Hartwick Rule and utilitarianism when population changes

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1 Introduction

The main point of this paper is to introduce a non-constant population in a DHSS model. The canonical DHSS model follows the work of Dasgupta and Heal [6], Solow [15] and Stiglitz [16] who study a closed economy with two assets: man-made capital and a non-renewable natural resource. They characterize optimal production and resource extraction paths, which follow the Hotelling rule [10]: along the optimal path, the marginal productivity of the resource (its rent) grows at the real interest rate. In this context, Hartwick [9] gives prominence to an investment rule in man-made capital allowing an optimal path to satisfy intertemporal equity (according to Solow [15] view): per capita consumption is constant whenever the rents from resource extraction are invested in man-made capital. We will consider here a slightly different view of intertemporal equity as we consider amenity services from the natural resource stock to individuals. Sustainability is then to maintain at least constant individual utility, not solely individual consumption. Here we also take into account non constant population, which then becomes another asset of the economy (see Arrow *et al.* [1]), and the Hartwick Rule has to take into account the value of population changes as it is for the natural asset.

The purpose of the paper is then to analyse sustainability along optimal growth paths. For the notion of sustainability used I refer to Pezzey [13] clarification and I focus on *sustained* paths i.e. paths along which individual utility is non-decreasing at each period (in this classification, *sustainable* paths are paths where utility is below the maximum level of utility that can be sustained). In this context - a non-renewable natural resource extracting economy with non-constant population and man-made capital - I characterize sustained paths and clarify the meaning of the Hartwick Rule. Indeed investing "rents" (here of natural resource stock *and* of population changes) in man-made capital neither provides constant consumption paths nor constant individual utility paths but constant *well-being* paths. Traditionaly (and naturaly) in the literature when population is constant the objective of the problem refer to one individual, either per capita consumption or individual utility (it is criteria belonging to individual utilitarianism). And most of the time the criterion is about the whole generation when population is varying (the total utilitarianism). I stress the point that these two concepts of utilitarianism lead up to different result when the Hartwick Rule applies. Indeed when population changes the Hartwick Rule implies that individual utility is constant when the criterion is individual utilitarianism whereas individual utility declines at the rate of population growth when the criterion is total utilitarianism. To show that I use a more general utility function that includes both form of utilitarianism, total and individual. The Hartwick Rule implies the constance of this well-being functional. At last, I formalize a social welfare function where the individual utility is explicitely separated: it is the individual utility function times the weight granted to population size in the welfare criterion (I refer to this criterion as *separable* individual utility well-being function). It allows me to give an investment rule *à la* Hartwick that caracterizes sustained paths for any form of utilitarianism (total, individual or even average utilitarianism).

There exist today a large litterature about sustainability. To characterize sustained paths and measure sustainability I refer to the concept of Genuine Saving. This definition of savings takes into account the changes in value of all the assets in the economy, and its nullity corresponds to the Hartwick Rule. We can refer to Vellinga and Withagen [18] and Hamilton and Withagen [8] for an analysis of the Hartwick Rule in a very general context, and Asheim and Withagen [3] for the converse of the rule. It is the point of this paper to focus the case of non-constant population and amenity services, and thus to go into welfare criteria details. In this it also adresses a complement to the work of Dasgupta [5]. As stressed there the literature is often built on three assumptions : a constant population without exogenous technical change, the economy is at optimum, and the commodity transformation possibility set is convex. Whereas the two last assumptions are relaxed, the model experiences no population changes. In this paper I stick with optimality but do not assume constant population.

