Optimal Financial Regulation and Bailouts in Presence of Regulatory Forbearance and Systemic Risk

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Abstract

We consider a moral hazard economy with the potential for collusion between bankers and borrowers to study how incentives for risk taking are affected by the quality of supervision. We show that a low cost of capital or low return on investment may generate excessive risk taking. Because of a pecuniary externality, the market equilibrium is inefficient, therefore bank capital ratio should be regulated. We extend the model by introducing time inconsistency due to systemic bailout guarantees, externalities in production, and endogenize the political economy of supervision quality. In each case, we characterize the optimal financial regulation which we show should be conditioned on various macroeconomic or institutional factors. Overall, our theory supports the need for macro-prudential regulations of bank capital, but demonstrates that optimal policies may be complex.

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1 Introduction

The financial crisis has ignited an intense policy debate on the determinants of incentives in the financial industry, and has resulted in substantial efforts to improve financial regulations to tame risk taking during booms and prepare capital buffers for downturns.[1] Some observers have argued that lax monetary policy was responsible for fueling an asset market boom through low interest rates that caused excessive risk taking and leverage by financial intermediaries.[2] Others instead remarked that it was the responsability of bank supervisors to ensure high quality lending decisions.[3] A related concern, however, is that regulations may have been influenced by the political environment which may have created immense pressures to relax lending standards.[4]

In spite of the very active policy debate, there is to date relatively few theoretical contributions studying the role of financial regulations in constraining excessive risk taking by financial institutions. The macroeconomic literature has traditionally focused on factors affecting the supply and demand of credit during the business cycle, and has rarely studied how banks actively choose the risk profile of their portfolios. The microeconomic literature has analyzed the role of regulations in enhancing the quality and size of the financial system in presence of moral hazard and asymmetric information, as well as the trade-offs associated with the internationalization of banking supervision and regulations (see for instance recent contributions by Morrison and White, 2005, 2009; Acharya, 2003; Dell' Ariccia and Marquez, 2005). But the notion of equilibrium "excessice risk taking" by intermediaries and the role played by supervision quality remain unsufficiently

[1] See for instance BCBS, 2010, "The Basel Committee's response to the financial crisis: report to the G20".

[2] See for instance Taylor (2008) and Rajan (2010).

[3] In his address at the 2010 Annual meeting of the American Economic Association, Fed Chairman Ben Bernanke argued that, based on evidence of declining lending standards during the boom, "stronger regulation and supervision aimed at problems with underwriting practices and lenders' risk management would have been a more effective and surgical approach to constraining the housing bubble than a general increase in interest rates".

[4] See for example Johnson and Kwak (2010), and Rajan (2010) for two views on the role of political factors in shaping the regulatory environment of the US financial sector; the first one emphasizing the political influence of the financial industry, and the second one stressing the role of politicians and of the government in pushing credit to low income households who could not afford it.

studied.[5]

In this paper, we present a model to study the incentives of financial intermediaries and borrowers in taking excessive risks, and rely on this framework to characterize optimal financial regulations.[6] There are two novelties in our analysis. First, we highlight the central role of imperfections in banking supervision and of its dependence on the political process in tilting incentives towards taking more risks. Second, because of regulatory forbearance by the supervisor, negative net present value projects may be undertaken in equilibrium – which justifies ex-ante policy interventions constraining the leverage of financial institutions and of borrowers. Crucially, optimal financial regulations will depend on institutional characteristics.

We consider a moral hazard economy in which banks monitor borrowers' efforts, but must be incentivized by investing their own capital in the project, in addition to the entrepreneur and investors' capital. There are two incentive problems: first, banks must monitor projects; second, they must be prevented from colluding with borrowers – which they do at the expense of uninformed investors by (sometimes) investing in non-productive projects which only generate non-verifiable benefits. Collusion can be prevented by auditing bank accounts. However, if bank audits are imperfect and stochastic, preventing collusion requires promising higher financial returns to the bank to ensure it will not collude with the borrower in the event the audit quality turns out to be poor. If however, audit quality is high ex-post, the bank enjoys a pure net rent equal to the private benefit of control enjoyed when audit quality is poor. The possibility of collusion and imperfect and stochastic supervision thus interact by creating an environment in which banks have pure economic rents.

Because bank capital is more costly than uninformed capital, collusion-proof contracts leaving rents to banks are not always in the best advantage of borrowers who would like to maximize leverage by minimizing the share of investment financed out of bank capital. When the differential in the costs of capital is large enough, private agents may prefer a contract relaxing the incentive constraint of the bank, by ensuring monitoring only when the audit by the supervisor is of good quality. The benefit is that a larger share of the financial return can be pledged to uninformed investors. This enhances the borrowing capacity ex-ante, and

[5] Morrison and White (2005) study the role of capital adequacy rule as a substitute to screening of bank applications when the supervisor has low reputation.

[6] We abstract from maturity mistmatches in bank balance sheet, hence we do not analyze funding liquidity issues, even if those have played a central role in the propagation of the financial crisis.

increases the leverage of the borrower and of the bank. The cost of such contracts is that bad projects are sometimes undertaken when the quality of supervision turns out to be low. This tends to reduce the average expected return on projects. Hence, there is "excessive risk taking" by a subset of financial intermediaries.

The market outcome is not necessarily optimal because of a pecuniary externality. Indeed, when choosing collusion contracts, borrowers do not internalize the general equilibrium effect on the return on bank capital which is depressed when more and more agents turn to collusion contracts. This provides a rationale for a capital adequacy rule (which is equivalent to a leverage constraint in our framework) preventing collusion contracts between intermediaries and borrowers.[7] We show that an adequately chosen fixed capital adequacy rule is sufficient to rule out equilibria with collusion, but such rules are not optimal. Indeed, a rule that always prevent collusion will also be excessively tight for some values of the cost of capital for which there is no collusion. We then characterize the optimal capital adequacy rule. We show that it should depend on the cost of capital in a *pro-cyclical* manner (because some increase in leverage is optimal when the cost of capital falls), even if the rule becomes binding for lower costs of capital. But it depends *counter-cyclically* on investment opportunities (e.g. the profitability of projects). This is because incentives to choose bad projects are stronger when the return on good projects fall. This suggests that, in the context of the current debate on the risk taking channels of monetary policy and the role of countercyclical capital rules in taming risk taking, it is essential to distinguish how the cost of capital and the return on productive investment vary during the credit cycle.[8] Moreover, the optimal capital rule also depends on broad institutional characteristics: it should be tighter if banks are less efficient, if supervision quality is lower, and if corporate governance is of worse quality.

We then consider three extensions of the model and study their implications for risk taking and for financial regulation.

In the first extension, we endogenize systemic bailout guarantees, and show that the existence of such

[7] Recent papers providing a rationale for financial regulation include Chari and Kehoe (2009) and Ranciere and Tornell (2010).

[8] Dell' Ariccia, Laeven and Marquez (2010) provide a framework to study the risk taking channels of monetary policy; Drehman et al. (2010) provide general lessons for the design of counter-cyclical capital buffer, and suggest that the deviation of credit to GDP from trend as a macro-indicator to calibrate counter-cyclical buffers.

expected bailout guarantees increases incentives for risk taking, but also lowers the cost of capital which stimulates investment. Capital rules should be tightened in presence of bailout guarantees only if the guarantees are large enough so that the first effect dominates the second effect.[9]

In the second extension, we introduce productive externalities. We assume that the return on individual projects depends on how successful other projects are. With such externalities, multiple equilibria become possible and investment in bad projects become more likely to take place. We also show that the previous capital adequacy rule becomes either ineffective or excessively tight. We suggest several alternatives rules, including (i) a rule making the capital rule explicitely dependent on the return on productive projects; or (ii) a rule combining a capital adequacy ratio with a constraint on asset allocation.

In the final extension, we study the political economy of the quality of banking supervision. We show that, during periods of low interest rates or low return on productive investments, there is pressure from financial intermediaries to worsen the quality of bank audits. This makes collusion less costly, and raises the rent received by the bank. We show that investors and borrower do not oppose such pressure because a lower cost of collusion tends to increase borowing capacity in the partial equilibrium (in the general equilibrium, this beneficial effect of lower supervision quality is offset by the increase in the cost of bank capital). In contrast, during periods of high interest rates and high return on investment, investors and borrowers unambiguously prefer high supervision quality to reduce the bank's economic rent under collusion-proof contracts, suggesting that the political pendulum is reversed to high quality supervision. Hence, in sum, the politcal process tends to exacerbate the phenomenon of excessive risk taking by weakening banking supervision precisely when it should be strenghtened. The implication for regulation is that the optimal capital adequacy rule will need to be tightened during the boom, relative to the situation in which the quality of supervision is immune to political pressures and does not worsen when interest rates fall.

The paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes financial contracts. The market equilibrium is solved in section 4. Section 5 solves the optimal capital regulation, and analyzes how it is affected by systemic bailout guarantees. Section 6 considers an economy with productive

[9] Farhi and Tirole (2009) and Diamond and Rajan (2009) show how the expectation of a monetary bailout induces banks to take correlated risks. Morrison and White (2010) show that regulatory forbearance can be ex-post efficient when the supervisor's reputation is not high enough.

externalities. In section 7, we endogenize the political process through which banking supervision quality is chosen. Section 8 concludes.

2 A Model of Bank Finance

We consider a single good moral hazard economy with four types of risk neutral agents: (a) investors, who supply capital elastically; (b) bankers who have the ability to monitor borrowers; (c) entrepreneurs who have investment opportunities and are endowed with an aggregate capital stock normalized to one; and (d) a banking supervisor who audits banks and enforces regulations.[10]

The economy lasts for three periods and there is no aggregate uncertainty. In period 1, agents write financial contracts. In period 2, agents discover the extent to which individual banks are audited, audits take place and projects are undertaken. In period 3, outcomes are realized, including the ex-post bail-out/closure of failed banks, and the payments to financiers, investors and entrepreneurs. Investment I in the first period is financed by a combination of internal funds (the entrepreneur's endowment 1), bank loans and direct borrowing from uninformed investors.

- **Production and External Financing Technologies**

All agents have access to a storage technology with a rate of return γ. In addition, there are also two types of projects: good and bad projects that can be undertaken by entrepreneurs only. A good project generates a verifiable financial return equal to R per unit of capital invested (if it succeeds) or to 0 (if it fails). A bad project yields only a non pledgeable private benefit (not verifiable) with probability 1 and whose value is determined by bankers' monitoring.

Formally, the return per unit of capital invested is given by:

$$\text{Good project:} \begin{cases} Y = R \text{ with probability } p \\ Y = 0 \text{ with probability } 1-p \end{cases}$$

[10] There is a unit mass of investors, of bankers and of entrepreneurs

$$\text{Bad project: } \begin{cases} Y = B \text{ with probability 1 if no banking monitoring} \\ Y = b \text{ with probability 1 if banking monitoring} \end{cases}$$

with $\Delta B = B - b > 0$. We assume that only good projects are socially efficient:

Assumption A: $pR - c - \gamma > 0 > B - \gamma > b - \gamma$

The banking sector, endowed with an aggregate stock of capital K_B, consists of many competitive intermediaries who monitor firms by paying a non verifiable cost c per unit of capital invested in the project. The market rate of return on bank capital is given by β. Monitoring reduces the entrepreneur's private benefit from B to b when choosing bad projects. This reduces moral hazard in production, and thus enhances the entrepreneur's borrowing capacity. To analyze the role of bank capital in mitigating moral hazard in banking, we assume that each bank finances only one project[11].

Investors do not monitor firms to which they lend, and supply capital elastically at the rate of return γ.[12] Uninformed investors can also be interpreted as bank depositors or bank creditors.

- **Collusion and the Quality of Banking Supervision**

As we shall see in the next section, an entrepreneur and a bank may have an incentive to collude after signature of the financial contract so that monitoring does not take place. The bank has all the bargaining power: if a bribe is paid to her, the benefits of collusion are transferred in the form of a non-verifiable side payment S that leaves the entrepreneur indifferent between colluding and not colluding[13]. Collusion requires a costly non-verifiable transfer from the former to the latter: the benefit to the bank of a side payment of 1 takes only a value k_C, with $0 \leq k_C < 1$.

The cost of the illicit transfer k_C is determined by the audit technology of the banking supervisor and is subject to *idiosyncratic* uncertainty which is revealed *after* the financial contract is signed, but *before* the

[11] In practise, banks often have large exposures to a small numbers of borrowers See for example, La Porta, Lopez-de-Silanes and Zamarripa (2003) for evidence of large related lending exposures in Mexico, Acharya, Hasan and Saunders (2006) for evidence of undiversified bank portfolios in Italy, or Dahiya, Saunders and Srinivasan (2003) on the sharp negative effects of defaults by major corporate borrowers in the U.S. on their lead lending bank.

[12] A possible justification for the fact that uninformed investors do not monitor is that they are atomistic and therefore do not have the monetary incentives to incur the cost of monitoring.

[13] Firms cannot default on promised side payments to banks contingent on the state of nature realized.

entrepreneur's choice of project. Specifically, the supervisor can audit banks in period 2 and impose sanctions if banks and entrepreneurs are investing in bad projects. However, because the supervisor cannot audit all banks or all projects perfectly, the technology of the banking supervisor is stochastic. With probability q, the audit is perfect and collusion becomes verifiable by a court. As a result, the bank cannot extract collusive rents from the borrower and $k_C = 0$. However, with probability $1 - q$, the audit is not perfect and a fraction $k > 0$ of the collusive rent of the bank is not observed by the bank supervisor. Hence $k_C = k$. Therefore, $1 - k$ represents the strength or quality of the banking supervisor, measured by the fraction of the collusive rent that is lost when an audit takes place.

There are two justifications for the stochastic auditing technology which introduces relationship-specific uncertainty in the contracting environment. First, because cost effectivenes limits the capacity of supervisory agencies, they are likely to randomly select and focus their auditing efforts on a subset of banks only. If a bank is selected for an audit, parties involved in a loan contract are then likely to be under close scrutiny, and therefore subject to high costs of hiding illicit transactions. Second, even if controls are not stochastic, there may be a relationship-specific component in the effectiveness of bank inspections and controls. This component will depend on the corruptibility of banking supervisors and the extent to which they are susceptible to political influence. For instance, it is well known that, in weak institutional environments, political connections facilitate bank regulatory forbearance, increase connected lending, and provide politically connected firms easier access to domestic bank credit or ex-post bail-out.[14]

We shall initially take the quality of banking supervisor as a given. In an extension we will investigate the political economy determinants of the quality of banking supervision.

- **Ex-Post Bailout Policies**

We assume that there is an ex-post bail-out/closure policy of banks which is reflected by an exogenous probability θ that banks' assets benefit of a public guarantee ex-post if the project fails. Given this, the probability that creditors (banks or investors) will be repaid ex-post is: $p(\theta) = p + (1 - p)\theta$.[15] To abstract

[14]See Claessens, Feijen and Laeven (2006), Faccio, Masulis, and McConnell (2005). As suggested by Fisman (2001)'s work on Indonesia, the value and effectiveness of these political connections may also change over time, hence generating some relation-specific uncertainty on the feasibility and costs of illicit transactions.

[15]Later on we will consider the possibility of an endogenous bail-out rule. See section ?

from taxation issues, we assume that the bailout is financed by lump-sum taxes.

3 Firms' Financial Contracts

For a project of total size I, financial contracts specify the maximum borrowing capacity of the entrepreneur $(I-1)$, the amount borrowed from bankers (I_B) from uninformed lenders (I_I), as well as the payments to each party if the project succeeds: the return $R \cdot I$ is shared between the bank (R_B), the uninformed investors (R_I) and the entrepreneur (R_E): $R \cdot I = R_E + R_B + R_I$.

Given that internal funds of the entrepreneur are equal to 1, I also measures entrepreneurial leverage, and I/I_B measures the leverage of banks. Two types of contracts can be analyzed depending on whether they allow for collusion or not between the entrepreneur and a bank. We first write the incentive and participation constraints for each of these contracts.

- **Incentives and Participation Constraints:**

(1) Collusion-Proof Contracts

Let us start with the contracts that prevent any investment in bad projects.

- Incentive compatibility constraints:

The entrepreneur must obtain an expected return equal to his private benefits:

$$pR_E \geq bI \tag{1}$$

Given a transaction cost of collusion k, a potential bribe SI and the promised financial return θ in the event of a bail-out, the bank's net expected return in absence of collusion must be greater of equal to the expected bailout payment plus the bribe if collusion occurs:

$$p(\theta)R_B - cI \geq \theta R_B + kSI \tag{2}$$

where the maximum side payment SI that the entrepreneur is willing to transfer to the bank is equal to

ΔBI, under the assumption that the bank has all the bargaining power and appropriate the full rent from collusion.

- Participation constraints:

The bank expected return net of monitoring cost must exceed the expected return on bank capital β:

$$p(\theta)R_B - cI \geq \beta I_B \tag{3}$$

while investors must break even on average:

$$p(\theta)R_I \geq \gamma I_I \tag{4}$$

(2) Partial collusion contracts

Consider now contracts that allow for partial collusion. The rationale for considering such contracts is that the incentive constraint in a collusion-proof contract is too tight if the audit technology turns out to be perfect with probability q, as it leaves an additional rent to the bank equal to $k\Delta BI$. A partial collusion contract aims at eliminating this pure rent, and does so by incentivizing the bank only when the audit technology is perfect. The cost is that, when the audit technology is not perfect, the bank is not incentivized to monitor.

- Incentive compatibility constraints:

The incentive compatibility constraint of the entrepreneur remains the same, he must choose the good project when the bank supervisor has perfect audit capacity.

Similarly, the bank must be incentivized to monitor when the bank supervisor has perfect audit capacity, but is not incentivized when collusion is feasible:

$$p(\theta)R_B - cI \geq \theta R_B \tag{5a}$$

- Participation constraints:

The bank must now break even if collusion occurs when auditing is imperfect. The overall bank return now includes the net financial payment if audit is perfect (with probability q), and the expected bailout and bribe when audit is imperfect (with probability $1-q$): $q \cdot (p(\theta)R_B - cI) + (1-q) \cdot (\theta R_B + kSI) \geq \beta I_B$. Hence the condition:

$$\widetilde{p}(\theta)R_B - qcI + (1-q)\, k\Delta BI \geq \beta I_B \tag{6}$$

where $\widetilde{p}(\theta) = qp(\theta) + (1-q)\theta$ is the probability of a repayment (taking place because the project succeeds or because a bailout takes place). The constraint shows that the bank saves on monitoring costs cI which are paid only with probability q, enjoys a bribe $k\Delta BI$ with probability $1-q$ (this replaces the same face value financial payment received with probability one to always ensure monitoring in a collusion-proof contract) but receives the financial return R_B with a lower probability $\widetilde{p}(\theta) < p(\theta)$.

Finally, uninformed investors must also break even on average, but a lower probability of payment $\widetilde{p}(\theta)$:

$$\widetilde{p}(\theta)R_I \geq \gamma I_I \tag{7}$$

- **The Borrower's Maximization Program**

Given rates of return γ and β, and the fact that the entrepreneur's participation constraint is binding, the collusion-proof contract or the partial collusion contract chosen by an entrepreneur with initial internal funds 1 is then the solution of the following maximization program:

Maximize: $U_E = b \cdot I$

subject to: - $1 + I_B + I_I = I$ (resource constraint);

- $R \cdot I = R_E + R_B + R_I$ (profit sharing rule);

- Incentive constraints (1) and (2), or (**??**) and (5a), and participation constraints (3) and (4), or (6) and (7)

4 Market Equilibrium

We are now ready to characterize the market equilibrium under various parameters. Note first that the incentive constraint of the bank is binding because bank capital is more costly than uninformed investors capital, hence the entrepreneur will minimize both the share of bank capital in external finance and the amount repaid to the bank for a given project size. The incentive constraint of the entrepreneur is binding because, to achieve maximum leverage, the entrepreneur will maximize the share of profits pledged to external providers of finance, and retain the minimum share of profits necessary to have incentives to choose the productive project (the "non-pledgeable income"). We assume the following:

$$\textbf{Assumption B}: \; R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)} \geq 0 \;\text{ and } c < k\Delta B$$

The first part of Assumption B states that the project's return is large enough so that the pledgeable income that uninformed investors get in case of success is positive, ensuring therefore the existence of an active credit market in the economy. The second part of assumption B follows Holmstrom and Tirole (1997) and ensures that monitoring by banks is socially valuable.

4.1 Choice of Financial Contracts

The optimal project size in collusion-proof contracts, and in partial-collusion contracts are then characterized in the following proposition.

Proposition 1 *Consider an entrepreneur with initial internal funds* 1.

(1) Define $\Phi_{NC}(\theta) = \gamma \cdot \frac{I_I}{I}$, *and* $\Lambda_{NC}(\theta) = \beta \cdot \frac{I_B}{I}$ *the net expected return the expected return to investors and to the bank per unit of capital invested in the project in a collusion-proof contract. The project size* I_{NC} *in a collusion-proof contract is given by:*

$$I_{NC} = \frac{1}{1 - \frac{\Phi_{NC}(\theta)}{\gamma} - \frac{\Lambda_{NC}(\theta)}{\beta}} \equiv \frac{1}{V_{NC}(\gamma, \beta)} \tag{8}$$

(2) Similarly, define $\Phi_C(\theta) = \gamma \cdot \frac{I_I}{I}$, *and* $\Lambda_C(\theta) = \beta \cdot \frac{I_B}{I}$ *the net expected return the expected return to investors and to the bank per unit of capital invested in the project in a partial collusion contract. The project size* I_C *of the optimal partial collusion contract is given by:*

$$I_C = \frac{1}{1 - \frac{\Phi_C(\theta)}{\gamma} - \frac{\Lambda_C(\theta)}{\beta}} \equiv \frac{1}{V_C(\gamma, \beta)} \tag{9}$$

Proof. See the appendix ■

A detailed interpretation of the parameters $\Lambda_{NC}(\theta)$ and $\Phi_{NC}(\theta)$ for the collusion-proof contract, and of $\Lambda_C(\theta)$ and $\Phi_C(\theta)$ for the partial-collusion contract is as follows. The expected return per unit of investment and *net* of monitoring costs provided to the bank to ensure monitoring and collusion proofness is:

$$\Lambda_{NC}(\theta) = \frac{1}{p(1-\theta)}(\theta c + p(\theta) k\Delta B)$$

Indeed, the bank must be compensated for not engaging in collusion ($p(\theta)k\Delta B$) and for monitoring the entrepreneur (θc). Monitoring generates a rent only in presence of bailouts which generate a payment (with probability θ) only if the project fails; in absence of bailout, there is no net rent associated with monitoring.

