Vintage Capital in Matching Models with Credible Bargaining *

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Abstract

A Hornstein, Krusell, and Violante (2007) vintage capital matching model with two sources of business cycles fluctuations, namely in labor productivity and in investment-specific technical change (ISTC), is analyzed. The defining feature is that once a firm purchases the capital, there is no possibility to adjust the capital content of a machine vintage in the future. I show that augmenting such model with the credible bargaining à la Hall and Milgrom (2008) improves the results with respect to the unemployment volatility puzzle on two dimensions. First, the model does not require to induce lower than observed persistence of the driving forces to close the gap between the data and the model. Second, credible bargaining solution smoothes the excessive real wage fluctuations and brings them in line with the data.

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1 Introduction

Shimer (2005) and Costain and Reiter (2008) have recently argued that the equilibrium search model of unemployment is quantitatively inconsistent. They argue that under common parameter values the textbook matching model fails to generate the observed cyclical behavior of unemployment and vacancies in the U.S. economy in response to shocks of plausible magnitude. The critique produced a significant amount of research addressing the problem.[1]

In the canonical Mortensen and Pissarides (1994) model, search and matching result in a surplus which is split between a firm and a worker using the Nash solution to obtain the wage bargain. A worker's wage is consequently a linear combination of his or her productivity and his or her value of unemployment. The latter value takes into account productivity of other jobs in the market. As a result, aggregate productivity shocks alter both of the terms defining the bargaining set in the same direction and almost by the same amount. This makes wages near-proportional to productivity. For this reason, an employer's expected profit from posting a vacancy does not fluctuate with productivity shocks. The Job Creation equation derived from the free-entry condition states that the expected cost of posting vacancy must equal the expected profit from hiring a worker. Such a condition implies that changes in market tightness (the ratio of vacancies to unemployment), are driven by changes in the expected profit from hiring. Consequently, the reason why the canonical model cannot replicate fluctuations in market tightness is that firms do not change their recruiting effort when their expected profit from hiring a worker does not fluctuate.

One of the earliest approaches to solve this puzzle was to challenge the wage determination mechanism by introducing extensions which create wage rigidities. Hall (2005) provided evidence that the model generates real data volatility in excess of what is observed if the wage is completely rigid. He shows that under certain conditions wage stickiness is an equilibrium and it produces substantial fluctuations in a firm's profit from hiring due to shocks in productivity.[2] However wage-stickiness based approach is subject to a criticism raised by Pissarides (forthcoming) and Haefke, Sonntag, and van Rens (2008). They argue that micro-evidence imply a near-proportionality of wages with respect to labor productivity in new matches.

[1] See Cardullo (forthcoming) for an excellent survey. *Scandinavian Journal of Economics* has published a special issue (Vol.109 2007, No.4) featuring macro-labor dynamics including four papers that respond directly to critique.

[2] Brugemann and Moscarini (2008), Gertler and Trigari (forthcoming), Rudanko (2009), Shimer (2004) suggest other wage-stickiness based solutions.

Hornstein, Krusell, and Violante (2007) (HKV) suggest including another source of business cycle fluctuations, namely investment-specific technical change (ISTC). It's importance is analyzed by Greenwood, Hercowitz, and Krusell (1997), Fisher (2006). HKV augment the standard matching model to allow for vintage capital. The defining feature is that once a firm purchases the capital, there is no possibility to adjust the capital content of a machine vintage in the future. Following Greenwood, Hercowitz, and Krusell (1997), ISTC is identified with changes in the relative price of capital. These fluctuations have important implications on the behavior of firms: a below-trend cost of new capital induces firms to enter the labor market and make capital gain.

HKV show that their model is successful in reducing the gap between the data and the model but not without a cost. On the one hand, the consistency of the generated volatilities depends largely on the persistance of the driving forces. The observed persistence is too high to be able to generate the observed fluctuations in unemployment and vacancies. On the other hand, the HKV model results in wage variations that are much larger than in the data. Their result gives motivation for a wage-stickiness based solution of their model, which will not be subject to Pissarides (forthcoming) and Haefke, Sonntag, and van Rens (2008) criticism.

This paper fixes both of the HKV issues. For this purpose, I replace the Nash model of bargaining with the so-called credible bargaining model from Hall and Milgrom (2008). The theory, pioneered by Binmore, Rubinstein, and Wolinsky (1986), argues that job-seeker's and employer's threat points to take outside options (unemployment value for a worker and zero for a firm) are not credible, since new matches produce a positive surplus, which "glues" two parties together. Instead, the bargaining game is extended to allow for the possibility that the agreement is delayed. In such case the job-seeker and employer receive fixed disagreement payoffs. As a consequence, the outside labor market is of limited influence on the outcome of the bargain. Modeling bargaining in this manner delivers more rigidity in the wages than the Nash bargaining model.

