

## Which Decision Theory?

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**Abstract.** A new laboratory experiment is designed to identify the best theories for describing decisions under risk. The experimental design has two noteworthy features: a representative sample of binary choice problems (for fair comparison across theories) and a lottery set with a small number of outcomes and probabilities (for ease of non-parametric estimation). We find that a simple heuristic, rank-dependent utility and expected utility theory provide the best goodness of fit. Each theory can account for about a quarter of individual choice patterns. Most of choice patterns best rationalized by expected utility theory can be equally well described by Yaari's dual model. Some of choice patterns best rationalized by rank-dependent utility can be equally well described by modified mean-variance approach.

**Key Words:** Decision Theory, Risk, Expected Utility Theory, Rank-Dependent Utility, Heuristic

**JEL Classification Codes:** C91, D03, D81

The aim of this paper is to identify descriptive decision theories that provide the best goodness of fit to experimental data. Several papers already addressed this question (*e.g.*, Hey and Orme, 1994; Hey, 2001). The present paper contributes to this literature in four aspects.

First, we use a representative sample of binary choice problems in our experiment. In contrast, in the existing literature, an experimenter hand-picks binary choice questions that subjects face in an experiment. Such design might not be optimal for comparing different theories. For example, expected utility theory (EUT) is known to be violated in pairs of common consequence problems (*e.g.*, Allais, 1953) but not when these problems are located in the interior of the probability triangle (*e.g.*, Conlisk, 1989). Hence, by picking too many (few) common consequence problems located in the interior of the probability triangle, an experimenter overestimates (underestimates) EUT's goodness of fit.

Second, we use a set of lotteries with a small number of outcomes and probabilities. In contrast, lottery sets used in the existing literature typically have only a small number of outcomes but many possible probabilities. This allows non-parametric estimation of decision theories with subjective parameters defined on the outcome space (such as EUT). Yet, theories with subjective parameters defined on the probability space (such as Yaari (1987) dual model) can be estimated only with additional parametric assumptions. By using a lottery set with a small number of outcomes and probabilities we can estimate all decision theories without any parametric assumptions.

Third, we use advanced econometric methods that are not biased to any particular theory. In contrast, the existing literature typically uses a strong utility model of probabilistic choice (*e.g.*, Fechner, 1860). EUT embedded in a strong utility model can account for certain types of the common ratio effect (*e.g.*, Loomes, 2005) and violations of betweenness (*e.g.*, Blavatskyy, 2006) that are simply not possible in deterministic EUT. Thus, a strong utility model overestimates EUT's goodness of fit compared to non-expected utility theories.

Fourth, we compare the performance of newly developed decision theories with that of classical theories. In particular, we consider the modified mean–variance approach (Blavatsky, 2010). We also consider the possibility of subjects using some simple rules of thumb in line with a recent psychological literature on heuristics (*e.g.*, Brandstätter et al., 2006).

The paper is organized as follows. Section 1 describes the design and implementation of our experiment. Section 2 summarizes ten decision theories considered in this paper. Section 3 presents our econometric model of discrete choice based on a latent dependent variable. Section 4 outlines our estimation procedure. Section 5 summarizes the results. Section 6 concludes.

## **1. Experiment**

We designed our experiment to facilitate non–parametric estimation of various theories. Specifically, all risky alternatives used in the experiment have a small number of outcomes and probabilities. Similar as Hey and Orme (1994) and Hey (2001), we deliberately restrict risky alternatives to have no more than four possible outcomes. These four outcomes are €5, €20, €25 and €40.

Using only probability values 0, 0.25, 0.5, 0.75 and 1, it is possible to construct 23 distinct probability distributions over these outcomes. Using only these 23 lotteries, it is possible to construct a total of 140 binary choice problems where none of the alternatives stochastically dominates the other. These 140 problems are presented in Table 1 in the Appendix.

The experiment was conducted as a paper–and–pencil classroom experiment. Subjects received a booklet with 140 decision problems (listed in Table 1 in the Appendix). Each problem was printed on a separate page. For each subject, pages with 140 problems were rearranged in a random order. Probability information was explained through the distribution of standard playing cards. Figure 1 shows an example of one decision problem as it was displayed in the experiment.

[INSERT FIGURE 1 HERE]

The experiment was conducted in the University of Innsbruck. Altogether, 38 undergraduate students took part in two experimental sessions, which were conducted on the same afternoon. Twenty out of 38 subjects (52.6%) were female. The average age of experimental participants was 21.5 years (minimum age was 18, maximum age was 34). Fourteen out of 38 subjects (36.8%) were economics majors. All subjects had no previous experience with economic experiments.

