

International Contagion through Leveraged Financial Institutions¹

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Abstract

The 2008-2009 financial crises, while originating in the United States, witnessed a drop in asset prices and output that was at least as large in the rest of the world as in the United States. In the context of a simple two-country model we investigate to what extent this may be the result of transmission through leveraged financial institutions. The paper highlights what the various transmission mechanisms associated with balance sheet losses are, how they operate, what their magnitudes are and what the role is of different types of borrowing constraints faced by leveraged institutions. For realistic parameters we find that the model cannot account for the global nature of the crisis, both in terms of the size of the impact and the extent of transmission.

1 Introduction

In response to the 2008 financial crisis a debate has reignited about channels of international transmission. The drop in asset prices was of similar magnitude all around the world. The decline in real GDP growth was also of similar magnitude in the rest of the world as in the United States.¹ This happened even though clearly this was a U.S. crisis that started with substantial losses on mortgage backed securities, which significantly deteriorated balance sheets of U.S. leveraged financial institutions. This naturally leads to the question of what can account for the nearly one-to-one transmission.

A natural hypothesis is that the shock, originating in the United States, was transmitted across countries through leveraged financial institutions. This hypothesis is consistent with the large increase in cross-border asset holding of leveraged institutions over the past two decades. In this paper we will develop a simple two-country model with leveraged financial institutions in order to address two questions. First, we aim to understand through what channels of transmission involving leveraged financial institutions a financial shock in the Home country impacts the Foreign country. Second, we want to get a sense of the magnitude of transmission for a realistic degree of cross-border financial integration.

The paper is related to a recent literature that has introduced leveraged financial institutions into open economy macro models. We discuss this literature in Section 5. The approach that we take in this paper is quite different in three respects. First, the model we consider is much simpler than the dynamic stochastic general equilibrium models considered in the literature. While this simplicity limits the breadth of our results (focusing for example on asset prices and not on real variables), it allows us to obtain simple analytical results. This makes it quite transparent what the various transmission channels are, how they operate, and what their magnitudes are.

Second, the literature so far has not addressed the second objective mentioned above, which is to measure the size of transmission through leveraged financial

¹See for example Perri and Quadrini (2011) for both GDP and stock prices. It shows that if anything stock prices and GDP fell slightly more in the G6 (the G7 minus the U.S.) than in the United States. Emerging market growth, while starting from a higher level, dropped about as much as industrialized country growth (see for example the 2011 World Economic Outlook, Figure 1.6.).

institutions for a realistic degree of cross-border financial integration. Several papers only consider the extremes where leveraged institutions can either only invest in domestic assets or can perfectly diversify across countries. Some papers consider intermediate cases, but do not focus on the specific question of how large transmission would be for realistic parameter assumptions.

Third, while the literature generally introduces borrowing constraints under which leveraged institutions operate, it is not clear what exactly the role of these constraints is in transmission and how transmission depends on the nature of these constraints. To help address this we will first consider the case where leveraged institutions are not subject to borrowing constraints. Then we consider the impact of adding a constant leverage constraint that is often adopted in the literature. And finally we consider a margin constraint that is relevant for collateralized borrowing. In contrast to a constant leverage constraint, borrowing is then limited not by the current value of the institution's assets, but rather by the expected future value of these assets and risk associated with the return on these assets.

The remainder of the paper is organized as follows. In Section 2 we describe the model for each of the three different assumptions about the nature of the borrowing constraints. Section 3 then considers at a theoretical level what determines the impact on asset prices of marginal defaults in the Home country. Section 4 calibrates the model in order to quantify the extent of transmission. Various extensions of the model are considered as well and the results are related to the 2008 crisis. Section 5 relates our findings to the existing literature and Section 6 concludes.

2 The Model

We first discuss the basic setup that applies under all three assumptions about borrowing constraints. After that we describe equilibrium under the different assumptions about borrowing constraints.

2.1 Basic Setup

The model has two countries, Home and Foreign. There are both leveraged financial institutions and non-leveraged investors in each country. There are two periods, 1 and 2. However, leveraged institutions inherit assets from a previous

period, which we call period 0, which affects their net worth at time 1.

We start with a description of the leveraged institutions. They purchase risky assets, financed through their net worth and borrowing by issuing bonds. Before describing the assets, a couple of points about the borrowing are in order. We make two simplifications. First, we keep the interest rate on the bond constant at R . We can think of this for example as an interest rate target of the central bank that accommodates any excess demand or supply in the bond market. Second, we assume that the leveraged institutions will make the full payment on their debt. In the absence of borrowing constraints this reflects a commitment mechanism that avoids default. In the presence of the borrowing constraints, these constraints are exactly meant to avoid a default outcome.²

Next consider the assets on the balance sheet of the leveraged institutions. Of the assets that they inherit from period 0, there are short-term assets that come due in period 1 and long-term assets with a singular payoff in period 2. The assets that come due in period 1 are introduced in order to generate balance sheet losses, which are associated with a partial default on these assets in the Home country.

We assume an initial balance sheet for Home leveraged institutions in period 0 that looks as follows. The net worth is W_0 and borrowing is B_0 . The value of the assets that will come due in period 1 is L_0 . The value of the other assets, whose payments will occur in period 2, is then $W_0 + B_0 - L_0$. For both short and long-term assets it is assumed that a fraction α is invested in Home assets and a fraction $1 - \alpha$ in Foreign assets. We assume $\alpha > 0.5$ as a result of portfolio home bias.

In the absence of default it is assumed that the payment on the short term assets in period 1 is $(1 + R)L_0$, for simplicity setting the return equal to the borrowing rate. The shock that we will consider in the model is where there is default on a fraction δ of the Home short-term assets. In the context of the 2007-2008 crisis one

²Even with the borrowing constraints, it is still feasible that the net worth of leveraged institutions turns negative in the model. For simplicity we assume that lenders are able to enforce payments through the courts. We therefore abstract from limited liability and from risk premia that lenders might charge to compensate for the costs of such legal proceedings. Particularly with the margin constraints, the entire point is to make the probability of such an outcome very small, so that any risk premia that might result are not large anyway. Lenders then respond to increased risk by demanding more collateral as opposed to raising the lending rate. In Section 4.6 we consider some of the implications that may arise when we relax the no default assumption.

can think of this as related for example to mortgage defaults or losses on mortgage backed securities.

In period 1 the Home leveraged institutions then receive

$$(1 + R)(\alpha(1 - \delta) + (1 - \alpha))L_0 = (1 + R)(1 - \alpha\delta)L_0$$

Foreign leveraged institutions inherit the same holdings from period 0, except that we assume that they invest a fraction $1 - \alpha$ in Home assets and α in Foreign assets, which gives rise to a symmetric home bias. The payment that they receive in period 1 on the short-term assets is then

$$(1 + R)(\alpha + (1 - \alpha)(1 - \delta))L_0 = (1 + R)(1 - (1 - \alpha)\delta)L_0$$

With $\alpha > 0.5$ the losses experienced by Home leveraged institutions will be larger as they have more exposure to the Home defaults.

From here on the focus will be on the long-term assets, which we will simply refer to as the Home and Foreign assets. The period 0 price of these assets is Q_0 . The quantities of the Home and Foreign assets held in period 0 by Home leveraged institutions are therefore $\alpha(W_0 + B_0 - L_0)/Q_0$ and $(1 - \alpha)(W_0 + B_0 - L_0)/Q_0$. Let Q_H and Q_F be the prices of the Home and Foreign assets in period 1. The net worth of Home leveraged institutions in period 1 is then

$$W_H = \frac{1}{Q(0)}(W_0 + B_0 - L_0)(\alpha Q_H + (1 - \alpha)Q_F) + (1 + R)((1 - \alpha\delta)L_0 - B_0) \quad (1)$$

where $\delta = 0$ without defaults and $\delta > 0$ with defaults. Analogously, the period 1 net worth of Foreign leveraged institutions is

$$W_F = \frac{1}{Q(0)}(W_0 + B_0 - L_0)((1 - \alpha)Q_H + \alpha Q_F) + (1 + R)(1 - (1 - \alpha)\delta)L_0 - B_0 \quad (2)$$

In period 2 the Home and Foreign (long-term) assets have a payoff of respectively D_H and D_F . These payoffs are stochastic. For now we assume that they are uncorrelated across countries, although in Section 4 we consider a generalization with correlated payoffs. We introduce home bias in the period 1 optimal holdings by assuming that domestic leveraged institutions are better informed about domestic asset payoffs than foreign leveraged institutions. Specifically, the perceived variance of D_H is σ^2 for Home leveraged institutions and $\sigma^2/(1 - \tau)$ for Foreign leveraged institutions, with $\tau > 0$ measuring the extent of information asymmetry generating portfolio home bias. Analogously, the perceived variance of D_F is

σ^2 and $\sigma^2/(1 - \tau)$ for respectively Foreign and Home leveraged institutions. The expected payoffs in both countries are D .

In period 1 the Home leveraged institutions purchase respectively K_{HH} and K_{HF} of Home and Foreign assets and borrow $K_{HH}Q_H + K_{HF}Q_F - W_H$. Their gross portfolio return is then

$$R_H^p = 1 + R + \frac{K_{HH}}{W_H}(D_H - (1 + R)Q_H) + \frac{K_{HF}}{W_H}(D_F - (1 + R)Q_F) \quad (3)$$

They maximize a simple mean-variance utility function $ER_H^p - 0.5\gamma var(R_H^p)$.³ The problem is analogous for Foreign leveraged institutions.

It is useful to point out that the Home and Foreign assets could in principle be either standard securities (stocks, bonds), asset backed securities, or regular loans. When they are loans, the price is related to the interest rate on the loan. For example, let \bar{D} be the upper bound of the payoffs D_H and D_F . This is the payment on the loans in the absence of default. Lower values are a result of partial default in period 2. The two-period interest rate at time 0 is then \bar{D}/Q_0 . In period 1 it is \bar{D}/Q_H for the Home loans and \bar{D}/Q_F for the Foreign loans.

Non-leveraged investors face an analogous portfolio maximization problem, except that for now we assume that they start period 1 with a given wealth W_{NL} in both countries. We therefore abstract from a feedback from asset prices back to the wealth of these other investors. This is meant to focus on the role of leveraged institutions for which Krugman (2008) and others emphasized such feedback effects. One way to interpret this is that any capital gains are simply consumed by the non-leveraged agents. In Section 4 we consider an extension where the wealth of non-leveraged investors does depend on asset prices.

The rate of risk-aversion is assumed to be much higher for the non-leveraged investors, which is exactly what makes them non-leveraged. We denote their risk-aversion as γ_{NL} , which is the same in both countries. We assume that non-leveraged investors have the same perceived risk of the asset payoffs as the leveraged institutions, with the same information asymmetry across countries.

³Assuming simple mean-variance preferences as opposed to expected utility preferences is not critical to any of the results. It has the advantage of allowing for a closed form solution to the portfolio problem, which helps in making the results more transparent. If instead we assume constant relative risk-aversion preferences and take a linear approximation of the portfolio Euler equation, we get the same portfolio expression.

The description of the model so far is the same whether the leveraged institutions face balance sheet constraints or not. We now complete the model by considering optimal portfolios both with and without balance sheet constraints and imposing market equilibrium.

2.2 Equilibrium without Balance Sheet Constraints

In the absence of balance sheet constraints, optimization leads to simple mean-variance portfolios. The optimal holdings of Home and Foreign assets by Home leveraged institutions are

$$K_{HH} = \frac{D - (1 + R)Q_H}{\gamma\sigma^2} W_H \quad (4)$$

$$K_{HF} = (1 - \tau) \frac{D - (1 + R)Q_F}{\gamma\sigma^2} W_H \quad (5)$$

The portfolios for the non-leveraged Home investors are exactly the same, with risk-aversion replaced by γ_{NL} and wealth by W_{NL} .

