

Source-dependence of utility and loss aversion: A critical test of ambiguity models

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Abstract:

This paper tests whether utility is the same for risk and for uncertainty. This test is critical to distinguish models that capture ambiguity aversion through a difference in utility between risk and uncertainty (like the smooth ambiguity model) and models that capture ambiguity aversion through a difference in event weighting between risk and uncertainty (like multiple priors and prospect theory). We designed a new method to measure utility and loss aversion under uncertainty without the need to introduce simplifying parametric assumptions. Our method extends Wakker and Deneffe's (1996) trade-off method by allowing for standard sequences that include gains, losses, and the reference point. It provides an efficient way to measure loss aversion and a useful tool for practical applications of ambiguity models. We could not reject the hypothesis that utility and loss aversion were the same for risk and uncertainty suggesting that utility reflects attitudes towards outcomes and not attitudes towards ambiguity. Sign-dependence was important both for risk and for uncertainty. Utility was S-shaped, concave for gains and convex for losses and there was substantial loss aversion. Our findings support models that explain ambiguity aversion through a difference in event weighting and suggest that descriptive models of ambiguity aversion should allow for reference-dependence.

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1. Introduction

An extensive amount of empirical work, originating from Ellsberg's (1961) famous thought experiment, shows that people are not neutral towards ambiguity, as assumed by subjective expected utility, but usually dislike ambiguity. New models have been proposed to explain this ambiguity aversion. Broadly speaking, these ambiguity models can be subdivided into two classes. The first class models ambiguity aversion through a difference in utility between risk and uncertainty. The best-known of these models is the smooth ambiguity model of Klibanoff et al. (2005), which is increasingly popular in economic applications (e.g. Treich 2010, Gollier 2011). Other models that belong to this class were proposed by Nau (2006), Chew et al. (2008), Seo (2009), and Neilson (2010). The second class assumes that utility does not depend on the source of uncertainty and is the same for risk and uncertainty. Instead, ambiguity aversion is modeled through a difference in event weighting. This class includes the multiple priors models (Gilboa and Schmeidler 1989, Jaffray 1989, Ghirardato et al. 2004) and modifications thereof (Gajdos et al. 2008, Maccheroni et al. 2006), vector expected utility (Siniscalchi 2009), Choquet expected utility (Gilboa 1987, Schmeidler 1989), and prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992).

This paper tests whether utility is the same for risk and uncertainty. Empirical evidence indicates that ambiguity attitudes differ between gains and losses (e.g. Cohen et al. 1987, Hogarth and Kunreuther 1989, Abdellaoui et al. 2005, Du and Budescu 2005) and that loss aversion is crucial in explaining attitudes towards both risk (Rabin 2000) and ambiguity (Roca et al. 2006). We, therefore, measured utility both for gains and for losses and we also measured loss aversion. We assume a general utility model, previously suggested by Miyamoto (1988), Luce (1991), and Ghirardato and Marinacci (2001) that includes all the models of the second class as special cases, and generalize it to include sign-dependence. An empirical test supported the central condition underlying this model.

Measuring loss aversion is complex, in particular if event weighting is allowed to differ for gains and losses, as we do. Previous measurements of loss aversion sidestepped this problem by introducing simplifying assumptions. We introduce a new method to measure loss aversion that imposes no simplifying assumptions. It can easily be applied, which may encourage the use of ambiguity models in practical decision problems where simple measurement methods are required. Our method extends the trade-off method of Wakker and Deneffe (1996) by allowing standard sequences (sequences of outcomes for which the utility difference between successive elements is constant) to pass through the reference point. This makes it possible to measure utility on its entire domain and to quantify loss aversion. It provides a useful tool for practical applications of ambiguity models. It also simplifies the axiomatization of ambiguity models as there is a close connection between

measurements of utility using the trade-off method and preference conditions (Köbberling and Wakker 2003). Our method provides a straightforward way to measure loss aversion without the need to fully measure utility. In particular, we provide an efficient way to operationalize Köbberling and Wakker's (2005) measure of loss aversion.

Our data support the assumption that utility and loss aversion are the same for risk and uncertainty. This suggests that utility primarily reflects attitudes towards outcomes. We also found evidence of ambiguity aversion for mixed acts. Taken together, these findings support models that explain ambiguity aversion through a difference in event weighting. This is the first message of our paper.

The second message is that descriptive ambiguity models should allow for reference-dependence. We obtained clear evidence that utility differed for gains and losses and there was sizeable loss aversion. Most ambiguity models do not allow for reference-dependence and assume that ambiguity attitudes are the same for gains and losses. This assumption may be adequate for normative purposes, but not for descriptive purposes.

2. Background

2.1. Theory

Consider a decision maker who has to make a choice in the face of uncertainty. Uncertainty is modeled through a *state space* S . Exactly one of the states will obtain, but the decision maker does not know which one. Subsets E of S are called *events* and E^c denotes the complement of E .

Acts map states to outcomes. Outcomes are money amounts and more money is preferred to less. In our measurements we will only need two-outcome acts $x_E y$, signifying that the decision maker obtains $\text{€ } x$ if event E occurs and $\text{€ } y$ otherwise. We use conventional notation to express the preference of the decision maker, letting \succ , \succcurlyeq , and \sim represent strict preference, weak preference, and indifference.

Preferences are defined relative to a reference point x_0 . *Gains* are outcomes strictly preferred to x_0 and *losses* are outcomes strictly less preferred than x_0 . An act is *mixed* if it involves both a gain and a loss. For mixed acts the notation $x_E y$ signifies that x is a gain and y is a loss. A *gain act* involves no losses (i.e. both x and y are nonnegative) and a *loss act* involves no gains. For gain and loss acts the notation $x_E y$ signifies that the absolute value of x exceeds the absolute value of y , i.e. if x and y are gains then $x > y$ and if x and y are losses then $x < y$. If probabilities are known, we will denote by $x_p y$ the act that pays $\text{€ } x$ with probability p and $\text{€ } y$ with probability $1 - p$. We will refer to $x_E y$ as

an *uncertain act* (meaning that probabilities are unknown) and to $x_p y$ as a *risky act* (meaning that probabilities are known).

