Buzz Management

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Abstract

We study how a firm uses structured information release, word of mouth marketing and advertising to maximize the diffusion of information about its product. Individuals are either a high or a low social type. During social interactions it is valuable for any individual to increase another person's posterior that they are a high social type. We develop a model to study how a firm interacts with this motive for maximizing the diffusion of information. We find that a firm will increase the cost for low types to acquire information and restrict the ability of low types from passing on information. A commitment by the firm not to advertise is beneficial which may take the form of releasing information a sufficient amount of time before the product is released.

Keywords: Word of mouth, advertising, buzz, diffusion.

VERY PRELIMINARY DRAFT: DO NOT CITE WITHOUT PERMISSION

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1 Introduction

In July of 2011, the European music site Spotify launched in the US. At first, its free version was available by invitation only.[1] Obtaining the invitation was not trivial: consumers could receive either through current users or through other channels. For example, Coca Cola,[2] gave out invitations to users who submitted their email address, and interacting with Spotify over Twitter could also result in an invitation. Eventually, anyone could download the free version of Spotify through the company's Web site. By November of 2011, 4 million users adopted Spotify.[3] Interestingly, in the same month Google launched its new social network site, Google+, using the same invitation-only method. Users could join either by receiving an invitation from Google or by receiving one from a friend. For a brief period in the summer of 2011, receiving an invitation from Google+ became a status symbol. Here is what Ken Hess wrote on the tech news site ZDNet on June 30, 2011, "Dear Google, I want a Google+ account. I'm an avid Googler and have always been an early adopter of all things Google. Please give me an account before you give them to anyone else on my list so I can gain some real street cred with my fellow ZDNetters. Come on, I really want one."[4] On October 2011, Google announced Google+'s user base to be at more than 40 million users.[5] Given the high rate of adoption for both of these sites, we can speculate that the initial scarcity successfully generated high amounts of word of mouth and invitations for these sites.

The tactic of using limited release of information to spur word of mouth has of course been around prior to July of 2011. For example, Hughes (2005) states, "Sometimes withholding can work better than flooding. Limit supply and everybody's interested. Limit those in the know of a secret, those not in the know want the currency of knowing - they want to be part of the exclusive circle." Hence, in the nineties, Ty used the exclusivity in its inital launch of bean-stuffed toys[6].

According to industry research, 54% of purchase decisions are influenced by word of mouth.[7] This insight has made the management of word of mouth a priority for marketing managers. The effect of word of mouth on sales has received a fair amount of attention over the past decade: see Godes and Mayzlin (2004), Chevalier and Mayzlin (2006), Chintagunta, Gopinath, and Venkataraman (2010), and Luca (2011). However, the issue of what motivates consumers to spread information has received less attention. For a firm trying to manage word of mouth, understanding how these motivations impact marketing strategy is essential.

[1] http://www.nytimes.com/2011/07/14/technology/spotify-music-streaming-service-comes-to-us.html

[2] http://news.cnet.com/8301-13845_3-20081418-58/get-a-quick-and-easy-invitation-to-spotify/

[3] http://articles.latimes.com/2011/nov/10/business/la-fi-ct-facetime-spotify-20111110

[4] http://www.zdnet.com/blog/btl/dear-google-where-the-hell-is-my-google-invitation/51640

[5] http://www.pcmag.com/article2/0,2817,2398114,00.asp

[6] See Dye (2000)

[7] Word of Mouth Marketing Association infographic: http://womma.org/word/wp-content/uploads/2011/10/word_of_mouth_marketing_impact_and_influence_womma.png

In this paper we focus on a particular motive behind the spread of word of mouth as self-enhancement (Baumeister et al. 1989) or the idea that people talk to make themselves look good in a social setting. We develop a model in which to study how a firm may structure its information release and advertising strategies to influence consumers' incentive to both acquire and pass on information. We find that if the initial spread of information about the product is asymmetric, in that the high-status consumers are more likely to initially obtain the information, word of mouth can serve as a credible signal of consumer's status. Consumers thus have an incentive to spread the information. as the information spreads through the population the signaling benefit of the information deteriorates over time (as more consumers become aware of the information), at some point consumers choose to stop talking about the product. Furthermore we find that a firm may increase the total diffusion of information by restricting who can pass on information about the product and through a credibly commitment not to undertake adverstising during the diffusion.

We highlight a basic trade-off faced by marketers who are trying to maximize the spread of information about the product: while confining the initial spread of information to the high status group maximizes the incentive to talk on the part of the each exposed consumer, the diffusion process takes a longer amount of time since the number of exposed individuals is small early on. We also show that advertising decreases the asymmetry of information across the different status groups, and hence decreases the incentive to talk on the part of the consumer.

Marketers are often urged to reach out to opinion leaders since they are more likely to persuade others to adopt the product. We point out an additional benefit of reaching out to an exclusive group of opinion leaders early on: if that group is higher-status than the rest of the population, the fact that the firm reaches out exclusively to them makes them more motivated to talk. Finally, our result on advertising displacing the incentive to talk explains why companies who try to generate buzz often wait for several months before conducting a mass advertising campaign: in the case of Mountain Dew Code Red, early on the company engaged in actions such as reaching out to influentials and organizing parties at college campuses, and did not engage in mass advertising until a few months later.[8]

2 Literature Review

Our paper deals with the incentives of consumers to talk, and how the firm's information-release strategy and advertising affect these incentives. In particular, we assume that consumers may derive benefits during social interactions from making themselves look good. Our model is motivated by the psychological theory of "self-enhancement" or the "tendency to affirm the self" (Baumeister 1998). In particular, we focus on the role of word of mouth conversations signaling the sender's high status.

Two recent papers provide indirect empirical evidence that word of mouth is influenced

[8] Kotler (2002).

by motives related to self-presentation. Berger and Milkman (2011) find that positive content is more likely to be shared, as is content that evoked high-arousal emotions. The authors speculate that the sharing of positive content may be due to impression-management. Berger and Schwartz (2011) find that in the short run conversations are influenced by how interesting the product is: consumers do not want to appear to be dull.

More direct empirical evidence of self-enhancement as a motive behind word of mouth has been demonstrated in several studies: In a survey conducted by Hennig-Thurau, et. al. (2004) respondents indicate self-enhancement as one of the primary motivation behind WOM. Sundaram, Mitra and Webster (1998) infer that self-enhancement accounts for 20% of positive word of mouth, and Wojnicki and Godes (2011) show in a series of experiments that experts are less likely to talk about their negative experiences since a negative outcome reflects badly on their ability to make choices (the experts want to signal success). Also, Berger and Heath (2007) find that consumers are more likely to diverge from majorities in product categories that consumers relate to identity (such as music or fashion, for example). Note that while the first three studies deal directly with word of mouth, the last study deals with observational learning, which is of course relates to word of mouth since both are forms of consumer interactions.

The current model studies how a firm's information release and advertising strategy interacts with social interactions between consumers. It is related to Pesendorfer (1995) where the focus is on the interaction between consumers' social matching behavior and the pricing strategy and design innovations of a firm. In that model owning a particular product serves as a signal for a consumer to others that they are a high social type. This is valuable for high types to identify one another during a matching process in order to ensure they are matched with another high type. The premise that there are socially desirable types and consumers may benefit from interacting with high types, and thus the ability to signal their types in a social context does connect both the models. However our focus is very different as we consider how a firm's information release and advertising strategy interacts with social concerns.

Similarly to our work, several recent papers have analytically modeled the effect of the desire to signal to others on word of mouth (Wojnicki and Godes 2011) or the firm's communication strategy (Yoganarasimhan 2012).[9] Wojnicki and Godes (2011) present a model where the desire for impression management motivates experts to suppress conversations about their negative experiences. A major differencs between our work and Wojnicki and Godes (2011) is the nature of the signal: while in Wojnicki and Godes (2011) the valence of experts' experiences signals their ability to choose, in our model the signal involves early access to information. Our work also relates to Yoganarasimhan (2012), which models a fashion firm's desire to withhold the identity of its "hottest" product in order to enable consumers to signal to each other that they are in "the know" in a static setting. Besides

[9]Kuksov (2007) also studies the incentives of consumers to reveal or conceal information about themselves to others through brand choices in the consumer matching context.

the differences in context - while Yoganarasimhan (2012) studies fashion goods, our model concerns itself with all products - one notable difference in the models is our focus on the effect of initial exclusive release on the extent of diffusion in a dynamic setting.

3 Model

3.1 A Model of Buzz

Suppose that a monopolist is selling a product to a mass 1 of agents. An agent i may be one of two types: high and low $\theta = h, l$ where $\Pr[\theta_i = h] = \phi < \frac{1}{2}$ where high types are relatively scarce. We can alternately think of these as high- and low-status consumers. For example, Google+ supposedly targeted opinion leaders or experts (high-status) with its early invitations, which is one reason that people were so desperate for an invitation. Similarly, the tech-savvy heavy users of Twitter were able to get a Spotify invitation early but the consumers who did not use Twitter were not able to obtain an invitation.