Simple inspection shows that $\Lambda_{NC}(\theta)$ is increasing in the cost of monitoring c, and in the potential for collusion as measured by $k\Delta B$. It is also increasing in the expected probability of bailout θ for two reasons. First, the probability of receiving a payment is increased. Second, bailouts reduce the incentives for the bank to monitor, increasing therefore the pledgeable income it should receive to satisfy the incentive compatibility constraint of monitoring.

The expected pledgeable income per unit of investment that is left to uninformed investors in the collusion-proof contract is:

$$\Phi_{NC}(\theta) = p(\theta)\left(R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)} \right)$$

Assumption B ensures that it is positive and therefore that there is an active credit market in this economy. $\Phi_{NC}(\theta)$ depends positively on the profitability of investment projects R, and negatively on the extend of the moral hazard problem in production b. Because of moral hazard and collusion in banking, $\Phi_{NC}(\theta)$ also depends negatively on the banking cost of monitoring c and on the potential for collusion $k\Delta B$. The effect of the bailout θ on the expected payment to investors $\Phi_{NC}(\theta)$ is ambiguous. On the one hand, investors receive payments with a higher probability because of the bailout ($p(\theta)$ increases with θ). On the other hand, a larger bailout θ reduces the banks'monitoring incentives (as reflected in the term $-\frac{c+k\Delta B}{p(1-\theta)}$ in the bracket), and therefore increases the expected payment to the bank.

Similar comparative statistics can be realized for $\Lambda_C(\theta)$ and $\Phi_C(\theta)$ in the case of the partial collusion contract.

A comparison between $\Lambda_{NC}(\theta)$ and $\Lambda_C(\theta)$ on the one hand, and between $\Phi_{NC}(\theta)$ and $\Phi_C(\theta)$ on the other hand, illustrates the basic trade-offs associated with collusion. Indeed:

$$\Lambda_C(\theta) - \Lambda_{NC}(\theta) = -qk\Delta B - \frac{\theta}{p(1-\theta)}k\Delta B < 0 \tag{10}$$

Allowing for collusion reduces the expected payment to the bank. There are two effects at hand. First, with a partial-collusion contract, the bank receives a collusion rent with probability $1-q$ but does not receive the equivalent financial rent if the quality of supervision is high (in contrast, in the collusion-proof contract, the financial rent equivalent to the private benefits is received in both states of nature to always incentivize the bank). Hence, the savings generated is equal to the financial rent $k\Delta B$ that is received with probability q when the quality of supervision is high. Second, in presence of bailout, the promised financial return is also paid to the bank with probability θ if the project fails. With partial collusion, the payment in case of a bailout is reduced.

The flipside of the lower expected financial return to the bank is that the expected financial return to the investors may increase if a partial collusion contract is signed. This can be written as:

$$\Phi_C - \Phi_{NC} = \underbrace{\left(\Lambda_{NC}(\theta) - \Lambda_C(\theta)\right)}_{+} \underbrace{- \left(p(\theta) - \widetilde{p}(\theta)\right)\left(R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)}\right)}_{-} \tag{11}$$

The first term is positive and represents the financial savings in the payment to the bank that can be transferred to the investor. The second (negative) term is the expected reduction in the financial payment resulting from lower expected probability of generating a return associated with the project, net of monitoring costs and income of the entrepreneur.

To summarize, allowing for partial collusion has the following effects on the borrowing capacity of the entrepreneur. First, it lowers the financial return promised to banks in case of success and it reduces the monitoring intensity by relaxing the incentive constraint of banks. As a result, the optimal partial-collusion contract leaves a lower expected pledgeable income per unit of investment to the bank compared to the situation without collusion (ie. $\Lambda_C(\theta) < \Lambda_{NC}(\theta)$). This allows the expected pledgeable income of uninformed investors to increase by a corresponding amount (the positive term on the LHS of (11), and therefore tends to improve the borrowing capacity of the entrepreneur. Second, with partial collusion, the probability of success of the project falls from $p(\theta)$ to $\widetilde{p}(\theta)$. This in turn leads to a reduced profitability of projects and a negative effect on the expected pledgeable income that uninformed investors can get (the negative term on the LHS of (11)). This tends to reduce the borrowing capacity of the entrepreneur.

Overall, if observed in equilibrium, collusion must improve the borrowing capacity of the entrepreneur. This is possible if and only if partial-collusion increases the financial return to uninformed investors (e.g. $\Phi_C(\theta) \geq \Phi_{NC}(\theta)$) who require a lower return on capital than banks ($\gamma < \beta$). As we shall see in the next section, a necessary condition is that the probability of collusion is not too high (assumption C):

$$\textbf{Assumption C: } \Phi_C(\theta) \geq \Phi_{NC}(\theta) \Leftrightarrow \frac{\widetilde{p}(\theta)}{p(\theta)} \geq 1 - \frac{\frac{k\Delta B}{p(1-\theta)}}{R - \frac{b}{p} - \frac{c}{p(1-\theta)}}$$

Note that this assumption is more likely to hold when the bailout θ is increased. The reason is that the LHS is an increasing function of θ while the RHS is decreasing in θ.

The overall effect of collusion on the entrepreneur's borrowing capacity depends on parameters' values and on the relative opportunity cost β/γ of "banking finance" instead of "market finance". We now turn to this choice.

The contract chosen ex-ante is the contract that maximizes the expected utility of the entrepreneur per unit of capital invested $U_E = \frac{b}{V_j(\gamma,\beta)}$with $j \in \{NC, C\}$, conditional on the participation and incentive constraints of the bank and the uniformed investors, given the return on bank capital β and the cost of funds γ. This is equivalent to maximizing the total present value of external financiers' expected returns – discounted with the correct interest rates:

$$\frac{\Phi_j(\theta)}{\gamma} + \frac{\Lambda_j(\theta)}{\beta} \tag{12}$$

From this inequality we derive the following proposition:

Proposition 2 *Under Assumptions A-C, partial collusion occurs if and only if the cost of bank capital β exceeds the return on investors capital γ by a margin Ψ, that is if and only if $\beta \geq \gamma \cdot \Psi$ where $\Psi = \frac{\Lambda_{NC}(\theta)-\Lambda_C(\theta)}{\Phi_C(\theta)-\Phi_{NC}(\theta)}$. This margin Ψ is increasing in the quality of bank supervision $\left(\frac{\partial\Psi}{\partial k} < 0\right)$, decreasing in the size of bailouts $\left(\frac{\partial\Psi}{\partial\theta} < 0\right)$, decreasing in the private benefits of control $\left(\frac{\partial\Psi}{\partial\Delta B} < 0\right)$ and increasing in the efficiency of bank monitoring $\left(\frac{\partial\Psi}{\partial c} < 0\right)$.*

Proof. See the Appendix. ■

This conditions states that the contract allowing for collusion is chosen if the present value of the increased payment to the uninformed investors $\frac{\Phi_C(\theta)-\Phi_{NC}(\theta)}{\gamma}$ exceeds the present value of the reduction $\frac{\Lambda_{NC}(\theta)-\Lambda_C(\theta)}{\beta}$ in the expected return to the bank.

The basic trade-off involved in the contract chooice can be obtained from the following equation derived from (11) and (12):

$$(\Lambda_{NC}(\theta) - \Lambda_C(\theta))(1 - \frac{\gamma}{\beta}) \geq (p(\theta) - \widetilde{p}(\theta)) \left(R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)}\right) \tag{13}$$

The LHS of (13) reflects the gain in financial leverage obtained by moving from a collusion-proof contract to a partial-collusion contract, if projects were equally successful under partial-collusion contracts. As said, with collusion the expected pledgeable income per unit of investment to the bank is reduced by $\Lambda_{NC}(\theta) - \Lambda_C(\theta)$ while that to uninformed investors is increased by the same amount. As "uninformed finance" is less costly than "banking finance" (ie. $\gamma < \beta$), this improves the financial leverage of the entrepreneur and increases the equilibrium investment size. Note that the larger the relative cost of banking capital relative to uninformed capital β/γ is, the higher is the leverage gain from shifting one unit of pledgeable income from bankers to nonbankers.The RHS side of (13) reflects the cost of the partial-collusion contract. As bad projects get implemented in some states of nature, the average profitability of projects is reduced by $p(\theta) - \widetilde{p}(\theta)$. Consequently, the expected pledgeable income of uninformed investors is also smaller. This in turn makes it more difficult to get cheaper loans from this "uninformed finance". It follows that, when bank capital becomes relatively expensive relative to uninformed capital and condition (13) is met, an entrepreneur can increase his expected utility by substituting away from bank capital towards uninformed capital. This is more likely to happen the larger β/γ is relative to the threshold Ψ.

Finally, inspection of the threshold Ψ provides the comparative statics on the parameters. Intuitively, a lower quality of bank supervision (ie larger value of k) and a larger value of the potential private benefits of collusion ΔB, will increase the pledgeable income $\Lambda_{NC}(\theta) - \Lambda_C(\theta)$ that can be shifted from the bank to uninformed investors by having a collusion contract, increasing therefore the LHS of (13). At the same time, such changes will reduce $(p(\theta) - \widetilde{p}(\theta)) \left(R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}\right)$, the loss of expected pledgeable income of uninformed investors that is induced by collusion (ie. the RHS of (13)). At a given value of β/γ, both effects make it easier to have an equilibrium collusion contract (and therefore a lower value of the threshold Ψ).

Similarly, a reduced efficiency of bank monitoring (larger value of c) reduce the pledgeable income $R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}$ that can be left to uninformed investors under the collusion proof contract. This again reduces the RHS of (13) and leads to a lower threshold level Ψ above which collusion is chosen in equilibrium. Finally, an increase in the size of the bailout θ has a positive effect on the LHS of (13) $\Lambda_{NC}(\theta) - \Lambda_C(\theta) = \widetilde{p}(\theta)\frac{k\Delta B}{p(1-\theta)}$, as positive bailouts increase relatively more the costs to incentivize banks to monitor under a collusion proof contract than under a partial collusion contract. Hence the leverage gain to shift to a collusion contract is enhanced. At the same time a larger θ also reduces the cost of the partial-collusion contract (the RHS of (13)). First when θ goes up, the difference in terms of probability of payments to investors $p(\theta) - \widetilde{p}(\theta)$ is reduced. Second, the pledgeable income $\left(R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}\right)$ received by uniformed investors is also reduced. In the end, both on the benefit side and the cost side, an increase in θ makes the collusion contract more likely to be implemented in equilibrium. This is consequently associated to a decrease of the threshold Ψ.

4.2 Bank Capital Market Equilibrium

The equilibrium return on bank capital $\beta^*(\gamma)$ is given by $K_B = I_B(\beta,\gamma)$, where the aggregate demand of bank capital $I_B(\beta,\gamma)$ depends on the type of financial contracts chosen by entrepreneurs:

$$\begin{aligned} I_B(\beta,\gamma) &= I_B^{NC}(\beta,\gamma) = \frac{1}{\beta}\frac{\Lambda_{NC}(\theta)}{V_{NC}(\gamma,\beta)} \quad \text{when } \beta < \gamma\cdot\Psi \\ &= I_B^{C}(\beta,\gamma) = \frac{1}{\beta}\frac{\Lambda_{C}(\theta)}{V_{C}(\gamma,\beta)} \quad \text{when } \beta > \gamma\cdot\Psi \\ &= \nu I_B^{NC}(\beta,\gamma) + (1-\nu) I_B^{C}(\beta,\gamma) \text{ with } \nu\in[0,1] \text{ when } \beta = \gamma\cdot\Psi \end{aligned}$$

Note that when $\beta = \gamma\cdot\Psi$, the two types of contracts are optimal and we should therefore consider a mixed equilibrium with $\nu \in [0,1]$, the (endogenous) fraction of contracts which are collusion free.

Inspection of (**??**) provides immediately that in regime $j \in \{NC, C\}$, the equilibrium return on bank capital $\beta_j(\gamma)$ is given by the following expression:

$$\beta_j(\gamma) = \frac{\Lambda_j(\theta)}{1 - \frac{\Phi_j(\theta)}{\gamma}}\left[1 + \frac{1}{K_B}\right]$$

The return on bank capital (i) decreases with the return on the storage technology: a lower γ improves the borrowing capacity of the entrepreneur, and therefore increases the demand for bank capital; (ii) increases with the expected payment $\Lambda_j(\theta)$ to the bank per unit of capital invested; (iii) increases with the expected payment $\Phi_j(\theta)$ to uninformed investors (because a higher expected payment improves the borrowing capacity of the entrepreneur, and therefore raises the demand for bank capital); (iv) decreases with the supply of bank capital.

In equilibrium, some or all firms will prefer partial-collusion contracts if and only if the cost of bank capital relative to uninformed capital is high enough: $\frac{\beta}{\gamma} \geq \Psi$. To characterize the nature of the bank capital market equilibrium, it is useful to define two thresholds $\overline{\gamma}$ and $\underline{\gamma}$ given respectively by

$$\beta_{NC}(\overline{\gamma}) = \overline{\gamma} \cdot \Psi$$

$$\beta_C(\underline{\gamma}) = \underline{\gamma} \cdot \Psi$$

$\overline{\gamma}$ is the cost of uninformed capital below which, starting from an equilibrium without collusion, some firms will start accepting contracts with collusion. Similarly, $\underline{\gamma}$ is the cost of uninformed capital above which some firms accept contracts that are collusion-proof. Note that there exists $\widehat{\gamma} < \underline{\gamma}$ such that for all $\gamma > \widehat{\gamma}$[16], the return on bank capital is lower in a partial-collusion regime than in a collusion-proof regime: $\beta_C(\gamma) < \beta_{NC}(\gamma)$.The following proposition characterizes then the bank capital equilibrium.

Proposition 3 ***Bank capital market equilibrium.*** *There exist $\overline{\gamma}$, $\underline{\gamma}$, with $\widehat{\gamma} < \underline{\gamma} < \overline{\gamma}$ such that: (1) if $\gamma > \overline{\gamma}$, all credit contracts are collusion-proof contracts; (2) if $\gamma < \underline{\gamma}$, all credit contracts are partial-collusion contracts; (3) if $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, a unique mixed equilibrium exists in which a proportion ν of firms chooses contracts that are collusion-proof, and a proportion $1 - \nu^*(\gamma)$ chooses partial-collusion contracts, where $\nu^*(\gamma)$ is an increasing function of γ with $\nu^*(\overline{\gamma}) = 1$, and $\nu^*(\underline{\gamma}) = 0$. In the mixed equilibrium region, domestic bank capital and uninformed capital become substitutes: the return on bank capital β falls as the cost of uninformed finance γ goes down.*

Proof. See the Appendix. ■

Corollary 4 *(1) For all $\gamma > \overline{\gamma}$, or $\gamma < \underline{\gamma}$, the cost of bank capital β is a decreasing function of the cost of external finance γ. (2) For all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, the cost of capital is an increasing function of the cost of external finance γ.*

Proof. See the appendix ■

[16] The value of $\widehat{\gamma}$ is given by :

$$\widehat{\gamma} = \frac{\Phi_C(\theta)\Lambda_{NC}(\theta) - \Phi_{NC}(\theta)\Lambda_C(\theta)}{\Lambda_{NC}(\theta) - \Lambda_C(\theta)}$$

which is positive under assumption C.

The equilibrium return on bank capital β is plotted on Figure 1. We are now equiped to characterize the social optimum and optimal financial regulations.

(Figure 1 about here)

5 Optimal Regulation of Bank Capital

Note that, so far, we have not introduced any externality on the financial side or on the real side of the economy. However, we shall see that, because of a pecuniary externality–individual agents do not internalize the impact of contract choices on the equilibrium return on bank capital– the social optimum may differ from the decentralized outcome, and therefore may justify imposing a capital adequacy ratio.

We characterize two different capital adequacy rules: a fixed capital adequacy ratio, and the optimal capital adequacy ratio. Since the project choice is not verifiable, the regulator can only impose a capital adequacy ratio in which risk weights are all equal, i.e. a leverage ratio. We show that the optimal rule implies a capital adequacy ratio that is pro-cyclical with respect to the return on the storage technology γ but countercyclical with respect to the return on capital R. However, even if the optimal ratio of bank capital to total investment decreases as the return on the storage technology γ falls, the discrepancy between the ratio chosen by the market and the optimal one instead increases as more and more agents choose financial contracts that leave some room for collusion.

5.1 Social Optimum

Formally, the second best socially optimal contract is the one that maximizes the sum of agents' expected utilities under the resource constraints and incentive constraints associated with each type of contract:

$$\max_{j\in\{C,NC\}}\left[bI_j+\beta_j K_B+\gamma I_I^j\right]$$

under the incentives constraints and participation constraints associated with each contract, and the relationship that determines the equilibrium return on bank capital.

From section III, we know that the maximization program above can be simplified into:

$$\max_{j\in\{C,NC\}}\left[b+\Lambda_j\left(\theta\right)+\Phi_j\left(\theta\right)\right]I_j(\gamma,\beta_j(\gamma))$$

with $I_j(\gamma, \beta_j(\gamma))$ the equilibrium level of project size under regime j and the equilibrium return on bank capital given by $\beta_j(\gamma) = \frac{\Lambda_j(\theta)}{1-\frac{\Phi_j(\theta)}{\gamma}}\left[1 + \frac{1}{K_B}\right]$.

We then have

Proposition 5 ***Social Optimality:*** *Under assumption C: i) social optimality implies that contracts allowing some collusion should be adopted when the funding cost falls below a threshold $\gamma^* > 0$. ii) This threshold is strictly below $\underline{\gamma}$.*

Proof. See the appendix ■

When assumption C holds, proposition C says that there exists a rate of return below which collusion is socially optimal. In such a case, the increase in leverage that is allowed by collusion more than outweigh the lower social rate of return associated to these contracts. The social optimum differs from the decentralized equilibrium because of a pecuniary externality: when switching to collusion contracts, agents do not internalize the fact that the return on bank capital is going to fall – because the social return on the project falls, the return per unit of capital invested must fall for some agents in equilibrium, and this is not internalized by the entrepreneur who maximizes leverage (which is equivalent to maximizing the present value of financiers' expected return at *given* rates of return γ and β: see condition 12).

In the following we shall assume that $\gamma > \gamma^*$: partial collusion contracts are never socially optimal.

5.2 Fixed Capital Adequacy Rule

First we consider a fixed capital adequacy rule. The choice of contract is now constrained by the additional condition:

$$\frac{I_B}{I} \geq CAR$$

- Consider first a collusion-proof contract. Combining the capital adequacy rule with the participation constraint

$$p(\theta)R_B - cI \geq \beta I_B$$

implies $p(\theta)R_B - cI \geq \beta CAR \bullet I$ or that $R_B \geq R_B^1 = \frac{\beta CAR + c}{p(\theta)} I$. At the same time, the incentive condition

((2)) implied that $R_B \geq R_B^2 = \frac{k\Delta B+c}{p(\theta)-\theta} I$. Hence the payment to the bank must be such that:

$$R_B = \max\left[\frac{k\Delta B + c}{p(\theta)-\theta} I; \frac{\beta CAR + c}{p(\theta)} I\right]$$

Hence, the capital adequacy ratio rule is binding if and only if:

$$CAR \geq \frac{1}{\beta}\left[\frac{p(\theta)}{p(\theta)-\theta}\left[k\Delta B + c\right] - c\right] = \frac{1}{\beta}\Lambda_{NC}$$

This implies that, in that case, the incentive constraint of the bank is not binding, and that the bank receives an additional rent over and above the payment necessary to avoid collusion. Since the return on bank capital β exceeds the cost of funds γ, this implies that the borrowing capacity of the entrepreneur goes down, and that the size of the investment will decline. More precisely an optimal collusion proof contract with a CAR is characterized as follows:

Proposition 6 ***Optimal collusion proof contract with a*** CAR : *i) For* $\beta \geq \frac{\Lambda_{NC}}{CAR}$, *the* CAR *is binding and the optimal size of investment* $I_{CAR}^N(\beta,\gamma)$ *for a collusion proof contract is such that:*

$$I_{CAR}^N(\beta,\gamma) < I_{NC}(\beta,\gamma)$$

ii)) For $\beta < \frac{\Lambda_{NC}}{CAR}$, *the* CAR *is not binding and the optimal size of investment is* $I_{NC}(\beta,\gamma)$

Proof. See the appendix. ■

- Consider now a collusion contract. Again combining the capital adequacy rule with the participation constraint

$$\widetilde{p}(\theta)R_B - qcI + (1-q)\, k\Delta BI \geq \beta I_B$$

implies $\widetilde{p}(\theta)R_B - qcI + (1-q)\, k\Delta BI \geq \beta CAR \bullet I$ or that $R_B \geq R_B^1 = \frac{\beta CAR + qc - (1-q)k\Delta B}{\widetilde{p}(\theta)} I$. At the same time, the incentive condition ((5a)) implied that $R_B \geq R_B^2 = \frac{c}{p(\theta)-\theta} I$. Hence the payment to the bank must be such that:

$$R_B = \max\left[\frac{c}{p(\theta)-\theta} I; \frac{\beta CAR + qc - (1-q)\, k\Delta B}{\widetilde{p}(\theta)} I\right]$$

Hence, the capital adequacy ratio rule is binding if and only if:

$$CAR \geq \frac{1}{\beta}\left[\frac{1}{p(1-\theta)}\theta c + (1-q)k\Delta B\right] = \frac{1}{\beta}\Lambda_C$$

We have therefore a similar proposition:

Proposition 7 ***Optimal collusion contract under a*** CAR***:*** *i) For* $\beta \geq \frac{\Lambda_C}{CAR}$*, the* CAR *is binding and the optimal size of investment* $I_{CAR}^C(\beta,\gamma)$ *for a collusion contract is such that:*

$$I_{CAR}^C(\beta,\gamma) \leq I_C(\beta,\gamma)$$

ii)) For $\beta < \frac{\Lambda_C}{CAR}$*, the* CAR *is not binding and the optimal size of investment is* $I_C(\beta,\gamma)$

Proof. See the appendix. ■

One may compare now the two types of contracts. We have

Proposition 8 *i) When* $\beta \geq \frac{\Lambda_{NC}}{CAR}$ *the optimal contract is a constrained collusion proof contract.*

ii) When $\beta < \frac{\Lambda_{NC}}{CAR}$ *and* $\beta < \Psi\gamma$*, the optimal contract is a non constrained collusion proof contract*

iii) When $\beta < \frac{\Lambda_C}{CAR}$ *and* $\beta > \Psi\gamma$*, the optimal contract is a non constrained collusion contract*

iv) When $\beta \in \left]\frac{\Lambda_C}{CAR};\frac{\Lambda_{NC}}{CAR}\right[>$ *and* $\beta > \Psi\gamma$*, the optimal contract a constrained collusion contract if*

$$\frac{(1-q)\,p(1-\theta)}{\gamma}\left[R-\frac{b}{p}-\frac{(c+k\Delta B)}{p(1-\theta)}\right]+\left(CAR-\frac{\Lambda_{NC}}{\beta}\right)\left(\frac{\beta}{\gamma}-1\right)>0 \tag{14}$$

otherwise it is a non constrained collusion proof contract.