Under *binary rank-dependent utility* the decision maker's preferences over mixed acts $x_E y$ are evaluated by:

$$W^+(E)U(x) + W^-(E^c)U(y), \quad (1a)$$

and preferences over gain or loss acts by:

$$W^i(E)U(x) + (1 - W^i(E))U(y), \quad (1b)$$

where $i = +$ for gains and $i = -$ for losses. U is a strictly increasing, real-valued *utility function* that satisfies $U(x_0) = 0$. U is an overall utility function that includes loss aversion. In empirical application U is often decomposed in a basic utility function, capturing the decision maker's attitudes towards final outcomes, and a loss aversion coefficient λ , capturing attitudes towards gains and losses (Sugden 2003, Köbberling and Wakker 2005, Köszegi and Rabin 2006). Our method does not require this decomposition. The utility function is a ratio scale and we can choose the utility of one outcome other than the reference point. The *event weighting functions* $W^i, i = +, -$, assign a number $W^i(E)$ to each event E such that

- (i) $W^i(\emptyset) = 0$
- (ii) $W^i(S) = 1$
- (iii) W^i is *monotonic*: $E \supset F$ implies $W^i(E) \geq W^i(F)$.

The event weighting functions W^i depend on the sign of the outcomes and may be different for gains and losses. They need not be additive. For gains, binary RDU contains most transitive ambiguity models as special cases, as was pointed out by Miyamoto (1988), Luce (1991), and Ghirardato and Marinacci (2001). The ambiguity models only differ when the number of outcomes is at least three. Equations (1a) and (1b) represent the extension of these models to include sign-dependence.

Binary RDU evaluates mixed risky acts $x_p y$ as

$$w^+(p)U(x) + w^-(1 - p)U(y) \quad (2a)$$

and gain and loss risky acts $x_p y$ as

$$w^i(p)U(x) + (1 - w^i(p))U(y). \quad (2b)$$

w^i is a strictly increasing *probability weighting function* that satisfies $w^i(0) = 0$ and $w^i(1) = 1$ and again may differ between gains and losses. Hence, in the evaluation of risky acts the event weighting functions W^i are replaced by probability weighting functions w^i . Binary RDU assumes that utility is the same for risk and uncertainty. Ambiguity aversion is modeled through a difference between W^i and w^i .

2.2. Previous evidence

Tversky and Kahneman (1992) assumed that utility differs between gains and losses and is S-shaped, concave for gains and convex for losses, and steeper for losses than for gains, reflecting loss aversion. Nearly all the empirical evidence on utility comes from decision under risk. There is much evidence that utility for gains is concave (Wakker 2010). For losses the evidence is more equivocal. While most studies found convex utility, some have also found linear or concave utility (e.g. Bruhin et al. 2010). The utility for losses was usually closer to linear than the utility for gains.

Empirical evidence on utility under uncertainty is scarce. Abdellaoui et al. (2005) measured utility under uncertainty and confirmed that it was concave for gains and convex but close to linear for losses. Their power coefficient estimates were close to those previously obtained under risk, but they did not directly measure utility under risk. Abdellaoui et al. (2011) measured utility under risk and under uncertainty for small gains assuming power utility. They found that utility for gains was linear for both risk and uncertainty. Neither of these studies measured loss aversion.

Several studies measured loss aversion in decision under risk. Nearly all these studies made simplifying assumptions about utility and probability weighting, typically assuming linear utility and either ignoring probability weighting (Pennings and Smidts 2003, Booij and van de Kuilen 2009, Baltussen et al. 2012)¹ or assuming equal weighting for gains and losses (Gaechter et al. 2007). The only exception is Abdellaoui et al. (2007) who imposed no simplifying assumptions on either probability weighting or utility.

Previous studies on loss aversion are hard to compare, because they adopted different definitions of loss aversion. Abdellaoui et al. (2007) compared several of the definitions of loss aversion that have been proposed in the literature and concluded that the definitions by Kahneman and Tversky (1979)

¹Booij and van de Kuilen (2009) tested for the robustness of their findings by using probability weights estimated in other studies.

and Köbberling and Wakker (2005) were empirically most useful. For these definitions, the loss aversion coefficient was usually close to two, meaning that losses weight approximately twice as much as absolutely commensurate gains (Booij et al. 2010).

Finally, even though binary RDU is consistent with much of the empirical data that has been collected on decision under risk and uncertainty and includes many ambiguity models as special cases, there is some evidence challenging it. For example, Starmer and Sugden (1993) and Birnbaum (2008) reported event-splitting effects that violate binary RDU and Birnbaum and Bahra (2007) and Wu and Markle (2008) obtained violations of binary RDU for mixed acts. We, therefore, included a test of the main condition underlying binary RDU in our experiment. This test is explained below.

3. Measurement method

Our method for measuring utility and loss aversion consists of three stages and is summarized in Table 1. In the first stage, a gain and a loss are elicited that connect utility for gains (measured in the second stage) with utility for losses (measured in the third stage). The measurements in the second and in the third stage employ the trade-off method of Wakker and Deneffe (1996). Within each domain, we determine a *standard sequence* of outcomes such that the utility difference between successive elements of the sequence is constant. The trade-off method is commonly used in decision theory (Fox and Poldrack 2008), but thus far it could only be used to measure utility for gains and utility for losses separately. It could not be used to measure loss aversion, which requires that the utility for gains and the utility for losses can be compared. Our method allows measuring utility for gains and utility for losses jointly and, consequently, it permits the measurement of loss aversion. In all the derivations presented below we imposed no parametric assumptions on utility and the weighting functions W^i and $w^i, i = +, -$ in binary RDU as can easily be checked. Hence, our method is parameter-free.

Table 1: Three-stage procedure to measure utility

The third column shows the quantity that was assessed in each of the three stages of the procedure. The fourth column shows the indifference that was elicited. The fifth column shows the stimuli used in the experiment. ℓ_{alt} and $k_{L,alt}$ were used to test for consistency (see Section 4 for explanation).