Similarly, let K_{FH} and K_{FF} be the fractions invested in Home and Foreign assets by the Foreign leveraged institutions. Their optimal portfolios are then

$$K_{FH} = (1 - \tau) \frac{D - (1 + R)Q_H}{\gamma\sigma^2} W_F \quad (6)$$

$$K_{FF} = \frac{D - (1 + R)Q_F}{\gamma\sigma^2} W_F \quad (7)$$

Again, analogous expressions hold for Foreign non-leveraged investors.

Market clearing implies that the total demand for Home assets is equal to the supply K , and similarly for Foreign assets. Using the portfolio expressions we can write these market clearing conditions as

$$\frac{D - (1 + R)Q_H}{\sigma^2} \left(\frac{1}{\gamma} (W_H + (1 - \tau)W_F) + \frac{1}{\gamma_{NL}} (2 - \tau)W_{NL} \right) = K \quad (8)$$

$$\frac{D - (1 + R)Q_F}{\sigma^2} \left(\frac{1}{\gamma} ((1 - \tau)W_H + W_F) + \frac{1}{\gamma_{NL}} (2 - \tau)W_{NL} \right) = K \quad (9)$$

2.3 Constant Leverage Constraint

Next consider a constant leverage constraint. Leverage, which is the ratio of assets to net worth, can be no larger than κ . For Home and Foreign leveraged institutions

this implies respectively

$$Q_H K_{HH} + Q_F K_{HF} \leq \kappa W_H \quad (10)$$

$$Q_H K_{FH} + Q_F K_{FF} \leq \kappa W_F \quad (11)$$

Since borrowing is equal to the assets minus net worth, we can also write this in the form of borrowing constraints:

$$B_H \leq \frac{\kappa - 1}{\kappa} (Q_H K_{HH} + Q_F K_{HF}) \quad (12)$$

$$B_F \leq \frac{\kappa - 1}{\kappa} (Q_H K_{FH} + Q_F K_{FF}) \quad (13)$$

where B_H and B_F are borrowing by Home and Foreign leveraged institutions in period 1.

These types of borrowing constraints are by now standard fare in the literature. Sometimes they are motivated by assuming that the leveraged institutions can run away with a fraction $1/\kappa$ of the assets. The constraint is then imposed to make sure that the institutions have no incentive to do so. A more sensible interpretation though is to think of these constraints as capital requirements that are imposed by regulatory institutions, with $1/\kappa$ the required capital as a fraction of assets.

Under these constant leverage constraints, the expressions for the optimal portfolios remain the same as before, with the only difference that $1 + R$ is replaced by $1 + R + \lambda_H$ and $1 + R + \lambda_F$ for respectively Home and Foreign leveraged institutions. Here λ_i is the Lagrange multiplier associated with the leverage constraint in country i . λ_i is positive if the constraint is binding in country i . The leverage constraint, if it becomes binding, therefore has an effect that is equivalent to an increase in the borrowing rate. We denote the effective borrowing rates as $R_H = R + \lambda_H$ and $R_F = R + \lambda_F$.

If the constraint is binding, we can solve for the Lagrange multipliers by substituting the optimal portfolios in the constraints with an equality sign. This gives

$$1 + R_H = \frac{(Q_H + (1 - \tau)Q_F)D - \kappa\gamma\sigma^2}{Q_H^2 + (1 - \tau)Q_F^2} \quad (14)$$

$$1 + R_F = \frac{((1 - \tau)Q_H + Q_F)D - \kappa\gamma\sigma^2}{(1 - \tau)Q_H^2 + Q_F^2} \quad (15)$$

Equilibrium in the asset markets is now represented by

$$\begin{aligned} & \frac{D - (1 + R_H)Q_H}{\gamma\sigma^2}W_H + \frac{D - (1 + R_F)Q_H}{\gamma\sigma^2}(1 - \tau)W_F \\ & + \frac{D - (1 + R)Q_H}{\gamma_{NL}\sigma^2}(2 - \tau)W_{NL} = K \end{aligned} \quad (16)$$

$$\begin{aligned} & \frac{D - (1 + R_H)Q_F}{\gamma\sigma^2}(1 - \tau)W_H + \frac{D - (1 + R_F)Q_F}{\gamma\sigma^2}W_F \\ & + \frac{D - (1 + R)Q_F}{\gamma_{NL}\sigma^2}(2 - \tau)W_{NL} = K \end{aligned} \quad (17)$$

2.4 Margin Constraints

We finally consider risk based constraints in the form of margin constraints. Such constraints are valid for collateralized lending. Most of the so-called shadow banking system (e.g. broker-dealers and hedge funds) uses primarily collateralized borrowing, especially in the form of repos contracts. We adopt standard margin constraints that are widely used in the literature and in everyday practice, limiting the risk that the collateral will be insufficient to pay the debt to a small probability π .⁴

We consider the case where the entire value of the assets is put up as collateral for the borrowing. The constraint then says that the probability that the value of the assets next period is less than what is owed on the debt should be no larger than π . Recall that total borrowing of Home leveraged institutions is $K_{HH}Q_H + K_{HF}Q_F - W_H$. Therefore the constraint is

$$Prob(K_{HH}D_H + K_{HF}D_F < (1 + R)(K_{HH}Q_H + K_{HF}Q_F - W_H)) \leq \pi \quad (18)$$

or

$$Prob(K_{HH}(D_H - (1 + R)Q_H) + K_{HF}(D_F - (1 + R)Q_F) + (1 + R)W_H < 0) \leq \pi \quad (19)$$

This is the case when

$$\begin{aligned} & K_{HH}(D - (1 + R)Q_H) + K_{HF}(D - (1 + R)Q_F) + (1 + R)W_H \geq \\ & z \left(K_{HH}^2\sigma^2 + K_{HF}^2\frac{\sigma^2}{1 - \tau} \right)^{0.5} \end{aligned} \quad (20)$$

⁴See Brunnermeier and Pedersen (2009) for a detailed discussion of the institutional features leading to these margin constraints.

where $z = -\Psi^{-1}(\pi)$ and $\Psi(\cdot)$ is the cumulative standard normal distribution.⁵ z is positive and approaches infinity as $\pi \rightarrow 0$. (20) says that portfolio risk (its standard deviation) needs to be less than or equal to a fraction $1/z$ of the expected value of the portfolio.

Note that this can also be written as a borrowing constraint. With borrowing by the Home financial institution equal to $B_H = K_{HH}Q_H + K_{HF}Q_F - W_H$, the constraint becomes

$$B_H \leq \frac{1}{1+R} \left((K_{HH} + K_{HF})D - z \left(K_{HH}^2 \sigma^2 + K_{HF}^2 \frac{\sigma^2}{1-\tau} \right)^{0.5} \right) \quad (21)$$

Importantly, the borrowing constraint limits borrowing not to the value of the collateral today, but the expected value of the collateral tomorrow adjusted for risk. The risk gets a higher weight the smaller π and therefore the larger z .

The optimal holdings of Home and Foreign assets by leveraged Home investors are now

$$K_{HH} = \frac{D - (1+R)Q_H}{\gamma_H \sigma^2} W_H \quad (22)$$

$$K_{HF} = (1-\tau) \frac{D - (1+R)Q_F}{\gamma_H \sigma^2} W_H \quad (23)$$

where

$$\gamma_H = \frac{\gamma - \lambda_H z \left((K_{HH}/W_H)^2 \sigma^2 + (K_{HF}/W_H)^2 \frac{\sigma^2}{1-\tau} \right)^{-0.5}}{1 - \lambda_H} \quad (24)$$

and λ_H is the Lagrange multiplier associated with the margin constraint.

The only impact of the margin constraint on the optimal portfolios of leveraged institutions is to affect their effective rate of risk-aversion. The rate of risk-aversion γ is replaced by the effective rate of risk-aversion γ_H in the optimal portfolios of leveraged Home institutions. When the margin constraint does not bind, so that $\lambda_H = 0$, it is immediate that $\gamma_H = \gamma$ and there is no change. When the margin constraint does bind, γ_H can be computed by making the constraint (20) an equality. This gives

$$\gamma_H = \frac{1}{1+R} \left(z(s_H^2 + (1-\tau)s_F^2)^{0.5} - s_H^2 - (1-\tau)s_F^2 \right) \quad (25)$$

where $s_H = (D - (1+R)Q_H)/\sigma$ and $s_F = (D - (1+R)Q_F)/\sigma$ are Sharpe ratios.

⁵This implicitly assumes that the asset payoffs are normally distributed.

Two opposite forces affect γ_H in response to a shock that reduces asset prices. On the one hand, expected excess returns $D - (1 + R)Q_i$ rise, which weaken the constraint. On the other hand, these higher expected excess returns increase leverage, which increases risk. In the calibration in Section 4 it is this second factor that strongly dominates, leading to an increase in risk-aversion.

The results for Foreign leveraged institutions are analogous, leading to an effective rate of risk-aversion of γ_F that is equal to

$$\gamma_F = \frac{1}{1 + R} \left(z((1 - \tau)s_H^2 + s_F^2)^{0.5} - (1 - \tau)s_H^2 - s_F^2 \right) \quad (26)$$

Market clearing conditions now become

$$\frac{D - (1 + R)Q_H}{\sigma^2} \left(\frac{1}{\gamma_H} W_H + \frac{1}{\gamma_F} (1 - \tau) W_F + \frac{1}{\gamma_{NL}} (2 - \tau) W_{NL} \right) = K \quad (27)$$

$$\frac{D - (1 + R)Q_F}{\sigma^2} \left(\frac{1}{\gamma_H} (1 - \tau) W_H + \frac{1}{\gamma_F} W_F + \frac{1}{\gamma_{NL}} (2 - \tau) W_{NL} \right) = K \quad (28)$$

3 Impact of Home Defaults

We now consider the impact on Home and Foreign asset prices of balance sheet losses due to Home defaults in period 1. We start from a symmetric equilibrium where $\delta = 0$ and then consider the impact of Home defaults by considering a marginal increase in δ . We compute the impact on asset prices by differentiating the market equilibrium conditions around the point where $\delta = 0$.

3.1 Symmetric Equilibrium

It is useful to first discuss the symmetric equilibrium before introducing the impact of the defaults. We will assume that in the presence of balance sheet constraints, these constraints are on the margin of starting to bind in the symmetric equilibrium. They will strictly bind once the economy is hit by the shock. Therefore the symmetric equilibrium is exactly the same for the three cases discussed in the previous section, with and without balance sheet constraints.

Without loss of generality, we normalize the mean dividend D such that $Q_H = Q_F = 1$ in this symmetric equilibrium. We set $Q_0 = 1/(1 + R)$, so that the net return on the (long-term) assets from period 0 to 1 is R . This is just a simplification, which is not important to the results. Define $W = W_H = W_F$,

which is wealth of leveraged institutions at the beginning of period 1. We then have $W = (1 + R)W_0$. Define leverage as the ratio of the value of Home plus Foreign assets relative to net worth. Leverage in period 0 is equal to⁶

$$LEV = \frac{W_0 + B_0 - L_0}{W_0} \quad (29)$$

Define $\bar{W} = W_{NL}\gamma/\gamma_{NL}$. This is a risk-aversion adjusted level of wealth of non-leveraged investors that has the same impact on asset demand as the wealth W of leveraged investors. Imposing asset market equilibrium gives the equilibrium expected excess return:

$$PREM = D - (1 + R) = \frac{\gamma\sigma^2}{2 - \tau} \frac{K}{W + \bar{W}} \quad (30)$$

Using this, leverage in period 1 after new portfolio decisions are made is equal to

$$LEV = \frac{2 - \tau}{\gamma\sigma^2} PREM = \frac{K}{W + \bar{W}} \quad (31)$$

We set K such that this is equal to leverage (29) in period 0.