Proof. See the appendix. ■

The optimal choice contract structure is depicted in figure (2) in the space $\{\gamma,\beta\}$. The capital adequacy ratio CAR is binding for both the collusion contract and the collusion proof contract in region 1 where $\beta \geq \frac{\Lambda_{NC}}{CAR}$. In such a region, collusion is dominated. In region 2 corresponding to $\beta < \frac{\Lambda_{NC}}{CAR}$ and $\beta < \Psi\gamma$, the capital adequacy ratio is not binding for the collusion proof contract, it may or may not be binding for the collusion contract. But as $\beta < \Psi\gamma$ the result of proposition (?) tells us that the collusion proof contract dominates an unconstrained collusion contract. Therefore it should dominate any type of collusion contract (whether it is constrained or not). Region 3 corresponds to the case where $\beta < \frac{\Lambda_C}{CAR}$ and $\beta > \Psi\gamma$. In such a situation, the CAR is binding for neither for the collusion contract nor the collusion proof contract. It follows that because $\beta > \Psi\gamma$ we can be sure that the unconstrained collusion contract dominates the unconstrained collusion proof contract. Finally, there is the last region 4 where $\beta \in \left]\frac{\Lambda_C}{CAR}; \frac{\Lambda_{NC}}{CAR}\right[$ > and $\beta > \Psi\gamma$. In such a region the CAR is binding for the collusion contract but not for the collusion proof contract. The determination of the optimal contract hinges therefore on the comparison between $I^C_{CAR}(\beta,\gamma)$ and $I^C_{NC}(\beta,\gamma)$, that is reflected in the condition (14) that characterizes when the constrained collusion contract dominates the collusion proof contract. Note that because we are in a region where $\beta \geq \Psi\gamma$, the unconstrained collusion contract dominates the collusion proof contract and therefore it is also possible for the constrained collusion contract to also eventually dominate no collusion.

- **Characterization of the banking capital market equilibrium:**

We are now in position to characterize the equilibrium on the banking capital market. The following proposition shows that a sufficiently restrictive fixed capital adequacy ratio eliminates the possibility of financial contracts with collusion.

Proposition 9 ***Banking capital market equilibrium under fixed capital adequacy ratio:*** *i) When* $CAR = \frac{\Lambda_{NC}}{\Psi\underline{\gamma}}$ *the banking capital market equilibrium is associated with collusion-proof contracts only. The equilibrium banking rate of return* $\beta^e_{NC}(\gamma)$ *is given by*

$$\begin{aligned}\beta^e_{NC}(\gamma) &= \beta_{NC}(\gamma) \quad \text{when } \gamma \geq \gamma^{CAR} \\ &= \beta^{CAR}_{NC}(\gamma) \quad \text{when } \gamma < \gamma^{CAR}\end{aligned}$$

with:

$$\gamma^{CAR} = \frac{\Phi_{NC}}{1 - CAR\left[1 + \frac{1}{K_B}\right]}$$

and

$$\beta^{CAR}_{NC}(\gamma) = -\lambda\gamma + \frac{p(\theta)}{CAR}\left[R - \frac{b}{p} - c\right] \text{ and } \lambda = \frac{1}{CAR} - \left[1 + \frac{1}{K_B}\right]$$

Proof. See the appendix ■

A restrictive capital adequacy ratio $CAR \geq \frac{\Lambda_C}{\Psi\underline{\gamma}}$ depresses investment as $I_{CAR} = \frac{1}{V^C_{CAR}(\beta,\gamma)}$ is a declining function of CAR. Hence avoiding collusion may generate high costs in terms of potential output. From a normative point of view, a fixed capital adequacy rule that eliminates collusion depresses total investment and is clearly welfare decreasing for high and low external costs of funds. Indeed for $\gamma \geq \overline{\gamma}$, the market would already provide collusion proof contracts without the eventually binding constraint on bank capital. On the other hand, as proposition (?) showed, for $\gamma \leq \gamma^*$, collusion contracts are socially optimal and therefore one should not eliminate them. For intermediate values of the opportunity cost of funds, (ie. $\gamma^* < \gamma < \overline{\gamma}$), the previous fixed capital adequacy rule eliminates the contracts with collusion. It has therefore the beneficial effects of reducing excessive risk taking but at the cost of a reduced size on total investment. The net effect depends on which effect dominates.

A less tight fixed capital adequacy rule, such that for instance $CAR = \frac{\Lambda_{NC}}{\Psi\overline{\gamma}}$ would not be binding when all agents choose collusion proof contracts (for $\gamma \geq \overline{\gamma}$). However, such a contract will not prevent collusion to occur for some $\gamma < \overline{\gamma}$. Indeed, one can show that for $\gamma = \underline{\gamma}$, the share of bank capital in total investment will exceed the capital adequacy ratio in a collusion contract: $\frac{\Lambda_C}{\Psi\underline{\gamma}} > \frac{\Lambda_{NC}}{\Psi\overline{\gamma}}$. Indeed, making use of the bank capital market equilibrium condition, the condition is equivalent to: $\overline{\gamma}\Phi_C > \underline{\gamma}\Phi_{NC}$, which is always verified. Hence, this capital adequacy rule will not be binding for collusion contracts and γ close to $\underline{\gamma}$.

5.3 Optimal Capital Adequacy Rule

The preceding discussion suggests that an optimal capital adequacy rule should be flexible enough to take into account the macro conditions related to the cost of external fundings. An optimal rule should be such that, for $\gamma > \gamma^*$, investment is maximized under the constraint that no collusion contracts are signed.

For $\gamma > \gamma^*$, an optimal capital adequacy rule must therefore verify:

$$CAR \leq \frac{\Lambda_{NC}}{\beta_{NC}(\gamma)} = \frac{I_B^{NC}}{I_{NC}}$$

while also satisfying

$$CAR > \frac{\Lambda_C}{\beta_C(\gamma)}$$

(to ensure that there is no market equilibrium consistent with collusion contracts). Note that:

$$\frac{\Lambda_C}{\beta_C(\gamma)} < \frac{\Lambda_{NC}}{\beta_{NC}(\gamma)}$$

is equivalent to $\Phi_C > \Phi_{NC}$ which is true under assumption C. Since the optimal CAR should minimize the distortion of investment size under collusion proofness, the optimal capital adequacy rule must therefore be:

$$CAR = \frac{\Lambda_{NC}}{\beta_{NC}(\gamma)} = \frac{1}{1+\frac{1}{K_B}}\left(1-\frac{\Phi_{NC}}{\gamma}\right) \quad \text{for } \gamma > \gamma^* \tag{15}$$

From this we have the following proposition:

Proposition 10 *The optimal capital adequacy rule that prevents collusion (ie; when $\gamma > \gamma^*$) $CAR_{opt} = CAR(\gamma, K_B, R, c, k\Delta B, \theta)$ is i) increasing in the cost of external funds γ, and the stock of banking capital K_B, ii) decreasing in the return on investment R, and iii) increasing in the cost of monitoring c, the rent associated with regulatory forbearance $k\Delta B$, and the non-pledgeable income of the entrepreneur b.*

Proof. See the appendix. ■

Hence, the optimal capital adequacy ratio should be procyclical with respect to the return on uninformed capital γ but countercyclical with respect to the return R on projects in which banks' invest, and should also depend negatively on the quality of banking supervision, on efficiency of banks, and on the quality of corporate governance.

5.4 Optimal Capital Adequacy ratio and Expected Systemic bailouts

Typically bailouts have two effects on the expected financial return to the investors. First, the probability of a financial payment rises, which tends to raise the expected financial return of uninformed investors. Second, bailouts make it more difficult to incentivize the banks. This tends to raise the payment of the bank, and decrease the one of investors. These two effects can be seen in the following derivative:

$$\frac{\partial \Phi_{NC}}{\partial \theta} = \underbrace{(1-p)\left(R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}\right)}_{+} \underbrace{- \frac{p(\theta)}{p}\frac{c+k\Delta B}{(1-\theta)^2}}_{-} \tag{16}$$

The first effect dominates for small bailout guarantees while the second effect dominates for large bailout guarantees. This implies that the optimal capital adequacy rule $CAR(\gamma, K_B, R, c, k\Delta B, \theta)$ is non monotonic in θ. It decreases with small bailouts and increases with large bailouts. More precisely we have the following result :

Proposition 11 *There exists* $\widetilde{\theta} \in]0,1[$ *such that the optimal flexible capital adequacy rule* $CAR(\gamma, K_B, R, c, k\Delta B, \theta)$ *is decreasing in* θ *in the range* $\theta \leq \widetilde{\theta}$ *and increasing in* θ *in the range* $\theta \geq \widetilde{\theta}$.

Proof. See the appendix. ■

We have so far assumed that the probability of a bailout is exogenous and does not depend on individual agents' actions. In the aggregate, however, the more risks agents take, the higher the probability that a systemic bailout takes place ex-post. Formally, we assume that a bailout of size θ takes place if a sufficiently large share of projects fail. That is a bailout is triggered when the value of failed projects to GDP is above a certain threshold x such that

$$\frac{(1-p)\left[R-\frac{b}{p}\right]}{pR} < x < \frac{[q(1-p)+(1-q)]\left[R-\frac{b}{p}\right]}{pqR}$$

The left hand side inequality states that, if all agents choose collusion-proof contracts, the value of failed projects will be below the threshold x, and there will not be a systemic bailout. The right hand side inequality states that if all agents choose partial-collusion contracts, the value of failed projects will exceed the threshold x, and a systemic bailout will take place. We assume that agents have the following beliefs (which are validated in equilibrium): if all contracts are collusion-proofs, there are no systemic bailouts $\theta^e = 0$; if all contracts are collusion contracts, a bailout of size $\theta^e = \theta$ will take place. Under these conditions, we can show, that there exists a range of values of the return on uninformed capital γ within which both equilibrium are possible: an equilibrium with a high proportion of collusion-proof contracts and no systemic bailouts, and and equilibrium with a high proportion of collusion contracts and a systemic bailout guarantee of size θ.

Proposition 12 *Let* $\widehat{\gamma}(\theta^e)$ *be the return on capital below which the market induces a systemic bailout guarantee of size* θ^e. *If* $\theta \leq \widehat{\theta} < 1$, *we have that* $\widehat{\gamma}(\theta) > \widehat{\gamma}(0)$ *so that the two types of equilibria coexist over the range* $]\widehat{\gamma}(0), \widehat{\gamma}(\theta)[$.

Proof. See the appendix. ■

Figure 3 about here

As can be seen immediately from the graph, there is a range of external costs of funds $\gamma \in]\widehat{\gamma}_0, \widehat{\gamma}(\theta)[$ such that both types of market equilibria coexist: one equilibrium with a large fraction of collusive contracts and a systemic bailout and one with mainly collusion proof contracts and no bailout. The intuition for this is simply the fact that the potential for a systemic bailout introduces a basic complementarity between the choices of financial contracts. The larger the fraction of collusive contracts in the economy, the larger the fraction of failed projects and the more likely the triggering of a systemic bailout. The higher expectation of a bailout in turn makes it more difficult to incentivize banks to monitor projects, leading to a higher banking rate of return in equilibrium. This larger rate of return on banking capital in turn makes it more likely to select collusive financial contracts at the individual level. It follows that for intermediate ranges of external funds, the potential for systemic bailouts induces the existence of multiple expectation driven equilibria. When no bailout is expected, the banking capital rate of return is sufficiently low so as to induce firms to choose collusion proof contracts, leading consequently to a high proportion of successful projects and therefore no implementation of a systemic bailout. Conversely when a bailout is anticipated, the equilibrium rate of return offered to banks is increased sufficiently to make it worthwhile for agents to choose collusion financial contracts. The higher proportion of failed contracts in turn triggers the implementation of the bailout.

How should should capital adequacy rules be affected when bailout guarantees are expected?

Two effects are present. On the one hand, bailout guarantees make it more difficult to incentivize banks, therefore a higher financial return is required for banks. This effect tends to reduce the expected financial return to investors, and therefore reduces the overall borrowing capacity of the entrepreneur, leading to a higher share of investment provided by the bank under collusion. This tends to raise the capital to asset ratio with or without collusion, and therefore requires a tighter capital adequacy ratio to deter collusion. On the other hand, bailout guarantees, everything else equal, raise the probability of a payment to investors. This raises the borrowing capacity of the entrepreneur, and therefore the demand for bank capital at any given bank return β. In equilibrium this tends to raise the return on bank capital, reducing therefore the share of investment provided by the bank with collusion. As the share of investment provided by the bank falls, this lowers admissible capital adequacy ratios that can deter collusion.

In the case of fixed capital adequacy rules, applying proposition (?), respectively for a no bailout economy (ie. $\theta = 0$) and an anticipated bailout $\theta > 0$, the minimum fixed capital adequacy ratios detering for sure the collusive market equilibrium are respectively :

$$CAR(0) = \frac{\Lambda_C(0)}{\beta_C(\underline{\gamma}(0), 0)} = \frac{1 - \frac{\Phi_C(0)}{\underline{\gamma}(0)}}{1 + \frac{1}{K_B}}$$

and

$$CAR(\theta) = \frac{\Lambda_C(\theta)}{\beta_C(\underline{\gamma}(\theta), \theta)} = \frac{1 - \frac{\Phi_C(\theta)}{\underline{\gamma}(\theta)}}{1 + \frac{1}{K_B}}$$

The comparison between $CAR(0)$ and $CAR(\theta)$ is equivalent to the comparison between $\frac{\underline{\gamma}(0)}{\Phi_C(0)}$ and $\frac{\underline{\gamma}(\theta)}{\Phi_C(\theta)}$. While $\underline{\gamma}(0) < \underline{\gamma}(\theta)$, the comparison between $\Phi_C(0)$ and $\Phi_C(\theta)$ depends on the size of the bailout. However, when the monitoring cost of banks c is high enough, it can be shown the minimum fixed capital adequacy ratio that deters collusion has to be tightened when a bailout is expected. More precisely, one has the following result:

Proposition 13 *Suppose $(1-q)\, k\Delta B < qc$, then an expected bailout of size θ increases the miminum fixed capital adequacy ratio that deters collusion for all value of the external cost γ (ie. $CAR(\theta) > CAR(0)$).*

Proof. See the appendix. ■

Consider now an optimal flexible capital adequacy rule $CAR(\gamma)$. As already discussed, for any value of $\gamma > \gamma^*$ the optimal flexible capital adequacy rule must verify:

$$\overline{CAR}(\gamma, \theta) = \frac{\Lambda_{NC}(\theta)}{\beta_{NC}(\gamma, \theta)} = \frac{1 - \frac{\Phi_{NC}(\theta)}{\gamma}}{1 + \frac{1}{K_B}}$$

while also satisfying

$$\overline{CAR}(\gamma, 0) = \frac{\Lambda_{NC}(0)}{\beta_{NC}(\gamma, 0)} = \frac{1 - \frac{\Phi_{NC}(0)}{\gamma}}{1 + \frac{1}{K_B}}$$

where θ is the value of the expected bailout. As said bailout guarantees have two opposite effects on the bank capital share of investment I_B^C / I^C under collusion. First a positive value of θ increases that ratio through the effect $\Lambda_C(\theta) > \Lambda_C(0)$). As it is more difficult to incentivize banks, this requires a higher pledgeable income for banks, reducing overall borrowing capacity and therefore a higher bank investment share under collusion. Second bailout guarantees, everything else equal, raise the probability of a payment to investors. ex-ante. This conversely raises the borrowing capacity of the entrepreneur and the demand for bank capital at any given bank return β. In equilibrium the return on bank capital $\beta_C(\gamma, \theta)$ is increased which reduces the share of investment provided by the bank.

It follows that bailout guarantees may not always imply tighter capital requirement to prevent collusion.

Proposition 14 *There exists a threshold* $\theta_f < 1$ *such that for bailouts of size* $\theta \leq \theta_f$: $\overline{CAR}(\gamma, \theta) < \overline{CAR}(\gamma, 0)$ *while for admissible bailouts of size* $\theta > \theta_f$, $\overline{CAR}(\gamma, \theta) > \overline{CAR}(\gamma, 0)$

Proof. See the appendix. ■

Hence it is only when bailouts are of a significant size that the minimum flexible capital adequacy ratio should be tighter.

6 Optimal Financial Regulation with Productive Externalities

6.1 An Economy with Externalities

Consider now that because of production or demand externalities (which could also be financial sector externalities), the return on the project if successful, depends on the number of other successful projects in the economy. Formally, let us assume that:

$$R = \widetilde{R}(X) = R_0 \left(\Omega X\right)^{\epsilon} \quad \text{with } \epsilon \geq 0 \text{ and } \Omega > 0$$

where X is the proportion of successful projects. Given our dichotomous outcomes for projects and using the law of large numbers, it follows that

$$\begin{aligned} R &= R_0 \left(\Omega p\right)^{\epsilon} \text{ if there is no collusion} \\ &= R_0 \left(\Omega pq\right)^{\epsilon} \text{ if there is collusion} \\ &= R_0 \left[\Omega \left(\nu p + (1-\nu) qp\right)\right]^{\epsilon} \text{ in a mixed equilibrium} \\ &\quad \text{with } \nu \text{ is the proportion of projects with no collusion} \end{aligned}$$

Denote then $\overline{R}(\epsilon) = R_0 (\Omega p)^{\epsilon}$ the return if there is no collusion, $\underline{R}(\epsilon) = R_0 (\Omega pq)^{\epsilon}$ the return if no collusion, and $\widetilde{R}(\epsilon, \nu) = R_0 [\Omega (\nu p + (1-\nu) qp)]^{\epsilon}$ the return in the mixed region. Note that $\epsilon \geq 0$ parametrizes the degree of productive externalities in the economy (ie.$\epsilon = 0$ corresponds to an economy with no externalities where and $\overline{R}(\epsilon) = \underline{R}(\epsilon) = R_0$).

Also we assume $p\Omega > 1 > pq\Omega$, that is an increase in the degree of externality ϵ pushes up the return to investment in a no collusion regime while it tends to decrease it in a collusion regime. Finally, to simplify the exposition, assume that there are no bailout guarantees: ie. $\theta = 0$.

For each regime $j \in \{NC, C\}$, we define $\overline{\Lambda}_j$, $\overline{\Phi}_j$, $\overline{V_j}$ respectively the return per unit invested for the bank and for the investor, and the investment multiplier if agents anticipate that all other agents will choose non collusion contracts (and therefore anticipate that the return on a successful project will be $\overline{R}(\epsilon)$. Similarly define $\underline{\Lambda}_j$, $\underline{\Phi}_j$, $\underline{V}_j$ the returns and investment multiplier if the expectation is that all other projects will be collusive projects with an expected return on a successful project equal to $\underline{R}(\epsilon)$.

What is the effect of productive externalities on the likelihood of collusion market equilibria?

Define $\overline{\gamma}(R^e)$ the cost of uninformed capital below which, starting from an equilibrium without collusion, some firms will start accepting contracts with collusion:

$$\beta_{NC}(\overline{\gamma}, R^e) = \overline{\gamma} \cdot \Psi(R^e)$$

where $\beta_{NC}(\gamma, R^e)$ is the equilibrium return on bank capital, $\Psi(R^e) = \frac{\Lambda_{NC} - \Lambda_C}{\Phi_C(R^e) - \Phi_{NC}(R^e)}$, and R^e is the expected return of successful projects.

Similarly, define $\underline{\gamma}(R^e)$ the cost of uninformed capital above which, starting from an equilibrium with collusion, some firms will start accepting contracts with no collusion. $\underline{\gamma}(R)$ is given by the following condition:

$$\beta_C\left(\underline{\gamma}, R^e\right) = \underline{\gamma} \cdot \Psi(R^e)$$

Proposition 15 *Suppose that $q < 1/2$, then there exists a threshold ϵ^* such that for $\epsilon \geq \epsilon^*$ an economy with productive externalities is more likely to generate collusion equilibria than the benchmark economy:*

$$\overline{\gamma}(\overline{R}(\epsilon)) < \underline{\gamma}(\underline{R}(\epsilon))$$

Moreover in the region $\gamma \in \left[\overline{\gamma}(\overline{R}(\epsilon)), \underline{\gamma}(\underline{R}(\epsilon))\right]$, there are multiple equilibria with both types of regimes (collusion and no collusion regimes) possible depending on agents' expectations

Proof. See the appendix. ■

The intuition of the previous proposition is the following. Two effects are at play when the economy exhibits productive externalities. First, at a given return on bank capital, the expectation of many failed projects (which is more likely to happen when there are collusion contracts) lowers the return on good productive projects and therefore worsens the moral hazard problem. Hence, it is more difficult to incentivize banks who must get a higher share of the pledgeable income. This in turn makes monitoring more expensive and increases the benefit of collusion contracts that relax the bank incentive constraint and raise the share of the pledgeable income to uninformed investors. Second, when other contracts are anticipated to be collusion contracts, the overall borrowing capacity of an entrepreneur is lower. This tends to lower the aggregate demand for bank capital, and therefore the equilibrium return on bank capital.