Subjects were allowed to go through experimental questions at their own pace with no time restriction. After answering all 140 questions, each subject was asked to spin a roulette wheel. The number of sectors on the roulette wheel corresponded to the total number of questions asked in the experiment. The question, which was randomly selected on the roulette wheel, was played out for real money.

Subjects who opted for a sure monetary payoff in the selected question simply received this amount in cash. Subjects who opted for a lottery were shown the corresponding composition of playing cards. The cards were subsequently reshuffled and subjects had to draw one card. Depending on the suit of their drawn card, they received the corresponding payoff. Upon observing the suit of their drawn card, subjects inspected all remaining cards to verify that card composition did not change after reshuffling.

Each experimental session lasted about one hour and a half. About one third of this time was spent on using physical randomization devices at the end of the experiment. On average, subjects earned €25. Two subjects earned €5, 19 subjects earned €20, 8 subjects earned €25 and 9 subjects earned €40.

## 2. Decision Theories

Let  $X = \{\text{€5}, \text{€20}, \text{€25}, \text{€40}\}$  denote the set of possible outcomes and let  $Q = \{0, 0.25, 0.5, 0.75, 1\}$  denote the set of probability values. Let  $L: X \rightarrow Q$  denote a typical lottery used in the experiment, *i.e.*,  $L(x) \in Q$  for all  $x \in X$  and  $\sum_{x \in X} L(x) = 1$ . For any lottery  $L$ , cumulative distribution function  $F_L(x)$  is defined

as  $F_L(x) = \sum_{y \in X, x \geq y} L(y)$ , for all  $x \in X$ . Similarly, decumulative distribution function  $G_L(x)$  of lottery  $L$  is defined as  $G_L(x) = \sum_{y \in X, y \geq x} L(y)$ , for all  $x \in X$ .

For each subject we estimated 10 decision theories. First, we consider maximization of expected value (EV). The utility of lottery  $L$  is then given by

$$(1) \quad U(L) = \sum_{x \in X} L(x) \cdot x$$

There are no subjective parameters to be estimated in this theory.

Second, we consider expected utility theory (EUT). In EUT, the utility of lottery  $L$  is given by

$$(2) \quad U(L) = \sum_{x \in X} L(x) \cdot u(x),$$

where  $u: X \rightarrow \mathbb{R}$  is (Bernoulli) utility function. Bernoulli utility function can be normalized for any two outcomes. We use normalization  $u(\text{€}5)=0$  and  $u(\text{€}40)=1$ . Utilities of two other outcomes  $u(\text{€}20)$  and  $u(\text{€}25)$  remain subjective parameters to be estimated. Note that EV is nested within EUT with two parameter restrictions:  $u(\text{€}20)=4/7$  and  $u(\text{€}25)=5/7$ .

Third, we consider Yaari (1987) dual model (Y). In this model the utility of lottery  $L$  is given by

$$(3) \quad U(L) = \sum_{x \in X} [\pi(G_L(x)) - \pi(1 - F_L(x))] \cdot x,$$

where  $\pi: Q \rightarrow [0,1]$  is a probability weighting function satisfying  $\pi(0)=0$  and  $\pi(1)=1$ . There are three subjective parameters to be estimated in Yaari's dual model:  $\pi(0.25)$ ,  $\pi(0.5)$  and  $\pi(0.75)$ . EV is nested within Y with three parameter restrictions  $\pi(0.25)=0.25$ ,  $\pi(0.5)=0.5$  and  $\pi(0.75)=0.75$ .

Fourth, we consider Quiggin (1981) rank-dependent utility (RDU). In the context of our experiment, RDU coincides with cumulative prospect theory (Tversky and Kahneman, 1992). In RDU the utility of lottery  $L$  is given by

$$(4) \quad U(L) = \sum_{x \in X} [\pi(G_L(x)) - \pi(1 - F_L(x))] \cdot u(x),$$

where  $\pi: Q \rightarrow [0,1]$  is a probability weighting function satisfying  $\pi(0)=0$  and  $\pi(1)=1$  and  $u: X \rightarrow \mathbb{R}$  is (Bernoulli) utility function normalized so that  $u(\text{€}5)=0$  and  $u(\text{€}40)=1$ . There are five subjective parameters to be estimated in RDU:  $\pi(0.25)$ ,  $\pi(0.5)$ ,  $\pi(0.75)$ ,  $u(\text{€}20)$  and  $u(\text{€}25)$ . EUT is nested within RDU with

three parameter restrictions  $u(0.25)=0.25$ ,  $u(0.5)=0.5$  and  $u(0.75)=0.75$ . Y is nested within RDU with two parameter restrictions:  $u(€20)=4/7$  and  $u(€25)=5/7$ .

