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FINANCIAL FRICTIONS AND BUSINESS CYCLES IN A MONETARY MARCO-MODEL

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1 Introduction

One of the main research subjects in macroeconomics during the 1990's has been the transmission mechanism of monetary policy. This interest was noteworthy since a large variety of work was published, both theoretical and empirical, due to the fact that many possible explanations of how money affects real activity in the economy have been risen. Therefore the way how money *matters* is still a controversial issue.

The "conventional wisdom" in monetary economics is that an expansionary monetary policy generates a short term decrease in nominal interest rates and an increase in the level of output. Some papers confirmed this reasoning, notably Lucas (1990) and Fuerst (1992) who showed that this outcome is possible in a monetary model with a cash- in-advance (CIA) constraint and if there is *limited participation* in financial markets in response to shocks. Such an assumption emphasizes the frequent recourse of entrepreneurs and financial intermediaries to the adjustment of their financial positions, which is not the case for households. For this reason, the impact of unanticipated monetary disturbances is likely to be felt first by entrepreneurs and intermediaries. This implies that expansionary monetary shocks produce a downward pressure on nominal interest rates by raising the amount of loanable funds that banks have available for lending to entrepreneurs, thereby leading to an increase in real activity, called *the liquidity effect* of monetary policy [1]. Using the Lucas-Fuerst framework, Christiano (1991) and Christiano and Eichenbaum (1992a, b) have introduced the liquidity effect in quantitative general equilibrium models.

In recent years many economists argued that frictions in credit markets are crucial to understand how this monetary shock works, that is to understand the monetary transmission mechanism which leads to an examination of the influence of financial factors on business cycle dynamics. For many decades, this issue was ignored in the business cycle research and the main reason for

1. The existence of the liquidity effect of monetary policy is well treated in Bernanke and Blinder (1992) and Christiano and Eichenbaum (1992a)

that is the difficulty of introducing heterogeneity across agents (e.g., borrowers and lenders) and informational problems (e.g., moral hazard) into dynamic equilibrium models. However, Gertler (1986) rectified this omission in his survey by arguing that, in comparison to the basic real business cycle (RBC) model, financial factors amplify and make more persistent the response to an aggregate shock. This trial was followed by many models that appeared and were based essentially on the idea of *the credit channel* of monetary policy [2] which consists on *the Bank Lending Channel* and *the Balance Sheet Channel*. The former emphasizes the fact that borrowers are Şbank-dependentŤ in the sense that they cannot directly access public debt markets but instead have to finance their projects by loans from banks. The latter emphasizes the role of borrowers' net worth as, due to problems of adverse selection and moral hazard following from informational asymmetries, the amount borrowed does not only depend on the price of credit but also on borrowers net worth. But until recently, there is no monetary stochastic dynamic general equilibrium (SDGE) model that takes into the accout both of the just mentioned points simultaneously and rigorously : First, in the existing models, the special role of banks is often purely ad hoc since the assumption that all borrowers can only borrow from banks is not further justified. Such an assumption should, however, be based on first principles as well as on explicitly modeled informational problems. Second and even more important, the special role of borrowersŠ net worth is not examined. Either there is no net worth of borrowers at all, or it is set constant.

Most of the time, the credit contractual arrangements cannot take into account the role of a changing net worth of borrowers. Theoretically, such a problem can be solved on two stages : first, by endogenizing the special role of banks and allowing for endogenous borrowers' net worth following by that Fuerst (1995), Fisher (1999), and Cooley and Nam (1998). Second, by embedding the costly state verification framework introduced by Townsend (1979) and Gale and Hellwig (1985) into a monetary business cycle model with cash-in-advance constraint and limited participation assumption. Diamond (1984) showed the presence of a clear role for banks which is to interme-

2. See e.g. Bernanke and Gertler (1995), Morris and sellon (1995) and Hubbard (1995).

diate between borrowers and households in order to minimize the aggregate monitoring costs.

Aiming to implement this theoretical solution, the current study is based essentially on the Carlstrom and Fuerst (1997) framework where the authors attempt to formulate a new context in which the heterogeneity and informational problems can be easily handled in an environment that is otherwise quite similar to the canonical real business cycle model. Built on Bernanke and Gertler work (1989) and Fuerst (1995), this new construction assumed that agency costs arise endogenously in the production of investment goods. This assumption was extended in their subsequent paper i.e Carlstrom and Fuesrt (1998) where they assumed that agency costs are more encompassing in the sense that the informational problems arise in the production of aggregate output which leads to a distinction between the "*output model*" and the "*investment model*. This new experiment made possible the investigation of the importance of the investment assumption vs. the output one for business cycle dynamics, and the conditions under which these agency models can deliver amplification and/or persistence. This approach was followed by Cooley and Nam (1998) and Fisher (1999) where they took into account the standard assumption of the real business cycle literature that consumption and investment goods are identical and therefore the relative price of investment is identical to unity.

Using the same framework of Carlstrom and Fuerst (1997), we build its monetary counterpart model where money is introduced via a cash-in-advance constraint on households purchases of consumption goods. Prices are perfectly flexible in the model so that money alters real behavior only via expected inflation effects and the monetary policy is a money supply growth rate rule. The remainder of this study is organized as follows : the next section describes the model and the main assumptions. Section 3 develops the optimal financial contract in a partial equilibrium setting and the properties of the lender's and borrower's shares of income. Section 4 deals with the general equilibrium environment. Section 5 presents the numerical analysis and results. We conclude in section 6.

2 The Model : Overview and Basic Assumptions

This model is essentially the monetary counterpart to the model proposed by Carlstrom and Fuerst (1997) in their famous article "Agency costs, net worth and business fluctuations : a computable general equilibrium model" [3]. Our analysis is carried out in an economy in which the real side is the standard real business cycle model. This work extends their original "investment model", where agency costs arose in the creation of new capital from an additional production sector, by introducing money via a cash-in-advance constraint on household purchases [4]. Prices are supposed to be perfectly flexible in the model so that money alters real behavior only via expected inflation effects. However, this model can be seen as the monetized real business cycle model with state cost verification (hereafter denoted as MSCV).

Extending any type of contemporary business cycle model to incorporate financial frictions is not straightforward. First, because these agency cost problems arise only in a setting in which the Modigliani-Miller theorem does not hold. Second, the main problem is the heterogeneity among agents which is crucial to make lending and borrowing occur among them at the equilibrium. Therefore, we cannot use the representative agent paradigm any more and we should grapple with the complication introduced by the agents' heterogeneity. In our model, we supposed that borrowers and lenders are distinct and we assume that borrowers (entrepreneurs) constitute together a unit mass. The other kind of heterogeneity that one should mention here is the one that characterizes the borrowers themselves. Since at any point in time, there will be a great deal of net worth heterogeneity across them (entrepreneurs), and taking into account the net worth distribution and the way by which it affects the real economy is quite difficult. This problem is settled by assuming the linearity of investment and monitoring technologies, which allows for an exploitation of the aggregation results.

3. Built on B. Bernanke and M. Gertler (1989) and on T. Fuerst (1995).
4. See Cooley and Hansen (1989), Stockman (1981).

The basic structure of our model is as follows : There are two types of principal agents called households and entrepreneurs. They are distinct from one another in order to explicitly motivate lending and borrowing. Households live forever ; they work, consume, save and hold real money balances. We provide more details on households behavior below.

In order to include the agency costs in the model, entrepreneurs play a key role. Following Carlstrom and Fuerst, these individuals are risk-neutral and long-lived for finite horizons [5]. In fact, the assumption of long-lived entrepreneurs allows the net worth effect on the cost of capital to persist and by the way, the net worth becomes a state variable, contributing to the dynamics of the model. Gertler (1995) stated in his comments on monetary and financial interactions in the business cycle the cost of assuming single-period-lived borrowers.

In spite its contribution in the simplification of the analytics, this assumption makes the variation in capital stock the only source of internal dynamics. In addition, because of agency costs and the long-lived borrowers assumption there is a strong tendency for entrepreneurs to accumulate net worth in order to avoid their need for external financing. This behavior is encouraged by the fact that the return to internal funds is greater than the return to external funds, therefore, entrepreneurs will postpone consumption and accumulate enough capital to be self-financed and the agency costs disappear. To keep agency costs operative within the model we need some assumption that will dampen this accumulation. This problem can be handled through two ways. First, following Carlstrom and Fuerst (1996), we can impose an exogenous death probability where dying means liquidating the entrepreneur's net worth, consuming the proceeds and exiting the model. In this context, the assumption of finite horizons for entrepreneurs intends to capture their ongoing births and deaths as well as to prevent any accumulation of capital leading to a self-finance. This same approach was used by Bernanke, Gertler and Gilchrist (2000). Second, we can assume that entrepreneurs optimize their consumption across time in such an extent that self-financing does not

5. Kiyotaki and Moore (1994) also supposed long-lived entrepreneurs.

arise. The most direct way is to assume that the entrepreneurs' personal discount rate is higher than the one of households. For our model, we take the second possibility as a solution since it leads to much more rapid net worth movements in response to chocks, in comparison to the ”death” assumption.

In each period t, entrepreneurs are involved in producing the investment good using a stochastic, constant-returns-to-scale technology that contemporaneously transforms consumption goods into capital. On the other hand, the acquisitions of the input which is the consumption good is financed by entrepreneurial wealth (net worth) and borrowing. Entrepreneurs receive their external financing from households via intermediaries. Following Carlstrom and Fuerst again, we will refer to these intermediaries as capital mutual funds (CMFs). The CMFs cope with the financial and the monetary intermediation in this economy.The model is populated also by a number of firms producing the consumption good utilizing capital and labor as input and they are not subject to any agency costs.

The entrepreneur gathers his net worth by supplying inelastically his labor to firms, buying his non depreciated capital from the last period and receiving the income of capital rent in the previous period to firms. The entrepreneur uses this net worth as the basis for the loan agreement that he will enter into with the lender. In this context, the entrepreneur's net worth plays a critical role in the dynamics of the model since the cost of the external finance is mainly determined by the borrower's financial position. According to the literature, higher levels of net worth allow for increased self-financing and reduce the need for external funds, mitigating the agency problems associated with external finance and reducing the external finance premium faced by the entrepreneur in equilibrium.