When the equilibrium on bank capital falls (relative to the situation in which only good projects are expected to be undertaken), this tends to reduce the likelihood of observing collusion contracts. This second general equilibrium effect goes therefore in the opposite direction and tends to offset the first direct effect mentionned above. The proposition shows conditions under which the first effect clearly dominate (ie. when q is smaller than 1/2).

The situation is illustrated in figure (?) where we show for each value of $R \in \left\{\underline{R}(\epsilon), \overline{R}(\epsilon)\right\}$, the value of $\Psi(R)\gamma$ (the value of β above which a collusion contract is chosen) and the equilibrium banking capital rates of returns $\beta_j(\gamma, R)$ for $j \in \{NC, C\}$. The bold lines CC and NN show the equilibrium rates of return on bank finance in the regime with collusion and without collusion. Note that there is also a set of mixed equilibria with a positive fraction of collusive and collusion proof contracts as shown by the dotted line that links the two bold parts CC and NN.

When the productive externality is large enough, there are multiple equilibria for the range of external costs of returns $\gamma \in \left[\overline{\gamma}(\overline{R}(\epsilon)); \underline{\gamma}(\underline{R}(\epsilon))\right]$. Intuitively, the productive externality introduces a strategic complementarity between financial contracts. In an environment with collusive (resp. collusion proof) contracts, the individual incentives to choose a collusive (resp. a collusion proof) contract are enhanced. When the strategic complementarity is strong enough, it may countervail the usual market forces of adjustments towards a unique equilibrium in an economy without productive externalities.

6.2 Optimal Capital Adequacy Ratio

How do externalities modify our optimal capital adequacy rule? With externalities, this rule becomes:

$$CAR = \frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma)}$$

where $\overline{\beta}_{NC}(\gamma) = \beta_{NC}\left(\gamma, \overline{R}(\epsilon)\right)$.

Hence, it must be equal to the share of bank finance in total investment under a collusion-proof contract if agents anticipate that other agents will choose the collusion-proof contract and that the return on projects will be high.

The optimal capital adequacy rule will prevent collusion if and only if the required capital ratio is above the share of bank capital under a collusion contracts if all other projects are expected to be non-productive:

$$\frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma)} > \frac{\Lambda_C}{\underline{\beta}_C(\gamma)}$$

where $\underline{\beta}_C(\gamma) = \beta_C\left(\gamma, \underline{R}(\epsilon)\right)$

This condition is equivalent to:

$$\Phi_{NC}\left(\overline{R}(\epsilon)\right) < \Phi_C\left(\underline{R}(\epsilon)\right)$$

i.e. the collusion contract must still be such that it allows to raise the financial return to investors even when bad projects are expected to be undertaken. It is obvious then, that when externalities are large enough, this condition will not hold, therefore the optimal capital adequacy rule will not be sufficient to prevent collusion to occur.

The key problem is that, with lower expected return on projects, the borrowing capacity of entrepreneurs falls, total investment falls, and as the return on bank capital is depressed, this tends to increase the proportion of investment that is, in equilibrium, financed by the bank. If this effect is large enough, we may observe in equilibrium a higher share of investment financed by banks even if contracts are collusion contracts. In that case, the CAR rule becomes consistent with collusion contracts, and is therefore ineffective.

Formally, the CAR rule will become ineffective at preventing collusion if and only if:

$$\Phi_C\left(\underline{R}\left(\epsilon\right)\right) - \Phi_{NC}\left(\overline{R}\left(\epsilon\right)\right) = -(1-q)p\left[\overline{R}\left(\epsilon\right) - \frac{b+c}{p}\right] + pq\left[\underline{R}\left(\epsilon\right) - \overline{R}\left(\epsilon\right)\right] + k\Delta B < 0 \tag{17}$$

This is possible and not inconsistent with the condition:

$$\Phi_C\left(\overline{R}\left(\epsilon\right)\right) - \Phi_{NC}\left(\overline{R}\left(\epsilon\right)\right) = -(1-q)p\left[\overline{R}\left(\epsilon\right) - \frac{b+c}{p}\right] + k\Delta B > 0$$

as $\underline{R}\left(\epsilon\right) - \overline{R}\left(\epsilon\right) < 0$.

The first and the last term of (17) were present before: the first term is the reduction in the expected pledgeable income net of monitoring costs resulting from collusion. the last term is the gain resulting from lower financial return to the bank. The middle term is the externality effect which tends to lower the expected pledgeable income further. If this term is large enough, the CAR becomes ineffective, and does not prevent collusion.

What are the possible solutions?

A first option is to make collusion contract infeasible with good quality audits by the supervisor ($k = 0$). However one can argue the quality of audit and forbearance of the regulator may be something endogenous that may not be politically feasible (see more on this below).

A second option is to make the CAR rule tighter when $\gamma \leq \underline{\gamma}(\underline{R}\left(\epsilon\right))$ (ie that is in the region multiple equilibria start to be possible). Typically a rule such that:

$$CAR = \frac{\Lambda_{NC}}{\underline{\beta}_{NC}(\gamma)}$$

where

$$\underline{\beta}_{NC}(\gamma) = \beta_{NC}\left(\gamma, \underline{R}\left(\epsilon\right)\right)$$

will deter collusion. This rule is however excessively tight if all agents choose the right project (and expect others to do so), and therefore is not optimal.

A third option would be to extend the the flexibility of the CAR rule to make it conditional on the average return on capital (if the latter can be estimated). Define the estimated return on capital $\widehat{R}$ and consider the following "extended" flexible CAR rule:

$$\begin{aligned}
\text{if } \widehat{R} &\approx \overline{R}(\epsilon) \text{ use } CAR = \frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma)} \\
\text{if } \widehat{R} &\approx \underline{R}(\epsilon) \text{ use } CAR = \frac{\Lambda_{NC}}{\underline{\beta}_{NC}(\gamma)}
\end{aligned}$$

This rule will deter the collusion equilibrium and will not be excessively restrictive. Indeed it makes the capital adequacy requirement tighter when the return on capital is lower (which may happen towards the end of a boom). The fundamental reason for making the capital regulation conditional on the return on capital is general in moral hazard economies. Indeed, moral hazard is higher when the return on capital is lower, making it more likely to generate collusive contracts. This depresses further the return to capital when productive externalities are present in the economy, leading to an evenmore severe moral hazard problem at the level of individual contracts and therefore the necessity of tighter constraints on banks to eliminate the collusive behaviors.

A fourth option would be to keep the initial capital requirement, but to impose that a portion μ of bank capital is invested in an alternative technology such as T bills if $\widehat{R} \approx \underline{R}(\epsilon)$. Indeed , taking explicitely the dependence of the equilibrium banking rates on the stock of bank capital K_B consider the portion $\overline{\mu}$ of bank capital invested in T bills is such that

$$\frac{\Lambda_C}{\underline{\beta}_C(\gamma, (1-\overline{\mu})K_B)} = \frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma, K_B)}$$

Then the flexible CAR rule:

$$CAR = \frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma, K_B)}$$

plus the imposition of a portion $\mu > \overline{\mu}$ of bank capital is invested in T bills if $\widehat{R} \approx \underline{R}(\epsilon)$, will also deter the collusive equilibria.

7 Political Economy of Banking Supervision

In this section we characterize agents' preferences over the quality of banking supervision. Let us return to an economy with no externalities (ie. $\epsilon = 0$) and no bailout guarantees (ie. $\theta = 0$). This will give us some insights into the political economy of banking supervision quality.

Assume that the quality of banking supervision $1-k$ is constrained to be in an interval $[1-k_{\max}, 1-k_{\min}]$, or alternatively, that the degree of regulatory forbearance $k \in [k_{\min}, k_{\max}]$. We characteriz the equilibrium utility of each type of agents: $U_E(k)$ for entrepreneurs, $U_B(k)$ for banks and $U_I(k)$ for the uninformed investors as function of k (the degree of regulatory forbearance). We show that the structure of preferences depends on the type of market equilibrium regimes that agents anticipate. To understand the basic intuition, note that:

$$\frac{\partial \Lambda_{NC}}{\partial k} = -\frac{\partial \Phi_{NC}}{\partial k} = \Delta B > 0 \tag{18}$$

When contracts are collusion-proof, a higher degree of regulatory forbearance redistributes the financial return from uninformed investors to the bank. Furthermore, in a partial equilibrium at a given β, a higher financial return for the bank reduces the borrowing capacity of the entrepreneur (because the cost of bank capital β exceeds the market cost of capital γ):

$$\frac{\partial V_{NC}}{\partial k} = \frac{\partial \Lambda_{NC}}{\partial k} \cdot \left(\frac{1}{\gamma} - \frac{1}{\beta}\right) > 0$$

Consider now a collusion contract. A higher degree of regulatory forbearance increases the private benefits of undertaking the bad project received by the bank. This has however no impact on the financial return received by uninformed investors:

$$\frac{\partial \Lambda_C}{\partial k} = (1-q)\Delta B > 0 \text{ and } \frac{\partial \Phi_C}{\partial k} = 0 \tag{19}$$

The partial equilibrium effect is thus to increase the borrowing capacity of the entrepreneur:

$$\frac{\partial V_C}{\partial k} = -\frac{1}{\beta} \cdot \frac{\partial \Lambda_C}{\partial k} < 0$$

Finally, we also know from proposition 2 that a lower supervision quality makes collusion more likely:

$$\Psi'(k) < 0$$

We now have the main ingredients to solve the political general equilibrium, where $\beta_j(\gamma) = \frac{\Lambda_j}{1-\frac{\Phi_j}{\gamma}}\left[1 + \frac{1}{K_B}\right]$.

- *Collusion proof market equilibrium regime*

Consider first the case where agents anticipate to be in a collusion-proof regime. This will occur when the cost of funds γ is high and that all financial contracts are collusion proof. This will occur indeed when $\gamma \geq \overline{\gamma}(k)$ with $\overline{\gamma}(k)$ the lowest cost of funds such that an equilibrium with collusion proof contracts prevails:

$$\Psi \cdot \overline{\gamma}(k) = \beta^{NC}\left(\overline{\gamma}(k)\right)$$

It is possible to show that, for a given value of γ, a collusion proof regime occurs when (see appendix for a formal proof):

$$k < \overline{k}(\gamma) \quad \text{with } \overline{k}'(\gamma) > 0$$

Hence, when the cost of capital is higher, a collusion-proof regime can be sustained for a higher degree of regulatory forbearance.

Taking into account the general equilibrium effect on the cost of bank capital, the utilities of each category of agents become:

$$\begin{aligned} U_E(k) &= b \cdot I_{NC} = \frac{b}{1-\frac{\Phi_{NC}}{\gamma}}\left[K_B+1\right] \\ U_B(k) &= \Lambda_{NC} \cdot I_{NC} = \frac{\Lambda_{NC}}{1-\frac{\Phi_{NC}}{\gamma}}\left[K_B+1\right] \\ U_I(k) &= \Phi_{NC} \cdot I_{NC} = \frac{\Phi_{NC}}{1-\frac{\Phi_{NC}}{\gamma}}\left[K_B+1\right] \end{aligned} \tag{20}$$

Using (20), and (18) simple differentiation immediately implies that $\frac{U_E'(k)}{U_E(k)} < 0$, $\frac{U_B'(k)}{U_B(k)} \gtrless 0$, and $\frac{U_I'(k)}{U_I(k)} < 0$ (see appendix for a formal proof).

Typically better supervision quality (ie. a lower value of k) unambiguously improves the borrowing capacity I_{NC} of the entrepreneur by reducing the rent that must be left to the bank.

Indeed

$$\frac{1}{I_{NC}}\frac{\partial I_{NC}}{\partial k} = \frac{\frac{1}{\gamma}\frac{\partial \Phi_{NC}}{\partial k}}{1-\frac{\Phi_{NC}}{\gamma}} < 0$$

Hence entrepreneurs unambiguously prefer a higher supervision quality.

Investors also prefer a higher supervision quality, because better supervision, besides increasing total investment, also improves the proportion of profits pledged to uninformed investors as:

$$\frac{\partial \Phi_{NC}}{\partial k} = -\Delta B < 0$$

In the case of banks, the overall effect of better supervision is ambiguous:

$$\frac{U_B'(k)}{U_B(k)} = \underbrace{\frac{\frac{\partial \Lambda_{NC}}{\partial k}}{\Lambda_{NC}}}_{+} + \underbrace{\frac{\frac{1}{\gamma}\frac{\partial \Phi_{NC}}{\partial k}}{1-\frac{\Phi_{NC}}{\gamma}}}_{-} \tag{21}$$

Hence, higher supervision (associated to a lower value of k) first reduces the rents left to banks to prevent collusion (the first term in (21)). This has a negative effect on the banking sector payoff. On the other hand, higher supervision also increases the borrowing capacity of entrepreneurs, leading to a higher demand for banking capital and a positve effect on the equilibrium return $\beta_{NC}(\gamma)$ for banks (the second term in (21)). This second effect therefore increases the banking sector payoff.

When the return on physical capital R is large enough, the expected pledgeable return on the project net of monitoring cost $\Lambda_{NC} + \Phi_{NC}$ exceeds the opportunity cost of funds γ and the positive effect of supervision quality on overall investment (which is positively related to the return on bank capital) outweighs the negative effect on the share of profits pledged to the bank. In such a case banks also prefer a high supervision quality.

- *Collusion contracts market equilibrium regime:*

Consider now the case where agents anticipate to be in a collusion regime. This will occur when the cost of funds γ is relatively low and collusion-contracts may enhance the borrowing capacity.

More specifically, this will occur when the cost of funds γ is such that $\gamma \leq \underline{\gamma}(k)$ with $\underline{\gamma}(k)$ the highest cost of funds such that an equilibrium with collusion contracts prevails:

$$\Psi \cdot \underline{\gamma}(k) = \beta^C \left(\underline{\gamma}(k)\right)$$

It follows that for a given value of γ, a collusion regime occurs when (see appendix for a formal proof):

$$k > \underline{k}(\gamma) \quad \text{with } \underline{k}'(\gamma) > 0$$

Consider again the utility of each group of agents as function of k (the degree of regulatory forbearance) when taking into account the equilibrium return on bank capital $\beta_C(\gamma)$:

$$\begin{aligned} U_E(k) &= bI_C = \frac{b}{1-\frac{\Phi_C}{\gamma}}\cdot[K_B+1] \\ U_B(k) &= \Lambda_C I_C = \frac{\Lambda_C}{1-\frac{\Phi_C}{\gamma}}\cdot[K_B+1] \\ U_I(k) &= \Phi_{NC} I_{NC} = \frac{\Phi_C}{1-\frac{\Phi_C}{\gamma}}\cdot[K_B+1] \end{aligned} \quad (22)$$

It follows immediately from (22) that: $\frac{U_E'(k)}{U_E(k)}=0$, $\frac{U_B'(k)}{U_B(k)}=\frac{\frac{\partial\Lambda_C}{\partial k}}{\Lambda_C}>0$, and: $\frac{U_I'(k)}{U_I(k)}=0$ (see appendix for a formal proof).

In the general equilibrium, entrepreneurs and uniformed investors are indifferent with respect to the quality of banking supervision while banks are opposed to better supervision.

As already discussed, conditional on a collusion contract, an entrepreneur would like to reduce the costs of adopting collusion contracts, including the cost of a bank audit, at a given cost of banking capital. This indeed allows more financial leverage and a higher borrowing capacity. In equilibrium though, the higher investment capacity leads to a higher demand for banking capital which in turn leads to an increase in the return to banking capital. Given a fixed supply of banking capital, this general equilibrium effect exactly offsets the benefit of higher financial leverage, and entrepreneurs are indifferent about the quality of banking supervision.

Similarly, in the case of uninformed investors, these higher private benefits of banks associated with a higher k do not lower their expected financial return as the bank is incentivized only when the quality of supervision is high. Hence the financial return of external finance Φ_C per unit of investment is not affected.

Finally, it is clear that banks will prefer a low quality of supervision as they obtain larger private benefits while the total investment is unchanged.

- *Mixed market equilibrium regime:*

When $k \in \left[\overline{k}(\gamma); \underline{k}(\gamma)\right]$, the banking market equilibrium is such that $\gamma \in \left[\underline{\gamma}(k), \overline{\gamma}(k)\right]$, and a unique mixed equilibrium exists in which a proportion $\nu(k)$ of firms chooses contracts that are collusion-proof, and a proportion $1-\nu(k)$ chooses partial-collusion contracts. In such an equilibrium the equilibrium rate of return of banks is $\beta = \Psi(k)\gamma$ and $\nu(k)$ is given by the banking capital market equilibrium :

$$\begin{aligned} \frac{\nu\Lambda_{NC}(k) + (1-\nu)\Lambda_C(k)}{\beta} I(k) &= K_B, \text{ with } I(k) = \frac{1}{1 - \frac{\Phi_{NC}(k)}{\gamma} - \frac{\Lambda_{NC}(k)}{\beta}} \\ \text{and } \beta &= \Psi(k)\gamma \end{aligned}$$

[17]. It is then immediate to see that

$$\begin{aligned} U_E(k) &= bI(k) = \frac{b}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C(k)}{\Psi(k)\gamma}} \\ U_B(k) &= \beta K_B = \Psi(k)\gamma K_B \\ U_I(k) &= \frac{\nu(k)\Phi_{NC}(k) + (1-\nu(k))\Phi_C}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C(k)}{\Psi(k)\gamma}} \end{aligned} \tag{23}$$

Note that

$$\begin{aligned} \frac{1}{I(k)} &= 1 - \frac{1}{\gamma}\left[\Phi_{NC}(k) + \frac{\Lambda_{NC}(k)}{\Psi(k)}\right] \\ &= 1 - \frac{1}{\gamma}\frac{\Lambda_{NC}(k)\Phi_C - \Lambda_C(k)\Phi_{NC}(k)}{\Lambda_{NC}(k) - \Lambda_C(k)} \end{aligned}$$

Therefore $I(k)$ is increasing in k. It follows that

$$U_E'(k) = bI'(k) > 0$$

In the mixed regime, entrepreneurs are in favor of more relaxed banking supervision (ie. larger values of k) as this increases their financial leverage.

For banks, we immediately have

$$U_B'(k) = \Psi'(k)\gamma K_B < 0$$

Interestingly, in the mixed equilibrium, banks are in favor of better banking supervision. To get an intuition of this result, it is interesting to rewrite the banks' payoffs as

$$U_B(k) = [\nu(k)\Lambda_{NC}(k) + (1-\nu(k))\Lambda_C(k)]I(k) = \beta(k)K_B$$

[17] Note that by definition of $\Psi(k)$, at $\beta = \Psi(k)\gamma$ one has also

$$\begin{aligned} I(k) &= \frac{1}{1 - \frac{\Phi_{NC}(k)}{\gamma} - \frac{\Lambda_{NC}(k)}{\beta}} \\ &= \frac{1}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C(k)}{\beta}} \end{aligned}$$

In this regime, a reduction of k associated to better banking supervision has three effects on banks' payoffs. First, better supervision reduces the financial leverage and scale of investment $I(k)$ and therefore leads to a reduced payoff to the banks. Also increased costs of audits reduce the private benefits of banks both for collusion proof contracts $\Lambda_{NC}(k)$ and for collusion contracts $\Lambda_C(k)$. Finally, better banking supervision leads also to a larger proportion of collusion proof contracts $\nu(k)$ which provide in turn higher pledgeable income per unit of investment to the banks than under collusion contracts (as $\Lambda_{NC}(k) - \Lambda_C(k) > 0$). Indeed, it is simple to see that $\nu(k)$ is $\nu(k)$ is decreasing in k (see the appendix) It turns out that in the mixed regime, conditions are such that this last compositional effect more than offset the first two effects and banks are in favour of better supervision in this regime.

Finally, consider the position of uniformed investors in the mixed regime. One gets

$$U_I(k) = \left[\nu(k)\Phi_{NC}(k) + (1 - \nu(k))\Phi_C\right] I(k)$$

It can be shown that in the mixed regime $U_I(k)$ is increasing in k and uniformed investors are in favor of relaxed supervision on banks (see the appendix). The intuition for this is again the fact that a better quality in banking supervision (ie. a reduced value of k) again has three effects on the investor's payoff. First, there is the positive effect that it increases the return to investment $\Phi_{NC}(k)$ for collusion proof contracts. Second however, there is the negative effect that it reduces the financial leverage and the scale of investment $I(k)$. Finally there is the composition effect that it increases the proportion of collusion proof contracts. As $\Phi_{NC}(k) < \Phi_C(k)$, this compositional effect also affects negatively the utility of the investor. It turns out that the two negative effects (scale and compositional) offset the first positive return effect. Investors are therefore in favor of relaxed banking supervision and a larger value of k.

Taking together the previous discussion, one has the following proposition on the different groups political preferences for the quality of banking regulation.

