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METHODS

The shadow price of assimilative capacity in optimal flow pollution control

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ABSTRACT

We present a model of optimal flow pollution control considering explicitly the dynamics of the corresponding assimilative capacity. We focus first on the degradation of this assimilative capacity triggered by pollution excesses and determine the intertemporal efficient pollution path, taking into account this ecological feedback. Our analysis shows that a minimum level of initial assimilative capacity is necessary to prevent its optimal extinction. We then allow for the restoration of assimilative capacity and characterize the conditions under which this option frees the optimal policy from the dependency on the initial conditions. In both cases our results call for environmental standards based on the shadow price of assimilative capacity that are stricter than the static optimum commonly used in flow pollution control.

1. Introduction

The assimilative capacity of an ecosystem receiving pollution can be defined as the ability "to receive a determined level of residues, to degrade them and to convert them in non-damaging and even beneficial products" (Pearce and Turner, 1990, p.38). This environmental sink function is at work in both stock and flow pollution. The assimilation of CO2 by oceans and forests and the protection of watercourses from lixiviated nutrient flows[1] by riparian buffer zones (Correll, 1996) illustrate these respective cases[2]. The level of assimilative capacity is not constant over time and depends either on the current stock of pollution (the concentration of greenhouse gases in the atmosphere) or on the "history" of pollution flows (periodic emissions of nitrates originating from fertilizers).

These dynamics are all the more important in flow pollution problems in that the level of assimilative capacity reflects the maximum amount of pollution that does not cause any social damage and that does not trigger any permanent alteration of the ecosystem functions. For instance, as long as the flows of lixiviated nitrates remain below the assimilative capacity threshold of riparian buffer ecosystems, no social damage is sustained and this capacity remains unaffected for future use. If the emissions exceed this threshold, not only will there will be contamination of the watercourses but the riparian buffers' assimilative capacity will be impaired by temporary nitrogen saturation (Hanson

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[1] In this setting, flow damages consist in increased costs of artificial purification for drinking water, health problems, temporary loss of recreational amenities and commercial benefits due to the temporary clogging of estuaries by seaweed.

[2] Noise can be considered as another example of flow pollution involving assimilative capacity. In that case the assimilative capacity at work is the human ability to cope with noise without suffering from stress.

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et al., 1994; Fromm, 2000). Therefore a merely static economic analysis of optimal flow pollution control will prove inappropriate when assimilative capacity is involved. Since the pollution optima serve as theoretical landmarks for environmental regulation, an economic instrument such as a pigouvian tax can lead to the extinction of the assimilative capacity if it is not calibrated properly. Indeed, if the static optimal level of pollution exceeds the assimilative capacity, it will cause damage and lower the threshold at which this social damage occurs in the future. At the next period, the same constant amount of pollution will thus be even more in excess of the assimilative capacity and will cause even more social damage and more degradation of assimilative capacity. This vicious cycle, first highlighted by Pearce (1976), can continue until the assimilative capacity is extinguished[3]. That is why it is crucial to carry out the economic analysis of flow pollution with assimilative capacity in an adequate dynamic framework.

The flow pollution control models found in the economic literature are either set in a static framework (Perman et al., 2003, p.171) or they do not allow for actual ecological dynamics (Schou, 2002). Meanwhile, the seminal articles on optimal stock pollution acknowledge the role played by assimilative capacity and its evolution over time (Forster, 1975). A survey of the different representations of assimilative capacity in the literature can be found in Pezzey (1996). Recently some authors such as Cesar and de Zeeuw (1994), Tahvonen and Salo (1996), Tahvonen and Withagen (1996), Toman and Withagen (2000), Chevé (2002), Hediger (2006) and Prieur (in press) have improved the specification of the assimilative capacity in various models of stock pollution control. However these contributions neither address the case of flow pollution nor allow for assimilative capacity restoration. The latter can provide a useful tool to a society that wishes to offset the degradation of assimilative capacity. For instance, CO_2 assimilation can be increased by afforestation while the assimilative capacity of riparian ecosystems can be restored through expansion and revegetation of buffer strips (Anderson and Ohmart, 1985; Goodwin et al., 1997). Although there exist significant work on environmental quality restoration (Phillips and Zeckhauser, 1998; Keohane et al., 2007) little attention has been paid specifically to the restoration of assimilative capacity (d'Arge, 1971; Pearce and Common, 1973) and to our knowledge this policy option has never been represented in a stylized model.

We therefore propose to build an optimal flow pollution control model, based on an intuition of Pearce (1976) extended later by Pezzey (1996) and Godard (2006), that takes into account the role and dynamics of assimilative capacity. We treat this assimilative capacity as an autonomous state variable that follows its own dynamics. This dynamic flow pollution model allows for a more comprehensive view of the economy–ecology interactions at stake and enables us to consider explicitly the option of restoring the assimilative capacity. After specifying in Section 2 our original pollution control model, we characterize in Section 3 the optimal pollution path and compare it to the static optimum. We introduce in Section 4 the possibility of restoring the assimilative capacity and we determine the new optimal path corresponding to this enhanced version of the model. In Section 5 we discuss the policy applications of our set of results. Section 6 concludes and points out potential extensions of our model.

2. The modified flow pollution model

As in most social optimization problems, we use a discounted utilitarian framework with a social welfare function including both the private benefit and the environmental damage with δ, $\delta \in]0,1[$, the social discount rate, supposed constant. We adopt a simplified pollution control model without capital accumulation similar to Ulph and Ulph (1994). The social planner problem amounts to

$$\max_p W = \int_0^{+\infty} U(p(t), A(t))e^{-\delta.t}dt = \int_0^{+\infty} (f(p(t)) - D(p(t), A(t)))e^{-\delta.t}dt \quad (1)$$

subject to $\dot{A}(t) = -h(p(t),A(t))$ and $A(0) = A_0$
where $U(p(t), A(t))$ is the utility derived from an economic activity emitting a flow of pollution[4] $p(t)$ while benefiting from a level $A(t)$ of assimilative capacity and $h(p(t), A(t))$ is the degradation function of the assimilative capacity. A_0 denotes the initial level of assimilative capacity supposedly known. U is an concave function that can be separated into the private benefit function from polluting activity f and the socio-environmental damage D function triggered by this flow of pollution such that $U(p(t), A(t)) = f(p(t)) - D(p(t), A(t))$.

We work with a private-benefit function characterized by the standard properties of the literature: f positive, non-decreasing, concave, defined over R^+, $f_p \geq 0$, $f_{pp} \leq 0$. As we assume that the polluting firm ignores the externality it imposes on society, its private pollution optimum x_p is such that $f_p(x_p) = 0$. We exclude the possibility of a technical change that would allow to yield the same benefit while polluting less. There is no particular need to give an essential dimension to this production, the benefit function should thus not impose an "infinite penalty" on a zero level of production, and therefore we shall reject the Inada conditions (see Heal, 2000, p.37). In particular, if the environmental conditions are such that any strictly positive level of emissions will have negative welfare effects, then the economy will switch to any backstop production solution yielding positive welfare effects:

$$\lim_{p \to 0} f_p(p) < +\infty \quad and \quad f(0) = 0 > -\infty.$$

We use a flow damage function $D(p(t), A(t))$ that depends not only on the level of emissions $p(t)$ but also on the level of assimilative capacity $A(t)$. Indeed, when there is a neutralizing assimilative capacity at work in the ecosystem, the environmental damage is nil for any flow of pollution below the current assimilative capacity level. Conversely, the higher the

[3] A similar cycle degrades soil productivity when farmers fail to consider the intertemporal impact of their activity on soil quality (Barbier, 1990).