The existence of an external finance premium and its negative relation with net worth do also appear in our framework. We postulate a simple agency problem that induces an informational asymmetry between the borrower and his respective lenders. In fact, we are assuming that the entrepreneur's technology is observed only by himself. Others can privately observe it only at a monitoring cost. This situation leads to a moral hazard problem since the entrepreneurs may misreport the true value of their production. Therefore, the optimal contract is designed in such a way that entrepreneurs have no incentive to deviate and they truthfully report the production realization. Following Carlstrom and Fuerst and for tractability we assume that there is enough inter-period anonymity in the financial market that only one-period contracts between lenders and borrowers are feasible [6]. Again this assumption is crucial since the entrepreneurs are long-lived and the contracting problem between lenders and entrepreneurs may take the features of a repeated game with moral hazard. However, the anonymity assumption insures that financial contracts depend only on the entrepreneur's level of net worth, and not on his entire financial past history (past debt repayment).

In the following analysis, we will proceed in two steps. First, we derive the key microeconomic relationship of the model : the dependence of the entrepreneur's demand for consumption good on his potential net worth. To do so, we consider the financial contract in a partial equilibrium setting and we derive an upwardly sloped supply curve for investment goods. Second, we embed these relations in an otherwise monetized standard RBC model. Our main objective is to show how agency costs can act to amplify and to propagate exogenous shocks to the system, particularly a money growth shock. Hereafter, we will try assess the impact of introducing financial frictions on the real variables responses to a monetary shock.

3 The Financial Contract and the Role of Net Worth

In this section we consider the financial contract in a partial equilibrium setting. We separated the consideration of the financial contract from the rest of the general equilibrium because the contract is only one-period in length. It is negotiated in the beginning of each period and is resolved by the end of

6. See Gertler (1992) for the theoretical analysis of an agency-cost model with two-period contract.

that same period. Here it is a two-goods economy, with consumption goods and capital goods. On competitive markets, the consumption good can be bought for the price P and the produced output of the capital good can be sold for the price q. General equilibrium issues affect the contract through the firm's net worth, $n > 0$, price of capital, $q > 0$ and consumption good price, $P > 0$. For instance and for the purpose of this section, net worth and both prices are taken parametrically. In the subsequent section these variables will be endogenized as part of a general equilibrium solution.

There are two types of agents, households (potential lenders) and entrepreneurs (potential borrowers). Both are assumed to be risk neutral. In the next section, households will be assumed to be risk averse and will be considered as a source of outside fund suppliers. However, in terms of the financial contract, they will be effectively risk neutral. Carlstrom and Fuerst (1997) denotes two conditions which are sufficient for this risk neutrality. Namely, (i) the absence of an aggregate uncertainty over the duration of the contract, and (ii) the financial intermediary can take advantage of the law of large numbers to eliminate idiosyncratic risks. These two properties allow the financial intermediary to assure deterministic return to consumers. At time t, the entrepreneur has access to a stochastic constant-returns-to-scale technology that transforms an amount i of consumption goods into ωi of capital, where ω is an idiosyncratic disturbance to the entrepreneur's production. The random variable ω is i.i.d across time and across entrepreneurs, with a continuously differentiable cumulative distribution function $\Phi(\omega)$ over a nonnegative support, a density function $\phi(\omega)$ and a mean of unity. The corresponding hazard rate $h(\omega)$ is given by

$$h(\omega) = \frac{\phi(\omega)}{1 - \Phi(\omega)} \tag{3.1}$$

At the beginning of the period, the investment level i is chosen. The entrepreneur possesses a real wealth of value n, measured as same terms as i, to finance his project. The entrepreneur is supposed to have a sufficiently small amount of net worth such that he would like to borrow some external financing from the financial intermediary, that obtains its funds from households, at a riskless interest rate. The gross interest rate, henceforth denoted by $R > 1$, is therefore the fixed opportunity cost of funds for the investor. Here, it is taken parametrically and it will be later endogenized. To finance the difference between his expenditures on consumption goods and his net worth, the entrepreneur borrow an amount of $(i - n)$ and agree to reimburse $R\,P\,(i - n)$ capital goods to the lender.

In order to allow for a non trivial role for financial structure, we follow Townsend (1979) and we assume a costly state verification problem, in which lenders must pay a fixed monitoring cost to be able to observe ω. This cost is assumed to equal a proportion μ of the value of capital output i.e. the monitoring cost equals $\mu q i$. However, an auditing attempt results in the destruction of μi units of capital. This specification allowed Townsend to show that the presence of these costs explains why uncollateralized external finance may be more expensive than internal finance.

With the absence of any aggregate uncertainty, the optimal contract between the lender and the entrepreneurs under costly state verification is equivalent to a standard risky debt [7] : In particular, the entrepreneur chooses the initial scale of the project i and the associated level of borrowing $P\,(i - n)$ prior to the realization of the idiosyncratic shock ω. According to the description of the contract, the entrepreneur will default if the realization of ω is not high enough to reach the threshold value $\bar{\omega}$ which is determined by :

$$RP(i - n) = qi\,\bar{\omega} \tag{3.2}$$

The entrepreneur has the following decision rule :

- If $\omega \geq \bar{\omega}$, i.e. the entrepreneur's revenue from selling his produced capital is more than the repayment. Under the optimal contract the entrepreneur repays the lender the promised amount $R\,P\,(i - n)$ and he keeps the difference.
- If $\omega < \bar{\omega}$, the entrepreneur cannot pay the contractual return and thus declares bankruptcy. In this situation the lending intermediary pays the auditing cost and then confiscates all the returns from the project. That

7. In addition, it must be assumed that monitoring is a deterministic function of the state and that a commitment device exists.

is, the intermediary net receipts are $i\,q\,(\omega-\mu)$ and the entrepreneur receives nothing.

Note that the contract is completely defined by the pair $(i,\bar{\omega})$ and that it is convenient to consider the optimization problem over these two arguments. The loan volume, $P\,(i-n)$, and the implied interest rate of loans R, can then be calculated given the optimal values of $\bar{\omega})$ and i. But before moving to the optimization process, we describe both entrepreneurs and lenders expected income.

3.1 The expected entrepreneurial income

Since the entrepreneur defaults with a probability $\Phi(\bar{\omega}) = P(\omega \leq \bar{\omega})$ then he reimburses his debt with a probability $1-\Phi(\bar{\omega})$. Therefore the expected entrepreneurial income is :

$$q\left[\int_{\bar{\omega}}^{\infty} \omega i \Phi\,(d\omega) - (1-\Phi) R P (i-n)\right] \tag{3.3}$$

But since $\bar{\omega} = \frac{RP(i-n)}{qi}$ then (3.3) can be written as :

$$\begin{aligned} q\left[\int_{\{\omega}^{\infty} \omega\, i\, \Phi\,(d\omega) - [1-\Phi(\bar{\omega})]\, i\, \bar{\omega}\right] &= q\, i\left[\int_{\bar{\omega}}^{\infty} \omega\, \Phi\,(d\omega) - [1-\Phi(\bar{\omega})]\; \bar{\omega}\right] \\ &= q\, i f(\bar{\omega}) \end{aligned} \tag{3.4}$$

Where $f(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega\, \Phi\,(d\omega) - [1-\Phi(\bar{\omega})]\, \bar{\omega}$.

$f(\bar{\omega})$ is interpreted as the fraction of the expected net capital output received by the entrepreneur.

3.2 The lender's expected income

If the entrepreneur defaults then the lender is obliged to incur a monitoring cost of $\mu q i$ with a probability Φ and he confiscates the total project's return $\omega q i$. But if the entrepreneur reports the true value of ω and reimburses his debt, the lender receives $R\,P\,(i-n)$ with probability $(1-\Phi)$. Then, using the definition of $\bar{\omega}$ the lender's net expected income is :

$$\begin{aligned} q\left[\int_0^{\bar{\omega}} \omega\, i\, \Phi\,(d\omega) - \Phi\,\mu\, i + (1-\Phi(\bar{\omega}))i\,\bar{\omega}\right] &= q\, i\left[\int_0^{\bar{\omega}} \omega\, \Phi\,(d\omega) - \Phi(\bar{\omega})\,\mu + [1-\Phi(\bar{\omega})]\,\bar{\omega}\right] \\ &= q\, i\, g(\bar{\omega}) \qquad (3.5) \end{aligned}$$

Here $g(\bar{\omega})$ is interpreted as the fraction of the expected net capital output received by the lender. Note that the cutoff $\bar{\omega}$ determines the division of the expected gross income qi between borrower and lender, since according to equations (3.4) and (3.5) we have that $f(\bar{\omega}) + g(\bar{\omega}) = 1 - \Phi(\bar{\omega})\,\mu$. So that on average an amount of $\Phi(\bar{\omega})\,\mu$ of the produced capital is destroyed by monitoring.

We define $G_1(\bar{\omega})$ as the expected gross share of income going to the lender :

$$G_1(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega\, \Phi\,(d\omega) + [1-\Phi(\bar{\omega})]\,\bar{\omega}$$

with

$$\begin{aligned} G_1'(\bar{\omega}) &= 1 - \Phi(\bar{\omega}) \\ G_1''(\bar{\omega}) &= -\phi(\bar{\omega}) \end{aligned}$$

These relations imply that the gross lender's income is an increasing, strictly concave function of the cutoff value $\bar{\omega}$. Similarly, we define $G_2(\bar{\omega})$ as the expected monitoring costs :

$$G_2(\bar{\omega}) \equiv -\Phi(\bar{\omega})\,\mu$$

with

$$G_2'(\bar{\omega}) = -\phi(\bar{\omega})\,\mu$$

Note that the net lender's expected income share $g(\bar{\omega})$ equals the gross share of income from which we deduce the monitoring costs paid by the the lender in case of defaults i.e. $g(\bar{\omega}) = G_1(\bar{\omega}) + G_2(\bar{\omega})$ and the share going to the entrepreneur is $1 - G_1(\bar{\omega})$.
From the assumptions made on the distribution function, we can see that $G_1(\bar{\omega}) + G_2(\bar{\omega}) > 0$ for all $\bar{\omega} > 0$ and

$$\begin{aligned} \lim_{\bar{\omega}\to 0} G_1(\bar{\omega}) + G_2(\bar{\omega}) &= 0 \\ \lim_{\bar{\omega}\to \infty} G_1(\bar{\omega}) + G_2(\bar{\omega}) &= 1 - \mu \end{aligned}$$

For this reason and following Bernanke and Gertler (1998), we should impose a new assumption on the riskless interest rate fixed in the beginning of the period for the contract so that $(1 - \mu < R - 1)$, otherwise the entrepreneur would be willing to prevent any default [8] every period by a decrease of the threshold value $\bar{\omega}$ via an increase of the capital price q. So, that he would be able to obtain unbounded profits under the existence of agency costs with a probability one.