I follow the literature and analyze the US labor market using Shimer (2005)'s data set for unemployment, labor-market tightness, job-finding rates and labor productivity. The relative price of capital, which is identified as the price of equipment and software investment relative to the price of personal consumption expenditures, has been falling since the late 1950s, i.e. the relative productivity of the capital goods producing sector has been increasing.

The stochastic model is calibrated to match US data and then simulated. I replicate the unemployment volatility puzzle with the basic matching model. The results are compared with the Hall and Milgrom (2008) matching model and the HKV model. Simulations suggest that the vintage capital model with credible bargaining reproduces the relevant US business cycle facts successfully. As intended, the model improves the HKV economy on two dimensions. First, the model does not require to induce lower than observed persistence of the driving forces to close the gap between the data and the model. Second, credible bargaining solution smoothes the excessive real wage fluctuations and brings them in line with the data. Besides, wage stickiness works as an amplification mechanism on its own.

Section 2 builds the theoretical search model. Section 3 describes the US labor market facts, model's calibration as well as the results. Finally, Section 4 concludes.

2 Search Model

In describing the model I closely follow Hornstein, Krusell, and Violante (2007) except for using discrete time and discrete random driving force.

I consider two alternative models of job creation. The first is the textbook labor matching model where a firm pays a fixed flow cost until it finds a worker and starts production. If a match separate for exogenous reasons, the firm actively engages in hiring at the same fixed cost. Let's call this framework the *vacancy cost* model.

The alternative setting, constructed by Hornstein, Krusell, and Violante (2007), considers that before firms can enter the labor market, they need to purchase a machine which is necessary for the production. Once a firm makes such investment, it can open a vacancy at no additional cost. Should a match separate, the machine can be returned to the matching pool at no cost again. Jobs are considered as durable capital. Let's call this framework the *investment cost* model. It is identical to the vacancy cost economy with the cost of investment equal to zero.

In addition, I will combine the two economies with two alternative rules of wage formation. The first is the standard in standard in the literature - the Nash bargain. In this model both parties receive the values of their threat point and given fractions of the surplus that results from the match. The threat points are firm's and worker's outside options, i.e. the values received by disclaiming the current opportunity and continuing to search.

The alternative wage formation model I use is an alternating offer bargaining model with disagreement payoffs à la Binmore, Rubinstein, and Wolinsky (1986). The main difference is that the threats from both sides are to extend bargaining rather than terminate it. The credible bargaining framework loosens the connection between wages and outside options, thus making wages more rigid.

The combination of two alternative job creation models and two alternative methods of wage bargaining gives me four economies, which I compare in what follows.

2.1 A frictional labor market and values

Workers and employers are risk-neutral and discount the future at rate equal to the interest rate, r. Let's note their discount factor $\xi = \frac{1}{1+r}$. There are two independent driving forces: productivity, p_j, and relative price of capital, q_k, where j and k are discrete stationary state variables $j \in [1, ..., N]$ and $k \in [1, ..., M]$ with transition matrices π^p and π^q respectively. If the economy is subject to both driving forces, the set of states changes to $N \times M$ with the state variable $i \in [1, ..., N \times M]$. Each state i represents a pair (p_j, q_k) with $i = (j-1) * N + k$. The transition matrix for this process is thus calculated as the Kronecker product: $\pi = \pi^p \otimes \pi^q$.

In the basic matching model of unemployment workers and firms meet randomly. The rate of creation of new matches is given by the Cobb-Douglas matching function

$$M_i = A u_i^{\alpha} v_i^{1-\alpha} \tag{1}$$

where u_i is the number of unemployed workers, v_i is the number of open vacancies in the market, A is the matching efficiency and α denotes the matching function elasticity. The market tightness, $\theta_i = v_i/u_i$, is the key variable which determines the equilibrium in the economy. The job finding rate for the unemployed workers is $\lambda_w(\theta_i) = M_i/u_i = A\theta_i^{1-\alpha}$ and the job filling rate for the firms is $\lambda_f(\theta_i) = M_i/v_i = A\theta_i^{-\alpha}$. I assume that the matches separate at a constant rate equal to σ.

I begin with the worker's values of staying unemployed and holding a job, because these equations are the same for both job creation alternatives. Three values characterize the job-seeker's bargaining position. Let's note U_i worker's present value of being unemployed. While searching, the worker receives some return z. I assume that z is constant and does not depend on market returns. After accepting the job offer, unemployed worker receives a wage contract with a present value of W_i and also enjoys a continuation value of E_i. U_i, W_i and E_i thus satisfy:

$$U_i = z + \xi \mathbb{E}_{i'}[\lambda_w(\theta_i)(W_{i'} + E_{i'}) + (1 - \lambda_w(\theta_i))U_{i'}], \tag{2}$$

and

$$E_i = \xi \mathbb{E}_{i'}[\sigma U_{i'} + (1-\sigma) E_{i'}]. \tag{3}$$

The present value, W_i, of the wages received by a worker over the course of a job can be expressed as:

$$W_i = w_i + \xi \mathbb{E}_{i'}[(1-\sigma) W_{i'}] \tag{4}$$

As the model solves for W_i, I will make use of equation (4) to find the equilibrium monthly wages w_i.