[4] Pollution is an input in production, and the firm must necessarily increase its polluting emissions if it wants to increase its profit, through either a greater production of goods or a reduction of its pollution control costs.

excess of pollution vis-à-vis the assimilative capacity, the higher the environmental damage sustained by society. Since we have stressed that the assimilative capacity is not constant but can evolve over time, a given amount of pollution that was harmless (harmful) before can thus become harmful (harmless) if the assimilation threshold of the ecosystem is reduced (increased). In this configuration, the damage function still reflects flow damages but the ecological features determining the shape of this damage function are not static. Formally this is reflected by a slight modification of the standard damage function found in the literature: D depends on the excess of pollution vis-à-vis the assimilative capacity and not only on the absolute amount of pollution $p(t)$. We can specify this environmental damage function as follows, measuring the social damage suffered by society when pollution exceeds the assimilative capacity. D is increasing and convex with respect to the excess of pollution $(p(t)-A(t))$, $D(p(t), A(t))=0\ \forall\ p(t)\leq A(t)$ and $D(p(t), A(t))>0\ \forall p(t)>A(t)$. Given the mechanisms described above, the following properties are straightforward[5]

$$\begin{array}{ll}\forall\ p(t)<A(t) & D_p(p(t),A(t))=-D_A(p(t),A(t))=0\\ \forall\ p(t)\geq A(t) & D_p(p(t),A(t))=-D_A(p(t),A(t))>0\\ \forall\ p(t)\geq A(t) & D_{pA}(p(t),A(t))=D_{Ap}(p(t),A(t))<0\\ \forall\ p(t)\geq A(t) & D_{AA}(p(t),A(t))=D_{pp}(p(t),A(t))>0.\end{array} \quad (2)$$

Relation (2) reflects the fact that when the assimilative capacity is strictly respected, an incremental change in the pollution level or in the assimilative capacity level does not trigger an ecological reaction. The function D displays a continuity problem in the neighborhood of $p(t)=A(t)$ but this problem will be dealt with later on. We also make the following reasonable assumption on the behavior of D

$$\forall(p(t),A(t))\ \lim_{A\to 0} D_p(p(t),A(t))=\lim_{A(t)\to 0} -D_A(p(t),A(t))=L>0. \quad (3)$$

L can be either finite or infinite. This assumption is quite realistic as the assimilative capacity tends towards extinction, the marginal damage imposed by an additional unit of pollution is equal to a high enough positive value L.

The dynamics of the assimilative capacity subject to anthropogenic stress are accounted for[6] by the function h. We begin here by assuming that restoring the assimilative capacity is not possible before relaxing this hypothesis in Section 4. For now h can be thought of as a degradation function such that:

$$\dot{A}(t)=-h(p(t),A(t))\leq 0. \quad (4)$$

The waste assimilation properties of an ecosystem are obviously subject to change if the ecosystem is disrupted by external stress. It is asserted ecologically that above a certain threshold of flow pollution, the ecosystem's equilibrium is affected and functions such as the assimilative capacity are altered. We will consider here, based on Pearce's arguments (1976), that this threshold can be identified in some cases with the level of assimilative capacity itself, although, as noted by Pezzey (1996), there is no systematic evidence to back up this assumption. According to Pearce (1988, p. 61) "the act of excessive pollution produces a negative feedback, making the ecosystem even less capable of dealing with waste"[7]. This phenomenon is quite clear in the case of microbial assimilation as the biological organisms degrading the pollutant can be harmed by the excess of pollution they are unable to digest immediately (Pezzey, 1996). Since it is this "moving" threshold effect that is of interest here and the economic model would hardly be tractable with an additional "degradation threshold" variable, we will consider that the thresholds for assimilative capacity degradation and the occurrence of environmental damage are identical and measured by the current level of assimilative capacity. We can therefore state that an excess of pollution in comparison to the assimilative capacity reduces the assimilative capacity available in the future. Conversely if the assimilative capacity is respected, *i.e.* not exceeded, the ecosystem remains unharmed and the assimilative capacity stays constant or can be increased if restoration is available. According to these mechanisms we have $h(p(t), A(t))=0\ \forall\ p(t)\leq A(t)$ and $h(p(t), A(t))>0\ \forall\ p(t)>A(t)$. As with the damage function, the following properties of h can be established

$$\begin{array}{ll}\forall\ p(t)<A(t) & h_p(p(t),A(t))=-h_A(p(t),A(t))=0\\ \forall\ p(t)\geq A(t) & h_p(p(t),A(t))=-h_A(p(t),A(t))>0\\ \forall\ p(t)\geq A(t) & h_{pA}(p(t),A(t))=h_{Ap}(p(t),A(t))<0\\ \forall\ p(t)\geq A(t) & h_{AA}(p(t),A(t))=h_{pp}(p(t),A(t))>0.\end{array}$$

It must be noted that h displays a continuity problem similar to the one affecting the damage function D in the neighborhood of $p(t)=A(t)$. We must also establish an upper bound equal to 0 on the marginal degradation of $A(t)$ in order to avoid negative values of $A(t)$.

$$\lim_{A\to 0} h_A=\lim_{A\to 0} h_p=0.$$

We finally assume that $A_0<x_p$. If this condition were not established, then the polluting producer would spontaneously choose a production-pollution level below the assimilative capacity and no external damage would ever occur.

Considering these dynamics, we have

$$\begin{cases}\dot{A}\leq 0\\ A(0)=A_0\\ 0\leq A(t)\leq A_0<x_p \quad \forall t\geq 0\end{cases}.$$

Finally, in order to overcome some mathematical complexities that would be beyond the scope of this paper, we need to make the following assumption

$$-h_A(p(t),A(t))<\delta \quad \forall(p(t),A(t)). \quad (5)$$

Although the degradation processes are far from thoroughly understood by ecological science, this assumption does not seem counter-intuitive. Indeed, for standard "low values" of δ such as $\delta<1$, Eq. (5) implies that a unit of pollution

[5] They can easily be verified by thinking of D as a direct function of the excess of pollution $(p-A)$. Thus $D(p(t), A(t))=D(p(t)-A(t))$ yields $D_p=\frac{dD}{dp(t)}=D'(p(t)-A(t)), D_A=\frac{dD}{dA(t)}=-D'(p(t)-A(t))=-D_p$ and so forth.

[6] We will work here in a deterministic framework, assuming that the effects of pollution excess on assimilative capacity are not subject to uncertainty.

[7] This is also stated by Pethig (1994, p. 218): "the environment has a limited capacity of assimilating pollutants. As long as the flow of released pollutants exceeds that capacity, the environmental quality is reduced until eventually nature's assimilative services are exhausted".

in excess reduces the assimilative capacity by an amount less than a unit.