We study here an augmented DHSS model, where labour follows its own exogenous growth path; amenity services from the natural resource are provided to individuals in order to introduce in a simple way the externality on welfare of environmental degradation, and we consider a general form of utilitarianism. I will study the meaning of the Genuine Saving and review the significance of the Hartwick Rule, as it characterizes constant well-being paths. I will formalize its expression in order to characterize sustained paths as it is not obvious that the standard rule works. In the second section I present the model and the optimality conditions. The third section analyses Genuine Saving, thus enlights the meaning of the Hartwick Rule. I study Genuine Saving beyond the standard Hartwick Rule (i.e. when Genuine Saving is not constant and nill) in the fourth section. The fifth section focuses on separable individual utility well-being functions to characterize a more general investment rule for sustained paths. And the sixth section concludes.

2 Optimality conditions and the value of population

We consider a general form of the utility function V. It represents the instantaneous well-being of society and depends at each period t on total consumption C_t, the remaining resource stock S_t and the size of the population N_t. We do not specify yet how population affects instantaneous well-being. The function has classical properties: we assume it is continuous and differentiable where V_C and V_S, denoting the first derivative with respect to C_t and S_t respectively, are positive. The sign of V_N is not clear as it depends on how N_t weighs individual utilities.

For example the standard case of total utilitarianism (assuming homogeneity of the population) leads to set $V(C_t, S_t, N_t) = N_t U\left(\frac{C_t}{N_t}, \frac{S_t}{N_t}\right)$ where U is the individual well-being function. If U is homogeneous of degree 1 then V_N is nill. In the case of individual utilitarianism i.e. when $V(C_t, S_t, N_t) = U\left(\frac{C_t}{N_t}, \frac{S_t}{N_t}\right)$, we have V_N negative.

The remaining resource stock enters in the evaluation of welfare as amenity services (see Krautkraemer [11]). As regard population size, one could ask if it is the per capita or the total size of the natural resource stock that matter for individuals. Further on I consider the per capita case, the focus is thus much more about the share between population of the aenity services than about the global scale effect of resource depletion (as it is when studying global warming).

The social planner is supposed to be utilitarian so intertemporal social welfare in this general formulation is written as:

$$W = \int_0^{\infty} e^{-\rho t} V(C_t, S_t, N_t) dt \tag{1}$$

with $\rho > 0$ the social discount rate, exogenous and constant.

We are interested here in exogenous population changes. Agents offer inelastically one unit of labor so that we will consider both indistinctly. We think more about logistic curves for population evolution, meaning indirectly that the position on the curve informs about the level of development of the economy. But it does not depend directly on the level of development (as considering the influence of the capital-labor ratio, representing the level of industrialization) so that the social planner does not trade off between population and economic growth *a priori*. We keep a general form of population growth :

$$\dot{N}_t = \varphi(N_t) \tag{2}$$

We only assume that φ is continuous and differentiable.

To solve the model, population will be treated as a state variable (see Arrow *et al.* [1]). This does not affect optimality conditions but allows to set an accounting price on population which is very helpful for sustainability measurement tools.

We have the two others state variables in common with the standard DHSS model: man-made capital and natural capital evolving according to:

$$\dot{K}_t = F(K_t, R_t, N_t) - C_t \tag{3}$$

$$\dot{S}_t = -R_t \tag{4}$$

where we denote by F the production technology whose inputs are at each date t man-made capital K_t, the resource extracted R_t and labor. K_0, S_0 and N_0 are given. F is continuous and verifies the law of decreasing marginal productivity : F_K, F_R and F_N are positive whereas F_{KK}, F_{RR} and F_{NN} are negative. I do not presuppose the sign of the cross effects.

The optimal control problem is then to maximise (1) subject to the dynamic constraints (2), (3) and (4). Control variables are C_t and R_t, state variables are K_t, S_t and N_t, and we denote λ_t, μ_t and ν_t the associated co-state variables. We write by $\mathcal{H}_t$ the Hamiltonian of the problem in present value at t as:

$$\mathcal{H}_t = V(C_t, S_t, N_t) + \lambda_t \left(F(K_t, R_t, N_t) - C_t\right) - \mu_t R_t + \nu_t \varphi(N_t)$$