Hornstein, Krusell, and Violante (2007) assume that to produce in the investment cost economy firms must purchase a machine at cost q_i. This investment constitutes the only cost of opening a vacancy. The difference with respect to the vacancy cost economy is that the vintage is not always destroyed when the match is dissolved. In fact a match may separate for two reasons. First, the match can be separated with probability ε without destroying the vintage. It allows firms to return their machines to the matching pool at zero cost. Second, whether the vintage is idle or employed in the match, it depreciates at the rate δ. Total rate of separations is thus $\varepsilon + \delta = \sigma$, the exogenous separation rate in the vacancy cost economy.

Let's note P_i and V_i the present value of the flow value of p_i, the marginal product of labor, and the present value of posting a vacancy. They must satisfy

$$P_i = p_i + \xi \mathbb{E}_{i'}[\varepsilon V_{i'} + (1-\sigma) P_{i'}] \tag{5}$$

and

$$V_i = -c + \xi \mathbb{E}_{i'}[\lambda_f(\theta_i)(P_{i'} - W_{i'}) + (1 - \lambda_f(\theta_i) - \delta) V_{i'}]. \tag{6}$$

Here, recruiting to fill a vacancy costs c per period.

The evolution of unemployment for both economies is given by the following flow balance equation

$$u_i = \frac{\sigma}{\sigma + \lambda_w(\theta_i)}. \tag{7}$$

To explain the dynamics of the unemployment, one needs to describe the dynamics of both, the job separations and the job finding probabilities. The consensus in the literature estimate that the contribution of job exit rate is about one third. As I assume the job exit rate to be exogenous, it is more appropriate to evaluate model's performance by comparing the generated and the observed fluctuations in the job finding probability. Alternatively, one can compare the generated and the observed volatility in tightness directly, because it is the only source of fluctuations in the job finding rate.

2.2 Free entry conditions

In equilibrium all profit opportunities from new jobs are exhausted. This condition has different implications depending on the job creation framework.

The Vacancy Cost Economy

In the vacancy posting economy the free entry condition condition requires that the value of posting a vacancy is driven to zero

$$V_i = 0, \forall i \tag{8}$$

Substituting this value into the expressions (6) and (5) gives

$$P_i = p_i + \xi \mathbb{E}_{i'}[(1-\sigma)P_{i'}] \tag{9}$$

and

$$c = \xi \mathbb{E}_{i'}[\lambda_f(\theta_i)(P_{i'} - W_{i'})] \tag{10}$$

(10) is the standard zero-profit condition of the MP matching model. The vacancy cost economy is subject to one driving force, the productivity shocks. Thus the set of states reduces to N. We can rewrite (10) as

$$\lambda_f(\theta_i) = \frac{c}{\xi \mathbb{E}_{i'}[(P_{i'} - W_{i'})]} \tag{11}$$

As noticed by Hall and Milgrom (2008), equation (11) implies that in the standard MP model there is only one source of fluctuations in the market tightness: the employer's expected surplus, $\mathbb{E}_{i'}[(P_{i'} - W_{i'})]$. As a consequence, the wage stickiness is a key issue. If wages instantly increase after a positive productivity shock, vacancy creation is dampened as firms change little their recruiting efforts. This results in a low market tightness variability. In the next section, I show that this is what happens when wages are modeled as a Nash bargain.

The Investment Cost Economy

In the investment cost economy the free entry condition requires

$$V_i = q_i, \forall i. \tag{12}$$

where q_i is the price at which a firm can purchase a machine. If $V_i > q_i$, i.e. the value of opening a vacancy is higher than the investment needed to start the firm, there will be no limit for entry. And also if $V_i < q_i$, no firm would enter because it would be impossible to gain back the invested sum. In the investment cost economy, the cost of opening a vacancy is zero, $c = 0$.

Making use of (12) changes (6) and (5) to

$$P_i = p_i + \xi \mathbb{E}_{i'}[\varepsilon q_{i'} + (1 - \sigma) P_{i'}] \tag{13}$$

and

$$q_i = \xi \mathbb{E}_{i'}[\lambda_f(\theta_i)(P_{i'} - W_{i'}) + (1 - \lambda_f(\theta_i) - \delta) q_{i'}] \tag{14}$$

A comparison of the free entry conditions (10) and (14) reveals that the latter allows for more channels that amplify tightness. Here, in addition to the wage-stickiness based source of fluctuations, the shocks to machinery cost have a direct impact on the equilibrium value of tightness.

In fact, it appears that fluctuations in the cost of posting a vacancy, c, in the equation (10) might result in similar amplification effects in the standard MP model. However, even assuming that they exist, we have little knowledge on the nature and the size of such shocks.