The firm's objective is to maximize the fraction of the population which receives information m about its product. We shall denote the fraction of the population that has received the information by S. Initially we assume no advertising before introducing it in Section 4.2. Without advertising, consumers can find out about the product in two ways. First, they may undertake costly search to learn about the product themselves. Second, they may costlessly hear about the product from another person who themselves will incur a cost to pass the information to them. Note once an agent has found out about the product through either channel, they are able to pass on information about it themselves. In the case of Spotify and Google+, a consumer could only learn about the product by using it. However, this generally may not be the case. We first consider the general setting where anyone can pass on information and then consider in Section 4.1, how a firm can increase the spread of information by restricting who can pass along information as it may be able to do if adopting the product is required in order to learn the information or credibly pass on the information (in the sense that it does not perfectly reveal the individual as someone who heard the information from somoeone else).

Timing

At time $t = -1$ each type chooses whether to obtain information m about a firm's product. We assume that this information is hard and verifiable: a consumer is not able to fabricate information. The costs for obtaining information for each type, c_h, c_l, are distributed uniformly on $[0, \bar{c}]$. However, the firm can costlessly (or at very small cost) increase but not decrease the costs for each type. For example, the firm can put the information about its upcoming product releases on a technical blog that is frequented only by category experts or it can email some consumers with information about the product. Again, using the Spotify example, the firm could allow anyone who registered on its website to obtain an invitation or it could only send an invitation to those who interacted with it on Twitter.

From time $t = 0$ onwards individuals mix at rate λ. During each meeting an individual may pass on the hard information m at a cost k, where we assume $\phi < k < 1 - \phi$, or pass on no information $\varnothing$ at zero cost. We assume that this is done simultaneously during the meeting so that each individual has the ability to do so without seeing the other individuals information first. This assumption makes the analysis more tractable.

Social Utility

The motivation for our analysis is that individuals derive a benefit from word of mouth due to "self-enhancement." We capture this idea through a social utility U_{ij}, that an individual i receives from an interaction with another individual j, where the utility is an increasing function of the beliefs the other agent has about the agent's type. In particular, agent i receives utility

$$U_{ij}\left(b_j\left(\theta_i = h|m, t\right)\right) \tag{1}$$

if agent i passes a message m at time t, where $b_j\left(\theta_i = h|m, t\right)$ is the other agent $j's$ belief that agent i is a high type upon receiving the information m. And similarly,

$$U_i\left(b_j\left(\theta_i = h|\varnothing, t\right)\right) \tag{2}$$

if the agent does not pass information, where $b_j\left(\theta_i = h|\varnothing, t\right)$ is the belief if no signal (denoted by $\varnothing$) is sent. Given our notions of high and low types we assume

$$\frac{dU}{db_j} > 0$$

Also note the signaling benefit at a time t is

$$\Delta U\left(t\right) = U_i\left(b_j\left(\theta_i = h|m, t\right)\right) - U_i\left(b_j\left(\theta_i = h|\varnothing, t\right)\right) \tag{3}$$

which is the difference between sending a signal and not sending a signal at that time t. We assume that utility is linear in beliefs thus

$$\begin{aligned} & U\left(b_j\left(\theta_i = h|m, t\right)\right) - U\left(b_j\left(\theta_i = h|\varnothing, 0\right)\right) \\ & = \overline{u}\left[b_j\left(\theta_i = h|m, t\right) - b_j\left(\theta_i = h|\varnothing, t\right)\right] \end{aligned} \tag{4}$$

where we normalize $\bar{u} = 1$. A simple model of this reduced form model is to assume the utility to an individual i from an interaction with another individual j is given by $U_{ij} = 1\left[\theta_j = h\right] e_i + e_j - \frac{e_i^2}{2}$ where e_i and e_j are investments made by each individual at personal cost $\frac{e^2}{2}$. Expected utility maximization results in an individual making an investment equal to the posterior about the other agent's type $e_j = \Pr_j\left[\theta_i = h\right]$. A individual i may increase the posterior of individual j through the message she sends $\Pr_j[\theta_i = h|m] > \Pr_j[\theta_i = h|\varnothing]$ and thus increases the investment e_j the agent makes.

Growth of the informed population

We now map the incentive to talk to the size of the informed population. Denote the fraction of types who become informed at $t = -1$ by φ_h, φ_l where these are going to be endogenously determined in equilibrium. The fraction of the population which are informed $S(t)$ evolves over time as agents mix at rate λ and pass on information. The initial condition is $S_0 = \varphi_h \phi + \varphi_l (1-\phi)$ and the rate of change of the informed population is given by:

$$\frac{dS}{dt} = \lambda S(t)(1 - S(t)) \tag{5}$$

This results in the following path for $S(t)$:

$$S(t) = \frac{1}{1 + ae^{-\lambda t}}, \text{ where } a = \frac{1 - S_0}{S_0} \tag{6}$$

which continues to grow until the beneficial impact of passing the firm's message is less than the cost of doing so. Hence $S(t)$ stops growing when $\Delta U(t^*) = k$ which defines the extent of the diffusion $S^* = S(t^*)$. In other words, the consumer stops talking when the signaling benefit of information . We assume the firm's profits are positively related to the extent of the diffusion and it will endeavor to maximize the extent of this diffusion.

Beliefs

At $t = 0$ beliefs are

$$b_j(\theta_i = h|m, 0) = \frac{\varphi_h \phi}{\varphi_h \phi + \varphi_l (1-\phi)} \tag{7}$$

and

$$b_j(\theta_i = h|\varnothing, 0) = \frac{(1-\varphi_h)\phi}{(1-\varphi_h)\phi + (1-\varphi_l)(1-\phi)}. \tag{8}$$

Beliefs change over time as the message diffuses through the population. The belief when a person receives a signal at a time t is given by

$$\begin{aligned} b_j(\theta_i = h|m, t) &= \frac{S(t) - S_0}{S(t)}[b_j(\theta_i = h|\varnothing, 0)] + \frac{S_0}{S(t)}[b_j(\theta_i = h|m, 0)] \\ &= b_j(\theta_i = h|\varnothing, 0) + \frac{S_0}{S(t)}[b_j(\theta_i = h|m, 0) - b_j(\theta_i = h|\varnothing, 0)]. \end{aligned} \tag{9}$$

The beliefs upon not receiving a signal do not change over time and hence are given by

$$b_j(\theta_i = h|\varnothing, t) = b_j(\theta_i = h|\varnothing, 0) = \frac{(1-\varphi_h)\phi}{(1-\varphi_h)\phi + (1-\varphi_l)(1-\phi)}. \tag{10}$$

Extent of diffusion

The diffusion of the signal thus stops when the marginal value of signaling equals the marginal cost of passing on the information:

$$\begin{aligned} & U\left(b_j\left(\theta_i=h|m,t\right)\right)-U\left(b_j\left(\theta_i=h|\varnothing,t\right)\right) \\ & =\frac{S_0}{S\left(t\right)}\left[b_j\left(\theta_i=h|m,0\right)-b_j\left(\theta_i=h|\varnothing,0\right)\right]=k \end{aligned} \tag{11}$$

Hence, the diffusion stops at

$$S^*=\frac{S_0}{k}\left[b_j\left(\theta_i=h|m,0\right)-b_j\left(\theta_i=h|\varnothing,0\right)\right] \tag{12}$$

We can now replace $b_j\left(\theta_i=h|m,0\right)$ and $b_j\left(\theta_i=h|\varnothing,0\right)$,

$$\begin{aligned} S^*\left(\varphi_h,\varphi_l\right) &= \frac{1}{k}\left(\varphi_h\phi+\varphi_l\left(1-\phi\right)\right)\left[\frac{\varphi_h\phi}{\varphi_h\phi+\varphi_l\left(1-\phi\right)}-\frac{\left(1-\varphi_h\right)\phi}{\left(1-\varphi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)}\right] \\ &= \frac{\phi}{k}\left[\varphi_h-\left(1-\varphi_h\right)\frac{\varphi_h\phi+\varphi_l\left(1-\phi\right)}{\left(1-\varphi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)}\right] \end{aligned} \tag{13}$$

The derivative of S^* with respect to φ_h,φ_l are:

$$\begin{aligned} \frac{dS^*}{d\varphi_h} &= \frac{1}{k}\left[\begin{array}{c} 1-\frac{(1-\varphi_h)\phi}{(1-\varphi_h)\phi+(1-\varphi_l)(1-\phi)}+ \\ \frac{\varphi_h\phi+\varphi_l(1-\phi)}{(1-\varphi_h)\phi+(1-\varphi_l)(1-\phi)}-\frac{\phi(1-\varphi_h)(\varphi_h\phi+\varphi_l(1-\phi))}{((1-\varphi_h)\phi+(1-\varphi_l)(1-\phi))^2} \end{array}\right] \\ &= \frac{1}{k}\left[\frac{\left(1-\varphi_l\right)\left(1-\phi\right)}{\left(\left(1-\varphi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)\right)^2}\right]\geq 0 \text{ if } \varphi_l<1 \text{ then } >0 \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{dS^*}{d\varphi_l} &= -\frac{\phi}{k}\left(1-\varphi_h\right)\left[\frac{\left(1-\phi\right)\left[\left(1-\phi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)+\varphi_h\phi+\varphi_l\left(1-\phi\right)\right]}{\left(\left(1-\varphi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)\right)^2}\right] \\ &= -\frac{\phi}{k}\frac{\left(1-\varphi_h\right)\left(1-\phi\right)}{\left(\left(1-\varphi_h\right)\phi+\left(1-\varphi_l\right)\left(1-\phi\right)\right)^2}\leq 0 \text{ if } \varphi_h<1 \text{ then } <0 \end{aligned} \tag{15}$$

Without considering the ex ante incentives for consumers to acquire information at $t=-1$, the diffusion of the firm's message is increasing in the number of high type infomred consumers (φ_h) and decreasing in the number of low type informed customers (φ_l). It is, therefore, easy to see that the diffusion of information is maximized at $\varphi_h=1$ and $\varphi_l=0$ in the absence of ex ante incentives for consumers to acquire information. However this may not be a feasible solution if the ex ante information acquisition constraints bind for some high types.