We get the following first order conditions[1]:

$$V_C = \lambda_t \tag{5}$$

$$F_R = \frac{\mu_t}{\lambda_t} \tag{6}$$

$$F_K = \rho - \frac{\dot{\lambda}_t}{\lambda_t} \tag{7}$$

$$\rho\mu_t - V_S = \dot{\mu}_t \tag{8}$$

$$\frac{1}{\nu_t} V_N + \frac{\lambda_t}{\nu_t} F_N + \varphi'(N_t) = \rho - \frac{\dot{\nu}_t}{\nu_t} \tag{9}$$

We denote by $\frac{\mu_t}{\lambda_t} = q_t$, $\frac{\nu_t}{\lambda_t} = \pi_t$ the relative prices (or more correctly shadow values) of natural resource extracted and population growth; and we denote $F_N = w_t$ and $F_K = r_t$ for convenient notations. Using condition (5) in conditions (7), (8) and (9) leads to:

$$\rho - r_t = \frac{\dot{V}_C}{V_C} \tag{10}$$

$$\dot{q}_t = q_t r_t - \frac{V_S}{V_C} \tag{11}$$

$$\dot{\pi}_t = \pi_t \left(r_t - \varphi'\left(N_t\right)\right) - w_t - \frac{V_N}{V_C} \tag{12}$$

[1]the last condition (9) is not a first order **optimality** condition but the adjoint associated to N_t. It is not used to solve the optimal control problem but to value population changes in the model. We will hear as optimality conditions the relations (6), (10) and (11) in the following.

The previous relations generalize the results we usually get. Condition (10) is the well-known Keynes-Ramsey condition where we have the rate of change of marginal social well-being of consumption, instead of growth rate of consumption or marginal individual utility solely. Condition (11) is the Hotelling Rule: when remaining natural resource stock do affects well-being, the standard Hotelling Rule is affected (see Krautkraemer [11]). The change in (relative) resource price q_t increases with the marginal rate of substitution between remaining resource stock and consumption (as the stock is decreasing, V_S is negative).

The relation of greatest interest is relation (12), the evolution of the (relative) value of population. It increases with its current value discounted at the rate $r_t - \varphi'$ and decreases both with the marginal productivity of labor and the marginal rate of substitution between population and consumption. The generalization of the social well-being function allows us to understand more correctly that the value of population varies with the social benefits of one more individual. If the current discounted value of population is larger than this gain, then the value increases. It becomes stable when the two are equal. This result is very intuitive, and the benefits of an individual are evaluated at his productivity and his "net price" in utility terms: it is the sum of his value in term of production and utility.

Since Arrow *et al.* [1] the last term - the marginal rate of substitution between population and consumption - is often interpreted as the "value of life". This expression comes from the fact that setting $V(C_t, S_t, N_t) = N_t U(\frac{C_t}{N_t})$ - the total utilitarianism case without amenities, leads to $\frac{V_N}{V_C} = \frac{U}{U_c}$. This term can be seen as a measure of the value of life: considering the indiffence curves of $U(c_t)$ with a survival probability p, i.e. curves where $pU(c_t)$ is constant, the willingness to pay for a marginal increase in the probability of survival $p\frac{dc}{dp}$ equals $-\frac{U}{U_c}$ (as $d\,(pU(c_t))$ is nill). Starting from a situation where the survival probability is 1 we get that the value of life is $\frac{U}{U_c}$.

3 The sustainability of the well-being

We now turn to the presentation of our formal definitions of the notions about sustainability presented in the introduction. To our concern, we can define sustained paths as follows:

Definition 1 *A growth path is sustained whenever the individual utility is non decreasing at each period i.e.* $\dot{U} \geq 0$

And we will focus on the following measurement tool to help us to characterize such paths:

Definition 2 *The Genuine Saving Index at date t, denoted* G_t*, is given by:*

$$G_t = \dot{K}_t + q_t \dot{S}_t + \pi_t \dot{N}_t \qquad (13)$$

Genuine Savings gives us at each date the change in value of every assets of the economy.