Fifth, we consider modified mean–variance approach (MV) recently proposed by Blavatsky (2010). In MV the utility of lottery  $L$  is given by

$$(5) \quad U(L) = \sum_{x \in X} L(x) \cdot u(x) - 0.5\rho \cdot \sum_{y \in X} L(y) \cdot |\sum_{x \in X} L(x) \cdot u(x) - u(y)|,$$

where  $u : X \rightarrow \mathbb{R}$  is utility function normalized so that  $u(€5)=0$  and  $u(€40)=1$  and  $\rho \in [-1, 1]$  is a parameter capturing individual's attitude to utility dispersion.

There are three subjective parameters to be estimated in MV:  $u(€20)$ ,  $u(€25)$  and  $\rho$ . EUT is nested within MV with one parameter restriction:  $\rho=0$ .

Sixth, we consider Chew (1983) weighted utility (WU). In the context of our experiment (where lotteries are independent random variables), WU is mathematically equivalent to regret theory (Bell, 1982; Loomes and Sugden, 1982) and skew–symmetric bilinear utility theory (Fishburn, 1982). In WU the utility of lottery  $L$  is given by

$$(6) \quad U(L) = \frac{\sum_{x \in X} L(x) \cdot u(x) \cdot v(x)}{\sum_{x \in X} L(x) \cdot v(x)},$$

where  $u : X \rightarrow \mathbb{R}$  is utility function normalized so that  $u(€5)=0$  and  $u(€40)=1$  and  $v : X \rightarrow \mathbb{R}$  is a weighting function satisfying  $v(€5)=1$  and  $v(€40)=1$ . There are four subjective parameters to be estimated in WU:  $u(€20)$ ,  $u(€25)$ ,  $v(€20)$  and  $v(€25)$ . EUT is nested within WU with two parameter restrictions:  $v(€20)=1$  and  $v(€25)=1$ .

Seventh, we consider quadratic utility (QU) theory proposed by Chew et al. (1991). In QU the utility of lottery  $L$  is given by

$$(7) \quad U(L) = \sum_{x \in X} \sum_{y \in X} L(x) \cdot L(y) \cdot \phi(x, y),$$

where function  $\phi : X \times X \rightarrow \mathbb{R}$  is normalized so that  $\phi(€5, €5)=0$  and  $\phi(€40, €40)=1$ . There are eight subjective parameters to be estimated in QU. EUT is nested within QU with six parameter restrictions:  $\phi(x, y)=0.5u(x)+0.5u(y)$ .

Eighth, we consider disappointment aversion (DA) theory proposed by Gul (1991). In DA the utility of lottery  $L$  is given by

$$(8) U(L) = \begin{cases} \frac{L(\text{€}20) \cdot u(\text{€}20) \cdot (1 + \beta) + L(\text{€}25) \cdot u(\text{€}25) \cdot (1 + \beta) + L(\text{€}40)}{1 + \beta \cdot [L(\text{€}5) + L(\text{€}20) + L(\text{€}25)]}, & \text{if } 0 \leq U(L) \leq u(\text{€}20) \\ \frac{L(\text{€}20) \cdot u(\text{€}20) \cdot (1 + \beta) + L(\text{€}25) \cdot u(\text{€}25) + L(\text{€}40)}{1 + \beta \cdot [L(\text{€}5) + L(\text{€}20)]}, & \text{if } u(\text{€}20) \leq U(L) \leq u(\text{€}25) \\ \frac{L(\text{€}20) \cdot u(\text{€}20) + L(\text{€}25) \cdot u(\text{€}25) + L(\text{€}40)}{1 + \beta \cdot L(\text{€}5)}, & \text{if } u(\text{€}25) \leq U(L) \leq 1 \end{cases}$$

where  $\beta > -1$  is a parameter capturing disappointment aversion or seeking.