Ex Ante Incentives

Consumers decide whether to acquire information at $t = -1$ (for example, to scour the Internet for the Spotify invitation). The decision to acquire the information at $t = -1$ depends on the total signaling benefit the agent will acquire during the diffusion process. If this benefit is above an individual's cost c then the agent will acquire information. Denoting the time at which the diffusion process ends by t^*. The signaling benefit for an agent is then

$$V = \lambda \left(\int_0^{t^*} \left(\frac{1 - S(t)}{1 - S_0} \right) \left(\frac{S_0}{S(t)} \left[b_j \left(\theta_i = h | m, 0 \right) - b_j \left(\theta_i = h | \varnothing, 0 \right) \right] - k \right) dt \right) \tag{16}$$

where $\frac{1-S(t)}{1-S_0}$ is the probability of remaining uninformed at time t for an individual uninformed at time 0. It may be further simplified by making a change of variable using

$$\frac{dS}{dt} = \lambda S(t) \left(1 - S(t)\right)$$
$$dt = \frac{dS}{\lambda S(t) \left(1 - S(t)\right)}$$

making this substitution for dt and substituting $\triangle b = b_j \left(\theta_i | m, 0\right) - b_j \left(\theta_i | \varnothing, 0\right)$, where $\triangle b = \frac{S^*}{S_0} k$ (from Equation 12), we get:

$$\begin{aligned} V &= \int_{S_0}^{S^*} \left(\frac{1}{1 - S_0} \right) \frac{1}{S(t)} \left(\frac{S_0}{S(t)} \left[b_j \left(\theta_i = h | m, 0 \right) - b_j \left(\theta_i = h | \varnothing, 0 \right) \right] - k \right) dS \qquad (17) \\ &= \frac{1}{1 - S_0} \int_{S_0}^{S^*} \left[\frac{\triangle b \cdot S_0}{S^2(t)} - \frac{k}{S(t)} \right] dS = \left(\frac{k}{1 - S_0} \right) \left(S^* \left[-\frac{1}{S} \right]_{S_0}^{S^*} - \left[\ln S \right]_{S_0}^{S^*} \right) \\ &= \left(\frac{k}{1 - S_0} \right) \left(\frac{S^* - S_0}{S_0} + \ln \frac{S_0}{S^*} \right) \end{aligned}$$

The total signaling benefit is a function of the initial diffusion state (S_0) and the total extent of information diffusion (S^*).

Lemma 1. *The total signaling benefit is increasing in S^* and decreasing in S_0:*

$$\frac{\partial V}{\partial S_0} < 0 \text{ and } \frac{\partial V}{\partial S^*} > 0.$$

Proof. The first order conditions of V with respect to S^* and S_0 are:

$$\frac{\partial V}{\partial S^*} = \left(\frac{k}{1-S_0}\right)\left[\frac{1}{S_0} - \frac{1}{S^*}\right],$$

$$\frac{\partial V}{\partial S_0} = \left[\begin{array}{c} k\left(\frac{1}{1-S_0}\right)^2\left(S^*\frac{1}{S_0} - 1 + \ln S_0 - \ln S^*\right) \\ +\left(\frac{k}{1-S_0}\right)\left(-\frac{1}{S_0^2}\frac{BS_0}{k} + \frac{1}{S_0}\right) \end{array}\right]$$

$$= \left[\frac{k}{(1-S_0)^2 S_0^2}\left\{(S^* - S_0)(2S_0 - 1) + S_0^2\left(\ln\frac{S_0}{S^*}\right)\right\}\right].$$

We have that $\left(\frac{k}{1-S_0}\right)\left[\frac{1}{S_0} - \frac{1}{S^*}\right] > 0$. Hence, $\frac{\partial V}{\partial S^*} > 0$.

Next, we show that $\frac{\partial V}{\partial S_0} < 0$. Note that it is immediate that $\frac{\partial V}{\partial S_0} < 0$ when

$$2S_0 - 1 < 0,$$

which is true for $S_0 < \frac{1}{2}$.exist

If $S_0 \geq \frac{1}{2}$, we need that

$$(S^* - S_0)(2S_0 - 1) + S_0^2\left(\ln\frac{S_0}{S^*}\right) < 0$$

$$\Leftrightarrow S_0^2\left(\ln\frac{S_0}{S^*}\right) < (S^* - S_0)(1 - 2S_0)$$

$$\Leftrightarrow \frac{\ln\frac{S_0}{S^*}}{\frac{S^*}{S_0} - 1} < \left(\frac{1}{S_0} - 2\right).$$

Now consider the left hand side, where $x = \frac{S^*}{S_0} > 1$.

$$\frac{\ln\frac{1}{x}}{x-1} = \frac{-\ln x}{x-1}$$

$$\Leftrightarrow \frac{d\left(\frac{-\ln x}{x-1}\right)}{dx} = \frac{1}{x-1}\left(\frac{\ln x}{x-1} - \frac{1}{x}\right)$$

which is greater than 0 for $x > 1$ if

$$\frac{\ln x}{x-1} - \frac{1}{x} > 0$$

$$\Leftrightarrow \ln x > 1 - \frac{1}{x}$$

which is known relation for the natural log. Hence, the left-hand side of the above is increasing in $\frac{S^*}{S_0}$ and an upper-bound on the left-hand side is given by $-\frac{\ln\frac{1}{S_0}}{\frac{1}{S_0}-1}$ and we need only check that

$$-\frac{\ln \frac{1}{S_0}}{\frac{1}{S_0}-1} < \frac{1}{S_0} - 2$$
$$-\ln y - (y-2)(y-1) < 0, \text{ where } y = \tfrac{1}{s_0}.$$

And now we show that it is a decreasing function of y for $1 \leq y \leq 2\,(\leftrightarrow \frac{1}{2} \leq S_0 \leq 1)$

$$\begin{aligned}\frac{d\left(-\ln y - (y-2)(y-1)\right)}{dy} &= -\frac{1}{y} - 2y + 3 \\ &= \frac{-2y^2+3y-1}{y} \\ &= \frac{(1-2y)(y-1)}{y} \\ &< 0 \text{ for } 1 \leq y \leq 2\end{aligned}$$

and note that

$$\lim_{y \to 1}\left[-\ln y - (y-2)(y-1)\right] = 0.$$

Hence, $-\ln y - (y-2)(y-1) < 0$, which shows that $\frac{\partial V}{\partial S_0} < 0$. □

The lemma shows that the total benefit of signaling from passing the firm's message is increasing in S^*. As the information will spread greater portion of population, the more benefit the consumer can enjoy over longer period time. Interestingly, the lemma also finds that the more people are informed about the message at the initial stage, the less benefit the consumer gains from acquiring information. **AC - not sure we have the right intuition here. Somewhat strange comparative static as we hold S_0, or S^* fixed while doing a comparative static on the other these are jointly determined so it makes it hard to interpret that comparative static really means. An alternative might be to do comparative statics on k and $\bar{c}$.

Firm's problem

We assume that the firm can increase the costs of the high and low types for acquiring information. Denote the amount by which the firm increases the cost of each type by $\tau_l \geq 0$ and $\tau_h \geq 0$. The cutoff type for each is then given by

$$\varphi_h \bar{c} + \tau_h \leq V(\varphi_h, \varphi_l)$$
$$\varphi_l \bar{c} + \tau_l \leq V(\varphi_h, \varphi_l)$$

This is equivalent to the firm choosing φ_l and φ_h subject to the feasibility constraints

$$\varphi_h \bar{c} \leq V(\varphi_h, \varphi_l)$$
$$\varphi_l \bar{c} \leq V(\varphi_h, \varphi_l)$$

$$0 \leq \varphi_h \leq 1$$
$$0 \leq \varphi_l \leq 1$$

We can now write the firm's optimization problem as maximizing the spread of information through the choice of φ_l and φ_h subject to these feasibility constraints:

$$\max_{\varphi_h, \varphi_l} S^* \left(\varphi_h, \varphi_l\right)$$

subject to

$$\varphi_h \bar{c} \leq V \left(\varphi_h, \varphi_l\right)$$
$$\varphi_l \bar{c} \leq V \left(\varphi_h, \varphi_l\right)$$

$$0 \leq \varphi_h \leq 1$$
$$0 \leq \varphi_l \leq 1$$

Proposition 1. *The optimal strategy for the firm has the following characteristics:*

$$0 < \varphi_h \leq 1 \text{ and } \varphi_l = 0.$$

Moreover,

$$\varphi_h < 1 \text{ if } \bar{c} \geq \frac{1 - k + k \ln k}{1 - \phi}$$
$$\varphi_h = 1 \text{ if } \bar{c} < \frac{1 - k + k \ln k}{1 - \phi}$$

