narrative would continue as follows: sometime in the middle of 2007, the world economy entered a recessionary phase. Firms suddenly seemed to have lost their good investment opportunities (low π) and this started a perverse investment cycle. Net worth is low and the best firms cannot borrow against future profits. Intermediation has collapsed and this has severely reduced the efficiency at which the world economy is operating. As a result, consumption is low and investment has fallen even further. As a result of this perverse cycle, the world economy is rapidly moving towards a new steady state characterized by a lower capital stock and consumption.

This narrative has some appealing aspects to it. It is clear that intermediation has collapsed, and that this is due to a drop in the net worth of firms. But we are skeptical that a research strategy based on this view will eventually deliver an accurate description of the current crisis and help us design the policies that are needed to cope with it. The main reason for this skepticism, of course, is that we do not see what is the specific technological shock that could have caused such a large change in the investment opportunities faced by firms.

One possible answer is that financial frictions have become more severe. That is, perhaps we should model the crisis as a drop in ϕ. Technically, transforming ϕ from a 'parameter' into a 'shock' does not entail great difficulty. The perceptive reader will notice immediately that the Equations of the model can be appropriately modified to deal with this change by replacing ϕ with $E_t\phi_{t+1}$. Nothing fundamental changes, since what matters is the share of future profits that are pledgeable and constitute net worth. Fluctuations in ϕ simply provide a source of fluctuation in the 'ex-post' cost of financing, i.e. now R^f_{t+1} will be made contingent on both ϕ_{t+1}, in such a way that $R_{t+1} = E_t R^f_{t+1}$. Assume, for instance, that this variable can take two values: $\phi_t \in \{\phi_H, \phi_L\}$, with $\phi_H > \phi_L$; and exhibits persistence: $\Pr\left(\phi_{t+1} \neq \phi_t\right) = p < 0.5$. Then, we can re-interpret the expansions and recessions in Figure 1 as shocks to the financial friction rather than to productivity.

According to this view, the expansion was driven by a shock to the quality of the financial system (high ϕ) that improved creditors ability to extract payments from debtors. This raised intermediation and the economy's average efficiency, starting the virtuous cycle that moved us towards the high steady state. But a few bankruptcies showed that the situation had changed, and that the ability of creditors to extract payments from debtors had declined again. The recession was the consequence of this (low ϕ). Intermediation collapsed, reducing the average efficiency of investment and starting the perverse cycle that is moving us towards the low steady state.

This view of the crisis as a drop in ϕ has led many to argue that a full understanding of the crisis is not going to come from macroeconomics alone. In macroeconomic models, ϕ is a very crude stand-in for a very rich and complex set of frictions that impede financial markets to carry out all useful intermediation. If we want to truly understand the current crisis, the argument goes, we need to go back to the microeconomic level and find out what is wrong with the financial system. That is, we need to stop using ϕ as a parameter or a shock and instead develop theories of it. We strongly endorse this strategy as an avenue to make progress. But we are also skeptical that this strategy will shed much light on the cause of the current crisis and the policies needed to handle it. The reason is that we find it difficult to see what is the specific change in the institutional and/or technological framework of financial markets that has so suddenly left them so impaired to do their job. Surely, some banks have failed and some investments (mostly real-state related) turned out to be much worse than expected. But this is not the first time it happens, and everything else looks pretty much the same as before the crisis. What is different now?

In the remainder of this paper, we explore an alternative view that focuses on a piece of the net worth of firms that has been overlooked by standard macroeconomic models. These models always assume that the value of a firm is the net present value of profits and that its collateralizable net worth is the fraction of these profits that can be pledged today. But often, these same macroeconomic models have equilibria in which firms are valued above the present value of their profits. We shall explore these equilibria and model cycles as transitions from fundamental to bubbly equilibria and vice versa. This change in perspective has profound implications for our understanding of the current crisis and the correct policies to handle them. To show this, we need to go back to theory and introduce bubbles into the canonical model.

4 Bubbles as pyramid schemes

Up to this point, we have not even discussed the life of a firm as a separate entity from its manager. In the canonical model, there is nothing wrong in assuming that firms last only one period and close down when the individual that builds them dies. After all, once production has taken place and all profits have been distributed, the capital has depreciated and there is nothing left in the firm. It does not make any difference whether the young purchase old firms at prize zero or, alternatively, they create new firms.

Perhaps it will come as a surprise to some readers that the canonical model has many other equilibria in which this indifference result breaks down, and individuals are willing to purchase old firms at a positive price even if these firms contain neither output nor capital. These equilibria capture the notion of firms being overvalued or having a bubble. Individuals are willing to buy firms that contain a bubble if their price grows fast enough. That is, bubbles are pyramid schemes by which individuals buy overvalued firms in the hope of selling them later at an even higher price. Somewhat paradoxically, we shall see that the presence of overvalued or bubbly firms will help the economy work better.

To study this phenomenon, we need additional notation. Let b_{it} be the overvaluation of the firm managed by individual i at time t. The presence of this overvaluation does not affect technology (i.e. Equations (1), (2), (3) still apply), but it does change the resources available today and tomorrow. In particular, we need to replace Equations (4) and (5) by the following generalizations:

$$I_{it} = \begin{cases} w_t + f_{it} - b_{it} & \text{if the firm is old or purchased} \\ w_t + f_{it} & \text{if the firm is new or newly created} \end{cases} \tag{11}$$

$$R^f_{t+1} \cdot f_{it} \leq \phi \cdot \left[F\left(l_{it+1}, k_{it+1}\right) - w_{t+1} \cdot l_{it+1}\right] + E_t b_{it+1} \tag{12}$$

Equations (11) and (12 show the two key effects of overvaluations or bubbles: (i) if the firm is currently overvalued, i.e. $b_{it} > 0$, there are less funds available for investment because the individual must use some of his/her internal and/or external funds to purchase the firm; and (ii) if the firm is expected to be overvalued in the future, i.e. $E_t b_{it+1}$, the firm's net worth increases and this allows it to borrow more today. Note that we have assumed that the entire future price of the firm is pledgeable, instead of only a fraction ϕ as is the case with profits. We make this assumption because we think it is more natural. After all, the sale price does not seem to be subject to the usual agency problems that reduce profits and motivate the parameter ϕ. In any case, none of the results that follow would change if we assumed instead that only a fraction ϕ of the future sale price can be pledged to creditors.