There are three subjective parameters to be estimated in DA:  $u(\text{€}20)$ ,  $u(\text{€}25)$  and  $\beta$ . EUT is nested within DA with one parameter restriction:  $\beta=0$ .

Ninth, we consider prospective reference (PR) theory proposed by Viscusi (1989). In PR the utility of lottery  $L$  is given by

$$(9) \quad U(L) = \lambda \cdot \sum_{x \in X} L(x) \cdot u(x) + (1 - \lambda) \cdot \sum_{x \in X} \text{sign}(L(x)) \cdot u(x) / \sum_{x \in X} \text{sign}(L(x)),$$

where  $u: X \rightarrow \mathbb{R}$  is utility function normalized so that  $u(\text{€}5)=0$  and  $u(\text{€}40)=1$ ,  $\lambda \in [0,1]$ ,  $\text{sign}(q)=1$  if  $q>0$  and  $\text{sign}(q)=0$  if  $q=0$ . There are three subjective parameters to be estimated in PR:  $u(\text{€}20)$ ,  $u(\text{€}25)$  and  $\lambda$ . EUT is nested within PR with one parameter restriction:  $\lambda=1$ .

Tenth, we consider the possibility of decisions to be driven by some simple heuristic. At least two observations from our experiment and earlier studies (Hey and Orme, 1994; Hey, 2001) point in this direction. On the one hand, despite a large number of questions, subjects cope with the experiment extremely quickly. Typically, they need less than 30 seconds for each decision. Only fast and frugal heuristics can result in such speedy decision making.

On the other hand, when examining the best fitting parameters of EUT and RDU we noticed an interesting fact. Quite a few subjects behaved as if they maximized an extremely risk averse utility function  $u(\text{€}5)=0$  and  $u(\text{€}20)=u(\text{€}25)=u(\text{€}40)=1$ . In other words, these subjects apparently minimized the probability of the lowest outcome. This is the second decision criterion in the recently proposed priority heuristic (Brandstätter et al., 2006). Unfortunately, the priority heuristic itself cannot be estimated on our data set. For example, the

priority heuristic is inconclusive in a decision problem depicted on Figure 1.

In the context of our experiment, it is very easy (*i.e.*, with little cognitive effort) to apply the following simple rule of thumb (abbreviated as H):

- a) pick a lottery with a smaller probability of the lowest outcome €5;
- b) if two lotteries yield the lowest outcome €5 with the same probability, then pick a lottery with the highest probability of the greatest outcome €40.

Note that there is no concept of utility value in H (as it is typical in the psychological literature on heuristics). There are no subjective parameters to be estimated in H. H is not nested in any other decision theory.

### 3. Econometric Model of Discrete Choice

Before presenting our econometric model of discrete choice based on a latent dependent variable we need to introduce the following notation. Each of 140 decision problems used in the experiment is a binary choice between two lotteries  $L$  and  $R$ . For any two lotteries  $L$  and  $R$ , lottery  $L \vee R$  yields outcome  $x \in X$  with a probability

$$(10) \quad \min\{F_L(x), F_R(x)\} + \max\{G_L(x), G_R(x)\} - 1.$$

Lottery  $L \vee R$  is the least upper bound on lotteries  $L$  and  $R$  in terms of first order stochastic dominance. Lottery  $L \vee R$  stochastically dominates both  $L$  and  $R$  and there is no other lottery that stochastically dominates both  $L$  and  $R$  but that is stochastically dominated by  $L \vee R$ .

For any two lotteries  $L$  and  $R$ , lottery  $L \wedge R$  yields outcome  $x \in X$  with a probability

$$(11) \quad \max\{F_L(x), F_R(x)\} + \min\{G_L(x), G_R(x)\} - 1.$$

Lottery  $L \wedge R$  is the greatest lower bound on lotteries  $L$  and  $R$  in terms of first order stochastic dominance. Both  $L$  and  $R$  stochastically dominate lottery  $L \wedge R$  and there is no other lottery that is stochastically dominated by both  $L$  and  $R$  but that stochastically dominates  $L \wedge R$ .

Existing literature (e.g., Hey and Orme, 1994; Hey, 2001) typically employs the following econometric model of discrete choice based on a latent

dependent variable. A decision maker chooses lottery  $L$  over lottery  $R$  if

$$(12) \quad U(L) - U(R) \geq \xi,$$

where  $\xi$  is a random variable (with zero mean) that is independently and identically distributed across all lottery pairs. Model (12) has at least three shortcomings:

- a) the distribution of random error  $\xi$  is affected by arbitrary affine transformation of utility function;
- b) standard microeconomic notion of risk aversion is not defined (see Wilcox, 2011);
- c) first-order stochastic dominance is violated.