Proof. First, consider V as a function of S_0 and S^*, $V\left(S^*, S_0\right) = \left(\frac{k}{1-S_0}\right)\left(\frac{S^*-S_0}{S_0} + \ln \frac{S_0}{S^*}\right)$. Taking the derivative $\frac{dV}{d\varphi_l}$,

$$\frac{dV}{d\varphi_l} = \frac{\partial V}{\partial S_0}\frac{dS_0}{d\varphi_l} + \frac{\partial V}{\partial S^*}\frac{dS^*}{d\varphi_l}.$$

We note that $\frac{\partial S_0}{\partial \varphi_l} > 0$, $\frac{dS^*}{d\varphi_l} \leq 0$ (equation 15), $\frac{\partial V}{\partial S_0} < 0$ and $\frac{\partial V}{\partial S^*} > 0$ (from Lemma 1). Hence,

$$\frac{dV}{d\varphi_l} = \underbrace{\frac{\partial V}{\partial S_0}}_{-}\underbrace{\frac{dS_0}{d\varphi_l}}_{+} + \underbrace{\frac{\partial V}{\partial S^*}}_{+}\underbrace{\frac{dS^*}{d\varphi_l}}_{-} < 0.$$

We already know that see S^* is maximized when $\varphi_h = 1$, independent of φ_l. However, this may not be a feasible solution if the ex ante information acquisition constraints bind for some high types. When $\varphi_h < 1$, we have that $\frac{dS^*}{d\varphi_l} < 0$. Hence, the optimal policy will result in $\varphi_l = 0$ if φ_l does not help alleviate the ex ante incentives for high type consumers to acquire information $\frac{dV}{d\varphi_l} < 0$, and $V(\varphi_h, 0) \geq 0$ for $\forall \varphi_h \geq 0$.

Finally, $0 \leq S_0(\varphi_h^*, 0) \leq S^*$ and $\frac{dV}{dS_0} < 0$.

$$\begin{aligned}\lim_{S_0 \to 0} V &= \lim_{S_0 \to 0} \left(\frac{k}{1-S_0}\right)\left(\frac{BS_0}{k}\frac{1}{S_0} - 1 + \ln S_0 - \ln \frac{BS_0}{k}\right) \\ &= k\left(\frac{B}{k} - 1 - \ln \frac{B}{k}\right) > 0. \\ \lim_{S_0 \to S^*} V &= 0\end{aligned}$$

Hence, $V(\varphi_h^*, 0) \geq 0$ for $\forall \varphi_h \geq 0$. This proves that $\varphi_l^* = 0$.

Next, when $\varphi_l^* = 0$;

$$\begin{aligned}S_0(\varphi_h^*, 0) &= \varphi_h^* \phi \\ S^*(\varphi_h^*, 0) &= \frac{\phi}{k}\left[\frac{\varphi_h^*(1-\phi)}{1-\varphi_h^*\phi}\right]\end{aligned}$$

$$\begin{aligned}V(\varphi_h^*, 0) = \left(\frac{k}{1-S_0}\right)\left(\frac{S^*-S_0}{S_0} + \ln \frac{S_0}{S^*}\right) &= \left(\frac{k}{1-\varphi_h^*\phi}\right)\left(\frac{\frac{1}{k}\left[\frac{\varphi_h^*(1-\phi)}{1-\varphi_h^*\phi}\right] - \varphi_h^*}{\varphi_h^*} + \ln \frac{\varphi_h^*}{\frac{1}{k}\left[\frac{\varphi_h^*(1-\phi)}{1-\varphi_h^*\phi}\right]}\right) \\ &= \left(\frac{k}{1-\varphi_h^*\phi}\right)\left(\frac{1}{k}\left[\frac{1-\phi}{1-\varphi_h^*\phi}\right] - 1 + \ln \frac{k(1-\varphi_h^*\phi)}{1-\phi}\right) \\ &= \frac{1-\phi}{(1-\varphi_h^*\phi)^2} - \left(\frac{k}{1-\varphi_h^*\phi}\right)\left(1 - \ln \frac{k(1-\varphi_h^*\phi)}{1-\phi}\right)\end{aligned}$$

In particular, when $\varphi_h^* = 1$, $V(1, 0) = \frac{1-k+k\ln k}{1-\phi}$. We now verify that an equilibrium exists where $\varphi_h > 0$. First, when $1 \cdot \bar{c} \geq \frac{1-k+k\ln k}{1-\phi}$, we note $\lim_{\varphi_h \to 0} V(\varphi_h, 0) = k\left(\frac{1-\phi}{k} - 1 - \ln \frac{1-\phi}{k}\right) > 0$, and hence there exists $0 < \varphi_h^* < 1$ such that $\varphi_h^* \bar{c} = V(\varphi_h^*, 0)$. Second, when $1 \cdot \bar{c} < \frac{1-k+k\ln k}{1-\phi}$, the cutoff type is $\varphi_h = 1$ and in this case the optimum only requires that $\varphi_h^* = 1$. □

Summarizing the result, we find that even with ex ante incentives for consumers to acquire information, the optimal strategy for the firm that maximizes the diffusion of the firm's message is to restrict information to the socially low type agents by choosing $\varphi_l = 0$ and minimizing the costs for the socially high type agents such that $0 < \varphi_h \leq 1$. In particular, for a given $\bar{c}$, when the cost of communication k becomes larger (i.e., $\frac{1-k+k\ln k}{1-\phi}$ becomes smaller such that $\bar{c} < \frac{1-k+k\ln k}{1-\phi}$), it is optimal to restirct information to even some of high type customers in order to increase the incentive to talk for each customer who acquired the information ($\frac{\partial V}{\partial S_0} < 0$). (**AC - I don't think this statement is correct. The firm will never increase the costs for high types to acquire information.) Otherwise (when the cost of communication is relatively low), it is optimal to set $\varphi_h = 1$.

3.2 Time Discounting

Next we show that the timing of the information diffusion matters. We revisit the firm's optimization problem with the discount factor $\beta^t = \exp(-rt)$, where r is the discount rate. Writing out the firm's optimization problem:

$$\max_{\varphi_h,\varphi_l} \int_0^{t^*} \frac{dS}{dt} e^{-rt} dt$$

subject to

$$\varphi_h \bar{c} \leq V(\varphi_h, \varphi_l)$$
$$\varphi_l \bar{c} \leq V(\varphi_h, \varphi_l)$$

$$0 \leq \varphi_h \leq 1$$
$$0 \leq \varphi_l \leq 1$$

Proposition 2. *When discount rate r is large enough, $\varphi_l > 0$.*

Proof. Let

$$R(\varphi_l, \varphi_h) = \int_0^{t^*(\varphi_h,\varphi_l)} \left(S_0 + \frac{dS}{dt} e^{-rt} \right) dt$$

We have immediately that

$$\lim_{r\to 0} R(\varphi_l, \varphi_h) = S^*(\varphi_l, \varphi_h)$$
$$\lim_{r\to\infty} R(\varphi_l, \varphi_h) = S_0(\varphi_l, \varphi_h).$$

When $r = 0$; $R(\varphi_l = 0, \varphi_h) > R(\varphi_l, \varphi_h)$ for all $\varphi_l > 0$ since $\frac{dS^*}{d\varphi_l} \leq 0$ (Equations 15). When $r = \infty$; $R = S_0(\varphi_l = 0, \varphi_h) < R = S_0(\varphi_l, \varphi_h)$ for any $0 < \varphi_h \leq 1$.

Furthermore,

$$\frac{dR(\varphi_l, \varphi_h)}{dr} = -\int_0^{t^*(\varphi_h,\varphi_l)} \frac{dS}{dt} t e^{-rt} dt < 0 \text{ for all } 0 \leq \varphi_l, \varphi_h \leq 1.$$

Hence, for any $\varphi_l > 0$, there exists $r^*(\varphi_l)$ such that for all $r > r^*(\varphi_l)$, $R(\varphi_l = 0, \varphi_h) < R(\varphi_l > 0, \varphi_h)$. □

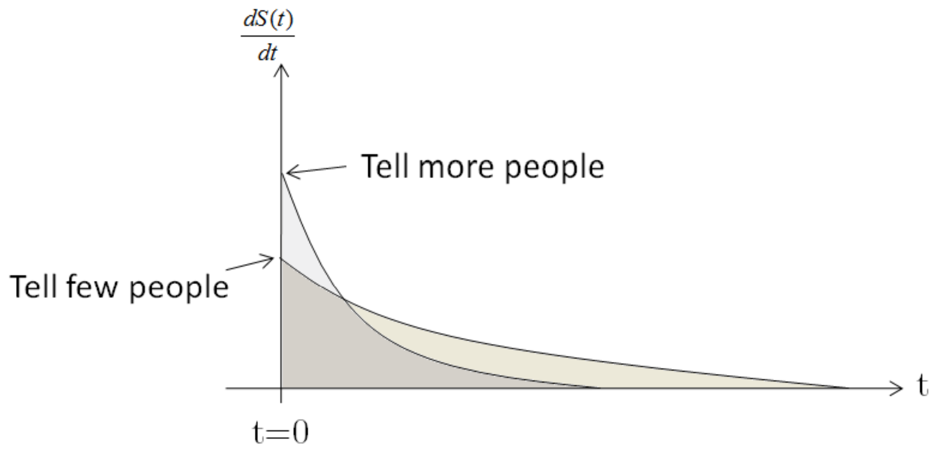


Figure 1: Diffusion of Information Based on Firm's Strategy

Hence, there is a tradeoff between the amount of information diffusion (S^*) and the timing of diffusion – as long as the firm is not too patient (r is sufficiently large), it may want to disseminate the information even to the low type customers by choosing $\varphi_l > 0$ to achieve a higher level of information diffusion at early stage.