With this notation at hand, we can write down the value of a firm managed by individual i as follows:

$$E_t V_{it+1} = \max_{\langle l_{it+1}, k_{it+1}, f_{it} \rangle} E_t \left\{ F(l_{it}, k_{it}) - w_{t+1} \cdot l_{it+1} - R^f_{t+1} \cdot f_{it} \right\} + E_t b_{it+1} \tag{13}$$

$$s.t. \text{ Equations (1), (2), (3), (11) and (12)}$$

Before solving the model, we shall introduce some structure to the types of overvaluations or bubbles that we shall consider:[6]

Assumption 2 (bubble dynamics): *Let z_t be investor confidence. Then, the economy fluctuates between two states $z_t \in \{F, B\}$ with transition probabilities:* $\Pr(z_{t+1} = B | z_t = F) = q$ *and* $\Pr(z_{t+1} = B | z_t = F) = p$. *If $z_t = F$, then*

$$b_{it} = 0 \quad \text{and} \quad \frac{E_t b_{it+1}}{R_{t+1}} = \begin{cases} q \cdot \delta \cdot (1-\alpha) \cdot k_t^{\alpha} & \text{if the firm is managed by } i \in I_t^L \\ 0 & \text{if the firm is managed by } i \notin I_t^L \end{cases}$$

where δ is assumed to be small. *If $z_t = B$, then*

$$b_{it} = b_t \quad \text{and} \quad E_t b_{it+1} = \begin{cases} E_t b_{t+1} & \text{if the firm is old or the firm is new and managed by } i \in I_t^L \\ 0 & \text{if the firm is new and is managed by } i \notin I_t^L \end{cases}.$$

The set $I_t^L \subseteq I_t$ contains $n_E \in [0, \varepsilon]$ and $n_I \in [0, 1-\varepsilon]$ measures of efficient and inefficient investors.

Assumption 2 says that investor sentiment fluctuates between pessimism, i.e. $z_t = F$; and optimism, i.e. $z_t = B$. We shall say that a bubble pops up or a bubbly episode starts in period t if $z_t = B$ and $z_{t-1} = F$. We shall say that a bubble bursts or a bubbly episode ends in period t if $z_t = F$ and $z_{t-1} = B$. When investors are pessimistic, firms have no bubble. There is

[6] This is not without loss of generality. In particular, Assumption 2 imposes strong symmetry in many dimensions. The model has many other equilibria that we disregard here.

however a probability q that a change in investor sentiment takes place and a small bubble starts, i.e. proportional to δ. When investors are optimistic, old bubbles are passed across generations. Moreover, new bubbles appear each period. Each generation contains a subset of lucky individuals that are able to create new bubbles. The rest of the generation cannot do so. Note that our assumptions ensure that b_t is the aggregate bubble, i.e. $b_t \equiv \int_{i \in I} b_{it}$.

Once we allow for overvalued firms or bubbles, individuals need to make one additional choice. Namely, whether to purchase an old firm or to create a new one. It is clear that all lucky individuals prefer to start new firms, since those firms offer them the same expected bubble tomorrow at zero price today. Unlucky individuals will be indifferent between starting a new firm or buying an old one if and only if:

$$(1 - n_E - n_I) \cdot \frac{E_t b_{t+1}}{b_t} = R_{t+1} \tag{14}$$

Equation (14) equates the return to holding a bubble to the interest rate, which is the cost of an additional unit of financing. The left-hand side describes the expected return to holding a bubble between period t and period $t+1$. This return is the increase in price. Since $n_E + n_I$ new bubbles are created in period $t+1$, this return falls short of the growth rate of the aggregate bubble. The right-hand side is the interest rate, which is also the expected return to external funds. At first sight, it might be surprising that this condition for indifference between purchasing bubbles or reducing external financing does not depend on whether the firm is credit constrained or not. But this turns out to be the case, since future bubbles can serve as a collateral firms only care about their net present value.[7]

We are ready now to describe the equilibria of this economy. Once again, this implies solving for the equilibrium in labor and financial markets. The presence of bubbles does not affect the functioning of the labor market and the wage is still given by Equation (7). The presence of bubbles does affect the functioning of the financial market, though. As before, we restrict our attention to the case in which the financial system is unable to do all the useful intermediation and inefficient investments are made. This case now requires that:

$$\phi \cdot \pi_t \cdot [(1-\alpha) \cdot k_t^{\alpha} - b_t] + \frac{n_E \cdot E_t b_{t+1}}{R_{t+1}} < (1-\varepsilon) \cdot (1-\alpha) \cdot k_t^{\alpha} - b_t \tag{15}$$

Without bubbles, Assumption 1 would ensure that this condition is always satisfied. But bubbles

[7] This would not be the case if ϕ applied to the expected sale price in Equation (12). In this case, constrained firms would have a higher threshold to buy the bubble. This is an interesting complication that we disregard here.

make this condition more stringent as they crowds out inefficient investments for two reasons: (i) the purchase of overvalued firms directly reduces the funds available for investing in inefficient investments (this is akin to an increase in ε); and (ii) the bubble increases the net worth of efficient firms and the additional credit given to these firms further reduces the funds available for investing in inefficient investments (this is akin to an increase in ϕ). As the bubble increases, inefficient investments decline and eventually disappear.