If we differentiate the net share of payoff to the lender, one gets :

$$\begin{aligned} g'(\bar{\omega}) &= G_1'(\bar{\omega}) + G_2'(\bar{\omega}) \\ &= (1 - \Phi(\bar{\omega}))(1 - \mu h(\bar{\omega})) \end{aligned}$$

Where $h(\bar{\omega})$ is the hazard function at the cutoff value $\bar{\omega}$. At this stage, in order to guarantee the existence of a unique interior maximum of the lender's income, we impose another condition on the density function ϕ, concerning its elasticity e_ϕ.

Proposition 1 *The lender's income maximization problem has a unique interior solution iff* $e_\phi > -\frac{\overline{\omega}}{\mu}$.

This last condition is relatively weaker than the one imposed by Bernanke et al. (1998) and is particularly satisfied by most conventional distributions including the uniform one under the parameters' calibration retained below. In addition, it is very important when proving the positive and the negative slopes of the investment curve as a function of the capital price and interest rate, respectively (See appendix A.1 for the proof). Therefore, it exists $\bar{\omega}^*$

8. Kiyotaki and Moore contract (1995) is that borrowing is constrained by the level of net worth that default never occurs in equilibrium. In contrast, in our model, default is an equilibrium phenomenon.

such that

$$\begin{aligned}(1-\Phi(\bar{\omega}))(1-\mu h(\bar{\omega})) &< 0 \; for \; \bar{\omega} > \bar{\omega}^* \\ &= 0 \; for \; \bar{\omega} = \bar{\omega}^* \\ &> 0 \; for \; \bar{\omega} < \bar{\omega}^*\end{aligned}$$

implying that the net payoff to the lender reaches a global maximum at $\bar{\omega}^*$. This implication with the restriction imposed on the elasticity of the density function are used to guarantee a non-rationing outcome.

3.3 The optimal contract and the aggregate supply

The optimal contracting problem with non-stochastic monitoring is given by the pair $(i, \bar{\omega})$ that maximizes the entrepreneurial expected income under the constraint that the lender is indifferent between loaning the funds and retaining them. The optimization problem is given by

$$\max_{(\bar{\omega},i)} q\, i\, f(\bar{\omega})$$

$$\text{subject to} \qquad q\, i\, g(\bar{\omega}) \geq R\, P\, (i-n)$$

We may here add the entrepreneur's participation constraint, which will always be satisfied below [9].

$$qif(\bar{\omega}) \geq n \tag{3.6}$$

The Lagrange equation of this problem is given by :

$$L = qif(\bar{\omega}) + \lambda(qig(\bar{\omega}) - RP(i-n))$$

9. It is straightforward to show that entrepreneurs will be willing to invest their net worth, since the expected the return on internal funds is higher than the expected return to external funds

Where λ is the Lagrange multiplier.
Assuming an interior solution and by computing the derivative of L with respect to i and $\bar{\omega}$ we get :

$$\begin{aligned} qf(\bar{\omega}) &= \lambda(RP - qg(\bar{\omega})) \\ qif'(\bar{\omega}) &= -\lambda qig'(\bar{\omega}) \end{aligned}$$

These two first order conditions are summarized in

$$\frac{f(\bar{\omega})}{if'(\overline{\omega})} = \frac{qg(\overline{\omega}) - RP}{qig'(\bar{\omega})} \tag{3.7}$$

We know that

$$g'(\bar{\omega}) = -\phi(\bar{\omega})\mu - f'(\bar{\omega}) \tag{3.8}$$

By plugging this result in (3.7) we get

$$q\left\{1 - \Phi(\bar{\omega})\mu + \phi(\bar{\omega})\mu \frac{f(\bar{\omega})}{f'(\bar{\omega})}\right\} = R\,P \tag{3.9}$$

This last equation can be written as :

$$\begin{aligned} q &= RP\left[1 - \Phi(\bar{\omega})\mu + \phi(\bar{\omega})\mu\,\frac{f(\bar{\omega})}{f'(\bar{\omega})}\right]^{-1} \\ &= RP\left[1 - D(\bar{\omega})\right]^{-1} \end{aligned}$$

(3.10)

Where $D(\bar{\omega})$ can be interpreted as the total default costs. Equation (3.10) defines an implicit function $\bar{\omega}(q/P, R)$ that is increasing in (q/P) and decreasing in R (See appendix A.2). The relative capital price, q/P, differs from R due to the presence of the credit market friction. That is, to compensate for the bankruptcy (monitoring) costs, there must be a premium on the price of the capital. And this premium is set by the amount of monitoring costs and the probability of bankruptcy. (Note that $f' = \Phi(\bar{\omega}) - 1 < 0$).
At the optimum, the constraint is binding which gives us the second first

order equation in i :

$$i = \frac{R\,P\,n}{R\,P - q\,g(\bar{\omega})} \tag{3.11}$$

By substituting $\bar{\omega}(q/P, R)$ into equation (3.11), we have again an implicit function $i(q/P, R, n)$, which represents the units of consumption goods invested in the capital technology. So, we get :

$$\begin{aligned} i &= \frac{R\,P}{R\,P - q\,g(\bar{\omega}(q/P, R))}\,n \\ &\equiv \psi(\frac{q}{P}, R)\,n \end{aligned}$$

This investment is linear in n with a factor of proportionality of $\psi(q/P, R)$, which exceeds one. Many similar multipliers to $\psi(q/P, R)$ can be found in the literature and in many other models with agency costs. For instance, Holmstrom and Tirole (1998) found the same multiplier in their model and they called it *equity multiplier*. In fact, it is a peculiar and common feature for those imperfect information models that investment requires a *down payment*. Given the infinite number of entrepreneurs, equation (3.11) can be interpreted as the new-capital supply function, increasing with the capital relative price and decreasing with the interest rate, since we have :

$$\psi_{q/P} > 0$$

and

$$\psi_R < 0$$

(See Appendix A.2 for the proof).

The expected capital output is given by :

$$I^s(q, n) \equiv i(\frac{q}{P}, R, n)\;1 - \mu\Phi\left[\bar{\omega}(q/P, R)\right] \tag{3.12}$$

This equation can be interpreted as the investment function. By deducing $\bar{\omega}$ from equation (3.9) and using the linearity of equation (3.11), we can sum up I^s given by (3.12) across all entrepreneurs. Thanks to the linearity of the stochastic and monitoring technologies, the aggregate investment depends only on the economy-wide relative price of capital (q/P), the economy-wide

identical opportunity costs R and on the aggregate net worth.
The expected return to internal funds is given by

$$\frac{qf(\bar{\omega})i}{n} = \frac{qf(\bar{\omega})RP}{RP - qg(\bar{\omega})} \tag{3.13}$$

This equation is equivalent to

$$\frac{i}{n} = \frac{RP}{RP - qg(\overline{\omega})} \tag{3.14}$$

Intuitively, the entrepreneurial net worth of size n is leveraged in a project of size i, with output (capital) is priced at (q/P) consumption goods.

4 Recursive Competitive Equilibrium

In this section we will embed the contracting problem of the previous section into a monetary business cycle model with limited participation assumption. Among other things, this will permit us to endogenize the riskless interest rate and prices of capital and consumption, which were taken as given in the partial equilibrium setting.

As it was mentioned previously, the model is populated by four types of agents : consumers, entrepreneurs, firms and a financial intermediary. In addition, new money is injected into the economy every period by a monetary authority. At the beginning of period t, the households decide on the way by which their cash will be allocated. In fact ,they should fix the amount to be deposited in the (CMFs) in one hand and the amount to be used for purchases of the consumption good on the other hand. Here, we are assuming that nominal consumption is fully financed with cash. After this saving decision, the monetary shock and the aggregate shock are realized. Note that the timing assumption is very important. However, under the limited participation or "sluggish cash flow" assumption, the asset market opens first and then the goods market opens [10]. Therefore, the household selects its level of deposits before knowing the current monetary and technological innovations.

10. See Walsh C. E (2003). Monetary theory and policy.

This assumption summarizes a portfolio rigidity in the typical household's cash saving choice. After the innovations' realization, the entrepreneurs and the (CMFs) write the credit contracts. In order to exploit the static costly state verification framework described above, we assume that the idiosyncratic productivity shock to the entrepreneurs, ω, is realized after the contractual negotiations and it is learned costlessly by the respective entrepreneur.

After all the shocks are realized, the household sells his labor and rents his capital stock to the firms and the entrepreneur supplies inelastically his unit of labor also to the firms. These latter produce the consumption good which is used in part by the entrepreneurs through the financial contract in order to produce the capital good.