Existence of two driving forces increase the state space to $N \times M$ states.

2.3 The importance of Wage Stickiness

Shimer (2005) showed that the elasticity of market tightness with respect to labor productivity can be obtained analytically in the economy with no shocks. Mortensen and Nagypál (2007) proves the validity of such approximation of the stochastic environment as long as the changes in productivity are small. I follow this technique in what follows.

It is straightforward to show from expressions (4), (5) and (3) that W, P and E can be expressed as follows

$$W = \frac{1+r}{r+\sigma} w \tag{15}$$

$$P = \frac{1+r}{r+\sigma} p + \frac{\varepsilon}{r+\sigma} V \tag{16}$$

$$E = \frac{\sigma}{r+\sigma} U \tag{17}$$

Replacing these values in equation (6) and taking $V = q$ for now, I get

$$q = -c + \frac{\lambda_f(\theta)}{r+\delta}(p - w + \frac{\varepsilon q}{1+r}) + \frac{q}{1+r}(1 - \lambda_f(\theta) - \delta) \tag{18}$$

Let $\epsilon_{\theta/p}$ be the elasticity of tightness with respect to p and $\epsilon_{\theta/w}$ the one of the wage. Assuming that p and q are independent, from (18) I get,

$$\epsilon_{\theta/p} = \frac{1}{\alpha} \frac{p - \epsilon_{\theta/w} w}{p - w - q\frac{r+\delta}{1+r}} \tag{19}$$

The equation (19) shows the sensitivity of $\epsilon_{\theta/p}$ with respect to the $\epsilon_{w/p}$. Considering the vacancy cost economy ($q = 0$) with wages nearly proportional to productivity, i.e. $\epsilon_{w/p} = 1$, we have $\epsilon_{\theta/p} = 1/\alpha$. The usual calibration value for α is 0.5, which results in $\epsilon_{\theta/p} = 2$. This reflects essentially the critique of Shimer (2005).

If $\epsilon_{w/p} < 1$, i.e. the wage is stickier, the volatility of tightness increases and Shimer's critique can be reconciled. However Pissarides (forthcoming) argues that empirically $\epsilon_{w/p}$ is close to 1. Therefore, a matching model needs to keep this property.

In the investment cost economy, the existance of the machinery cost, q, reduces the importance of wage stickiness. A positive value of q increases $\epsilon_{\theta/p}$ all other things being equal.

2.4 Wage rule as a Nash Bargain

The original MP model uses the Nash Bargaining solution to determine how the surplus is split between the employer and the job-seeker. The total surplus of a match is $S_i = (P_i - W_i - V_i) + (E_i + W_i - U_i) = P_i - V_i + E_i - U_i$, i.e. the sum of the firm's and the worker's net value of being in the match. This surplus is split between the two parties according to their bargaining power $1-\beta$ and β respectively:

$$W_i + E_i - U_i = \beta S_i \tag{20}$$

$$P_i - W_i - V_i = (1-\beta) S_i \tag{21}$$

Let's consider again a stationary deterministic economy with no shocks to obtain the properties of the latter wage formation rule. Using the same expressions for W, P and E, the wage equation $W = \beta(P - V) + (1-\beta)(U - E)$ gives

$$w = \beta(p - V\frac{r+\delta}{1+r}) + (1-\beta)\frac{r}{1+r}U \tag{22}$$

In the vacancy cost economy the equation (22) reduces to

$$w = \beta p + (1-\beta)\frac{r}{1+r}U \tag{23}$$

It is now more intuitive to see why the wage is nearly proportional to productivity. The wage is a weighted average of the flow return of productivity, p, and the worker's net opportunity cost, U. The latter depends on the wages in the job market and thus is highly sensitive to the movements in productivity.

In the investment cost economy, the wage equation becomes:

$$w = \beta(p - q\frac{r+\delta}{1+r}) + (1-\beta)\frac{r}{1+r}U \tag{24}$$

The difference is that the flow cost of the machine is subtracted from the flow return of production. Apart from the fact that the price of capital, q, is scaled down by $\frac{r+\delta}{1+r}$, it has the same direct impact on the wage as the productivity. For this reason, the second source of fluctuations will increase significantly the volatility of wages in the simulations.

The investment cost and the vacancy cost economies with Nash wage bargain have $5 \times N \times M$ and $5 \times N$ endogenous variables respectively, the worker's value of being unemployed, U_i, the present value of a wage contract, W_i, additional value enjoyed by a worker, E_i, the present value of productivity, P_i, and the market tightness θ_i. The investment cost economy has $5 \times N \times M$ equations, (2), (3), (13), (14), (20). The vacancy cost model has $5 \times N$ equations, (2), (3), (9), (10), (20).