Finally, we set $\alpha = 1/(2 - \tau)$, so that the fraction invested in assets of the domestic country is the same in periods 0 and 1. We also define the share of assets held by leveraged institutions in the symmetric equilibrium as $SHARE$, which is equal to $W/(W + \bar{W})$.

3.2 Impact of Shock without Balance Sheet Constraints

We now consider the impact of marginal Home defaults. Define $LOSS = L_0\delta/W_0$. This is the value of Home defaults, scaled by initial net worth. Define $d\bar{Q}_H$ and $d\bar{Q}_F$ as the asset prices changes in the absence of balance sheet constraints. Fully differentiating the asset market clearing conditions, we get

$$d\bar{Q}_H = -0.5 \left(\frac{1}{d_1} + \left(\frac{\tau}{2 - \tau} \right)^2 \frac{1}{d_2} \right) PREM * SHARE * LOSS \quad (32)$$

$$d\bar{Q}_F = -0.5 \left(\frac{1}{d_1} - \left(\frac{\tau}{2 - \tau} \right)^2 \frac{1}{d_2} \right) PREM * SHARE * LOSS \quad (33)$$

⁶Here we do not include the short-term assets in the definition of leverage to be consistent with period 1, where there are only long-term assets. Including them makes little difference to leverage in the application in the next section as we need only a small amount of the short-term assets coming due in period 1 in order to generate a large drop in net worth due to defaults.

where

$$d_1 = 1 + R - PREM * SHARE * LEV \quad (34)$$

$$d_2 = 1 + R - \frac{\tau^2}{(2 - \tau)^2} * PREM * SHARE * LEV \quad (35)$$

The algebra behind this result, as well as others in this section, can be found in the Appendix.

The Home asset price clearly falls, while the Foreign asset price falls as long as $\tau < 1$. The case of $\tau = 1$ is an extreme of financial autarky, where only domestic assets are held and there is no transmission to the Foreign country ($d\bar{Q}_F = 0$). The other extreme case is $\tau = 0$, where there is perfect portfolio diversification. In that case Home and Foreign asset prices drop by the same amount, so that there is one-to-one transmission to the Foreign country. The more interesting and realistic cases though lie in between, where $0 < \tau < 1$ and portfolios are only partially diversified across countries. Transmission is then partial in that the Foreign asset prices drops by less than the Home asset price.

There are three channels of transmission of the shock to the Foreign country. In order to see this, it is useful to disentangle the various exposures that the countries have to each other. There are three types. Consider the Foreign leveraged institutions. First, they inherit claims from period 0 on Home short-terms assets on which the defaults take place. Second, they inherit claims from period 0 on Home long-terms assets. And finally, they partially invest their portfolio in period 1 in Home assets.

These three types of exposures lead to three different transmission mechanisms through which the Foreign country is affected. The first is through balance sheet losses associated with the Home assets on which defaults take place. This is a direct exposure channel. The second is through further balance sheet losses due to a drop in the prices of Home (long-term) assets to which Foreign leveraged institutions are exposed. This is a standard balance sheet valuation channel. And finally there is a portfolio growth channel. The drop in net worth of Home leveraged institutions leads to a drop in their demand for Foreign assets in period 1. One can also think of this as a lending channel to the extent that the assets consist of loans rather than securities.

In the model we have assumed that these three types of cross-border financial exposures are identical and can be summarized with a single τ . But in order to

understand their separate roles in transmission, it is useful to disentangle them. First consider the direct exposure channel. In order to isolate this, assume that there are no cross-border holdings of the long-term assets, either in period 0 or 1. It is easy to show that in this case

$$dQ_H = -\frac{\frac{1}{2-\tau} * PREM * SHARE * LOSS}{1 + R - PREM * SHARE * LEV} \quad (36)$$

$$dQ_F = -\frac{\frac{1-\tau}{2-\tau} * PREM * SHARE * LOSS}{1 + R - PREM * SHARE * LEV} \quad (37)$$

Since the portfolio shares invested in Home short-term assets are respectively $\alpha = 1/(2 - \tau)$ and $1 - \alpha = (1 - \tau)/(2 - \tau)$ for Home and Foreign leveraged institutions, the exposure of Foreign institutions to the Home assets on which the defaults take place is a fraction $1 - \tau$ of the exposure by Home institutions. Corresponding to that, (36)-(37) show that the drop in the Foreign asset price is a fraction $1 - \tau$ of the drop in the Home asset price. Transmission only depends on τ . The closer it is to 1 (the bigger the home bias), the lower the transmission. Higher leverage and a larger asset share held by leveraged institutions only affect the overall drop in asset prices, not the relative drop of the Foreign to the Home asset price.⁷

In what follows it is useful to also write (36)-(37) in terms of changes in the average asset price and the difference in asset prices, denoted $Q_A = 0.5(Q_H + Q_F)$ and $Q_D = Q_H - Q_F$. When there is only transmission through direct exposure, we have

$$dQ_A = -0.5 \frac{PREM * SHARE * LOSS}{1 + R - PREM * SHARE * LEV} \quad (38)$$

$$dQ_D = -\frac{\frac{\tau}{2-\tau} * PREM * SHARE * LOSS}{1 + R - PREM * SHARE * LEV} \quad (39)$$

Next we bring on board the balance sheet valuation channel by assuming that leveraged institutions inherit diversified claims on the long-term assets from period

⁷A higher asset share of leveraged institutions raises the response of asset prices to the shock in two ways. First, the shock itself matters more the larger the relative size of the leveraged institutions that are hit by the shock. Second, there is an amplification effect when asset prices go down as it reduces the net worth of leveraged institutions more. The larger the relative size of leveraged institutions, the more this amplification matters for equilibrium prices. This latter effect is also enhanced the more leveraged the institutions are as a given drop in asset prices reduces their net worth more when they are more leveraged. Also note that leverage matters indirectly by affecting the asset share of leveraged institutions, which can be written as $SHARE = (W/\gamma)/[(W/\gamma) + (W_{NL}/\gamma_{NL})]$. More leverage is the result of a drop in γ .

0. Institutions invest a fraction $1 - \alpha = (1 - \tau)/(2 - \tau)$ in the asset of the other country. In that case the drop in the Home asset price leads to a further balance sheet loss for Foreign leveraged institutions, providing an additional transmission mechanism. The change in the average asset price remains the same as in (38) because we have simply reshuffled the losses from the Home price decline away from Home leveraged institutions and towards Foreign leveraged institutions. The additional transmission to the Foreign country reduces the difference between the decline in Home and Foreign asset prices, which is now

$$dQ_D = -\frac{\frac{\tau}{2-\tau} * PREM * SHARE * LOSS}{1 + R - \frac{\tau}{2-\tau} * PREM * SHARE * LEV} \quad (40)$$

This is smaller than in (39), which implies a larger decline in the Foreign asset price relative to the decline in the Home asset price.

We finally introduce the third transmission channel, through optimal portfolio allocation in period 1. This leads to additional transmission to the Foreign country as the lower net worth of Home leveraged institutions leads to a drop in their demand for Foreign assets. The change in the average asset price remains the same as in (38) because the change here involves a reshuffling of portfolio allocation, with a larger decline in demand now falling on Foreign assets and a smaller decline on Home assets. This third transmission mechanism leads to a further reduction in the difference between the decline in Home and Foreign asset prices, which is now⁸

$$dQ_D = -\frac{\left(\frac{\tau}{2-\tau}\right)^2 * PREM * SHARE * LOSS}{1 + R - \left(\frac{\tau}{2-\tau}\right)^2 * PREM * SHARE * LEV} \quad (41)$$

The bottom line from all of this is that the transmission to the Foreign country may be larger than suggested by the financial exposures themselves. Even though Foreign leveraged institutions have an exposure to Home assets that is only a fraction $1 - \tau$ of the exposure by Home leveraged institutions, the relative drop in the Foreign asset price is clearly larger than $1 - \tau$. The reason for this is the cumulative effect of the various transmission channels.

We can provide further insight into the magnitude of these transmission channels by considering the results in terms of order calculus. A shock in the model, or a standard deviation of shocks, is first-order. Therefore $d\delta$ and σ are first-order.

⁸It is easily checked that (32)-(33) correspond to (38) and (41) when using $Q_H = Q_A + 0.5Q_D$ and $Q_F = Q_A - 0.5Q_D$.

Analogously, σ^2 is second-order and $\sigma^2 d\delta$ is third-order. The zero-order component of a variable is its value in the absence of shocks ($\sigma \rightarrow 0$ in the symmetric equilibrium). $SHARE = W/(W + \bar{W})$ and $LEV = K/(W + \bar{W})$ are zero-order as they do not depend on shocks or σ . $LOSS$ is first-order as it is proportional to $d\delta$. $PREM$ is second-order as it is proportional to σ^2 from (30).

It is now easy to check that changes in asset prices are third-order through the product of $PREM$ and $LOSS$ in the numerator of all the expressions above. There is also a term that depends on $PREM$ in the denominators, as well as in d_1 and d_2 . These contribute to a fifth-order component of the change in asset prices, which tends to be quite small. If we focus on the third-order component, which is the dominant component of asset price changes, we can drop the terms in $PREM$ in the denominators and in d_1 and d_2 .

If transmission only takes place through direct exposure, the changes in asset prices are then

$$dQ_H = -\frac{1}{2 - \tau} \frac{1}{1 + R} PREM * SHARE * LOSS \quad (42)$$

$$dQ_F = -\frac{1 - \tau}{2 - \tau} \frac{1}{1 + R} PREM * SHARE * LOSS \quad (43)$$

This again shows that the drop in the Foreign asset price is a fraction $1 - \tau$ of the drop in the Home asset price. It is useful for what follows to understand why the changes in asset prices are third-order. The defaults lead to a first-order drop in net worth of leveraged institutions, which leads to a first-order drop in demand for assets. In order to generate equilibrium it is sufficient to have a third-order drop in asset prices. The resulting third-order increase in the expected excess return leads to a first-order increase in demand for the risky assets as the expected excess return is divided by σ^2 in the optimal portfolios.

These changes in asset prices remain unchanged when we add transmission through balance sheet valuation effects. Balance sheet valuation effects, while theoretically present, are very small. The reason is that the changes in equilibrium asset prices, which are third-order, have a third-order effect on net worth. This is two orders of magnitude smaller than the impact of the defaults on net worth, which is first-order. The additional drop in asset prices that is needed to clear the market is then of fifth-order, which tends to be quite small.

If we finally also bring on board transmission through portfolio allocation, we

have

$$dQ_H = -0.5 \left(1 + \left(\frac{\tau}{2-\tau} \right)^2 \right) \frac{1}{1+R} PREM * SHARE * LOSS \quad (44)$$

$$dQ_F = -0.5 \left(1 - \left(\frac{\tau}{2-\tau} \right)^2 \right) \frac{1}{1+R} PREM * SHARE * LOSS \quad (45)$$

Transmission is now increased as

$$\frac{dQ_F}{dQ_H} = \frac{1 - \left(\frac{\tau}{2-\tau} \right)^2}{1 + \left(\frac{\tau}{2-\tau} \right)^2} > 1 - \tau$$

There is now a larger decline in demand for Foreign assets, which is first-order, and lower decline in demand for Home assets, accounting for the additional transmission.