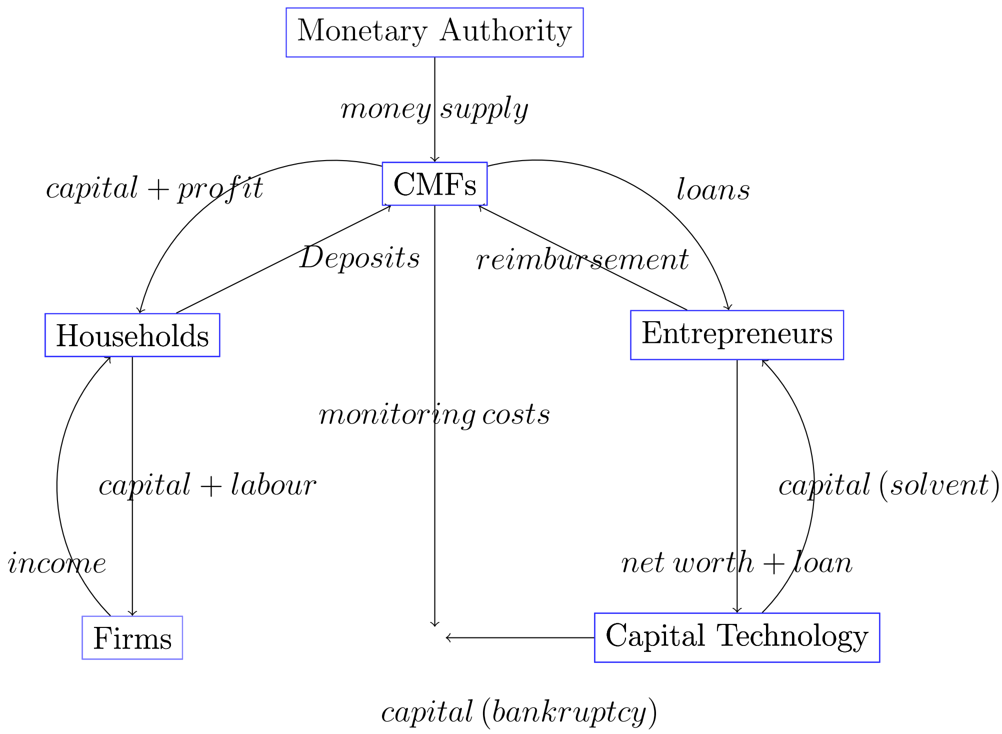

Fig. 1. Flow of funds in credit channel model.

Then the entrepreneurs reimburse their credit or claim bankruptcy. In the latter case, the faulty entrepreneur will be monitored by the (CMFs). But if the entrepreneur produces enough capital and still has resources, he will decide his level of consumption and the amount of money to be kept for the next period. The main reason for assuming that entrepreneurs supply labor and carry a money balance over to the next period is that if a non solvent entrepreneur going bankrupt in the current period, his net worth for the next period is different from zero anyway since he will have at least an amount equals to the labour income for the current period. This assumption is very important for two reasons : First the optimal contract is not well defined for zero levels of net worth. As we can see from equation (3.11), the investment level i is linear in the net worth. Second, the faulty entrepreneurs in the current period should have at least some amount of internal wealth in order to finance an other project in the next period. Otherwise a non solvent entrepreneur in the current period will quit the economy in the next one. Therefore, the fraction of active entrepreneurs in this economy will decrease over time, which will make the computations more complicated.
At the end of the period, the financial intermediary pays back the deposits to the households with their interests. Note that households own the (CMFs) and the firms ; then, they get profits from them and carry their monetary balance over to the next period as cash holding. Figure 1 describes the implied flows and the main transactions in this economy.
In the following, we will present a formal description of the model and the optimization problem for each agent. The details of the corresponding computations are presented in Appendix B.1.

4.1 Households

We assume a continuum of identical, infinitely lived households with preferences given by the following utility function :

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

Where E_0 denotes the expectation operator conditional on time-0 information ; $\beta \in (0,1)$ is the personal discount factor ; C_t is time-t consumption ;

L_t is time-t labor and the leisure endowment is normalized to unity, so that $L_t \in (0,1)$. In the course of any given period, households are engaged in accumulating capital which they rent to firms at a real rental rate r_t. We assume that households enter period t with a nominal money balances equal to ${M_t}^h$ that are carried over from the previous period. In the beginning of each period, the household should take its saving decision i.e. he must decide how much of this cash to keep on hand for contemporaneous consumption and capital purchasing and how much to deposit in the intermediary where a gross nominal return R_t can be earned. Consumption is fully financed by cash, so that it is a cash good while investment and leisure are credit goods [11]. Let N_t denote the amount of cash deposited in the intermediary. N_t is supposed to be fixed until the next period. As it was mentioned above, we assume a portfolio rigidity framework i.e. N_t should be selected before any current shock. Therefore, the cash-in-advance constraint is

$$P_t C_t \leq {M_t}^h - N_t \tag{4.1}$$

Where P_t is the nominal price of the consumption good.
In the end of each period, Households receive their labor income earned by offering labor to firms at nominal wage rate W_t, then this income is equal to $W_t L_t$. Furthermore, they receive the nominal capital income $r_t P_t K_t$ and the savings paid back in the form of interest plus principal, represented by $R_t N_t$. Since, each household owns an equal equity share in each of the firms and in the CMFs, then he receives cash dividend payment from both the firms and intermediary. Hence, the households' budget constraint in nominal terms is given by :

$${M_{t+1}}^h = W_t L_t + (R_t - 1)N_t + r_t P_t K_t + {\Pi^F}_t + {\Pi^B}_t + {M_t}^h - P_t C_t - q_t i_t \tag{4.2}$$

Where ${\Pi^F}_t$ and ${\Pi^B}_t$ are the profits coming from firms and CMFs , respectively. q_t is the nominal capital price, i_t is the level of investment and K_t is the aggregate capital stock.
The level of investment i is undertaken in order to augment the capital stock K owned by the household. Then the capital stock obeys to the following

11. Following the terminology of Lucas and Stockey (1983, 1987).

law of motion :

$$K_{t+1} = (1-\delta)K_t + i \tag{4.3}$$

where $\delta \in (0,1)$ is the depreciation rate on capital.

Formally, the representative household maximizes his utility function subject to constraints (4.2) and (4.3) by choosing the efficient levels of consumption, savings, the fraction of labour offered by the household and investment. Since the timing in our model is very important, the resolution of the maximization problem for every variable is conditioned on the available information when the variable was decided. Therefore, we distinguish two sets of information : $\mathcal{I}_{0t}$ and $\mathcal{I}_{1t}$. The set $\mathcal{I}_{0t}$ includes the aggregate capital stock K_t, the amounts of cash holdings in the beginning of the period for households and entrepreneurs and the values of all-economy wide variables decided at (t-1) and earlier. While $\mathcal{I}_{1t}$ includes $\mathcal{I}_{0t}$ and the period t realizations of the aggregate technology and monetary shock. Then, the optimal households' decisions are summarized in the following Euler-equations :

$$E\left\{U_{L,t} + \beta U_{c,t+1}\frac{W_t}{P_{t+1}}|\mathcal{I}_{1t}\right\} = 0, \tag{4.4}$$

$$E\left\{U_{c,t+1}\frac{q_t}{P_{t+1}} - \beta U_{c,t+2}\frac{r_{t+1}P_{t+1} + (1-\delta)q_{t+1}}{P_{t+2}}|\mathcal{I}_{1t}\right\} = 0, \tag{4.5}$$

$$E\left\{U_{c,t} - \beta R_t U_{c,t+1}\frac{P_t}{P_{t+1}}|\mathcal{I}_{0t}\right\} = 0. \tag{4.6}$$

4.2 Firms

The firms in the model produce a homogeneous consumption good in a competitive market context, using a constant-returns-to-scale production function. As an aggregation device, we assume that all the firms in the economy are represented by one price-taker firm which maximizes its profits and produces the aggregate quantity Y_t of consumption good using the production function $f(.)$ given by :

$$Y_t = \theta_t f(K_t, H_t, {H_t}^e),$$

where K_t is the aggregate capital stock, H_t is the labour supplied by the households, ${H_t}^e$ is the labour input of the entrepreneurs and θ_t is the aggregate productivity shock. We assume that θ_t evolves according to the following law of motion :

$$\theta_t = \bar{\theta}^{1-\rho}{\theta^\rho}_{t-1}\varepsilon_t$$

where ε_t is an i.i.d random variable with mean 0, $\rho \in (0,1)$ is the autocorrelation coefficient and $\bar{\theta}$ is the non stochastic steady state of θ. Hereafter, the representative firm will aim to maximize its profit ${\Pi^F}_t$ taking the factor prices as given.

$${\Pi^F}_t = P_tY_t - W_tH_t - W_t^eH_t^e - r_tP_tK_t.$$

The profit maximizing representative firm's first order conditions are given by the factors markets competition conditions ; that wage and rental rates are equal to their respective marginal productivities i.e.

$$W_t = P_t\theta_t f_H(t) \tag{4.7}$$
$$W_t^e = P_t\theta_t f_{H^e}(t) \tag{4.8}$$
$$r_t = \theta_t f_K(t) \tag{4.9}$$

4.3 Monetary policy

The monetary policy in this economy is conducted by the government through the central bank. Along with the model, we refer to it by the monetary authority. At the beginning of every period, the government supplies a quantity of money to the economy, taking the form of a lump-sum transfer [12]. This latter is equal to $(\tau_t - 1){M_t}^s$, where M_t^s is the per capita money supply in period t and τ_t is the gross money growth rate and it follows a law of motion given by :

$$\tau_t = \bar{\tau}^{1-\gamma}{\tau^\gamma}_{t-1}u_t$$

where u_t is an i.i. d random variable with mean 0, $\gamma \in (0,1)$ is the autocorrelation coefficient, and $\bar{\tau}$ is the non-stochastic steady state of τ_t.

12. Called by some authors "the helicopter drop of new money, see Carlstrom and Fuerst (2001)"

4.4 Financial intermediary

In this economy we refer to the financial intermediary by the capital mutual funds (CMFs) and it copes with the financial and monetary operations. At the beginning of period t, the CMFs' balances are augmented with the new money coming from the monetary authorities. In addition, the intermediary receives also deposits N_t from households so, the accumulated cash coming from households and central bank is loaned to entrepreneurs using the debt contract outlined in the previous section. The intermediary's main role is to coordinate lending from consumers to entrepreneurs. We assume here that the financial intermediary is the only way to get a loan i.e. all the lending in this economy must be handled by the CMFs [13]. Therefore, This implies that the CMFs' profit is given by

$$\Pi^B{}_t = R_t M_t{}^s(\tau_t - 1)$$

4.5 Entrepreneurs

The key innovation of this paper is to allow for the heterogeneity among entrepreneurs and for endogenous net worth accumulation. Each entrepreneur is indexed by i with $i \in [0, 1]$.