3. Optimal pollution path without restoration

In this section we characterize the optimal pollution path when the degradation of assimilative capacity is irreversible. Let us start by establishing immediately a useful result:

Proposition 1. *At any time t along the optimal path, $p(t)=A(t)$ must always be preferred to $p(t)<A(t)$.*

Indeed, choosing a pollution level $p(t)=A(t)$ yields a higher private benefit $f(p(t))$ than any $p(t)$ such that $p(t)<A(t)$ while triggering the same amount of damage and assimilative-capacity depletion (both equal to zero). Thanks to this restriction of the definition set of $p(t)$ on the optimal path, we can avoid the difficulties caused by the continuity problem of h and D in the neighborhood of $p(t)=A(t)$.

The current value Hamiltonian H of our problem, according to Eqs. (1) and (4) and under the assumption of no technological change, is

$$H(t)=f(p(t))-D(p(t),A(t))-\lambda(t)h(p(t),A(t))$$

where $\lambda(t)$ is the co-state variable representing the shadow price of the assimilative capacity. Given that the contribution of the latter to the social welfare function is always positive, this price λ will necessarily be positive along the optimal path: $\lambda(t)\geq 0\ \forall\ t\geq 0$.

3.1. *First-order conditions*

Let us establish the first-order conditions[8] determining this optimal path. These conditions are

$$\frac{dH}{dp}=0$$
$$\frac{dH}{dA}=\delta\lambda-\dot{\lambda}$$
$$\frac{dH}{d\lambda}=\dot{A}$$

hence

$$\begin{aligned} &f_p(p)-D_p(p,A)-\lambda h_p(p,A)=0\\ &\delta\lambda-\dot{\lambda}=-D_A(p,A)-\lambda h_A(p,A)\end{aligned} \tag{6}$$

which yields

$$f_p(p)=D_p(p,A)+\lambda h_p(p,A) \tag{7}$$

$$\dot{\lambda}=\lambda(\delta+h_A(p,A))+D_A(p,A). \tag{8}$$

In addition there is the transversality condition:

$$\lim_{t\to+\infty} e^{-\delta t}\lambda(t)A(t)=0. \tag{9}$$

Eq. (7) establishes a very intuitive result: the net private benefit from an additional unit of pollution must be equal to the total marginal damage caused by this unit. This marginal damage includes the standard flow marginal damage D_p as well as the marginal loss of assimilative capacity, valued by the product of the shadow price of assimilative capacity, λ, and the amount of assimilative capacity depleted by this incremental unit of pollution $h_p(p, A)$. Rewriting Eq. (8) and using (7) we get:

$$\frac{\dot{\lambda}}{\lambda}=\delta+\left(h_A+\frac{D_A h_p}{D_p-f_p}\right). \tag{10}$$

The rate of change in the shadow price of the undepleted assimilative capacity is determined not only by the depletion-adjusted social discount rate[9] $(\delta+h_A)$ but also by an additional factor indicating the social value of *in situ* assimilative capacity. Eq. (10) reads as a modified version of the Hotelling rule. If we treat the assimilative capacity as an exhaustible resource, its productivity, which in a perfectly competitive market must be equal to its price at the equilibrium, must grow at a rate given by the right-hand side of Eq. (10). This term needs to be compared with the standard discount rate δ but unfortunately the sign of the additional term $h_A+\frac{D_A h_p}{D_p-f_p}$ cannot be determined unambiguously. However, it can be shown that when A is low enough, the additional term is negative and the shadow price of assimilative capacity must grow at a rate lower than the value of the discount rate, which implies a slower depletion rate of the resource itself.

3.2. *Comparison with the static optimum*

From now on we shall denote $(p^*(t), A^*(t))$ the set of optimal values of p and A on the optimal path at time t. Relation (6) can be written for A and λ given with the function $\pi_{A,\lambda}$ such that

$$\pi_{A,\lambda}(p)=f_p(p)-D_p(p,A)-\lambda h_p(p,A)$$

$\pi_{A,\lambda}$ is decreasing in p according to the properties of f, D and h for a given A and λ. For $p^*(t)$ the pollution optimum at any time t we have, according to Eq. (6)

$$\pi_{A,\lambda}\left(p^*\right)=f_p\left(p^*\right)-D_p\left(p^*,A\right)-\lambda h_p\left(p^*,A\right)=0. \tag{11}$$

Let us define $\bar{p}(A)$ as the static Turvey optimum for a given assimilative capacity A. $\bar{p}(A)$ is determined by standard static internalization of external effects such that $f_p(\bar{p}(A))=D_p(\bar{p}(A), A)$. This optimum fails to consider any dynamic evolution of the problem since it statically equalizes the marginal benefit and the marginal damage for a given level of A. We assume for simplicity of exposition and to focus on the most interesting case that f and D are such that

$$\forall\quad A\geq 0\quad \exists\quad \bar{p}(A)>0\quad \text{s.t.}\quad f_p(\bar{p}(A))=D_p(\bar{p}(A),A). \tag{12}$$

We can now compare this Turvey optimum with the dynamic optima by using our function $\pi_{A,\lambda}$. We have thus

$$\begin{aligned}&\pi_{A,\lambda}(\bar{p}(A))=f_p(\bar{p}(A))-D_p(\bar{p}(A),A)-\lambda h_p(\bar{p}(A),A)\\ &\pi_{A,\lambda}(\bar{p}(A))=-\lambda h_p(\bar{p}(A),A)\leq 0\end{aligned} \tag{13}$$

[8] For notational ease, the time index of variables will be omitted from now on whenever no ambiguity can arise.

[9] This adjusted discount rate is lower than the initial discount rate and similar to the pollution-adjusted discount rate found in the literature, see Hediger (2006).

hence, given the decreasing nature of $\pi_{A,\lambda}$, Eqs. (11) and (13) yield, for any given A and any t:

$$p^*(t) \leq \bar{p}(A(t)). \tag{14}$$

Proposition 2. *In the no restoration case, the level of optimal pollution must be lower than the static Turvey optimum at all times.*

We establish here a very intuitive result since it is natural that the dynamic optimum p^* accounting for intertemporal externalities should be lower than the static optimum $\bar{p}(A)$.