It is also useful to note that the extent of transmission only depends on τ and not on leverage or the share of wealth held by leveraged institutions. To get some sense of the numbers, consider $\alpha = 0.85$, so that 85% is invested in domestic assets. In Section 4 we will argue that this is pretty close to reality. In that case $\tau = 0.8235$. The drop in the Foreign asset price relative to the drop in the Home asset prices is then a fraction 0.18 with only the direct exposure channel present and 0.34 when the portfolio allocation channel is added. Total transmission is therefore about one third, which is not very big. But we have not yet considered the impact of the borrowing constraints.

3.3 Impact of Shock with Constant Leverage Constraints

Next consider the case where there is a constant leverage constraint. Fully differentiating in this case yields

$$dQ_H = \frac{1}{e_1} (\psi d\bar{Q}_H + (1 - \psi) d\bar{Q}_F) \quad (46)$$

$$dQ_F = \frac{1}{e_1} ((1 - \psi) d\bar{Q}_H + \psi d\bar{Q}_F) \quad (47)$$

where

$$\psi = 0.5 + 0.5 \frac{e_1}{e_2} \quad (48)$$

$$e_1 = 1 - \frac{1 + R - PREM}{d_1} SHARE \quad (49)$$

$$e_2 = 1 - \frac{\tau^2}{(2 - \tau)^2} \frac{1 + R - PREM}{d_2} SHARE \quad (50)$$

We will again consider the case where $0 < \tau < 1$ as the extremes of $\tau = 1$ (financial autarky) and $\tau = 0$ (perfect diversification) again give the previous results of no transmission ($dQ_F = 0$) and perfect transmission ($dQ_F = dQ_H$). When $0 < \tau < 1$, we have $0 < \psi < 1$, so that the changes in the asset prices are a weighted average of the changes in the two asset prices in the absence of balance sheet constraints, times an amplification factor. These results imply more transmission in that the ratio of dQ_F to dQ_H is bigger, as well as a larger overall impact of the shock on asset prices.

The larger overall drop in asset prices, as well as the bigger relative drop in the Foreign asset price, are a result of the balance sheet constraint that becomes binding. To see this, we have

$$dR_H = -\frac{1 + R - PREM}{2 - \tau} (dQ_H + (1 - \tau)dQ_F) \quad (51)$$

$$dR_F = -\frac{1 + R - PREM}{2 - \tau} ((1 - \tau)dQ_H + dQ_F) \quad (52)$$

A drop in asset prices raises the effective borrowing rates. The reason for this is that lower asset prices lead to higher expected returns and therefore higher optimal leverage. The leverage constraints then become binding, which is equivalent to an increase in the borrowing rate. Higher borrowing rates imply lower asset demand, which is now an additional amplification mechanism.

There is now also a fourth transmission mechanism. The lower Home asset price raises the expected excess return on the Home asset, which raises the demand for Home assets by Foreign leveraged institutions. This increases their leverage and makes the balance sheet constraint of the Foreign leveraged institutions more binding, raising their effective borrowing rate. This explains the further increase in the relative drop of the Foreign asset price.

To get a sense of the magnitude of this additional transmission channel, we can write the third-order component of the change in asset prices as (49)-(50) with $d\bar{Q}_H$ and $d\bar{Q}_F$ being the third-order components in the absence of the leverage constraint (in (44)-(45)), $e_1 = 1 - SHARE$ and

$$\psi = 0.5 + 0.5 \frac{1 - SHARE}{1 - \left(\frac{\tau}{2-\tau}\right)^2 SHARE}$$

Clearly $\psi < 1$, so that transmission is larger. Just like the balance sheet valuation channel, the leverage constraint channel operates through changes in asset prices.

But the leverage constraint channel is stronger. The third-order drop in asset prices leads to a third-order drop in net worth through the balance sheet valuation channel and a third-order increase in effective borrowing rates through the leverage constraint channel. But while the former affects asset demand only to the third-order, the latter leads to a first-order change in asset demand as the expected excess return is divided by σ^2 in optimal portfolios.

The extent of transmission now depends not only on τ , but also on *SHARE*. While a drop in asset prices raises the expected excess return for all investors (both leveraged and non-leveraged), the additional impact on the excess return through effective borrowing rates is only relevant for leveraged investors. The larger their relative size, the more this affects the equilibrium. In comparison to the case with no borrowing constraints, an increase in *SHARE* raises both the overall drop in asset prices and the transmission to the Foreign country. If *SHARE* becomes very small, the additional transmission through the leverage constraint vanishes.

3.4 Impact of Shock with Margin Constraints

Finally consider the case of margin constraints. Fully differentiating in this case yields

$$dQ_H = \frac{1}{h_1} (\omega d\bar{Q}_H + (1 - \omega)d\bar{Q}_F) \quad (53)$$

$$dQ_F = \frac{1}{h_1} ((1 - \omega)d\bar{Q}_H + \omega d\bar{Q}_F) \quad (54)$$

where

$$\omega = 0.5 + 0.5 \frac{h_1}{h_2} \quad (55)$$

$$h_1 = 1 - \frac{1 + R - PREM * LEV}{d_1} SHARE \quad (56)$$

$$h_2 = 1 - \frac{\tau^2}{(2 - \tau)^2} \frac{1 + R - PREM * LEV}{d_2} SHARE \quad (57)$$

In what follows we assume that $1 + R > PREM * LEV$, which is the case for reasonable parameterization (see Section 4).

The extremes of financial autarky ($\tau = 1$) and perfect diversification ($\tau = 0$) again imply respectively perfect transmission and no transmission. When $0 < \tau < 1$, we have $0 < \omega < 1$, so that the changes in the asset prices are a weighted average

of the changes in the two asset prices in the absence of balance sheet constraints, times an amplification factor. This is analogous to the results under a constant leverage constraint. These results again imply larger transmission and a bigger overall impact of the shock on asset prices.

The larger overall drop in asset prices, as well as the bigger transmission to the Foreign country, are again the result of the balance sheet constraint that becomes binding. We have

$$\frac{d\gamma_H}{\gamma} = -\frac{1}{PREM} \frac{1 + R - PREM * LEV}{2 - \tau} (dQ_H + (1 - \tau)dQ_F) \quad (58)$$

$$\frac{d\gamma_F}{\gamma} = -\frac{1}{PREM} \frac{1 + R - PREM * LEV}{2 - \tau} ((1 - \tau)dQ_H + dQ_F) \quad (59)$$

A drop in asset prices raises the effective rates of risk-aversion. The reason for this is that lower asset prices lead to higher expected returns and therefore higher optimal leverage. This in turn leads to increased balance sheet risk, so that the margin constraints become binding. As discussed in Section 2, there is one offsetting factor. Holding leverage constant, the higher expected returns themselves make the margin constraints less binding. This is especially the case when leverage is high to begin with. However, as long as $1 + R > PREM * LEV$, the increase in risk dominates. The constraints then become more binding, which implies an increase in effective risk-aversion.

Higher effective rates of risk-aversion reduce asset demand, which accounts for the further drop in asset prices. Just as was the case for the constant leverage constraint, there is now also a fourth transmission channel. The lower Home asset price raises the expected excess return on Home assets, which raises demand for Home assets by the Foreign leveraged institutions and makes them more leveraged. This leads the margin constraint to bind more and therefore the effective rate of risk-aversion to rise. This leads to a further drop in the relative demand for Foreign assets and therefore a larger relative decline in the price of the Foreign asset.

If we consider the third-order component of the change in asset prices in this case, it is easy to see that it is exactly the same as in the case of a constant leverage constraint. This is because the zero-order components of h_1 and h_2 are the same as those for respectively e_1 and e_2 . Therefore the zero-order component of ω is the same as that for ψ . Surprisingly therefore, while the nature of the constraint is a very different one, up to third-order they have the same impact on the asset prices.

4 Numerical Results

We next calibrate the model parameters in order to quantify the magnitude of the overall transmission of the shock to the Foreign country. In contrast to the theoretical exercise in the previous section, we now consider a large default shock. We set $\delta = 0.565$ and $L_0/W = 1$, which under the benchmark parameterization discussed below implies that the net worth of Home leveraged institutions is cut exactly in half due to the Home defaults.

4.1 Calibration

We calibrate the parameters to the solution of the model under the symmetric equilibrium where $\delta = 0$ (no defaults). First consider the values of LEV , $SHARE$ and $PREM$. As discussed below, these are related to structural model parameters. We set leverage in period 0 and 1 equal to $LEV=12$. This number is based on an estimate by Greenlaw et.al. (2008), which is based on the entire leveraged financial sector (commercial banks, savings institutions, credit unions, finance companies, brokers/hedge funds and GSEs) at the end of 2007. Based on the same definition of leveraged funds, we set $SHARE=0.15$. The value of financial assets held by leveraged financial institutions in the last quarter of 2007, based on Flow of Funds data, was 22.9 trillion dollars. The value of total financial assets was 148.8 trillion dollars. We set $PREM=0.02$. Based on FDIC data for U.S. commercial banks from 2000 to 2007, the average net operating profits as a fraction of assets was 1.22%. Since other less regulated leveraged institutions (such as broker/dealers and hedge funds) surely earn higher average returns, we assume an average excess return of 2%.

The values of LEV , $SHARE$ and $PREM$ translate into values of various structural parameters. For the balance sheet variables what matters is their relative values, not their absolute size. Note that $W = (1+R)W_0$ from the previous section. LEV at time 0 gives us a value of $(W_0 + B_0 - L_0)/W_0$, which in turn gives a value of B_0/W_0 as we already assumed $L_0/W = 1$. $SHARE$ gives us a value of W/\bar{W} . LEV at time 1 then gives us a value of K/W . Finally, $PREM$ and LEV are used to set γ from (30):

$$\gamma\sigma^2 = (2 - \tau)PREM \frac{1}{LEV} \quad (60)$$

This also uses τ , which we discuss below. Note that only the product $\gamma\sigma^2$ affects

the equilibrium. We can therefore set σ at any arbitrary level and then choose γ such that this equation is satisfied. The breakdown between σ and γ is irrelevant for the results.

We will report our results in the form of pictures that relate the percentage drop in asset prices to values of $\alpha = 1/(1 - \tau)$ ranging from 0.5 (full diversification) to 1 (complete home bias). But it is critical to know where we are in this range, which varies all the way from perfect transmission to no transmission. Fidora, Fratzscher and Thimann (2007) report that the United States invests 86% in domestic equity and 95% in domestic debt securities. This is based on data over the period 2001-2003. The numbers are not much different for financial institutions. Buch et.al. (2010) reports that 89% of the assets of U.S. banks in 2004 are domestic. This abstracts from foreign subsidiaries. But García-Herrero and Vazquez (2007) report that U.S. bank holding companies hold only 6% of assets in foreign subsidiaries. This is actually an overstatement as it includes only those banks that are large and have at least 3 foreign subsidiaries. So overall the fraction of assets held at home is probably somewhere around 85%. This implies $\alpha = 0.85$ and $\tau = 0.8235$.

It is useful to point out that $\alpha = 0.85$ is also consistent with data on direct exposure to U.S. asset backed securities by foreign leveraged institutions. $\alpha = 0.85$ implies that 85% of the exposure to asset backed securities is by U.S. leveraged institutions and 15% by Foreign leveraged institutions. Estimates by Beltran, Pounder and Thomas (2008) of foreign exposure to U.S. asset backed securities as of June 2007 are equal to 19% of all U.S. asset backed securities (see also Kamin and Pounder Demarco (2010)). Similarly, Greenlaw et. al. (2008) estimate that foreign leveraged institutions held 16% of the total U.S. subprime mortgage exposure.

We set the riskfree rate R at 0.008, based on Mehra and Prescott (1985). Also, as mentioned in the previous section, without loss of generality we set D such that the asset prices are equal to 1 in the symmetric equilibrium. There is one additional parameter for the constant leverage constraint, which is κ . We set it such that the constraint just binds in the symmetric equilibrium. This is the case for $\kappa = LEV$. Similarly, under margin constraints z is set such that the constraint just binds in the symmetric equilibrium, which is the case when $\sigma z = (2 - \tau)^{0.5}((1 + r)/LEV + PREM)$.