Other existing models of probabilistic choice share some of these shortcomings. For example, problem a) applies also to Luce's choice model (Luce, 1957; Holt and Laury, 2002) and the model of Blavatskyy (2009, 2011) with a non-homogeneous sensitivity function  $\phi(\cdot)$ . Problem c) applies also to a tremble model (Harless and Camerer, 1994), heteroscedastic Fechner model (*e.g.*, Hey, 1995; Buschena and Zilberman, 2000; Blavatskyy, 2007) and a contextual utility model of Wilcox (2008, 2010).

Another popular econometric model is a random preference approach (*e.g.*, Falmagne, 1985; Loomes and Sugden, 1995) including random utility (*e.g.*, Gul and Pesendorfer, 2006). Unfortunately, random preference/utility approach allows for intransitive choice cycles (similar to the Condorcet's paradox). Such cycles are normatively unappealing and rarely observed in the data (*e.g.*, Rieskamp et al., 2006, p. 648).

In this paper we use a modification of model (12) which avoids problems a)–c). First, consider the case when lottery  $L$  stochastically dominates lottery  $R$ . In this case,  $U(L) - U(R) = U(L \vee R) - U(L \wedge R)$ . Thus, to avoid violations of stochastic dominance, we need to make sure that the realization of a random variable  $\xi$  is never greater than the difference  $U(L \vee R) - U(L \wedge R)$ . In other words, inequality (12) must be always satisfied if  $L$  stochastically dominates  $R$ . Thus,

stochastic dominance imposes an upper bound on possible errors:

$$(13) \quad \xi \leq U(L \vee R) - U(L \wedge R).$$

Second, consider the case when  $R$  stochastically dominates  $L$ . In this case,  $U(L) - U(R) = U(L \wedge R) - U(L \vee R)$ . To avoid violations of stochastic dominance, we need to make sure that the realization of a random variable  $\xi$  is never less than the difference  $U(L \wedge R) - U(L \vee R)$ . In other words, inequality (12) must always hold with a reversed sign if  $R$  stochastically dominates  $L$ . Thus, stochastic dominance also imposes a lower bound on possible errors:

$$(14) \quad \xi \geq U(L \wedge R) - U(L \vee R).$$

Inequalities (13) and (14) imply that random variable  $\xi$  must be distributed on a bounded interval. In general, this interval varies across lottery pairs. Thus, random variable  $\xi$  cannot be independently and identically distributed across all lottery pairs. Yet, it is possible to write random error  $\xi$  as  $\epsilon \cdot [U(L \vee R) - U(L \wedge R)]$ . Inequalities (13) and (14) are then both satisfied if random variable  $\epsilon$  is independently and identically distributed on the interval  $[-1, 1]$ . Note that we also solve shortcoming a) of model (12). Multiplying utility function  $U(\cdot)$  by an arbitrary positive constant does not affect the distribution of random error  $\epsilon$ .

To summarize, we use the following econometric model of discrete choice. A decision maker chooses lottery  $L$  over lottery  $R$  if

$$(15) \quad U(L) - U(R) \geq \epsilon \cdot [U(L \vee R) - U(L \wedge R)],$$

where  $\epsilon$  is a random variable symmetrically distributed around zero on the interval  $[-1, 1]$ . Let  $\Phi: [-1, 1] \rightarrow [0, 1]$  be the cumulative distribution function of random error  $\epsilon$ . A decision maker then chooses  $L$  over  $R$  with probability (16).

$$(16) \quad P(L, R) = \Phi \left( \frac{U(L) - U(R)}{U(L \vee R) - U(L \wedge R)} \right)$$

We assume that  $\Phi(v) = I_{\eta, \eta}(0.5 + 0.5v)$  for all  $v \in [-1, 1]$ , where  $I_{\eta, \eta}(\cdot)$  is the cumulative distribution function (aka the regularized incomplete beta function) of a symmetric beta distribution with parameters  $\eta$  and  $\eta$ . Beta distribution is

quite flexible and includes the uniform distribution ( $\eta=1$ ), unimodal distribution ( $\eta>1$ ) and bimodal (U-shaped) distribution ( $\eta<1$ ) as special cases. Subjective parameter  $\eta \in \mathbb{R}_+$  can be interpreted as a measure of noise. If  $\eta \rightarrow +\infty$  then model (16) converges to a deterministic decision theory.