Proposition 16 *The political preferences of agents for the quality of banking regulation are the following:*

- *For entrepreneurs:*

$$\begin{aligned}
\textit{collusion proof regime (ie. } k &< \overline{k}(\gamma)\textit{)}: \ U_E'(k) \leq 0 \\
\textit{mixed equilibrium regime (ie. } k &\in \left[\overline{k}(\gamma); \underline{k}(\gamma)\right] \textit{)}: \ U_E'(k) \geq 0 \\
\textit{collusion regime (ie. } k &> \underline{k}(\gamma)\textit{)}: \ U_E'(k) = 0
\end{aligned}$$

- *For uniformed investors:*

$$\begin{aligned}
\textit{collusion proof regime (ie. } k &< \overline{k}(\gamma)\textit{)}: \ U_I'(k) \leq 0 \\
\textit{mixed equilibrium regime (ie. } k &\in \left[\overline{k}(\gamma); \underline{k}(\gamma)\right] \textit{)}: \ U_I'(k) \geq 0 \\
\textit{collusion regime (ie. } k &> \underline{k}(\gamma)\textit{)}: \ U_I'(k) = 0
\end{aligned}$$

- *For banks :*

$$\begin{aligned}
\textit{collusion proof regime (ie. } k &< \overline{k}(\gamma)\textit{)}: \ U_B'(k) < 0 \textit{ when } R > R^* \\
\textit{mixed equilibrium regime (ie. } k &\in \left[\overline{k}(\gamma); \underline{k}(\gamma)\right] \textit{)}: \ U_B'(k) \leq 0 \\
\textit{collusion regime (ie. } k &> \underline{k}(\gamma)\textit{)}: \ U_B'(k) > 0
\end{aligned}$$

The different preferences are depicted in figure (7a) (7b) and (7c). It follows immediately from their inspection that agents do not have unimodal preferences about the quality of banking regulation. An interesting implication of this is the fact that depending on the structure of the audit technology $[k_{\min}, k_{\max}]$ and the value of the cost of funds γ, one may end up with very different political support for or against better quality of banking supervision.

For instance when $k_{\min} > \underline{k}(\gamma)$, then the political incentives in the economy are strongly in favor of relaxed banking auditing, as two groups of agents(entrepreneurs and investors) are indifferent and banks are in favor of the minimum possible cost of auditing $1 - k_{\max}$. Note that such situation can also occur when $k_{\min} > \overline{k}(\gamma)$ and that entrepreneurs and investors have enough political power to impose their political positions. In such a case again, the political outcome is likely to be a weak quality of banking supervision $k_{\max}$. The economy will end up in a market equilibrium with collusion financial contracts (full or partial).

On the opposite if $k_{\max} < \overline{k}(\gamma)$, and the return to physical capital R is large enough, there is again a consensus in society to pick the most stringent banking supervision level $k_{\min}$. In such an economy the market equilibrium will only have collusion proof contracts.

Interestingly, whether we end up in a situation with political support for relaxed banking supervision or a situation with stricter financial supervision, depends on the level of the external cost of fundings (the interest rate γ).

Specifically for low interest rates $\gamma < \underline{k}^{-1}(k_{\min})$, the political equilibrium is likely to support weak banking supervision and a collusive equilibrium. On the opposite, for high interest rates $\gamma > \overline{k}^{-1}(k_{\max})$, the economy will be in favor of stricter banking supervision and the banking capital market is characterized by collusion proof contracts. Such political economy arguments therefore reinforce the effect of lower interest rates on risk taking ad discussed in the previous sections.

What are the implications for financial regulation? We have shown that, in absence of productive externalities, the optimal capital adequacy rule is given by $CAR^* = \frac{\Lambda_{NC}}{\overline{\beta}_{NC}(\gamma, K_B)}$ which depends positively on k. Since the political process will tend to weaken the quality of supervision when interest rates are low, this implies that the optimal capital adequacy rule will have to be tightened as supervision quality worsens during the boom.

8 Conclusion

The financial crisis has resulted in an intense policy effort to better regulate and supervise financial systems. This paper presents a theory of excessive risk taking by financial institutions resulting from collusion and imperfections in banking supervision. When the cost of capital or the return on investment are low, financial institutions and borrowers may collude and undertake projects with negative net present value. This is possible because auditing of bank accounts is stochastic and imperfect. Because individual agents do not take into account the effect of collusion on the equilibrium return on bank capital (a pecuniary externality), the market outcome is not necessarily efficient.

Under such conditions we show that the optimal capital adequacy rule (which is a leverage constraint) is *pro-cyclical* with respect to the cost of capital but *counter-cyclical* with respect to the return on productive activities. This suggests that the Basel III proposal for countercyclical capital buffer is not as conceptually straightforward as it may seem.

We make the following arguments. First, policy-makers cannot commit not to bailout financiers ex-post. The anticipation of systemic bailout guarantees requires tighter capital adequacy rules ex-ante, if the size of the bailout is sufficiently large. Second, in presence of productive externalities, banks and borrowers are more likely to collude. In that case, the optimal financial regulation must explicitely depend on the estimated return on projects. If this is not possible, the capital adequacy ratio must be complemented by a asset allocation constraint. Third, the political economy of banking supervision results in a weakening of supervision quality when it should instead be strenghtened (i.e. when interest rates or the return on capital are low). Consequently, capital adequacy ratio should be tightened, which could be possible if regulations are less sensitive to the political environment than supervision quality.

Appendix

- **Proof of proposition 1:**

(A) Consider the optimal collusion proof contract with investment size I_{NC}. Combining the incentive constraints of the entrepreneur (1) and of the bank (2), the minimum ex-post payoff that needs to be left to the bank in order to induce monitoring with no collusion is given by:

$$R_B = \frac{c + k\Delta B}{p(1-\theta)} \cdot I_{NC}$$

Hence the expected pledgeable amount that has to be left to the bank is:

$$p(\theta)R_B - cI_{NC} = \frac{\{\theta c + p(\theta)k\Delta B\}}{p(1-\theta)} \cdot I_{NC} = \Lambda_{NC}(\theta) \cdot I_{NC}$$

Using the participation constraint of the bank (3), one then obtains the size of bank loans:

$$I_B = \frac{\Lambda_{NC}(\theta)}{\beta} \cdot I_{NC}$$

The pledgeable income left to uniformed investors is:

$$R_I = RI_{NC} - R_B - R_E = \left(R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)} \right) I_{NC}$$

The size of the uninformed investors investment is obtained from the participation constraint of the uninformed investors (4):

$$I_I = \frac{p(\theta)}{\gamma} = \frac{\Phi_{NC}(\theta)}{\gamma} \cdot I_{NC}$$

The project size under the optimal collusion contract satisfies:

$$\begin{aligned} I_{NC} &= 1 + I_B + I_I \\ &= 1 + \frac{\Lambda_{NC}(\theta)}{\beta} \cdot I_{NC} + \frac{\Phi_{NC}(\theta)}{\gamma} \cdot I_{NC} \end{aligned}$$

From which we get:

$$I_{NC} = \frac{1}{1 - \frac{\Phi_{NC}(\theta)}{\gamma} - \frac{\Lambda_{NC}(\theta)}{\beta}} \equiv \frac{1}{V_{NC}(\beta, \gamma)}$$

(B) Consider now the optimal partial collusion contract with investment size I_C. Following the same line of reasoning, and using the incentive constraints of the entrepreneur (**??**) and of the bank (5a), we characterize the minimum ex-post payoff that needs to be left to the bank in order to induce monitoring in the state of with perfect auditing:

$$R_B = \frac{c}{p(1-\theta)} \cdot I_C$$

Now the expected pledgeable amount that has to be left to the bank is given by $\widetilde{p}(\theta) R_B - qcI_C + (1-q) k\Delta BI_C$. The bank is paying the monitoring cost cI_C only in the state of nature with perfect auditing while it enjoys bribes $k\Delta BI_C$ in the state of nature without perfect auditing. This can be written as:

$$\begin{aligned} \widetilde{p}(\theta) R_B - qcI_C + (1-q) k\Delta BI_C &= \left[\frac{1}{p(1-\theta)}\theta c + (1-q)k\Delta B\right] \cdot I_C \\ &= \Lambda_C(\theta) \cdot I_C \end{aligned}$$

Using then (6), the size of bank loans is given by:

$$I_B = \frac{\Lambda_C(\theta)}{\beta} \cdot I_C$$

Similarly, under partial collusion, the pledgeable income that is left to uniformed investors is:

$$R_I = RI_C - R_B - R_E = \left[R - \frac{c + b + k_L\Delta B}{\Delta p}\right] \cdot I_C$$

From (7), one obtains the size of the uninformed investors investment:

$$I_I = \frac{\widetilde{p}(\theta)}{\gamma} R_I = \frac{\Phi_C(\theta)}{\gamma} \cdot I_{NC}$$

Using $I_C = 1 + I_B + I_I$ provides immediately:

$$I_C = \frac{1}{1 - \frac{\Phi_{NC}(\theta)}{\gamma} - \frac{\Lambda_{NC}(\theta)}{\beta}}$$

QED.

- **Proof of proposition 2:** To be written

- **Proof of Proposition 3 and Corollary 1:**

In regime $j \in \{NC, C\}$, the equilibrium return on bank capital $\beta_j(\gamma)$ must be given by the following expression:

$$\beta_j(\gamma) = \frac{\Lambda_j(\theta)}{1 - \frac{\Phi_j(\theta)}{\gamma}} \left[1 + \frac{1}{K_B}\right]$$

which is a decreasing function of the return on uninformed capital γ.

Define then the two thresholds $\overline{\gamma}$ and $\underline{\gamma}$ given respectively by

$$\begin{aligned} \beta_{NC}\left(\overline{\gamma}\right) &= \overline{\gamma}\cdot\Psi \\ \beta_{C}\left(\underline{\gamma}\right) &= \underline{\gamma}\cdot\Psi \end{aligned}$$

It follows that

$$\begin{aligned} \overline{\gamma} &= \Phi_{NC}\left(\theta\right)+\frac{\Lambda_{NC}\left(\theta\right)\left[1+\frac{1}{K_B}\right]}{\Psi}, \\ \underline{\gamma} &= \Phi_{C}\left(\theta\right)+\frac{\Lambda_{C}\left(\theta\right)\left[1+\frac{1}{K_B}\right]}{\Psi}. \end{aligned}$$

i) First, note that $\widetilde{\gamma}<\underline{\gamma}$. Indeed:

$$\widetilde{\gamma}<\underline{\gamma}\Leftrightarrow\Phi_{C}\left(\theta\right)+\frac{(\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right))\Lambda_{C}\left(\theta\right)}{\Lambda_{NC}\left(\theta\right)-\Lambda_{C}\left(\theta\right)}<\Phi_{C}\left(\theta\right)+\frac{\Lambda_{C}\left(\theta\right)\left[1+\frac{1}{K_B}\right]}{\Psi}$$

It is easy to see that the second inequality is satisfied as it is equivalent to

$$\frac{(\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right))\Lambda_{C}\left(\theta\right)}{\Lambda_{NC}\left(\theta\right)-\Lambda_{C}\left(\theta\right)}<\frac{(\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right))\Lambda_{C}\left(\theta\right)}{\Lambda_{NC}\left(\theta\right)-\Lambda_{C}\left(\theta\right)}\left[1+\frac{1}{K_B}\right]$$

which is always true.

Similarly it is easy to show that $\overline{\gamma}>\underline{\gamma}$. Indeed,

$$\overline{\gamma}>\underline{\gamma}\Leftrightarrow\Phi_{NC}\left(\theta\right)+\frac{\Lambda_{NC}\left(\theta\right)\left[1+\frac{1}{K_B}\right]}{\Psi}>\Phi_{C}\left(\theta\right)+\frac{\Lambda_{C}\left(\theta\right)\left[1+\frac{1}{K_B}\right]}{\Psi}$$

Again the second inequality is equivalent to

$$\frac{(\Lambda_{NC}\left(\theta\right)-\Lambda_{C}\left(\theta\right))\left[1+\frac{1}{K_B}\right]}{\Psi}>\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right)$$

or

$$(\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right))\left[1+\frac{1}{K_B}\right]>\Phi_{C}\left(\theta\right)-\Phi_{NC}\left(\theta\right)$$

which is always true.

ii) if $\gamma>\overline{\gamma}$, then $\beta_{C}\left(\gamma\right)<\gamma\Psi$. It follows that only a collusion proof equilibrium is possible in such region with a bank return $\beta_{NC}\left(\gamma\right)<\gamma\Psi$.

iii) Similarly when $\gamma < \underline{\gamma}$, then as $\overline{\gamma} > \underline{\gamma}$ one has $\beta_{NC}(\gamma) > \gamma\Psi$. It follows that only a collusion equilibrium is possible in such region with a bank return $\beta_C(\gamma) > \gamma\Psi$.

iv) Assume now that $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. The proportion of firms ν of firms choosing collusion-proof contracts is given by the equilibrium on the credit market, and the condition that firms must be indifferent between the collusion-proof contract and the partial collusion contract in equilibrium:

$$
\begin{aligned}
K_B &= \left[\nu I_B^{NC} + (1-\nu) I_B^C\right] \\
\beta &= \gamma \cdot \Psi
\end{aligned}
$$

From $\beta = \gamma \cdot \Psi$, we get $V_{NC} = V_C$. Hence in the mixed regime, total investment size is the same under both types of contracts

$$
I^{NC} = I^C = \frac{1}{1 - \frac{\Phi_{NC}(\theta)}{\gamma} - \frac{\Lambda_{NC}(\theta)}{\gamma\Psi}} \tag{24}
$$

Substituting (24), the equilibrium condition on the bank capital market writes as:

$$
K_B = \frac{\nu\Lambda_{NC}(\theta) + (1-\nu)\Lambda_C(\theta)}{\gamma\Psi - \Psi\Phi_{NC}(\theta) - \Lambda_{NC}(\theta)}
$$

which gives

$$
\nu = \nu^*(\gamma) = \frac{\left[\gamma\Psi - \Psi\Phi_{NC}(\theta) - \Lambda_{NC}(\theta)\right] K_B - \Lambda_C(\theta)}{\Lambda_{NC}(\theta) - \Lambda_C(\theta)}
$$

which is an increasing function of γ. The mixed equilibrium prevails when $\nu^*(\gamma) \in [0,1]$.Straightforward computations show that $\nu^*(\underline{\gamma}) = 0$ while $\nu^*(\overline{\gamma}) = 1$. Hence the mixed equilibrium prevails for $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

v) Note also that for all $\gamma > \overline{\gamma}$, or $\gamma < \underline{\gamma}$, the equilibrium interest rate β is given by

$$
\beta_j(\gamma) = \frac{\Lambda_j(\theta)}{1 - \frac{\Phi_j(\theta)}{\gamma}} \left[1 + \frac{1}{K_B}\right] \quad \text{for } j \in \{C, NC\}
$$

which is a decreasing function of the cost of external finance γ

and ii) for $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, the equilibrium interest rate $\beta = \gamma \cdot \Psi$ which is an increasing function of the cost of external finance γ. **QED.**

- **Proof of proposition 5 on social optimality**

i) To characterize whether collusion contracts are socially better than collusion proof contracts, one needs to compare

$$\begin{aligned}\frac{\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b}{V_{NC}(\gamma,\beta_{NC}(\gamma))} &= (\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b)\frac{\beta_{NC}(\gamma)}{\Lambda_{NC}(\theta))}K_B \\ &= \frac{(\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b)}{1-\frac{\Phi_{NC}(\theta)}{\gamma}}(1+K_B)\end{aligned}$$

to

$$\begin{aligned}\frac{\Lambda_{C}(\theta)+\Phi_{C}(\theta)+b}{V_{C}(\gamma,\beta_{C}(\gamma))} &= (\Lambda_{C}(\theta)+\Phi_{C}(\theta)+b)\frac{\beta_{C}(\gamma)}{\Lambda_{C}(\theta))}K_B \\ &= \frac{(\Lambda_{C}(\theta)+\Phi_{C}(\theta)+b)}{1-\frac{\Phi_{C}(\theta)}{\gamma}}(1+K_B)\end{aligned}$$

therefore the collusion contract is socially optimal if and only if

$$\frac{(\Lambda_{C}(\theta)+\Phi_{C}(\theta)+b)}{1-\frac{\Phi_{C}(\theta)}{\gamma}} \geq \frac{(\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b)}{1-\frac{\Phi_{NC}(\theta)}{\gamma}}$$

or

$$[\gamma-\Phi_C(\theta)](\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b) \leq [\gamma-\Phi_{NC}(\theta)](\Lambda_C(\theta)+\Phi_C(\theta)+b)$$

or

$$\gamma \leq \gamma^* = \frac{\Phi_C(\theta)\Lambda_{NC}(\theta)-\Phi_{NC}(\theta)\Lambda_C(\theta)+b[\Phi_C(\theta)-\Phi_{NC}(\theta)]}{[\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)]-[\Lambda_C(\theta)+\Phi_C(\theta)]}$$

given that $\Phi_C(\theta) > \Phi_{NC}(\theta)$ and $\Lambda_{NC}(\theta) > \Lambda_C(\theta)$, it follows that $\gamma^* > 0$.

ii) Now we now that:

$$\underline{\gamma} = \Phi_C(\theta) + \frac{\Lambda_C(\theta)\left[1+\frac{1}{K_B}\right][\Phi_C(\theta)-\Phi_{NC}(\theta)]}{\Lambda_{NC}(\theta)-\Lambda_C(\theta)}$$

From this it follow that $\gamma^* < \underline{\gamma}$ if and only if

$$\left[\underline{\gamma}-\Phi_C(\theta)\right](\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)+b) > \left[\underline{\gamma}-\Phi_{NC}(\theta)\right](\Lambda_C(\theta)+\Phi_C(\theta)+b)$$

or after substitutions:

$$\left[1+\frac{1}{K_B}\right]\frac{\Lambda_C(\theta)}{\Lambda_{NC}(\theta)-\Lambda_C(\theta)}(\Lambda_{NC}(\theta)+\Phi_{NC}(\theta)-\Lambda_C(\theta)-\Phi_C(\theta)) > (\Lambda_C(\theta)+\Phi_C(\theta)+b)$$

This is satisfied when K_B is small enough (Bank capital is sufficiently scarce) **QED.**

- **Proof of proposition 6:**

i) Suppose that $\beta \geq \frac{\Lambda_{NC}}{CAR}$ then the CAR is binding and in such a case, the maximum payment to the investor declines is given by:

$$R_I = \left[R - \frac{b}{p} - R_B^1\right] I < \left[R - \frac{b}{p} - R_B^2\right] I$$

The size of the investment is given by:

$$I_{CAR}^N = \frac{1}{V_{CAR}^N(\beta,\gamma)}$$

where:

$$V_{CAR}^N(\beta,\gamma) = 1 - \frac{\Phi_{CAR}}{\gamma} - CAR$$

and

$$\Phi_{CAR}^{NC} = p(\theta)\left[R - \frac{b}{p} - \frac{c + \beta CAR}{p(\theta)}\right]$$

Formally, the optimal size of the investment declines (compared to the case without the CAR) if and only if: $V_{CAR}^N(\beta,\gamma) > V_{NC}(\beta,\gamma)$

This equivalent to :

$$CAR + \frac{\Phi_{CAR}^{NC}}{\gamma} < \frac{\Phi_{NC}}{\gamma} + \frac{\Lambda_{NC}}{\beta} \tag{25}$$

The condition is met if the capital adequacy ratio is binding. Indeed:

$$CAR + \frac{\Phi_{CAR}}{\gamma} = \frac{p(\theta)}{\gamma}\left[R - \frac{b}{p} - \frac{c}{p(\theta)}\right] + CAR\left(1 - \frac{\beta}{\gamma}\right)$$

And:

$$\frac{\Phi_{NC}}{\gamma} + \frac{\Lambda_{NC}}{\beta} = \frac{p(\theta)}{\gamma}\left[R - \frac{b}{p} - \frac{c}{p(\theta)}\right] + \frac{\Lambda_{NC}}{\beta}\left(1 - \frac{\beta}{\gamma}\right)$$

As $\beta \geq \frac{\Lambda_{NC}}{CAR}$, the capital adequacy ratio rule is binding (ie. $\frac{\Lambda_{NC}}{\beta} < CAR$), and (25) holds.

ii) When $\beta < \frac{\Lambda_{NC}}{CAR}$, the CAR is not binding and therefore the optimal collusion proof contract is just as if the CAR does not exist (ie. the optimal investment scale is $I_{NC}(\beta,\gamma) = \frac{1}{V_{NC}(\beta,\gamma)}$. **QED.**

- **Proof of proposition 7:**

i) Suppose that $\beta > \frac{\Lambda_C}{CAR}$ then the optimal collusion proof contract is constrained by the CAR and the optimal size of the investment under such contract is then given by:

$$I^C_{CAR} = \frac{1}{V^C_{CAR}(\beta,\gamma)}$$

where:

$$V^C_{CAR}(\beta,\gamma) = 1 - \frac{\Phi^C_{CAR}}{\gamma} - CAR$$

and

$$\Phi^C_{CAR} = \widetilde{p}(\theta)\left[R - \frac{b}{p} - \frac{qc-(1-q)k\Delta B + \beta CAR}{\widetilde{p}(\theta)}\right]$$

Formally, the optimal size of the investment declines (compared to the case without the CAR) if and only if: $V^C_{CAR}(\beta,\gamma) > V_C(\beta,\gamma)$

This is equivalent to :

$$CAR + \frac{\Phi^C_{CAR}}{\gamma} < \frac{\Phi_C}{\gamma} + \frac{\Lambda_C}{\beta} \tag{26}$$

The condition is met if the capital adequacy ratio is binding. Indeed:

$$CAR + \frac{\Phi^C_{CAR}}{\gamma} = \frac{\widetilde{p}(\theta)}{\gamma}\left[R - \frac{b}{p} - \frac{qc-(1-q)k\Delta B}{\widetilde{p}(\theta)}\right] + CAR\left(1 - \frac{\beta}{\gamma}\right)$$

And:

$$\frac{\Phi_C}{\gamma} + \frac{\Lambda_C}{\beta} = \frac{\widetilde{p}(\theta)}{\gamma}\left[R - \frac{b}{p} - \frac{qc-(1-q)k\Delta B}{\widetilde{p}(\theta)}\right] + \frac{\Lambda_C}{\beta}\left(1 - \frac{\beta}{\gamma}\right)$$