4.2 Graphical Results

Figure 1 shows the percentage drop in the Home and Foreign asset prices as a function of $\alpha = 1/(2 - \tau)$, the fraction invested in domestic assets. Under the benchmark parameterization we assume $\alpha = 0.85$.

Figure 1 shows that as we increase home bias, the Home price drops more while the Foreign price drops less. A rise in τ implies that the losses from the defaults fall more on Home leveraged institutions. In addition, for given relative losses of Home leveraged institutions, increased home bias in period 1 implies that more of the drop in asset demand affects the Home assets. The same factors imply that the Foreign asset price is less affected when home bias increases, up to the point where $\alpha = 1$ and the Foreign asset price is unaffected.

Two key conclusions that can be drawn from Figure 1 are that transmission is relatively small under the benchmark parameterization where $\alpha = 0.85$ and the role of borrowing constraints is quite limited. Consistent with the findings in the previous section for a marginal Home default, in the absence of borrowing constraints the drop in the Foreign asset price is one third of the drop in the Home asset price. It is only slightly higher with leverage constraints (fraction 0.38 for constant leverage and 0.37 with margin constraints). We have seen that the additional transmission through the borrowing constraints depends critically on *SHARE*. Since the share of assets held by leveraged institutions is relatively small (15%), the additional transmission through the borrowing constraints also tends to be small. Overall therefore we can conclude that transmission, while not negligible, is far from perfect under the benchmark parameterization.

Figure 2 shows what happens if we increase *SHARE* to 0.5, so that now leveraged financial institutions hold half of all assets. This is more than three times as big as observed in the data. It is therefore not meant to be realistic, but rather to shed light on the role of leveraged institutions. Consistent with the results in Section 3, increasing the asset share of leveraged institutions has two implications. First, it substantially increases the overall impact of the shock on asset prices. Second, it increases transmission with binding borrowing constraints.

Transmission in the absence of borrowing constraints remains about one third when $\alpha = 0.85$. With a constant leverage constraint transmission is increased from 0.38 to 0.52. With margin constraints it is increased from 0.37 to 0.44.⁹ Therefore

⁹The fact that transmission is larger under constant leverage constraints than margin con-

even with this large asset share of leveraged institutions, transmission is at most one half.

Two other key parameters are *LEV* and *PREM*. Consistent with the results in Section 3, changing these parameters mainly impacts the magnitude of the asset price changes, with little effect on transmission.

4.3 Two Extensions

We finally consider two extensions: correlated asset payoffs and feedback effects from asset prices to the wealth of non-leveraged investors.

We introduce a positive correlation in a way analogous to Okawa and van Wincoop (2010). The Home and Foreign dividends are respectively $D_H = D + \epsilon_H + \epsilon_W$ and $D_F = D + \epsilon_F + \epsilon_W$, where ϵ_H and ϵ_F are country specific dividend innovations and ϵ_W is a global innovation. The global and country-specific innovations are uncorrelated. The standard deviation of the global innovation is σ_w^2 . For the country-specific innovations we continue to assume the information asymmetry. For example, the variance of ϵ_H is σ^2 from the perspective of Home investors and $\sigma^2/(1 - \tau)$ from the perspective of Foreign investors. The variance-covariance matrix for of the asset payoffs from the perspective of Home and Foreign agents is then

$$\Sigma_H = \begin{pmatrix} \sigma^2 + \sigma_w^2 & \sigma_w^2 \\ \sigma_w^2 & \frac{\sigma^2}{1-\tau} + \sigma_w^2 \end{pmatrix} \quad \Sigma_F = \begin{pmatrix} \frac{\sigma^2}{1-\tau} + \sigma_w^2 & \sigma_w^2 \\ \sigma_w^2 & \sigma^2 + \sigma_w^2 \end{pmatrix} \quad (61)$$

This of course affects asset demand. For example, in the absence of borrowing constraints demand for Home and Foreign assets by Home leveraged institutions is

$$\begin{pmatrix} K_{HH} \\ K_{HF} \end{pmatrix} = \frac{1}{\gamma} \Sigma_H^{-1} \begin{pmatrix} D - (1 + R)Q_H \\ D - (1 + R)Q_F \end{pmatrix} W_H \quad (62)$$

For data purposes we treat the standard deviation of the country-specific shocks as σ^2 , so that the correlation between the asset returns is $1/(1 + \sigma_w^2/\sigma^2)$. In Figure 3 we report results when we set σ_w^2/σ^2 such that this correlation is 0.3. This is

 strains to a non-negligible degree suggests that effects higher than third-order play some role. We found in Section 3 that the third-order price impact under margin constraints is the same as that under constant leverage constraints.

probably an upper-bound of what is reasonable based on cross-country correlations of stock and bond returns.¹⁰

Introducing a positive correlation leads to a fifth transmission channel, which is an arbitrage channel. When the correlation between Home and Foreign assets is zero, a change in the expected return on the Home asset has no effect on the demand for the Foreign asset. There is only a switch between the Home asset and the bond. With a positive correlation, the Home and Foreign assets become substitutes. A drop in the Home asset price, which raises the expected excess return on the Home asset, now leads to a portfolio shift away from Foreign assets to Home assets. This leads to a larger drop in the Foreign asset price than before and therefore larger transmission. Relative to the benchmark parameterization the transmission coefficient is increased from 0.34 to 0.44 without borrowing constraints, from 0.38 to 0.48 with constant leverage constraints and from 0.37 to 0.47 with margin constraints. It therefore remains the case that the Foreign asset price drops by less than half as much as the Home asset price.

The second extension is to allow the wealth of non-leveraged investors to depend on asset prices. So far we have assumed that the wealth of non-leveraged institutions at the start of period 1 is a given W_{NL} that does not depend on asset prices. Instead now assume that for Home non-leveraged investors it is

$$W_{NL}(\eta(\alpha Q_H + (1 - \alpha)Q_F) + 1 - \eta)$$

It therefore remains equal to W_{NL} in the symmetric equilibrium where $Q_H = Q_F = 1$. But now wealth drops in response to the shock as asset prices fall. It is assumed that a fraction η of wealth is sensitive to asset prices and of that a fraction α to the Home asset price and $1 - \alpha$ to the Foreign asset price. This is analogous to the assumption for leveraged institutions, with the only exception that there $\eta > 1$ due to leverage, while here we assume $\eta < 1$ as investors inherit non-leveraged positions from the previous period.

¹⁰Buch et.al. (2010) approximate cross-country bank returns with cross-country government bond returns. Using cross-country correlations for 5-year government bond returns among 13 industrialized countries from Cappiello et.al. (2006), together with data on the relative size of these government bond markets, we find a correlation between the U.S. bond return and an aggregate of non-U.S. bond returns of 0.18. This is less than the correlation between stock returns. For example, Dumas et.al. (2002) report a correlation of 0.54 between the U.S. stock return and the aggregate stock return in the rest of the world. But leveraged financial institutions do not hold a lot of stock.

The impact of this change on the results turns out to be negligible. Even when we set η equal to 1 (the maximum without leverage), transmission remains the same without borrowing constraints, rises from 0.38 to 0.39 with constant leverage constraints and from 0.37 to 0.38 with margin constraints. It has an effect analogous to the balance sheet valuation channel for leveraged institutions. As we have seen in Section 3, this only has a small fifth-order effect on the change in asset prices.

We should also point out that making the wealth of non-leveraged investors a positive function of asset prices has an effect similar to that of making the supply of capital K in the two countries a negative function of the asset prices. This would be the case if for example we introduce investment to the model, which depends negatively on asset prices through a Tobin's q effect. Or alternatively, as discussed in Section 2, we could interpret the assets as loans with the interest rates inversely related to the asset prices. Then a negative relationship between the demand for loans and the interest rate also implies a positive relationship between K and Q .

Since this is all similar to making W_{NL} a negative function of asset prices, the impact on the changes of equilibrium asset prices remains virtually identical. One new result develops though if we do this. Investment, or more generally the demand for loans, will drop. The drop will be of third-order, proportional to the third-order drop in asset prices. The impact of the shock on the real side of the two economies will then be proportional to the impact on their asset prices. Limited transmission to the Foreign asset price will then translate into limited transmission to the Foreign real economy.

4.4 Magnitude of the Impact of the Balance Sheet Shock

So far we have focused on the extent of transmission. But at least as interesting is the magnitude of the impact, which is quite small. Even though the shock cuts the total net worth of leveraged institutions in half, the Foreign asset price in Figure 3 drops by only 0.05% when $\alpha = 0.85$. This is clearly tiny relative to the large drop in asset prices seen during the crisis. If we think of the asset as a loan, for which the gross lending rate is \bar{D}/Q_F , when it translates into an increase in the Foreign lending rate by about 5 basis points. This is again very small. Even if we double the shock, which would reduce the net worth of Home leveraged institutions to zero, the impact on the Foreign lending rate is still only 10 basis points.

If we were to extend the model by introducing output, saving and investment decisions, the impact of such a small increase in the lending rate on output would obviously be very small. This is also consistent with Hebling, Huiddrom, Kose and Otrok (2010), who find that a global credit shock during the crisis can account for only about one tenth of the overall drop in world GDP.

Two factors play a key role in this small effect. The first is that changes in asset prices are third order in the model for reasons discussed above. The second is that the share of assets held by leveraged institutions is only 15%.

4.5 Transmission Outside of Europe

Kamin and Pounder Demarco (2010) document that the lion share of the foreign exposure to U.S. asset backed securities (84% of it) was held in European and offshore banking centers during the crisis. This means that only about 3% of U.S. ABS are held outside of the U.S., Europe and Carribean. This makes it even more remarkable that the rest of the world (outside of the U.S. and Europe) was similarly affected in terms of GDP growth and stock price declines. In order to consider the transmission outside of Europe, we can consider an application of the model where the Home country combines the United States, Europe and the Caribbean. This has several other advantages as well over the approach takes so far. The United States and the European Union combined have a GDP that is 49% of world GDP in 2010, which better reflects the two equally sized countries in our model. In addition, several European countries have had their own mortgage market problems independent of the United States.

Based on the BIS Consolidated banking statistics, of the foreign assets held by European banks outside of Europe, 52% is held in the U.S. and the Caribbean. Similarly, 58% of foreign assets held by U.S. banks are in Europe and the Caribbean. Assuming, as before, that 85% of U.S. banking assets are domestic, and similarly for Europe, and that about 55% of their foreign assets are within the U.S./Europe/Caribbean, this implies that about 93% of assets of banks within the expanded Home country are claims on the expanded Home country itself.

Setting $\alpha = 0.93$, Figure 3 implies that, independent of the type of borrowing constraints, transmission to the Foreign country is just short of 25%. The impact on the Foreign country is therefore at most one fourth of the impact on the Home country. This is even more at odds with the data, which shows that the world

outside of the U.S. and Europe was similarly impacted overall (in terms of GDP and stock prices).

4.6 Discussion and Connection to the 2008 Crisis

The model clearly has a hard time accounting for the global nature of the 2008-2009 crisis, both in terms of transmission and the magnitude of the impact. It is quite possible that there are other important transmission mechanisms that are not captured by our simple model. Perhaps the greatest shortcoming, which this paper shares with the related recent literature reviewed in the next section, is that we have not allowed for the possibility of default of leveraged institutions. Introducing the possibility of default by itself would not change the results much when we only consider collateralized borrowing, which leads to the margin constraints. This is because lenders can then minimize the probability of any loss by demanding sufficient collateral.