3.3. Phase-diagram analysis

The existence of a steady state is guaranteed by the joint concavity of our Hamiltonian (Theorem 13, Seierstad and Sydsaeter, 1987, p.234), proven in Appendix A[10]. Along the optimal path, the level of emissions must be adjusted continuously to satisfy the first-order conditions. The optimal level of pollution can thus be represented as an implicit function of λ and A where

$$p = p(\lambda, A).$$

Rewriting Eq. (7) with the utility function U we get

$$U_p(p(\lambda, A), A) = \lambda h_p(p(\lambda, A), A). \tag{15}$$

Differentiating each side with respect to λ yields (after simplifications)

$$\frac{dp(\lambda, A)}{d\lambda} = \frac{h_p}{U_{pp} - \lambda h_{pp}}.$$

According to the properties of f, D, h and λ we have

$$\frac{dp(\lambda, A)}{d\lambda} < 0 \tag{16}$$

Similarly, differentiating Eq. (15) with respect to A gives us after simplifications

$$\frac{dp(\lambda, A)}{dA} = \frac{\lambda h_{pA} - U_{pA}}{U_{pp} - \lambda h_{pp}}.$$

Given the properties of f and D we have $U_{pA}>0$. Combining with the properties of h and λ we have $\lambda h_{pA} - U_{pA}<0$. Hence

$$\frac{dp(\lambda, A)}{dA} > 0. \tag{17}$$

It is straightforward from Eqs. (4) and (8) that the behavior of the system from any initial point (A_0, λ_0) is governed by

$$\dot{\lambda} \gtreqless 0 \text{ as } \quad \lambda(\delta + h_A(p, A)) \gtreqless -D_A(p, A) \tag{18}$$

$$\dot{A} \gtreqless 0 \quad \text{as} \quad -h(p, A) \gtreqless 0. \tag{19}$$

According to the properties of h, the space where $h(p, A)=0$ should not be a curve (the standard isocline) but a plane since $h(p, A)=0\ \forall (p, A)$ s.t. $A \geq p$. However, we have proven previously that $p<A$ is never optimal, so the potential steady states on the optimal path are necessarily on the $[A=p]$-isocline.

The slopes of the stationary loci satisfying Eqs. (18) and (19) with equality are given by (see Appendix B for calculation details)

$$\frac{d\lambda}{dA}\Big|_{\dot{\lambda}=0} = -\frac{\frac{dp(\lambda,A)}{dA}(D_{Ap} + \lambda h_{Ap}) + \lambda h_{AA} + D_{AA}}{\delta + h_A + \frac{dp(\lambda,A)}{d\lambda}(\lambda h_{Ap} + D_{Ap})} \tag{20}$$

and

$$\frac{d\lambda}{dA}\Big|_{\dot{A}=0} = -\frac{\frac{dp(\lambda,A)}{dA} h_p + h_A}{\frac{dp(\lambda,A)}{d\lambda} h_p}. \tag{21}$$

Inequalities (16) and (17), Eqs. (20) and (21) respectively yield (see Appendix B)

$$\operatorname{sgn}\left(\frac{d\lambda}{dA}\Big|_{\dot{\lambda}=0}\right) = -\operatorname{sgn}(\delta + h_A)$$
$$\frac{d\lambda}{dA}\Big|_{\dot{A}=0} < 0. \tag{22}$$

Since according to Eq. (5) we have $\delta + h_A>0$, we can write

$$\frac{d\lambda}{dA}\Big|_{\dot{\lambda}=0} < 0. \tag{23}$$

Let us now characterize the optimal paths in the A–λ plane. In this plane, A is bounded below by 0 and above by x_p while λ is only bounded below by 0. According to Eq. (23), the isocline I_λ, where $\dot{\lambda}=0$, is monotonically decreasing in the A–λ plane. Based on Eq. (8) and the properties of h_A and D_A[11] we can easily show that $\dot{\lambda}>0$ on the right of I_λ and $\dot{\lambda}<0$ on the left of I_λ. In addition, we know from Eq. (22) that the isocline I_A, where $\dot{A}=0$, is decreasing. We obviously have $\dot{A}<0$ below I_A and must exclude the gray area where $A>p$ from our phase analysis. Note that the absence of restoration rules out the case $\dot{A}>0$.

The geometrical properties of the two isoclines, established in Appendix C, imply the following graphical representation in the diagram (note that we arbitrarily draw the I_A curve in a linear manner and set L to infinity for simplicity of exposition). Considering the geometrical properties characterized in Appendix C, it is straightforward that there is a unique interior solution for the steady state. The two isoclines intersect at a unique equilibrium candidate (A_e, λ_e) that is a saddle point(see Fig. 1). As A necessarily decreases along the optimal path, we can conclude from Eq. (17) that p also decreases along this path. The different cases depending on the initial level of assimilative capacity A_0 are discussed in Proposition 3.

Proposition 3. *Case (1): If $A_0 \geq A_e$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium (A_e, λ_e). As $\dot{A}<0$ along this path, the level of polluting emissions exceeds the assimilative capacity until it reaches the equilibrium and stabilizes with $p^* = A_e$. The shadow price of the assimilative capacity rises along this path while the pollution level decreases according to the properties of $p(\lambda, A)$.*

Case (2): If $A_0<A_e$, then the optimal path will never reach the steady state as the assimilative capacity cannot be increased. The optimal path in this quadrant will lead to extinction of the resource (see details below).

[10] The Appendix is available from the author upon request.

[11] An increase in A *ceteris paribus* induces an increase in $\dot{\lambda}$.

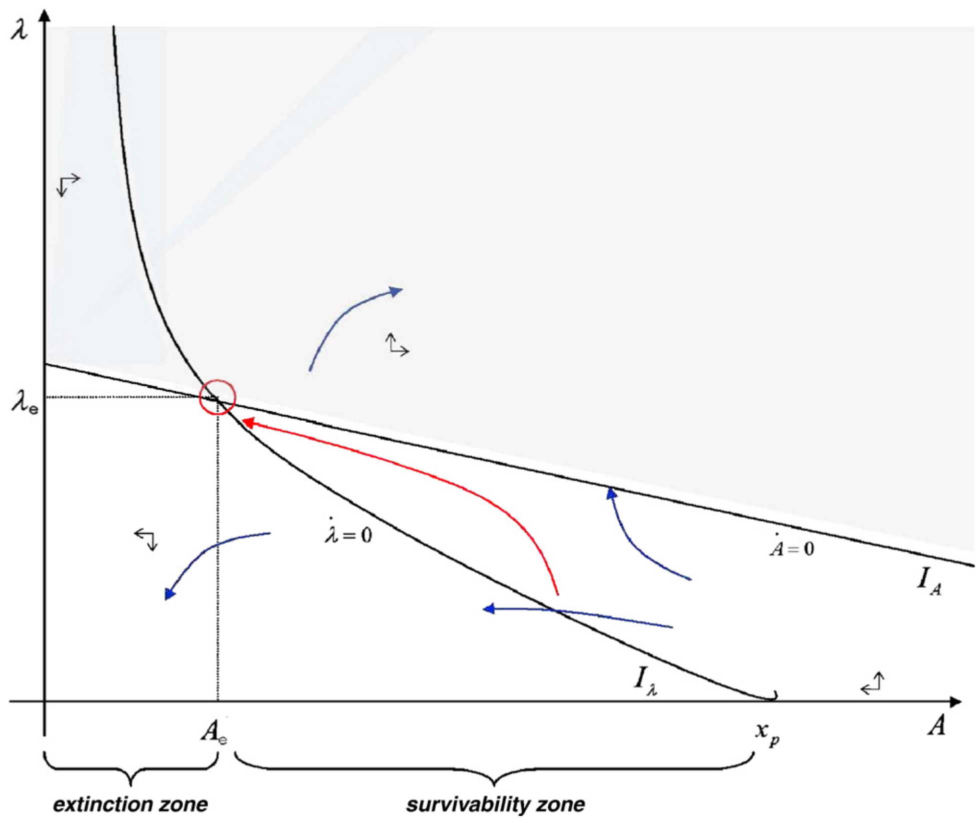

Fig. 1 – Optimal pollution path.