2.5 Wage rule as a Credible bargain

As we have seen, wage stickiness is the sole determinant of the unemployment volatility in the standard MP model. This is the central point of Hall and Milgrom (2008). As a consequence they come up with a micro-based more rigid wage formation which results in higher unemployment volatility. However, Pissarides (forthcoming) noticed that if the wage stickiness can deliver more volatility, it does not mean that it is the answer to it. The investment cost framework allows for more channels to generate volatility in the unemployment.

Hornstein, Krusell, and Violante (2007) have shown that the vintage capital model creates excessive variability of wages compared to the data. To deal with this problem, I change the wage formation in both economies by adding the credible bargaining solution à la Hall and Milgrom (2008).

They challenge bargaining threats used by a Nash bargain. They argue that once a qualified worker meets an employer, a threat to walk away is not credible from neither side. Instead, the authors suggest using an alternative wage-setting mechanism based on non-cooperative alternating offer model based on Binmore, Rubinstein, and Wolinsky (1986).

The bargaining takes place over time, the parties follow a predetermined bargaining agreement. The offers are made at points of time, the first period beginning with the proposal by the employer. Following such proposal, the job-seeker has three possibilities: to accept it, to make a counter-offer or to exit the bargaining. Exiting bargaining is never optimal but occurs at rate η. In such case employer and job-seeker receives their outside options, i.e. the job-seeker receives U_i and the employer gets the value of a vacant position which is equal to the relative price of capital in the investment cost economy. If responding party is making a counter-proposal, both parties receive disagreement payoff for that period, i.e. the employer incurs a loss of $\gamma > 0$ and the worker receives the flow benefit z. The main point is that these payoffs are fixed and do not depend on the values while searching. As a consequence, the wage agreed is more rigid.

The values of all the three options are uniquely defined so that worker's strategy is to accept an offer only if it is better off than both continuing or exiting the bargaining. As a consequence, there is some lowest wage contract value W that the worker will accept and the highest wage contract value W' that the employer will accept.

In equilibrium, the worker will be indifferent from accepting W or making a counter-offer W'. This condition thus satisfy:

$$W_i + E_i = \eta U_i + (1-\eta)[z + \xi \mathbb{E}_{i'}(W'_{i'} + E_{i'})]. \tag{25}$$

Similarly, the employer must be indifferent from making an offer W' and then receiving a counter-proposal W:

$$P_i - W_i' = \eta V_i + (1-\eta)[-\gamma + \xi \mathbb{E}_{i'}(P_{i'} - W_{i'})]. \tag{26}$$

Since the employer is beginning the equilibrium wage is W_i, which Hall and Milgrom (2008) have proved to be unique.

The properties of the strategic bargaining wage rule can be obtained again by considering the economy with no shocks. However here, as in Hall and Milgrom (2008) the wage W is somewhat complicated, so I will solve for the average $\frac{1}{2}(W + W')$, which is highly correlated with W in my calibration.

Subtracting (26) from (25) and rearranging gives the following expression for the average wage:

$$\frac{1}{2}\{W + W'\} = \frac{1}{2}\{P + \frac{1+r}{r}[\eta(U - V) + (1-\eta)(z+\gamma)] - E\} \tag{27}$$

The vacancy cost economy with the credible bargaining solution is characterized by the same equation except for the fact in the equation (26) $V_i = 0, \forall i$. The equation (27) becomes

$$\frac{1}{2}\{W + W'\} = \frac{1}{2}\{P + \frac{1+r}{r}[\eta(U) + (1-\eta)(z+\gamma)] - E\} \tag{28}$$

The shocks in productivity have a direct impact on the wage equation. However the value of unemployment,U, is scaled down by δ giving it less weight. The lower δ, the more weight is given to the fixed part $z+\gamma$ and consequently the more rigid the wage. This flexible form allows us consequently to vary the wage stickiness by changing the coefficient δ. The continuation value E is not very sensitive to the productivity shocks because it is composed of the future values of unemployment which are scaled down by σ.

Since the free entry condition requires $V = q$ in the investment cost economy, equation (27) can be rewritten as

$$\frac{1}{2}\{W + W'\} = \frac{1}{2}\{P + \frac{1+r}{r}[\eta(U - q) + (1-\eta)(z+\gamma)] - E\} \tag{29}$$

Here, the shocks to the cost of machinery, q, are scaled down by the same parameter δ. In the investment cost economy with the strategic bargaining solution, δ smoothes the impact of both driving forces.

The investment cost and the vacancy cost economies with credible bargaining have $6 \times$

$N \times M$ and $6 \times N$ endogenous variables respectively, because W' needs to be calculated.