We are exploring now its use to characterize optimal paths that are sustained when population changes exogenously. Genuine Saving becomes relevant thanks to the Hartwick Rule [9], saying that the nullity of Genuine Savings means sustainability. It takes the form of an investment rule consisting, in the standard DHSS model, in accumulating man-made capital at the level of the value of the resource depleted. Our extension allows the standard investment rule to be relaxed in the sense that the presence of an increase of population implies that less than the value of resource extracted has to be invested.

As proved in a general setting in Hamilton and Withangen [8] the Hartwick Rule is a necessary condition for sustainability. I give the details of the proff in my setting in the appendix ; and I can state the following: We get the following result:

Property 1 *Along an optimal path,*

$$\dot{V} = 0 \textit{ if } \dot{K}_t = q_t R_t - \pi_t \dot{N}_t$$

It expresses the Hartwick Rule and its converse. Using a general expression of social well-being allows us to understand in a different manner the meaning of the Hartwick Rule. As mentionned before it is often said to identify sustained individual utility paths or constant consumption path whereas we see here it identifies constant *instantaneous well-being* paths. I stress the point that it generalizes standard literature result for the social welfare function as it consists in specifying the social well-being function V to get the standard approaches; for example in the constant population case with no amenity services we get the constant consumption path.

In the following section I will discuss the sustainability of non-zero Genuine Saving paths and I go into more details about the implications on standard utilitarianism criteria in section 5.

The converse of the rule can be less obvious (see Asheim and Withagen [3], it may need more restrictive assumptions to apply). I give the proof for this model in the appendix.

Property 2 *Along an optimal path,*

$$\dot{K}_t = q_t R_t - \pi_t \dot{N}_t \textit{ if } \dot{V} = 0$$

Another result is in the line of Buchholz *et al.* [4]: we obtain a kind of *myopic efficiency.* The Hartwick Rule is myopic in the sense that along a sustained path, if the Hartwick Rule is verified it ensures also the path to be efficient.

Property 3 *For the pricing conditions (6) and (12) to hold, any two of the three followings that is verified imply the third to be :*

(i) *the path is efficient in the sense that the Hotelling Rule holds*

(ii) $\dot{V} = 0$

(iii) *Hartwick Rule holds*

The appendix proves the last implication (Hartwick Rule and equity imply efficiency).

It shows both that a constant well-being path following Hartwick rule is necessarily efficient, or if it is efficient it necessarily follows the Hartwick Rule. This point stresses one more time (see Toman *et al* [17]) that the Hartwick Rule is desciptive and not prescriptive : it characterizes optimal paths where the objective function is constant over time.

4 Dynamics of Genuine Savings and well-being

In this section I focus on cases where Genuine Saving is not necessarily constant nor nill. The question I always adress is to know if, along an optimal path i.e. when (6), (10) and (11) are verified, the instantaneous well-being is non-decreasing; I am interested in the investment rule (not its converse). Well-being and Genuine Savings are linked, my point is to clarify in which way. Knowing Genuine Saving, its positive or negative value and its dynamics, would allow us to know how well-being is affected i.e. to identify an unsustainable path.

To study the dynamics of well-being with respect to Genuine Saving, there is a relation between $\dot{G}_t$ and $\dot{V}$ of great usefulness. By equalling (3) and (13), differentiating with respect to time and using (6), (10) and (11) we get :

$$\dot{G}_t - r_t G_t = -\frac{\dot{V}}{V_C}$$

It is the same relation obtained in Hamilton and Withagen [8], here expressed in capital-value.

We get straightly the following result:

Property 4 *Along an optimal path,*

(i) *if* $G_t > 0$ *and* $\frac{\dot{G}_t}{G_t} \leq r_t$*, then* $\dot{V} \geq 0$

(ii) *if* $G_t < 0$ *and* $\frac{\dot{G}_t}{G_t} \leq r_t$*, then* $\dot{V} \geq 0$

In most cases a positive Genuine Saving allows non-decreasing instantaneous well-being. Conditional on G_t to grow at a rate less than or equal to r_t it is always the case: the investment in man-made capital (or population growth) is enough to allow future generations to prevent the welfare loss due to decreasing amenity services and resource depletion.