At the period t, entrepreneurs are endowed with one unit of leisure that they inelastically supply to firms for a wage rate W^e. As it was mentioned earlier, the entrepreneurs carry over cash holdings M^e_{it} from the last period in order to guarantee a non null net worth for the actual period. Therefore, the individual net worth of an entrepreneur, used in the financial contract negotiations, is given by :

$$n_{it} = \frac{1}{P_t}(M^e_{it} + W^e_t) \tag{4.10}$$

At the end of the period, solvent entrepreneurs spend their income for consumption C^e_t and for net worth accumulation, taking the form of a strictly positive cash holding M^e_{it+1} carried over into the next period. While, cash holding is

13. Diamond (1984) and Williamson (1986) show that intermediation dominates borrowing and lending between individuals. Hellwig (1994) explained extensively the underlying frictions in markets that lead to intermediation.

equal to 0 for faulty entrepreneurs since all their production is confiscated by the CMFs. Formally, each entrepreneur i maximizes his inter-temporal utility

$$E_0 \sum_{t=0}^{\infty} (\beta\lambda)^t C_{it}^e$$

Subject to the following budget constraint

$$\begin{aligned} M_{it+1}^e = q_t \omega_{it}\, i_t - R_t P_t (i_t - n_{it}) - P_t C_{it}^e, \quad \text{if} \quad \omega_{it} \geq \bar{\omega}_i \\ M_{it+1}^e = 0 \text{ and } C_{it}^e = 0 \quad \text{if} \quad \omega_{it} < \bar{\omega}_i \end{aligned} \tag{4.11}$$

By integrating this budget constraint over all entrepreneurs, we get the law of motion for aggregate entrepreneurial cash holding, M_t^e, and it is given by :

$$M_{t+1}^e = i_t q_t f(\bar{\omega}) - P_t C_t^e, \tag{4.12}$$

Where C_t^e is the aggregate entrepreneurial consumption. This result is deduced from the financial contract optimization problem. In fact, at the equilibrium the entrepreneurial total income is given by $i_t q_t f(\bar{\omega})$ which corresponds exactly to the aggregated individual entrepreneurial profit given by $q_t \omega_{it}\, i_t - R_t P_t (i_t - n_{it})$. It is straightforward to recognize that equation (4.12) depends only on the aggregate amount of net worth through the investment level. It was shown in section 2 that the aggregate investment level is also independent of the distribution of net worth across the entrepreneurs and does only depend on the aggregate level of net worth n_t. The heterogeneity introduced in equation (4.11) is altered by the linearity properties of the financial contract.

The maximization of the entrepreneur's utility by choosing his level of consumption and his cash holding for the next period, under the budget constraint given by (4.12), leads to the following entrepreneur's Euler-equation :

$$E \left\{ 1 - \beta\lambda \frac{q_{t+1}}{P_{t+1}} \frac{R_{t+1} p_t f(\bar{\omega})}{R_{t+1} P_{t+1} - q_{t+1} g(\bar{\omega})} | \mathcal{I}_{1t} \right\} = 0. \tag{4.13}$$

This Euler-equation is independent of the individual level of net worth and it is satisfied for all solvent entrepreneurs. Therefore, it summarizes the all behavior of the entrepreneurial sector and it will be considered solely in the general equilibrium analysis.

4.6 The general equilibrium

The stochastic trend in money raised a non-stationarity problem. Therefore, the model should be detrended and made stationary. The most direct solution is to normalize it relative to the aggregate money stock [14]. Therefore, we propose the following transformation which consists on dividing the nominal prices and quantities by the money supply at the beginning of the period, ${M^s}_t$, so we get :
$\widetilde{P}_t = P_t/{M_t}^s$, $\widetilde{q}_t = q_t/{M_t}^s$, $\widetilde{N}_t = N_t/{M_t}^s$, $\widetilde{W}_t = W_t/{M_t}^s$, $\widetilde{W^e}_t = {W_t}^e/{M_t}^s$, $\widetilde{\Pi_t}^F = {\Pi_t}^F/{M_t}^s$, $\widetilde{\Pi_t^B} = {\Pi_t}^B/{M_t}^s$, $\widetilde{M_t}^h = {M_t}^h/{M_t}^s$, $\widetilde{M_t}^e = {M_t}^e/{M_t}^s$.

A rational equilibrium is defined in the usual way. It consists of time invariant aggregate allocation and price functions of the relevant state such that given these rules agents' optimization satisfies market clearing. Thus, it is defined in our setting by the stationary competitive equilibrium which consists on a set of policy functions, C, K, L, i, $\widetilde{N}_t$, $\widetilde{M_t}^h$, C^e, $\widetilde{M_t}^e$, $\bar{\omega}$ and pricing functions $\widetilde{P}_t$, $\widetilde{q}_t$, $\widetilde{W^e}_t$, $\widetilde{W}_t$, R, r, that satisfy the following efficiency and market clearing conditions :

14. Cooley and Hansen (1989) used the same method and they discussed it extensively in Cooley and Hansen (1995).

Efficiency conditions and financial contract

$$E\left\{U_{L,t}+\beta U_{c,t+1}\frac{\widetilde{W}_t}{\widetilde{P}_{t+1}\tau_t}|\mathcal{I}_{1t}\right\} = 0, \tag{4.14}$$

$$E\left\{U_{c,t+1}\frac{\widetilde{q}_t}{\widetilde{P}_{t+1}\tau_t}-\beta U_{c,t+2}\frac{r_{t+1}\widetilde{P}_{t+1}+(1-\delta)\widetilde{q}_{t+1}}{\widetilde{P}_{t+2}\tau_{t+1}}|\mathcal{I}_{1t}\right\} = 0, \tag{4.15}$$

$$E\left\{U_{c,t}-\beta R_t U_{c,t+1}\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}\tau_t}|\mathcal{I}_{0t}\right\} = 0, \tag{4.16}$$

$$E\left\{1-\beta\lambda\frac{\widetilde{q}_{t+1}}{\widetilde{P}_{t+1}}\frac{R_{t+1}\widetilde{P}_t f(\bar{\omega}_t)}{R_{t+1}\widetilde{P}_{t+1}-\widetilde{q}_{t+1}g(\bar{\omega}_t)}|\mathcal{I}_{1t}\right\} = 0, \tag{4.17}$$

$$\widetilde{q}_t\left\{1-\Phi(\bar{\omega}_t)\mu+\phi(\bar{\omega}_t)\mu\frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)}\right\} = R\,\widetilde{P}_t, \tag{4.18}$$

$$\frac{R_t\,\widetilde{P}_t\,n_t}{R_t\,\widetilde{P}_t-\widetilde{q}_t\,g(\bar{\omega}_t)} = i_t, \tag{4.19}$$

$$\widetilde{P}_t\,\theta_t\,f_H(t) = \widetilde{W}_t, \tag{4.20}$$

$$\widetilde{P}_t\,\theta_t\,f_{H^e}(t) = \widetilde{M_t}^e, \tag{4.21}$$

$$\theta_t\,f_K(t) = r_t \tag{4.22}$$

combined with the following **market clearing conditions**

$$K_{t+1}=(1-\delta)K_t+i_t, \tag{4.23}$$

$$C_t+C_t^e+i_t=Y_t, \tag{4.24}$$

$$H_t=L_t, \tag{4.25}$$

$$H_t^e=1, \tag{4.26}$$

$$\widetilde{M_t}^h+\widetilde{M_t}^e=\widetilde{M}_t, \tag{4.27}$$

$$\widetilde{M}_t=1, \tag{4.28}$$

$$\widetilde{P}_t(i_t-n_t)=\widetilde{N}_t+\widetilde{M}_{t+1}\tau_t-\widetilde{M}_t, \tag{4.29}$$

And the other **complementary conditions**

$$(\widetilde{W}_t^e + \widetilde{M}_t^e)/\widetilde{P}_t = n_t, \tag{4.30}$$

$$\widetilde{M}_{t+1}^e \tau_t = \widetilde{q}_t i_t f(\bar{\omega}_t), \tag{4.31}$$

$$\theta_t K_t^{\alpha_1} H_t^{\alpha_2} (H_t^e)^{1-\alpha_1-\alpha_2} = Y_t, \tag{4.32}$$

$$\widetilde{M}_t^h - \widetilde{N}_t = \widetilde{P}_t C_t, \tag{4.33}$$

$$\theta_t = \bar{\theta}^{1-\rho} \theta^{\rho}{}_{t-1} \varepsilon_t, \tag{4.34}$$

$$\tau_t = \bar{\tau}^{1-\gamma} \tau^{\gamma}{}_{t-1} u_t. \tag{4.35}$$

4.7 The Steady state analysis

The steady state is defined by time-invariant quantities
$C_t = C^*, C_t^e = C^{e*}, Y_t = Y^*, \widetilde{M_t}^h = \widetilde{M^h}^*, \widetilde{M_t}^e = \widetilde{M^e}^*, \widetilde{P}_t = \widetilde{P}^*, i_t = i^*,$
$n_t = n^*, h_t = h^*, \widetilde{q}_t = \widetilde{q}^*, R_t = R^*, r_t = r^*, \widetilde{W}_t = \widetilde{W}^*, \widetilde{W_t}^e = \widetilde{W}^{e*}, K_t = K^*,$
$\widetilde{N}_t = \widetilde{N}^*, \bar{\omega}_t = \bar{\omega}^*$
So there are seventeen unknowns. While we have nineteen equilibrium conditions.
From equation (4.15) we have

$$\widetilde{q}^* = \beta(r^* \widetilde{P}^* + (1-\delta)\widetilde{q}^*) \tag{4.36}$$

