# Participating insurance contracts and the Rothschild-Stiglitz equilibrium puzzle

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**Abstract**

We show that an equilibrium always exists in the Rothschild-Stiglitz insurance market model with adverse selection when insurers can offer either non-participating or participating policies, i.e. insurance contracts with policy dividends or supplementary calls for premium. The equilibrium allocation coincides with the Miyazaki-Spence-Wilson equilibrium allocation, which may involve cross-subsidization between contracts. The paper establishes that participating policies act as an implicit threat that dissuades deviant insurers who aim at attracting low risk individuals only. The model predicts that the mutual corporate form should be prevalent in insurance markets where second-best Pareto efficiency requires cross-subsidization between risk types. Stock insurers and mutuals may coexist, with stock insurers offering insurance coverage at actuarial price and mutuals cross-subsidizing risks. The same line of argument extends to other markets such as retail and commercial banking or agricultural services where mutuals and cooperative entreprises have substantial market shares.

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# 1 Introduction

The paper of Rothschild and Stiglitz (1976) on competitive insurance markets under adverse selection is widely considered as one of the most important contributions to the insurance economics literature. In this famous article, Rothschild and Stiglitz analyse the equilibrium of an insurance market where policyholders have private information on their risk and they reveal this information through their insurance contract choice. High risk individuals choose to be fully covered, while low risks choose partial coverage. In other words, the menu of contracts offered in the market separates risk types. The influence of the Rothschild-Stiglitz paper on the academic research in insurance economics has been and is still extremely important. In particular, it shows how hidden information may lead to efficiency losses in competitive markets and it yields a way to understand why insurance markets are so deeply associated with contractual or legal mechanisms that reduce the intensity or the consequences of hidden information, such as risk categorization, risk auditing or experience rating.

The Rothschild-Stiglitz (RS) model nevertheless includes an enigma that has puzzled and annoyed many economists during the last three decades : in this model, the market equilibrium may not exist! When it is assumed that each insurer only offers a single contract there is no pooling equilibrium, at which all individuals would take out the same contract offered by all insurers. Indeed, at a candidate pooling equilibrium insurers make zero profit, but any insurer can deviate to a profitable contract with lower coverage and lower premium by attracting only low risk individuals. The only type of equilibrium that may exist is separating, with full coverage for high risks and partial coverage for low risks. However, it turns out that such a separating equilibrium only exists when the proportion of high risk individuals is sufficiently large, because otherwise insurers could make profit by deviating to another contract that would attract high risks and low risks simultaneously. As observed by Rothschild and Stiglitz (1976) themselves, allowing each insurer to offer a menu of contracts makes the condition under which an equilibrium exists even more restrictive. Relaxing the two-type assumption would not alleviate the difficulty. In particular, as shown by Riley (1979), a competitive equilibrium never exists in the Rothschild-Stiglitz environment when there is a continuum of risk types.

Rather than a theoretical oddity, a model without equilibrium is like a map with *terra incognita* showing some unexplored territories. In other words, observing that no equilibrium may exist is just a way to acknowledge that the model does not always predict where market forces are leading us. In game theory words, an equilibrium of the RS model is a pure strategy subgame perfect Nash equilibrium of a two stage game, in which insurers simultaneously offer insurance contracts at stage 1 and then at stage 2 individuals choose the contract they prefer in the menu of available offers. Technically speaking, the nonexistence of equilibrium is related to the discontinuity of insurers' payoff functions[1]. Dasgupta and Maskin (1986a,b) have established existence theorems for mixed-strategy equilibrium in a class of games with discontinuous payoff functions and Rosenthal and Weiss (1984) have illustrated these results by constructing a mixed strategy equilibrium for the Spence's model of education choices. Such an equilibrium exists for the RS model. However, assuming that firms play a mixed-strategy at the contract offer stage has not been retained as a reasonable assumption in the subsequent literature on markets with asymmetric information. Furthermore at a mixed-strategy equilibrium there is a strategy a potential entrant could use to earn positive expected profit. As empasized by Rosenthal and Weiss (1984), this result confirm the Rothschild-Stiglitz intuition about the nonexistence of an entry-deterring equilibrium.

Actually, faced with these difficulties, most theorists who have tackled the equilibrium nonexistence problem have strayed away from the simple and most natural timing of the RS model. The "anticipatory equilibrium" of Miyazaki (1977), Spence (1978) and Wilson (1977), the "reactive equilibrium" of Riley (1979) and the variations on the equilibrium concept introduced by Hellwig (1987) and Engers and Fernandez (1987) share this common strategy, which consists in introducing some interactive dynamics among insurers[2]. The existence of equilibrium can then be established, but at the cost of much arbitraryness in the structure of the game and thus of its predictions. Dubey and Geanakoplos (2002) give up the setting of the RS model with managers of insurance firms who design contracts. In their model, the forces of perfect competition lead to the creation of pools and individuals signal their risk type by choosing which pool to join. As a special case, the RS separation allocation is always sustained as an equilibrium. Analyzing contracts as the outcome of perfect competition's invisible hand or as decisions of firms' managers is a question of paradigmatic preference. However, we may question the empirical relevance of a market allocation that would be second-best inefficient[3]. It is indeed doubtful that competitive forces do not destabilize this allocation in a way or another. Put it differently, we may consider that focusing only on risk signalling provides a partial view on competitive interactions in markets with adverse selection.

In this paper we will not move away from the basic ingredients of the RS model, but we will focus attention on the nature of contracts which are traded in the insurance market. It is indeed striking to observe that almost all the papers which have focused attention on the equilibrium existence issue seem to have taken for granted that the insurance contracts should take the simple form of non-participating contracts postu-

[1] Indeed small deviations at the contract offer stage may lead all individuals of a given type to switch to another insurer, hence a jump in insurers expected profits.

[2] See also Ania *et al.*(2002) on an evolutionary approach to insurance markets with adverse selection.

[3] The RS separating allocation is second-best inefficient (i.e. it is Pareto-dominated among allocations that satisfy incentive compatibility conditions) when high risk individuals account for a small part of the whole population.

lated by Rothschild and Stiglitz (1976) and this simple form only. A non-participating insurance contract specifies a fixed premium and an indemnity to be paid should a loss occurs, but we should keep in mind that insurers may also offer participating contracts, i.e. contracts with policy dividend when risk underwriting proved to be profitable and supplementary call when it was in deficit. After all, in the real world, mutuals and sometimes stock insurers offer participating contracts. Focusing attention on non-participating contracts only amounts to restricting the scope of insurance contracts in an unjustified way. As we will see in this paper, eliminating this restriction on contractual arrangements restores the existence of the equilibrium in the RS model. Note however, as it will clearly appear later, that our starting point is not an *ex ante* institutional distinction between corporate forms (stock insurers and mutuals). We will consider an insurance market where insurers (entrepreneurs) trade with risk averse insurance seekers and possibly with risk neutral capitalists. The nature of contracts, and thus the corporate form, are endogenous. If an insurer only trades with insurance seekers by offering them participating contracts, we may call it a mutual. If an insurer offers non-participating contracts to insurance seekers and transfers its profit to capitalists, it is a stock insurer. In other words, we will lift a restriction on risk sharing contracts in the economic environment of the RS model.

Theoretical consistency but also empirical realism will end up better off and ultimately explaining how participating insurance contract solve the RS equilibrium nonexistence puzzle is also a way to better understand how competitive forces, risk sharing and private information interact. Indeed, from a factual standpoint, statistical evidence suggests that mutuals play a major role in the insurance sector and forgetting them just comes down to a truncated version of how insurance markets actually work. The mutual market share at the end of 2008 was 24% worldwide (and 27% for non-life business). Of the largest ten insurance countries, representing 77% of the world market, five of them have over 30% of their markets in mutual and cooperative business, namely Germany 44%, France 39%; Japan 38%, Netherlands 33% and USA 30% [4]. Mutuals usually charge their members a premium, known as an advance call, at the start of each policy period. However, they have the right to charge additional premium, known as a supplementary call, if they need additional income to pay claims or increase the reserves. They may also refund part of the advance call if the fortunes of the financial year are better than expected[5]. Note however that the present paper

[4] Data are drawn from ICMIF (2010).

[5] Policy dividends are steadily distributed to holders of participating life insurance policies (be they underwritten by mutuals or by stock insurers) but, as regards mutuals, they are not restricted to life policies. For instance, State Farm Mutual Automobile Insurance Company, the largest automobile insurer in the United States and the world's largest mutual property and casualty insurer, distributes dividends to its policyholders almost every year. The policy dividend mechanism is most explicitly illustrated by maritime mutual insurance : P&I Clubs (which stands for **P**rotection and **I**ndemnity mutuals) pass back good underwriting years to shipowners through returned premiums or ask them to pay supplementary premiums when the financial year turns out to be less favorable. See www.igpandi.org on P&I Clubs. Mutuals actually differ according to the way policyholders share in

will conclude that policy dividends and supplementary calls act as implicit threats against competitors, which does not mean that they should be frequently observed in practice. After all efficient threats have not to be carried out ! We should also keep in mind that mutuals smooth the distribution of surpluses and the allocation of shortfalls between participating members by making transfers to or from their reserves[6]. In other words, mutuals also act as an intertemporal resource allocation mechanism. Mutuals will ask the participating members for a supplementary call when liabilities exceed assets. When assets exceed liabilities, then the balance may be either transferred to the mutual's reserve or returned to the members, which is usually done in proportion of their respective premiums and subscriptions.

It would be unfair not to acknowledge the contribution of the few papers that have addressed the role of mutuals offering participating contracts in the RS environnement. Boyd, Prescott and Smith (1988) have used a cooperative game theoretic approach in which individuals are viewed as forming coalitions for the purpose of pooling risk. Any coalition decides upon an "arrangement" that specifies the risk sharing within the coalition, as a function of its membership. A coalition may be interpreted as a mutual and an arrangement as a participating contract. Boyd, Prescott and Smith (1988) show that the Miyazaki-Spence-Wilson (MSW) equilibrium allocation[7] is a core arrangement associated with an unblocked incumbent (grand) coalition that would include all individuals. However they do not analyse the competition between mutuals. By contrast, the present paper shows that the MSW allocation can be sustained as a subgame perfect equilibrium of a non-cooperative game played by insurers who interact with individual agents in a competitive market. Smith and Stutzer (1990) analyse how participating policies serve as a self-selection device when there is exogenous aggregate uncertainty. They interpret mutuals as insurance firms that share undiversifiable aggregate risk with policyholders through participating contract, contrary to stock insurers who share this risk with shareholders. However they do not observe that a pooling equilibrium may occur under participating contracts, and consequently they do not say anything about equilibrium existence issues. Ligon and Thistle (2005) study the coexistence between mutuals or between mutuals and stock insurers They show that, under certain conditions, a separating equilibrium exists in which high risks form

the experience of the insurer. *Advance premium mutuals* (like State Farm) set premium rates at a level that is expected to be sufficient to pay expected losses and expenses and that provide a margin for contingencies, and usually policyholders receive dividends. On the contrary a*ssessment mutuals* (like P&I Clubs) collect an initial premium which is just sufficient to pay typical losses and expenses, and they levy supplementary premiums whenever unusual losses occur. The mutual insurance industry also include reciprocals, fraternal societies, risk retention groups and group captives. See Williams, Smith and Young (1998) for details.

[6] See Section 5 on deferred premium variations as substitutes to policy dividends and supplementary calls.

[7] When there are two risk types, the MSW allocation maximizes the expected utility of low risk individuals in the set of second-best feasible allocations, i.e. allocations that break even on aggregate and satisfy incentive compatibility constraints.

large mutuals and low risks form small mutuals. The conditions under which this separating equilibrium exists are analogous to those under which a separating equilibrium exists in the standard RS model.

The central argument of this paper may be set out in a few words. In our model, when there is cross-subsidization between risk types, participating policies act as an implicit threat against deviant insurers who would like to attract low risks only. This is actually a very intuitive result. Indeed, assume that high risk individuals have taken out a participating policy which is cross-subsidized by low risk individuals. In such a case, when low risk types move to another insurer, the situation of high risk types deteriorates (because of the participating nature of their insurance contract), which means that it is more difficult for the deviant insurer to attract low risk individuals without attracting also high risk individuals. An equilibrium with cross-subsidization is thus possible because of this implicit threat. Our model thus predicts that we should observe participating contracts when there is cross-subsidization between risk types, and non-participating contracts otherwise[8]. In the two risk type case, allocations with cross-subsidization Pareto-dominate the Rosthchild-Stiglitz pair of contracts when the proportion of large risks is small. In that case there is no equilibrium in the RS model with non-participating contracts, while an equilibrium with cross-subsidized participating contracts actually exists. Since participating contracts are mainly offered by mutuals, and non-participating contracts by stock insurers[9], we deduce that the mutual corporate form should be prevalent in markets or segments of markets with cross-subsidization between risk types, while there should be stock insurers in other cases.

The model is presented in Section 2, with a brief reminder of the RS model. Sections 3 and 4 characterize the market equilibrium when insurers can offer participating or non-participating contracts. Section 3 restricts attention to the two-risk type case, as in the RS model. It starts with the most simple case where each insurer is supposed to offer only one contract, and then it considers the more realistic setting where each insurer can offer several contracts (say a menu of insurance policies). Most developments in Section 3 are based on figures, with an intuitive game theory framework. Section 4 extends our results to the case of an arbitrary number of risk types, within a more formal game theory setting. The two-risk type case has been very popular in the literature following the seminal paper of Rothschild and Stiglitz (1976) because most of the analysis can then be developed by means of simple graphical representations. Here we need a broader setting in order to explain why various types of contracts and

[8] More precisely, it turns out that participating contracts are of particular interest when risk cross-subsidization improves the efficiency in insurance markets. Otherwise, non-participating contracts do the job as well. Note also that no policy dividend or supplementary call may be paid or made on the equilibrium path of our model. In that case, they are just implicit threats which are not carried out at equilibrium.

[9] This distinction is valid for property-casualty insurance. Stock insurers also offer participating life insurance contracts.

corporate forms may coexist in the same market, and that's what is done in Section 4. In this section we first introduce the market game, which is a two stage game where insurers offer menus of (participating or non-participating) contracts at stage 1 and individuals react at stage 2 by choosing the contract they prefer among the offers available in the market. An equilibrium allocation is sustained by a subgame perfect Nash equilibrium of the market game. We define a candidate equilibrium allocation as done by Spence (1978) in his extension of the Miyazaki-Wilson equilibrium. We then show that this candidate equilibrium allocation is sustained by (subgame perfect) equilibrium strategies of the market game where insurers offer participating contracts in the segments of markets (i.e. for subgroups of risk types) with cross-subsidization, and non-participating contracts in the other segments. Section 5 sketches a dynamic extension of our model. It shows that transferring underwriting profits to reserves and increasing or decreasing premiums according to the level of accumated surplus may act as a substitute to policy dividend or supplementary call with similar strategic effects. Such deferred premium variations involve more complex competitive mechanisms with a signalling dimension. Final comments follow in Section 6. We compare our analysis to other approaches to financial corporate form, including insurance and banking. We conclude that adverse selection is a powerful motive for the development of various forms of cooperative enterprises, including mutual insurance, but also mutual banks and agricultural service cooperatives. Mathematical proofs are in the Appendix.