But it does not ensure non-decreasing well-being forever. For example it may be that Genuine Saving is positive and decreasing then relates an increase in well-being. It is a decrease in investment in the large sense (investment accounting for every assets) that allows to devote more resources to consumption. However it may not hold forever: Genuine Saving becoming negative implies that the economy is disinvesting, and loses welfare possibilities.

In the cases where Genuine Saving is negative, it seems natural that well-being declines. When Genuine Saving decreases it is obvious that future welfare posssibilities are neglected: either the product is not devoted enough to man-made capital but to current consumption, or natural resource depletion is too high (or even both). But also if Genuine Saving increases (and still is negative), consumption must be sacrificed to capital investment thus well-being declines; but then the path is getting closer to one where well-being will stop its decline.

These results are quite intuitive, Genuine Saving in this model represents the path the well-being will follow. It informs the ability of the economy to compensate welfare losses from natural resource exploitation with final good consumption i.e. to allow some future generations to prevent resource exhaustion. But of course the information Genuine Saving relates at one point in time may not hold for any future period.

5 On the sustainability of utilitarian criteria

This section is devoted to apply the result of property 1 about the Hartwick Rule to more specified utilitarian criteria. And we will focus on sustained path in the sense of definition 1 (constant **individual utility** paths, not constant well-being paths).

The criterion traditionaly used in growth model is total utilitarianism where the social planner seeks to maximize the sum of the total utility of each generation. The total utility consists in summing every individual utility functions, which gives us the following well-being function as seen in section 2:

$$V(C_t, S_t, N_t) = N_t U(c_t, s_t)$$

We denote now and for the following per capita variables with small letters. I recall that U is the individual utility function.

We have the following, from property 1:

Property 5 *Along an optimal path, in the total utilitarianism case :*

$$\text{If } \dot{K}_t = q_t R_t - \pi_t \dot{N}_t \text{ then } \frac{\dot{U}}{U} = -\frac{\dot{N}_t}{N_t}$$

When population varies over time, the standard Hartwick result does not provide sustained paths. The intuition behind this result is clear: well-being is sustained i.e. total utility is. Thus an increase in population means a decrease in individual utility. We get here a kind of *repugnant conclusion* (see Parfit [12]) even if the social planner does not "choose" the level of population: he takes advantage of the growing population (which would increase social well-being all other things being equal) to decrease individual well-being in order to maintain the total level of well-being of the generation.

We focus now on another interpretation of utilitarianism where only the utility of one individual of each generation matters. We can call this criterion individual utilitarianism, and the social well-being function takes the form:

$$V(C_t, S_t, N_t) = U(c_t, s_t)$$

Thus it is straightforward with property 1:

Property 6 *Along an optimal path, in the individual utilitarianism case:*

$$\textit{If } \dot{K}_t = q_t R_t - \pi_t \dot{N}_t \textit{ then } \dot{U} = 0$$

This result give back usefulness to individual utilitarianism for sustainability analysis : the path is sustained in the sense of definition 1. The fundamental difference between individual and total utilitarianism is captured in the value of the population through the implicit price of population in term of utility $\frac{V_N}{V_C}$.

As V_N is larger in the total utilitarianism case (V_C, and also V_S do not change with the criterion considered) the value of population π_t is smaller. It induces a lower capital accumulation in the case of growing population when we follow the Hartwick Rule. In this way we understand that capital does not accumulate enough to provide future generation the same amount of per capita consumption good as regard population increase.