We shall show later that Assumption 1 and our assumption that the initial bubble is small (i.e. of size δ) jointly ensure that Condition (15) holds. As a result, the credit constraint is binding and efficient firms do not have enough net worth to absorb all the savings of the economy. As a result, the interest rate equals the return to investing in inefficient firms and intermediation is less than optimal:

$$R_{t+1} = E_t R^f_{t+1} = \alpha \cdot k_{t+1}^{\alpha-1} \tag{16}$$

$$m_t = \frac{\phi \cdot \pi_t}{1 - \phi \cdot \pi_t} \cdot \varepsilon \cdot (1-\alpha) \cdot k_t^{\alpha} + \frac{1}{1 - \phi \cdot \pi_t} \cdot \frac{n_E \cdot E_t b_{t+1}}{\alpha \cdot k_{t+1}^{\alpha-1}} \tag{17}$$

Comparing Equations (8) and (16) we find that the interest rate has not changed, but now it is the expected cost of financing that equals the interest rate. The reason, of course, is that it is optimal to make the cost of financing contingent on the future bubble so that the credit constraint is binding in all states of nature. Comparing Equations (9) and (17), we find that the expected bubble constitutes net worth and this allows for more intermediation. This is captured by the second term in the right-hand side of Equation (17). Despite the optimality of contracts and the additional net worth, remember that Condition (15) applies and this means that some inefficient firms invest.

Straightforward algebra allows us to write down the law of motion of the aggregate capital stock as follows:

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_t}\right] \cdot (1-\alpha) \cdot k_t^{\alpha} + \frac{\pi_t - 1}{1 - \phi \cdot \pi_t} \cdot \frac{n_E \cdot E_t b_{t+1}}{\alpha \cdot k_{t+1}^{\alpha-1}} - b_t \tag{18}$$

A comparison of Equations (10) and (18) illustrates that, in principle, the effect of bubbles on capital accumulation is ambiguous. The last term of Equation (18) shows that purchasing the existing bubble reduces capital accumulation by diverting resources away from investment. Since only inefficient investors purchase the bubble and their investment efficiency is one, the existing bubble crowds out capital one-to-one. The second term of Equation (18) shows that the expected bubble expands capital accumulation by relaxing credit constraints, increasing intermediation and the average efficiency of investment. To understand this term, note that the expected bubble raises the net worth of efficient investors by $\frac{n_E \cdot E_t b_{t+1}}{\alpha \cdot k_{t+1}^{\alpha-1}}$, which enables them to expand borrowing by a factor of $\frac{1}{1-\phi \cdot \pi_t}$, and each unit borrowed entails an efficiency gain of $\pi_t - 1$.

A difference between Equations (10) and (18) is that the former provides a complete description of the dynamics of the economy as a function of productivity shocks; while the latter does not. To obtain a complete description of the dynamics of the economy, we need to characterize the behavior of the aggregate bubble b_t. We turn to this task next.

5 Back to the crisis

We shall now explore the possibility that the current crisis was caused by a change in investor sentiment rather than a shock to productivity. To provide a characterization of bubbly episodes in the model, we shall exploit its nice recursive structure. Define x_t as the bubble's share of wealth or savings, i.e. $x_t \equiv \frac{b_t}{(1-\alpha) \cdot k_t^{\alpha}}$. Then, combining Equations (14), (16) and (18), we find that:

$$x_t = \begin{cases} 0 & \text{if } z_t = F \\ \dfrac{(1-p)^{-1} \cdot (1-n_E-n_I)^{-1} \cdot \alpha \cdot x_{t-1}}{(1-\alpha) \cdot \left[1 - x_{t-1} + \dfrac{\pi_t - 1}{1-\phi \cdot \pi_t} \cdot \left(\varepsilon + \dfrac{n_E}{1-n_E-n_I} \cdot x_{t-1}\right)\right]} & \text{if } z_t = B \end{cases} \tag{19}$$

with the additional condition that $x_t = x_{0,t}$ if $z_t = B$ and $z_{t-1} = F$.[8] Equation (19) provides a full characterization of the set of bubbly episodes that are feasible under Assumptions 1 and 2. Naturally, in the derivation of Equation (19) we have assumed that Condition (15) holds, and we will need to check that this is the case. In particular, we will need to make sure that the bubble does not become larger than:

$$\bar{x}_t \equiv \frac{1-\varepsilon-\phi \cdot \pi_t}{\dfrac{n_E}{1-n_E-n_I} + 1 - \phi \cdot \pi_t} \leq 1 \tag{20}$$

[8] The initial size of the bubble is as follows:

$$x_{0,t} \equiv \frac{(n_E + n_I) \cdot \delta \cdot \alpha}{(1-\alpha) \cdot \left[1 + \dfrac{\pi_{t-1} - 1}{1-\phi \cdot \pi_{t-1}} \cdot (\varepsilon + q \cdot n_E \cdot \delta)\right]}$$

where $\bar{x}_t$ is the largest possible bubble that is consistent with Condition (15). With the equilibrium path of the bubble at hand, we can then compute the dynamics of the capital stock as follows:

$$k_{t+1} = \begin{cases} \left[1 + \dfrac{\pi_t - 1}{1 - \phi \cdot \pi_t} \cdot (\varepsilon + n_E \cdot q \cdot \delta)\right] \cdot (1-\alpha) \cdot k_t^{\alpha} & \text{if } z_t = F \\ \\ \left[1 - x_t + \dfrac{\pi_t - 1}{1 - \phi \cdot \pi_t} \cdot \left(\varepsilon + \dfrac{n_E}{1 - n_E - n_I} \cdot x_t\right)\right] \cdot (1-\alpha) \cdot k_t^{\alpha} & \text{if } z_t = B \end{cases} \tag{21}$$