But from equation (4.22) we have

$$\begin{aligned} r^* &= \bar{\theta}\alpha_1 K^{*\alpha_1 - 1} H^{*\alpha_2} \\ &= \alpha_1 K^{*\alpha_1 - 1} H^{*\alpha_2}. \end{aligned} \tag{4.37}$$

then (4.36) gives :

$$\begin{aligned} \widetilde{q}^* &= \frac{\beta \widetilde{P}^*}{1-\beta(1-\delta)} \alpha_1 K^{*\alpha_1 - 1} H^{*\alpha_2} \\ &= \frac{\alpha_1 \beta \widetilde{P}^*}{1-\beta(1-\delta)} \frac{Y^*}{K^*}. \end{aligned}$$

with

$$Y^* = K^{*\alpha_1} H^{*\alpha_2}$$

From equation (4.14) we have

$$
\begin{aligned}
\beta\frac{\widetilde{W}^*}{\bar{\tau}C^*\widetilde{P}^*} &= \nu \\
\beta\frac{K^{*\alpha_1}\,\alpha_2\widetilde{P}^*H^{*\alpha_2-1}}{\bar{\tau}C^*\widetilde{P}^*} &= \nu \qquad (4.40)
\end{aligned}
$$

Which gives finally,

$$H^* = \frac{\beta\alpha_1}{\nu\bar{\tau}}\frac{Y^*}{C^*} \qquad (4.41)$$

From equation (4.23) we have

$$i^* = \delta K^* \qquad (4.42)$$

In a typical standard monetary RBC model, these previous equations are used to find the steady state $(C^*, K^*, H^*, \widetilde{P}^*\ )$ because $q^* = 1$. Here since the price of capital is endogenous, we have more unknowns than equations. From equation (4.16) we have

$$R^* = \frac{\bar{\tau}}{\beta} \qquad (4.43)$$

We have two conditions from the financial contract

$$\widetilde{q}^* = \frac{R^*\widetilde{P}^*}{1 - 1 - \Phi(\bar{\omega}^*)\mu + \phi(\bar{\omega}^*)\mu\frac{f(\bar{\omega}^*)}{f'(\bar{\omega}^*)}} \qquad (4.44)$$

and

$$i^* = \frac{R^*\widetilde{P}^*}{R^*\widetilde{P}^* - q^*(1 - \Phi(\bar{\omega}^*)\mu - f(\bar{\omega}^*))}n^* \qquad (4.45)$$

While from equation (4.30) we have

$$\widetilde{P}^*n^* = \widetilde{W}^{e*} + \widetilde{M}^{e*} \qquad (4.46)$$

and from equation (4.47)

$$\widetilde{M^e}^* = \frac{1}{\bar{\tau}}\widetilde{q}^*i^*f(\bar{\omega}^*), \qquad (4.47)$$

And therefore,

$$\widetilde{M^h}^* = 1 - \widetilde{M^e}^* \qquad (4.48)$$

From equation (4.33) we have

$$\widetilde{N}^{*} = \widetilde{M^{h}}^{*} - \widetilde{P}^{*}C^{*} \tag{4.49}$$

Therefore, using this last equation and equations (4.46) and (4.29) we get

$$\widetilde{P}^{*} = \frac{\widetilde{M^{h}}^{*} - \widetilde{P}^{*}C^{*} + \bar{\tau} - 1}{\widetilde{P}^{*}(i^{*} - n^{*})}. \tag{4.50}$$

Finally, we have the following resource constraint

$$C^{*} + C^{e*} + i^{*} = K^{*\alpha_1}\, H^{*\alpha_2}. \tag{4.51}$$

5 The numerical analysis

As the model is too complicated to be solved analytically, we will follow a numerical approach using the optimization conditions derived above. In order to come up with a numerical solution to the model, the values of the parameters have to be specified. Then the solution technique can be applied. Thereafter, the quantitative properties will be discussed using impulse response functions derived later.

5.1 Calibration

The model is parametrized at the non-stochastic steady state in order to match empirical results. Similarly to many empirical models, the period considered here is assumed to be one quarter. As it is mentioned in the Appendix B.1, the household's utility function is given by $U(C_t, L_t) = logC_t + \nu(1 - L_t)$ [15]. The constant ν is chose such that the empirical ratio of $L/(1-L)$ matches its empirical value i.e. 0.28. Therefore, ν is set to be equal to 3.985. The riskless interest rate and the growth rate $\bar{\tau}$ of money supply are supposed to be equal to 8.05% and 1.0119, respectively [16].The consumption technology is supposed to be Cobb-Douglas as in Appendix B.2, with a capital share of 0.36 and household labour share of 0.6399. Thus, the entrepreneurial labour

15. Te same specification was used by Hansen (1985) and Rogerson (1988).
16. These values are taken from Christiano and Eichenbaum (1995).

share is very small and it is equal to 0.0001. Carlstrom and Fuerst (1997) used the same values in order to guarantee a strictly positive net worth for the entrepreneur. On the hand in our context the entrepreneur's share is chose arbitrary small so that the model without agency costs collapses to the monetized standard RBC model [17]. The discount rate β is fixed to be 0.99. Following Christiano (1991) who estimated the capital depreciation rate δ and we set it to 0.0212.

Concerning the autocorrelation coefficients of the aggregate shocks, we follow the standard business cycle literature by setting $\rho = 0.95$ and Christiano (1991) by setting $\gamma = 0.32$. The value of $\bar{\theta}$ is equal to unity for a normalization purpose.

As for monitoring technology, since the monitoring costs are incurred by the financial intermediary only if the entrepreneur declares bankruptcy, μ is determined using empirical evidence on bankruptcy costs. There is a great deal of controversy within the empirical literature about the amount of these costs. They range from 1% to 36% . In fact, Warner (1977) argued that agency costs range from 1% to 5.3% of total firm assets. But Alderson and Becker (1995) found a higher amount which is around 36% . In our model we will set $\mu = 0.2$ which is a choice roughly in the middle. Concerning the distribution Φ, we assume that it is uniform on the interval [0, 2]. The range of the distribution is used for benchmark results and to guarantee a unit mean.

Finally, we must fix the value of the extra discount factor for entrepreneurs λ. This parameter will be treated as unobservable. As reported in Carlstrom and Fuerst (1997), it is computed indirectly and chose to match two measures of risk : the annual risk premium and the quarterly bankruptcy rate. This latter is estimated from Dun and Bradstreet data set 1984-1992 and it is equal to 0.998%, while the former is computed by taking the average spread between the prime rate and th three month commercial paper rate, which is

17.

equal to 187 basis point during the period of April 1971 to June 1996. These empirical risk measures give a value for λ which corresponds to 0.96683.

5.2 Simulation

The numerical resolution is done by following many steps : First the equilibrium conditions are log-linearized around the non-stochastic steady state of the economy (see Appendix B.2 for the steady state analysis). Then decision rules are computed using a perturbation method introduced for the first time by Klein (2000) and Sims (2002).

The main goal of this section is to study the dynamic properties of the model and to answer to the main question raised in this article : How can financial frictions alter the dynamic properties of a standard monetary business cycle model ? Note that financial frictions are introduced through the agency costs hitting the debt contract between entrepreneurs and households. Therefore, it remains interesting to compare the impulse responses of the our (MCSV) model to a monetary and technology shock with an other variant of it. It is the standard monetary real business cycle model which corresponds to the benchmark model i.e. MCSV model without agency problems : we set all ω_i to be equal to their mean value of 1 and we avoid the monitoring costs i.e. $\mu = 0$ [18]. Hereafter, we refer to this model by MRBC model.

5.2.1 Monetary shock

This section will cope with the analysis of the behavior of real economy as a response to a temporary monetary shock. Appendix 3.1 reports the results of a one-time shock to the money growth rate for the two models. This shock takes place in period 0 and it is of size 1% which means that $u_0 = 0.01$. After this shock, u_t is supposed to be 0 again. Given the specification of the money growth rate which is supposed to be AR(1) with a positive autocorrelation

18. According to Robert G. King et al. (1988), the only important difference between our model with $\mu = 0$ and monetary RBC is the small share of labor income following to entrepreneurs. In our model, for the case $\mu = 0$, the higher entrepreneurial discount rate prevent entrepreneurs to accumulate capital in the steady state.

coefficient, the shock does not have a one-time effect on the growth rate. Therefore, τ_t stays above the trend for several quarters.

An increase of the money supplied to this economy has a noteworthy impact on the two contract parties, namely the households and the entrepreneurs. Since the households under the limited participation assumption choose their level of consumption and deposits before the monetary shock, they are not able to change their decision and particularly to reduce deposits. Therefore, the monetary shock can be interpreted as a wealth shock driven to the entrepreneurs : they are able to borrow more and the financial intermediary which is owned by households is able to lend more. Note that, as mentioned earlier, borrowers are obliged to deal with the CMFs for any signed debt contract in this economy. This situation explains the increase of the loans demand by entrepreneurs. At this stage, we can distinguish two effects related to this wealth transfer. First, we can see from the impulse response functions in appendix 3.1 that the threshold value $\bar{\omega}$ under which the entrepreneur defaults increased considerably from the first period after the shock. It reaches a maximum in the fifth quarter and declines again towards the steady state. This increase means that the entrepreneurial productivity should be high enough in order to be able to reimburse the loan, which is equivalent to higher probability of default. This last observation sheds the light on the moral hazard problem in the model. In fact, a higher probability of default induces higher monitoring costs which prevents entrepreneurs to misreport their level of production and avoids any deviation from the truth at the equilibrium. Therefore, an increase in the borrowed amounts leads to smaller shares of income for lenders and borrowers. In order to maximize his income share, the lender will increase the costs of the debt which corresponds to an increase of the interest rate. On the other hand, the entrepreneur will aim to maximize his income also by increasing the capital price. This feature is noticed in the impulse response functions reported below. The second effect is related to the behavior of the investment function and its response to the interest rate and the capital price dynamics. As it was expected, the correspondent impulse response functions show a decrease of the investment level and the capital demand at the same time.