However, it is possible to get an investment rule *à la* Hartwick which provides in any case sustained individual utility paths:

Property 7 *Along an optimal path, we consider a separable individual utility social well-being function with the homogeneous individual utility function $U(c_t, s_t)$ and the general form of population weight $P(N_t)$ on individual utility so that $V(C_t, S_t, N_t) = P(N_t)U(c_t, s_t)$. We thus have:*

$$\dot{U} = 0 \textit{ if and only if } G_t = e^{D_t} \left[G_0 - \int_0^t \dot{P}(N_u) \frac{U}{\lambda_t} e^{-D_u} du \right]$$

where $D_t = \int_0^t r_u du$

See the appendix for the proof.

This property provides a quite general investment rule to follow in order to characterize sustained paths. For example it goes back to the standard Hartwick Rule when the criterion is individual utilitarianism. The case of total utilitarianism is much more complex, the level of Genuine Saving to reach at each period is given by :

$$G_t = e^{D_t} \left[G_0 - \int_0^t \dot{N}_u \frac{U}{\lambda_t} e^{D_u} du \right]$$

Usually, investment in man-made capital must be equal to the value of the natural and population stock depreciation. Population increase implies the reverse, it diminishes the amount of investment required. In addition, in the case of total utilitarianism another term appears. It tells us that the present value of Genuine Saving must equal initial Genuine Saving minus the discounted value of the utility of every new born (in capital-value) for individual utility to be sustained. The greater the gap the more Genuine Saving must be high.

6 Conclusion

In this paper I apply the general model of Hamilton and Withagen [8] to the case of the D.H.S.S. model (two capital goods, man-made and natural ones) extended with amenity services and population growth. The key point I stress is about the implication of the extended Hartwick Rule (i.e. investing in man-made capital the rents from the other two kind of capital, the natural asset and population): it allows sustainability of the well-being. Thus it does not ensure sustained path (in the sense of constance of individual utility) as for the standard utilitarian criterion, total individualism. In this case individual utility is sustained if and only if Genuine Saving equals an amount that depends particularly on the utility in value of new borns.

Appendix

. About properties 1, 2 and 3: Equalling (3) and (13), differentiating with respect to time, using (6) and (12) gives:

$$\dot{G}_t - r_t G_t = -\left(\frac{V_N}{V_C}\dot{N}_t + \dot{C}_t\right) + \dot{S}_t(\dot{q}_t - r_t q_t) \tag{14}$$

- If the Hartwick Rule is verified on a time-interval around t, and the Hotelling Rule is too, then it follows that $\dot{V} = 0$. One of the implication of property 1 and the implication (i)+(iii)⇒(ii) of property 3 are proved
- If Hartwick Rule and $\dot{V} = 0$ hold, then it is straight that the Hotelling Rule holds too. It states the implication (ii)+(iii)⇒(i)
- The property 2 and the last relation of property 3 are a bit longer to prove. Assuming that the Hottelling Rule and $\dot{V} = 0$ are verified, we have:

$$\dot{G}_t = r_t G_t$$

that is

$$G_t = e^{D_t} G_0$$

with $D_t = \int_0^t r_u du$

We obtain the Hartwick Rule as a special case of the Dixit-Hammond-Hoel Rule (see Dixit *et al.* [7]) where Genuine Savings are nill. Indeed G_0 is necessary nill:

by definition of Genuine Saving and using the previous relation, we get:

$$e^{D_t} G_0 = \dot{K}_t + q_t \dot{S}_t + \pi_t \dot{N}_t$$

Log-differentiating, using (3) then again the definition of Genuine Saving, and (11) and (12) lead to:

$$\dot{G}_0 = -e^{-D_t} \frac{\dot{V}}{V_C}$$

That is, as $\dot{V} = 0$

$$\dot{G}_0 = 0$$

At the same time we know that $\dot{G}_t = r_t G_t$ i.e. $\dot{G}_0 = r_0 G_0$ thus $G_0 = 0$.
■