3.4. *Survivability analysis and optimal extinction*

Considering the essential service provided by the environment's assimilative capacity, its degradation due to pollution excesses raises serious sustainability concerns. Since our model does not allow for substitution between manufactured and natural capital, it is beyond the scope of this paper to discuss the general sustainability of the optimal path obtained above. We can nevertheless determine under what condition this path is compatible with an *a minima* environmental sustainability[12] condition requiring "the use of environmental services at rates which can hold on for very long time periods, and in theory, indefinitely" (Pearce, 1988, p.58). Assuming the absence of technological change in this activity, a requirement to maintain the assimilative capacity above a minimum threshold for future generations can be assimilated to a survivability criteria. We can thus give an additional interpretation of Proposition 3 in the light of this survivability concern. The set $[A_e, x_p[$ of Case (1) can be defined as the survivability zone, keeping in mind that the sustained level of assimilative capacity A_e might be low. Conversely, since the path corresponding to Case (2) does not respect such a survivability condition we call the $[0, A_e[$ set the extinction zone. A_e is thus the minimum initial level of assimilative capacity required to ensure a survivable optimal pollution path.

[12] In terms of "strong sustainability" (Ayres et al., 1998), it is clear that any pollution path degrading, even slightly, the assimilative capacity of an ecosystem is unsustainable.

Let us shed some light on the extinction trajectory. Considering the dynamics of A we know that an optimal path initiated for $A<A_e$ will lead to the complete depletion of the assimilative capacity (in finite or infinite time depending on the exact specification of h) if $p^*(t)>A^*(t)$ for all $A^*(t)\geq 0$ and especially if $p^*(t)>0$ when A tends towards 0. This is the case if and only if the marginal degradation of the assimilative capacity triggered by a strictly positive level of pollution has a positive welfare effect for all $A\geq 0$. According to the first-order condition (7), this implies

$$f_p(p)\geq D_p(p,A)+\lambda h_p(p,A) \quad \forall \quad A\geq 0.$$

We know from Eq. (5) that $h_p(p, A)$ tends towards 0 when A tends towards 0. The depletion path leads to extinction if and only if

$$\lim_{A\to 0}\left(f_p(p)-D_p(p,A)\right)\geq 0 \tag{24}$$

Let us focus on the most interesting case where Assumption (12) holds. In that case we know from Eq. (14) that $p^*(t)\leq\bar{p}(t)$ for all t. Given the concavity of U we have $U_p(p^*(t), A^*(t))>U_p(\bar{p}(t), A^*(t))$. Assuming that Assumption (12) holds, this yields

$$\lim_{A\to 0}\left(f_p\left(p^*(t)\right)-D_p\left(p^*(t),A^*(t)\right)\right)\geq\left(f_p(\bar{p}(t))-D_p\left(\bar{p}(t),A^*(t)\right)\right)=0.$$

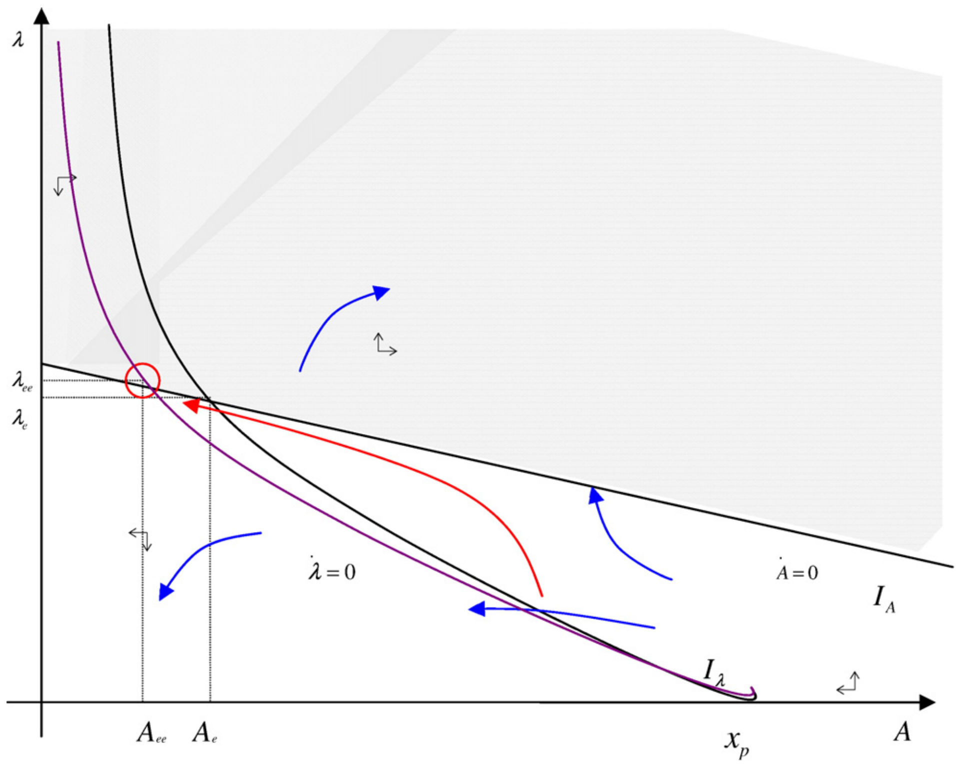

Fig. 2 – The effects of a higher discount rate.

Condition (24) is thus verified for all $A \in [0, A_0[$. The optimal path originating in the extinction zone leads systematically to the extinction of the assimilative capacity. It will depend on the functional forms of f, D and h to determine if this extinction takes place in a finite time or not. A possible economic interpretation of this behavior is that if a strictly positive pollution level has always a positive marginal welfare effect, it is not worthwhile to preserve a low initial stock of assimilative capacity. It is more efficient, on pure economic grounds, to totally deplete the assimilative capacity in order to reap the benefits of pollution today rather than allowing for a higher damage-free pollution in the future.

3.5. *Comparative statics*

The qualitative conclusions drawn in the previous section do not provide us with a quantitative definition of the equilibrium A_e. However, comparative statics can help us describe how the exogenous parameters of the model affect this equilibrium level. In Eq. (8) an increase in the social rate of discount δ for a constant λ must be compensated by a lower value for h_A and/or D_A, which implies a lower value for A. Graphically this means that the I_λ isocline will shift to the left for a higher δ. Fig. 2 shows the twofold ambiguous effect of a higher discount rate on the survivability of the optimal path. On the one hand, it diminishes the level of A_e, leaving future generations with less natural capital. On the other hand, it widens the survivability zone, increasing the range of initial ecological conditions that are compatible with a survivable path.