3 Quantitative Analysis

3.1 The Data

I reproduce Shimer (2005)'s statistical analysis of the US economy, which covers the time period 1951-2004, the data are quarterly. Following Hornstein, Krusell, and Violante (2007), I measure the relative price of capital as the ratio of the price of business equipment and software investments to the price of personal consumption expenditures.[3]

Since the model is concentrated on the business cycle fluctuations, I detrend the data. For this purpose I use the standard Hodrick-Prescott (HP) filter. The results are first obtained with the usual smoothing parameter for quaterly data, $\lambda = 1600$. Recently Shimer (2007) has argued that a more appropriate smoothing parameter is $\lambda = 10^5$. Table 1 reports the resulting standard deviations and autocorrelation coefficients of the key US labor market variables for both filters. The choice of the filter does not affect substantially the relative standard deviations.

The statistics with the more usual smoothing parameter 10^5 are chosen as the benchmark.

3.2 Calibration

This section explains how the driving forces are constructed as well as the choice of model's other parameters to match the time series behavior of the US labor market.

Using the time series for the productivity and the relative price of capital, I construct discrete versions of the two driving forces as the deviations of their logarithms from HP trend with the smoothing parameter 10^5. The number of states for both processes is chosen to be $N = 5$. The resulting first order Markov chains are presented in Table 2. The ergodic probabilities of the five states in both cases are equal to 0.2.

The investment-cost economies are subject to both driving forces. As a result of the independence hypothesis, a set of possible states changes to $N \times N = 25$ with the transition matrix calculated as the following Kronecker product: $\pi = \pi^p \otimes \pi^q$. The ergodic probabilities of the twenty five states equal 1/25.

I first calibrate the vacancy-cost economy. The unit of time is chosen to be one month. It makes the results directly comparable with Shimer's stylized facts. The annual interest rate

[3] Both price indices are from the National Income and Product Accounts of the US (NIPA).

Table 1: The US economy, 1951-2004

	u	θ	λ_w	w	p	q
	(1) HP(1600)-filter					
Std Dv	12.49	25.71	10.71	1.58	1.32	1.14
AR(1)	0.8692	0.8951	0.8026	0.6126	0.7589	0.8860
	(2) HP(10^5)-filter					
Std Dv	18.86	37.96	16.37	2.26	1.99	2.32
AR(1)	0.9379	0.9466	0.9110	0.8030	0.8895	0.9676
	(3) Cross Correlations for HP(10^5)-filter					
u	1.000	-0.978	-0.965	-0.347	-0.455	0.405
θ		1.000	0.961	0.300	0.407	-0.364
λ_w			1.000	0.277	0.388	-0.426
w				1.000	0.474	-0.314
p					1.000	-0.344

Note: This data except the relative price of capital, q, was constructed by Robert Shimer. For additional details, please see Shimer (2007) and his webpage http://robert.shimer.googlepages.com/flows.

Table 2: Markov Chains for the driving forces

Productivity						
State	Percent dev.	Monthly transition probabilities				
		1	2	3	4	5
1	-3.07	0.8488	0.1395	0.0116	0.0000	0.0000
2	-0.98	0.1395	0.7558	0.0698	0.0349	0.0000
3	0.14	0.0114	0.0682	0.7614	0.1364	0.0227
4	1.09	0.0000	0.0357	0.1667	0.6667	0.1310
5	2.81	0.0000	0.0000	0.0000	0.1628	0.8372
Relative price of capital						
State	Percent dev.	Monthly transition probabilities				
		1	2	3	4	5
1	-3.41	0.9419	0.0581	0.0000	0.0000	0.0000
2	-1.37	0.0595	0.8690	0.0714	0.0000	0.0000
3	-0.02	0.0000	0.0682	0.8636	0.0682	0.0000
4	1.39	0.0000	0.0116	0.0581	0.8488	0.0814
5	3.40	0.0000	0.0000	0.0116	0.0814	0.9070

is 0.05, it corresponds to a monthly interest rate of 0.42 percent, $r = 0.0042$. Shimer (2007) finds that the average monthly separation rate is $\sigma = 0.035$, *i.e.* the jobs last on average 2.5 years. The unemployment rate is set to 5.5%. Given the job-separation rate and the unemployment rate, I use equation (7) to solve for the monthly job-finding rate of $\lambda_w = 0.60$, which is in line with Shimer's findings using the data from the Current Population Survey.

Pissarides (forthcoming) estimate that the sample mean for the tightness in $1960 - 2006$ was 0.72.[4] The matching function elasticity is set to $\alpha = 0.525$ to fit the the relative volatility of the tightness with respect to the job-finding rate. This value is in line with the findings of Petrongolo and Pissarides (2001). Given the matching function elasticity, α, the labor-market tightness, θ, and the job-finding rate, λ_w, using the matching function we can solve for the scale parameter, A.

I use Hall and Milgrom (2008) calculations to set the unemployment benefits $z = 0.71$. The cost of posting a vacancy is set to 0.4, which is also close to the findings of Hall and Milgrom (2008). I obtain the worker's bargaining power by solving the zero-profit condition to match unemployment rate. The estimated $\beta = 0.47$ is close to economy's elasticity of the matching function, $\alpha = 0.57$, meaning that the steady state is not far away from the so-called "Hosios (1990) condition", which makes the equilibrium search efficient.