Discussion

The analysis above demonstrates that the firm faces the following trade-off: by confining the initial spread of information to high-status consumers only, it maximizes the individual's incentive to spread the information. On the other hand, by spreading the information more widely (allowing some low-types access to it), the firm is guaranteeing itself an initial high level of adoption, but the total number of conversations over time decreases. Of course, if the firm is impatient, it prefers to make the information more widely accessible to consumers, even if they happen to be low-status. The trade-off can be summarized in Figure 1. That is, telling more people initially results in a lower over-all diffusion but a higher level of early diffusion of information, while telling few people results in higher overall level of diffusion in the long run.

4 Extensions

4.1 Restricted Diffusion

In this subsection we consider a technology that the firm can employ which restricts who can credibly spread buzz about the product. For instance, Facebook initially only allowed individuals with email addresses from a small number of elite schools to adopt its product and without adopting the product, one cannot learn the information. We find that the firm can increase the diffusion of information by restricting who can spread information about the product. There are two potentially beneficial effects. The first is that if the restriction is biased towards high types in that they are more able to spread the information, then this will improve the inference from being able to spread the information. The second is that provided that the initial adopters aren't restricted by the policy, then restricting the subsequent imitators during the diffusion of the information improves ex ante incentives to acquire information.

We assume that the firm can exclude some low types from being able to spread information about the product. The subset of the population who can spread information conditional on acquiring it is denoted by Ω. We assume that all high types are in Ω, $\Pr(i \in \Omega \mid \theta_i = h) = 1$; however, the firm excludes some low types, $\Pr(i \in \Omega \mid \theta_i = l) < 1$, and hence $\phi \leq \Pr(i \in \Omega) = \omega < 1$. We distinguish the fraction of the population which are informed by $S^I(t)$ from the fraction which can diffuse information by $S^R(t) = S^I(t) \bigcap \Omega$. The informed population $S^I(t)$ increases due to the word of mouth behavior of the population $S^R(t)$. In this section we take as given the results of the previous section that the firm increases the costs for low social types to acquire information ex ante. Rather we ask whether the firm can do better by restricting the set of individuals who may diffuse information about the product to Ω and answer it in the affirmative, thus even if the firm strictly prefers to change its information acquisition structure it will only strengthen the result of this section. We denote the cutoff type of high value consumer by φ^Ω and φ^* for the cases when the firm does and does not restrict the set of individuals who can diffuse the information respectively. Similarly define for each case the total diffusion by $S^\Omega(\varphi_h)$ and $S^*(\varphi_h)$ conditional on a level of ex ante information acquisition and denote beliefs by b^Ω and b.

The equations describing the evolution of these variables over time as agents mix at rate λ are given by:

$$\begin{aligned}\frac{dS^I}{dt} &= \lambda S^R(t)\left(1 - S^I(t)\right)\\ \frac{dS^R}{dt} &= \lambda \omega S^R(t)\left(\omega - S^R(t)\right)\end{aligned}$$

we proceed by proving three lemmas.

Lemma 2. *Under initial conditions $S_0^I = S_0^B = \varphi_h \phi$. The diffusion path for $S^B(t)$ and*

$S^I(t)$ are

$$\begin{aligned} S^B(t) &= \frac{\omega}{1+ae^{-\lambda\omega t}} \\ S^I(t) &= \frac{1}{1+ae^{-\lambda\omega t}} - \frac{(1-\omega)\,e^{-\lambda\omega t}}{1+ae^{-\lambda\omega t}} \end{aligned}$$

where $a = \frac{\omega-\varphi_h\phi}{\varphi_h\phi}$.

Proof. The derivation of $S^B(t)$ is the same as earlier for $S(t)$ taking it as given substitute it into the equation for $S^I(t)$:

$$\begin{aligned} \frac{dS^I}{dt} &= \lambda\frac{\omega}{1+ae^{-\lambda\omega t}}\left(1-S^I(t)\right) \\ \frac{dS^I}{(1-S^I(t))} &= \lambda\frac{\omega}{1+ae^{-\lambda\omega t}}dt \\ -ln\left(1-S^I(t)\right) &= ln\left(-a-e^{\lambda\omega t}\right)+ln(d) \\ \frac{1}{(1-S^I(t))} &= -d\left(a+e^{\lambda\omega t}\right) \\ 1-S^I(t) &= \frac{-1}{d\left(a+e^{\lambda\omega t}\right)} \end{aligned}$$

redefine $d = \frac{-1}{d}$ hence

$$S^I(t) = 1-\frac{d}{a+e^{\lambda\omega t}}$$

solving for d using the initial condition and $a = \frac{\omega-\varphi_h\phi}{\varphi_h\phi}$ we find

$$\begin{aligned} \varphi_h\phi &= 1-\frac{d}{\frac{\omega-\varphi_h\phi}{\varphi_h\phi}+1} \\ \frac{d\varphi_h\phi}{\omega} &= 1-\varphi_h\phi \\ d &= \omega\frac{1-\varphi_h\phi}{\varphi_h\phi} \end{aligned}$$

substituting back in

$$
\begin{aligned}
S^{I}(t) &= \frac{a-d+e^{\lambda\omega t}}{a+e^{\lambda\omega t}} \\
S^{I}(t) &= \frac{e^{\lambda\omega t}-1+\omega}{a+e^{\lambda\omega t}} \\
S^{I}(t) &= \frac{1-(1-\omega)e^{-\lambda\omega t}}{1+ae^{-\lambda\omega t}} \\
S^{I}(t) &= \frac{1}{1+ae^{-\lambda\omega t}}-\frac{(1-\omega)e^{-\lambda\omega t}}{1+ae^{-\lambda\omega t}}
\end{aligned}
$$

□

Lemma 3. *Conditional on φ_h and the level of diffusion, $S = S^I$, the signaling value of information is greater when diffusion is restricted to Ω:*

$$b^{\Omega}(\theta_i = h|m, S, \varphi_h) - b^{\Omega}(\theta_i = h|\varnothing, S, \varphi_h) > b(\theta_i = h|m, S, \varphi_h) - b(\theta_i = h|\varnothing, S, \varphi_h)$$

Proof. Note that

$$b^{\Omega}(\theta_i = h|m, S)\, S^B + b^{\Omega}(\theta_i = h|\varnothing, S)\left(1-S^B\right) = b(\theta_i = h|m, S)\, S + b(\theta_i = h|\varnothing, S)(1-S) = \phi$$

where $S^B < S$, $b^{\Omega}(\theta_i = h|m, S) > b^{\Omega}(\theta_i = h|\varnothing, S)$, and $b(\theta_i = h|m, S) > b(\theta_i = h|\varnothing, S)$ so it suffices to show that

$$b^{\Omega}(\theta_i = h|\varnothing, S) < b(\theta_i = h|\varnothing, S)$$

The inference from not receiving a signal is:

$$b^{\Omega}(\theta_i = h|\varnothing, S) = \frac{1-S^I(t)}{1-S^B(t)} \times \frac{\phi-\varphi_h\phi}{1-\varphi_h\phi}$$

and we already know from earlier that $b(\theta_i = h|\varnothing, S) = \frac{\phi-\varphi_h\phi}{1-\varphi_h\phi}$ Hence conditional on $S^I = S$ the result follows immediately. □

Lemma 4. *Conditional on φ_h the value of acquiring the signal ex ante is greater when diffusion is restricted to Ω*

$$V^{\Omega}(\varphi_h) > V(\varphi_h)$$

Proof. The value of acquiring a signal ex ante when diffusion is unrestricted is

$$V(\varphi_h) = \lambda\left(\int_{S_0}^{S^*(\varphi_h)}\left(\frac{1}{1-\varphi_h\phi}\right)\frac{1}{S}\left(\left[b(\theta_i = h|m, S, \varphi_h) - b(\theta_i = h|\varnothing, S, \varphi_h)\right]-k\right)dS\right)$$

the value of acquiring a signal when diffusion of information is restricted is

$$V^{\Omega}(\varphi_h) = \lambda \left(\int_{S_0}^{S^{\Omega}(\varphi_h)} \left(\frac{1}{1-\varphi_h \phi} \right) \frac{1}{S} \left(\left[b^{\Omega}(\theta_i = h|m, S, \varphi_h) - b^{\Omega}(\theta_i = h|\varnothing, S, \varphi_h) \right] - k \right) dS \right)$$

where the term inside the integral is positive at all points along the path. Now from lemma 2 we have $b^{\Omega}(\theta_i = h|m, S, \varphi_h) - b^{\Omega}(\theta_i = h|\varnothing, S, \varphi_h) > b(\theta_i = h|m, S, \varphi_h) - b(\theta_i = h|\varnothing, S, \varphi_h)$ and hence $S^{\Omega}(\varphi_h) > S^*(\varphi_h)$. Thus $V^{\Omega}(\varphi_h) > V(\varphi_h)$. □

We can now present our main result of this section. When the firm restricts the set of people who can diffuse information to Ω then the extent of the diffusion is greater.