However, this abstracts from unsecured lending, for example in the form of interbank lending. In that case default by a leveraged institution leads to losses by the lender, which may possibly be a leveraged institution itself. In that case it is possible to get a domino effect, where one bankruptcy leads to other bankruptcies of leveraged institutions. This is the case in the bank run model of Allen and Gale (2000).

In addition, a lemons problem can arise as well in this case when lenders do not know what is on the balance sheet of leveraged institutions. Consider for example a leveraged institution that does not have any exposure to risky asset backed securities, sometime referred to as “toxic assets”. With collateralized borrowing this institution should have no problem as it only has good collateral to offer. However, the story is different with unsecured lending. The lender then needs to make a judgment about the balance sheet overall. Even if the borrowing institution has no toxic assets, the lender does not know this. This may cause such unsecured lending to freeze up, consistent with the drying up of interbank lending and the commercial paper market during the crisis.

It remains to be seen how much such additional transmission channels can account for the impact of the crisis outside of the United States and Europe. Several separate pieces of information though give us pause in interpreting the global nature of the crisis as resulting from transmission through leveraged institutions.

First, evidence reported by Kamin and Pounder Demarco (2010) and Rose and Spiegel (2010) shows that there is no relation between financial linkages that countries have with the United States (including exposure to U.S. mortgage backed securities) and the decline in their asset prices and GDP growth. Indeed, it is particularly puzzling that Japan and emerging markets, which have had very limited exposure to U.S. ABS, were as much affected as the United States. In fact, Japanese GDP growth dropped even more than in the United States.

Second, Kahle and Stulz (2010) provide evidence suggesting that a global credit shock alone is at odds with the facts. Less credit would have implied that non-financial firms issue more equity and reduce cash holdings, the exact opposite of what we saw in the data. It would also imply that investment drops more for firms that are more bank dependent, which is not what we see in the data. They conclude that the global nature of the crisis is more easily explained by a negative demand shock or (possibly related) a risk shock.

This brings us to a third and final piece of evidence that appears hard to explain with a bank transmission channel alone, which is the spike in risk seen in the data. Bacchetta and van Wincoop (2010) show that the VIX (measure of stock price risk) approximately quadrupled across all industrialized countries, and even in emerging markets.

It is possible to have an endogenous increase in asset price risk in a more dynamic version of the model where asset returns depend on asset prices changes. The contraction of leveraged institutions implies a drop in liquidity as they are more sensitive to asset price changes than non-leveraged investors. This in turn can increase asset price volatility in response to future shocks, thus increasing risk today. This link between balance sheets, liquidity and risk has received a lot of attention in recent contributions such as Adrian and Shin (2008), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2011), Gromb and Vayanos (2008), He and Krishnamurthy (2008a, b).¹¹ But none have been able to generate the huge spike in risk seen in the data.

Even if you could successfully generate a huge spike in risk this way, it is not clear how this could generate an equal spike in risk in other countries. This is particularly a concern as it is hard to separate the sharp drop in asset prices from the spike in risk. In order to explain the global nature of the crisis in terms of asset

¹¹See also Kyle and Xiong (2001) and Xiong (2001).

prices we therefore need to understand the global nature of the spike in risk.¹²

A related weakness of the model is that it cannot account for the sharp drop in leverage during the Fall of 2008, especially among brokers and dealers. In the absence of borrowing constraints, as well as with margin constraints, leverage increases as the lower asset prices raise the expected excess return, which increase optimal leverage. This is related to the constant asset return risk in the model. An increase in risk would reduce optimal leverage.

5 Connection to Existing Literature

As mentioned in the introduction, there are several related papers that have investigated the role of leveraged financial institutions in the international transmission of balance sheet shocks. We now discuss how the findings in these papers relate to the one in this paper.

An early contribution, in a middle of the crisis itself, came from Krugman (2008). With only a very sketchy model he shows how a drop in the Home asset price leads to a drop in the Foreign asset price through the balance sheets of leveraged institutions. This happens both because the Foreign leveraged institutions have exposure to Home assets (the balance sheet valuation channel) and because the lower net worth of Home leveraged institutions reduces their demand for Foreign assets. Regarding this portfolio growth or lending channel, Krugman credits Calvo (1998) for originally proposing this transmission channel in the context of contagion from Brazil to Russia in 1998 through the balance sheets of hedge funds.

Perhaps most closely related to our paper is Devereux and Sutherland (2010), who build on Devereux and Yetman (2010). They consider a two-country model where investors borrow from savers to invest in risky assets. The investors, which are similar to our leveraged financial institutions, face a constant leverage constraint. It is shown that when financial markets are perfectly integrated (no frictions associated with investing in foreign assets), a shock that tightens the leverage constraint of Home investors leads to an equal drop of Home and Foreign asset prices. This is consistent with our results under perfect diversification ($\tau = 0$, so

¹²An explanation that is quite different from transmission through leveraged institutions is offered in Bacchetta and van Wincoop (2010). They develop a model where there is a common self-fulfilling spike in risk in both the Home and Foreign country that is very large in magnitude and accompanied by a very sharp drop in asset prices.

that $\alpha = 0.5$).

Devereux and Sutherland (2010) find that in the absence of international trade in risky assets, the Home and Foreign asset prices actually move in opposite directions. This is similar in spirit to our finding that there is no positive transmission when $\alpha = 1$. In their model the Foreign asset price actually rises instead of remaining unchanged as in our model. The reason for this is a fall in the equilibrium world interest rate in their model, which we kept constant.

Devereux and Sutherland (2010) do not consider the intermediate cases. Note that in our model borrowing constraints only affect the extent of transmission when the degree of financial integration is partial, in between the extremes of zero and perfect integration. Under perfect portfolio diversification our model suggests that the shock is perfectly transmitted to the rest of the world independent of whether the financial institutions face balance sheet constraints.

Also related is Dedola and Lombardo (2010). While their model is quite complicated, they also focus on the role of leveraged institutions in the transmission of a financial shock across countries. They distinguish between two aspects of international financial integration that can lead to transmission: balance sheet exposure to foreign assets and the ability to arbitrage returns of Home and Foreign assets. We have referred to these as the balance sheet valuation channel and the portfolio allocation channel. Their key point is that transmission of a financial shock can be perfect (in that the Foreign asset price is affected just as much as the Home price) even in the absence of any balance sheet exposure to foreign assets. In other words, in order to get complete transmission the portfolio allocation channel is sufficient. One does not need the balance sheet valuation channel.

This is consistent with our findings that the balance sheet valuation channel is infinitesimal in size. If we get rid of the balance sheet valuation channel by assuming that agents do not inherit any exposure to foreign (long-term) assets from period 0, then indeed transmission would be perfect if agents can freely diversify among Home and Foreign assets in period 1. Note that this case is a bit odd because it is hard to see why there would be no diversification one period and perfect diversification the next, but it helps to zero in on the key channel of transmission.

While Dedola and Lombardo (2010) do not consider the direct exposure channel, it is sufficient for complete transmission that only the portfolio growth channel is fully operational without any financial frictions. However, shutting down the

direct exposure channel our model implies that transmission to the Foreign asset price is only a fraction $1 - \tau$. While this implies full transmission when $\tau = 0$ (essentially the case considered by Dedola and Lombardo), for a realistic value of τ of around 0.82 it implies only very limited transmission where the drop in the Foreign asset price is only a fraction 0.18 of the drop in the Home asset price.

There are also a number of papers that have considered the role of leveraged financial institutions in transmitting Home financial shocks to the Foreign country through credit channels. This is not significantly different from transmission through asset prices as a drop in lending usually entails a rise in lending rates, which is analogous to the drop in the price of assets. Kollmann et.al. (2010) develops a model with a banking sector that is perfectly integrated across countries. There is one global bank. In that case a negative balance sheet shock to the bank leads to an equal drop in lending to entrepreneurs of both countries (which takes place through a higher landing rate) and Home and Foreign output drop equally. Transmission is again perfect because of the assumption that financial markets are perfectly integrated across countries. Perri and Quadrini (2011) also have a model where a decline in credit leads to the same impact on two countries when financial markets are perfectly integrated.

Ueda (2010) and Kalemli-Ozcan et.al. (2011) consider models that allow for partial international integration. Ueda (2010) has a quite complicated model in which financial intermediaries and entrepreneurs all face borrowing constraints, there are 4 parameters that measure different aspects of financial integration across the two countries and 4 different types of shocks. The paper compares financial autarky to partial financial integration and finds partial transmission under a shock to the balance sheet of the Home financial intermediaries. This is consistent with our findings, but there is no attempt to assess the extent of transmission under calibrated values of the various financial integration parameters.

Kalemli-Ozcan et.al. (2011) consider a model where the extent of cross-country banking integration is measured by a parameter λ . There are two sectors. In sector 1 banks intermediate between consumers and firms in the domestic country only, while in sector 2 banks operate at a global level without any friction. The relative size of sector 2 is λ , which can be seen as a measure of banking integration. The paper focuses on business cycle synchronization under a combination of technology shocks and bank balance sheet shocks. It finds that bank balance sheet shocks contribute to higher business cycle synchronization and more so the larger λ . But

the paper does not report the extent of transmission of balance sheet shocks as a function of λ .

6 Conclusion

We have developed a very simple two-country model with leveraged financial institutions in order to consider various channels through which a balance sheet shock to leveraged institutions in one-country can affect the other country. We have identified five transmission channels: a direct exposure channel, a balance sheet valuation channel, a portfolio growth or lending channel, a balance sheet constraint channel and an arbitrage channel.

Even though there are quite a few transmission channels, we have seen that both transmission and the magnitude of the impact on asset prices are well below those in the data. The limited transmission results from home bias in assets of leveraged financial institutions, especially when the Home country combines the United States and Europe. The small share of total financial assets that is held by leveraged financial institutions significantly limits the magnitude of the impact balance sheet shocks.

Future research most productively can go in two directions. First, one could consider additional transmission channels. As discussed in Section 4, a substantial limitation of our model (shared with the related literature) is that we do not consider unsecured lending in the context of the possibility of default of leveraged institutions. Doing so may generate additional transmission channels. This is especially the case when lenders have imperfect information about the assets on the balance sheet of the borrower.

Second, we need to consider other explanations for the global nature of the crisis. Even if we could get close to 100% transmission, which is a big if, it is hard to see how balance sheet shocks alone could generate a common drop in asset prices and output of a magnitude seen in the data. The quantitative impact is far too small in our model. A good model should account for the sharp increase in risk seen across all countries during the crisis as it is hard to disconnect this from the similarly sharp drop in asset prices around the globe. A good model also needs to account for the lack of a relationship between financial linkages and transmission that is evident in the data.

Appendix

In this Appendix we derive the theoretical results from Section 3 under the three different assumptions about borrowing constraints.