4. Optimal pollution path with restoration

4.1. *Modeling assimilative capacity restoration*

We now wish to consider the situation where it is possible to restore the assimilative capacity of a given ecosystem through either natural or artificial means. Natural restoration requires to grant this assimilative capacity a "rest", similar to a fallow period, by ensuring that the pollution emitted is strictly lower than this assimilative capacity. The latter will then increase proportionally to the "rest" granted[13]. Artificial restoration consists in investing a part of the economic benefit in natural capital to regenerate the assimilative function. We show in the next subsection that this investment can be accounted for exactly like the natural restoration process. We can thus proceed to the following modification of the function h to allow restoration:

$$\begin{aligned} &\forall \quad p(t) < A(t) \quad h(p(t), A(t)) < 0 \Rightarrow \dot{A}(t) > 0 \\ &\forall \quad p(t) > A(t) \quad h(p(t), A(t)) > 0 \Rightarrow \dot{A}(t) < 0 \\ &p = A \Rightarrow h(p(t), A(t)) = 0 \Rightarrow \dot{A}(t) = 0. \end{aligned} \quad (25)$$

Eq. (25) states that if the assimilation capacity is strictly respected, it increases by an amount $-h(p(t), A(t))$. Given the convexity of h, the restoration efforts display decreasing

[13] Here we assume that the level of restoration obtained can be determined in a continuous way. However, works on restoration of the environment's quality such as Keohane et al. (2007) consider restoration as a destination-driven threshold process.

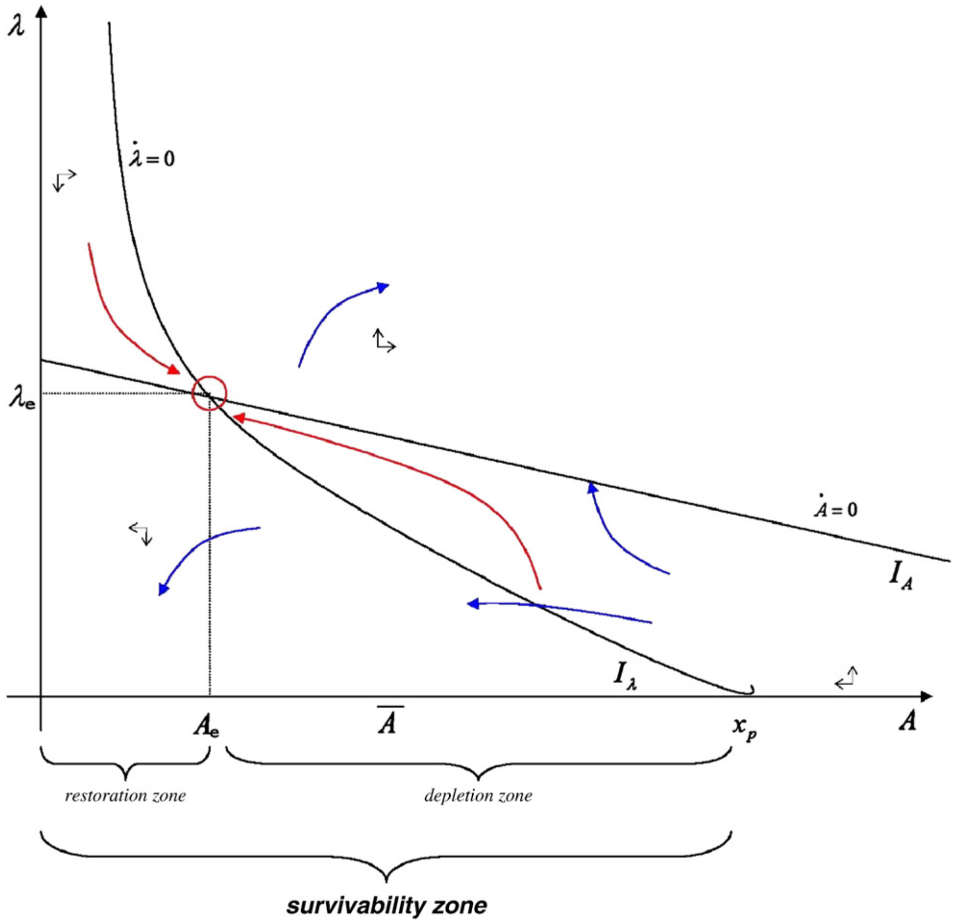

Fig. 3 – Optimal pollution path with restoration ($\bar{A} > A_e$).

returns, which seems like the most plausible assumption in such a context. Each additional unit of pollution "given up" yields a smaller restoration effect than the previous one. Now we must also have

$$\forall \quad p(t)\ h_p(p(t), A(t)) > 0$$
$$\forall \quad p(t)\ h_A(p(t), A(t)) < 0.$$

This new specification (strict inequalities) frees us from the continuity problem that we pointed out above for h in the neighborhood of $p(t) = A(t)$ in the previous case. The other properties of h remain valid.

There is naturally an upper bound $\bar{A}$ above which A cannot be restored: $A(t) \le \bar{A}\ \forall\ t$. We shall assume here that this upper limit is at least equal to the initial assimilative capacity, *i.e.* $A_0 \le \bar{A}$. This implies that it is always possible to restore the ecosystem's assimilative capacity to its initial level[14]. We also restrict logically our analysis to the case where $\bar{A} \le x_p$ as it would yield no additional social benefit to restore assimilative capacity beyond the private pollution optimum. We have now

$$\dot{A}(t) \gtreqless 0\ \forall\ t \ge 0$$
$$A(t) \le \bar{A} \le x_p \quad \forall\ t \ge 0.$$

Assumption (5) still holds.

We shall specify the cost of restoration in a negative way, treating it as a foregone benefit. We have seen in Section 3 that in the no-restoration case it is always optimal to pollute at least as much as the level of assimilative capacity. Polluting less than this level to let the assimilative capacity "rest" means giving up an economic benefit. We shall use this forgone benefit to account for the cost of restoration without introducing an additional cost function in the model. There are two ways to interpret a level of pollution $p(t) < A(t)$. If the social planner resorts to "natural restoration" it imposes on the polluters a pollution level strictly below the assimilative capacity and prevents them (and society as a whole) from enjoying the highest "damage-free" feasible benefit for a given $A(t)$, $f(A(t))$. In that case, the cost of restoring the assimilative capacity by an amount $|h(p(t), A(t))|$ is equal to the difference between the maximum "damage-free" benefit $f(A(t))$ that could have been achieved and the actual benefit $f(p(t))$. Given the concavity of f, the cost of restoration $f(A(t)) - f(p(t))$ is a

[14] This assumption might be challenged empirically in some local cases but is robust for the wide range of problems where the assimilative capacity can increased globally without restoring the exact spot where it had been depleted previously. This is the case when deforestation in Brazil is offset by afforestation in Europe to increase the planet's carbon assimilative capacity or when a degraded buffer strip protecting a stream from nutrient runoffs is extended or widened.

convex function of p which is coherent. In that case the social planner's objective function writes simply

$$f(p(t)) - D(p(t), A(t)). \tag{26}$$

If the social planner resorts to “artificial restoration”, it will let the polluters enjoy the maximal damage-free level of pollution[15] $p(t)=A(t)$ yielding a private benefit $f(A(t))$ and spend an amount $f(A(t))-f(p(t))$ (the cost of restoration assessed above) in order to restore the assimilative capacity by an amount $|h(p(t), A(t))|$. In that case the social planner's objective function writes

$$f(A(t)) - D(p(t), A(t)) - (f(A(t)) - f(p(t))) \tag{27}$$
$$= f(p(t)) - D(p(t), A(t)).$$

The two interpretations amount to equivalent relations (26) and (27) that can fit various cases of restoration depending on local conditions.