Calibrating the investment cost economy is essentially the same with some exceptions. Worker's bargaining power is set again to match the observed average unemployment level, $\beta = 0.73$. The mean relative price of capital is set to $q = 6$. The same value is kept in the economy with the strategic bargaining wage formation.

Hornstein, Krusell, and Violante (2007) document that the average annual depreciation rate for business equipment is 14% or a monthly rate of about 1.2%, $\delta = 0.012$. Hence the rate at which the matches separate without the vintage being destroyed is $\varepsilon = \sigma - \delta = 0.023$.

The economies with the credible bargaining wage formation need to calibrate two more parameters: the disagreement payoff for the firm, γ, and the rate at which the job opportunity ends, η. I follow Hall and Milgrom (2008) and choose the employer's cost of delay so that it generates the observed average level of unemployment. I obtain that $\gamma_{vc} = 0.37$ in the vacancy cost economy and $\gamma_{ic} = 0.79$ in the investment cost economy. Higher values of η lead to more flexible wage and thus lower unemployment volatility. Since our knowledge on this parameter is limited, it is set to match the observed volatility. This method gives $\eta_{vc} = 0.25$ and $\eta_{ic} = 0.49$ in the vacancy and investment cost economies respectively.

The summary of parameters and variables values can be found in Table 3.

[4] The data is from The Job Openings and Labor Turnover Survey (JOLTS) and the Help-Wanted index.

Table 3: Parameter values

Variables and parameters common to all economies				
interest rate	$r = 0.0042$			
scale of matching function	$A = 0.5823$			
matching function elast.	$\alpha = 0.57$			
job separation rate, total	$\sigma = 0.035$			
job finding rate	$\lambda_w = 0.60$			
unemp. flow payment	$z = 0.71$			
unemp. rate	$u = 0.055$			
market tightness	$\theta = 0.72$			
Variables and parameters that differ across economies				
	VC+NB	VC+CB	IC+NB	IC+CB
worker surplus share	$\beta = 0.47$	-	$\beta = 0.73$	-
job separation rate	$\varepsilon = \sigma$	$\varepsilon = \sigma$	$\varepsilon = 0.023$	$\varepsilon = 0.023$
depreciation rate	$\delta = 0$	$\delta = 0$	$\delta = 0.012$	$\delta = 0.012$
employer's cost of delay	-	$\gamma_{vc} = 0.37$	-	$\gamma_{ic} = 0.71$
opportunity ending rate	-	$\eta_{vc} = 0.25$	-	$\eta_{ic} = 0.56$
entry cost	$c = 0.4$	$c = 0.4$	$q = 6.0$	$q = 6.0$

Note: VC is vacancy posting, NB is Nash bargaining, IC is investment cost economy, CB is credible bargaining and LP is less persistence.

3.3 Results

The model solves for the equilibrium market tightness, θ_i, in all states of the economy. The matching function allows to find the equilibrium job-finding rates, $\lambda_w(\theta_i)$, and the equilibrium unemployment, u_i, is obtained from the flow balance equation (7). Monthly wages, w_i are obtained by using the equation (4). It is straightforward to calculate the moments of interest in the key labor market variables by making use of the ergodic probabilities over the states. I obtain the standard deviations and the contemporaneous cross correlations of the log-levels of the generated variables to make the results comparable with the analysis of the US labor market facts.

The $AR(1)$ coefficients are obtained using simulations. For each economy, I generate a time series with 4000 realizations of the driving force process. Then I take the last 1000 realizations and construct a time series of the key variables. The results are reported in Table 4.

Table 4.1 reproduces the unemployment volatility puzzle. Labor productivity shocks alone are able to generate 17% of the observed market tightness volatility. This is consistent with Shimer (2005), although he estimates that the standard MP model generates only

Table 4: Model Simulations

	u	θ	λ_w	w	p	q
	1. Vacancy Cost + Nash Bargaining					
Std Dv	2.67	6.57	2.83	1.81	1.98	0.00
AR(1)	0.9210	0.9210	0.9210	0.9194	0.9157	0.0000
	2. Vacancy Cost + Credible Bargaining					
Std Dv	6.55	16.14	6.94	1.47	1.98	0.00
AR(1)	0.9257	0.9257	0.9257	0.9230	0.9157	0.0000
	3. Vacancy Cost + Nash Barg. (shocks to q only)					
Std Dv	5.70	14.00	6.02	0.93	0.00	2.32
AR(1)	0.8958	0.8959	0.8959	0.8976	0.0000	0.9622
	4. Investment Cost + Nash Bargaining					
Std Dv	6.90	16397	7.30	2.31	1.98	2.32
AR(1)	0.8982	0.8983	0.8983	0.9087	0.9109	0.9631
	5. Invest. Cost + Nash Barg. + Less Pers.					
Std Dv	11.26	27.63	11.88	2.66	1.98	2.32
AR(1)	0.7858	0.7857	0.7857	0.8159	0.8419	0.9270
	6. Investment Cost + Credible Bargaining					
Std Dv	14.46	35.65	15.33	2.12	1.98	2.32
AR(1)	0.9102	0.9102	0.9102	0.9146	0.9109	0.9631
	7. Cross Correlations of 6.					
u	1.000	-1.000	-1.000	-0.9877	-0.8341	0.4834
θ		1.000	1.000	0.9877	0.8346	-0.4828
λ_w			1.000	0.9877	0.8346	-0.4828
w				1.000	0.9085	-0.3701
p					1.000	0.000