		Assessed quantity	Indifference	Choice variables
Stage 1		L	$G_E L \sim x_0$	$G = \text{€}2000$ $E = \text{color of a ball drawn from an unknown Ellsberg urn, } p = \frac{1}{2}$
		x_1^+	$x_1^+ \sim G_E x_0$	

		x_1^-	$x_1^- \sim L_{E^c} x_0$	
Stage 2	Step 1	\mathcal{L}	$x_1^+ \mathcal{L} \sim \ell_{E^c} x_0$	$\ell = -\text{€}300; k_G = 6$
	Step 2 to k_G	x_j^+	$x_j^+ \mathcal{L} \sim x_{j-1}^+ \ell$	$\ell_{alt} = \text{€}0; k_{Galt} = 3$
Stage 3	Step 1	\mathcal{G}	$\mathcal{G}_E x_1^- \sim \mathcal{G}_E x_0$	
	Step 2 to k_L	x_j^-	$\mathcal{G}_E x_j^- \sim \mathcal{G}_E x_{j-1}^-$	$\mathcal{G} = \text{€}300; k_L = 6$

3.1 First stage: elicitation of the gauge outcomes

We start by selecting an event E that will be kept constant throughout the first stage and a gain G . Then we elicit the loss L for which $G_E L \sim x_0$. It follows from equation (1a) that:

$$W^+(E)U(G) + W^-(E^c)U(L) = U(x_0) = 0. \quad (3)$$

We next elicit certainty equivalents x_1^+ and x_1^- such that $x_1^+ \sim G_E x_0$ and $x_1^- \sim L_{E^c} x_0$. The indifference $x_1^+ \sim G_E x_0$ implies that

$$U(x_1^+) = W^+(E)U(G). \quad (4)$$

The indifference $x_1^- \sim L_{E^c} x_0$ implies that

$$U(x_1^-) = W^-(E^c)U(L). \quad (5)$$

Combining Eqs. (3), (4), and (5) gives

$$U(x_1^+) = -U(x_1^-). \quad (6)$$

Equation (6) defines the first elements x_1^+ and x_1^- of the standard sequences for gains and losses that we will construct in the second and third stages.

For choice under risk, the elicitation of x_1^+ and x_1^- is similar except that the event E is replaced by a known probability p , and the weights $W^+(E)$ and $W^-(E^c)$ are replaced by $w^+(p)$ and $w^-(1-p)$, respectively.

3.2 Second and third stage: elicitation of utility for gains and losses

In the second stage, we elicit a standard sequence of gains. Let ℓ be a prespecified loss. We first elicit the loss \mathcal{L} such that the decision maker is indifferent between the acts $x_1^+ \mathcal{L}$ and $\ell_{E^c} x_0$. The outcome x_1^+ was elicited in the first stage. We could take an event E' different from the event E used in the first stage, but, for simplicity, we used in our experiment the same event in all three stages. The indifference $x_1^+ \mathcal{L} \sim \ell_{E^c} x_0$ implies that

$$W^+(E)U(x_1^+) + W^-(E^c)U(\mathcal{L}) = W^-(E^c)U(\ell). \quad (7)$$

Rearranging Eq. (7) and using $U(x_0) = 0$ gives,

$$U(x_1^+) - U(x_0) = \frac{W^-(E^c)}{W^+(E)} (U(\ell) - U(\mathcal{L})). \quad (8)$$

Next, we elicit the gain x_2^+ such that $x_2^+ \mathcal{L} \sim x_1^+ \ell$ from which we obtain after rearranging

$$U(x_2^+) - U(x_1^+) = \frac{W^-(E^c)}{W^+(E)} (U(\ell) - U(\mathcal{L})). \quad (9)$$

Combining Eqs. (8) and (9) gives :

$$U(x_2^+) - U(x_1^+) = U(x_1^+) - U(x_0). \quad (10)$$

We proceed by eliciting a series of indifferences $x_j^+ \mathcal{L} \sim x_{j-1}^+ \ell, j = 2, \dots, k_G$, to obtain the sequence $\{x_0, x_1^+, x_2^+, \dots, x_{k_G}^+\}$. It is easy to see that $U(x_j^+) - U(x_{j-1}^+) = U(x_1^+) - U(x_0)$ for all j . When probabilities are known, we can apply the above procedure with the event E replaced by a probability p (which may be different from the probability used in the first stage, but in our experiment we used the same probability in all three stages).

The standard sequence of losses is constructed similarly. We select a gain \mathcal{G} and an event E and elicit the gain \mathcal{G} such that $\mathcal{G}_E x_1^- \sim \mathcal{G}_E x_0$.² We then proceed to elicit a standard sequence $\{x_0, x_1^-, x_2^-, \dots, x_{k_L}^-\}$ by eliciting a series of indifferences $\mathcal{G}_E x_j^- \sim \mathcal{G}_E x_{j-1}^-, j = 2, \dots, k_L$. For risk, we can replace the event E by a probability p .

By combining the second and the third stages we have elicited a sequence $\{x_{k_L}^-, \dots, x_1^-, x_0, x_1^+, \dots, x_{k_G}^+\}$ that runs from the domain of losses through the reference point to the domain of gains and for which it is true that the utility difference between successive elements is constant. We can scale utility by selecting the utility of an arbitrary element. In the analyses reported below, we set $U(x_{k_G}^+) = 1$ from which it follows that $U(x_j^+) = j/k_G$ for $j = 1, \dots, k_G$, and $U(x_j^-) = -j/k_G$, for $j = 1, \dots, k_L$.

4. Experiment

4.1 Experimental set-up

² As in the second stage, we could have selected an event E'' different from the events used in the first two stages, but we used the same event in our experiment.

Subjects were 75 economics students of the Erasmus School of Economics, Rotterdam (29 female, mean age of 20.7 years). Each subject was paid a flat fee of €10 for participation in the experiment. Before conducting the actual experiment, the experimental protocol was extensively tested in pilot sessions.

The experiment was run on computers. Subjects answered the questions individually in sessions of at most two subjects. They first received instructions about the tasks and then completed five training questions. After these questions, the experimenter asked them whether they had understood the tasks and gave the final instructions. Subjects were told that there were no correct or wrong answers and that they should go through the experiment at their own pace. They were instructed to approach the experimenter if they needed any advice concerning the experiment. A session lasted 40 minutes on average.

The order in which utility under risk and uncertainty were measured was randomized between sessions. When a subject had completed the first part of the experiment, the experimenter would approach her to explain the subsequent part of the experiment. Within the risk and uncertainty elicitation, the second and third stage were also randomized; some subjects started with the elicitation of the gain sequence, others with the elicitation of the loss sequence. The first stage always had to come first because it served as an input for the other two stages.