Equation (21) describes the law of motion of the capital stock as a function of the path of the bubble. There are two pieces of this law of motion corresponding to whether the economy is experiencing a bubbly episode or not. Interestingly, we find that even when there is no bubbly episode the world economy intermediates more than in the canonical model. The reason is that the possibility that a bubbly period arises in the future increases the net worth of firms and allows them to borrow more. The presence of a bubble will of course change the behavior of the economy, as discussed after Equation (18). In particular, we saw that the effects of the bubble on capital accumulation were ambiguous. We find now that bubbly episodes are expansionary if $\dfrac{\pi_t - 1}{1 - \phi \cdot \pi_t} \cdot \dfrac{n_E}{1 - n_E - n_I} > 1$, and contractionary otherwise.

Equations (19) and (21) provide a full characterization of the dynamics of the world economy. We define the bubbleless economy as the special case in which bubbly episodes never occur, i.e. $q = 0$ and the economy starts in period 0 with $z_0 = F$. Thus, the bubbleless economy is nothing but the canonical model of Section 2.

We are ready to study the bubble episodes that can take place in this world economy. In all of them two conditions must concur:

1. The bubble must be expected to grow fast enough. Otherwise, holding the bubble would not be attractive and nobody would purchase it. This requirement is embodied in Equation (19), which is nothing but a transformation of Equation (14), i.e. the first-order condition of unlucky individuals to hold the bubble.

2. The bubble cannot be expected to grow too fast. Otherwise, it would eventually exceed available funds and it could not be purchased. Knowing this, standard backward induction arguments would rule out the bubble today. This means that any proposed path for the bubble must be such that $x_t \leq 1$ in all dates and states of nature.

Thus, any path of x_t that satisfies Equation (19) and ensures that $x_t \leq \bar{x}_t \, (< 1)$ in all dates and states of nature is an equilibrium under Assumptions 1 and 2. It turns out that this simple model can give rise to a wide array of equilibrium dynamics with bubbly episodes of different sorts. While a full treatment of these episodes lies outside the scope of this paper, we can however say that there are two types that are feasible.[9]

The first type is the conventional or contractionary bubbly episode emphasized by Tirole (1985). Episodes of this type take place in economies in which investments are dynamically inefficient. Bubbles raise the interest rate and reduce the capital stock. An important feature of these episodes is how they unfold. Each episode starts with a large bubble that slowly disappears over time. Therefore, conventional bubbly episodes start with a bang and go away quietly.

The second type of bubbly episode that is feasible is the non-conventional or expansionary one analyzed by Martin and Ventura (2010). Episodes of this type arise in economies with financial frictions, and exist even if investments are dynamically efficient. These bubbles reduce the interest rate and increase the capital stock. Moreover, they look quite different from the conventional ones. In particular, episodes start with a small bubble that gains momentum over time. These bubbles can become very large before suddenly bursting. Thus, non-conventional bubbles start quietly and end up with a bang.

Non-conventional bubbles are the only ones that stand a chance to be empirically relevant in the present situation. Therefore, we rule out conventional bubbles by assuming that the economy is dynamically efficient:[10]

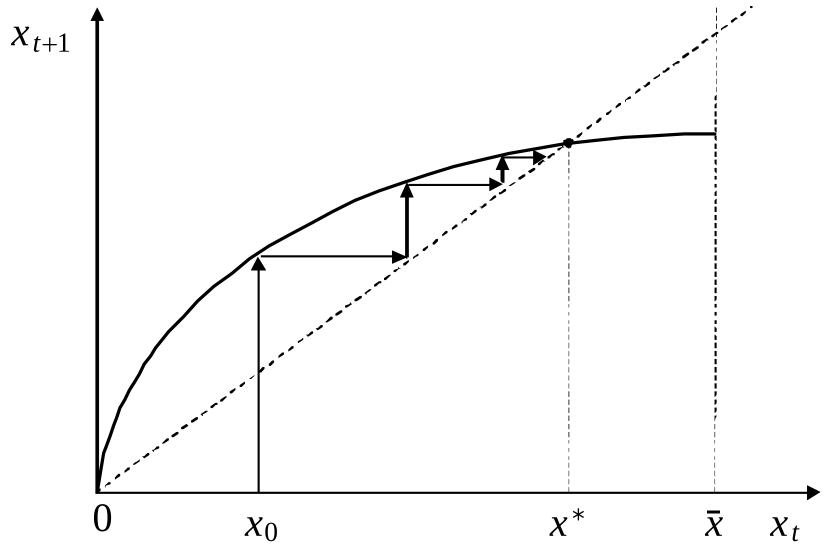
Assumption 3 (dynamic efficiency): $1+\dfrac{(\pi_t-1)\cdot\varepsilon}{1-\phi\cdot\pi_t}<\dfrac{\alpha}{1-\alpha}$ in all dates and states.

Figure 2 shows the dynamics of x_t during a bubbly episode, i.e. conditional on $z_t = B$. This Figure has been drawn under the assumption that π_t is constant. There are two stationary equilibria, $x = 0$ and $x = x^*$. Any initial bubble between 0 and $\bar{x}$, converges to x^*. The figure shows the path of a small bubble that starts with size x_0. Under our assumptions, all bubbly episodes look like those in Figure 2.[11]

[9] The interested reader is referred to Martin and Ventura (2010) who provide a full analysis of the set of equilibria for the special case in which $\phi = n_I = 0$ and $\pi_t = \pi$ for all dates and states of nature.