4.2. *Optimal restoration*

We will now determine the optimal path of pollution when restoration is an option available to the social planner. It is straightforward that this new problem is very similar to the irreversible case addressed previously. Indeed, the only modifications to the formal specification is the extension of the degradation function h which can now take negative values and the introduction of an upper bound $\bar{A}$ on A.

$$\max_p W = \int_0^{+\infty} (f(p(t)) - D(p(t), A(t)))e^{-\delta.t}dt$$

subject to $\dot{A}(t) = -h(p(t), A(t))$ $A(0) = A_0$, $A(t) \le \bar{A}$ $\forall t$

Adding a multiplier ω to ensure that the constraint on A holds at all times, we obtain the following Lagrangian:

$$L(t) = f(p(t)) - D(p(t), A(t)) - \lambda(t)h(p(t), A(t)) + \omega(\bar{A} - A(t)).$$

The transversality condition (9) still holds and we can extend the interpretation of the first order conditions.

4.2.1. *First-order conditions*

$$\begin{aligned} &f_p(p) - D_p(p, A) - \lambda h_p(p, A) = 0 \\ &\dot{\lambda} = \lambda(\delta + h_A(p, A)) + D_A(p, A) + \omega \\ &\omega(\bar{A} - A) = 0,\ \omega \ge 0,\ \bar{A} - A \ge 0. \end{aligned} \tag{28}$$

Eq. (28) can be reinterpreted in a very interesting way in the light of the restoration possibility. If $p<A$, the marginal damage is nil (Eq. (2)) and we have $f_p(p) = \lambda h_p(p, A)$. If at any time restoration is the optimal choice along the optimal path, it must be carried out until the marginal cost of the restoration effort $f_p(p)$ equals the value of an additional unit of *in situ* assimilative capacity. This value corresponds to the product of the marginal increase of assimilative capacity $h_p(p,A)$ and the shadow price of assimilative capacity λ. Regarding the comparison with the Turvey optimum, it is easy to verify that in the case with restoration, Eq. (13) and Proposition 2 still hold.

4.2.2. *Survivability analysis*

Since $A(t) \le x_p$ in the restoration setting as well, x_p remains the upper bound of A but in a weaker way. The phase analysis remains very similar to the irreversible case (see Appendix C). The introduction of the multiplier ω affects the optimal path when the constraint bites but we shall focus here on the most interesting case where $\bar{A}$ is large enough. The isoclines display the same properties and this time also there is a unique stable equilibrium (A_e, λ_e) that is a saddle point (see Fig. 3).

It is now possible to move from left to right on the optimal path (*i.e.* increasing the assimilative capacity). This possibility grants access to the area on the diagram that was off-limits in the irreversible case and changes the “extinction zone” into a "restoration" zone. However, this restoration of the assimilative capacity up to the steady state (A_e, λ_e) is obviously feasible only if $\bar{A} \ge A_e$. Fig. 3 illustrates this case and the general conclusions are drawn in Proposition 4.

Proposition 4. *If $A_0 \ge A_e$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium (A_e, λ_e). As $\dot{A}<0$ along this path, the level of polluting emissions exceeds the assimilative capacity until it reaches the equilibrium and stabilizes with $p^* = A_e$. The shadow price of the assimilative capacity rises along this path while the pollution level decreases, just like in the no-restoration case.*

If $A_0 < A_e$, then the optimal path will increase the assimilative capacity up to A_e if $\bar{A} \ge A_e$. It is thus optimal for the social planner to restore the assimilative capacity up to A_e. Along such a restoration path, the shadow price of assimilative capacity decreases while the optimal level of pollution, initially strictly lower than the assimilative capacity level, increases.

It must be noted that if $\bar{A} < A_e$, it is then optimal to immediately deplete the assimilative capacity until extinction in order to get the maximum social benefit. Indeed it cannot be optimal in a discounted framework to restore A up to $\bar{A}$ and then start depleting it again since the steady state cannot be reached.

The restoration option, when the maximum restoration threshold is sufficiently high, can free the economy from the depletion path imposed by low initial conditions.

The sensibility to the discount rate remains exactly the same as the comparative statics analysis showed in the no-restoration case.

5. Policy interpretation of the results

5.1. *Stricter regulation to prevent vicious environmental degradation cycles*

Proposition 2 shows, in both cases, that the dynamic pollution optima ought to be stricter than those obtained under classic static optimization. This result highlights the need to take into considerations the dynamics of the assimilative capacity in order to avoid the vicious environmental degradation cycle

[15] Given the decreasing returns of restoration and the convexity of the degradation function, it would be inconsistent to let the polluter degrade the assimilative capacity while restoring it through costly investments at the same time.

described in the Introduction. Although social and political factors might incline the policymaker to guarantee a steady level of production/pollution, if this level is in significant excess of the ecosystem's assimilative capacity it will trigger a degradation cycle that will affect society as a whole. Considering this threat, it is crucial to calibrate the economic instruments of environmental regulation with a set of dynamic optima.

5.2. Shadow price and optimal pigouvian tax

The results stated in Propositions 3 and 4) are very similar to the fundamental result found in the seminal literature on optimal environmental quality (Barbier and Markandya, 1990) and optimal resource extinction (Cropper et al., 1979). Low initial stocks can justify an optimal extinction of the resource and a high discount rate reduces the steady-state level. Assuming the initial conditions are met, our analysis allows the determination of the shadow price of the assimilative capacity λ as a means ensuring that the economy will follow the optimal path leading to the steady state. As such, this shadow price plays an important role in setting the pigouvian tax that could implement the optimal pollution policy. Whereas a tax in a standard flow pollution problem would be based exclusively on the marginal flow damage, in our dynamic framework this tax needs to internalize the detrimental effect of assimilative capacity depletion.

Another interesting result of our model is that when the restoration option is available it can be optimal to initiate an optimal path with net restoration. Our model thus outlines the fact that the optimality of restoring the assimilative capacity instead of depleting it is tantamount to the optimality of investing in (natural) capital instead of depleting it. In this case, future flows of services from this capital stock more than offset the short-term benefit of consuming it beyond its regeneration threshold. From this standpoint, we suggest that our model helps to restore symmetry between natural and physical capital in the standard economic analysis framework that tends to treat them in an asymmetrical manner (Godard, 2006).

6. Conclusion

The main contribution of this work is to build a stylized model that more precisely accounts for the ecological processes at work in flow pollution problems and to draw some policy conclusions in terms of environmental regulation. Our model draws attention to the necessary inclusion of the environment's regenerative conditions in economic frameworks that tend to ignore them. Furthermore, we show that the introduction of the restoration process of the assimilative capacity enables the economy to avoid extinction paths linked to a low initial capacity. An interesting extension of this model is to introduce uncertainty into the dynamics of the assimilative capacity. Although a vast body of literature addresses optimal pollution within a stochastic frameworks (uncertainty on the intensity of damages or on the evolution of abatement technologies), only Heal (1984) treats the assimilative capacity level itself as uncertain. Our original specification of assimilative capacity as a state variable provides an promising framework in which to explore this issue and enables us to apply uncertainty to both the initial level of assimilative capacity available A_0 and the degradation function h.