We used sizeable monetary amounts because we were interested in studying both utility curvature and loss aversion and utility is approximately linear over small intervals (Wakker and Deneffe 1996). Given that substantial losses were involved, all choices were hypothetical. It is impossible to find subjects willing to participate in an experiment where they can lose substantial amounts of money. We will provide a more detailed discussion of the use of incentives in the Discussion Section.

We did not directly ask subjects for their indifference values. Instead, we used a series of binary choice questions in order to zoom in at the value for which a subject was indifferent. These binary choice questions corresponded to iterations of a bisection process. Examples of such iterations can be found in the Appendix. We applied a choice-based elicitation procedure as previous research suggests that it leads to more reliable results than directly asking for indifference values (Bostic et al. 1990, Noussair et al. 2004).

4.2 Details

To perform the elicitation described in Section 3, we had to specify a number of parameters, which are depicted in the final column of Table 1. We made the common assumption that the reference

point x_0 is equal to 0. In the risk condition, the outcome of an act was determined by drawing a ball from an urn containing five red balls and five black balls. Subjects could state which color they preferred to bet on with the chance of winning always equal to 50 percent. In the uncertainty condition, the outcome of an act was determined by drawing a ball from an urn containing ten balls, which were either red or black in unknown proportions. Again, subjects could select the color they preferred to bet on.

Both for gains and for losses, we elicited six points of the utility function under both risk and uncertainty. Next to these elicitations, we performed a second smaller sequence in the domain of gains, varying the gauge amount ℓ . This second elicitation was meant to test sign-comonotonic trade-off consistency (Köbberling and Wakker 2003), the central condition underlying binary RDU. By definition ℓ needs to be smaller or equal to x_0 . In the main elicitation we set $\ell = -\text{€}300$. Asking the question whether the elicited amounts would depend on the value of ℓ , we also elicited x_2^+ and x_3^+ using an alternative gauge amount $\ell_{alt} = \text{€}0$. Under binary RDU the elicitations of x_2^+ and x_3^+ should not depend on the value we set for ℓ .

Figures A1-A3 in the Appendix show the displays used under uncertainty. The screens under risk were similar, except that the two branches would simply say 50% rather than “Red” or “Black”. Figure A1 displays the typical decision that subject had to make. Subjects were faced with a choice between two acts denoted as options A and B. They could not state indifference. By choosing between the two acts, the subject narrowed down the interval in which her indifference value should fall.

After narrowing down the interval thrice, we presented subjects with a scrollbar (Figure A2). The scrollbar allowed subjects to specify their indifference value up to €1 precision. The starting point of the scrollbar was in the middle of the interval determined by their previous answers. The range of the scrollbar was wider than this interval, so that subjects could correct any mistakes they might have made. The data on the use of the scrollbar also give an indication of the quality of the data. If many subjects would provide answers that did not align with their previous choices, possibly even violating stochastic dominance, this would signal that we would have to worry about subjects’ understanding of the task. After specifying a value with the scrollbar, subjects were asked to confirm their choice (Figure A3). If they cancelled their choice, the process started over. If subjects confirmed their choice, they moved on to the next choice.

We included a number of repetitions to test for consistency. First, in each of the six standard sequences (the short and the long gain sequences and the loss sequence for both risk and

uncertainty), we repeated the second-to-last iteration in the elicitation of $x_2^i, i = +, -$. Repeating the second-to-last iteration is a strong test of consistency, as subjects were probably close to indifference at the end of the iteration process. Furthermore, at the end of eliciting the long gain sequence, we elicited x_4^+ again, for both risk and uncertainty. Together, these repetitions and the way in which subjects used the scrollbar allowed us to gain insight into the quality of the data.

4.3 Analyses

4.3.1 Analyses of utility curvature

Two different methods were used to investigate utility curvature.³ For the first, nonparametric, method, we calculated the area under the utility function. The domain of U was normalized to $[0,1]$, by transforming every gain x_j^+ to the value x_j^+/x_6^+ and every loss x_j^- to x_j^-/x_6^- . If utility is linear, the area under this normalized curve equals $\frac{1}{2}$. For gains, we consider utility to be convex [concave] if the area under the curve is smaller [larger] than $\frac{1}{2}$. For losses, utility is considered to be convex [concave] if the area under the curve is larger [smaller] than $\frac{1}{2}$.

We also analyzed the utility function by parametric estimation. We employed the power family, x^α , as it is the most commonly employed parametric family. For gains [losses] $\alpha > 1$ corresponds to convex [concave] utility, $\alpha = 1$ corresponds to linear utility, and $\alpha < 1$ corresponds to concave [convex] utility. Estimation was done using nonlinear least squares. To test for robustness, we also performed a mixed-effects estimation in which each individual parameter was estimated as the sum of a fixed effect, common to all subjects, and an individual-specific random effect. The results were similar. A potential problem in estimating a model like binary RDU using nonlinear least squares is collinearity between utility and the event weights, which implies that the obtained estimates may not be uniquely identified. The trade-off method voids this problem by keeping event weighting fixed, while eliciting utility and, hence, the obtained estimates are uniquely identified.

4.3.2 Loss aversion

In the literature, loss aversion has been defined in a multitude of ways. Abdellaoui et al. (2007) compared different measures of loss aversion. They concluded that the definitions proposed by Kahneman and Tversky (1979) and Köbberling and Wakker (2005) were empirically most useful, and

³ We also used a third, nonparametric, method based on the evolution of the slope of utility. This analysis led to similar conclusions.

we will use these. Other definitions, proposed by Wakker and Tversky (1993), Bowman et al. (1999), and Neilson (2002), turned out to be too strict for empirical purposes, leaving many subjects unclassified.

Kahneman and Tversky (1979) defined loss aversion as $-U(-x) > U(x)$ for all $x > 0$. To measure loss aversion coefficients, we computed $-U(-x_j^+)/U(x_j^+)$ and $-U(-x_j^-)/U(x_j^-)$ for $j = 1, \dots, 6$, whenever possible.⁴ Usually $U(-x_j^+)$ and $U(-x_j^-)$ could not be observed directly and had to be determined through linear interpolation. Some subjects occasionally violated stochastic dominance. As a result, their utility was not unique and one amount could have multiple utilities. For these amounts, utility was undefined. A subject was classified as loss averse if $-U(-x)/U(x) > 1$ for all observations, as loss neutral if $-U(-x)/U(x) = 1$ for all observations, and as gain seeking if $-U(-x)/U(x) < 1$ for all observations. To account for response error, we also used more a lenient approach, classifying subjects as loss averse, loss neutral, or gain seeking if the above inequalities held for more than half of the observations.