[10] Assumption 3 ensures that the interest rate of the bubbleless economy is always above the long-run rate of growth. Since the return to investments carried out by inefficient firms equal the interest rate, even these investments are dynamically efficient. This rules out conventional bubbles and allows us to focus exclusively on non-conventional ones.

[11] To see this, note that Assumption 3 implies that the slope of the map is higher than one at the origin. For a bubble to exist, the map must therefore be concave. It is immediate to see that we can find many bubbles (i.e. combinations of p, n_E and n_I) that ensure that the map is concave and crosses the 45 degree line as depicted. The



We can now use the model to describe a bubble episode and provide a narrative of the crisis. It goes as follows: sometime in the mid 1990s the world economy entered a bubble episode and started an expansionary phase. There was a small overvaluation of firms (i.e. δ) that kept growing over time (i.e. n_E and n_I). This bubble had two positive effects on the macroeconomy. The first one was a positive wealth shock or transfer from the future. This is a central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, lucky individuals assigned themselves and sold the "rights" to the savings of a generation living in the very far future or, to be more exact, living at infinity. This appropriation of rights was a pure windfall or wealth gain for the lucky individuals that could start and sell overvalued firms.

A second positive effect of the bubble was an efficiency gain. The bubble increased net worth allowing efficient firms to borrow and invest more. In a very real sense, the bubble was the oil

set of all such bubbles constitutes the set of feasible bubbles, given our assumptions. We can characterize this set as those combinations of p, n_E and n_I that ensure that the following inequalities hold:

$$0 < x_t^* \equiv \frac{\dfrac{\alpha}{(1-p)\cdot(1-n_E-n_I)\cdot(1-\alpha)} - 1 - \dfrac{(\pi_t - 1)\cdot\varepsilon}{1-\phi\cdot\pi_t}}{\dfrac{\pi_t - 1}{1-\phi\cdot\pi_t}\cdot\dfrac{n_E}{1-n_E-n_I} - 1} \leq \bar{x}_t$$

Note that all the bubbles in this set must be expansionary. Since Assumption 3 implies that the numerator is always positive, the denominator must also be positive. Thus, it must be the case that $\dfrac{\pi_t - 1}{1-\phi\cdot\pi_t}\cdot\dfrac{n_E}{1-n_E-n_I} > 1$ and this is nothing but the condition for bubbles to be expansionary, as discussed after Equation (21).

that greased financial markets during this period. The rights to the future generated by the bubble provided the collateralizable net worth that financial markets needed to work efficiently. The result was that efficient investments displaced inefficient ones and the world economy became more efficient.

As the world economy started growing, so did the bubble and its two effects: increased wealth and efficiency gains. On the macroeconomic front, the capital stock increased and the interest rate fell, both fueling the demand for the bubble. New bubbles arose, further raising net worth and leading to even more intermediation, increased efficiency and growth. The world economy was rapidly moving towards a steady state with a high capital stock and consumption.

Unfortunately, some time in the middle of 2007 there was a change in investor sentiment and the world economy entered a recessionary phase. The bursting of the bubble was akin to losing the rights to the future all at once. All the wealth and efficiency gains accumulated over the expansionary phase suddenly evaporated. Those holding the bubble found themselves unable to sell it and experienced a dramatic wealth loss. Net worth collapsed and so did intermediation. Inefficient investments are now expanding at the expense of efficient ones. This perverse cycle has put the world economy on a path towards a new steady state with a low capital stock and consumption.

As a research strategy, viewing the current crisis as the collapse of a bubble episode is more attractive than alternatives that rely on productivity shocks in either the corporate (i.e. π_t) or the financial (i.e. ϕ_t) sectors. The main reason for this is that the model-based narrative of the crisis as a bubble episode seems to better conform with the facts. To start with, it does not require us to identify a large productivity shock (or amplify an unobservable one) to find a culprit for the current state of the world economy. In addition, it naturally explains why the expansionary phase was gradual and protracted, while the recessionary phase has been sudden and sharp. Furthermore, this view of the crisis helps us better understand the connection between financial and real economic activity, and the blazing speed with which the current crisis has propagated across the globe. We deal with each of these two issues in turn.

6 The connection between financial and real activity

As mentioned already, one of the most striking aspects of the world economy in recent years has been the massive growth in the financial sector during the expansion and the dramatic collapse that has followed since the outbreak of the crisis. The model can shed light on the determinants of financial and real activity and the connection between them. To see this, remember that funds are intermediated through the use of financial contracts. These contracts are optimal in the sense that they enable firms to borrow against all of their collateralizable net worth, specifying contingent payments if necessary. As a result, one can interpret these contracts either as risky debt, equity, or both. This means that m_t, which we have defined as the total amount of credit in our economy, is also the total market value of corporate debt and equity. It is therefore the most natural measure of financial activity. The most natural measure of real activity, in turn, is given by the market value of production which in the model equals k_t^{α}.