As $\beta \geq \frac{\Lambda_C}{CAR}$, the capital adequacy ratio rule is binding (ie. $\frac{\Lambda_C}{\beta} < CAR$), and (26) holds.

ii) When $\beta < \frac{\Lambda_C}{CAR}$, the CAR is not binding and therefore the optimal collusion proof contract is just as if the CAR does not exist (ie. the optimal investment scale is $I_C(\beta,\gamma) = \frac{1}{V_C(\beta,\gamma)}$). **QED.**

- **Proof of proposition 8:**

i)When $\beta \geq \frac{\Lambda_{NC}}{CAR}$, for both the collusion proof and the collusion contracts, the CAR is binding and the constrained collusion proof contract will dominate the constrained collusion contract if and only if:

$$I^C_{CAR} = \frac{1}{V^C_{CAR}(\beta,\gamma)} < I^N_{CAR} = \frac{1}{V^N_{CAR}(\beta,\gamma)}$$

or

$$\widetilde{p}(\theta)\left[R-\frac{b}{p}-\frac{qc-(1-q)k\Delta B+\beta CAR}{\widetilde{p}(\theta)}\right]<p(\theta)\left[R-\frac{b}{p}-\frac{c+\beta CAR}{p(\theta)}\right]$$

or

$$[-qc+(1-q)k\Delta B-\beta CAR]<[p(\theta)-\widetilde{p}(\theta)]\left[R-\frac{b}{p}\right]-c-\beta CAR$$

$$(1-q)\left[c+k\Delta B\right]<[p(\theta)-\widetilde{p}(\theta)]\left[R-\frac{b}{p}\right]$$

or

$$[c+k\Delta B]<p\left[1-\theta\right]\left[R-\frac{b}{p}\right]$$

or

$$R-\frac{b}{p}-\frac{c+k\Delta B}{p\left[1-\theta\right]}>0$$

which is always satisfied.

ii) When $\beta<\frac{\Lambda_{NC}}{CAR}$ and $\beta<\Psi\gamma$, The CAR is not binding for the collusion proof contract. Given that $\beta<\Psi\gamma$ such a contract dominates also a non constrained collusion contract and at fortiori a constrained collusion contract. Hence for this configuration of parameters, the collusion proof contract (which is non constrained) is the optimal contract.

iii) When $\beta<\frac{\Lambda_{C}}{CAR}$ and $\beta>\Psi\gamma$, the CAR is neither binding for a collusion proof contract nor a collusion contract. Given that $\beta>\Psi\gamma$, we know that a non constrained collusion contract dominates the collusion proof contract.

iv) Finally consider the case where $\beta\in\left]\frac{\Lambda_{C}}{CAR};\frac{\Lambda_{NC}}{CAR}\right[>$ and $\beta>\Psi\gamma$. Then under such configuration of parameters, the CAR is not binding for the optimal collusion proof contract while it is binding for the collusion contract. The (constrained) collusion contract dominates when

$$\frac{\Phi_{CAR}^{C}}{\gamma}+CAR>\frac{\Phi_{NC}}{\gamma}+\frac{\Lambda_{NC}}{\beta}=\frac{p(\theta)}{\gamma}\left[R-\frac{b}{p}-\frac{c}{p(\theta)}\right]+\frac{\Lambda_{NC}}{\beta}\left(1-\frac{\beta}{\gamma}\right)\tag{27}$$

where

$$\Phi_{CAR}^{C}=\widetilde{p}(\theta)\left[R-\frac{b}{p}-\frac{qc-(1-q)k\Delta B+\beta CAR}{\widetilde{p}(\theta)}\right]$$

(27) writes therefore as

$$\begin{aligned} & \frac{\widetilde{p}(\theta)}{\gamma}\left[R-\frac{b}{p}-\frac{qc-(1-q)k\Delta B+\beta CAR}{\widetilde{p}(\theta)}\right]+CAR \\ > \quad & \frac{p(\theta)}{\gamma}\left[R-\frac{b}{p}-\frac{c}{p(\theta)}\right]+\frac{\Lambda_{NC}}{\beta}\left(1-\frac{\beta}{\gamma}\right) \end{aligned}$$

or :

$$\frac{(1-q)\,p(1-\theta)}{\gamma}\left[R-\frac{b}{p}-\frac{(c+k\Delta B)}{p(1-\theta)}\right]+\left(CAR-\frac{\Lambda_{NC}}{\beta}\right)\left(\frac{\beta}{\gamma}-1\right)>0$$

when this inequality is reversed, we obviously have the region of parameters where the collusion proof contract dominates. **QED.**

- **Proof of proposition 9: Banking capital market equilibrium under fixed capital adequacy ratio**

- **Proof of proposition 10:** i) As is obvious from (15), the optimal CAR is increasing in γ and K_B.

ii) Also the optimal CAR is decreasing in Φ_{NC}. As Φ_{NC} is itself increasing in the value of R, c, and $k\Delta B$, the result follows immediately. **QED.**

- **Proof of proposition 11: Bailouts and flexible capital adequacy ratios:**

Note that

$$\frac{\partial \Phi_{NC}}{\partial \theta}=(1-p)\left(R-\frac{b}{p}-\frac{c+k\Delta B}{p(1-\theta)}\right)-\frac{p(\theta)}{p}\frac{c+k\Delta B}{(1-\theta)^2}$$

Also

$$\frac{\partial^2 \Phi_{NC}}{\partial \theta^2}<0$$

Note that when $\theta \to 0$ (small bailout guarantees)

$$\begin{aligned} \lim_{\theta\to 0}\frac{\partial \Phi_{NC}}{\partial \theta} &= (1-p)\left(R-\frac{b}{p}-\frac{c+k\Delta B}{p}\right)-(c+k\Delta B) \\ &= \left(R-\frac{b}{p}-\frac{c+k\Delta B}{p}\right)-p\left(R-\frac{b}{p}-\frac{2(c+k\Delta B)}{p}\right)>0 \end{aligned}$$

At the same time consider the smallest value $\overline{\theta}$ compatible with assumption C (that ensures the existence of a credit market).$\overline{\theta}$ is determined by

$$R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\overline{\theta})} = 0$$

Then it is easy to see from (16) that

$$\lim_{\theta \to \overline{\theta}} \frac{\partial \Phi_{NC}}{\partial \theta} < 0$$

It follows that when $(1-p)\left(R - \frac{b}{p}\right) - \frac{(c+k\Delta B)}{p} > 0$, there exists $\widetilde{\theta} \in \left[0, \overline{\theta}\right]$ such that $\frac{\partial \Phi_{NC}}{\partial \theta} \geq 0$ if and only if $\theta \leq \widetilde{\theta}$. Hence from (15), the optimal capital adequacy rule $CAR\left(\gamma, K_B, R, c, k\Delta B, \theta\right)$ is decreasing in θ in the range $\theta \leq \widetilde{\theta}$ and increasing in θ in the range $\theta \geq \widetilde{\theta}$.**QED**.

- **Proofs of proposition 12 and 13: Banking Capital market equilibrium with systemic bailouts:**

First, the following lemmata are helpful to characterize the structure of equilibrium contracts and bailouts.

Lemma 17 *i) For all $\theta \in [0,1]$, one has*

$$\frac{\partial \Lambda_C(\theta)}{\partial \theta} > 0$$

ii) There exists $\widehat{\theta} \in]0,1[$such that:

$$\begin{aligned} \frac{\partial \Phi_C(\theta)}{\partial \theta} &\geq 0 \quad \text{when} \quad \theta \leq \widehat{\theta} \\ \frac{\partial \Phi_C(\theta)}{\partial \theta} &\leq 0 \quad \text{when} \quad \theta \geq \widehat{\theta} \end{aligned}$$

iii) When $\theta \leq \widehat{\theta}$, the equilibrium bank return $\beta_C(\gamma, \theta)$ with collusion contracts and a bailout guarantee θ is an increasing function of θ :

$$\frac{\partial \beta_C(\gamma, \theta)}{\partial \theta} > 0$$

Proof: Note that $\begin{cases} \Phi_C(\theta) = \widetilde{p}(\theta)\left(R - \frac{b}{p} - \frac{c}{p(1-\theta)}\right) \\ \Lambda_C(\theta) = \frac{1}{p(1-\theta)}\theta c + (1-q)k\Delta B \end{cases}$, therefore:

$$\frac{\partial \Lambda_C(\theta)}{\partial \theta} = \frac{c}{p(1-\theta)^2}c > 0$$

while

$$\frac{\partial \Phi_C(\theta)}{\partial \theta} = \widetilde{p}'(\theta)\left(R - \frac{b}{p} - \frac{c}{p(1-\theta)}\right) - \widetilde{p}(\theta)\frac{c}{p(1-\theta)^2}$$

and

$$\frac{\partial^2 \Phi_C(\theta)}{\partial^2 \theta} = -2\left(q(1-p)+(1-q)\right)\frac{c}{p(1-\theta)^2} - \widetilde{p}(\theta)\frac{2c}{p(1-\theta)^3} < 0$$

Now define θ_0 such that:

$$R - \frac{b}{p} - \frac{c}{p(1-\theta_0)} = 0$$

then we have:

$$\begin{aligned}\lim_{\theta \to 0} \frac{\partial \Phi_C(\theta)}{\partial \theta} &= [q(1-p)+(1-q)]\left(R - \frac{b}{p} - \frac{c}{p}\right) - qc \\ &= \left(R - \frac{b+c}{p}\right) - qp\left(R - \frac{b+2c}{p}\right) > 0\end{aligned}$$

because of assumption B. Also

$$\lim_{\theta \to \theta_0} \frac{\partial \Phi_C(\theta)}{\partial \theta} = -\widetilde{p}(\theta_0)\frac{c}{p(1-\theta_0)^2} < 0$$

Hence as $\frac{\partial \Phi_C(\theta)}{\partial \theta}$ is decreasing in θ, there exists a unique $\widehat{\theta} \in]0, \theta_0[$ such that $\frac{\partial \Phi_C(\theta)}{\partial \theta} \geq 0$ if and only if $\theta \leq \widehat{\theta}$.

iii) The equilibrium bank return with collusion is given by:

$$\beta_C(\gamma, \theta) = \frac{\Lambda_C(\theta)}{1 - \frac{\Phi_C(\theta)}{\gamma}}\left[1 + \frac{1}{K_B}\right]$$

Hence

$$\frac{1}{\beta_C(\gamma,\theta)}\frac{\partial \beta_C(\gamma,\theta)}{\partial \theta} = \frac{\frac{\partial \Lambda_C(\theta)}{\partial \theta}}{\Lambda_C(\theta)} + \frac{\frac{\partial \Phi_C(\theta)}{\partial \theta}}{\gamma - \Phi_C(\theta)} > 0$$

when $\theta \leq \widehat{\theta}$. **QED.**

Lemma 18 *Collusion contracts are more likely to be chosen when the expected bailout guarantee θ^e increases.*

Proof: Recall simply that for a given expected bailout guarantee θ^e, a collusion contract is chosen when $\beta \geq \Psi(\theta^e)\gamma$ where

$$\begin{aligned}\Psi(\theta^e) &= \frac{\Lambda_{NC}(\theta^e) - \Lambda_C(\theta^e)}{\Phi_C(\theta^e) - \Phi_{NC}(\theta^e)} \\ &= \frac{\widetilde{p}(\theta^e)\frac{k\Delta B}{p(1-\theta^e)}}{\widetilde{p}(\theta^e)\frac{k\Delta B}{p(1-\theta^e)} - (1-q)p(1-\theta^e)\left(R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta^e)}\right)}\end{aligned}$$

simple differentiation shows that $\Psi'(\theta^e) < 0$. Therefore the range of relative returns β/γ for which collusion is chosen is enlarged. **QED.**

- **Proposition 12: Market equilibrium with systemic bailouts:**

- Consider the value of $\widehat{\gamma}_0$ such that in a mixed equilibrium with $\theta = 0$, at $\beta = \gamma \cdot \Psi(0)$, one has exactly an equilibrium fraction of collusion-proof contracts $\nu^* (\gamma)$ equal to $\widehat{\nu}$, the threshold above which there is no triggering of a systemic bailout. $\widehat{\gamma}_0$ is given by the banking capital market equilibrium with $\beta = \gamma \cdot \Psi(0)$. That is by the relationship:

$$K_B = \frac{\widehat{\nu}\Lambda_{NC}(0) + (1 - \widehat{\nu})\Lambda_C(0)}{\gamma\Psi(0) - \Psi(0)\Phi_{NC}(0) - \Lambda_{NC}(0)}$$

Substitution of $\Psi(0)$ and the fact that $0 < \widehat{\nu} < 1$ implies that $\widehat{\gamma}_0 \in \left]\underline{\gamma}(0), \overline{\gamma}(0)\right[$.

- Similary one may define $\widehat{\gamma}(\theta)$ the value of γ such that in a mixed equilibrium with $\theta > 0$, at $\beta = \gamma \cdot \Psi(\theta)$, one has exactly an equilibrium fraction of collusion-proof contracts ν equal to $\widehat{\nu}$, the threshold below which there is the triggering of a systemic bailout. $\widehat{\gamma}(\theta)$ is given by the banking capital market equilibrium with $\beta = \gamma \cdot \Psi(\theta)$, that is by the relationship:

$$K_B = \frac{\widehat{\nu}\Lambda_{NC}(\theta) + (1 - \widehat{\nu})\Lambda_C(\theta)}{\gamma\Psi(\theta) - \Psi(\theta)\Phi_{NC}(\theta) - \Lambda_{NC}(\theta)}$$

Again substitution of $\Psi(\theta)$ and the fact that $0 < \widehat{\nu} < 1$ implies that $\widehat{\gamma}(\theta) > \underline{\gamma}(\theta)$.

i) Now it is easy to check that for $\gamma < \widehat{\gamma}_0$, a market equilibrium with no bailout cannot exist. Indeed given that $\beta_{NC}(\gamma, 0) > \gamma \cdot \Psi(0)$, all financial contracts are with collusion and this triggers the implementation of a systemic bailout. Conversely, it is easy to check that for $\gamma \geq \widehat{\gamma}_0$, a no bailout market equilibrium (ie. with $\theta^* = 0$) is sustainable.

a) Indeed for $\gamma > \overline{\gamma}(0)$, if all agents expect no systemic bailout and $\theta^e = 0$, given that $\beta_{NC}(\gamma, 0) < \gamma \cdot \Psi(0)$, an equilibrium market rate of return $\beta^* = \beta_{NC}(\gamma, 0)$ will induce the implementation of collusion proof financial contracts. This in turn is consistent with a market equilibrium with no bailout $\theta^* = 0$.

b) Similarly when $\gamma \in \left]\widehat{\gamma}_0, \overline{\gamma}(0)\right[$, and all agents expect no systemic bailout, an equilibrium market rate of return $\beta^* = \gamma \cdot \Psi(0)$ will induce a mixed equilibrium with a fraction of collusion proof financial contracts $\nu^*(\gamma, 0) \geq \widehat{\nu}$, which again is consistent with the implementation of no equilibrium bailout $\theta^* = 0$.

ii) It is also easy to check that for $\gamma > \widehat{\gamma}(\theta)$ a market equilibrium with bailout cannot exist. Indeed given that $\beta_C(\gamma, \theta) < \gamma \cdot \Psi(\theta)$, all financial contracts will be collusion proof which would not trigger the implementation of a systemic bailout. Conversely, it is easy to check that for $\gamma \leq \widehat{\gamma}(\theta)$, a bailout market equilibrium (ie. with $\theta^* = \theta$) is sustainable.

a) Indeed for $\gamma \leq \underline{\gamma}(\theta)$, if all agents expect a systemic bailout and $\theta^e = \theta$, given that $\beta_C(\gamma, \theta) < \gamma \cdot \Psi(\theta)$, an equilibrium market rate of return $\beta^* = \beta_C(\gamma, \theta)$ will induce the full implementation of collusive financial contracts. This in turn will trigger the implementation of the bailout $\theta^* = \theta$.

b) Similarly when $\gamma \in]\underline{\gamma}(\theta), \widehat{\gamma}(\theta)]$, and all agents expect a systemic bailout, an equilibrium market rate of return $\beta^* = \gamma \cdot \Psi(\theta)$ will induce a mixed equilibrium with a fraction of collusion proof financial contracts $\nu^*(\gamma, \theta) \leq \widehat{\nu}$, which again is consistent with the implementation of the bailout $\theta^* = \theta$.**QED.**

- **Proposition 13: multiple equilibria with systemic bailouts.**

First denote by $\gamma_l(\theta)$ the value of γ such that $\beta_C(\gamma, 0) = \Psi(\theta)\gamma$. Lemma 2 gives $\Psi(\theta) < \Psi(0)$ and as $\beta_C(\gamma, 0)$ is decreasing in γ, this implies that $\gamma_l(\theta) > \widehat{\gamma}_0$. Now for $\theta \leq \widehat{\theta}$, from lemma 1 iii) we know that $\beta_C(\gamma, \theta)$ is increasing in θ. Hence

$$\beta_C(\gamma_l(\theta), \theta) > \beta_C(\gamma_l(\theta), 0) = \gamma_l(\theta)\Psi(\theta)$$

Hence

$$\frac{\beta_C(\gamma_l(\theta), \theta)}{\gamma_l(\theta)} > \Psi(\theta) = \frac{\beta_C(\widehat{\gamma}(\theta), \theta)}{\widehat{\gamma}(\theta)}$$

Given that the function while $\beta_C(\gamma, \theta)/\gamma$ is decreasing in γ, this implies that

$$\widehat{\gamma}(\theta) > \gamma_l(\theta) > \widehat{\gamma}_0$$

QED.

- **Proof of proposition 14**: **Capital adequacy ratios under systemic bailouts**

Note that

$$\underline{\gamma}(\theta) = \Phi_C(\theta) + \frac{\Lambda_C(\theta)\left[1 + \frac{1}{K_B}\right]}{\Psi(\theta)}$$

and

$$\begin{aligned}\frac{\underline{\gamma}(\theta)}{\Phi_C(\theta)} &= 1 + \frac{\Lambda_C(\theta)\left[1 + \frac{1}{K_B}\right]}{\Phi_C(\theta)\Psi(\theta)} \\ &= 1 + \left[1 + \frac{1}{K_B}\right]\frac{1 - \frac{\Phi_{NC}(\theta)}{\Phi_C(\theta)}}{\frac{\Lambda_{NC}(\theta)}{\Lambda_C(\theta)} - 1}\end{aligned}$$

Note also that

$$\frac{\Phi_{NC}(\theta)}{\Phi_C(\theta)} = \frac{p(\theta)}{\widetilde{p}(\theta)} \frac{R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}}{R - \frac{b}{p} - \frac{c}{p(1-\theta)}}$$

It is easy to see that $p(\theta)/\widetilde{p}(\theta)$ is decreasing in θ. As well the ratio:

$$\frac{R - \frac{b}{p} - \frac{c+k\Delta B}{p(1-\theta)}}{R - \frac{b}{p} - \frac{c}{p(1-\theta)}}$$

is declining in θ. Hence $\Phi_{NC}(\theta)/\Phi_C(\theta)$ is a decreasing function of θ.

$$\begin{aligned} \frac{\Lambda_{NC}(\theta)}{\Lambda_C(\theta)} &= \frac{\theta c + p(\theta)k\Delta B}{\theta c + p(1-\theta)(1-q)k\Delta B} \\ &= \frac{\theta c + [p + (1-p)\theta]k\Delta B}{\theta c + p(1-\theta)(1-q)k\Delta B} \end{aligned}$$

and $\Lambda_{NC}(\theta)/\Lambda_C(\theta)$ is a decreasing function of θ if and only if $(1-q)\,k\Delta B < qc$.Hence it follows that when $(1-q)\,k\Delta B < qc$, the function $\frac{\gamma(\theta)}{\Phi_C(\theta)}$ is an increasing function of θ and therefore $\frac{\gamma(0)}{\Phi_C(0)} < \frac{\gamma(\theta)}{\Phi_C(\theta)}$. This implies that $CAR(0) < CAR(\theta)$. One needs to have a tighter fixed capital adequacy ratio under an expected bailout θ. **QED.**

- **Proof of proposition 15: Flexible capital adequacy ratios and systemic bailouts**

- When $\theta \leq \widehat{\theta}$, $\Phi_C(\theta) > \Phi_C(0)$, thus $\overline{CAR}(\gamma, \theta) < \overline{CAR}(\gamma, 0)$

- When $\theta > \widehat{\theta}$, $\Phi_C(\theta)$ is a decreasing function of θ and we know that $\lim_{\theta \to 1} \Phi_C(\theta) = -\infty$. Hence there exists $\theta_0 > \widehat{\theta}$ such that $\Phi_C(\theta_0) = 0$. As well there exists $\theta_f \in]\widehat{\theta}, \theta_0[$ such that $\Phi_C(\theta_f) = \Phi_C(0)$. It follows that for all bailouts $\theta \in \left]\widehat{\theta}, \theta_f\right[$ one again has $\Phi_C(\theta) > \Phi_C(0)$, thus $\overline{CAR}(\gamma, \theta) < \overline{CAR}(\gamma, 0)$. Conversely for all admissible bailouts $\theta \in]\theta_f, \theta_0[$, $\Phi_C(\theta) < \Phi_C(0)$ and therefore $\overline{CAR}(\gamma, \theta) > \overline{CAR}(\gamma, 0)$. **QED.**

- **Proof of proposition 16: Market equilibrium with productive externalities**

The following lemma is useful to characterize the banking capital market equilibrium with productive externalities:We have the following lemma:

Lemma : *Suppose that* $q < 1/2$, *then* $\frac{\partial \overline{\gamma}}{\partial R} < 0$ and $\frac{\partial \overline{\gamma}}{\partial R} < 0$

proof: i) Note that

$$[\Phi_C - \Phi_{NC}]\,(R) = \widetilde{p}\,(\theta)\,\frac{k\Delta B}{p(1-\theta)} - (p(\theta) - \widetilde{p}\,(\theta))\left[R - \frac{b}{p} - \frac{c + k\Delta B}{p(1-\theta)}\right]$$

is a decreasing function of R while $\Lambda_{NC} - \Lambda_C = \widetilde{p}(\theta)\,\frac{k\Delta B}{p(1-\theta)}$ is independent from R. Thus

$$\Psi(R) = \frac{\Lambda_{NC} - \Lambda_C}{\left[\Phi_C - \Phi_{NC}\right](R)}$$

is an increasing function of R.

ii) Now we have:

$$\begin{aligned}\overline{\gamma}(R) &= \Phi_{NC}(R) + \frac{\Lambda_{NC}\left[1+\frac{1}{K_B}\right]}{\Psi(R)} \\ &= \frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C} + \frac{\Lambda_{NC}}{\Psi(R)K_B}\end{aligned}$$

and

$$\begin{aligned}\underline{\gamma}(R) &= \Phi_C(R) + \frac{\Lambda_C\left[1+\frac{1}{K_B}\right]}{\Psi(R)} \\ &= \frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C} + \frac{\Lambda_C}{\Psi(R)K_B}\end{aligned}$$

Therefore

$$\frac{\partial\overline{\gamma}}{\partial R} = \frac{\partial}{\partial R}\left[\frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C}\right] \underbrace{-\frac{\Lambda_{NC}}{\Psi^2 K_B}\frac{\partial\Psi}{\partial R}}_{-} \tag{28}$$

and

$$\frac{\partial\underline{\gamma}}{\partial R} = \frac{\partial}{\partial R}\left[\frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C}\right] \underbrace{-\frac{\Lambda_C}{\Psi^2 K_B}\frac{\partial\Psi}{\partial R}}_{-} \tag{29}$$

The second term of (28) and (29) is unambiguously a decreasing function of R. For the first term, one has:

$$\begin{aligned}\frac{\partial}{\partial R}\left[\frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C}\right] &\propto \left[\Lambda_{NC}\frac{\partial}{\partial R}\Phi_C(R) - \Lambda_C\frac{\partial}{\partial R}\Phi_{NC}(R)\right] \\ &\propto \left[\Lambda_{NC}\ qp - \Lambda_C p\right]\end{aligned}$$

But $[\Lambda_{NC}\ qp - \Lambda_C p]$ has the sign of $(pk\Delta B)\,qp - (1-q)pk\Delta Bp = k\Delta Bp^2\,[2q-1]$. Therefore when $2q-1<0$

$$\frac{\partial}{\partial R}\left[\frac{\Lambda_{NC}\Phi_C(R) - \Lambda_C\Phi_{NC}(R)}{\Lambda_{NC} - \Lambda_C}\right] < 0$$

From this it follows finally that

$$\frac{\partial\overline{\gamma}}{\partial R} < 0 \ \text{ and } \ \frac{\partial\underline{\gamma}}{\partial R} < 0$$

when $q < 1/2$. **QED.**

- **Proposition 16:**

i) define:

$$\overline{\Psi}(\epsilon) = \frac{\Lambda_{NC} - \Lambda_C}{\overline{\Phi}_C(\epsilon) - \overline{\Phi}_{NC}(\epsilon)} \text{ and } \underline{\Psi}(\epsilon) = \frac{\Lambda_{NC} - \Lambda_C}{\underline{\Phi}_C(\epsilon) - \underline{\Phi}_{NC}(\epsilon)}$$

with obvious notations: $\underline{\Phi}_C(\epsilon) = \Phi_C(\underline{R}(\epsilon))$, etc...

It follows from $\overline{R}(\epsilon) > \underline{R}(\epsilon)$ that $\overline{\Psi}(\epsilon) > \underline{\Psi}(\epsilon)$. Note also that through simple differentiation $\overline{\Psi}'(\epsilon) > \underline{\Psi}'(\epsilon) > 0$ and $\overline{\Psi}(0) = \underline{\Psi}(0)$

2) Define:

$$\Theta(\epsilon) = \overline{\gamma}(\overline{R}(\epsilon)) - \underline{\gamma}(\underline{R}(\epsilon))$$

Differentiation gives:

$$\Theta'(\epsilon) = \frac{\partial\overline{\gamma}}{\partial R}\overline{R}'(\epsilon) - \frac{\partial\underline{\gamma}}{\partial R}\underline{R}'(\epsilon)$$

Given that $p\Omega > 1 > pq\Omega$, we have $\overline{R}'(\epsilon) > 0 > \underline{R}'(\epsilon)$. Also when $q < 1/2$, $\frac{\partial\overline{\gamma}}{\partial R} < 0$ and $\frac{\partial\underline{\gamma}}{\partial R} < 0$; Hence it follows that

$$\Theta'(\epsilon) = \underbrace{\frac{\partial\overline{\gamma}}{\partial R}}_{-}\underbrace{\overline{R}'(\epsilon)}_{+} - \underbrace{\frac{\partial\underline{\gamma}}{\partial R}}_{-}\underbrace{\underline{R}'(\epsilon)}_{-} < 0$$

and $\Theta(\epsilon)$ is decreasing in ϵ. Note also that $\Theta(0) = \overline{\gamma}(\overline{R}(0)) - \underline{\gamma}(\underline{R}(0)) = \overline{\gamma}(R_0) - \underline{\gamma}(R_0) > 0$

3) Now note that for all $R \geq 0$

$$\begin{aligned}
\overline{\gamma}(R) - \underline{\gamma}(R) &= \frac{1}{K_B}\left[\Phi_C(R) - \Phi_{NC}(R)\right] \\
&= \frac{1}{K_B}\left[pq\left(R - \frac{b+c}{p}\right) - p\left(R - \frac{b+c+k\Delta B}{p}\right)\right] \\
&= \frac{1}{K_B}\left[(1-q)(b+c) + k\Delta B - p(1-q)R\right]
\end{aligned}$$

and there is a value R^* such that $\overline{\gamma}(R^*) - \underline{\gamma}(R^*) = 0$. Therefore there exists also a unique value $\widetilde{\epsilon} > 0$ such that $\overline{R}(\widetilde{\epsilon}) = R^*$as $\overline{R}(\epsilon)$ is an increasing function of ϵ such that $\overline{R}(0) = R_0 < R^*$ (R_0 satisfies assumption C (for $\theta = 0$) ensuring the existence of collusive regimes without externalities) and $\lim_{\epsilon\to\infty}\overline{R}(\epsilon) = +\infty$.

Simple inspection then shows that:

$$\begin{aligned}
\Theta(\widetilde{\epsilon}) &= \overline{\gamma}(\overline{R}(\widetilde{\epsilon})) - \underline{\gamma}(\underline{R}(\widetilde{\epsilon})) = \overline{\gamma}(R^*) - \underline{\gamma}(\underline{R}(\widetilde{\epsilon})) \\
&= \underline{\gamma}(R^*) - \underline{\gamma}(\underline{R}(\widetilde{\epsilon})) = \underline{\gamma}(\overline{R}(\widetilde{\epsilon})) - \underline{\gamma}(\underline{R}(\widetilde{\epsilon})) < 0
\end{aligned}$$

Given that $\overline{R}(\widetilde{\epsilon}) > \underline{R}(\widetilde{\epsilon})$ and $\underline{\gamma}(R)$ is a decreasing function of R when $q < 1/2$. Hence, given that $\Theta(\epsilon)$ is decreasing in ϵ and that $\Theta(0) > 0 > \Theta(\widetilde{\epsilon})$, there is a unique threshold value $\epsilon^* \in]0, \widetilde{\epsilon}[$ such that $\Theta(\epsilon^*) = 0$. Also for all values of $\epsilon > \epsilon^*$ $\Theta(\epsilon) < 0$ and therefore

$$\overline{\gamma}(\overline{R}(\epsilon)) \leq \underline{\gamma}(\underline{R}(\epsilon))$$

3) For $\gamma \in \left[\overline{\gamma}(\overline{R}(\epsilon)); \underline{\gamma}(\underline{R}(\epsilon))\right]$, when agents have expectations of a collusive market equilibrium, they expect the rate of return on successful productive projects to be $\underline{R}(\epsilon)$, Hence as $\gamma \leq \underline{\gamma}(\underline{R}(\epsilon))$, a market equilibrium with collusion contracts prevails. However, for the same value of γ, if agents have expectations of a collusion proof market equilibrium, then they expect the rate of return on successful productive projects to be $\overline{R}(\epsilon)$, Hence as $\gamma \geq \overline{\gamma}(\overline{R}(\epsilon))$, a market equilibrium with no collusion contracts also prevails.**QED.**

- **Political economy of banking supervision with collusion-proof contract**

$$\overline{\gamma}(k) = \Phi_{NC}(k) + \frac{\Lambda_{NC}(k)\left[1 + \frac{1}{K_B}\right]}{\Psi(k)}$$

Simple differentiation of this expression gives that:

$$\frac{\partial \overline{\gamma}}{\partial k} = \frac{\partial}{\partial k}\left[\frac{\Lambda_{NC}(k)\Phi_C - \Lambda_C(k)\Phi_{NC}(k)}{\Lambda_{NC}(k) - \Lambda_C(k)}\right] \underbrace{- \frac{\Lambda_{NC}(k)}{\Psi^2 K_B}\Psi'(k) + \frac{\Lambda'_{NC}(k)}{\Psi K_B}}_{+}$$

As

$$\frac{\Lambda_{NC}(k)\Phi_C - \Lambda_C(k)\Phi_{NC}(k)}{\Lambda_{NC}(k) - \Lambda_C(k)} = \frac{pq\left(R - \frac{b+c}{p}\right) - p(1-q)\left(R - \frac{b}{p} - \frac{c+k\Delta B}{p}\right)}{q}$$

it is increasing in k and $\frac{\partial \overline{\gamma}}{\partial k} > 0$. Therefore $\frac{\partial \overline{\gamma}}{\partial k} > 0$.

Using (20), and (18) simple differentiation immediately implies that:

$$\begin{aligned}
\frac{U'_E(k)}{U_E(k)} &= \frac{\frac{1}{\gamma}\frac{\partial \Phi_{NC}}{\partial k}}{1 - \frac{\Phi_{NC}}{\gamma}} < 0 \\
\frac{U'_B(k)}{U_B(k)} &= \frac{\frac{\partial \Lambda_{NC}}{\partial k}}{\Lambda_{NC}} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_{NC}}{\partial k}}{1 - \frac{\Phi_{NC}}{\gamma}} = \frac{\frac{\partial \Lambda_{NC}}{\partial k}}{\Lambda_{NC}\left[1 - \frac{\Phi_{NC}}{\gamma}\right]}\left[1 - \frac{\Lambda_{NC} + \Phi_{NC}}{\gamma}\right] \gtrless 0 \\
\frac{U'_I(k)}{U_I(k)} &= \frac{\frac{\partial \Phi_{NC}}{\partial k}}{\Phi_{NC}} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_{NC}}{\partial k}}{1 - \frac{\Phi_{NC}}{\gamma}} = \frac{\frac{\partial \Phi_{NC}}{\partial k}}{\Phi_{NC}\left[1 - \frac{\Phi_{NC}}{\gamma}\right]} < 0
\end{aligned}$$

- **Political Economy of banking supervision in the collusion region**

$$\underline{\gamma}(k) = \Phi_C(k) + \frac{\Lambda_C(k)\left[1 + \frac{1}{K_B}\right]}{\Psi(k)}$$

Again simple differentiation gives that

$$\frac{\partial \underline{\gamma}}{\partial k} = \underbrace{\frac{\partial}{\partial k}\left[\frac{\Lambda_{NC}(k)\Phi_C - \Lambda_C(k)\Phi_{NC}(k)}{\Lambda_{NC}(k) - \Lambda_C(k)}\right]}_{+} \underbrace{- \frac{\Lambda_C(k)}{\Psi^2 K_B}\Psi'(k) + \frac{\Lambda_C'(k)}{\Psi K_B}}_{+}$$

and $\underline{\gamma}(k)$ is also increasing in k.

One has:

$$\begin{aligned} U_E(k) &= bI_C = \frac{b}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C}{\beta_C(\gamma)}} = \frac{bK_B}{1 - \frac{\Phi_C}{\gamma}}\left[1 + \frac{1}{K_B}\right] \\ U_B(k) &= \Lambda_C I_C = \frac{\Lambda_C}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C}{\beta_C(\gamma)}} = \frac{\Lambda_C K_B}{1 - \frac{\Phi_C}{\gamma}}\left[1 + \frac{1}{K_B}\right] \\ U_I(k) &= \Phi_{NC} I_{NC} = \frac{\Phi_C}{1 - \frac{\Phi_C}{\gamma} - \frac{\Lambda_C}{\beta_C(\gamma)}} = \frac{\Phi_C K_B}{1 - \frac{\Phi_C}{\gamma}}\left[1 + \frac{1}{K_B}\right] \end{aligned} \tag{30}$$

with the equilibrium rate of return of banking capital given by:

$$\beta_C(\gamma) = \frac{\Lambda_C}{1 - \frac{\Phi_C}{\gamma}}\left[1 + \frac{1}{K_B}\right]$$

It follows immediately from (22) that:

$$\begin{aligned} \frac{U_E'(k)}{U_E(k)} &= \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = 0 \\ \frac{U_B'(k)}{U_B(k)} &= \frac{\frac{\partial \Lambda_C}{\partial k}}{\Lambda_C} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = \frac{\frac{\partial \Lambda_C}{\partial k}}{\Lambda_C} > 0 \\ \frac{U_I'(k)}{U_I(k)} &= \frac{\frac{\partial \Phi_C}{\partial k}}{\Phi_C} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = 0 \end{aligned}$$

The equilibrium return on bank capital is:

$$\beta_C(\gamma) = \frac{\Lambda_C}{1 - \frac{\Phi_C}{\gamma}}\left[1 + \frac{1}{K_B}\right]$$

It follows immediately from (22) that:

$$\begin{aligned} \frac{U_E'(k)}{U_E(k)} &= \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = 0 \\ \frac{U_B'(k)}{U_B(k)} &= \frac{\frac{\partial \Lambda_C}{\partial k}}{\Lambda_C} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = \frac{\frac{\partial \Lambda_C}{\partial k}}{\Lambda_C} > 0 \\ \frac{U_I'(k)}{U_I(k)} &= \frac{\frac{\partial \Phi_C}{\partial k}}{\Phi_C} + \frac{\frac{1}{\gamma}\frac{\partial \Phi_C}{\partial k}}{1 - \frac{\Phi_C}{\gamma}} = 0 \end{aligned}$$

- **Political economy of banking supervision in the mixed equilibrium region**

Lemma : *$\nu(k)$ is decreasing in k*

proof: $\nu(k)$ is determined by:

$$\nu(k)\Lambda_{NC}(k)+(1-\nu(k))\Lambda_C(k)=K_B\Psi(k)\gamma\left[1-\frac{1}{\gamma}\Delta(k)\right]$$

where

$$\Delta(k)=\frac{\Lambda_{NC}(k)\Phi_C-\Lambda_C(k)\Phi_{NC}(k)}{\Lambda_{NC}(k)-\Lambda_C(k)}$$

and $\Delta'(k)>0$. Therefore

$$\begin{aligned}\nu(k) &= K_B\frac{1}{\Phi_C-\Phi_{NC}(k)}\left[\gamma-\Delta(k)\right]-\frac{\Lambda_C(k)}{\Lambda_{NC}(k)-\Lambda_C(k)}\\ &= K_B\frac{1}{\Phi_C-\Phi_{NC}(k)}\left[\gamma-\Delta(k)\right]-\frac{(1-q)}{q}\end{aligned}$$

the function

$$\Xi(k)=K_B\frac{1}{\Phi_C-\Phi_{NC}(k)}\left[\gamma-\Delta(k)\right]$$

is decreasing in k implies immediately that $\nu(k)$ is decreasing in k. **QED.**

Lemma : *$U_I(k)$ is decreasing in k*

Proof: Indeed

$$U_I(k)=\left[\nu(k)\Phi_{NC}(k)+(1-\nu(k))\Phi_C\right]I(k)$$

which after substitution of $\nu(k)$ gives

$$\begin{aligned}U_I(k) &= \frac{\left[K_B\frac{1}{\Phi_C-\Phi_{NC}(k)}\left[\gamma-\Delta(k)\right]-\frac{(1-q)}{q}\right]\left[\Phi_{NC}(k)-\Phi_C\right]+\Phi_C}{\left[1-\frac{1}{\gamma}\Delta(k)\right]}\\ &= \frac{-K_B\left[\gamma-\Delta(k)\right]+\frac{(1-q)}{q}\left[\Phi_C-\Phi_{NC}(k)\right]+\Phi_C}{\left[1-\frac{1}{\gamma}\Delta(k)\right]}\end{aligned}$$

as $\Delta(k)$ is increasing in k and $\Phi_{NC}(k)$ is decreasing in k, the numerator is increasing in k while the denominator is decreasing in k. It results that in the mixed regime $U_I(k)$ is increasing in k and uniformed investors are in favor of relaxed supervision on banks **QED.**