Köbberling and Wakker (2005) defined loss aversion as the kink of utility at the reference point (Benartzi and Thaler (1995) suggested a similar definition). Formally, they defined loss aversion as $U'_l(0)/U'_r(0)$, where $U'_l(0)$ represents the left derivative and $U'_r(0)$ the right derivative of U at the reference point. To operationalize this empirically, we computed each subject's coefficient of loss aversion as the ratio of $U(x_1^-)/x_1^-$ over $U(x_1^+)/x_1^+$, because x_1^- and x_1^+ are the loss and gain closest to the reference point. Given that $U(x_1^-) = -U(x_1^+)$, this ratio is equal to $x_1^+/-x_1^-$. Hence, our method immediately gives Köbberling and Wakker's (2005) loss aversion coefficient without the need to further measure utility. A subject was classified as loss averse if this ratio exceeded 1, as loss neutral if it was equal to 1, and as gain seeking if it was smaller than 1.

5. Results

Three subjects violated stochastic dominance in the first stage of the measurement procedure. This undermines their subsequent answers and they were omitted from the analyses. For the remaining 72 subjects, we could determine the entire utility function, for both gains and losses and under both risk and uncertainty. Of these 72 subjects, 14 violated stochastic dominance at least once. Violations of stochastic dominance potentially signal a lower degree of understanding or a lower degree of effort put in the task. We, therefore, also analyzed the data including only the 58 subjects who never

⁴ These computations required that $-x_j^+$ was contained in $[x_6^-, 0]$ and $-x_j^-$ in $[0, x_6^+]$.

violated stochastic dominance, but this led to similar conclusions.

5.1 Consistency checks

Overall, consistency was satisfactory. Subjects made the same choice in 63.7% of the repeated choices. Reversal rates around $\frac{1}{3}$ are common in the literature (Stott 2006). Moreover, our consistency test was strict, as subjects were close to indifference in the repeated choice and, hence, reversals were more likely. There were no differences in consistency between risk and uncertainty.

The correlation between the original measurement and the repeated measurement of x_4^+ was almost perfect. For risk, Kendall's τ was 0.924, for uncertainty it was 0.938.

As a final indication of consistency, we compared whether the final answer provided by using the slider fell within the interval as set up by the bisection procedure. Subjects provided answers that aligned with their original choices. Furthermore, when a subject's final answer was outside the bisection interval, it typically only violated the final choice, probably indicating that they were close to indifference at this point.

5.2 A test of binary RDU

As explained in Section 4, we elicited two sequences of gains, a longer one based on $\ell = -\text{€}300$, which we use in the main analysis, and a shorter one based on $\ell_{alt} = \text{€}0$. If our subjects behaved according to binary RDU, then the values of x_2^+ and x_3^+ in the short sequence should be equal to those obtained in the long sequence.

We found support for binary RDU, both for risk and for uncertainty. The correlation between the obtained values was substantial. Under risk, Kendall's τ was 0.564 for x_2^+ and 0.518 for x_3^+ . Under uncertainty, these values were 0.694 for x_2^+ and 0.625 for x_3^+ . All correlation coefficients were different from 0 ($P < 0.001$). Moreover, for uncertainty, we could not reject the hypotheses that the values of x_2^+ and x_3^+ obtained in the short sequence were equal to those obtained in the long sequence (Wilcoxon test, both $P > 0.684$). For risk, the values of x_2^+ differed marginally ($P = 0.055$), but the values of x_3^+ did not differ ($P = 0.138$). Hence, even though x_3^+ was chained to x_2^+ , the marginal difference for x_2^+ did not carry over to x_3^+ .

5.3 Ambiguity aversion

The measurement of L and x_1^+ in stage 1 of our method provide insight into subjects' ambiguity attitudes. Let L_r and L_u denote the elicited values of L for risk and uncertainty, respectively. Then, $2000_{.5}L_r \sim 0$ and $2000_E L_u \sim 0$. A subject is ambiguity averse if $2000_{.5}L_r > 2000_E L_r$. By transitivity, $2000_E L_u > 2000_E L_r$ and, thus, $L_u > L_r$. This was true for 63.9% of our subjects (Binomial test, $p = 0.024$) and the median elicited value of L_u ($-\text{€}612.50$) indeed exceeded the median value of L_r ($-\text{€}750$) (Wilcoxon test, $P = 0.012$). Hence, we found evidence of ambiguity aversion in the measurement of L .

Ambiguity aversion also predicts that $x_{1,r}^+$, the value of x_1^+ measured under risk will exceed $x_{1,u}^+$, the value of x_1^+ measured under uncertainty. This follows by transitivity from $x_{1,r}^+ \sim 2000_{.5}0 > 2000_E 0 \sim x_{1,u}^+$. However, this was only true for 44.4% of our subjects and we could not reject the hypothesis that x_1^+ was the same for risk and for uncertainty (Wilcoxon test, $P = 0.807$).

5.4 The utility for gains and losses

Figure 1, Panel A displays the utility for gains and losses under risk, based on the median data. Figure 1, Panel B shows the same graph for uncertainty. Taken at face value, the utility functions seem similar. They are consistent with the typical finding of convex utility for losses and concave utility for gains. Furthermore, the utility function appears considerably steeper for losses than for gains, indicating loss aversion.