Proposition 3. *The diffusion of information is greater when the firm restricts the diffusion of information to individuals in* Ω, $S^{\Omega *} > S^*$

Proof. The result follows from noting that

$$\begin{aligned} S^*(\varphi_h^*) &< S^*(\varphi_h^{\Omega}) \\ S^*(\varphi_h^{\Omega}) &< S^{\Omega}(\varphi_h^{\Omega}) \end{aligned}$$

where the first inequality follows by noting that an immediate consequence of lemma 3 is that the cutoff type is greater when diffusion is restricted $\varphi_h^{\Omega} > \varphi_h^*$ and from earlier $\frac{dS^*}{d\varphi_h} > 0$. The second inequality follows immediately from lemma 2. □

The firm benefits from restricting who can spread information about the product because it strengthens the signaling value of the information increasing both the ex ante incentives to acquire the information and the ex post incentives to spread it. This form of information management is consistent with marketing campaigns by Facebook and Spotify which endeavored to create artificial scarcity for their products by restricting who can initially adopt them and credibly spread information about them as an initial adopter rather than an imitator.

4.2 Advertising

In this section we allow the firm to engage in untargeted advertising. We find that advertising crowds out the incentives for individuals to acquire information and engage in word of mouth. Advertising is a source of information which is not correlated with an individuals type and so weakens the signaling value of the information. We find that a commitment by the firm not to advertise can lead to a greater diffusion of information. We discuss a source of commitment available to a firm, to release information a sufficient amount of time prior to the product release, and find that it is optimal for a firm to release information a sufficient amount of time prior to the product release and undertake any advertising only after word of mouth has spread. This accords well with observed behavior of firms which successfully generate buzz for a product.

In this section consumers choose whether or not to acquire information at $t = -1$. The firm access to a costly advertising technology at $t = 0$. The technology is untargeted so is equally likely to inform high or low types. We assume that consumers only observe whether they themselves receive or don't receive the advertisement, in particular consumers do not observe the level of β but must infer it in equilibrium. Consumers inference of the level of advertising by the firm affects their choice whether to acquire information ex ante and how their beliefs evolve over time about the signaling value of information. At $t = 0$ the firm chooses the fraction β of the population to advertise to at cost $C(\beta) \geq 0$, $C'(\beta) > 1$, $C''(\beta) > 0$ for all $\beta \geq 0$. We show that a firm may be strictly better off when it can commit not to undertake advertising. We assume in this section that the firm maximizes φ_h subject to the incentive constraint by choosing $\tau_h = 0$ and minimizes φ_l by setting it equal to 0 by choosing a large τ_l. In this setting $(S_0 = \varphi_h)$, the incentives for a consumer to acquire information and a firm to advertise depend on one another. We work backwards from the period $t \geq 0$ where consumers choose the length of time $t^*\left(\varphi_h, \tilde{\beta}\right)$ to spread the word of mouth as a function of beliefs based on the conjectured level of advertising by the firm $\tilde{\beta}$ and the level of information acquisition φ_h at $t = -1$. The firm chooses a level of advertising $\beta^*\left(\varphi_h, \tilde{t}\right)$ as a function of the the conjectured amount of time individuals spread word of mouth $\tilde{t}$ and the level of information acquisition φ_h at $t = -1$. An equilibrium of the sub-game for a given level of information acquisition is a fixed point $(t^*(\varphi_h, \beta^*), \beta^*(\varphi_h, t^*))$. The equilibrium of the game is then given by the maximum level of information acquisition $\varphi_h^* \bar{c} = V\left(\varphi_h^*, t^*\left(\varphi_h^*, \tilde{\beta}\right), \beta^*\left(\varphi_h^*, \tilde{t}\right)\right)$.

Consumer's Decision

We consider S^* as the function of φ_h, φ_l, and advertising β, such that $S^* = S^*(\varphi_h, \varphi_l, \beta)$. Since in this section we assume that $\varphi_l = 0$, this simplifies to $S^*(\varphi_h, 0, \beta)$. In this setting $(S_0 = \varphi_h)$, note that the role of advertising is to change the initial state of $(\varphi_h^*, 0)$ to $(\varphi_h^* + \beta(1 - \varphi_h^*),\ \beta)$ at $t = 0$; in other words, given the advertising level of β the fraction of the informed population becomes $S_0 = \varphi_h \phi + \tilde{\beta}\left((1 - \varphi_h)\phi + 1 - \phi\right)$ at $t = 0$. We now show that for any given level of information acquisition by consumers, $S_0 = \varphi_h$, the conjectured level of diffusion $S^*(S_0, \beta)$ at which individuals cease to spread information is independent of the conjectured level of advertising $\tilde{\beta}$ in the following lemma.

Lemma 5. For any given $S_0 = \varphi_h$, $S^*(S_0, \beta) = S^*(S_0, 0)$ for all $\beta \geq 0$.

Proof. Suppose that at $t = -1$, the initial population of informed customers are $S_0 = (\varphi_h^*, 0)$. We know that when there is no advertising,

$$S^*(\varphi_h^*, 0, 0) = \frac{\varphi_h^* \phi}{k}\left[1 - \frac{\phi(1 - \varphi_h^*)}{1 - \varphi_h^* \phi}\right] = \frac{1}{k}\left[\varphi_h^* \phi - \frac{\phi^2 \varphi_h^* (1 - \varphi_h^*)}{1 - \varphi_h^* \phi}\right].$$

The role of advertising is to change the initial state of $(\varphi_h^*, 0)$ to $(\varphi_h^* + \beta(1-\varphi_h^*),\, \beta)$ at $t = 0$. Hence, $S^*(\varphi_h^*, 0, \beta) = S^*(\varphi_h^* + \beta(1-\varphi_h^*),\, \beta, 0)$.

Now we plug this into Equation (13);

$$\begin{aligned} S^*(\varphi_h^*, 0, \beta) &= S^*(\varphi_h^* + \beta(1-\varphi_h^*),\, \beta, 0) \\ &= \frac{\varphi_h^*\phi + \beta(1-\varphi_h^*\phi)}{k}\left[\frac{\varphi_h^*\phi + \beta(1-\varphi_h^*)\phi}{\varphi_h^*\phi + \beta\left(1-\varphi_h^*\phi\right)} - \frac{\phi(1-\varphi_h^*)}{1-\varphi_h^*\phi}\right] \\ &= \frac{1}{k}\left[\varphi_h^*\phi + \beta(1-\varphi_h^*)\phi - \frac{\phi^2\varphi_h^*(1-\varphi_h^*)}{1-\varphi_h^*\phi} - \beta(1-\varphi_h^*)\phi\right] \\ &= \frac{1}{k}\left[\varphi_h^*\phi - \frac{\phi^2\varphi_h^*(1-\varphi_h^*)}{1-\varphi_h^*\phi}\right] = S^*(\varphi_h^*, 0, 0) \end{aligned}$$

□

The lemma suggests that for any given level of information acquisition by consumers, $S_0 = \varphi_h$, the extent of diffusion $S^*(S_0, \beta)$ is independent of the level of advertising β. The information diffusion is governed by the signlaing value, which is again function of the initial composition of high and low types consumers who acquired the information at $t = -1$. Since the advertising is untargeted, the increased population through the advertising does not change the initial composition of customer types and thus does not affect the consumer inference about the type. In this sense, the role of advertising is the same as the word of mouth diffusion since both are untargeted. Hence, the customer's belief updating is govenered by the same mechanism and the advertising is purely substituting the word of mouth process and it does not affect the extent of diffusion ultimately.

Although the advertising does not affect S^* but it has effect on the time at which the diffusion stops. The time $t^*\left(\varphi_h, \tilde{\beta}\right)$ is given by

$$t^*\left(\varphi_h, \tilde{\beta}\right) = \frac{1}{\lambda}\ln\frac{S^*(\varphi_h^*, \beta)}{1 - S^*(\varphi_h^*, \beta)}\frac{1 - S_0\left(\varphi_h, \tilde{\beta}\right)}{S_0\left(\varphi_h, \tilde{\beta}\right)}$$

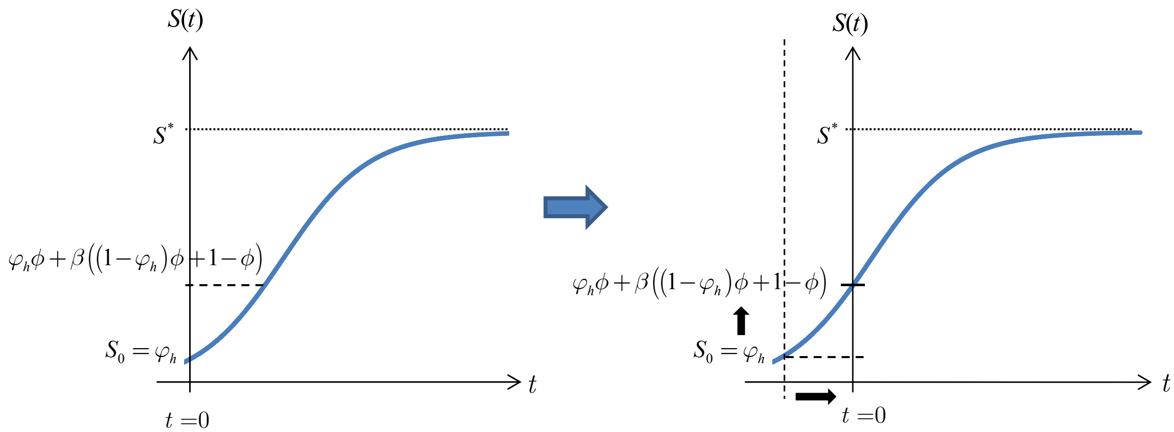
where

$$S_0\left(\varphi_h, \tilde{\beta}\right) = \varphi_h\phi + \tilde{\beta}\left((1-\varphi_h)\phi + 1 - \phi\right).$$

finding $\frac{dt^*}{d\tilde{\beta}}$:

$$\frac{dt^*}{d\tilde{\beta}} = -\frac{1-\varphi_h\phi}{\lambda}\left[\frac{1}{1-S_0} + \frac{1}{S_0}\right]$$

$t^*\left(\varphi_h, \tilde{\beta}\right)$ is strictly decreasing S_0 and hence is also strictly decreasing in the conjectured level of advertising $\tilde{\beta}$. The conjectured level of advertising $\tilde{\beta}$ has no direct effect on S^* and so has not effect on t^* through this term.