The increase of the capital price stimulated an increase of the relative price of capital q/p which induces a *substitution effect* from the households side. However, the capital good became more expensive than the consumption good then households shift from the former to the latter and their consumption increases considerably. In addition, households reduce their labour supply in order to maximize their utility and equilibrate the firms bill. The households consumption behavior can be explained also by an other common feature in monetary business cycle models [19] which is the *the anticipated inflation effect*. The prices of both goods increased but the capital prices raise was more important, that's why the relative price of capital augmented. In absolute terms, the consumption good price increased also but less proportionally than the capital one and since $\gamma > 0$, the monetary growth continues to be high relative to its steady state level after one-time shock. This increased household's expectation for future inflation may encourage them to shift again from the cash good (consumption) to the credit good (investment) and potentially reduces his labour supply. Therefore, the response of household's consumption and labour is a combination of the these two opposite effects. It is straightforward to recognize that at the equilibrium the substitution effect dominated the anticipated inflation effect for consumption and the opposite happened for labour which explains partly the wage increase. The drop of the total output was the final consequence.

The dynamics of the endogenous net worth confirmed our reasoning above. However, in spite the dramatic fall of the net worth and the investment level, the amount borrowed is still increasing since the absolute value of the net worth's variation is much higher than the one of the investment amount. These dynamics sparked off the capital price increase. The entrepreneurial consumption response is totally explained by these three facts. The increase of the mark up reflects the increase of the difference between the returns of external and internal funds. It becomes a common feature of the cost state verification models that the return of internal funds is higher than the one of the external funds. Furthermore, the increase of the difference encourages the entrepreneur to accumulate net worth and reduce his own consumption.

19. See Cooley and Hansen(1995).

We refer to this behavior by *the accumulation effect*. On the other hand, the higher probability of default influences negatively the consumption behavior of entrepreneurs since a faulty entrepreneur gives every thing to the CMFs and does not consume.

Christiano and Eichenbaum (1995) and Christiano et al. (1996) defined some stylized facts for monetary policy of the US economy. They are summarized in four main aspects : an expansionist monetary policy shows a decrease of the nominal interest rate and an increase of employment, output level and wages. The question here is : to what extent our agency model replicates these facts ? The only fact reproduced in our model is the increase of the wage rate. Unfortunately our (MCSV) model is not able to show the others. The total output, employment and nominal interest rate dropped following a positive money supply shock.

In order to understand the special features and the contribution of the endogenous wealth accumulation of the entrepreneurs in the monetary cost state verification (MCSV) model, we followed the comparative approach described above. The other model to be analyzed is the monetary real business cycle (MRBC) model, where monitoring costs are supposed to be equal to 0. The appendix 3.2 draws the impulse response functions of the (MRBC) model. The most important common feature is the substitution effect that is replicated in the two models. In the monetary business cycle model, the relative capital price increased but it returns more quickly to the steady state which makes its effect weaker. In contrast with the first model, the borrowed amount by entrepreneurs decreased despite the raise of the interest rate and the capital price. In addition, household consumption increased in the first quarter and returned quickly to its steady state. Therefore, one can say that the substitution effect disappears and the impact is weaker and shorter. By examining the impulse response functions, it is straightforward to notice that the anticipated inflation effect dominates in this case since the response of the consumption good price is higher than the one in (MCSV) model and the considerable decrease of the employment rate confirms this statement. Once again the monetary real business cycle model with flexible prices is unable to reproduce the stylized facts concerning the output and employment and the liquidity effect of monetary policy is absent [20].

The absence of monitoring costs did not alter significantly neither the behavior of the net worth nor the investment response. They still decrease. But the only difference consists on the size of the variation which induced a decrease in the borrowed amounts. In contrast with the first model, borrowing is more sensitive to its cost i.e. the interest rate. By examining impulse response functions, one can remark that an increase of the interest rate is followed by a decrease of the loans demand. Therefore, agency costs can be considered as *the loan adjustment costs.*.

In summary, the effect of agency costs is not that deep since it does not alter completely the model's response to a monetary shock. In fact, the main difference was in the lender's side where the substitution effect disappears and the expected inflation effect takes place. While , this latter is also weak since it influences only the labour decision. Therefore, embedding agency costs does not have large effects on the global model dynamics and features. The same conclusion was drawn by Fuerst (1995) and Carlstrom and Fuerst (1997) for real models.

5.2.2 Technology shock

Similarly, we consider a one-time technology shock of size 1% to the aggregate productivity in period 0 i.e. $\epsilon_0 = 0.01$. During the subsequent quarters, ϵ_t is supposed to be equal to 0 again. The productivity follows also an AR(1) then, the productivity does not return to the steady state in the first quarter after the shock since its autocorrelation coefficient ρ_t is positive. The response of the economy to such shock is reported in Appendix 4 and we adopt the same comparative approach where the impact is assessed for (MCSV) and (MRBC) models.

20. Christiano (1991) showed that pure models with a cash in advance constraint with sluggish cash flow are unable to generate liquidity effect of monetary policy. There is a need for more inflexibility and imperfection.

The main result deduced from the impulse response functions of (MCSV) and (MRBC) models is the appearance of the output response to the technology shock. In the MCSV, the aggregate output suddenly with sharp rise followed by monotone decline accordingly in the subsequent period to the shock and it starts decreasing immediately which contradicts the previous findings. Carlstrom and Fuerst (1997) and Carlstrom and Fuerst (1998) showed their agency costs model are subject to Cogley and Nason (1995) criticism. However, these latter criticized in their study that real business cycles models are unable to replicate the hump-shaped output response. This stylized fact was shown the first time by Blanchard and Quah (1989) and confirmed by Cochrane (1994) for US data and was called the *mean-reversion*. According to this notion, the response of output to a technology shock is hump-shaped and does not jump to its peak : it takes from two to four quarter to reach its maximum.

In our model, the hump-shaped appearance is not that clear since the impulse response functions start displaying the behavior of the variable from the subsequent period to the shock i.e. the behavior of the variable between the first and the second quarter is not observable. Independently on the appearance of the impulse response function, the total output increased behind the shock.

In the (MCSV) model, the investment increased immediately with the output while in the (MRBC) model the investment increased but for several quarters after the shock and the peak is reached in the fourth quarter. The size of the investment increase is more important for the (MRBC) model. With the presence of the monitoring costs, entrepreneurs hold in the beginning of the period 0 their net worth accumulated through cash holding, then the net worth is a fixed amount and its initial response is limited. On the other hand, the positive shock of technology encourages the households to invest more in the capital technology owned by entrepreneurs. Since the investment function depends only on capital price and net worth, investment is driven to jump by initial increase in capital prices. The high capital price leads to higher levels of capital production since entrepreneurs want to take advantage of the increased prices and improve their income. For both models, it is seen from the impulse response functions that capital and investment are increasing with different speeds. Entrepreneurs financed their increased capital production by higher cash holding in the beginning of the period. This reaction is equivalent to an increase the capital supply which shifts the investment supply function upwards and consequently, the relative price of capital decreases.

The raise of the capital price and the reduction of the excess demand improves the entrepreneurial consumption in the subsequent period and reduces the accumulation of capital and wealth. Comparing to the (MRBC) model, the (MCSV) model is characterized by an investment that does not increase for long time then, the capital stock raises less. In addition, the investment in the (MCSV) model decreases monotonically, back to its steady state value and therefore output did not show a hump-shaped response either.

6 Conclusion

We tried by this project to develop a monetary business cycle model with asymmetric information in the credit market. The moral hazard problem created by this informational imperfection in lending distorts the capital good production and stimulates the endogenization of the capital price. In our model, we took into account the entrepreneurs' heterogeneity by assuming different levels of net worth. This latter was endogenous and depends on the entrepreneurs money holdings which more realistic than supposing the absence of entrepreneurs heterogeneity.

The numerical analysis of the model showed that a positive money supply shock has different effects and the final response depends mainly on the size of each effect. In fact, such shock generates an increase in nominal interest rate followed by a decrease of the level of investment. In addition, the output and employment decreased. The same features were displayed by the standard monetary RBC model. In fact, the cost state verification model with ex-ante heterogeneous entrepreneurs produced less amplification than the standard one and a weaker propagation. Moreover, the (MCSV) model was not able to replicate the hump-shaped output response to the technology shock nor the stylized facts outlined by the cost state verification model with ex-post heterogeneity.

A possible extension of the actual model is to incorporate the time-varying uncertainty through assuming a variable variance of the entrepreneur's distribution and to explore how changes in uncertainty effect equilibrium characteristics. It is straightforward to recognize that the impact of the uncertainty on investment via the lending channel is fairly transparent since an increase of the uncertainty results in a shift of the investment supply. In particular, an increase of the uncertainty will cause an increase in the price of capital and a fall in investment activity. Therefore, it remains very interesting to provide an empirical investigation of this statements. An other way by which we can extent the actual work is to abandon the assumption that credit contract depends only on the entrepreneur's current net worth and does not depend on his historical financial situation. This will lead to a repeated game framework which is quite complicated to be embedded in a general equilibrium setting.

Appendices

Appendix A

1. **Concavity of the lender's expected income**

   We know that $g'(\bar{\omega}) = (1-\Phi(\bar{\omega}))(1-\mu h(\bar{\omega}))$, using the density function and the cf properties and we differentiate $g(\bar{\omega})$ twice, we get :

$$\begin{aligned} g''(\bar{\omega}) &= -\phi(\bar{\omega})\left[1-\mu h(\bar{\omega})\right] - \mu h'(\bar{\omega})\left[1-\Phi(\bar{\omega})\right] \\ &= -\phi(\bar{\omega}) - \mu\phi'(\bar{\omega}) \\ &= -\left[\phi(\bar{\omega}) + \mu\phi'(\bar{\omega})\right] \end{aligned}$$

   Then, $g''(\bar{\omega}) < 0$ iff $\phi(\bar{\omega}) + \mu\phi'(\bar{\omega}) > 0$.
   But since

$$\phi(\bar{\omega}) + \mu\phi'(\bar{\omega}) > 0 \Longleftrightarrow 1 + \mu\frac{e_\phi}{\bar{\omega}} > 0$$

   Where $e_\phi = \frac{\bar{\omega}\phi'(\bar{\omega})}{\phi(\bar{\omega})}$ Then :

$$g''(\bar{\omega}) < 0 \text{ iff } e_\phi > -\frac{\bar{\omega}}{\mu}$$

   . QED.