. About property 7 : Going back to (14) and using (11) we have:

$$\dot{G}_t - r_t G_t = -\frac{\dot{V}}{V_C}$$

i.e.

$$\dot{G}_t - r_t G_t = -P(N_t)\frac{\dot{U}}{\lambda_t} - \dot{P}(N_t)\frac{U}{\lambda_t} \tag{15}$$

as $V_C = \lambda_t$. Thus,

- If $\dot{U} = 0$ then $\dot{G}_t - r_t G_t = -\dot{P}(N_t)\frac{U}{\lambda_t}$ i.e.

$$G_t = e^{D_t}\left(G_0 - \int_0^t \dot{P}(N_t)\tfrac{U}{\lambda_t}e^{D_u}du\right)$$

- Conversely, if $G_t = e^{R_t}\left(G_0 - \int_0^t \frac{P'(N_t)\dot{N}_t N_t}{P(N_t)}\frac{U}{U_c}e^{R_u}du\right)$, equalizing with the definition of Genuine Saving, log-differentiating, using (3), using again the definition of Genuine Saving, and (11) and (12) lead to:

$$\dot{P}(N_t)U = \dot{V}$$

  i.e. $\dot{U} = 0$ as $\dot{V} = \dot{P}(N_t)U + P(N_t)\dot{U}$ by definition.

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References

[1] Arrow, K.J., P. Dasgupta, K.-G. Mäler: The Genuine savings and the value of population, *Economic theory*, 20, 217-225 (2003)

[2] Asheim, G.: Green national accounting with a changing population, *Economic theory*, 23, 601-619 (2004)

[3] Asheim, G., Withagen, C.: Characterizing sustainability: the converse of Hartwick's rule, *Journal of economic dynamics and control*, 23, 159-165 (1998)

[4] Buchholz, W., S. Dasgupta, T. Mitra: Intertemporal equity and Hartwick's rule in an exhaustible resource model, *Scandinavian Journal of Economics*, 107(3), 547-561 (2002)

[5] Dasgupta, P.: The welfare economic theory of Green National Accounts, *Environmental and Resource Economics*, 42(3), 3-38 (2009)

[6] Dasgupta, P,. G. Heal: The optimal depletion of exhaustible resources, *Review of Economic Studies* (symposium); 3-28 (1974)

[7] Dixit, A., Hammond, P., Hoel, M.: On Hartwicks rule for regular maximin paths of capital accumulation and resource depletion, *Review of Economic Studies*, 47, 551556 (1980)

[8] Hamilton, K., Withagen, C.: Savings growth and the path of utility, *Canadian Journal of Economics*, 40(2), 703-713 (2007)

[9] Hartwick, J.: Intergenerational equity and the investing of rents from exhaustible resources, *American Economic Review*, 66(5), 972-974 (1977)

[10] Hotelling, H.: The economics of exhaustible resources, *Journal of Politcal Economy*, 39, 137-175 (1931)

[11] Krautkraemer, J.A.: Optimal growth, resource amenities and the preservation of natural environments, *Review of Economic Studies*, 52, 153.170 (1985)

[12]

[13]

[14] Pezzey, J.: One-sided sustainability tests with amenities, and changes in technology, trade and population, *Journal of Environmental Economics and Management*, 48(1), 613-631 (2004)

[15] Solow, R.: Intergenerational equity and exhaustible resources, *Review of Economic Studies* (symposium), 29-45 (1974)

[16] Stiglitz, J.E.: Growth with exhaustible natural resources : efficient and optimal growths paths, *Review of economic studies*, 41, 123-137 (1974)

[17] Toman, M.A., J. Pezzey, J. Krautkraemer: Neoclassical economic growth theory and sustainability, in D. W. Bromley, ed., Handbook of Environmental Economics, Oxford UK and Cambridge USA: Blackwell, 139-165 (1995)

[18] Vellinga, N., Withagen, C.: On the concept of green national income, *Oxford Economic Papers*, 48, 499-514 (1996)