Combining Equations (17) and (18) together with Assumption 2, we obtain the following expression for the ratio of financial to real activity:

$$\frac{m_t}{k_t^{\alpha}} = \begin{cases} \left(\dfrac{\phi \cdot \pi_t}{1 - \phi \cdot \pi_t} \cdot \varepsilon + \dfrac{1}{1 - \phi \cdot \pi_t} \cdot n_E \cdot q \cdot \delta \right) \cdot (1 - \alpha) & \text{if } z_t = F \\ \\ \left(\dfrac{\phi \cdot \pi_t}{1 - \phi \cdot \pi_t} \cdot \varepsilon + \dfrac{1}{1 - \phi \cdot \pi_t} \cdot \dfrac{n_E}{1 - n_E - n_I} \cdot x_t \right) \cdot (1 - \alpha) & \text{if } z_t = B \end{cases} . \qquad (22)$$

Equation (22) breaks down this ratio in two components. The first term inside the brackets is the fundamental component, i.e. the part of the market value of assets that represents the net present value of dividends. The second term inside the brackets is the bubbly component, i.e. the part of the market value of assets that correspond to bubbles. Note that this second component is also positive, albeit small, even if the economy is in the fundamental state. We can use now this equation to compare the predictions of the canonical model of Section 2 and the bubbly economy of Section 4.[12]

In the canonical or bubbleless economy, asset values always equal their fundamental component. In this case, the ratio of financial to real activity increases with positive productivity shocks (i.e. increases in π) and with shocks that reduce the severity of the financial friction (i.e. increases in ϕ). Both types of shocks increase collateralizable net worth, allowing firms to issue more assets per unit of output. Note that the ratio of the financial to real activity inherits the properties of the

[12] To provide some perspective on the discussion that follows, we note that in the RBC model all useful intermediation takes place. In our setup, this implies:

$$\frac{m_t}{k_t^{\alpha}} = (1 - \varepsilon) \cdot (1 - \alpha),$$

so that financial and real activity move one-to-one.

underlying shocks. For instance, if the productivity shock follows the two-state process described in Section 2, m_t/k_t^α increases at the onset of the expansion and stays high throughout, only collapsing when the downturn begins.

The ratio of financial to real activity in the bubbleless economy may thus be volatile due to the presence of financial frictions. But it will always mirror the evolution of fundamentals. Absent any other ingredients, such a model cannot account for asset prices that fluctuate in ways that seem unrelated to fundamentals. And yet, these fluctuations in the stock and housing markets have been so large and unpredictable in recent years that it seems hard to think of the underlying fundamental shocks that caused them. Although these events are perhaps too close to us to draw any definitive conclusions in this regard, the recent past has given us a similar experience in the form of the asset boom and bust of the late 1990s. This episode, which has been widely studied, seems hard to attribute to movements in fundamentals.[13]

In a bubbly economy, asset values are always higher as they also include the bubble component. This exacerbates the sensitivity of the ratio of financial to real activity to changes in productivity, as these shocks now also affect the bubbly component of asset values. In addition, and due to changes in the bubble component, the ratio of financial to real activity fluctuates even in the absence of productivity shocks. Changes in investor sentiment cause the economy to transition between bubbly and fundamental states, inflating and deflating the value of assets in the process. But even if investor sentiment is high and the economy remains in a bubbly state, the equilibrium bubble may expand or contract over time dragging the value of assets with it.

The introduction of bubbles and investor sentiment therefore increases the volatility of asset values. But this is not all, since it does so in a very particular and appealing way. In the equilibrium based on Assumption 2, bubbly episodes start with very small departures of asset values from their fundamental component: throughout the bubbly episode, this departure becomes gradually larger over time until its abrupt disappearance at the time of the bubble bursting. This implies that asset values behave asymmetrically, rising slowly during the bubbly episode but falling sharply when the crisis hits. This behavior is reminiscent of many real-world asset markets. In the bubbleless economy, such a behavior could only arise in the unlikely event that the fundamentals themselves displayed such an asymmetry, improving gradually but deteriorating sharply to bring about a crisis.

[13]For a detailed discussion on this last point, see LeRoy (2004).

7 How are crises transmitted across countries?

[THIS SECTION IS VERY PRELIMINARY] A difference between the canonical and bubbly economy is how shocks are transmitted. to see this, assume the world has two identical countries, the US and ROW. Each of them is perfectly described by the model above. We shall assume that labor markets are local, but there is financial integration. This means that R_t is the same in both countries.

Consider first the canonical model. Now there are four states of nature: In one state both countries have π_L; in another state both countries have π_H; and in the remaining states one country has π_L and the other has π_L. If the current crisis is modelled as a global shock, i.e. both countries moving simultaneously from π_H to π_L. everything is as in the previous sections. Moreover, the net foreign asset positions remain zero throughout. The interesting case here is that in which initially the both countries have π_H and suddenly one of them, say the US, moves to π_L. We assume that, when one of the countries is in π_L, not all useful intermediation takes place and some inefficient firms invest. This means that:

$$R_{t+1} = \alpha \cdot k_{US,t+1}^{\alpha-1} = \alpha \cdot k_{ROW,t+1}^{\alpha-1} \tag{23}$$

We can now write the laws of motion of the capital stock as follows:

$$k_{US,t+1} = \left[1 + \frac{(\pi_{US,t} - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_{US,t}}\right] \cdot (1-\alpha) \cdot k_{US,t}^{\alpha} - nfa_t \tag{24}$$

$$k_{ROW,t+1} = \left[1 + \frac{(\pi_{ROW,t} - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_{ROW,t}}\right] \cdot (1-\alpha) \cdot k_{ROW,t}^{\alpha} + nfa_t \tag{25}$$

where nfa_t is the net foreign asset position of US. Combining these Equations, we find that:

$$k_{ROW,t} = k_{US,t} = k_t \tag{26}$$

$$nfa_t = \frac{1-\alpha}{2} \cdot k_t^{\alpha} \cdot \left(\frac{(\pi_{US,t} - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_{US,t}} - \frac{(\pi_{ROW,t} - 1) \cdot \varepsilon}{1 - \phi \cdot \pi_{ROW,t}}\right) \tag{27}$$

There is an interesting aspect to this theory that is currently unappreciated. We have that the country with the negative technology shock runs a deficit! Why is this so? A reduction in productivity lowers the capital stock and raises the interest rate. The shock is negatively transmitted through the interest rate and both countries experience a reduction in the capitals stock. The channel of transmission is the interest rate.