# 2 The model

We consider a large population represented by a continuum of individuals facing idiosyncratic risks of accident. All individuals are risk averse : they maximize the expected utility of wealth $u(W)$, where $W$ denotes wealth and the (twice continuously differentiable) utility function $u$ is such that $u' > 0$ and $u'' < 0$. If no insurance policy is taken out, we have $W = W_N$ in the no-accident state and $W = W_A$ in the accident state; $A = W_N - W_A$ is the loss from an accident[10]. Individuals differ according to their probability of accident $\pi$ and they have private information on their own accident probability. There are $n$ types of individuals, with $\pi = \pi_i$ for type $i$ with $0 < \pi_n < \pi_{n-1} < ... < \pi_1 < 1$. Hence the larger the index $i$ the lower the probability of an accident. $\lambda_i$ is the fraction of type $i$ individuals among the whole population with $\sum_{i=1}^{n} \lambda_i = 1$. This section and the following focus on the two risk type case, i.e. $n = 2$. Type 1 is a high risk and type 2 is a low risk and $\overline{\pi} = \lambda_1\pi_1 + (1 - \lambda_1)\pi_2$ denotes the average probability of loss.

Insurance contracts are offered by $m$ insurers ($m \geq 2$) indexed by $j = 1, ..., m$. They are entrepreneurs who may be stock insurers or mutual insurers. Stock insurers

[10] The word "accident" is taken in its generic meaning: it refers to any kind of insurable loss, such as health care expenditures or fire damages.

pool risks between policyholders through non-participating insurance contracts and they transfer underwriting profit to risk neutral shareholders. Mutual insurers have no shareholders : they share risks between their members only through participating contracts. The insurance corporate form is not given *ex ante* : it will be a consequence of the kind of insurance contracts offered at the equilibrium of the insurance market, and as we shall see this contract form (participating or non-participating) is the outcome of competitive pressures. The underwriting activity as well as all the other aspects of the insurance business (e.g. claims handling) are supposed to be costless. Insurers earn fixed fees in a competitive market. The mere fact that they may transfer risks to risk neutral investors lead them to maximize the expected residual profit which is the difference between underwriting profits and policy dividends[11].

We assume that each individual can take out only one contract. An insurance contract is written as $(k, x)$ where $k$ is the insurance premium, $x$ is the net payout in case of an accident. Hence $x + k$ is the indemnity. Participating insurance contracts also specify how policy dividends are paid or supplementary premiums are levied. For example, in the simple case where each insurer only offers a single contract, policy dividend $D$ may be written as a proportion $\gamma$ of profit per policyholder $P$ (or more generally as a function of $P$), with $\gamma = 0$ for a non-participating contract and $\gamma = 1$ for a full participating contract[12]. Since individual risks are independently distributed, when contract $(k, x)$ is taken out by a large population of individuals, its average profit may be written as

$$P = \alpha_1 P_1 + \alpha_2 P_2,$$

where $\alpha_i$ is the proportion of type $i$ individuals among the purchasers of this contract, with $\alpha_1 + \alpha_2 = 1$ and $P_1, P_2$ respectively denote the expected profit made on high and low risk policyholders[13]. Using $D = \gamma P$ then allows us to write the expected utility of a policyholder as

$$Eu = (1 - \pi)u(W_N - k + \gamma P) + \pi u(W_A + x + \gamma P),$$

with $\pi = \pi_1$ or $\pi_2$ according to the policyholder's type.

In Section 3 we will confine ourselves to such simple linear policy dividend rules in which a given proportion of profit is shared among policyholders, with different formulations depending on whether each insurer offers a single contract or a menu of

[11] Indeed if an insurer could increase its residual expected profit (i.e. the expected corporate earnings after dividends have been distributed) by offering other insurance policies, then it could contract with risk neutral investors and secure higher fixed fees. Note that the residual profit of a mutual is nil if profits are distributed as policy dividends or losses are absorbed through supplementary premiums. In that case, if the mutual insurer could make positive residual profit, then he would benefit from becoming a stock insurer.

[12] Formally, a supplementary premium is equivalent to $D < 0$. Note that $\gamma \in (0, 1)$ is observed in life insurance markets when stock insurers distribute a part of profit as policy dividend.

[13] In other words, we use the law of large number to identify the average profit with the expected profit made on a policyholder who is randomly drawn among the customers.

contracts. We assume that individuals observe the profit made by their insurers, which makes policy dividend rules feasible[14]. A more general definition of dividend policy rules and underlying informational assumptions will be introduced in Section 4.

As a reminder, let us begin with a brief presentation of the RS model. Rothschild and Stiglitz restrict attention to non-participating contracts. An equilibrium in the sense of Rothschild and Stiglitz consists of a set of contracts such that, when individuals choose contracts to maximize expected utility, $(i)$: Each contract in the equilibrium set makes non-negative expected profit, and $(ii)$: There is no contract outside the equilibrium set that, if offered in addition to those in the equilibrium set, would make strictly positive expected profits. This concept of equilibrium may be understood as a pure strategy subgame perfect Nash equilibrium of a game where insurers simultaneously offer contracts and individuals respond by choosing one of the contracts (or refusing them all). At equilibrium, each contract makes zero profit and there is no profitable deviation at the contract offering stage, given the subsequent reaction of the insurance purchasers.

Let $C_i^* = (k_i^*, x_i^*) = (\pi_i A, A - \pi_i A)$ be the actuarially fair full insurance contract for a type $i$. Rothschild and Stiglitz show that there cannot be a pooling equilibrium where both groups would buy the same contract. Only a separating equilibrium can exist : different types then choose different contracts. They establish that the only candidate separating equilibrium is such that high risk individuals (i.e. types 1) purchase full insurance at fair price, i.e. they choose $C_1^*$, and low risk individuals (types 2) purchase a contract $C_2^{**}$ with partial coverage. $C_2^{**}$ is the contract that low risk individuals most prefer in the set of (fairly priced) contracts that do not attract high risk individuals: $C_2^{**} = (k_2^{**}, x_2^{**}) = (\pi_2 A', A' - \pi_2 A')$ with $A' \in (0, A)$ given by

$$u(W_N - \pi_1 A) = (1 - \pi_1)u(W_N - \pi_2 A') + \pi_1 u(W_A + (1 - \pi_2)A'). \tag{1}$$

Rothschild and Stiglitz also show that the candidate equilibrium $C_1^*, C_2^{**}$ is actually an equilibrium (in the sense of the above definition) if and only if $\lambda_1$ is large enough. The RS equilibrium is illustrated in Figure 1, with state-dependent wealth on each axis[15]. $W^1 = W_N - k$ and $W^2 = W_A + x$ respectively denote final wealth in the no-accident state and in the accident state. The no-insurance situation corresponds to point $E$ with coordinates $W^1 = W_N$ and $W^2 = W_A$. The high risk and low risk fair-odds line are labelled $EF_1$ and $EF_2$ in the figures, with slopes (in absolute value) respectively equal to $(1 - \pi_1)/\pi_1$ and $(1 - \pi_2)/\pi_2$. At $C_1^*$ the type 1 indifference curve is tangent to the type 1 fair-odds line $EF_1$. Similarly, $C_2^*$ is at a tangency point between a type 2 indifference curve and the type 2 fair-odds line $EF_2$. $C_2^{**}$ is at the intersection

[14] Of course, in practice, policyholders do as shareholders do : they trust the profit announcement released by mutual insurers. To be viable, participating policies and shareholding require truthful financial accounting and efficient corporate control mechanisms.

[15] When no ambiguity occurs, we use the same notation for insurance contracts $(k, x)$ and their images in the $(W^1, W^2)$ plane.

between $EF_2$ and the type 1 indifference curve that goes through $C_1^*$. $E\overline{F}$ in Figure 1 corresponds to the average fair-odds line with slope $(1-\overline{\pi})/\overline{\pi}$.

**Figure 1**

A pooling allocation with zero profit would correspond to a contract located on $E\overline{F}$, such as $C$ in Figure 2. However a pooling equilibrium cannot exist in the RS model because offering a contract like $C'$ would be a profitable deviation that would attract low risks only.

**Figure 2**

When $\lambda_1 = \lambda^*$, the low risk indifference curve that goes through $C_2^{**}$ is tangent to $E\overline{F}$. Hence when $\lambda_1 \geq \lambda^*$, as in Figure 1, the allocation $C_1^*, C_2^{**}$ is a separating equilibrium of the RS model: type 1 individuals choose $C_1^*$, types 2 choose $C_2^{**}$ and no insurer can make profit by offering a contract that would attract either one type or both types of individuals. Conversely when $\lambda_1 < \lambda^*$, as in Figure 3, the separating allocation $C_1^*, C_2^{**}$ is Pareto-dominated by a pooling allocation like $C'$ where all individuals choose the same contract. Hence there exists a profitable deviation in which the deviant insurer would attract all individuals and no equilibrium exists in this case.

**Figure 3**

The above given definition of an equilibrium assumes that each insurer can only offer one contract. At equilibrium some insurers offer $C_1^*$ and others offer $C_2^{**}$. When insurers are allowed to offer a menu of contract, then the definition of an equilibrium in the sense of Rothschild and Stiglitz consists of a set of menus that break even on average, such that there is no menu of contracts outside the equilibrium set that, if offered in addition, would make strictly positive expected profits. In a game theory setting, the equilibrium is a pure strategy subgame perfect Nash equilibrium of a game where insurers simultaneously offer menus, and then individuals respond either by selecting the contract they prefer in the menus offered in the market or refusing them all. At equilibrium, each menu makes zero aggregate profit and there is no profitable deviation at the menu offering stage, given the subsequent reaction of the insurance purchasers. At an equilibrium, the menu $(C_1^*, C_2^{**})$ is offered by all insurers: types 1 choose $C_1^*$ and types 2 choose $C_2^{**}$. Hence the set of equilibrium contracts is unchanged, with zero profit made on each contract. An equilibrium exists if and only if $\lambda_1 \geq \lambda^{**}$ where $\lambda^{**} \in (\lambda^*, 1)$. When $\lambda_1 < \lambda^{**}$ there exists a menu of incentive compatible contracts with cross-subsidization that breaks even and that Pareto-dominates $(C_1^*, C_2^{**})$ : no equilibrium exists in such a case. Hence the possibility of offering a menu increases the critical proportion of high risk individuals above which an equilibrium exists, because it enlarges the set of potentially profitable deviations. It thus makes the existence of equilibrium less likely.

# 3 Equilibrium with participating contracts

We first focus on the case where each insurer offers a single contract, before considering the more general setting where insurers can offer menus.

## 3.1 Case where each insurer offers one contract

We characterize the subgame perfect equilibrium of a two stage game. At stage 1 each insurer $j$ offers a contract $C^j = (k^j, x^j)$ with policy dividend $D^j = \gamma^j P^j$, where $P^j$ denotes the profit per policyholder and $\gamma^j \in [0,1]$. The contractual offer of insurer $j$ is thus characterized by $(C^j, \gamma^j)$. At stage 2, individuals respond by choosing the contract they prefer among the offers made by the insurers[16]. In other words, the only difference with the RS model is that we allow insurers to offer either participating or non-participating contracts. Because this is just an extension of the RS model obtained by suppressing a restriction on the set of feasible contracts, we will call it the extended RS model.

When the population of individuals who choose $(C^j, \gamma^j)$ includes type $i$ individuals in proportion $\alpha_i^j$ with $\alpha_1^j + \alpha_2^j = 1$, then we may draw the corresponding average fair-odds line, labelled $E\overline{F}^j$ in Figure 4. It goes through $E$ and its slope is $(1-\overline{\pi}^j)/\overline{\pi}^j$ in absolute value, with $\overline{\pi}^j = \alpha_1^j \pi_1 + \alpha_2^j \pi_2$. It coincides with $EF_1, EF_2$ or $E\overline{F}$ if $\alpha_1^j = 1, 0$ or $\lambda_1$. Contract $(C^j, \gamma^j)$ and proportions $(\alpha_1^j, \alpha_2^j)$ generate a lottery on final wealth. In Figure 4, point $C^j$ corresponds to the lottery $(W_0^{j1}, W_0^{j2})$ when $\gamma^j = 0$ , i.e. $W_0^{j1} = W_N - k^j$ and $W_0^{j2} = W_A + x^j$ [17]. When $\gamma^j = 1$ then $C^j$ generates a lottery $C_1^j = (W_1^{j1}, W_1^{j2})$ which is at the crossing between $E\overline{F}^j$ and a 45° line which goes through $C^j$. When $0 < \gamma^j < 1$ then the lottery associated with $C^j$ is located in the interior of the line segment $C^j C_1^j$, like $C_2^j$ in Figure 4.

**Figure 4**

As in the RS model, two types of equilibrium have to be considered in the extended RS model, either a separating equilibrium or a pooling equilibrium[18]. Consider first a separating equilibrium : types 1 and types 2 would then choose different contracts. We

[16] For any set of contracts $C = (C^1, ..., C^m)$ offered at stage 1, individuals have expectations about the risk type distribution of individuals who choose $C^j$ and thus about policy dividends $D^j$, for all $j = 1, ..., m$. When $C^j$ is actually chosen by some individuals, then expectations coincide with equilibrium values, and otherwise there is no restriction on (out of equilibrium) expectations.

[17] Of course the lottery depends on the policyholder's type: $W^1 = W_0^{j1}$ (resp. $W_0^{j2}$) with probability $\pi_i$ (resp. $1 - \pi_i$) for a type $i$ individual.

[18] We can check that individuals do not randomize at equilibrium. In particular there is no semi-separating equilibrium where two different contracts would be offered and all type 1 (resp. type 2) individuals would choose the same contract while type 2 (resp. type 1) individuals would be shared between both contracts.

know from the RS model that if a separating equilibrium exists, then the corresponding lotteries on final wealth should be $C_1^*$ for type 1 and $C_2^{**}$ for type 2, for otherwise a profitable deviation would exist. We also know from the RS model that there does not exist any profitable deviation through non-participating contracts if and only if $\lambda_1 \geq \lambda^*$. Since any deviation through a participating contract can be replicated by a deviation through a non-participating contract[19], we deduce that $\lambda_1 \geq \lambda^*$ is a necessary and sufficient condition for a separating equilibrium to exist.

Consider now a pooling equilibrium: type 1 and type 2 individuals then choose the same contract. The equilibrium lottery on final wealth is necessarily located on $E\overline{F}$. More specifically, this lottery has to be located at the tangency point between $E\overline{F}$ and a type 2 indifference curve: this is $\widehat{C}$ in Figure 5. Indeed, at any other point on $E\overline{F}$ it would be possible to make profit by deviating to a non-participating contract that would attract types 1 (and also possibly types 2), which would contradict the definition of an equilibrium. We also know from the RS model (see Figure 2) that a non-participating contract cannot be offered at such a pooling equilibrium, for otherwise there would exist a profitable deviation attracting types 2, while types 1 would keep choosing the same contract. Let us focus on a symmetric pooling equilibrium with a participating contract offered by each insurer and such that $\gamma^1 = \gamma^2 = ... = \gamma^m = \gamma$.