No Borrowing Constraints

Differentiating (8)-(9) around $Q_H = Q_F = 1$ gives

$$-(1+R)(2-\tau)(W+\bar{W})dQ_H + PREM(dW_H + (1-\tau)dW_F) = 0 \quad (63)$$

$$-(1+R)(2-\tau)(W+\bar{W})dQ_F + PREM((1-\tau)dW_H + dW_F) = 0 \quad (64)$$

where we have used that $PREM = D - (1+R)$ is the excess return. It is useful to rewrite this in terms of sums and differences, giving

$$-(1+R)(W+\bar{W})(dQ_H + dQ_F) + PREM(dW_H + dW_F) = 0 \quad (65)$$

$$-(1+R)(2-\tau)(W+\bar{W})(dQ_H - dQ_F) + PREM\tau(dW_H - dW_F) = 0 \quad (66)$$

Using $LEV = (W_0 + B_0 - L_0)/W_0$, we have

$$dW_H = (1+R)W_0LEV(\alpha dQ_H + (1-\alpha)dQ_F) - (1+R)W_0\alpha LOSS \quad (67)$$

$$dW_F = (1+R)W_0LEV((1-\alpha)dQ_H + \alpha dQ_F) - (1+R)W_0(1-\alpha)LOSS \quad (68)$$

where $LOSS = L_0 d\delta/W_0$. (65) and (66) then become

$$\begin{aligned} &(-(W+\bar{W}) + PREM * LEV * W_0)(dQ_H + dQ_F) \\ &-PREM * W_0 * LOSS = 0 \end{aligned} \quad (69)$$

$$\begin{aligned} &(-(2-\tau)(W+\bar{W}) + PREM * LEV * W_0 * (2\alpha - 1)\tau)(dQ_H - dQ_F) \\ &-(2\alpha - 1)\tau * PREM * W_0 * LOSS = 0 \end{aligned} \quad (70)$$

Dividing (69) by $(W+\bar{W})/(1+R)$ and (70) by $(W+\bar{W})(2-\tau)/(1+R)$, and using $2\alpha - 1 = \tau/(2-\tau)$, this implies

$$dQ_H + dQ_F = -\frac{1}{d_1}PREM * SHARE * LOSS \quad (71)$$

$$dQ_H - dQ_F = -\frac{1}{d_2}\left(\frac{\tau}{2-\tau}\right)^2 PREM * SHARE * LOSS \quad (72)$$

where d_1 and d_2 are defined in (34)-(35). Taking the sum and difference of these equations gives the expressions for dQ_H and dQ_F in (32)-(33).

Constant Leverage Constraints

Differentiating (16)-(17) around $Q_H = Q_F = 1$ and $R_H = R_F = R$ gives the same expressions as (63) and (64) with respectively the terms $-W(dR_H + (1 - \tau)dR_F)$ and $-W((1 - \tau)dR_H + dR_F)$ added on the left hand side. (71)-(72) then become

$$\begin{aligned} dQ_H + dQ_F &= -\frac{1}{d_1} PREM * SHARE * LOSS \\ &\quad -\frac{1}{d_1} SHARE * (dR_H + dR_F) \end{aligned} \quad (73)$$

$$\begin{aligned} dQ_H - dQ_F &= -\frac{1}{d_2} \left(\frac{\tau}{2 - \tau} \right)^2 PREM * SHARE * LOSS \\ &\quad -\frac{1}{d_2} \frac{\tau}{2 - \tau} SHARE * (dR_H - dR_F) \end{aligned} \quad (74)$$

Differentiating (14)-(15) gives

$$dR_H = -\frac{1 + R - PREM}{2 - \tau} (dQ_H + (1 - \tau)dQ_F) \quad (75)$$

$$dR_F = -\frac{1 + R - PREM}{2 - \tau} ((1 - \tau)dQ_H + dQ_F) \quad (76)$$

Using these expressions in (73)-(74), we have

$$dQ_H + dQ_F = \frac{1}{e_1} (d\bar{Q}_H + d\bar{Q}_F) \quad (77)$$

$$dQ_H - dQ_F = \frac{1}{e_2} (d\bar{Q}_H + d\bar{Q}_F) \quad (78)$$

where $d\bar{Q}_H$ and $d\bar{Q}_F$ are the asset prices changes in the absence of balance sheet constraints and e_1 and e_2 are defined in (49)-(50). Taking the sum and difference of these equations then gives (46)-(47).

Margin Constraints

Differentiating (27)-(28) around $Q_H = Q_F = 1$ and $\gamma_H = \gamma_F = \gamma$ gives the same expressions as (63) and (64) with respectively the terms $-PREM * W(d\gamma_H + (1 - \tau)d\gamma_F)/\gamma$ and $-PREM * W((1 - \tau)d\gamma_H + d\gamma_F)/\gamma$ added on the left hand

side. (71)-(72) then become

$$dQ_H + dQ_F = -\frac{1}{d_1} PREM * SHARE * LOSS$$

$$-\frac{1}{d_1} SHARE * PREM * \frac{d\gamma_H + d\gamma_F}{\gamma} \quad (79)$$

$$dQ_H - dQ_F = -\frac{1}{d_2} \left(\frac{\tau}{2 - \tau} \right)^2 PREM * SHARE * LOSS$$

$$-\frac{1}{d_2} \frac{\tau}{2 - \tau} SHARE * PREM * \frac{d\gamma_H - d\gamma_F}{\gamma} \quad (80)$$

Differentiating (25)-(26) gives

$$\frac{d\gamma_H}{\gamma} = - \left(\frac{z(2 - \tau)^{-0.5}}{\gamma\sigma} - \frac{2 * PREM}{\gamma\sigma^2} \right) (dQ_H + (1 - \tau)dQ_F) \quad (81)$$

$$\frac{d\gamma_F}{\gamma} = - \left(\frac{z(2 - \tau)^{-0.5}}{\gamma\sigma} - \frac{2 * PREM}{\gamma\sigma^2} \right) ((1 - \tau)dQ_H + dQ_F) \quad (82)$$

From $\gamma_H = \gamma_F = \gamma$ we have

$$\frac{z(2 - \tau)^{-0.5}}{\gamma\sigma} = \frac{1 + R}{PREM * (2 - \tau)} + \frac{PREM}{\gamma\sigma^2} \quad (83)$$

(81)-(82) then become, using that from (30)-(31) $PREM * (2 - \tau) / (\gamma\sigma^2) = LEV$,

$$\frac{d\gamma_H}{\gamma} = -\frac{1}{PREM * (2 - \tau)} (1 + R - LEV * PREM) (dQ_H + (1 - \tau)dQ_F) \quad (84)$$

$$\frac{d\gamma_F}{\gamma} = -\frac{1}{PREM * (2 - \tau)} (1 + R - LEV * PREM) ((1 - \tau)dQ_H + dQ_F) \quad (85)$$

Substituting these results into (79)-(80) gives

$$dQ_H + dQ_F = \frac{1}{h_1} (d\bar{Q}_H + d\bar{Q}_F) \quad (86)$$

$$dQ_H - dQ_F = \frac{1}{h_2} (d\bar{Q}_H + d\bar{Q}_F) \quad (87)$$

where $d\bar{Q}_H$ and $d\bar{Q}_F$ are the asset prices changes in the absence of balance sheet constraints and h_1 and h_2 are defined in (56)-(57). Taking the sum and difference of these equations then gives (53)-(54).

References

- [1] Adrian, Tobias and Hyun S. Shin (2008), “Financial Intermediary Leverage and Value-at-Risk,” Federal Reserve Bank of New York Staff Reports no. 338.
- [2] Allen, Franklin and Douglas Gale (2000), “Financial Contagion,” *Journal of Political Economy* 108, 1-33.
- [3] Bacchetta, Philippe and Eric van Wincoop (2010), “Explaining Sudden Spikes in Global Risk,” mimeo.
- [4] Beltran, Daniel O., Laurie Pounder and Charles Thomas (2008), “Foreign Exposure to Asset-Backed Securities of U.S. Origin,” International Finance Discussion Papers, number 939.
- [5] Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999), “The financial accelerator in a quantitative business cycle framework,” in John B. Taylor and Michael Woodford, eds, *Handbook of Macroeconomics*, Elsevier.
- [6] Brunnermeier, Markus K. and Lasse H. Pedersen (2009), “Market Liquidity and Funding Liquidity,” *Review of Financial Studies* 22, 2201-2238.
- [7] Brunnermeier, Markus K. and Yuliy Sannikov (2011), “A Macroeconomic Model with a Financial Sector,” mimeo.
- [8] Buch, Claudia M., John C. Driscoll and Charlotte Ostergaard (2010), “Cross-Border Diversification in Bank Asset Portfolios,” *International Finance* 13(1), 79-108.3
- [9] Calvo, Guillermo A. (1998), “Capital Market Contagion and Recession: An Explanation of the Russian Virus,” mimeo.
- [10] Dedola, Luca and Giovanni Lombardo (2010), “Financial Frictions, Financial Integration and the International Propagation of Shocks,” mimeo.
- [11] Devereux, Michael B. and Alan Sutherland (2011), “Evaluating International Integration under Leverage Constraints,” *European Economic Review*, forthcoming.

- [12] Devereux, Michael B. and James Yetman (2010), “Financial Deleveraging and the International Transmission of Shocks,” mimeo.
- [13] Dumas, Bernard, Campbell R. Harvey and Pierre Ruiz (2003), “Are Correlations in Stock Returns Justified by Subsequent Changes in National Outputs?,” *Journal of International Money and Finance* 22(6), 777-811.
- [14] Eichholtz, Piet M.A. (1996), “Does International Diversification Work Better for real Estate than for Stocks and Bonds?,” *Financial Analysts Journal* 52(1), 56-62.
- [15] Fidora, Michael, Marcel Fratzscher and Christian Thimann (2007), “Home Bias in Global Bond and Equity Markets: The Role of Real Exchange Rate Volatility,” *Journal of International Money and Finance* 26, 631-655.
- [16] Garcia-Herrero, Alicia and Francisco Vazquez (2007), “International Diversification Gains and Home Bias in Banking,” IMF Working Paper 07/281.
- [17] Gourio, Francois, Michael Siemer and Adrien Verdelhan (2010), “International Risk Cycles,” mimeo.
- [18] Greenlaw, David, Jan Hatzius, Anil K. Kashyap and Hyun Song Shin (2008), “Leveraged Losses: Lessons from the Mortgage Market Meltdown,” report for the U.S. Monetary Policy Forum 2008.
- [19] Gromb, Denis and Dimitri Vayanos (2008), "Leverage and Liquidity Dry-Ups: A Framework and Policy Implications," mimeo.
- [20] He, Zhiguo and Arvind Krishnamurthy (2008a), "Intermediary Asset Pricing," NBER WP No. 14517.
- [21] He, Zhiguo and Arvind Krishnamurthy (2008b), "A Model of Capital and Crises," NBER WP No. 14366.
- [22] Helbling, Thomas, Raju Huidrom, Ayhan Kose and Christopher Otrok (2010), “Do Credit Shocks Matter? A Global Perspective,” IMF Working Paper 2010/261.
- [23] Kalemli-Ozcan, Sebnem, Elias Papaioannou and Fabrizio Perri (2011), “Global Banks and Crisis Transmission,” mimeo.

- [24] Kamin, Steven B. and Laurie Pounder DeMarco (2010), “How Did a Domestic Slump Turn into a Global Financial Crisis?” “International Finance Discussion Papers, number 994.
- [25] Kollmann, Robert, Zeno Enders and Gernot Muller (2010), “Global Banking and International Business Cycles,” ECARES Discussion Paper 2010/28.
- [26] Krugman, Paul (2008), “The International Finance Multiplier,” mimeo.
- [27] Mehra, Rajnish and Edward C. Prescott (1985), “The Equity Premium: A Puzzle,” *Journal of Monetary Economics* 15, 145-161.
- [28] Okawa, Yohei and Eric van Wincoop (2010), “Gravity in International Finance,” mimeo.
- [29] Perri, Fabrizio and Vincenzo Quadrini (2011), “International Recessions,” working paper.
- [30] Rose, Andrew and Mark Spiegel (2010), “The Causes and Consequences of the 2008 Crisis: International Linkages and American Exposure,” *Pacific Economic Review* 15, 340-363.
- [31] Ueda, Kozo (2010), “Banking Globalization and International Business Cycles,” Bank of Japan Working Paper 2010-E-16.
- Xiong, Wei (2001), "Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets," *Journal of Financial Economics* 62, 247-292.