Consider next the bubbly economy. Once again there are four states of nature: in one state both countries have 0; in another state both countries have x_H; and in the remaining states one country has 0 and the other has x_H. If the current crisis is modelled as a global shock, i.e. both countries moving simultaneously from x_H to 0 everything is as in the previous sections. Moreover, the net foreign asset positions remain zero throughout. The interesting case here is that in which initially the both countries have x_H and suddenly one of them, say the US, moves to 0. We assume that, when one of the countries is in 0, not all useful intermediation takes place and some inefficient firms invest. This means that Equations (23) and (26) still hold but the laws of motion of the capital stock are different:

$$k_{US,t+1} = \left[1 - x_{US,t} + \frac{\pi - 1}{1 - \phi \cdot \pi} \cdot \left(\varepsilon + \frac{n_E}{1 - n_E - n_I} \cdot x_{US,t}\right)\right] \cdot (1 - \alpha) \cdot k_{US,t}^{\alpha} - nfa_t \tag{28}$$

$$k_{ROW,t+1} = \left[1 - x_{ROIW,t} + \frac{\pi - 1}{1 - \phi \cdot \pi} \cdot \left(\varepsilon + \frac{n_E}{1 - n_E - n_I} \cdot x_{ROW,t}\right)\right] \cdot (1 - \alpha) \cdot k_{ROW,t}^{\alpha} + nfa_t \tag{29}$$

and we get:

$$nfa_t = \frac{1 - \alpha}{2} \cdot k_t^{\alpha} \cdot \left(\frac{\pi - 1}{1 - \phi \cdot \pi} \cdot \frac{n_E}{1 - n_E - n_I} - 1\right) \cdot (x_{US,t} - x_{ROW,t}) \tag{30}$$

Once again, we have the result that the country where the bubble collapses runs a deficit! But now the impact in the world economy is much larger because the bubble in the remaining country declines in value. That is, there are two channels of transmission: (i) the classic one through the interest rate as in the canonical model, and (ii) a further channel through the interaction between the bubbles. This creates a correlation in the bubbles that amplifies the transmission of shocks.

8 How can policy help deal with the crisis?

Another crucial difference between the canonical and the bubbly economies lies in the effectiveness of economic policy. Consider first the canonical model. As we have already mentioned, a crisis in that model can only be a consequence of an adverse shock to productivity, i.e. a decline in π. The negative impact of such a decline in productivity is then amplified by tightening credit constraints, which lead to further decreases in investment and output. This is essentially the way in which Gertler and Kiyotaki frame the recent crisis within the canonical model. Their model is slightly different to ours because the borrowing frictions operate at the bank, as opposed to the entrepreneurial, level: negative productivity shocks tighten the credit constraints of banks in the interbank market, which are therefore forced to cut back funding to entrepreneurs. This modification, which is meant to illustrate the key role of banks in the recent crisis, does not affect the model's main results and implications.

But once the crisis hits, what can the government do? If the crisis is aggravated by a decline in credit and investment, it seems natural to think that policies to restore credit should be useful. One direct policy along these lines is to implement reforms that alleviate credit market frictions, thereby increasing ϕ. Since binding credit constraint increase the payoffs to such reforms in times of crises, it might be optimal for governments to undertake them precisely in these times. In practice, though, this might be difficult as ϕ reflects the quality of the legal system and other institutional arrangements that take time to adjust. We shall thus assume, as Gertler and Kiyotaki do, that ϕ is exogenously given to the government and cannot be affected by policy in the short run.

If the government cannot adjust ϕ, is there anything else it can do to increase efficiency? Gertler and Kiyotaki analyze the effects of direct government interventions in credit markets, in which the government sustains credit by acting as a lender to constrained borrowers. These interventions are reminiscent of the policies adopted by the Federal Reserve during the current crisis. In the context of the canonical model, though, such policies can only be effective if the government has some type of advantage in lending vis-a-vis private creditors, which is what Gertler and Kiyotaki assume. In our context, the government may successfully orchestrate such an intervention if it is better at collecting from creditors than the market, i.e., if it is subject to a higher ϕ. In the event that it is not, the government is unable to expand total credit because borrowers are already constrained. In this last case, the government may still adopt redistributive policies that entail taxing one group of agents (i.e., inefficient investors, future generations) in order to benefit another group (efficient investors, current generations), but no Pareto-improving interventions will be possible.

If one adopts instead the view provided by the bubbly model, the role of policy looks quite different. In that case, the crisis is brought about by a decline in investor confidence that triggers a fall in asset prices and a tightening of credit constraints. The bubble, which was useful to intermediate resources, has disappeared. This creates two problems: on the one hand, it leads to a decline in the investment of efficient individuals because their collateral has contracted. On the other hand, it leads to an expansion of investment by inefficient individuals who no longer have a bubble to purchase. But note that the government can address both of these problems through an appropriate use of government debt.

In order to see this, we slightly modify the model of the previous section to incorporate government debt and taxation. We assume that the government is able to issue one-period non-contingent bonds, and that it can tax in a lump-sum fashion. Let d_t and T_{it} respectively denote the stock of government debt and the taxes levied from individual i at time t. Using R^d_{t+1} to depict the contractual rate of interest on public bonds, the government's budget constraint can be expressed as,

$$R^d_{t+1} \cdot d_t = d_{t+1} + \int_{i \in I_{t+1}} T_{i,t+1},$$

where it is implicitly assumed that the government does not default on its debt.