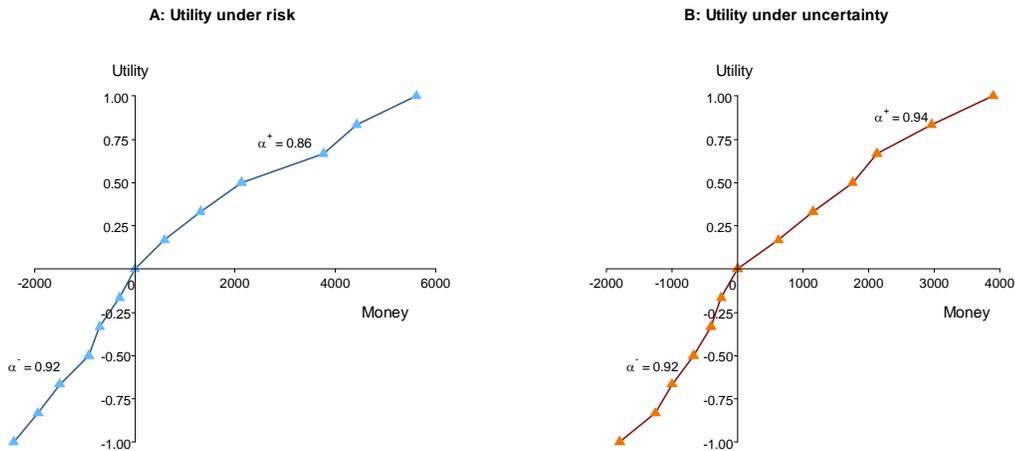


Figure 1: The utility for gains and losses under prospect theory based on the median data. The figure displays the utility for gains and losses under prospect theory based on the median responses of our subjects. $\alpha^+[\alpha^-]$ indicates the estimated power coefficient for gains [losses]. Panel A displays the utility function as measured under risk. Panel B displays utility as measured under uncertainty.

In order to investigate these patterns more thoroughly, we move to the individual level analysis. Table 2 shows that the classification of subjects according to the shape of their utility function was remarkably similar for risk and uncertainty. Performing Fisher's exact test, we indeed found that there were no differences in the overall distribution of classifications between conditions ($P = 0.943$). Utility under risk and uncertainty were related (Kendall's $\tau = 0.389$ for gains and 0.455 for losses, $P < 0.001$ in both cases) and the common pattern was that of an S-shaped utility function: concave for gains and convex for losses. Less than 20% of the subjects behaved according to the traditional assumption in economics that utility is concave throughout.

Table 2: Classification of subjects according to the shape of their utility function

The table classifies the subjects according to the shape of their utility function based on the area under the normalized utility function. Panel A displays the results under risk. Panel B displays the results under uncertainty.

Panel A: Risk				
Gains	Losses			Total
	Concave	Convex	Linear	
Concave	13	31	1	45
Convex	15	8	1	24
Linear	2	0	1	3
Total	30	39	3	72

Panel B: Uncertainty				
Gains	Losses			Total
	Concave	Convex	Linear	
Concave	13	30	0	43
Convex	18	10	2	30
Linear	1	0	0	1
Total	32	40	2	72

The parametric results confirmed the above conclusions. Table 3 shows the estimated power functions at the individual level. Utility was mostly concave for gains and convex for losses. Under risk, 32 subjects had S-shaped utility. Under uncertainty, this was the case for 30 subjects.

Wilcoxon signed rank tests on these power function estimates indicated that there was no difference in curvature for losses between risk and uncertainty ($P = 0.866$). There was some indication that utility for gains was more concave under risk ($P = 0.027$)⁵, but the difference was small. The power coefficients of utility under risk and under uncertainty were moderately correlated: Kendall's τ was 0.373 for gains and 0.423 for losses.

Table 3: Summary of individual parametric fittings of utility for gains

⁵ The difference was no longer significant if we restrict attention to the 58 subjects who never violated stochastic dominance.

The table depicts the results of fitting power functions on each subject's choices individually. Shown are the median and interquartile range (IQR) for the resulting estimates.

	Risk		Uncertainty	
	Gains	Losses	Gains	Losses
Median	0.857	0.924	0.937	0.898
IQR	[0.616-1.062]	[0.649-1.154]	[0.716-1.188]	[0.675-1.356]

Figure 2 shows the relationship between individual estimates for the power coefficients under risk and uncertainty. The dashed lines correspond to the case where subjects have exactly the same coefficients in the two domains. Most estimates were relatively close to the dashed lines and there was no strong indication that subjects had different curvature under risk than under uncertainty.

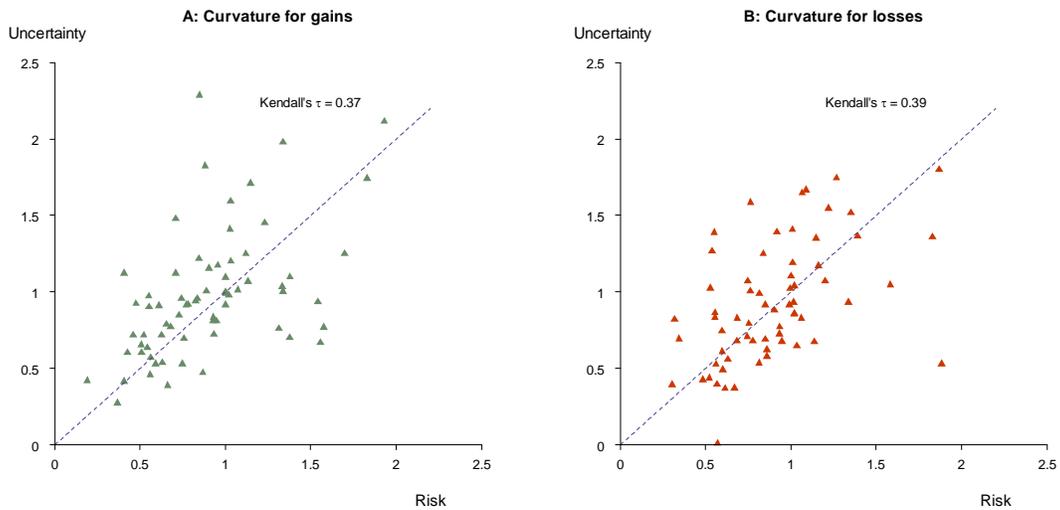


Figure 2: The relationship between individual power coefficients under risk and uncertainty. Panel A displays the power coefficients for gains. Panel B displays the power coefficients for losses. Subjects who had a power coefficient in excess of 2.5 are not shown in the graphs (4 for gains, 7 for losses). The dashed lines correspond to the case where subjects had exactly the same coefficients under risk and uncertainty.