**Figure 5**

W.l.o.g. consider a deviation where an insurer offers a non-participating contract $C'$, while other insurers keep offering $\widehat{C}$ [20]. $\widehat{C}$ is a pooling equilibrium if for any deviation $C'$ there exists a continuation equilibrium[21] which makes it unprofitable. We may restrict attention to deviations such that types 2 choose $C'$ and types 1 keep choosing $\widehat{C}$ [22]. Figures 5 and 6 represent what happens to type 1 in such a deviation. As illustrated in Figure 5, contract $\widehat{C}$ will then generate a lottery $\widehat{C}_1$ if $\gamma = 1$ and a

[19] Replicating a deviation means that there exists a non-participating contract that would induce an amount of profit for the deviant which is equal to the residual profit (i.e. profit after policy dividends have been paid) obtained with participating contract offered in deviation from equilibrium. Indeed assume for instance that insurer $j$ deviates from its equilibrium strategy to the participating contract $C^j = (k^j, x^j)$, with a proportion of profit $\gamma^j$ distributed as policy dividends. Let $P^j$ be the profit of insurer $j$ at a continuation equilibrium following this deviation to $C^j$ (i.e. at a stage 2 allocation where all individuals make optimal contract choices given the available contract offer) and consider the non-participating contract $C^{j\prime} = (k^j - \gamma^j P^j, x^j + \gamma^j P^j)$. At a continuation equilibrium following the deviation to $C^{j\prime}$, the profit of insurer $j$ would be equal to its residual profit after the deviation to $C^j$.

[20] Once again any deviation through a participating contract could be replicated by a deviation through a non-participating contract.

[21] A continuation equilibrium is a Nash equilibrium of the stage 2 subgame that follows some contract offers made by insurers at stage 1.

[22] Obviously $C'$ cannot be profitable if it attracts only type 1 individuals. Furthermore, there exist out of equilibrium expectations on the risk type of individuals who choose $\widehat{C}$ such that $C'$ cannot be profitable at a continuation equilibrium where $C'$ attracts both types : for instance expectations stipulating that $\widehat{C}$ is chosen by type 2 individuals only.

lottery like $\widehat{C}_2$ on the $\widehat{C}\widehat{C}_2$ line segment if $0 < \gamma < 1$. Hence, if $C'$ is in the grey area in Figure 5 , then any continuation equilibrium is such that types 2 choose $C'$, types 1 choose $\widehat{C}$, and the deviant insurer makes positive profit. Thus no pooling equilibrium exists in such a case. For $\lambda_1$ and $\lambda_2$ given (and thus for $\widehat{C}$ given), the grey area in Figure 5 is shrinking when $\gamma$ is increasing. When looking for the existence of a pooling equilibrium we may thus restrict attention to the case where $\gamma = 1$ since it corresponds to the smallest set of profitable deviations[23]. Observe that the type 2 expected utility is lower at any lottery in the grey area than at $C_2^{**}$. Thus $\lambda_1 \leq \lambda^*$ is a sufficient condition for the grey area to vanish. In that case (which is represented in Figure 6) there does not exists any deviation $C'$ with positive profit at all continuation equilibrium and thus a pooling equilibrium exists. In the case drawn in Figure 5 deviations located in the grey area are profitable at any continuation equilibrium and there is no pooling equilibrium. We may observe that a pooling equilibrium at $\widehat{C}$ coexists with a separating equilibrium at $C_1^*, C_2^{**}$ when $\lambda_1$ is larger but close to $\lambda^*$.

**Figure 6**

**Proposition 1** *When $n = 2$ an equilibrium always exists in the extended RS model with a single contract per insurer. A participating contract is offered at a pooling equilibrium, while contracts may be participating or non-participating at a separating equilibrium. A separating equilibrium exists when $\lambda_1 \geq \lambda^*$ with the same pair of contracts $C_1^*$,$C_2^{**}$ and the same individual choices as in the RS model. There exists a pooling equilibrium where all individuals choose $\widehat{C}$ when $\lambda_1 \leq \lambda^*$. When $\lambda_1$ is larger than $\lambda^*$ but close to $\lambda^*$, a pooling equilibrium at $\widehat{C}$ coexists with a separating equilibrium at $C_1^*$,$C_2^{**}$.*

Proposition 1 states that an equilibrium always exists in the extended RS model with a single contract per insurer. It coincides with the Rothschild-Stiglitz separating allocation when the proportion of high risks is large and it is a pooling allocation when this proportion is low, with an overlap of the two regimes. The pooling allocation maximizes the type 2 expected utility among the allocations that break even on aggregate and it should be sustained by a participating contract, contrary to the separating allocation that may be sustained by participating or non-participating contracts.

[23]Note that when $\gamma = 1$ the equilibrium contract (i.e. the lottery without any sharing of profit) could be located at any point on the 45° line that goes through $\widehat{C}$. In particular if the equilibrium contract were at $\widehat{C}_1$ then policy dividends would always be non-negative even if low risk individuals were attracted by deviant insurers. Setting premiums at levels that allow to pay dividends is the common practice of the socalled *advance premium mutuals* because (contrary to *assessment mutuals*) such mutuals cannot charge additional premiums if losses were greater than expected.

### 3.2 Case where insurers offer menus of contracts

Let us assume now that each insurer $j$ offers a menu of contracts $(C_1^j, C_2^j)$ where $C_i^j$ is chosen by type $i$ individuals. A menu is said to be participating if it involves some policy dividend rule for at least one contract[24]. Because of the information asymmetry between insurers and insureds, the lotteries on final wealth should satisfy incentives constraints, i.e. type 1 individuals should weakly prefer the lottery generated by $C_1^j$ to the lottery generated by $C_2^j$ and conversely for type 2.

The lotteries generated at a candidate equilibrium of the extended RS model with menus should maximize the expected utility of type 2 individuals in the set of lotteries that break even and that satisfy incentive compatibility constraints[25]. We may refer to these lotteries as the Miyazaki-Spence-Wilson (MSW) allocation because they correspond to the equilibrium contracts under the anticipatory equilibrium hypothesis introduced by Wilson (1977) and Miyazaki (1977) and further developped by Spence (1978). We know from Crocker and Snow (1985) that there exists a threshold $\lambda^{**}$ in $(\lambda^*, 1)$ such that the MSW allocation coincide with the Rothschild-Stiglitz pair of contracts $C_1^*$,$C_2^{**}$ without cross-subsidization when $\lambda_1 \geq \lambda^{**}$, while it involves cross-subsidization between contracts when $\lambda_1 \leq \lambda^{**}$. This is illustrated in Figures 7 and 8. $H$ is at the crossing between the average fair odds line $E\overline{F}$ and the 45° line and the set of second-best Pareto-optimal lotteries is represented by locus $HC_2^{**}$ [26]. More precisely, a part of the $HC_2^{**}$ curve corresponds to the contracts which are taken out by type 2 individuals at second-best Pareto optimal allocations. Type 1 individuals then get full coverage and their incentive compatibility constraint is binding. The type 2 lottery moves from $C_2^{**}$ to $H$ when the type 1 lottery moves from $C_1^{**}$ to $H$. Figure 7 corresponds to the case where $\lambda_1 \geq \lambda^{**}$. In that case, $C_2^{**}$ maximizes the type 2 expected utility on the $HC_2^{**}$ line, which means that $C_1^*$,$C_2^{**}$ is a second-best Pareto optimal allocation. In such a case, there exists an equilibrium in the RS model, and it is obviously still the case in the extended RS model: the equilibrium coincides with the Rothschild-Stiglitz allocation, with full coverage at $C_1^*$ for type 1 and partial coverage at $C_2^{**}$ for type 2. Risk type separation can then be obtained through menus

[24] For instance, policy dividends paid to the individuals who have chosen $C_1^j$ may depend on the profit made on $C_1^j$ only, or on the aggregate profit on $(C_1^j, C_2^j)$, while the holders of $C_2^j$ may have no right to dividends.

[25] Obviously the menu offered by insurers should break even, for otherwise insurers would deviate to a "zero contract" without indemnity and premium, i.e. they would exit the market. The equilibrium menu could neither make positive profit because in such a case there would exist a profitable deviation in which an insurer would attract all insureds by slightly decreasing the premiums of both contracts. If the expected utility of type 2 individuals were not maximized in the set of incentive compatible allocations that break even, then it would be possible to attract individuals from the type 2 group by offering a menu of non-participating contracts that would be profitable even if it also attracts type 1 individuals, hence a contradiction with the definition of an equilibrium.

[26] See Crocker and Snow (1985) and Dionne and Fombaron (1996).

of participating or non-participating contracts or a mixture of both contractual forms.

**Figure 7**

The case where $\lambda_1 < \lambda^{**}$ is drawn in Figure 8. The type 2 expected utility is maximized in the set of second-best allocations at $\widetilde{C}_2$ where a type 2 indifference curve is tangent to the $HC_2^{**}$ locus. Lottery $\widetilde{C}_1$ should then be attributed to type 1 individuals. $\widetilde{C}_1$ is a full coverage policy chosen by type 1 individuals. It is cross-subsidized by $\widetilde{C}_2$, which is a partial coverage policy chosen by types 2. $(\widetilde{C}_1, \widetilde{C}_2)$ is thus the candidate equilibrium menu. Assume that $(\widetilde{C}_1, \widetilde{C}_2)$ is a menu of participating contracts with full distribution of profits or repayment of losses uniformly among policyholders[27]. Any menu of contracts offered by a deviant insurer can be profitable only if it includes a contract $C'$ which only attracts individuals from the type 2 group and, here also, we may assume w.l.o.g. that $C'$ is a non-participating contract. When such an offer is made, the type 1 lottery shifts from $\widetilde{C}_1$ to $C_1^*$ since type 1 individuals are now the only customers of the non-deviant insurers and $\widetilde{C}_1$ is a participating contract with full repayment of losses by policyholders. If $C'$ is profitable when chosen by types 2 and not attractive for types 1, then type 2 individuals reach an expected utility which is (weakly) lower than at $C_2^{**}$, and thus lower than at $\widetilde{C}_2$ when $\lambda_1 < \lambda^{**}$. Hence, following such a deviation where $C'$ is offered, there exists a continuation equilibrium where type 2 individuals keep choosing $\widetilde{C}_2$ and the deviant insurer doesn't make any profit[28]. We conclude that $\widetilde{C}_1, \widetilde{C}_2$ is a separating equilibrium with cross-subsidization between contracts when $\lambda_1 < \lambda^{**}$.

**Figure 8**

**Proposition 2** *When $n = 2$ an equilibrium always exists in the extended RS model with menus. It induces an equilibrium allocation, which coincides with the MSW allo-*

[27] Other policy dividend distribution rules are possible without affecting our conclusions. For example dividends may be proportional to premiums. Note also that the equilibrium premium levels are defined up to an additive constant since a uniform increase in premiums for all policies would be compensated by additional dividends. In particular, for high enough premiums, policy dividends would remain positive for a set of deviations by competitors. This is a way to reproduce the behavior of *advance premium mutuals*, since these mutuals are not allowed to charge additional premiums (which would be equivalent to negative dividends in our model). Charging higher premiums and paying positive dividends is also a way of improving the credibility of the insurer-policyholder financial relationship, when the latter may claim (rightly or wrongly) to suffer liquidity constraints in order to postpone or cancel increases in premium.

[28] The same kind of argument extends to the case where only some type 2 individuals choose the deviating contract $C'$. In that case, the type 1 lottery shifts from $\widetilde{C}_1$ to an intermediate position between $C_1^*$ and $\widetilde{C}_1$. Here also there exists a continuation equilibrium where type 2 individuals do not choose $C'$. The existence of equilibrium thus goes through the fact that there always exists a continuation equilibrium where the deviant insurer does not make profit. We should not conclude that deviations are not profitable at *all* continuation equilibria. Section 4 will show that more stringent conditions on policy dividend rules are required to get this result.

*cation, i.e. it maximizes the type* 2 *expected utility under the zero-profit constraint and incentive compatibility conditions. When* $\lambda_1 \geq \lambda^{**}$*, the pair of separating contracts* $(C_1^*, C_2^{**})$ *is offered at equilibrium without cross-subsidization as in the RS model and it may be participating or non-participating. When* $\lambda_1 < \lambda^{**}$*, the equilibrium pair of separating contracts* $(\widetilde{C}_1, \widetilde{C}_2)$ *is participating with cross-subsidization :* $\widetilde{C}_1$ *is chosen by type* 1 *individuals and it is in deficit, while* $\widetilde{C}_2$ *is chosen by type* 2 *individuals and it is profitable.*

Proposition 2 states that an equilibrium exists in the extended RS model with menus. The equilibrium allocation coincides with the MSW allocation. When $\lambda_1 < \lambda^{**}$ high risks are cross-subsidized by low risks and the the equilibrium involves participating contracts, while there is no cross-subsidization and participating contracts are not required when $\lambda_1 \geq \lambda^{**}$.

# 4 The $n$-type problem

We now assume that there is an arbitrary number of $n$ risk types in the population. More heavy notations are required to precisely describe the market game. As before, each insurer offers a menu of participating or non-participating contracts at the first stage and individuals respond by choosing their prefered policy at the second stage. We assume that individuals can observe the profit and the market share of each contract, which makes a large scope of policy dividend rules possible athough, as we will see, policy dividend rules that are less demanding about information may well be offered at equilibrium.

A strategy of insurer $j$ is defined by a menu of $n$ contracts, one for each type of individual, written as $C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$ where $C_h^j = (k_h^j, x_h^j)$ specifies the premium $k_h^j$ and the net indemnity $x_h^j$. $D^j(.)$ is a policy dividend strategy, i.e. a way to distribute the net profits made on $C^j$, with $D^j(.) = (D_1^j(.), ..., D_n^j(.))$, where $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j)$ denotes the policy dividend paid to each individual who has chosen contract $C_h^j$ when $N_i^j$ individuals (expressed as a proportion of the whole population) have chosen contract $C_i^j$ with average profit per policyholder $P_i^j$, with $i = 1, ..., n$ and $\sum_{j=1}^{m}\sum_{i=1}^{n} N_i^j = 1$ [29]. $C^j$ is non-participating if $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv 0$ for all $h$ and otherwise it is said to be participating. In particular $C^j$ is fully participating if[30]

$$\sum_{h=1}^{n} N_h^j D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv \sum_{h=1}^{n} N_h^j P_h^j$$

[29] Once again $D_h^j < 0$ corresponds to a supplementary premium levied on $C_h^j$. Note that a policy dividend rule need not be linear in profits or losses.

[30] $C^j$ may be fully participating with $D_h^j \equiv 0$ for some $h$. In other words, a fully participating menu may include non-participating policies.

We will restrict attention to policy dividend rules such that $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j)$ is homogeneous of degree zero with respect to $(N_1^j, ..., N_n^j)$. Besides making the analysis easier[31], this homogeneity condition fits with the standard policy dividend rules we may think of, such as distributing the aggregate profits of the insurer to policyholders with distribution weights that depend on insurance contracts or distributing the profit made on a given contract evenly among its own policyholders. We may then write the policy dividend as $D_h^j = D_h^j(\theta_1^j, P_1^j, ..., \theta_n^j, P_n^j)$,where $\theta_h^j \equiv N_h^j / \sum_{i=1}^n N_i^j$ is the fraction of $C_h^j$ holders among insurer $j$'s customers, with $\sum_{h=1}^n \theta_h^j = 1$.