Consider that the economy is in a bubbly episode. Let t_0 denote the period in which the economy transitions from a bubbly to a fundamental state, so that there is a negative shock to investor sentiment that causes the bubble to burst. This shock hurts the consumption of the old generation because they were the sole owners of the bubble. But it also hurts the young because the disappearance of the bubble (i) deprives inefficient individuals of an attractive alternative for saving, forcing them to invest in their firms, and it (ii) decreases the expected net worth of efficient firms, tightening their borrowing constraint and contracting their investment. The disappearance of the bubble, then, leads to a decline in the interest rate and in the efficiency of investment. These adverse effects will disappear once investor confidence is high again and the bubble reappears: let t_1 denote the period in which this happens putting an end to the crisis ends.

The government can offset all of these effects through an appropriate debt policy. On impact, it can effectively replace the bubble by issuing an amount $d_{t_0} = x^* \cdot (1-\alpha) \cdot k^{\alpha}_{t_0}$ of government bonds that promise an interest rate $R^d_{t_0+1} = \alpha \cdot k^{\alpha-1}_{t_0+1}$. Inefficient individuals are indifferent between buying these bonds and investing in their firms, and the proceeds from the sale can be used by the government to compensate the old generation. In this manner, all of the effects of the bursting bubble are offset except one: the fall in credit that is associated to the decline in the expected net worth of entrepreneurs. But the government can offset this too by committing to the following debt policy until investor sentiment is high again:

- In each subsequent period $t_0 < t < t_1$ the government sets:

$$\begin{aligned} d_t &= d_{t-1} \cdot \frac{\alpha \cdot k_t^{\alpha-1}}{1-n_I-n_E}, \\ T_{i,t} &= \begin{cases} -\left(d_t \cdot \frac{n_E}{\varepsilon}\right) & \text{if } i \in I_t^E \\ -\left(d_t \cdot \frac{n_I}{1-\varepsilon}\right) & \text{if } i \in I_t^I \end{cases}. \end{aligned}$$

During the crisis, the government rolls over its debt and continues to replace the disappeared bubble through the sale of public bonds. A fraction $(1 - n_I - n_E)$ of the revenues obtained from this sale is used to pay off the outstanding debt. The remaining fraction is transferred as lump-sum subsidies to the efficient and inefficient old, who respectively receive shares n_E and n_I of the revenues.

- In periods $t > t_1$, when the bubble reappears, the government sets:

$$\begin{aligned} d_t &= \max\left\{d_{t-1} \cdot \frac{\alpha \cdot k_t^{\alpha-1}}{1-n_I-n_E} - b_t, 0\right\}, \\ T_{i,t} &= \begin{cases} -\left(d_t \cdot \frac{n_E}{\varepsilon}\right) & \text{if } i \in I_t^E \\ -\left(d_t \cdot \frac{n_I}{1-\varepsilon}\right) & \text{if } i \in I_t^I \end{cases}, \end{aligned}$$

so that public bonds are still used in the bubbly period to complement the growing bubble until it reaches x^*. In each period, the proceeds from the sale of government debt are used to pay off the outstanding debt and to complement the rents that entrepreneurs get from bubble creation. Note that the debt decreases as a share of output until a new crisis comes, in which case it increases once again.

By following a policy like the one described above, the government offsets the effects of the fluctuating bubble even though it has no superior enforcement technology relative to the market.[14],[15] The disappearance of the bubble is a coordination failure that is solved through the introduction of public debt: this debt is rolled over until the bubble reappears, at which point it is gradually withdrawn from the market. Throughout the crisis, the government gives out handouts to entrepreneurs,[16] which are instrumental in maintaining credit flows that sustain a high efficiency of

[14] We have however assumed that the announced government policy is fully credible. If this is not the case, the government's ability to effectively deal with a bursting bubble might be seriously compromised. This might happen, for example, if government debt is prone to roll-over crises.

[15] We have not sought to design an optimal policy in the current section, but rather to describe one way in which the government could undo the adverse effects of a bursting bubble.

[16] This policy can be thought of as an asset-purchase scheme, in which the government buys worthless assets from

investment. In this sense, government intervention effectively keeps interest rates low during crisis episodes. Ultimately, a policy like the one described replicates the allocations and welfare that would arise under a stationary bubble like x^* when its probability of bursting goes to zero.

9 Where do we go from here?

[UNDER CONSTRUCTION]

firms and gives them goods or public bonds in return.

References

Bernanke, B. and M. Gertler, 1989, Agency Costs, Net Worth and Business Fluctuations, *American Economic Review* 79, 14-31.

Caballero, R. and A. Krishnamurthy, 2006, Bubbles and Capital Flow Volatility: Causes and Risk Management, *Journal of Monetary Economics* 53(1), 33-53.

Farhi, E. and J. Tirole, 2009, Bubbly Liquidity, working paper, Harvard.

Gertler, M. and N. Kiyotaki, 2009, Financial Intermediation and Credit Policy, working paper, NYU.

Kiyotaki, N., and J. Moore, 1997, Credit Cycles, *Journal of Political Economy* 105, 211-248.

Kraay, A., and J. Ventura, 2007, The Dot-Com Bubble, the Bush Deficits, and the US Current Account, in *G7 Current Account Imbalances: Sustainability and Adjustment*, R. Clarida (eds.), The University of Chicago.

LeRoy, S., 2004, Rational Exhuberance, *Journal of Economic Literature* 42, 783-804.

Martin, A. and J. Ventura, 2010, Economic Growth with Bubbles, mimeo, CREI.

Samuelson, P., 1958, An Exact Consumption-loan Model of Interest with or without the Social Contrivance of Money, *Journal of Political Economy* 66, 467-482.

Tirole, J., 1985, Asset Bubbles and Overlapping Generations, *Econometrica* 53 (6), 1499-1528.