We can use model (16) for estimating all decision theories considered in section 2 except for a simple heuristic H which lacks utility function  $U(\cdot)$ . Since H already specifies a deterministic choice rule, we can easily extend it into a model of probabilistic choice as follows. A decision maker chooses lottery  $L$  over lottery  $R$  with probability (17).

$$(17) \quad P(L,R) = \begin{cases} \eta, & \text{if } L(\text{€}5) < R(\text{€}5) \quad \text{or } L(\text{€}5) = R(\text{€}5) \text{ and } L(\text{€}40) > R(\text{€}40) \\ 1-\eta, & \text{if } L(\text{€}5) > R(\text{€}5) \quad \text{or } L(\text{€}5) = R(\text{€}5) \text{ and } L(\text{€}40) < R(\text{€}40) \end{cases}$$

Again, subjective parameter  $\eta \in [0.5, 1]$  can be interpreted as a measure of noise. If  $\eta=1$  then a decision maker literally applies heuristic H in every decision problem. If  $\eta=0.5$  then a decision maker chooses at random.

#### 4. Estimation Procedure

Estimation is done separately for each subject. Subjective parameters of ten decision theories plus noise parameter  $\eta$  are estimated by maximizing total log-likelihood (formulas (16) and (17) show the likelihood of one decision). Non-linear optimization is solved in the Matlab 7.2 package (based on the Nelder-Mead simplex algorithm).

We begin by estimating two decision theories with no subjective parameters: EV and H. These two theories are then compared in terms of their goodness of fit to the revealed choice pattern of each subject. We use Vuong likelihood ratio test for strictly non-nested models (see Vuong (1989) and Appendix A.2 in Loomes et al. (2002) for technical details). If one of the theories provides a significantly better fit (at 5% significance level) than the other, it is tentatively labeled as the best descriptor for the corresponding subject. If two theories do not significantly differ in terms of their goodness of fit, both are tentatively labeled as the best descriptors.

Next, we consider EUT and Y and compare their goodness of fit with that of the best descriptor(s). We use standard likelihood ratio test for nested models and Vuong likelihood ratio test for strictly non-nested models. In the latter case, Akaike information criterion is used to penalize EUT or Y for a greater number of parameters. If EUT (or Y) significantly outperforms the best descriptor(s), it tentatively becomes the best descriptor. If both EUT and Y significantly outperform the best descriptor(s), then EUT and Y are compared with each other using Vuong likelihood ratio test for overlapping models.

Finally, we consider all remaining non-expected utility theories from section 2 and repeat the same routine as for EUT and Y. At the end of this exercise, for each subject we identify one or several decision theories such that none of the remaining theories provides a significantly better goodness of fit to the subject's revealed choice pattern.

## 5. Results

Figure 2 summarizes estimation results. There are three best-fitting decision theories: EUT, RDU and H. Each of these theories can account for about a quarter of individual choice patterns. Most of choice patterns best rationalized by EUT can be equally well described by Y. Some of choice patterns best rationalized by RDU can be equally well described by MV.

[INSERT FIGURE 2 HERE]

At the same time, there are three decision theories (EV, DA and PR) that always (*i.e.*, for every subject) provide a significantly worse goodness of fit than some other theory. Two more theories (QU and WU) provide the best description only for one or two subjects. Thus, we can confidently delete EV, DA, PR, QU and WU from the list of promising descriptive decision theories.

[INSERT FIGURE 3 HERE]

Figure 3 shows estimated best-fitting Bernoulli utility functions in EUT for those subjects for whom EUT turned out to be the best describing decision theory. For most of these subjects, the best-fitting utility function is concave,

i.e. subjects reveal risk averse behavior. Only one subject behaved as if maximizing a convex utility function.

[INSERT FIGURE 4 HERE]

Figure 4 shows estimated best-fitting Bernoulli utility functions in RDU for those subjects for whom RDU turned out to be the best describing decision theory. For many subjects the best-fitting utility function is concave but we observe a lot of heterogeneity. Three subjects (#3, #12 and #15) behave as if maximizing an extremely risk averse utility function  $u(\text{€}5)=0$  and  $u(\text{€}20)=u(\text{€}25)=u(\text{€}40)=1$ . They apparently minimized the probability of the lowest outcome but used some other heuristic than H.