Let $C \equiv (C^1, C^2, ..., C^m)$ be the profile of contract menus offered in the market. The strategy of a type $i$ individual specifies for all $j$ and all $h$ the probability $\sigma_{ih}^j(C)$ to choose $C_h^j$ as a function of $C$. The contract choice strategy of type $i$ individual is thus defined by $\sigma_i(C) \equiv \{\sigma_{ih}^j(C) \in [0,1]$ for $j = 1, ..., m$ and $h = 1, ..., n$ with$\displaystyle\sum_{j=1}^m \sum_{h=1}^n \sigma_{ih}^j(C) = 1\}$ for all $C$. Let $\sigma(.) \equiv (\sigma_1(.), \sigma_2(.), ..., \sigma_n(.))$ be a profile of individuals' strategies.

When an insurance contract $C_h^j = (k_h^j, x_h^j)$ is taken out by a type $i$ individual, with (non-random) policy dividend $D_h^j$, the policyholder's expected utility and the corresponding insurer's profit are respectively written as

$$\begin{aligned}
&U_i(C_h^j, D_h^j) \equiv (1 - \pi_i)u(W_N - k_h^j + D_h^j) + \pi_i u(W_A + x_h^j + D_h^j), \\
&\Pi_i(C_h^j) \equiv (1 - \pi_i)k_h^j - \pi_i x_h^j.
\end{aligned}$$

**Definition 1** *A profile of strategies* $\widetilde{\sigma}(.), \widetilde{C} \equiv (\widetilde{C}^1, \widetilde{C}^2, ..., \widetilde{C}^m)$, *where* $\widetilde{C}^j = (\widetilde{C}_1^j, \widetilde{C}_2^j, ..., \widetilde{C}_n^j, \widetilde{D}^j(.))$, *is a subgame perfect Nash equilibrium of the market game if :*

$$\sum_{j=1}^m \sum_{h=1}^n \widetilde{\sigma}_{ih}^j(C) U_i(C_h^j, \overline{D}_h^j(C)) = \max\{U_i(C_h^j, \overline{D}_h^j(C)); j = 1, ..., m, h = 1, ..., n\}$$

$$\text{for all } i = 1, ..., n \text{ and all } C \tag{2}$$

$$\overline{\Pi}^j(\widetilde{C}) \geq \overline{\Pi}^j(C^j, \widetilde{C}^{-j}) \text{ for all } C^j \text{ and all } j = 1, ..., m \tag{3}$$

*where* $C \equiv (C^1, C^2, ..., C^m)$,$C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$, $\widetilde{C}^{-j} = (\widetilde{C}^1, ..., \widetilde{C}^{j-1}, \widetilde{C}^{j+1}, ..., \widetilde{C}^m)$ *and*

$$\overline{D}_h^j(C) \equiv D_h^j(\overline{\theta}_1^j(C), \overline{P}_1^j(C), ..., \overline{\theta}_n^j(C), \overline{P}_n^j(C)), \tag{4}$$

$$\overline{\Pi}^j(C) \equiv \sum_{i=1}^n \sum_{h=1}^n \lambda_i \widetilde{\sigma}_{ih}^j(C)[\Pi_i(C_h^j) - \overline{D}_h^j(C)], \tag{5}$$

[31] Easiness relates to the modelling of (out of equilibrium) beliefs about policy dividends for contracts that are not chosen by any individual. See Definition 1 below.

$$
\begin{aligned}
\overline{\theta}^j_h(C) &= \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}^j_{ih}(C)}{\sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}^j_{ik}(C)} \quad \textit{for all } h \textit{ if } \quad \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}^j_{ik}(C) > 0, \\
\overline{P}^j_h(C) &= \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}^j_{ih}(C) \Pi_i(C^j_h)}{\sum_{i=1}^n \lambda_i \widetilde{\sigma}^j_{ih}(C)} \quad \textit{for all } h \textit{ if } \quad \sum_{i=1}^n \lambda_i \widetilde{\sigma}^j_{ih}(C) > 0, \\
\overline{\theta}^j_h(C) &\geq 0 \textit{ for all } h \textit{ and } \sum_{h=1}^n \overline{\theta}^j_h(C) = 1 \textit{ if } \quad \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}^j_{ik}(C) = 0, \\
\overline{P}^j_h(C) &\in [\Pi_1(C^j_h), ..., \Pi_n(C^j_h)] \quad \textit{if } \sum_{i=1}^n \lambda_i \widetilde{\sigma}^j_{ih}(C) = 0.
\end{aligned}
$$

The notations in Definition 1 are as follows. Consider a profile of contracts $C = (C^1, ..., C^m)$ where $C^j = (C^j_1, C^j_2, ..., C^j_n, D^j(.))$ is the menu offered by insurer $j$. $\overline{\theta}^j_h(C)$ is the proportion of insurer $j'$s policyholders who choose $C^j_h$ when $C$ is offered, with $\overline{P}^j_h(C)$ the corresponding profit per policyholder. When insurer $j$ attracts policyholders, then $\overline{\theta}^j_h(C)$ and $\overline{P}^j_h(C)$ are derived from individuals' contract choice strategy. Otherwise, $\overline{\theta}^j_h(C)$ and $\overline{P}^j_h(C)$ are beliefs that fullfill the coherency conditions stated in Definition 1. Then $\overline{D}^j_h(C)$ and $\overline{\Pi}^j(C)$ defined by (4) and (5) respectively denote the policy dividend for contract $C^j_h$ and the residual profit of insurer $j$. They depend on the set of contracts $C$ offered in the market and on the profile of individuals'contract choice strategy $\widetilde{\sigma}(.)$. In particular $\overline{D}^j_h(C) = \widetilde{D}^j_h(\overline{\theta}^j_1(C), P^j_1(C), ..., \overline{\theta}^j_n(C), P^j_n(C))$ if $C^j = \widetilde{C}^j$.

Keeping these notations in mind, (2) and (3) correspond to the standard definition of a subgame perfect Nash equilibrium. From (2), choosing $C^j_h$ with probability $\widetilde{\sigma}^j_{ih}(C)$ is an optimal contract choice for type $i$ individuals. (3) means that $\widetilde{C}^j$ is an optimal offer by insurer $j$ (i.e. an offer that maximizes residual profit) when $\widetilde{C}^{-j}$ is offered by the other insurers, given the contract choice strategy of individuals.

Let $C^*$ denote the menu of contracts at a *symmetric equilibrium of the market game* (defined as an equilibrium where all insurers offer the same menu and individuals are evenly shared between insurers), with $\widetilde{C}^1 = \widetilde{C}^2 = ... = \widetilde{C}^m = C^* = (C^*_1, C^*_2, ..., C^*_n, D^*(.))$ and $C^*_h = (k^*_h, x^*_h)$ for all $h = 1, ..., n$ and $D^*(.) \equiv (D^*_1(.), ..., D^*_n(.))$. If individuals do not randomize between contracts (i.e. all individuals of a given type choose the same contract), $C^*_i = (k^*_i, x^*_i)$ denotes the contract chosen by type $i$ individuals.

A symmetric equilibrium of the market game sustains an *equilibrium allocation* $\{(W^{1*}_i, W^{2*}_i), i = 1, ..., n\}$, where $(W^{1*}_i, W^{2*}_i)$ is the lottery on final wealth induced by the equilibrium strategies for type $i$ individuals (meaning that their final wealth is $W_i^{1*}$ with probability $1-\pi_i$ and $W_i^{2*}$ with probability $\pi_i$) with

$$\begin{aligned} W_i^{1*} &= W_N - k_i^* + D_i^*, \\ W_i^{2*} &= W_A + x_i^* + D_i^*, \end{aligned}$$

where

$$D_i^* \equiv D_i^*(\lambda_1, \Pi_1^*, ..., \lambda_n, \Pi_n^*) \text{ with } \Pi_i^* \equiv \Pi_i(C_i^*).$$

To establish the existence of such a symmetric equilibrium of the market game, we first characterize a candidate equilibrium allocation by following the Spence (1978) approach to the Miyazaki-Wilson equilibrium with an arbitrary number of types, and next we show that this allocation is sustained by strategy profiles which are a subgame perfect Nash equilibrium of the market game.

As Spence (1978), let us first define a sequence of expected utility levels $\overline{u}_i$ as follows :

$$\begin{aligned} &\overline{u}_1 = \max(1-\pi_1)u(W^1) + \pi_1 u(W^2) \\ &\text{with respect to } W^1, W^2, \text{ subject to} \\ &(1-\pi_1)W^1 + \pi_1(W^2 + A) = W_N \end{aligned}$$

and for $2 \leq k \leq n$, $\overline{u}_i$ is defined as

$$\begin{aligned} &\overline{u}_i = \max(1-\pi_i)u(W_i^1) + \pi_i u(W_i^2) \\ &\text{with respect to } W_h^1, W_h^2, h = 1, ..., i \text{ , subject to} \\ &(1-\pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq \overline{u}_h \text{ for } h < i, && (6) \\ &(1-\pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq (1-\pi_h)u(W_{h+1}^1) + \pi_h u(W_{h+1}^2) \text{ for } h < i, && (7) \\ &\sum_{h=1}^{i} [(1-\pi_h)W_h^1 + \pi_h(W_h^2 + A)] = W_N. && (8) \end{aligned}$$

Let $\mathbb{P}_i$ denote the problem which defines $\overline{u}_i$, with $i = 1, ..., n$. The objective function in $\mathbb{P}_i$ is the expected utility of type $i$ individuals by restricting attention to individuals with types 1 to $i$. Constraints (6) ensure that higher risk individuals (i.e. $h < i$) get expected utility no less than $\overline{u}_h$. (7) are incentive compatibility constraints : type $h$ individuals (with $h < i$) are detered from choosing the policy targeted for type $h+1$. (8) is the break-even constraint. For $n = 2$, the optimal solution to $\mathbb{P}_2$ is the Miyazaki-Wilson equilibrium allocation considered in Section 3. Let $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ be the optimal solution to $\mathbb{P}_n$.

The sequence $\overline{u}_i, i = 1, ..., n$ corresponds to reservation utilities. Indeed if type $i$'s expected utility were lower than $\overline{u}_i$, then it would be possible to make positive profit by attracting type $i$ individuals and all more risky types $h$. Maximizing type $i$ expected utility in $\mathbb{P}_i$ may require to increase the expected utility of more risky types over their reservation utility $\overline{u}_h$, in order to relax their incentive compatibility constraints. The optimal solution to $\mathbb{P}_i$ involves a trade off between reducing the cost of more risky types and relaxing their incentive compatibility constraints. This trade off may tip in favor of the reduction of cost or of the relaxation of incentive compatibility constraints. Constraint (6) is binding in the first case and it is slack in the second one. Lemma 1, which is adapted from Spence (1978), characterizes the optimal solution to this trade off in $\mathbb{P}_n$.

**Lemma 1** *There exists $\ell_t \in \{0,...,n\}$, $t = 0,...,\overline{t}+1$ with $\ell_0 = 0 \leq \ell_1 \leq \ell_2 ... \leq \ell_{\overline{t}} < \ell_{\overline{t}+1} = n$ such that for all $t = 0,...,\overline{t}$*

$$\begin{aligned}\sum_{i=\ell_t+1}^{h} \lambda_i[W_N - (1-\pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] &< 0 \quad \text{for all } h = \ell_t+1,...,\ell_{t+1}-1,\\ \sum_{i=\ell_t+1}^{\ell_{t+1}} \lambda_i[W_N - (1-\pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] &= 0.\end{aligned}$$

*Furthermore, we have*

$$\begin{aligned}(1-\pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) &= \overline{u}_i \text{ if } i \in \{\ell_1,\ell_2,...,n\},\\ (1-\pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) &> \overline{u}_i \quad \text{otherwise.}\end{aligned}$$

Lemma 1 states that risk types are pooled in $\overline{t}+1$ subgroups indexed by $t$. Subgroup $t$ includes risk types $h = \ell_t+1,...,\ell_{t+1}$ with $\ell_0 = 0$ and $\ell_{\overline{t}+1} = n$. Within each subgroup $t$, all types $h$ except the highest (i.e. $h = \ell_t+1,...,\ell_{t+1}-1$) get more than their reservation utility $\overline{u}_h$, with negative profit over this subset of individuals. They are cross-subsidized by the highest risk type, i.e. by type $\ell_{t+1}$). Type $\ell_{t+1}$ just reaches its reservation utility $\overline{u}_{\ell_{t+1}}$ with zero profit over the whole subgroup. Increasing the expected utility of type $\ell_{t+1}$ over $\overline{u}_{\ell_{t+1}}$ would be suboptimal.

In what follows $I$ will denote the set of risk types in subgroups with cross-subsidization, i.e.

$$\begin{aligned}i &\in I \subset \{1,...,n\} \text{ if } \ell_t < i \leq \ell_{t+1}\\ \text{for } t &\in \{0,...,\overline{t}\} \text{ such that } \ell_{t+1} - \ell_t \geq 2.\end{aligned}$$

Lemma 1 is illustrated in Figure 9, with $n = 5, \overline{t} = 2, \ell_1 = 3$ and $\ell_2 = 4$. There are three subgroups in this example: type $i = 3$ cross-subsidizes types 1 and 2, while the contracts offered to types 4 and 5 make zero profit. We thus have $I = \{1,2,3\}$ and $u_h > \overline{u}_h$ for $h = 1,2$ and $u_h = \overline{u}_h$ for $h = 3,4$ and 5, where $u_h$ is the type $h$ expected utility at the optimal solution to $\mathbb{P}_n$.

**Figure 9**

**Lemma 2** *There does not exist any incentive compatible allocation* $\{(W_h^1, W_h^2), h = 1,...,n\}$ *such that*

$$(1-\pi_{\ell_t})u(W_{\ell_t}^1) + \pi_{\ell_t}u(W_{\ell_t}^2) \geq \overline{u}_{\ell_t} \text{ for all } t = 1,...,\overline{t}+1 \tag{9}$$

*and*

$$\sum_{h=1}^{n} \lambda_h[W_N - (1-\pi_h)W_h^1 - \pi_h(W_h^2 + A)] > 0. \tag{10}$$

Lemma 2 states that no insurer can make positive profit by attracting all individuals and offering more than $\overline{u}_{\ell_t}$ to threshold types $\ell_t$. Indeed suppose that there exists a profitable allocation close to $\{(\widehat{W}_h^1, \widehat{W}_h^2), h = 1,...,n\}$ that provides more than $\overline{u}_{\ell_t}$ to types $\ell_t$. Such an allocation would provide an expected utility larger than $\overline{u}_h$ for all $h$ (this is just a consequence of the second part of Lemma 1), which would contradict the definition of $\overline{u}_n$. The proof of Lemma 2 - which follows Spence (1978) - shows that this argument extends to allocations that are not close to $\{(\widehat{W}_h^1, \widehat{W}_h^2), h = 1,...,n\}$. The main consequence of Lemma 2 is that it is impossible to make positive profit in a deviation from $\{(\widehat{W}_h^1, \widehat{W}_h^2), h = 1,...,n\}$ if threshold types $\ell_t$ are guaranteed to get at least $\overline{u}_{\ell_t}$.