5.5. Loss Aversion

Figure 3 displays the relationships between the medians of x_j^+ and $-x_j^-$ under risk and under uncertainty. Note that for a given j , x_j^+ and $-x_j^-$ have the same utility by construction, and, thus, the figure shows the relationships between gains and losses with the same absolute utility. The dashed line corresponds to equality of median gains and losses with the same absolute utility. As Figure 3 clearly shows, $-x_j^-$ was below x_j^+ for all j , both under risk and under uncertainty. The β 's display the coefficients from the linear regression of the median values of $(-x_j^-)$ on the median values of

(x_j^+) . Both β 's (for risk and uncertainty) differed from unity ($P < 0.001$) and we could not reject the hypothesis that they were equal ($P = 0.431$).

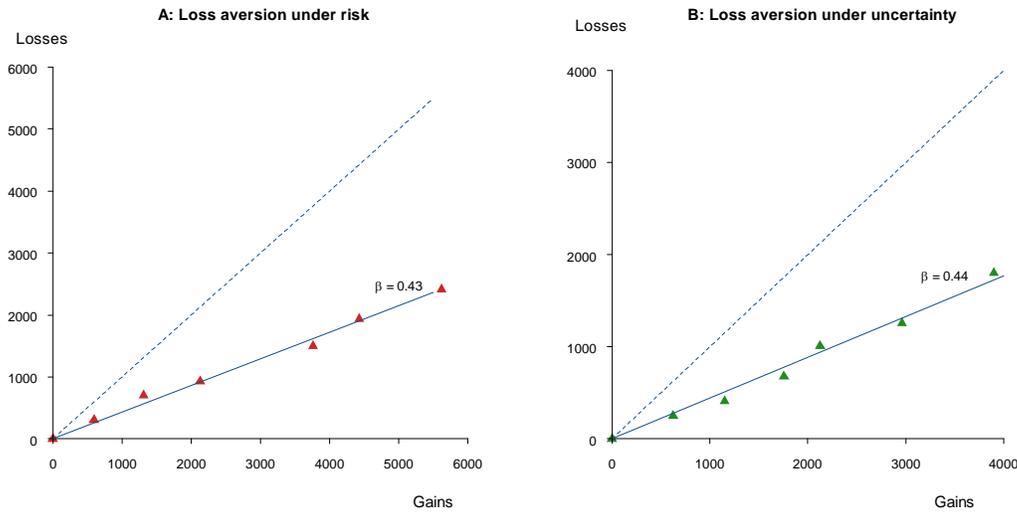


Figure 3: The relationship between median gains and median losses with the same absolute utility. Panel A displays the relationship between median gains and losses under risk. Panel B displays the same relationship under uncertainty. The dashed line corresponds to the case where gains and losses of the same absolute utility would be of equal size. The straight line with slope β corresponds to the best fitting linear equation through all points.

Moving to the individual level, we found that $x_j^+ > -x_j^-$ for all j (Wilcoxon tests, all $P < 0.001$). Furthermore, $x_j^+ / -x_j^-$ did not differ between risk and uncertainty for any j (Wilcoxon tests, all $P > 0.254$).

Figure 3 provides immediate evidence of loss aversion under Kahneman and Tversky's (1979) definition ($U(x) < -U(-x)$ for all $x > 0$). Given that $U(x_j^+) = -U(x_j^-)$, $x_j^+ > -x_j^-$ implies $U(x_j^+) < -U(-x_j^+)$. Moreover, the finding that there were no differences in the tendency of x_j^+ to exceed $-x_j^-$ between risk and uncertainty, can be taken as a first indication that loss aversion was similar under risk and uncertainty.

Table 4: Results under the various definitions of loss aversion

The table depicts results under the two definitions of loss aversion for both risk and uncertainty. The table displays how the coefficient is defined and the number of loss averse, gain seeking, and loss neutral subjects in both conditions. The numbers in parentheses for Kahneman and Tversky's definition correspond with the case where response errors are not taken into account. Furthermore, the table depicts the median and interquartile range (IQR) for each measure of loss aversion under both conditions.

Definition	Coefficient	Condition	Loss averse	Gain seeking	Loss neutral	Median [IQR]
Kahneman and Tversky (1979)	$\frac{-U(-x)}{U(x)}$	Risk	58(46)	10(6)	1(1)	2.19 [1.06, 5.59]
		Uncertainty	54(50)	16(10)	0(0)	2.48 [1.10, 7.16]
Köbberling and Wakker (2005)	x_1^+	Risk	56	13	3	1.86 [1.06, 4.47]

Table 4 shows the results for the individual analyses of loss aversion based on Kahneman and Tversky's (1979) and Köbberling and Wakker's (2005) measures. The table clearly shows evidence of loss aversion, irrespective of the definition used. Based on both definitions, most subjects could be classified as loss averse, regardless of whether we took response errors into account. According to both definitions, the median loss aversion coefficients for risk and uncertainty did not differ (Wilcoxon test, $P > 0.257$ in both tests) and were moderately correlated (Kendall's $\tau > 0.368$, $P < 0.001$ in both tests).

Finally, the two measures of loss aversion were substantially correlated. For risk, Kendall's τ was 0.740 and for uncertainty it was 0.799 (all $P < 0.001$ in both cases). It is comforting to observe that these two distinct measures, one of a local nature and relying on a single kink in the slope of the utility function, and the other global and relying on different absolute utilities associated with the same absolute money amounts in the positive and negative domain, showed a high degree of consistency in classifying subjects.

5.6 Reflection

The aggregate findings reported earlier suggest that the power coefficients were similar in the gain and loss domains. This implies that the utility for losses is the mirror image of the utility for gains and is usually referred to as *reflection*. It is of interest to test whether reflection also held at the individual level. Practically, this would allow us to infer utility for both gains and losses by only measuring it in one of these domains. Theoretically, it would provide support for the idea that utility in both domains is caused by the same psychophysical response to changes relative to the reference point. Reflection is a central result in Tversky and Kahneman (1992) and is widely adopted in theoretical and empirical analyses based on prospect theory (e.g. Barberis et al. 2001).