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Figures

Figure 1: Banking Market Equilibrium rate of return

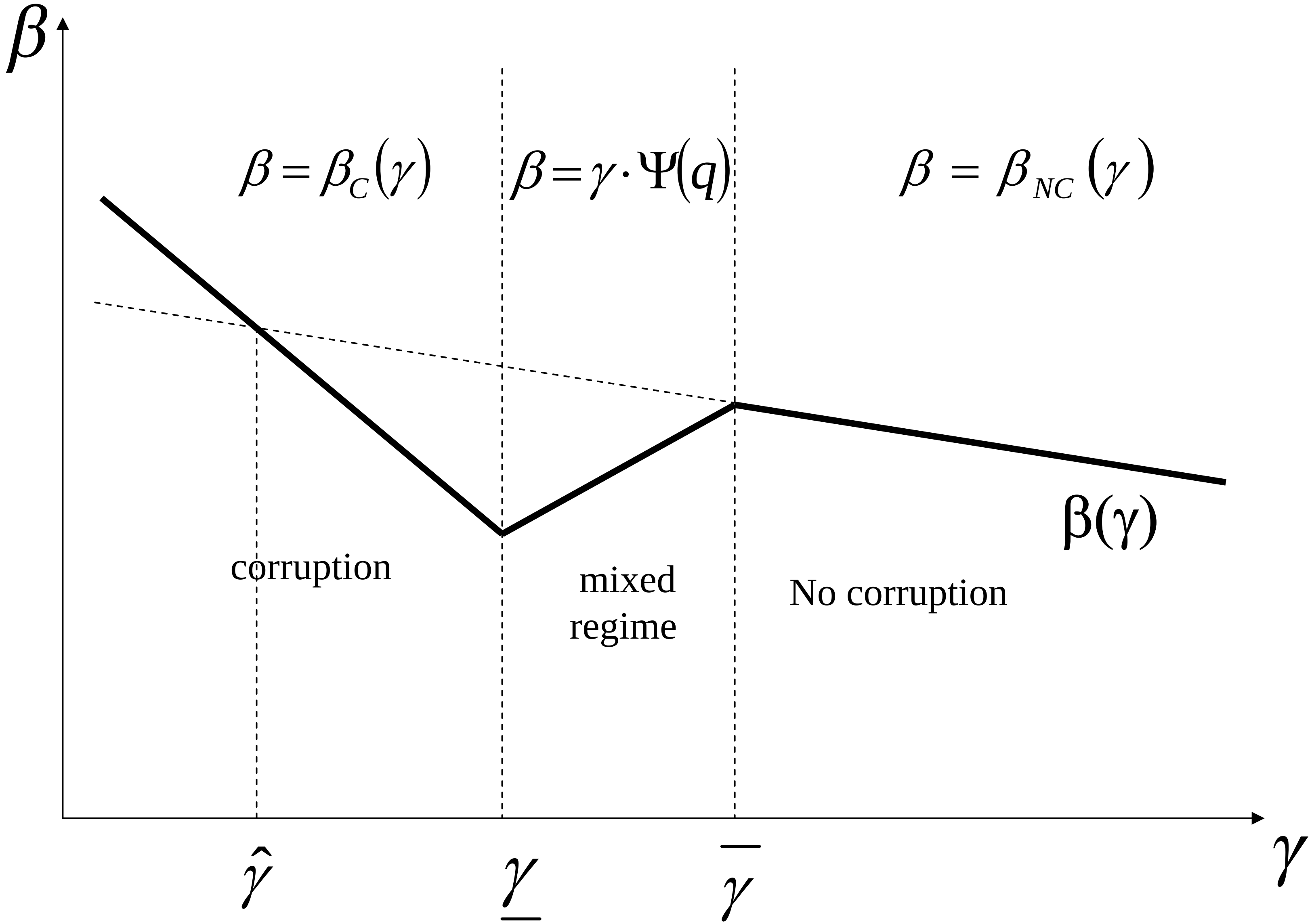

Figure 2: Choice of contracts under fixed capital adequacy rule

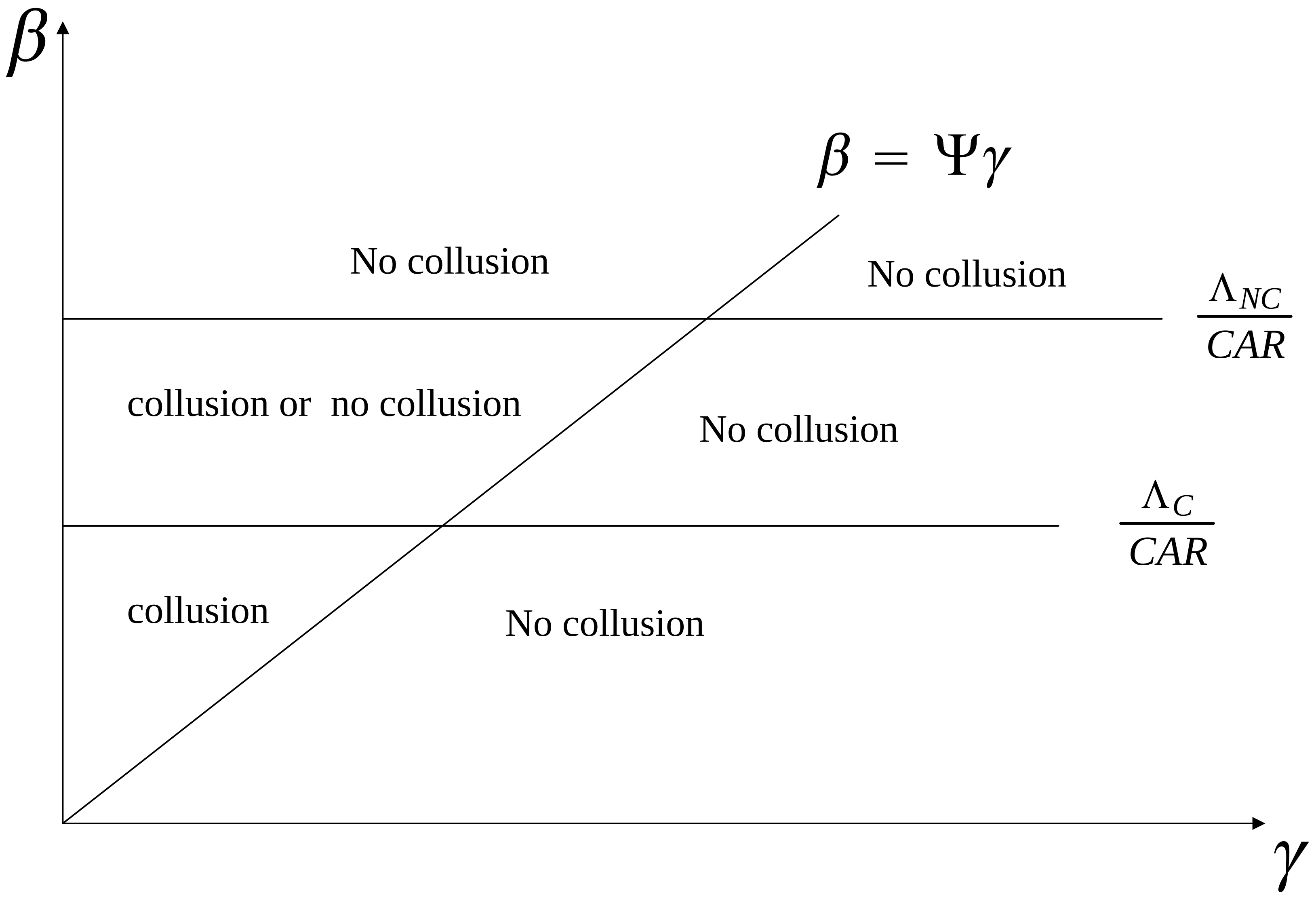

Figure 3: Elimination of collusion equilibria with fixed capital adequacy ratio

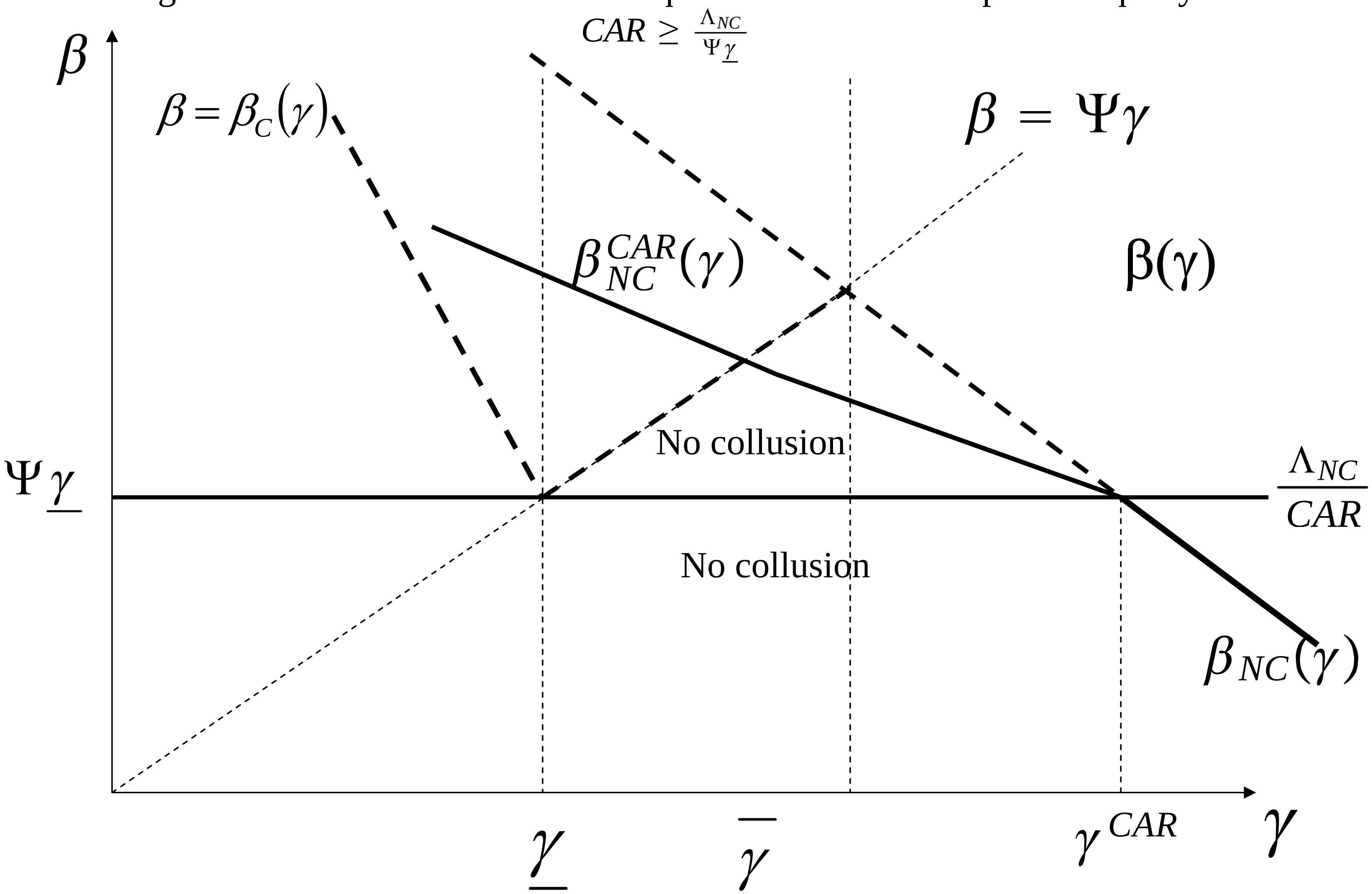

Figure 5 : Banking Market Equilibrium with systemic bailout

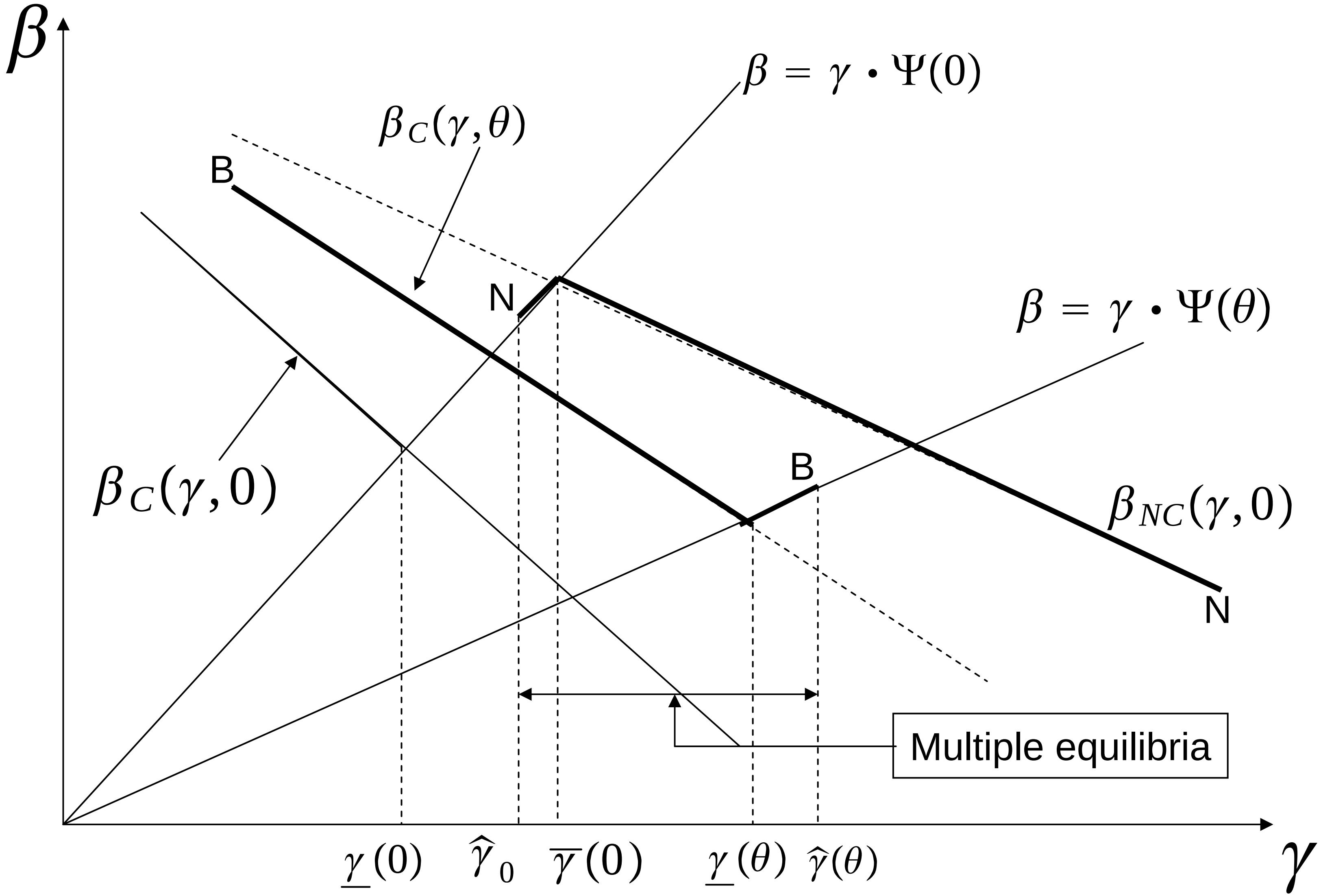

Figure 6: Banking Market Equilibrium with productive externalities ϵ

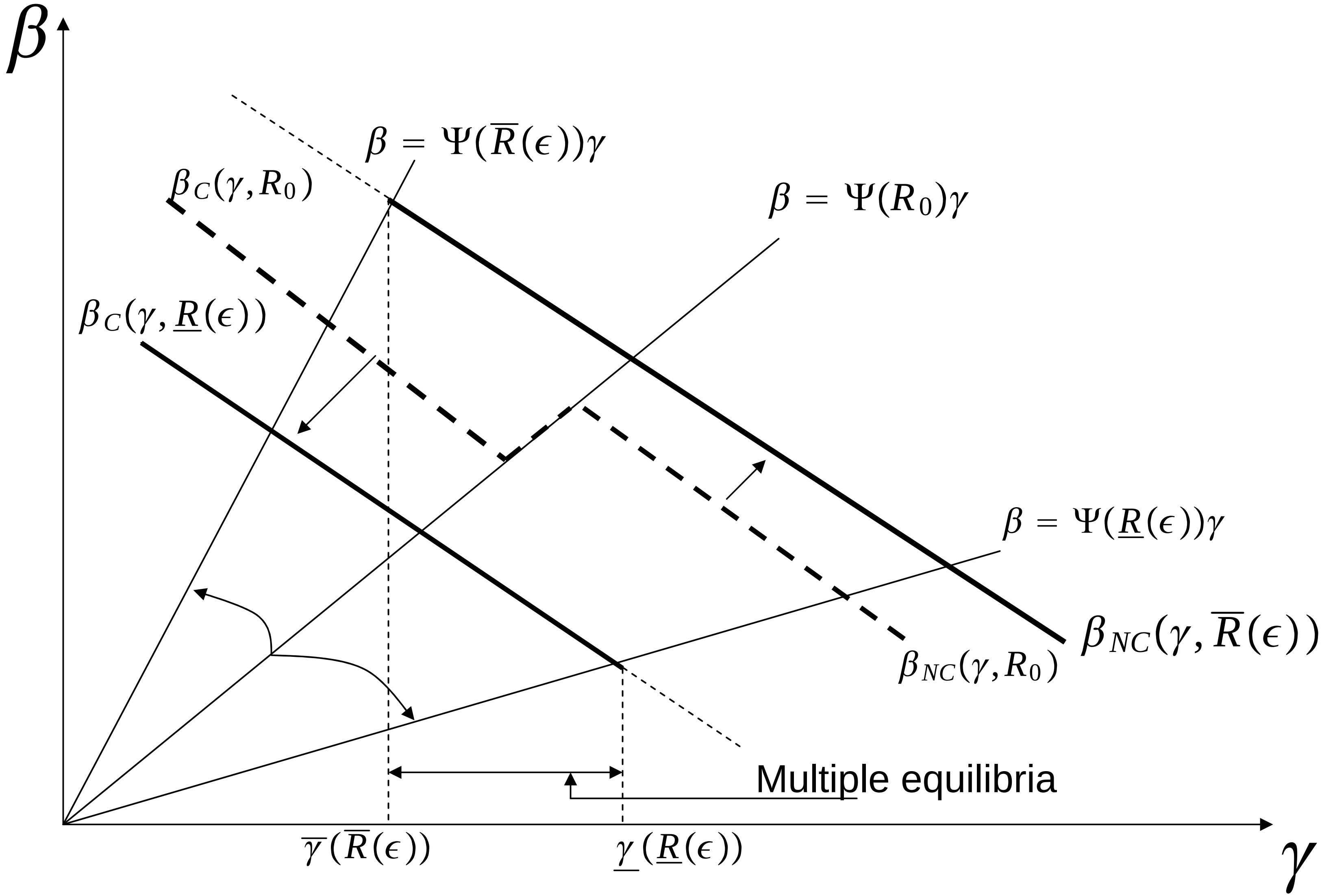

Figure 7a): Preferences of entrepreneurs for banking supervision

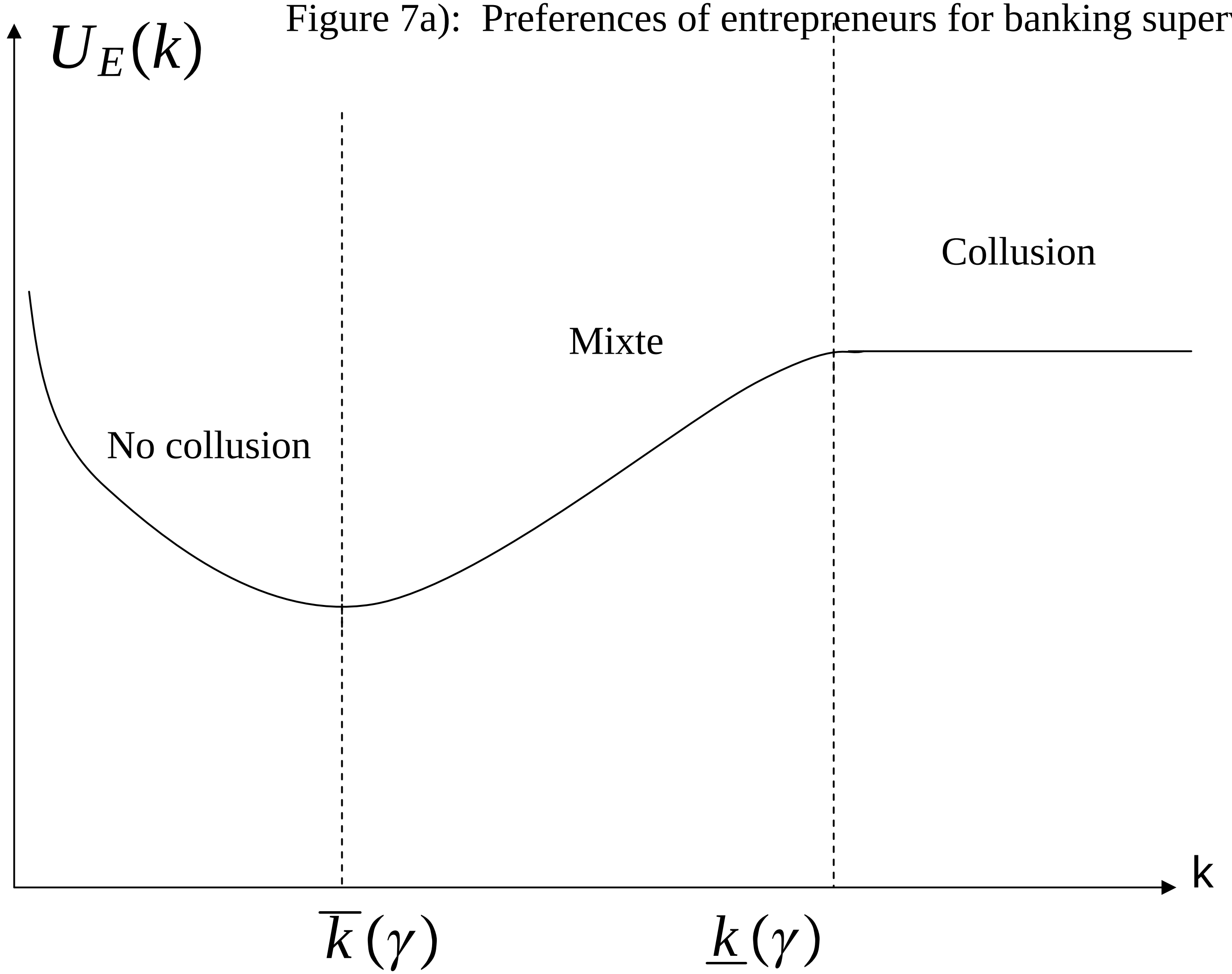

Figure 7b): Preferences of uniformed investors for banking supervision

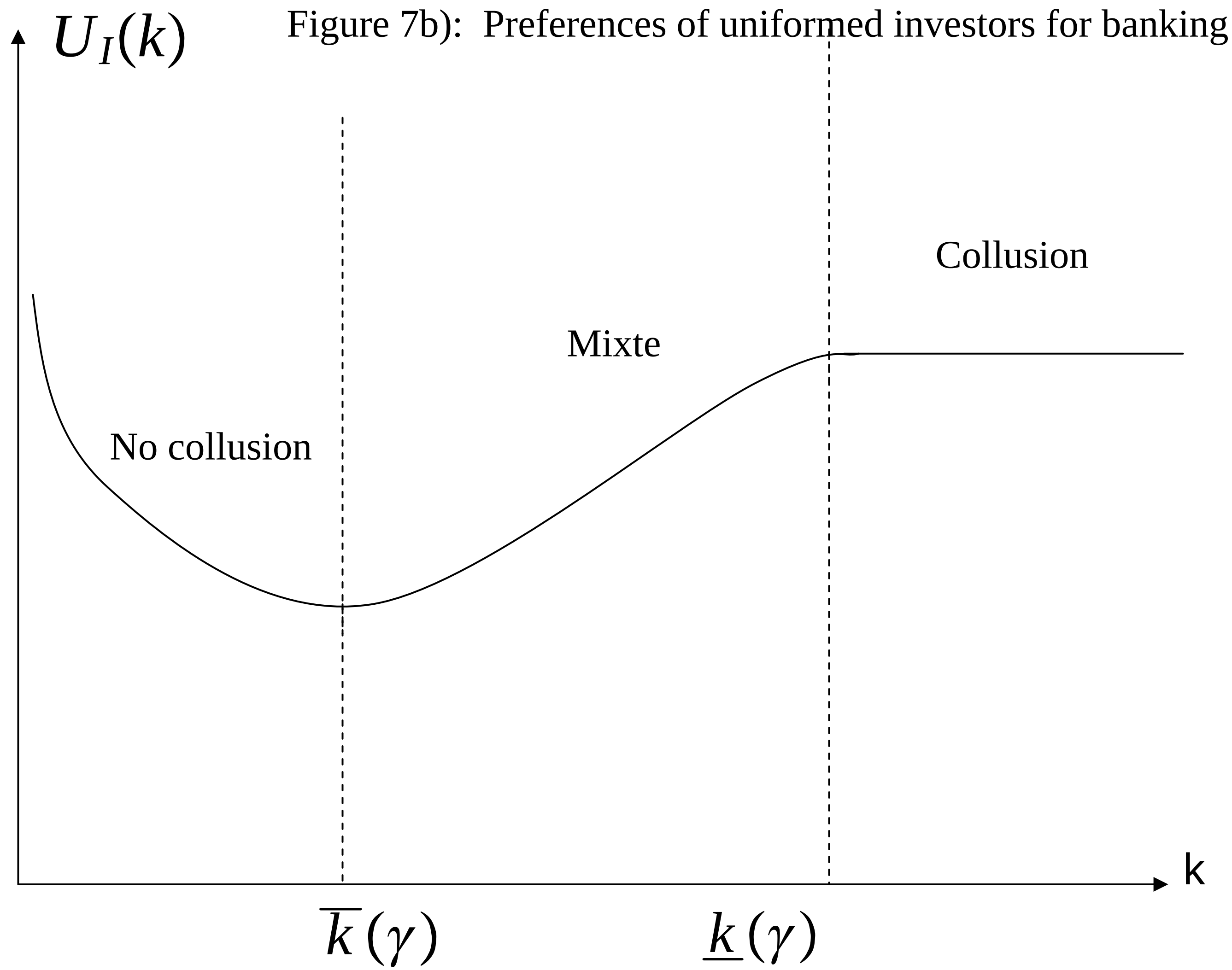

Figure 7c): Preferences of Banks for banking supervision