Lemmas 1 and 2 easily extend to allocations where individuals of a given type may randomize between contracts that are equivalent for him. An allocation is then a type-dependent randomization over a set of lotteries. Formally, an allocation is defined by a set of lotteries $\{(W_s^1, W_s^2), s = 1,...,N\}$ and individuals' choices $\sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)$ with $\sigma_i = (\sigma_{i1}, ..., \sigma_{iN})$, where $\sigma_{is}$ is the probability that a type $i$ individual chooses $(W_s^1, W_s^2)$, with $\sum_{s=1}^{N} \sigma_{is} = 1$. In other words, type $i$ individuals get a compound lottery $\widetilde{W}_i$ generated by their mixed strategy $\sigma_i$ over available lotteries $\{(W_s^1, W_s^2), s = 1,...,N\}$ [32]. An allocation is incentive compatible if

$$\sum_{s=1}^{N} \sigma_{is}[(1-\pi_i)u(W_s^1) + \pi_i u(W_s^2)] = \max\{(1-\pi_i)u(W_s^1) + \pi_i u(W_s^2), s = 1,...,N\},$$

for all $i = 1,...,n$. In words, an allocation is incentive compatible when individuals only choose their best contract with positive probability. The definition of Problem $\mathbb{P}_i$ for $i = 1,...,n$ can be extended straightforwardly to this more general setting, with unchanged definition of $\overline{u}_i$. In particular, individuals choose only one (non compound) lottery at the optimal solution to $\mathbb{P}_i$, and the optimal solution of $\mathbb{P}_n$ is still $\widetilde{W}_i = \widehat{W}_i$ for $i = 1,...,n$ [33]. Lemma 1 is thus still valid. Lemma 3 straighforwardly extends Lemma 2 to the case where individuals may randomize between contracts.

[32] $N$ is given but arbitrarily large.

[33] Consider a randomized allocation which is feasible in $\mathbb{P}_h$ and replace it by another allocation without randomization where individuals choose (with probability 1) the most profitable lottery

**Lemma 3** *There does not exist any incentive compatible allocation with randomization* $\{(W_s^1, W_s^2), s = 1, ..., N; \sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)\}$ *such that*

$$\sum_{s=1}^{N} \sigma_{\ell_t,s}[(1-\pi_{\ell_t})u(W_s^1) + \pi_{\ell_t}u(W_s^2)] \geq \overline{u}_{\ell_t} \quad \text{for all } t = 1, ..., \overline{t}+1 \tag{11}$$

*and*

$$\sum_{h=1}^{n} \lambda_h \{\sum_{s=1}^{N} \sigma_{hs}[W_N - (1-\pi_h)W_s^1 - \pi_h(W_s^2 + A)]\} > 0. \tag{12}$$

We are now in position to establish the existence of a symmetric equilibrium in the extended RS model with an arbitrary number of types.

**Proposition 3** $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ *is an equilibrium allocation sustained by a symmetric equilibrium of the market game where type* $i$ *individuals choose* $C_i^* = \widehat{C}_i \equiv (\widehat{k}_i, \widehat{x}_i)$ *with* $\widehat{k}_i = W_N - \widehat{W}_i^1, \widehat{x}_i = \widehat{W}_i^2 - W_A$ *and* $D_i^*(.)$ *is such that*

$$\sum_{i\in I} N_i D_i^*(N_1, P_1, ..., N_n, P_n) \equiv \sum_{i\in I} N_i P_i, \tag{13}$$

$$D_i^*(\lambda_1, \Pi_1(\widehat{C}_1), ..., \lambda_n, \Pi_n(\widehat{C}_n)) = 0 \quad \text{for all } i = 1, ..., n, \tag{14}$$

$$D_{\ell_t}^*(N_1, P_1, ..., N_n, P_n) \equiv 0 \text{ for all } t = 1, ..., \overline{t}+1 \tag{15}$$

At the symmetric equilibrium of the market game described in Proposition 3, each insurer offers $C^* = (\widehat{C}_1, ..., \widehat{C}_n, D^*(.))$ and type $i$ individuals choose $\widehat{C}_i$. The conditions on $D^*(.)$ are sufficient for $C^*$ to be an equilibrium contract offer. (13) means that profits are fully distributed among the individuals who choose a contract with cross-subsidization at equilibrium, and from (14) no policy dividend is paid on the equilibrium path. From (15) threshold types $\ell_t$ are excluded from the sharing of profits.

Proposition 3 requires some comments. First, to intuitively understand how it is deduced from Lemma 3, consider an allocation induced by $C^{j_0} \neq C^* = (C_1^*, ..., C_n^*)$ offered by a deviant insurer $j_0$. It corresponds to a compound lottery that mixes $C^{j_0}$ and $C^*$. The aggregate residual profit of this allocation is larger or equal to the profit made by on $C^{j_0}$ alone, because non-deviant insurers $j \neq j_0$ offer a menu of contracts with full distribution of profits or payment of losses on $\{\widehat{C}_i, i \in I\}$ and non-negative profits on $\{\widehat{C}_i, i \notin I\}$[34]. Furthermore, Condition (15) assures that all threshold types

which they choose with positive probability in the initial lottery. By doing that, we relax the profit constraint and other constraints still hold in $\mathbb{P}_h$. This shows that individuals do not randomize at an optimal solution to $\mathbb{P}_h$. This argument is also used in the proof of Lemma 3.

[34] From (15), no policy dividend is paid to the holders of $\widehat{C}_i$ and $\widehat{C}_{i-1}$ when $i \notin I$, which implies that types $h$, $h \leq i$, do not choose $\widehat{C}_i$ at a continuation equilibrium that follows the deviation by insurer $j_0$. Hence the fact that non-deviant insurers $j \neq j_0$ makes non negative profit on $\widehat{C}_i$. See the proof of Proposition 3 in the Appendix for details.

$\ell_t$ get at least $\overline{u}_{\ell_t}$. Lemma 3 shows that this allocation cannot be profitable, hence deviant insurer $j_0$ does not make positive profit.

Proposition 2, which was valid when $n = 2$, didn't require an assumption like (15), but its proof went through the existence of a non-profitable continuation equilibrium when new contracts are offered by a deviant insurer. The proof of Proposition 3 is of a different nature : it consists in showing that no profitable deviation can exist, which requires Condition (15), but it is valid for all $n$ [35]. Note however that equilibrium premiums are not uniquely defined, since insurers may compensate higher premium through higher dividends. More precisely, the equilibrium allocation $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ can also be sustained by an equilibrium of the market game where insurers offer contracts $\widehat{C}_i' \equiv (\widehat{k}_i', \widehat{x}_i')$ where $\widehat{k}_i' = \widehat{k}_i + \delta$ and $\widehat{x}_i' = \widehat{x}_i - \delta$ with policy dividend rule $D_i^{**}(N_1, P_1, ..., N_n, P_n) \equiv D_i^*(N_1, P_1 - \delta, ..., N_n, P_n - \delta) + \delta$, with $\delta > 0$. In that case, dividends include a fixed part $\delta$ paid to all policyholders and a variable part that does not concern threshold types. Hence the fundamental meaning of Condition (15) is not the fact that threshold types do not receive policy dividends (since they may actually receive such dividends according to the level of premiums) : Condition (15) assures us that threshold types cannot be penalized when deviant insurers offer new contracts.

Although no policy dividend (or dividend $\delta$) is paid on the equilibrium path, there may be variations in policy dividends when a deviant insurer $j^0$ offers a menu $C^{j_0}$ that differs from $C^* = (\widehat{C}_1, ..., \widehat{C}_n)$. Indeed such a deviation may affect the types distribution of individuals who still choose a contract in $C^*$, with possible variations in profits or losses of insurers $j \neq j_0$, and thus policy dividends or supplementary premiums. Variations in policy dividends can then act as an implicit threat that dissuades deviant insurers from undertaking competitive attacks. For the sake of example assume

$$D_i^*(N_1, P_1, ..., N_n, P_n) = \frac{\widehat{k}_i - \widehat{k}_{\ell_{t+1}}}{\sum_{h=\ell_t+1}^{\ell_{t+1}} N_h(\widehat{k}_h - \widehat{k}_{\ell_{t+1}})} \sum_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h \tag{16}$$

[35] To illustrate this difference, consider the case $\lambda_1 < \lambda_1^{**}$ when $n = 2$ in Section 3.2, with uniform distribution of profits or repayment of losses among policyholders. The candidate equilibrium allocation is $(\widetilde{C}_1, \widetilde{C}_2)$. When $\widetilde{C}_2$ doesn't attract any customer, consider beliefs according to which profits on $\widetilde{C}_2$ correspond to purchases by type 1 individuals only (i.e. out of equilibrium beliefs are highly pessimistic). Then there exist profitable deviations that attracts all type 2 individuals. The situation of type 2 individuals would deteriorate in this deviation, which could not occur under Condition (15). An assumption like (15) seems necessary to get any equilibrium existence result when $n > 2$. For the sake of illustration, assume $n = 3$ and consider a case where $\widehat{C}_1$ is in deficit and $\widehat{C}_2$ and $\widehat{C}_3$ are profitable when respectively chosen by types 1, 2 and 3 (a case where $I = \{1, 2, 3\}$ and $\bar{t} = 0$). Assume also that underwriting profit or losses are uniformly shared between policyholders, including type 3. In that case if $\lambda_2$ is small enough, there exist profitable non-participating contracts $C_2'$ closed to $\widehat{C}_2$ which would attract type 2 individuals if offered in deviations from equilibrium, while types 1 and 3 would keep choosing $\widehat{C}_1$ and $\widehat{C}_3$ and pay (small) supplementary premiums.

for all $i \in \{\ell_t + 1, ..., \ell_{t+1}\} \subset I$. Here $D^*(.)$ involves the sharing of profit within each subgroup $t$ with cross-subsidization. The total profit made within subgroup $t$ is $\sum_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h$. It is affected to policyholders within the same subgroup. Furthermore, according to the policy dividend rule, the larger the premium, the larger the policy dividend in absolute value. There is no right to receive policy dividend for the individuals who pay the smallest premium (i.e. for type $\ell_{t+1}$) while rights are larger for types $i$ who pay larger premiums. We have $\sum_{h=\ell_t+1}^{\ell_{t+1}} \lambda_h \Pi(\widehat{C}_h) = 0$ for all $t$, and thus this policy dividend rule satisfies conditions (13)-(15). If a deviant insurer $j_0$ attracts some individuals who cross-subsidize other risk types within subgroup $t$, then after the deviation we will have $\sum_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h < 0$ for non-deviant insurers $j \neq j_0$ and consequently the welfare of these other individuals will deteriorate if they keep choosing the same contract because they will have to pay supplementary premiums. It may then be impossible for insurer $j_0$ not to attract them also, which will make its offer non-profitable. The proof of Proposition 3 shows that this is indeed the case.

More generally, we may choose $D^*(.)$ such that

$$\sum_{\substack{h=\ell_t+1 \\ h \in I}}^{\ell_{t+1}} N_h D_h^*(N_1, P_1, ..., N_n, P_n) \equiv \sum_{\substack{h=\ell_t+1 \\ h \in I}}^{\ell_{t+1}} N_h P_h,$$

for all subgroup $t$ with cross-subsidization, which shows that the equilibrium allocation is also sustained by equilibrium strategies where each insurer sells insurance to a given subgroup of individuals (gathering risk types $h = \ell_t+1, ..., \ell_{t+1}$ in $I$) or to a combination of these subgroups. Insurers who sell insurance to subgroups with only one risk type (i.e. to types $i \notin I$) or to a combination of these subgroups do not cross-subsidize risks. They offer non-participating policies, and we may consider them as stock insurers. Insurers who sell insurance policies to individuals who belong to subgroups with cross-subsidization (i.e. to types $i \in I$) offer fully participating policies : they act as mutuals do. In the example illustrated in Figure 9, mutuals would offer participating contracts to subgroup $t = 1$ (that includes types 1,2 and 3) and stock insurers would offer non-participating contracts to subgroups $t = 2$ and 3. Hence, the model explains why stock insurers and mutuals may coexist : mutuals offer insurance contracts that are robust to competitive attacks when there is cross-susidization, while stock insurers offer insurance contracts at actuarial price. The following corollary recaps our results more compactly.

**Corollary 1** *The equilibrium allocation $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ is also sustained by an equilibrium of the market game where mutual insurers offer participating contracts* *to subgroups of individuals with types $i \in I$ and stock insurers offer non-participating contracts to types $i \notin I$.*

# 5 Deferred premium variations

More often than not mutuals shift the payment of underwriting profit to their members by transferring current profit to reserves and by later increasing or decreasing premiums according to the level of accumulated surplus. Reserves then act as a shock absorber and unforeseen supplementary calls occur only in case of large unexpected losses. Such deferred premium variations are substitutes to policy dividends and supplementary premiums paid or levied during the current period. They may have similar strategic effects, but they also entail specific dynamic issues that are more complex than the instantaneous contracting problem we have considered thus far. Although a comprehensive dynamic analysis is out of the scope of this paper, we may nevertheless sketch the similarities and differences between participating contracts and deferred premium variations.

Consider an overlapping generation setting in which each individual lives for two periods (1 and 2). Assume that money can be transferred costlessly over time with zero interest rate. Assume also that transaction costs prevent individuals from changing their insurers between periods 1 and 2, which is of course a very strong assumption. Type $i$ individuals suffer loss $A$ at each period of their life with probability $\pi_i$. Insurers offer intertemporal insurance contracts with variable premiums. Contractual agreements specify the net coverage $x$ in the case of an accident (be it at period 1 or 2). Premium may increase or decrease between periods 1 and 2 according to underwriting profits made at period 1. Policyholders pay $k$ at period 1 and $k - D$ at period 2, where $D$ depends on the profit made at period 1. $D$ is thus analogous to a policy dividend moved one period back[36]. For example, if the premium increase just covers the underwriting losses over the two periods, we should have $D = 2P$, where $P$ is the underwriting profit per period[37]

Since wealth can be transferred over time without cost, individuals maximize the expected utility of their cumulated final wealth, which is written as

$$\begin{aligned} Eu &= (1-\pi)^2 u(W_N - 2k + D) \\ &\quad + 2\pi(1-\pi)u(W_{A1} - k + x + D) + \pi^2 u(W_{A2} + 2x + D), \end{aligned}$$

where $W_{A1} = W_N - A, W_{A2} = W_N - 2A$ and $\pi = \pi_i$ for type $i$ individuals. An allocation is now written as $\{(W_i^1, W_{i1}^2, W_{i2}^2), i = 1, ..., n\}$, where $W_i^1$ denotes the final

[36]Experience rating (i.e. conditioning the premium paid at period 2 on the policyholder's loss experience at period 1) would improve the efficiency of market mechanisms. It is not considered here for notational simplicity.

[37]Note that profit remains constant across time for a given cohort since policyholders do not change insurers between the two periods of their life.

wealth of type $i$ individuals who do not suffer any accident and $W_{i1}^2$ (respect.$W_{i2}^2$) is the final wealth in case of one accident (respect. two accidents), with

$$\begin{aligned} W_i^1 &= W_N - 2k_i + D_i, \\ W_{i1}^2 &= W_{A1} - k_i + x_i + D_i, \\ W_{i2}^2 &= W_{A2} + 2x_i + D_i, \end{aligned}$$

where $k_i, x_i$ and $D_i$ refer to the insurance contract chosen by type $i$ individuals.

The developments and conclusions of Section 4 can be straightforwardly adapted to this new setting, with unchanged definitions of reservation utility $\overline{u}_i$ and of the candidate equilibrium allocation $\{(\widehat{W}_i^1, \widehat{W}_{i1}^2, \widehat{W}_{i2}^2), i = 1, ..., n\}$[38]. This allocation is sustained by a subgame perfect Nash equilibrium of the market game where insurers offer intertemporal insurance contracts with variable premiums to each cohort of individuals who enter the market.