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Appendix A

We assume that we have $U_{pp}U_{AA}-U_{pA}^2\geq 0$ to ensure the concavity of the Hamiltonian, *i.e.* $(f_{pp}-D_{pp})D_{AA}-D_{pA}^2\geq 0$. This property can be proved easily for a simple functional form such as $D(p, A)=D(p-A)$. Under this specification we can show, after simplifications, that

$$U_{pp}U_{AA}-U_{pA}^2 = -D''f''+D''^2-D''^2 = -D''f''\geq 0.$$

Appendix B

According to Eq. (18):

$$\dot{\lambda}=0\Rightarrow\lambda(\delta+h_A)+D_A=0.$$

Let us have the function $M(A, \lambda)$ such that $M(A, \lambda)=\lambda(\delta+h_A)+D_A$. The theorem of implicit functions gives us

$$\frac{d\lambda}{dA}\Big|_{\dot{\lambda}=0} = -\frac{M_A}{M_\lambda} = -\frac{\frac{dp(\lambda,A)}{dA}\left(D_{Ap}+\lambda h_{Ap}\right)+\lambda h_{AA}+D_{AA}}{\delta+h_A+\frac{dp(\lambda,A)}{d\lambda}\left(\lambda h_{Ap}+D_{Ap}\right)}.$$

From Eq. (6) and the properties of h and D, we can easily show that when $p>A$, $\lambda h_{Ap}+D_{Ap}=\lambda h_{pA}+D_{pA}f_{pA}=0$. When $p\leq A$, according to the properties of h and D, we know that $h_{Ap}=D_{Ap}=0$. Hence for any p and A along the optimal path we have

$$\frac{dp(\lambda,A)}{d\lambda}\left(\lambda h_{Ap}+D_{Ap}\right)=0.$$

As the denominator of the right-hand side of expression (30) is positive, we can write

$$\operatorname{sgn}\left(\frac{d\lambda}{dA}\Big|_{\dot{\lambda}=0}\right) = -\operatorname{sgn}(\delta+h_A).$$

Applying the same method to Eq. (19) gives us

$$\frac{d\lambda}{dA}\Big|_{\dot{A}=0} = -\frac{\frac{dp(\lambda,A)}{dA}h_p+h_A}{\frac{dp(\lambda,A)}{d\lambda}h_p}$$

and since $h_p = -h_A$ for any (A, p), we can write

$$\frac{d\lambda}{dA}\Big|_{\dot{A}=0} = \frac{\left(1 - \frac{dp(\lambda,A)}{dA}\right)h_p}{\frac{dp(\lambda,A)}{d\lambda}h_p}.$$

We can easily show that $\frac{dp(\lambda,A)}{dA} < 1$ for any A as Eq. (17) leads to a contradiction with the initial assumptions if $\frac{dp(\lambda,A)}{dA} \geq 1$. Hence

$$\frac{d\lambda}{dA}\Big|_{\dot{A}=0} < 0.$$

Appendix C

No-restoration case.

Behavior of I_λ.

Let us determine the y-axis and x-axis intercepts of I_λ. To estimate the y-axis intercept of I_λ, we must study the behavior of Eq. (8) when A tends towards 0:

$$\lim_{A\to 0} \lambda|_{\dot{\lambda}=0}$$

From the properties of h_A and D_A applied to Eq. (8) when A tends towards 0, we get

$$\lim_{A\to 0} \lambda|_{\dot{\lambda}=0} = \lim_{A\to 0} -D_A = L.$$

For the x-axis intercept, we define $p_\lambda(\lambda,A)^{-1}$ as the inverse function of $p(\lambda,A)$ for A given. Let us choose $\lambda = p_\lambda(\lambda,A)^{-1}(x_p)$. Eq. (8) with $p = p(\lambda,A)$ yields

$$\begin{aligned}\dot{\lambda} &= p_\lambda(\lambda,A)^{-1}(x_p)\left(\delta + h_A\left(p_\lambda(\lambda,A)^{-1}(x_p)\right), A\right)) \\ &+ D_A(p\left(\left(p_\lambda(\lambda,A)^{-1}, A\right)(x_p)\right) = p_\lambda(\lambda,A)^{-1}(x_p)\delta - \left(h_p(x_p, A)\right) \\ &+ D_A(x_p, A)) = p_\lambda(\lambda,A)^{-1}(x_p)\delta - f_p(xp) = p_\lambda(\lambda,A)^{-1}(x_p)\delta.\end{aligned}$$

On the isocline, this writes

$$\dot{\lambda} = p_\lambda(\lambda,A)^{-1}(x_p)\delta = 0$$

so

$$p_\lambda(\lambda,A)^{-1}(x_p) = \lambda = 0$$

and

$$\lambda|_{\{\dot{\lambda}=0\}\cap\{p((\lambda = x_p, A)\}} = 0.$$

The x-axis intercept of I_λ is x_p.

Behavior of I_A.

Given the economic meaning of λ, $\lambda = 0$ on the optimal path is equivalent to a situation where a marginal variation of A has no effect on the welfare. Taken at $A = p = x_p$, this would mean (see Eq. (7)) that

$$D_p(x_p, x_p) = -D_A(x_p, x_p) = 0. \quad \text{(A.1)}$$

The interpretation of this condition is that for a sufficiently high level of assimilative capacity ($A \geq x_p$ for example), a marginal variation of A or p, Δ_A or Δ_p will not trigger additional damage and since it will not modify the benefit function either ($f_p(x_p) = 0$ by definition), it will have no effect on the welfare and therefore its shadow value $\lambda\Delta_A$ will be nil.

- If (A.1) is not respected, I_A does not cross the x-axis at x_p, and in our restricted definition set $[0, x_p[$ it is always above the x-axis.
- If (A.1) is respected, then I_A crosses the x-axis at x_p and the two isoclines intersect at two equilibrium candidates (A_e, λ_e) and x_p. But since x_p can never be reached in the irreversible configuration, it boils down to the first case.

Regarding the y-axis intercept, since the limit of D_A when A tends towards 0 is finite (Eq. (3)), there is no reason for $\lambda|_{\dot{A}=0}$ to tend towards infinity when A tends towards 0, contrary to what happens with the I_λ curve. Therefore the y-axis intercept of I_A is finite.

Restoration case.

The geometrical properties are identical to those in the previous case. If condition (A.1) is not met, the two isoclines intersect at a unique equilibrium candidate (A_e, λ_e) that is a saddle point. If condition (A.1) is met the two isoclines intersect at two equilibrium candidates (A_e, λ_e) and x_p. In the reversible configuration, it is not impossible *a priori* to have $A(t) = x_p$. However the phase-diagram shows that given the motion vectors defined by the isoclines around x_p, x_p cannot be on an optimal path and it boils down to the first case again.

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