2. **Slope of the investment curve**

   Let F be the function defined as

$$F(q/P, R,) = q\left\{1 - \Phi(\bar{\omega})\mu + \phi(\bar{\omega})\mu\frac{f(\bar{\omega})}{f'(\bar{\omega})}\right\} - R\,P$$

   and let $x = (q/P, R)$ and $y = \bar{\omega}$.
   Suppose that $F : \mathcal{R}^2 \to \mathcal{R}$ is continuously differentiable in an open set A containing (x_0, y_0) such that $F(x_0, y_0) = 0$ and $F_y(x_0, y_0) \neq 0$. There exists an interval $I_1 = (x_0-\delta, x_0+\delta)$ and an interval $I_2 = (y_0-\epsilon, y_0+\epsilon)$ with $I_1 \times I_2 \subset A$ such that :
   – For every $x \in I_1$ the equation $F(x, y) = 0$ has a unique solution in I_2 which defines a function $y = \varphi(x)$.

– φ is continuously differentiable and :

$$\begin{aligned}\frac{\partial\bar{\omega}}{\partial q} &= -\frac{F_q(x,\varphi(x))}{F_{\bar{\omega}}(x,\varphi(x))} \\ &= -\frac{1-\Phi(\bar{\omega})\mu+\phi(\bar{\omega})\mu\left[f(\bar{\omega})/f'(\bar{\omega})\right]}{q\{\phi'(\bar{\omega})\mu\left[f(\bar{\omega})/f'(\bar{\omega})\right]-\phi(\bar{\omega})\mu\left[f(\bar{\omega})f''(\bar{\omega})/f'^2(\bar{\omega})\right]\}}\end{aligned}$$

But since
$1-\Phi(\bar{\omega})\mu+\phi(\bar{\omega})\mu\frac{f(\bar{\omega})}{f'(\bar{\omega})}>0$ and $f'=\Phi(\bar{\omega})-1<0$. So that :

$$\begin{aligned}f'' &= \phi(\bar{\omega}) \\ &\geq 0\end{aligned}$$

Then

$$\begin{aligned}\frac{\partial\bar{\omega}}{\partial q} &= -\frac{1-\Phi(\bar{\omega})\mu+\phi(\bar{\omega})\mu\left[f(\bar{\omega})/f'(\bar{\omega})\right]}{q\phi(\bar{\omega})\mu\left[\frac{e_\phi}{e_f}-\frac{e'_f}{e_f}\right]} \\ &= -\frac{1-\Phi(\bar{\omega})\mu+\phi(\bar{\omega})\mu\left[f(\bar{\omega})/f'(\bar{\omega})\right]}{q\mu\frac{\phi(\bar{\omega})}{e_f}\left[e_\phi-e'_f\right]}\end{aligned}$$

We know that

$$\begin{aligned}e_f &= \bar{\omega}f'(\bar{\omega})/f(\bar{\omega}) \\ &= \bar{\omega}\left[\Phi(\bar{\omega})-1\right]/f(\bar{\omega}) \\ &< 0\end{aligned}$$

Therefore, the sign of $\partial\bar{\omega}/\partial q$ is the one of $(e_\phi-e'_f)$. We know also that

$$\begin{aligned}e_{f'} &= \bar{\omega}f''(\bar{\omega})/f'(\bar{\omega}) \\ &= \bar{\omega}\phi(\bar{\omega})/\left[\Phi(\bar{\omega})-1\right] \\ &= -\bar{\omega}\,h(\bar{\omega}) \\ &< 0.\end{aligned}$$

Where $h(\overline{\omega})$ is the hazard function. According to our condition imposed above on e_ϕ i.e. $e_\phi>-\frac{\overline{\omega}}{\mu}$. Then, we have :

$$
\begin{aligned}
e_{f'} - e_\phi &= -\bar{\omega}\, h(\bar{\omega}) - e_\phi \\
&< -\bar{\omega}\, h(\bar{\omega}) + \frac{\bar{\omega}}{\mu} \\
&< \bar{\omega}\left[h(\bar{\omega}) - \frac{1}{\mu}\right] \\
&< 0. \text{ because } h \text{ reaches its maximum in } \frac{1}{\mu}
\end{aligned}
$$

So, $\partial\,\bar{\omega}/\partial q > 0$ because $e_\phi - e_{f'} > 0$. Therefore the tresholed $\bar{\omega}$ is upwardly sloping in q.
And,

$$
\begin{aligned}
\frac{\partial\bar{\omega}}{\partial R} &= \frac{p}{q\{\phi'(\bar{\omega})\mu\,[f(\bar{\omega})/f'(\bar{\omega})] - \phi(\bar{\omega})\mu\,\left[f(\bar{\omega})f''(\bar{\omega})/f'^2(\bar{\omega})\right]\}} \\
&= \frac{p}{q\mu\,\frac{\phi(\bar{\omega})}{e_f}\,[e_\phi - e_{f'}]} \\
&< 0.
\end{aligned}
$$

Then, the thresholf $\bar{\omega}$ is downward sloping in R.
Now, let us come back to the slope of the investment function $i(q/P, R, n)$.

$$
\begin{aligned}
\frac{\partial i(\frac{q}{P}, R, n)}{\partial q} &= \frac{\partial\psi(\frac{q}{P}, R)}{\partial q}\, n \\
&= \frac{RP\left[g(\bar{\omega}) + q\frac{\partial\bar{\omega}}{\partial q}\frac{\partial g(\bar{\omega})}{\partial\bar{\omega}}\right]}{\left[RP - qg(\bar{\omega})\right]^2}\, n \\
&> 0. \text{ because } \frac{\partial\bar{\omega}}{\partial q} > 0.
\end{aligned}
$$

Then, the new-capital supply function is upwardly sloping in the capital price q.

$$
\begin{aligned}
\frac{\partial i(\frac{q}{P}, R, n)}{\partial R} &= \frac{\partial\psi(\frac{q}{P}, R)}{\partial R}\, n \\
&= \frac{RPq\frac{\partial\bar{\omega}}{\partial R}\frac{\partial g(\bar{\omega})}{\partial\bar{\omega}} - Pq\, g(\bar{\omega})}{\left[RP - qg(\bar{\omega})\right]^2} \\
&= \frac{Pq\, g(\bar{\omega})\,[e_{\bar{\omega}}\, e_g - 1]}{\left[Rp - qg(\bar{\omega})\right]^2}
\end{aligned}
$$

But since $e_{\bar{\omega}}(R) < 0$, Then

$$\frac{\partial i(\frac{q}{P}, R, n)}{\partial R} < 0$$

Then, the investment function is downward-sloping in the interest rate R. QED.

Appendix B

1. **Equilibrium analysis**

We proceed by agent in the analysis of the first odrder conditions

– **Households** The representative household is infinitely lived and has expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ln(C_t) + \nu(1 - L_t)\right], \qquad \nu > 0.$$

The F.O.C for the household inter-temporal optimization problem are :

$$\eta_t = -\frac{U_{C,t}}{P_t} \quad (C_t) \qquad (.1)$$

$$U_{L,t} - \beta\eta_{t+1}W_t = 0 \quad (L_t) \qquad (.2)$$

$$\eta_{t+1}q_t - \beta\eta_{t+2}r_{t+1}P_{t+1} - \beta\eta_{t+2}(1-\delta)q_{t+1} = 0 \quad (K_{t+1}) \qquad (.3)$$

$$\eta_t - \beta\eta_{t+1}R_t = 0 \quad (N_t) \qquad (.4)$$

where η_t is the discounted lagrange multiplier. Its expression is given by the first equation and using the functional form of the utility function, we get

$$\eta_t = -\frac{1}{C_tP_t}$$

By replacing η_t in the subsequent equations, we get the following Euler-equations :

$$\beta\frac{W_t}{C_{t+1}P_{t+1}} - \nu = 0 \quad (L_t) \qquad (.5)$$

$$\frac{q_t}{C_{t+1}P_{t+1}} - \beta\frac{r_{t+1}P_{t+1} + (1-\delta)q_{t+1}}{C_{t+2}P_{t+2}} = 0 \quad (K_{t+1}) \qquad (.6)$$

$$\beta R_t\frac{1}{C_{t+1}P_{t+1}} - \frac{1}{C_tP_t} = 0 \quad (N_t) \qquad (.7)$$

– **Firms** The economy's consumption good is produced by firms using Cobb-Douglas technology with the following specification :

$$Y_t = \theta_t K_t^{\alpha_1} H_t^{\alpha_2} (H_t^e)^{1-\alpha_1-\alpha_2} \tag{.8}$$

F.O.C are :

$$\begin{aligned} W_t &= P_t \, \theta_t \, \alpha_2 \, K_t^{\alpha_1} \, H_t^{\alpha_2 - 1} \, (H_t^e)^{1-\alpha_1-\alpha_2} && (.9) \\ W_t^e &= P_t \, \theta_t \, (1-\alpha_1-\alpha_2) \, K_t^{\alpha_1} \, H_t^{\alpha_2} \, (H_t^e)^{-(\alpha_1+\alpha_2)} && (.10) \\ r_t &= \theta_t \, \alpha_1 \, K_t^{\alpha_1 - 1} \, H_t^{\alpha_2} \, (H_t^e)^{1-\alpha_1-\alpha_2} && (.11) \end{aligned}$$

– **Entrepreneurs** The entreprneurs maximize the utility function subject to the budget constraint given by (4.12). The two F.O.C are :

$$\eta_t = \frac{1}{P_t}\eta_t - \beta\lambda\eta_{t+1}\frac{R_{t+1}q_{t+1}f(\bar{\omega})}{R_{t+1}P_{t+1} - q_{t+1}g(\bar{\omega})} \tag{.12}$$

Where η_t is the discounted Lagrange multiplier. By replacing η_t in the second equation , we find the Euler-equation for the entrepreneurs i.e.

$$E\left\{\beta\lambda\frac{q_{t+1}}{P_{t+1}}\frac{R_{t+1}p_t f(\bar{\omega})}{R_{t+1}P_{t+1} - q_{t+1}g(\bar{\omega})}\right\} = 1.$$