(a) Without advertising

(b) With advertising

Figure 2: The Role of Untargeted Advertising

The Figure 2 above illustrates the role of untargeted advertising. The total diffusion of information is independent of the level of advertising. The advertising basically faciliate the diffusion of the information but do not change the mechanism which affects the incentive to talk for consumers.

Firm's Advertising Decision

A firm's incentive to undertake advertising balances marginal costs of advertising against the marginal impact on S^*.

$$\arg\max_{\beta} S^*\left(\varphi_h, \beta\right) - C\left(\beta\right)$$

where φ_h is the level of information acquired by consumers. The function $S^*\left(\varphi_h, \beta\right)$ may be written in terms of S_0 and the conjectured amount of time consumers will spread information $\tilde{t}$

$$S^*\left(\varphi_h, \beta\right) = \frac{1}{1 + ae^{-\lambda\tilde{t}}}, \text{ where } a = \frac{1 - S_0}{S_0} \tag{18}$$

the firm is able to influence S^* through the initial informed share of the population S_0 but can not directly influence t^* which is only affected by consumers' expectations of the level advertising $\tilde{\beta}$in equilibrium. The level of advertising affects S_0 through the following relationship:

$$S_0\left(\varphi_h, \beta\right) = \varphi_h\phi + \beta\left(\left(1 - \varphi_h\right)\phi + 1 - \phi\right) \tag{19}$$

hence the marginal effect of advertising on S_0 is

$$\frac{dS_0}{d\beta} = (1-\varphi_h)\,\phi + 1 - \phi$$

Now taking the derivative of equation 18 with respect to S_0 while holding $\tilde{t}$ constant:

$$\frac{dS^*}{dS_0} = e^{-\lambda\tilde{t}}\left(\frac{S^*}{S_0}\right)^2$$

and the marginal effect of advertising on S^* is therefore:

$$\frac{dS^*}{dS_0}\frac{dS_0}{d\beta} = \frac{dS^*}{d\beta} = e^{-\lambda\tilde{t}}\left(\frac{S^*}{S_0}\right)^2(1-\varphi_h\phi)$$

$$= \frac{S^*(1-S^*)}{S_0(1-S_0)}(1-\varphi_h\phi)$$

The firm's optimal choice of advertising $\beta\left(\varphi_h,\tilde{t}\right)$ for a level of information acquisition by consumers and conjectured spreading time $\tilde{t}$ satisfies:

$$\begin{aligned}
C'(\beta) &= \frac{S^*(\varphi_h,\beta)\,(1-S^*(\varphi_h,\beta))}{S_0(\varphi_h,\beta)\,(1-S_0(\varphi_h,\beta))}(1-\varphi_h\phi) \\
C'(\beta) &= \frac{(1-\varphi_h\phi)\,e^{-\lambda\tilde{t}}}{\left[\left(1-e^{-\lambda\tilde{t}}\right)S_0+e^{-\lambda\tilde{t}}\right]^2} \\
C'(\beta) &= \frac{(1-\varphi_h\phi)\,e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^2}
\end{aligned}$$

Implicitly differentiating to find $\frac{d\beta}{d\tilde{t}}$:

$$\frac{d\beta}{d\tilde{t}} = \frac{\frac{\lambda(1-\varphi_h\phi)e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^2} - 2S_0\lambda e^{\lambda\tilde{t}}\frac{(1-\varphi_h\phi)e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^3}}{C''(\beta)+2\left(e^{\lambda\tilde{t}}-1\right)\frac{(1-\varphi_h\phi)e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^3}}$$

$$\frac{d\beta}{d\tilde{t}} = \lambda\frac{1-S_0-e^{\lambda\tilde{t}}S_0}{\frac{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^3}{(1-\varphi_h\phi)e^{\lambda\tilde{t}}}C''(\beta)+2\left(e^{\lambda\tilde{t}}-1\right)}$$

$$\frac{d\beta}{d\tilde{t}} = \lambda(1-S_0)\frac{1-\frac{S^*}{1-S^*}}{\frac{\left[\frac{1-S_0}{1-S^*}\right]^3}{(1-\varphi_h\phi)\frac{S^*(1-S_0)}{S_0(1-S^*)}}C''(\beta)+2\frac{S^*-S_0}{S_0(1-S^*)}}$$

$$\frac{d\beta}{d\tilde{t}} = \lambda\left(1-\varphi_h\phi\right)\frac{1-2S^*}{\frac{S_0}{S^*}\frac{1-S_0}{1-S^*}C''\left(\beta\right)+2\left(\frac{S^*}{S_0}-1\right)\left(1-\varphi_h\phi\right)}$$

and $\frac{d\beta}{d\varphi_h}$:

$$\frac{d\beta}{d\varphi_h} = \frac{\phi\frac{e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^2}+2\left(e^{\lambda\tilde{t}}-1\right)\left(1-\varphi_h\phi\right)\frac{\left(1-\varphi_h\phi\right)e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^3}}{C''\left(\beta\right)+2\left(e^{\lambda\tilde{t}}-1\right)\frac{\left(1-\varphi_h\phi\right)e^{\lambda\tilde{t}}}{\left[\left(e^{\lambda\tilde{t}}-1\right)S_0+1\right]^3}} > 0$$

Lemma 6. *Suppose $\phi < \frac{1}{4}$ and $k > 2\phi$ then the equilibrium of the subgame $\left(t^*\left(\varphi_h,\beta^*\right),\beta^*\left(\varphi_h,t^*\right)\right)$ is stable and continuous in φ_h for $\varphi_h \in (0,1]$.*

Proof. When

$$\begin{aligned} 1 &> \frac{d\beta}{d\tilde{t}}\frac{dt^*}{d\tilde{\beta}} \\ -\frac{\lambda S_0\left(1-S_0\right)}{1-\varphi_h\phi} &< \lambda\left(1-\varphi_h\phi\right)\frac{1-2S^*}{\frac{S_0}{S^*}\frac{1-S_0}{1-S^*}C''\left(\beta\right)+2\left(\frac{S^*}{S_0}-1\right)\left(1-\varphi_h\phi\right)} \end{aligned}$$

there is a unique stable solution to the subgame. A sufficient condition is for $S^* < \frac{1}{2}$ which is true when $\phi < \frac{1}{4}$ and $k > 2\phi$. □

Lemma 7. *Suppose $\phi < \frac{1}{4}$ and $k > 2\phi$ then the value of information acquisition $V\left(\varphi_h,t^*\left(\varphi_h,\beta^*\right),\beta^*\left(\varphi_h,t^*\right)\right)$ is continuous in φ_h for $\varphi_h \in (0,1]$.*

Proof. The expression for the value of information acquisition is

$$V\left(\varphi_h,t^*\left(\varphi_h,\beta^*\right),\beta^*\left(\varphi_h,t^*\right)\right) = \left(1-\beta^*\right)\frac{k}{1-S_0}\left[\frac{S^*}{S_0}-1-\ln\frac{S^*}{S_0}\right]$$

which is continuous in S^* and S_0 for $0 < S_0 < 1$ and $S^* \geq S_0$. Continuity then is established by noting that both are continuous in t, β and φ_h, and from earlier $t^*\left(\varphi_h,\beta^*\right)$ and $\beta\left(\varphi_h,t^*\right)$ are continuous in φ_h. □