Matters would be much less obvious if individuals could move to another insurer between periods 1 and 2. Indeed consider the two risk type case and assume that there is cross-subsidization at the candidate equilibrium. Then if a deviant insurer attracts low risk individuals at the first period of their life, thereby leading to underwriting losses for non-deviant insurers, then high risks may choose to quit only at period 2 in order to escape from the increase in premium. However, by doing that they would signal themselves as high risk (because low risks are attracted from period 1), which would reduce the advantage they may get from moving to another insurer. This signalling effect may lead high risk individuals either to move to the deviant insurer over the two periods of their life or not to move at all. Whether it is sufficient to annihilate competitive attacks remains an open issue.

# 6 Concluding comments

The initial motivation of this paper was an inquiry on the nonexistence of equilibrium in the RS model, starting with the observation that this model restricts the set of insurance contracts to non-participating policies. The result of this inquiry is actually striking since it turns out that allowing insurers to offer participating policies guarantees the existence of an equilibrium in the RS model. The equilibrium allocation coincides with the MSW allocation. In the case of two groups of individuals, there is cross-subsidization between contracts when the proportion of high risks individuals is under a threshold, while the equilibrium allocation coincides with Rothschild-Stiglitz pair of contracts in the other case. In the general case with an arbitrary number of risk

[38] Note in particular that the marginal rate of substitution between $x$ and $k$ is increasing with $\pi$. Indifference curves thus cross only one time in this part of the $(x, k)$ plane, which implies that only upward adjacent incentive compatibility constraints are binding. The analysis of Section 4 can thus be replicated without substantial change.

types, the equilibrium alllocation is characterized by a classification of individuals into subgroups as done by Spence (1978), with cross-subsidization within each subgroup.

Participating policies act as an implicit threat which prevents deviant insurers to attract low risk individuals only. If a deviant insurer attracts individuals who cross-subsidize other risk types within a given subgroup, then these other individuals will have to pay supplementary premiums or receive lower dividends if they keep choosing the same contract from their non-deviant insurer. Consequently it will be impossible for the deviant insurer not to attract them also, which will make its offer non-profitable. This mechanism is similar to the logic of the MSW equilibrium. In both cases, a deviant insurer is detered to attract low risk individuals because it is expected that ultimately its offer would also attract higher risks, which would make it unprofitable. However, in the MSW equilibrium insurers are protected from these competitive attacks because they can react by withdrawing contracts that become unprofitable. This assumption is most unsatisfactory because it means that insurers are not committed to actually offer the announced contracts. It can also be legitimately argued that this description of the dynamic relationship between insurers is arbitrary. Other timings are possible, as shown by Riley (1979), Hellwig (1987) and others. The present model has not stepped away from the Nash equilibrium setting of the RS model, and we have just explored the consequences of deleting an exogenous restriction on the content of insurance policies.

More than the solution to a theoretical puzzle, the outcome of this inquiry on the extended RS model provides a new explanation about why mutuals are so widespread in insurance markets and why they coexist with stock insurers. Most explanations about why mutuals may be more efficient than stock insurers are either based on the reduction in agency costs made possible by the mutual corporate form (Mayers and Smith,1988), on the fact that mutuals may emerge as a risk screening mechanism (Smith and Stutzer,1990; Ligon and Thistle,2005), on their ability to cover undiversifiable risks (Doherty and Dionne,1993) or to provide incentives to policyholders (Smith and Stutzer,1995) or to optimize the insurer's financial strategy in the presence of frictional cost of capital and governance problems (Laux and Muermann, 2010). This paper has explored a different way. Starting from the RS equilibrium puzzle, we finally came to the conclusion that mutuals endogenously emerge in a competitive setting when second-best allocative efficiency requires cross-subsidization, while stock insurers can survive in the other cases. Comparing the empirical validity of these approaches remains an open issue, which is outside the scope of the present paper. We will thus confine ourselves to a couple of simple remarks on the comparative relevance of these theories.

Firstly the previous approaches aimed at establishing conditions under which either mutuals or stock insurers constitute the most efficient corporate form for a given line of business, with the logical implication that the less efficient pattern should be doomed to extinction. On the contrary, our analysis of the $n$ group case shows that mutuals and stock insurers may steadily coexist in the same line of business in accordance with factual observations. Secondly, reducing agency costs and managing undiversifiable risks are common objectives of many financial intermediaries including banks, but mutual banks play a much more significant part in the retail and commercial banking industry than in the investment banking sector. The existence of hidden information about customers is a common concern of insurance firms and retail and commercial banks, which reinforce the plausibility of the link between adverse selection and the existence of mutuals in competitive markets. Thirdly cooperative enterprises also play an important role in non-financial markets. In particular, agricultural service cooperatives act as machinery pools, as marketing cooperatives or as credit unions and, by doing that, they allow their members to reap the benefits of technical or informational economies of scale. Having said that, it remains to explain why agricultural cooperatives emerged as a major form of business organization, competing with investor-owned firms and sometimes replacing them. The extended RS model provides an answer to this question : farmers'efficiency is heterogenous, not easily observable and the financing of services to farmers may require some degree of cross-subsidization from more efficient farms to less efficient ones. This is what agricultural cooperative can do in a competitive agricultural service market.

Finally, let us mention the work of Lamm-Tennant and Starks (1993) who show that stock insurers bear higher undiversifiable risk than mutuals, when risk is measured by the intertemporal variance of the loss ratios (loss incurred/premium earned). They argue that this result supports the agency and risk screening approaches to the insurance company structure. The present analysis brings us to another empirical verification strategy. It suggests that we should relate the insurance corporate form to the cross-section variance of their loss ratios among contracts for a given line of business (e.g. conditioning the loss ratio on deductible levels or coinsurance rates). More cross-subsidization goes together with larger cross-section variance of the loss ratio, which paves the way for possible empirical tests of the theory developed in this paper. However, at this stage, the only conclusion we can draw is of a theoretical nature : mutuals are robust to competitive attacks in insurance markets characterized by adverse selection, which may not be the case for stock insurance companies.

# Appendix

**Proof of Lemma 1**

If $\sum_{h=1}^{i}\lambda_i[W_N-(1-\pi_i)\widehat{W}_i^1-\pi_i(\widehat{W}_i^2+A)]>0$ for $i\in\{1,...,n\}$, then it would be possible to provide a higher expected utility than $\overline{u}_\ell$ for all $\ell=1,...,i$, while breaking even over the subset of individuals $\ell=1,...,i$, which would contradict the definition of $\overline{u}_i$. We thus have $\sum_{h=1}^{i}\lambda_i[W_N-(1-\pi_i)\widehat{W}_i^1-\pi_i(\widehat{W}_i^2+A)]\leq 0$ for all $i\in\{1,...,n\}$, which yields the first part of the Lemma.

We have $(1-\pi_i)u(\widehat{W}_i^1)+\pi_i u(\widehat{W}_i^2)\geq\overline{u}_i$ for all $i$ from the definition of $\mathbb{P}_n$. If $i\in\{\ell_1,\ell_2,...,n\}$, we have $\sum_{h=1}^{i}\lambda_h[(1-\pi_h)\widehat{W}_h^1+\pi_h(\widehat{W}_h^2+A)]=W_N$ from the first part of the Lemma and we deduce $(1-\pi_i)u(\widehat{W}_i^1)+\pi_i u(\widehat{W}_i^2)=\overline{u}_i$, for otherwise we would have a contradiction with the definition of $\overline{u}_i$. Conversely, suppose we have $(1-\pi_i)u(\widehat{W}_i^1)+\pi_i u(\widehat{W}_i^2)=\overline{u}_i$ and $i \notin \{\ell_1,\ell_2,...,n\}$. We would then have $\sum_{h=1}^{i}\lambda_h[W_N-(1-\pi_h)\widehat{W}_h^1-\pi_h(\widehat{W}_h^2+A)]<0$. Hence the allocation $\{(\widehat{W}_h^1,\widehat{W}_h^2), h=1,...,i\}$ is in deficit. Let $\{(W_h^{1\prime},W_h^{2\prime}), h=1,...,i\}$ be the optimal solution to $\mathbb{P}_i$. Replacing $\{(\widehat{W}_h^1,\widehat{W}_h^2), h=1,...,i\}$ by $\{(W_h^{1\prime},W_h^{2\prime}), h=1,...,i\}$ allows us to improve the optimal solution to $\mathbb{P}_n$, since the same type $i$ expected utility $\overline{u}_i$ can be reached while breaking even on the set $h=1,...,i$, which provides additional resources that could be used to raise $(1-\pi_n)u(W_n^1)+\pi_n u(W_n^2)$ over $(1-\pi_n)u(\widehat{W}_n^1)+\pi_n u(\widehat{W}_n^2)$. We thus obtain a contradiction with the fact that $\{(\widehat{W}_i^1,\widehat{W}_i^2), i=1,...,n\}$ is the optimal solution to $\mathbb{P}_n$.

**Proof of Lemma 2**

We first restrict attention to incentive compatible allocations $\{(W_h^1,W_h^2), h=1,...,n\}$ located in a neighbourhood of $\{(\widehat{W}_h^1,\widehat{W}_h^2), i=1,...,n\}$. Suppose that such an allocation satisfies (9)-(10). Lemma 1 shows that

$$(1-\pi_h)u(W_h^1)+\pi_h u(W_h^2)\geq \overline{u}_h \text{ for all } h=1,...,n, \tag{17}$$

if $(W_h^1,W_h^2)$ is close enough to $(\widehat{W}_h^1,\widehat{W}_h^2)$. Hence $\{(W_h^1,W_h^2), h=1,...,n\}$ satisfies the constraints of $\mathbb{P}_n$ with positive profits and expected utility larger or equal to $\overline{u}_n$ for type $n$, hence a contradiction.

We now prove that there does not exist any incentive compatible allocation $\{(W_h^1,W_h^2), h=1,...,n\}$ that satisfies (9)-(10) even if we do not restrict attention to allocations close to $\{(\widehat{W}_h^1,\widehat{W}_h^2), i=1,...,n\}$. Let us define

$$\begin{aligned} q_h &\equiv \frac{1-\pi_h}{\pi_h}u(W_h^1), z_h\equiv u(W_h^2) \\ \widehat{q}_h &\equiv \frac{1-\pi_h}{\pi_h}u(\widehat{W}_h^1), \widehat{z}_h\equiv u(\widehat{W}_h^2). \end{aligned}$$

With this change of variable the Lemma states that there does not exist $\{(q_h,z_h), h=1,...,n\}$ such that

$$q_{\ell_\theta}+z_{\ell_\theta}\geq \frac{\overline{u}_{\ell_\theta}}{\pi_{\ell_\theta}} \text{ for all } \theta=1,...,\overline{\theta}+1\ , \tag{18}$$

$$q_h+z_h\geq \frac{1-\pi_h}{\pi_h}\frac{\pi_{h+1}}{1-\pi_{h+1}}q_{h+1}+z_{h+1} \text{ for } h=1,...,n-1\ , \tag{19}$$

$$\begin{aligned} &\sum_{h=1}^{n}\lambda_h\{(1-\pi_h)[W_N-u^{-1}(\frac{\pi_h q_h}{1-\pi_h})]-\pi_h[u^{-1}(z_h)-W_A]\} \\ &>\sum_{h=1}^{n}\lambda_h\{(1-\pi_h)[W_N-u^{-1}(\frac{\pi_h \widehat{q}_h}{1-\pi_h})]-\pi_h[u^{-1}(\widehat{z}_h)-W_A]\}. \end{aligned} \tag{20}$$

The set of $\{q_h, z_h, h = 1, ..., n\}$ that satisfies the conditions (18)-(20) is convex[39]. Hence if there is any allocation $\{q_h, z_h, h = 1, ..., n\}$ that satisfies conditions (18)-(20), there is an allocation in any neighbourhood of $\{\widehat{q}_h, \widehat{z}_h, h = 1, ..., n\}$ that satisfies them, which contradicts our previous result.

**Proof of Lemma 3**

For a given incentive compatible allocation with randomisation $\{(W_s^1, W_s^2), s = 1, ..., N; \sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)\}$, let $(\overline{W}_h^1, \overline{W}_h^2) = (W_{s(h)}^1, W_{s(h)}^2)$ be the most profitable lottery which is chosen by type $h$ individuals with positive probability, i.e. $s(h)$ is such that $\sigma_{h,s(h)} > 0$ and

$$(1 - \pi_h)W_{s(h)}^1 + \pi_h W_{s(h)}^2 W_{s(h)}^2 \leq (1 - \pi_h)W_{s'}^1 + \pi_h W_{s'}^2$$

for all $s'$ such that $\sigma_{h,s'} > 0$. If (11)-(12) hold for the initial allocation with randomization, then (9)-(10) also hold for the non-randomized incentive compatible allocation $\{(\overline{W}_h^1, \overline{W}_h^2), h = 1, ..., n\}$, which contradicts Lemma 2.

**Proof of Proposition 3**

Assume that each insurer offers $\widehat{C} = (\widehat{C}_1, \widehat{C}_2, ..., \widehat{C}_n, D^*(.))$ such that (13)-(15) hold. Then $\widehat{C}_i$ is an optimal choice of type $i$ individuals if no policy dividend is paid on any contract. (14) shows that this is actually the case when all individuals are evenly shared among insurers.

Suppose some insurer $j_0$ deviates from $\widehat{C}$ to another menu $C^{j_0} = \{C_1^{j_0}, C_2^{j_0}, ..., C_n^{j_0}, D^{j_0}(.)\}$ with $C_i^{j_0} = (k_i^{j_0}, x_i^{j_0})$. Let $\widetilde{\sigma}(C^{j_0}, \widehat{C}^{-j_0})$ be a continuation equilibrium following the deviation, i.e. equilibrium contract choices by individuals when $C^{j_0}$ and $\widehat{C}$ are simultaneously offered, respectively by insurer $j_0$ and by all the other insurers $j \neq j_0$. Such a continuation equilibrium exists since it is a mixed-strategy equilibrium of a finite strategic-form game[40]. We restrict the definition of this game by imposing $\widetilde{\sigma}_{i,i-1}^1 = 0$ for all $i \notin I$. From (15), type $i$ individuals weakly prefer $\widehat{C}_i$ to $\widehat{C}_{i-1}$ if $i \notin I$, so that any equilibrium of the restricted game is also an equilibrium of the original game. Let $\overline{P}_h^j$ be the profit per policyholder made by insurer $j \neq j_0$ on contract $\widehat{C}_h$ and $\overline{\theta}_h^j$ be the

[39] Note in particular that $u^{-1}(.)$ is convex because $u(.)$ is concave.

