

Stationary Markov Equilibria in Discounted Stochastic Games

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Abstract

While the existence of Nash equilibria in stationary Markov strategies for m -player, non-zero sum, discounted stochastic games with *countable* state spaces and compact metric action spaces has long been established (e.g., see Federgruen, 1978), the existence of such equilibria for the *uncountable* case has remained an open question since the problem was first analyzed by Himmelberg, Parthasarathy, Raghavan, and Van Vleck (1976). Beginning with Fudenberg and Levine (1983), Harris (1985), and Forges (1986), one of the striking insights to emerge from the literature on the existence of subgame perfect equilibria (SPE) in non-Markov (i.e., partly history-dependent) strategies in stage games with uncountable state spaces concerns the fundamental role played by public randomization devices in resolving existence problems in such games. The importance of public randomization devices for existence was then confirmed in an infinite horizon, stochastic game setting by Nowak and Raghavan (1992) and Duffie, Geanakoplos, Mas-Colell, and McLennan (1994) who showed that m -player, non-zero sum, uncountable-compact discounted stochastic games naturally possess stationary Markov correlated equilibria. Our main contribution is to establish the existence of stationary Markov equilibria (i.e., SPE in Markov stationary strategies) for this class of stochastic games, thus showing for the stationary Markov case that public randomization devices are not required for existence - and thus providing a positive resolution to a long-standing open question in stochastic games.

JEL Classification C73, E23

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