Figure 1 Percentage Drop in Asset Prices Due to Home Defaults

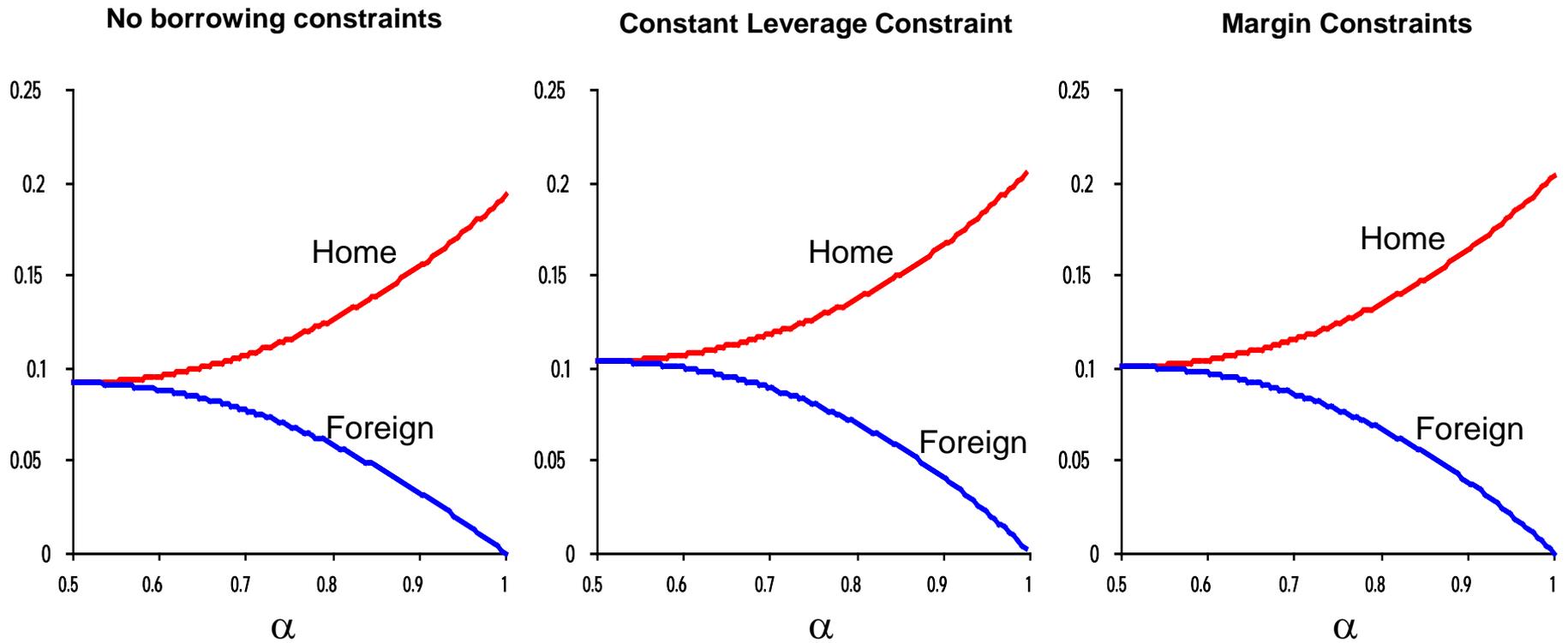
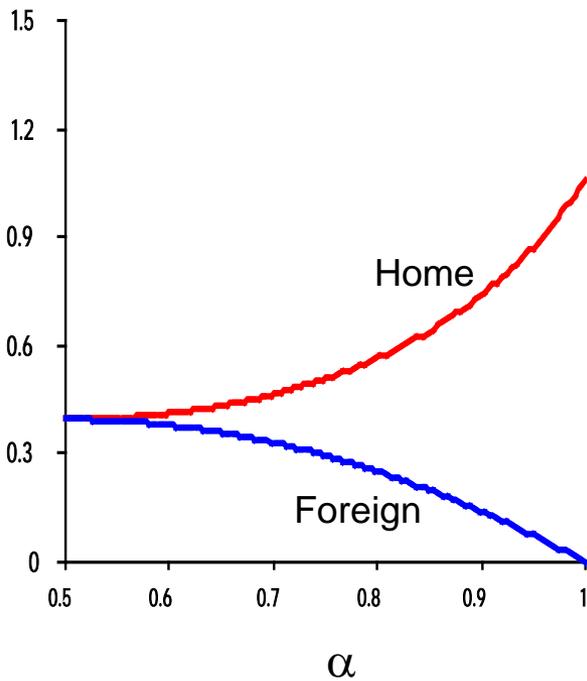
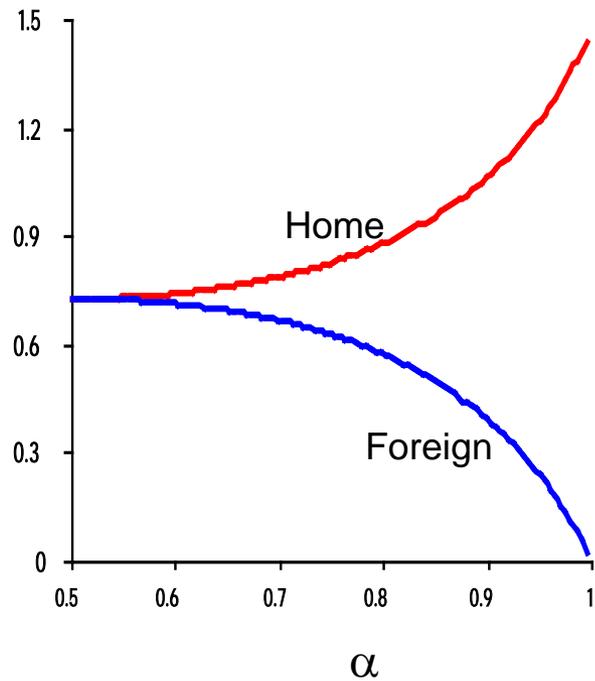


Figure 2 Percentage Drop in Asset Prices when Leveraged Institutions Own Half of all Assets

No borrowing constraints



Constant Leverage Constraint



Margin Constraints

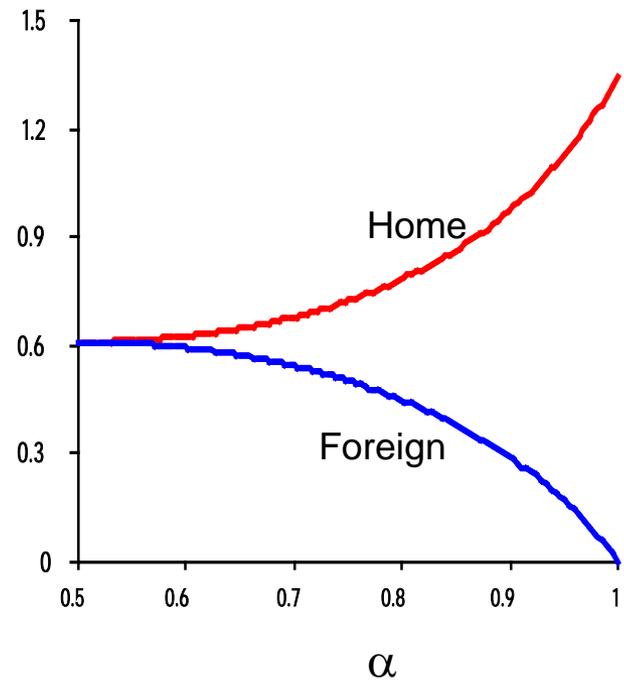
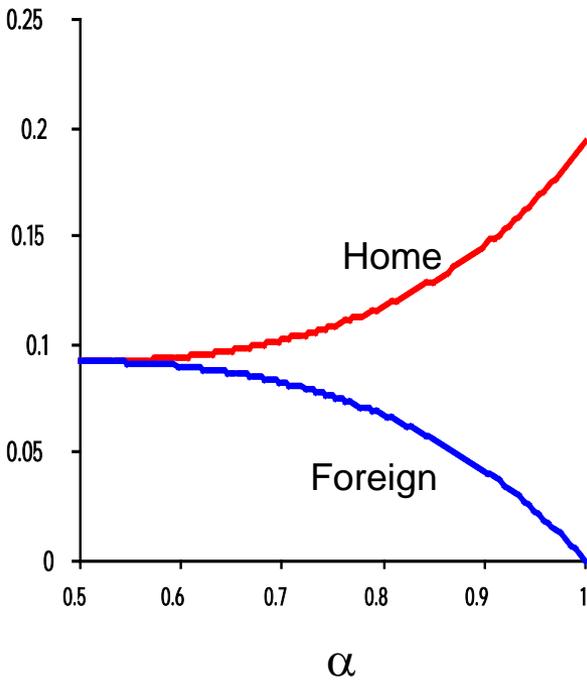
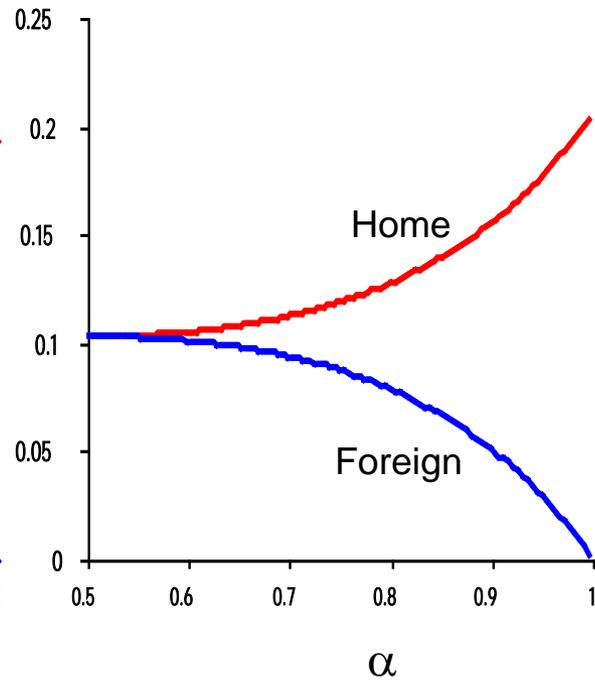


Figure 3 Percentage Drop in Asset Prices with Correlated Asset Returns*

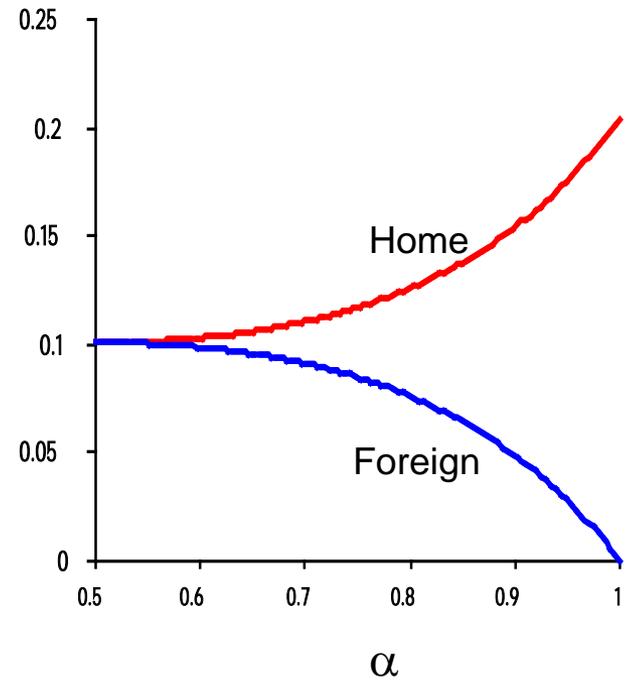
No borrowing constraints



Constant Leverage Constraint



Margin Constraints



* Assumes a correlation of asset returns of 0.3.