[40] Note that the conditions on $\overline{\theta}_h^j(C)$ and $\overline{P}_h^j(C)$ induce a discontinuity in the payoffs, but Dasgupta and Maskin (1986a) yields existence theorems for such discontinuous game.

proportion of insurer $j'$ s customers who choose $\widehat{C}_h$. We have

$$\overline{P}_h^j \equiv \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0}) \Pi_i(\widehat{C}_h)}{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0})} \quad \text{when } \sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0}) > 0,$$

$$\overline{\theta}_h^j = \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0})}{\sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^j (C^{j_0}, \widehat{C}^{-j_0})} \quad \text{when } \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^j (C^{j_0}, \widehat{C}^{-j_0}) > 0.$$

Let also

$$\overline{P}_h^j = \Pi_h(\widehat{C}_h) \quad \text{when } \sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0}) = 0,$$

$$\overline{\theta}_h^j = \lambda_h \quad \text{when } \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^j (C^{j_0}, \widehat{C}^{-j_0}) = 0.$$

Consider a continuation equilibrium where individuals of a given type are evenly shared between insurers $j \neq j_0$, i.e. where $\widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0}) = \widetilde{\sigma}_{ih}^{j'} (C^{j_0}, \widehat{C}^{-j_0})$ for all $h$ if $j \neq j'$, $j, j' \neq j_0$ [41]. We may then use more compact notations $\widetilde{\sigma}_{ih}^0 \equiv \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0})$ and $\widetilde{\sigma}_{ih}^1 \equiv \widetilde{\sigma}_{ih}^j (C^{j_0}, \widehat{C}^{-j_0}), \overline{P}_h^1 = \overline{P}_h^j, \overline{N}_h^1 = \overline{N}_h^j$ for all $j \neq j_0$. Let also

$$\overline{P}_h^0 \equiv \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0}) \Pi_i(C_h^{j_0})}{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0})} \quad \text{when } \sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0}) > 0 \ ,$$

$$\overline{\theta}_h^0 = \frac{\sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0})}{\sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^{j_0} (C^{j_0}, \widehat{C}^{-j_0})} \quad \text{when } \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^{j_0} (C^{j_0}, \widehat{C}^{-j_0}),$$

$$\overline{P}_h^0 = \Pi_h(C_h^{j_0}) \quad \text{when } \sum_{i=1}^n \lambda_i \widetilde{\sigma}_{ih}^{j_0} (C^{j_0}, \widehat{C}^{-j_0}) = 0 \ ,$$

$$\overline{\theta}_h^0 = \lambda_h \quad \text{when } \sum_{i=1}^n \sum_{k=1}^n \lambda_i \widetilde{\sigma}_{ik}^{j_0} (C^{j_0}, \widehat{C}^{-j_0}),$$

[41] Such a continuation equilibrium exists because it is a Nash equilibrium of an equivalent game with only two insurers that respectively offer $\widehat{C}^{-j_0}$ and $C^{j_0}$. Note that this equivalence is possible because $D_h^j(.)$ is homogeneous of degree 1 with respect to $(N_1^j, ..., N_n^j)$.

where $\overline{P}_h^0$ and $\overline{P}_h^0$ denote respectively the average profit made on $C_h^{j_0}$ by insurer $j_0$ and the proportion of insurer $j_0'$ customers who choose $C_h^{j_0}$.Hence, after the deviation by insurer $j_0$, type $i$ individuals get the following lottery on final wealth :

$$
\begin{aligned}
(W^1, W^2) &= (W_{0h}^1, W_{0h}^2) \equiv (W_N - k_h^{j_0} + \overline{D}_h^0, W_A + x_h^{j_0} + \overline{D}_h^0) \text{ with probabilpe ity } \widetilde{\sigma}_{ih}^0, \\
(W^1, W^2) &= (W_{1h}^1, W_{1h}^2) \equiv (\widehat{W}_h^1 + \overline{D}_h^1, \widehat{W}_h^2 + \overline{D}_h^1) \text{ with probability } \widetilde{\sigma}_{ih}^1(n-1),
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{D}_h^0 &= D_h^{j_0}(\overline{\theta}_1^0, \overline{P}_1^0, ..., \overline{\theta}_n^0, \overline{P}_n^0), \\
\overline{D}_h^1 &= D_h^*(\overline{\theta}_1^1, \overline{P}_1^1, ..., \overline{\theta}_n^1, \overline{P}_n^1),
\end{aligned}
$$

for $h = 1, ..., n$, with $\sum_{h=1}^{n}[\widetilde{\sigma}_{ih}^0 + \widetilde{\sigma}_{ih}^1(n-1)] = 1$. Let us denote this lottery by $\mathcal{L}$.

Let $\Delta$ denote the residual profit made by insurer $j_0$. We have

$$
\Delta = \sum_{i=1}^{n} \lambda_i \{\sum_{h=1}^{n} \widetilde{\sigma}_{ih}^0 [W_N - (1-\pi_i)W_{0h}^1 - \pi_i(W_{0h}^2 + A)]\}. \tag{21}
$$

We know from (13) that $D^*(.)$ involves the full distribution of profits made by non-deviant insurers on the set of contracts $\{\widehat{C}_i, i \in I\}$. Furthermore, we have $\widetilde{\sigma}_{ih}^1 = 0$ if $h < i-1$ when $i \notin I$ because types $h$ strongly prefer $\widehat{C}_{i-1}$ to $\widehat{C}_i$ for all $h < i-1$[42]. Consequently the profit made on $\widehat{C}_i$ by non-deviant insurers is non-negative when $i \notin I$. We deduce that non-deviant insurers $j$ make non-negative residual profit. We thus have

$$
\sum_{i=1}^{n} \lambda_i \{\sum_{h=1}^{\prime n} \widetilde{\sigma}_{ih}^1 [W_N - (1-\pi_i)W_{1h}^1 - \pi_i(W_{1h}^2 + A)]\} \geq 0. \tag{22}
$$

(21) and (22) then yield

$$
\begin{aligned}
\Delta \leq & \sum_{i=1}^{n} \lambda_i \{\sum_{h=1}^{n} \widetilde{\sigma}_{ih}^0 [W_N - (1-\pi_i)W_{0h}^1 - \pi_i(W_{0h}^2 + A)] \\
& +(n-1)\sum_{h=1}^{n} \widetilde{\sigma}_{ih}^1 [W_N - (1-\pi_i)W_{1h}^1 - \pi_i(W_{1h}^2 + A)]\}.
\end{aligned} \tag{23}
$$

Furthermore, we have

$$
\begin{aligned}
& \sum_{h=1}^{n} \widetilde{\sigma}_{\ell_\theta,h}^0 [(1-\pi_{\ell_\theta})u(W_{0h}^1) + \pi_{\ell_\theta}u(W_{0h}^2)] \\
& +(n-1)\sum_{h=1}^{n} \widetilde{\sigma}_{\ell_\theta,h}^1 [(1-\pi_{\ell_\theta})u(W_{1h}^1) + \pi_{\ell_\theta}u(W_{1h}^2) \\
\geq \ & \overline{u}_{\ell_\theta} \text{ for all } \theta = 1, ..., \overline{\theta}+1
\end{aligned} \tag{24}
$$

[42] Note that we here use $D_i^* \equiv 0$ and $D_{i-1}^* \equiv 0$ when $i \notin I$, which follows from (15).

because $(W^1_{1\ell_\theta}, W^2_{1\ell_\theta}) = (\widehat{W}^1_{1\ell_\theta}, \widehat{W}^2_{1\ell_\theta})$ since $\overline{D}^1_{\ell_\theta} = 0$ from (15) and $(1-\pi_{\ell_\theta})u(\widehat{W}^1_{1\ell_\theta}) + \pi_{\ell_\theta}u(\widehat{W}^2_{1\ell_\theta}) = \overline{u}_{\ell_\theta}$ and $\{\widetilde{\sigma}^0_{\ell_\theta,h}, \widetilde{\sigma}^1_{\ell_\theta,h}, h = 1,...,n\}$ is an optimal contract choice strategy of type $\ell_\theta$ individuals. The right-hand side of (23) is the expected profit associated with $\mathcal{L}$. Lemma 3 applied to lottery $\mathcal{L}$ then gives $\Delta \leq 0$. Hence the deviation is non-profitable, which completes the proof.

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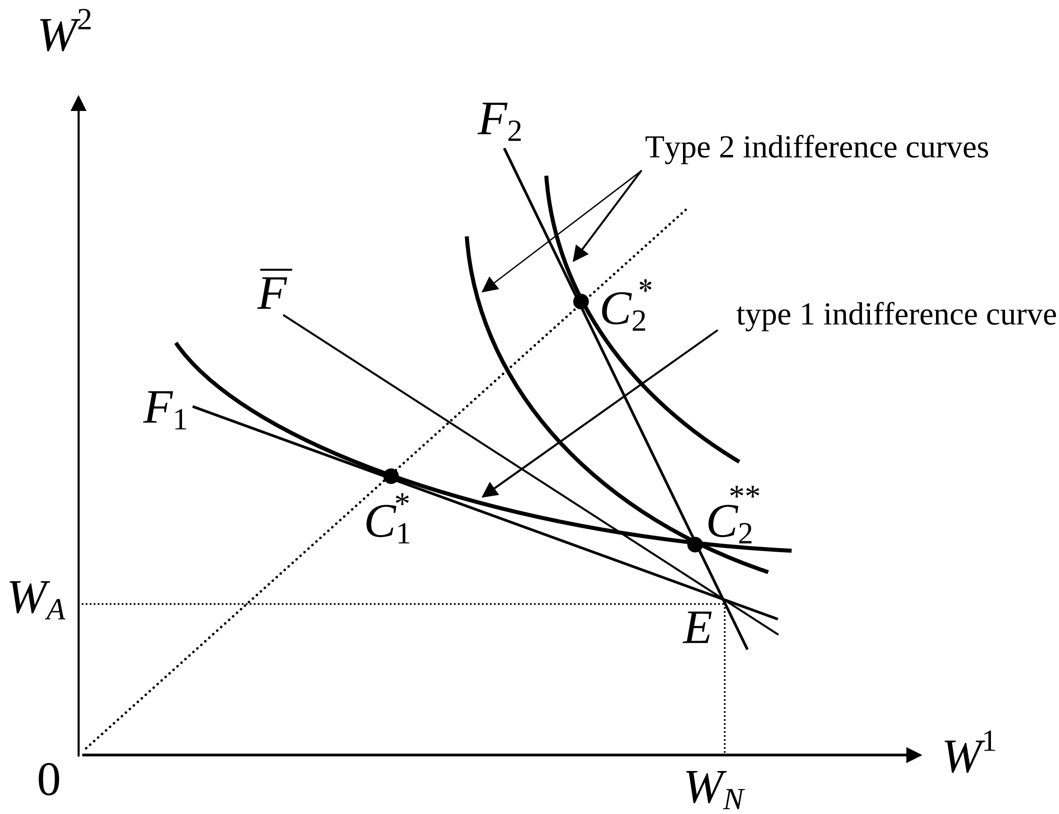
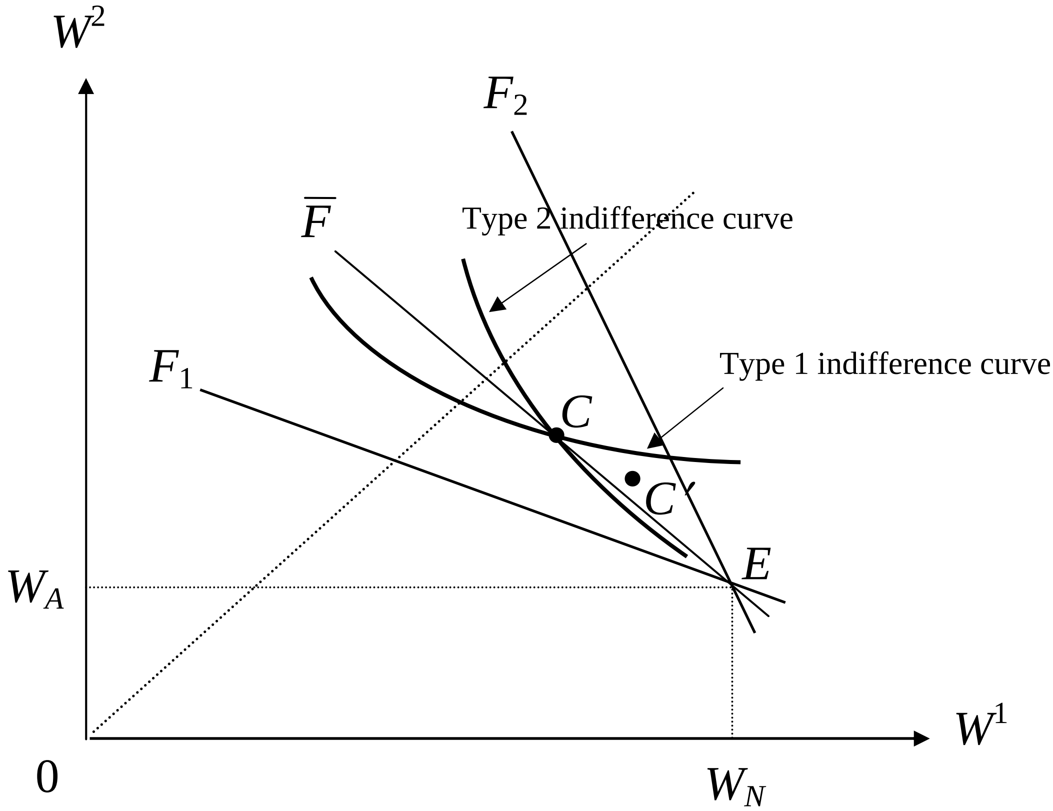


$W^2$
$F_2$
Type 2 indifference curves
$\overline{F}$
$C_2^*$
type 1 indifference curve
$F_1$
$C_1^*$
$C_2^{**}$
$W_A$
$E$
0
$W_N$
$W^1$