[INSERT FIGURE 5 HERE]

Figure 5 shows estimated best-fitting probability weighting functions in RDU. For many subjects the best-fitting probability weighting function turns out to be a concave function. Only one subject (#20) revealed a convex probability weighting function. A textbook inverse S-shaped probability weighting function is found only for one subject (#10). Two subjects (#12 and #34) behave as if they have an S-shaped probability weighting function.

## 6. Conclusion

This paper finds clear evidence that people use fast and frugal heuristics when making decisions under risk. Specifically, we identified one simple heuristic. First and foremost, people minimize the probability of the worst outcome. If risky alternatives yield the same chance of the worst outcome, then people maximize the probability of the best outcome. For 10 out of 38 subjects (26.32%) this simple heuristic correctly predicts at least 130 out of 140 revealed choices (92.86%). For one subject (#6) it even rationalizes 139 out of 140 revealed choices (99.29%). The heuristic achieves such astonishing goodness of fit despite the fact that it has no subjective parameters to be estimated.

Having successfully identified the rule of thumb that people use, we should not be surprised that it fits the data better than sophisticated

mathematical decision theories do. Yet, the real challenge is to find which heuristic people use. Different people might use different heuristics in the same decision problem. Even the same individual is likely to use different heuristics in different decision contexts. Thus, behavioral economists should perhaps answer the question: "Which heuristic?" rather than "Which decision theory?". In the meanwhile, when a correct heuristic in a specific decision context is not known, assuming that people behave as if maximizing utility function according to some mathematical decision theory remains the second-best solution.

Among standard decision theories, expected utility and rank-dependent utility provide the best goodness of fit. Each theory can best describe the revealed choices of about a quarter of all subjects. Most of choice patterns best rationalized by expected utility theory can be equally well described by Yaari's dual model. Some of choice patterns best rationalized by rank-dependent utility can be equally well described by modified mean-variance approach. At the same time, maximization of expected value, Gul's disappointment aversion theory and prospective reference theory do not fit any of revealed choice patterns. One can safely conclude that all three belong to the shelves of the history of economic thought.

A new experimental design introduced in this paper can be used for non-parametric estimation of various decision theories (cf. Figure 4 and 5 for rank-dependent utility). Traditional methods of utility elicitation rely on a revealed indifference relation (*e.g.*, Wakker and Deneffe, 1996). Yet, there is no incentive-compatible method to detect indifference using only a small number of simple binary choice questions. Thus, utility elicitation is typically conducted under hypothetical incentives (*e.g.*, Abdellaoui, 2000; Bleichrodt and Pinto, 2000). Our experimental design overcomes this problem. Instead of looking for few hard-to-detect indifference points, we ask subjects many binary choice questions that impose a maximum number of constraints on few unknown subjective parameters. The latter are then estimated by econometric methods.

## References

Abdellaoui, Mohammed (2000) "Parameter-free elicitation of utility and probability weighting functions" *Management Science* 46, 1497-1512

Allais, Maurice (1953) "Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulates et Axiomes de l'Ecole Américaine" *Econometrica* 21, 503-546

Bell, David (1982) "Regret in Decision Making under Uncertainty" *Operations Research* 20, 961-981

Blavatskyy, Pavlo (2006) "Violations of betweenness or random errors?" *Economics Letters*, Vol. 91, pp. 34-38

Blavatskyy, Pavlo (2007) "Stochastic Expected Utility Theory" *Journal of Risk and Uncertainty*, Vol. 34, pp. 259-286

Blavatskyy, Pavlo (2009) "Preference Reversals and Probabilistic Choice" *Journal of Risk and Uncertainty*, Vol. 39 (3), pp. 237-250

Blavatskyy, Pavlo (2010) "Modifying the Mean-Variance Approach to Avoid Violations of Stochastic Dominance" *Management Science*, 56, 2050-205

Blavatskyy, Pavlo (2011) "A Model of Probabilistic Choice Satisfying First-Order Stochastic Dominance" *Management Science*, forthcoming

Bleichrodt, Han and Jose-Luis Pinto (2000) "A parameter-free elicitation of the probability weighting function in medical decision analysis" *Management Science* 46, 1485-1496

Brandstätter, Eduard, Gerd Gigerenzer and Ralph Hertwig (2006) "The Priority Heuristic: Making Choices without Trade-offs" *Psychological Review*, Vol. 113 (2), pp. 409-432