about 10% of the US data volatility. The difference is due to a choice of the unemployment compensation, z. Following Hall and Milgrom (2008) estimation, I choose $z = 0.71$ instead of Shimer's 0.4.

The vacancy cost economy with credible bargaining mechanism reported in Table 4.2 is able to generate the required volatility in the market tightness, conditionally on the fact that productivity is the only driving force, however not without a cost. The result is achieved by inducing more wage stickiness. As we have seen in previous sections, the wage stickiness is the only source of the v-u ratio fluctuations in the model. The wage volatility is reduced to 1.47 and is lower then the wage volatility observed in the US data. This simulation illustrates the contribution of Hall and Milgrom (2008) paper.

Next, I shut down shocks to labor productivity and simulate the economy with investment-specific technology shocks only. The simulation is calibrated the same way as the economy in Table 4.1. Such economy generates about the same amount of volatility in labor market variables. The results confirms that shocks to q is an important source of business cycle fluctuations.

For the rest of the economies I assume that the productivity and the investment cost shocks are the only two shocks influencing tightness. As a consequence the target volatility is 37.96, the one we observe in the US data.

Table 4.4 reports the results of the investment cost economy with the wages modeled as the Nash bargain. The model is able to reproduce about the same volatility in key variables as the vacancy cost economy with the credible bargaining solution. The only difference is the wage volatility, which is now much close to the observed wage volatility. This is a step in a good direction, however given the model's target of reproducing the whole volatility in tightness, it is unable to close the gap between the data and the model.

Following, I perform the exercice in the same spirit as Hornstein, Krusell, and Violante (2007). They have shown that the persistance of the driving forces play an important role in the elasticities of labor market tightness with respect to productivity and investment cost: when investment cost and productivity are not persistent, the response of tightness increases. Consequently, Hornstein, Krusell, and Violante (2007) reduce the autocorrelation coefficient of their driving forces and obtain significantly higher volatility in tightness.

For the purpose of performing the same exercice, I build the Markov-process using the quarterly transition matrices but keep model's unit of time the same - one month. This naturally makes the process less persistent. The results are reported in the Table 4.4.[5] The driving forces have significantly higher impact, they are able to account for the 73% of the observed volatility in the v-u ratio. However the model runs into another difficulty. The wage volatility is nearly twice as high as in the data. The latter result is consistent with Hornstein, Krusell, and Violante (2007) and it is the main motivation of the next economy I consider.

The investment cost economy mixed with the credible bargaining wage formation replicates *all* the fluctuations as observed in the data. This is not surprising, because the model is calibrated to match the observed volatility of tightness. Here, the wage stickiness resulting from the alternating offers bargaining framework brings the wage volatility in line with the data.

[5] The economy is calibrated the same way as the previous case.

4 Conclusions

The literature following Shimer (2005) has established that the productivity shocks alone can explain about 40% of the tightness volatility. The only way to generate more volatility in labor market variables is to introduce other cyclical driving forces.

One of the few steps in this direction is Hornstein, Krusell, and Violante (2007). They augment the standard MP model with a second source of aggregate fluctuations: fluctuations in investment-specific productivity. The simulations revealed that this model is successful in reducing the gap between the model and the data. However Hornstein, Krusell, and Violante (2007) run into two problems. The persistence of the driving forces needed to generate the observed volatility is lower than observed in the data. Second, the real wage fluctuations are too high.

I have added the wage smoothing mechanism into the Hornstein, Krusell, and Violante (2007) framework. It improves the model on both of it's problems.

First, the model does not require to induce lower persistence of the driving forces than it is estimated to generate additional volatility in labor-market variables.

Second, wage formation à la Hall and Milgrom (2008) smoothes the real wage fluctuations and brings them in line with the data. This induces more fluctuations in the labor market variables, as the wage stickiness is the source of tightness volatility in the investment cost economy as well.

The extension of this paper will require calculating elasticities of tightness and wage with respect to productivity and investment cost. This will allow to quantify the impact of each driving force separately and to investigate if the wage is nearly proportional to productivity as in the data. Also, Hornstein, Krusell, and Violante (2007) framework assumes a fixed capital stock. Further research should investigate the effects of loosening this assumption.

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