Further we define $t^*\left(0,\beta^*\right) = \lim_{\varphi_h\to 0} t^*\left(\varphi_h,\beta^*\right)$, $\beta^*\left(0,t^*\right) = \lim_{\varphi_h\to 0}\beta^*\left(\varphi_h,t^*\right)$ and $V\left(0,t^*\left(0,\beta^*\right),\beta^*\left(0,t^*\right)\right) = \lim_{\varphi_h\to 0} V\left(\varphi_h,t^*\left(\varphi_h,\beta^*\right),\beta^*\left(\varphi_h,t^*\right)\right)$ to ensure that the value of information acquisition is continuous over the closed interval $\varphi_h \in [0,1]$. Agents are small relative to the size of population so take the fraction of agents who acquire information at $t = -1$ in equilibrium as given and anticipate their own action having zero effect on the play of the game at $t = 0$. We define an equilibrium level of information acquisition by a cutoff φ_h^* which satisfies:

$$\begin{aligned} \varphi_h^*\bar{c} = V\left(\varphi_h^*,t^*\left(\varphi_h^*,\beta^*\right),\beta^*\left(\varphi_h^*,t^*\right)\right) \quad &\text{if} \quad 0 \leq \varphi_h^* < 1 \\ \varphi_h^* = 1 \quad &\text{if} \quad V\left(1,t^*\left(1,\beta^*\right),\beta^*\left(1,t^*\right)\right) \geq \bar{c} \end{aligned}$$

further we also require that it is stable $\exists\epsilon > 0 : \forall\delta; 0 \leq \delta \leq \epsilon$

$$\begin{aligned}
(\varphi_h^* + \delta)\,\bar{c} &\geq V(\varphi_h^* + \delta, t^*(\varphi_h^* + \delta, \beta^*), \beta^*(\varphi_h^* + \delta, t^*)) \\
(\varphi_h^* - \delta)\,\bar{c} &\leq V(\varphi_h^* - \delta, t^*(\varphi_h^* - \delta, \beta^*), \beta^*(\varphi_h^* - \delta, t^*))
\end{aligned}$$

where only the first and then the second need to hold at $\varphi_h = 0, 1$ respectively. The following proposition establishes sufficient conditions under which there exists a sub-game perfect equilibrium of the model.

Proposition 4. *Suppose $\phi < \frac{1}{4}$ and $k > 2\phi$ then there exists a sub-game perfect equilibrium.*

Proof. $V(\varphi_h, t^*(\varphi_h, \beta^*), \beta^*(\varphi_h, t^*))$ is continuous in φ_h and bounded below by 0. In the case $V(0, t^*(0, \beta^*), \beta^*(0, t^*)) = 0$ and $\exists\epsilon > 0$ such that $\forall\delta, 0 \leq \delta \leq \epsilon$ then $\delta\bar{c} \geq V(\delta, t^*(\delta, \beta^*), \beta^*(\delta, t^*))$ and $\{\varphi_h^* = 0, t^*(\varphi_h^*), \beta^*(\varphi_h^*)\}$is a sub-game perfect equilibrium. If not $\exists\epsilon > 0$ such that $\forall\delta, 0 \leq \delta \leq \epsilon$ then $\delta\bar{c} < V(\delta, t^*(\delta, \beta^*), \beta^*(\delta, t^*))$. In this case $V(\varphi_h, t^*(\varphi_h, \beta^*), \beta^*(\varphi_h, t^*))$ will either cut the line $\varphi_h\bar{c}$ at a point $\varphi_h^* \in (0, 1)$ or will satisfying the conditions $V(1, t^*(1, \beta^*), \beta^*(1, t^*)) \geq \bar{c}$ and $\exists\epsilon > 0$ such that $\forall\delta, 0 \leq \delta \leq \epsilon$ then $(1 - \delta)\,\bar{c} < V(1 - \delta, t^*(1 - \delta, \beta^*), \beta^*(1 - \delta, t^*))$, hence $\varphi_h^* = 1$. □

The following proposition gives the main result of the section. It shows that the firm may be better off with a higher marginal cost of advertising which allows it to commit not to undertake advertising. It is useful to define a cutoff type φ_h^{**} which satisfies the following condition:

$$\varphi_h^{**}\bar{c} = \frac{1 - \phi}{(1 - \varphi_h^{**}\phi)^2} - \left(\frac{k}{1 - \varphi_h^{**}\phi}\right)\left(1 - \ln\frac{k(1 - \varphi_h^{**}\phi)}{1 - \phi}\right)$$

Proposition 5. *Suppose $\phi < \frac{1}{4}$ and $k > 2\phi$ and consider two cost functions of advertising C_1 and C_2 where*

$$1 < C_2'(0) < \frac{S^*(\varphi_h^{**}, 0)(1 - S^*(\varphi_h^{**}, 0))}{S_0(\varphi_h^{**}, 0)(1 - S_0(\varphi_h^{**}, 0))}((1 - \varphi_h^{**})\phi + 1 - \phi) < C_1'(0)$$

then $S_1^* > S_2^*$.

Proof. In the case of C_1 we have $C_1'(0) > \frac{S^*(\varphi_h^{**},0)(1-S^*(\varphi_h^{**},0))}{S_0(\varphi_h^{**},0)(1-S_0(\varphi_h^{**},0))}((1 - \varphi_h^{**})\phi + 1 - \phi)$, therefore $\beta_1^*(\varphi_h^{**}) = 0$, $t_1^*(\varphi_h^{**}) = \frac{1}{\lambda}\ln\frac{S^*(\varphi_h^{**})}{1-S^*(\varphi_h^{**})}\frac{1-\varphi_h^{**}\phi}{\varphi_h^{**}\phi}$ and $\varphi_{h1}^* = \varphi_h^{**}$.

In the case of C_2, we have $\beta_2(\varphi_h^{**}, t(\varphi_h^{**}, 0)) > 0$ because $C_2'(0) < \frac{S^*(\varphi_h^{**},0)(1-S^*(\varphi_h^{**},0))}{S_0(\varphi_h^{**},0)(1-S_0(\varphi_h^{**},0))}((1 - \varphi_h^{**})\phi + 1 - \phi$ Furthermore $\beta_2^*(\varphi_h^{**}) > 0$, $t_2^*(\varphi_h^{**}) < t_1^*(\varphi_h^{**})$ and $V(\varphi_h^{**}, t_2^*(\varphi_h^{**}), \beta_2^*(\varphi_h^{**})) < \varphi_h^{**}\bar{c}$. Hence the equilibrium in the case of C_1 is not an equilibrium for C_2. Now take $\hat{\varphi}_h > \varphi_h^{**}$, $V(\hat{\varphi}_h, t_2^*(\hat{\varphi}_h), \beta^*(\hat{\varphi}_h)) \leq V(\hat{\varphi}_h, t_2^*(\hat{\varphi}_h), 0) < \hat{\varphi}_h\bar{c}$. Hence $\hat{\varphi}_h$ can not be part of an equilibrium and $\varphi_{h2}^* < \varphi_{h1}^* = \varphi_h^{**}$. We know from earlier that $\frac{dS^*}{d\varphi_h} > 0$ hence $S_1^* > S_2^*$. □

The result shows that a higher marginal cost of advertising may lead to an unambiguously greater diffusion of information for the firm. The commitment not to advertise is therefore valuable to the firm. An immediate corollary of this proposition is that a source of commitment for a firm is the release date of the product. For instance movie theatres release movies on certain holidays during the year and technology companies often release information and announce the future release date for the product concurrently. The key is that the cost of advertising at the time of information release calculated in dollars calculated at the product release date is increased by e^{rT} where T is the amount of time between the information release and the product release and r is the interest rate.

Corollary 1. *Suppose* $T > \frac{1}{r} \ln \frac{S^*(\varphi_h^{**},0)(1-S^*(\varphi_h^{**},0))}{S_0(\varphi_h^{**},0)(1-S_0(\varphi_h^{**},0))} \frac{((1-\varphi_h^{**})\phi+1-\phi)}{C'(0)}$ *and* $T > t^*(\varphi_h^*, 0)$ *then* $\beta = 0$ *and* $\varphi_h = \varphi_h^*$.

Proof. First note that $T > t^*(\varphi_h^*, 0)$ ensures that the diffusion of word of mouth is competed prior to the product being released as to be consistent with our assumption about discounting. The corollary follows from noting that the marginal cost of advertising at the time of information release is $e^{rT}C'(\beta)$ and when $T > \frac{1}{r} \ln \frac{S^*(\varphi_h^{**},0)(1-S^*(\varphi_h^{**},0))}{S_0(\varphi_h^{**},0)(1-S_0(\varphi_h^{**},0))} \frac{((1-\varphi_h^{**})\phi+1-\phi)}{C'(0)}$ we have $e^{rT}C'(0) > \frac{S^*(\varphi_h^{**},0)(1-S^*(\varphi_h^{**},0))}{S_0(\varphi_h^{**},0)(1-S_0(\varphi_h^{**},0))} ((1-\varphi_h^{**})\phi + 1 - \phi)$ and from the proposition we have that $\beta = 0$ and $\varphi_h = \varphi_h^*$. □

The corollary highlights that early information release can serve as a commitment not to undertake advertising during the period of time that individuals are undertaking word of mouth. Of course the firm could also undertake advertising upon the product being released which does not affect the word of mouth, as we see with many products significant buzz/word of mouth usually occurs prior to any advertising campaign by a firm.

5 Conclusion

We study the implications of a self enhancement motivation for word of mouth communication between consumers for the information release and advertising strategy of a firm. A firm maximizes the diffusion of information, by structuring its information release strategy so that the act of passing on information, through word of mouth communication, can serve as a signal of a consumer's social type. In our model a firm constrains low-social types' access to information and their ability to pass on information to other consumers, in order to maintain the signaling value of passing on information. Even though these activities seem to restrict the spread of information in the immediate term, these in fact serve to maximize the total diffusion of information. The firm may also benefit from a commitment not to undertake advertising which serves to crowd out word of mouth as a source of information. We highlight that a potential source of this commitment is to coordinate the information release a sufficient amount of time prior to the product release.

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