Buschena, David and David Zilberman (2000) "Generalized Expected Utility, Heteroscedastic Error, and Path Dependence in Risky Choice" *Journal of Risk and Uncertainty*, Vol. 20, pp. 67-88

Chew, Soo Hong (1983) "A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory

Resolving the Allais Paradox" *Econometrica* **51**, 1065–1092

Chew, Soo Hong, Larry G. Epstein and Uzi Segal (1991) "Mixture Symmetry and Quadratic Utility" *Econometrica* **59**, 139–163

Conlisk, John (1989) "Three Variants on the Allais Example" *American Economic Review* **79** (3), 392–407

Falmagne, Jean–Claude (1985) "Elements of Psychophysical Theory" Oxford University Press, New York

Fechner, Gustav (1860) "Elements of Psychophysics" New York: Holt, Rinehart and Winston

Fishburn, Peter (1982) "Nontransitive Measurable Utility" *Journal of Mathematical Psychology* **26**, 31–67

Gul, Faruk (1991) "A Theory of Disappointment Aversion" *Econometrica* Vol. 59, pp. 667–686

Gul, Faruk and Wolfgang Pesendorfer (2006) "Random Expected Utility" *Econometrica*, Vol. 71(1), pp. 121–146

Harless, David and Colin Camerer (1994) "The Predictive Utility of Generalized Expected Utility Theories" *Econometrica* **62**: 1251–1289

Hey, John (1995) "Experimental investigations of errors in decision making under risk" *European Economic Review*, Vol. 39, pp. 633–640

Hey, John (2001) "Does repetition improve consistency?" *Experimental Economics*, Vol. 4, pp. 5–54

Hey, John and Chris Orme (1994) "Investigating Generalisations of Expected Utility Theory Using Experimental Data" *Econometrica*, Vol. 62, pp. 1291–1326

Holt, Charles and Susan Laury (2002) "Risk Aversion and Incentive Effects" *American Economic Review* **92** (5), 1644–1655

Loomes, Graham (2005) "Modelling the stochastic component of behaviour in experiments: some issues for the interpretation of data" *Experimental Economics*, Vol. 8, pp. 301–323

Loomes, Graham and Robert Sugden (1982) "Regret theory: An alternative theory of rational choice under uncertainty" *Economic Journal*, **92(4)**, 805–24

Loomes, Graham and Robert Sugden (1995) "Incorporating a stochastic element into decision theories" *European Economic Review*, Vol. 39, pp. 641–648

Loomes, Graham, Peter Moffatt and Robert Sugden (2002) "A microeconomic test of alternative stochastic theories of risky choice" *Journal of Risk and Uncertainty*, Vol. 24, pp. 103–130

Luce, Duncan (1959) "Individual choice behavior" New York: Wiley

Quiggin, John (1981) "Risk perception and risk aversion among Australian farmers" *Australian Journal of Agricultural Recourse Economics*, Vol. 25, pp. 160–169

Rieskamp, Joerg, Jerome Busemeyer and Barbara Mellers (2006) "Extending the Bounds of Rationality: Evidence and Theories of Preferential Choice" *Journal of Economic Literature*, Vol. XLIV, pp. 631–661

Tversky, Amos and Daniel Kahneman (1992) "Advances in prospect theory: Cumulative representation of uncertainty" *Journal of Risk and Uncertainty* **5**, 297–323

Viscusi, Kip (1989) "Prospective Reference Theory: Toward an Explanation of the Paradoxes" *Journal of Risk and Uncertainty* **2**, 235–264

Vuong, Quang (1989) "Likelihood ratio tests for model selection and non-nested hypotheses" *Econometrica*, Vol. 57, pp. 307–333

Wakker P. Peter and Daniel Deneffe (1996) "Eliciting von Neumann–Morgenstern utilities when probabilities are distorted or unknown" *Management Science* **42**, 1131–1150

Wilcox, Nathaniel (2008) "Stochastic models for binary discrete choice under risk: A critical primer and econometric comparison" in J. C. Cox and G. W. Harrison, eds., *Research in Experimental Economics Vol. 12: Risk Aversion in*

*Experiments* pp. 197–292. Bingley, UK: Emerald

Wilcox, Nathaniel (2011) “Stochastically more risk averse:’ A contextual theory of stochastic discrete choice under risk” *Journal of Econometrics* forthcoming

Yaari, Menahem (1987) “The Dual Theory of Choice under Risk” *Econometrica* 55, 95–115