Figure 1



W2
F2
F
Type 2 indifference curve
Type 1 indifference curve
F1
C
C′
E
WA
0
WN
W1


Figure 2

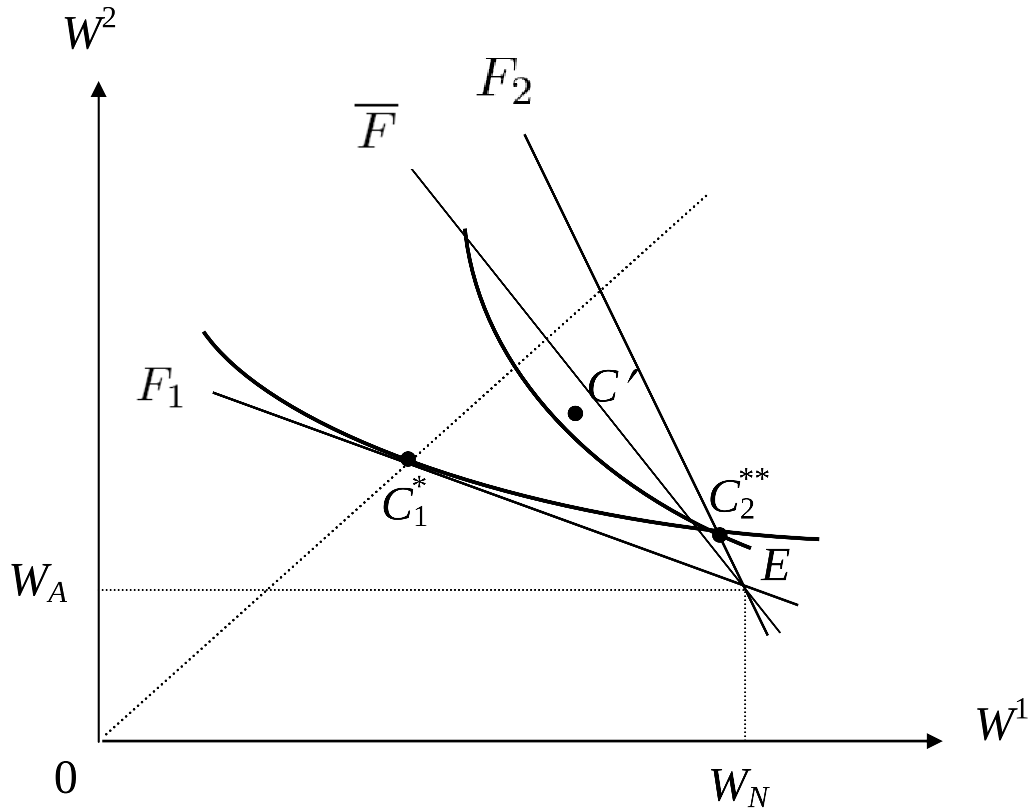



Figure 3

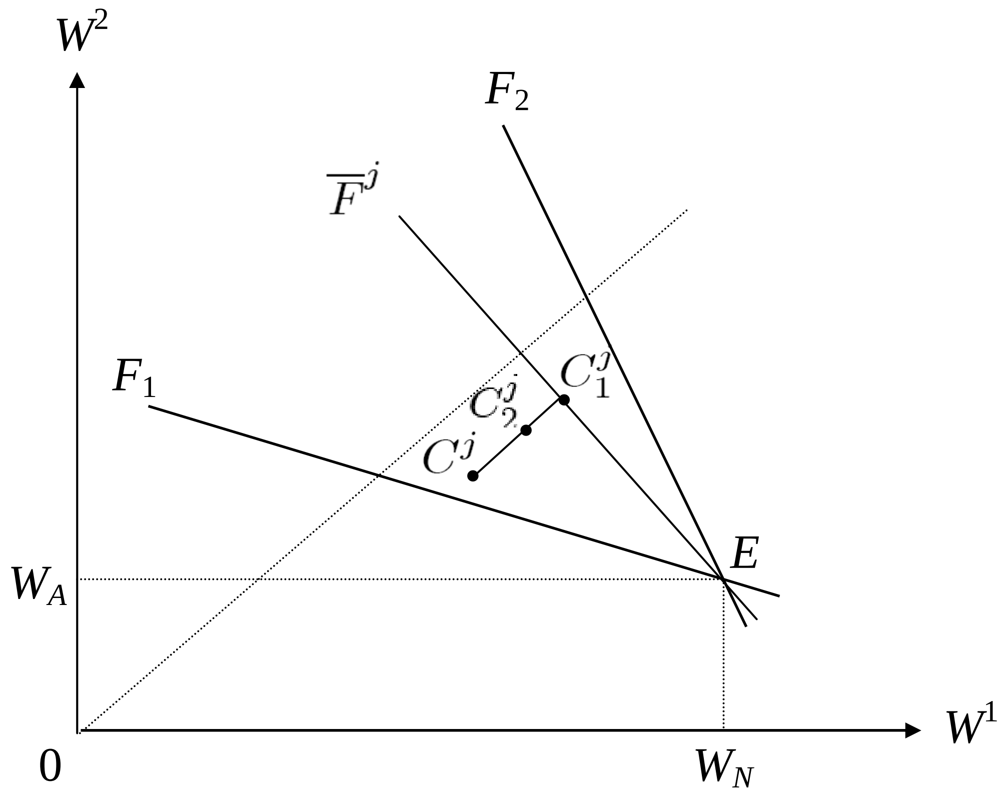
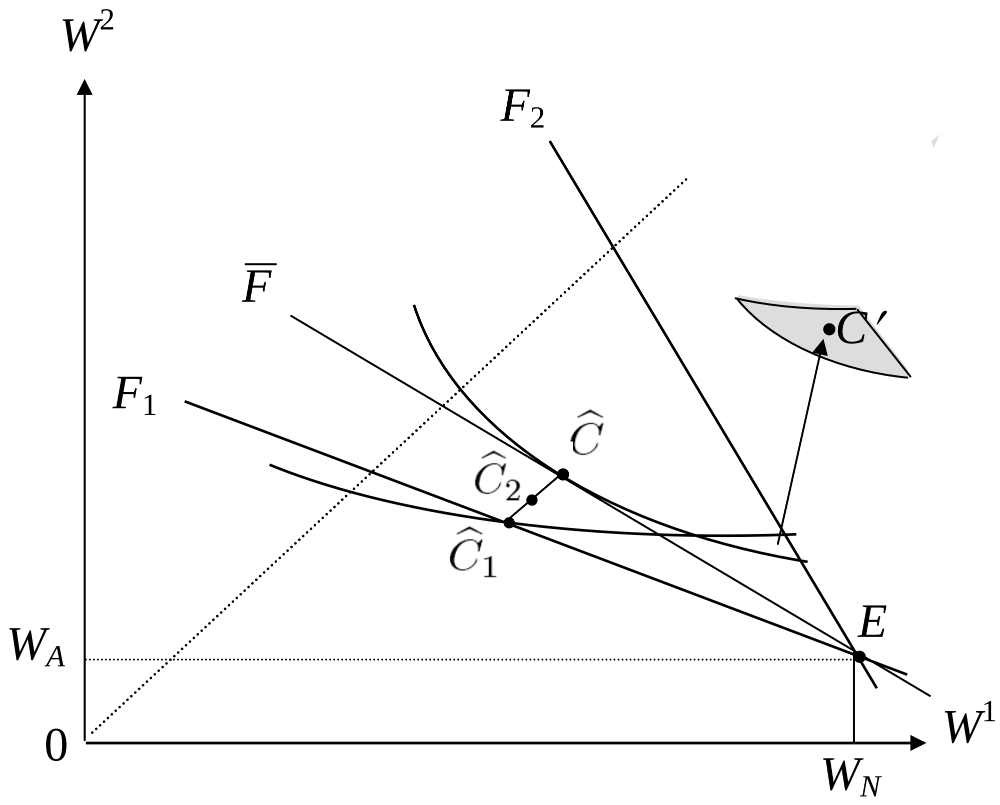



Figure 4



$W^2$
$F_2$
$\overline{F}$
$C'$
$F_1$
$\widehat{C}$
$\widehat{C}_2$
$\widehat{C}_1$
$E$
$W_A$
0
$W^1$
$W_N$


Figure 5

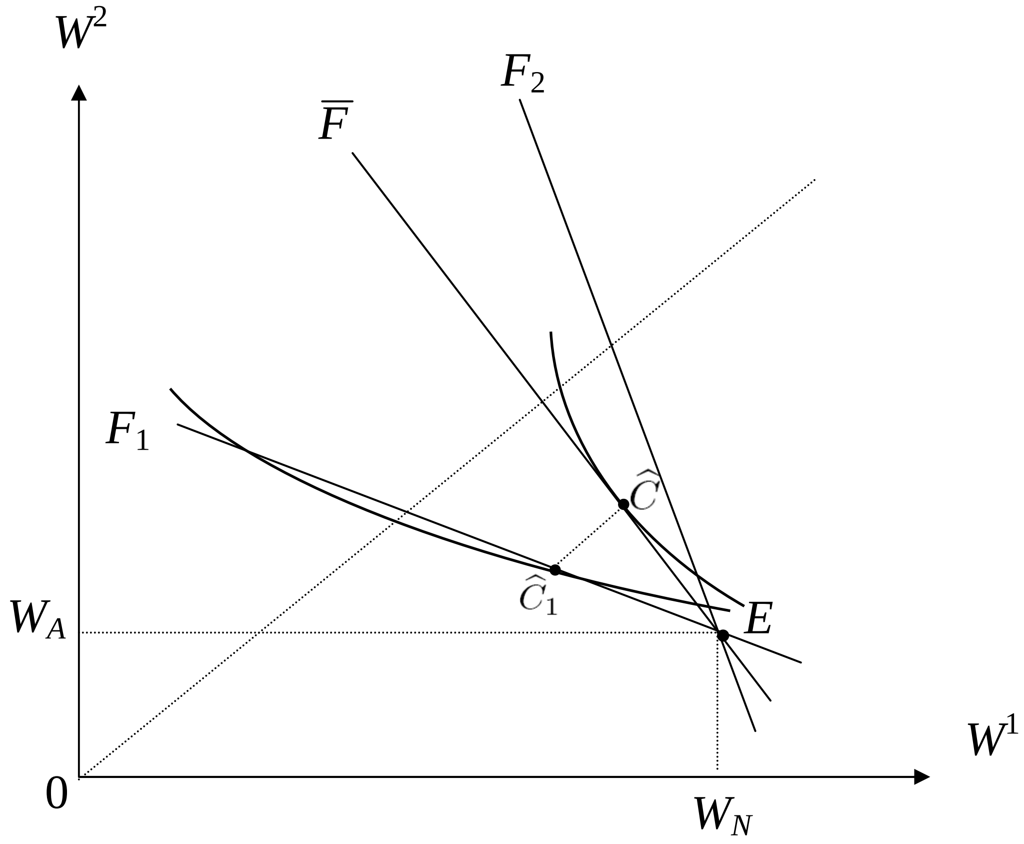



Figure 6

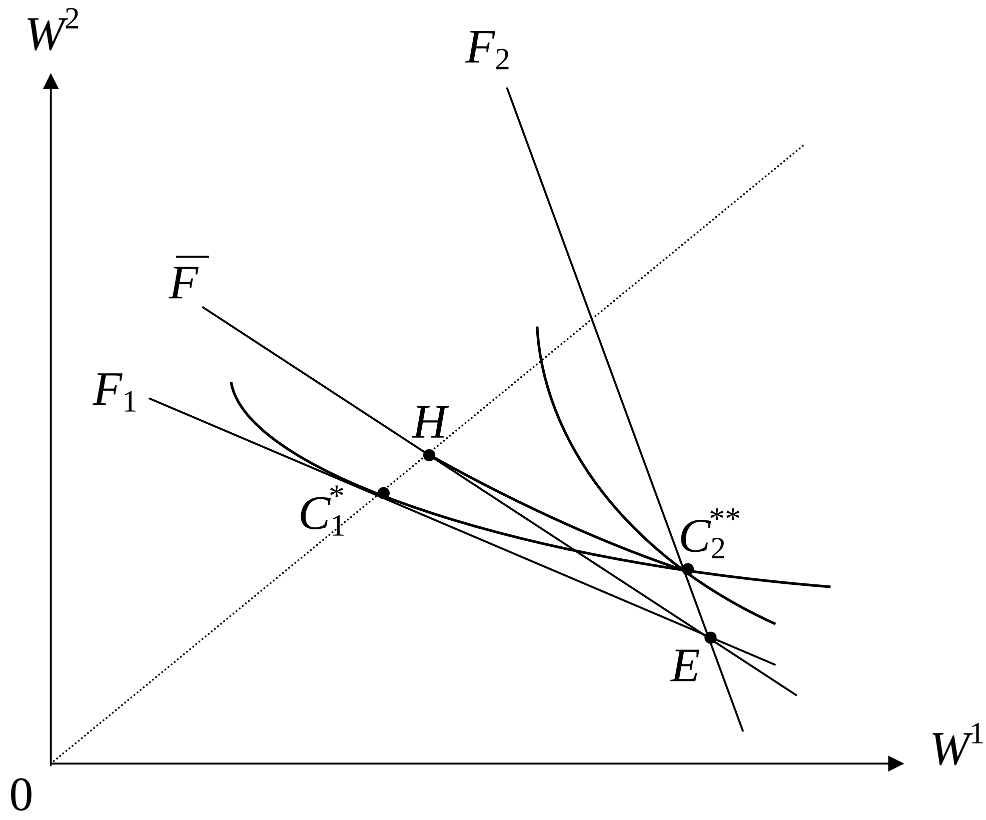



Figure 7

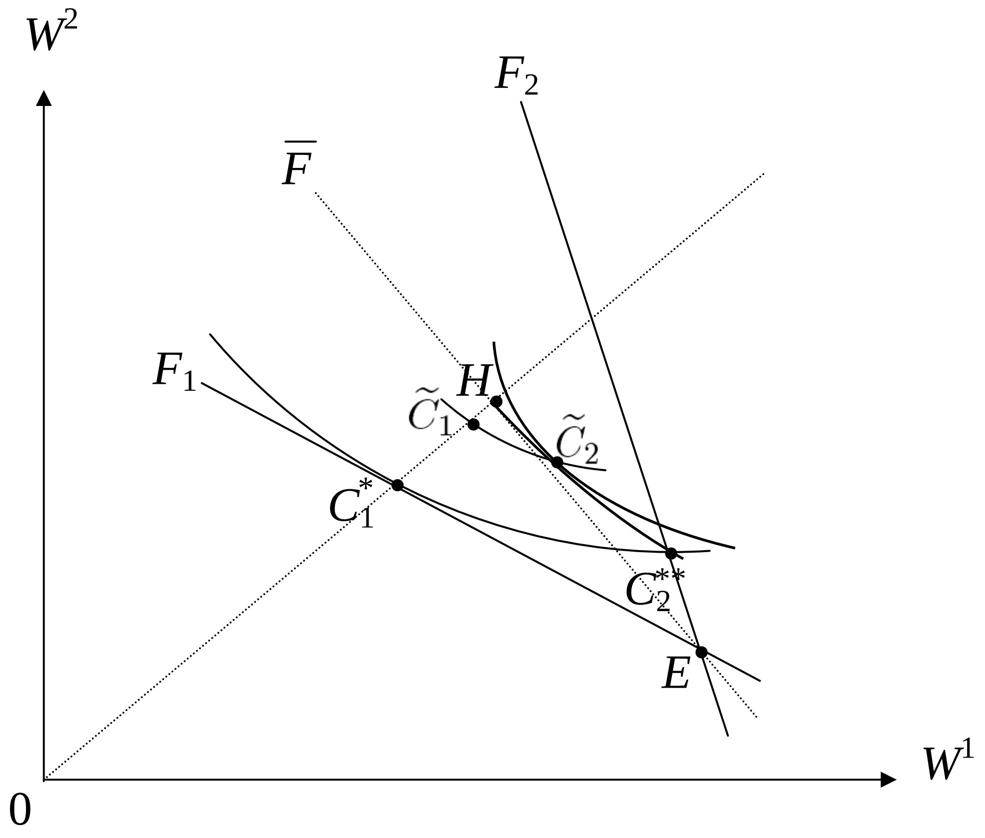



Figure 8

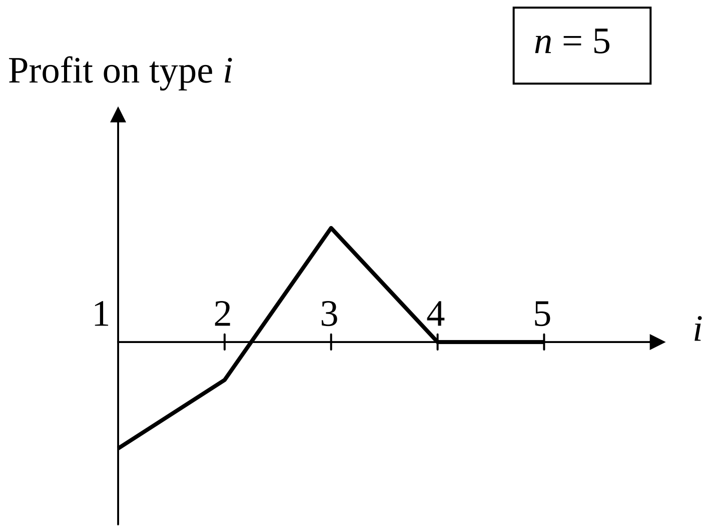



Figure 9