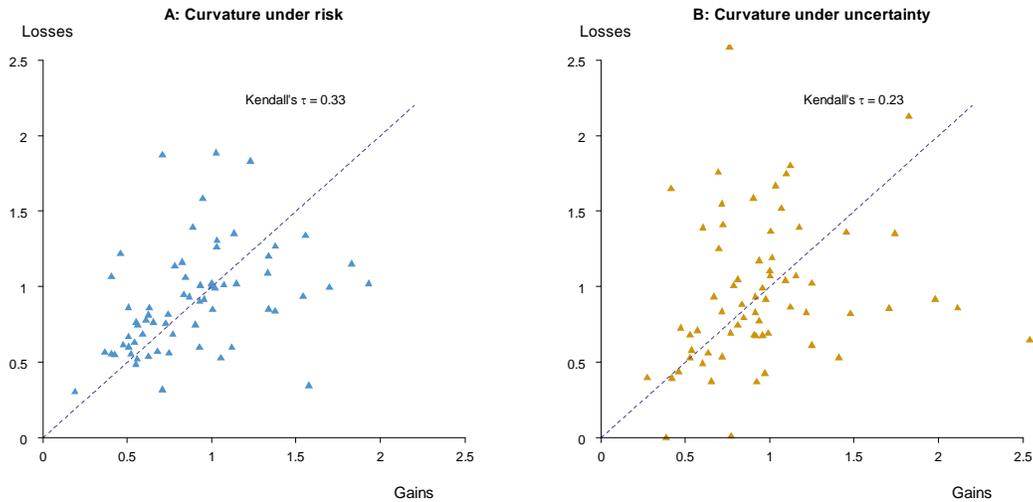


Figure 4: The relationship between individual power coefficients for gains and losses. Panel A displays the power coefficients under risk. Panel B displays the power coefficients under uncertainty. Subjects who had a power coefficient in excess of 2.5 are not shown in the graphs (6 for risk, 9 for uncertainty). The dashed lines correspond to the case where subjects had exactly the same coefficients for gains and losses.

We found little indication that reflection should be rejected. A nonparametric test of reflection is obtained by looking at the absolute deviation of the area under the utility function from $\frac{1}{2}$. Reflection implies that this deviation should be the same for gains and losses. For risk, there was some, albeit marginal, indication that this deviation was more pronounced for gains than for losses (Wilcoxon test: $P = 0.067$). For uncertainty, there was no difference (Wilcoxon test, $P = 0.724$). Reflection also implies that the power coefficients for gains and losses should be identical. We could not reject this hypothesis, neither for risk (Wilcoxon test: $P = 0.128$) nor for uncertainty ($P = 0.814$).

On the other hand, both the area measure and the power coefficients, were only slightly correlated under uncertainty, and moderately correlated under risk. For the area measure, Kendall's τ was 0.317 under risk ($P < 0.001$), and 0.191 under uncertainty ($P = 0.018$). For the power coefficients, Kendall's τ was 0.325 under risk ($P < 0.001$), and 0.231 under uncertainty ($P = 0.004$). Figure 5 displays the relation between the power coefficients for both risk and uncertainty. The straight line corresponds to reflection. Both for risk and for uncertainty, reflection approximately held for most subjects, but for some it was a poor working hypothesis, particularly under uncertainty.

6. Discussion

Ambiguity models differ in whether they allow different utility functions for risk and uncertainty.

Under binary rank-dependent utility, which includes the multiple priors models and prospect theory as special cases, utility is independent of the source of uncertainty and, hence, the same for risk and uncertainty. Ambiguity aversion is modeled through a difference in event weighting. We tested empirically whether the assumption of identical utility functions is justified and obtained support for it. We could not reject the hypothesis that utility and loss aversion were the same under risk and under uncertainty. We also obtained convincing evidence for reference-dependence: utility was concave for gains, but convex for losses and there was substantial loss aversion. Finally, the elicited standard sequences were similar for different stimuli supporting the central condition underlying binary rank-dependent utility (Köbberling and Wakker 2003), which had not been tested before.

Our findings pose a descriptive challenge for models that explain ambiguity aversion through a difference in utility curvature between risk and uncertainty alone, like the popular smooth ambiguity model. We observed that standard sequences were similar for risk and uncertainty. In particular, this implies that preference midpoints were similar for risk and uncertainty. Following Theorem 2.3 in Baillon et al. (2012, Theorem 2.3), this means that for any theory based on expected utility, the utility function for risk must be identical to the utility function under uncertainty, thereby leaving no room for explaining the ambiguity aversion for mixed acts that we observed in our experiment.

It is interesting that loss aversion under risk and under uncertainty were similar. If loss aversion reflects the psychological intuition that losses loom larger than gains then one would expect that measurements of loss aversion are related across domains. Previous evidence of this correlation gave mixed results. Gaechter et al. (2007) found a positive correlation between loss aversion in a risky and in a riskless task, but Abdellaoui et al. (forthcoming) found that loss aversion under risk and loss aversion in intertemporal choice were uncorrelated. Several studies have found that loss aversion is fickle and subject to framing (e.g. Novemsky and Kahneman 2005, Ert and Erev 2008, Abdellaoui et al. forthcoming). We found that loss aversion was stable under risk and uncertainty if the elicitation method is held constant.

In many decisions probabilities are unknown. People are often not neutral towards ambiguity and it is often important to take ambiguity attitudes into account. For example ambiguity aversion helps explain the overreaction to unknown risks such as terroristic threats and the swine flu. Our study contributes to the application of ambiguity models in empirical studies and practical applications by providing a new parameter-free method to measure utility and loss aversion under uncertainty that is robust to event weighting and that can easily be applied. Our method extends the trade-off method by allowing for standard sequences that contain both gains and losses and that go through the reference point. It provides a straightforward way of exploring whether decision makers are loss

averse without the need to elicit the entire utility function. As stage 1 of our method shows, three elicitation suffice to measure loss aversion in the sense of Köbberling and Wakker (2005) and with a few more measurements loss aversion in the sense of Kahneman and Tversky (1979) can be verified. We hope that the provision of such a simple procedure will foster the use of ambiguity models in decision analysis.

Our main conclusions, that both utility and loss aversion were the same for risk and for uncertainty, were not caused by the fact that subjects faced the same stimuli for risk and uncertainty. A simple heuristic that subjects might have used was to simplify the uncertain decision task by assuming that the probability of their preferred color in the ambiguous urn was $\frac{1}{2}$. Then, the decisions under risk and uncertainty would be the same and our conclusions would naturally follow. Our data did not corroborate this hypothesis. The value of the loss L stated in the first stage of our method was significantly lower under ambiguity (Wilcoxon test, $P < 0.001$), consistent with ambiguity aversion. Consequently, the subsequent choices that subjects faced were markedly different for risk and uncertainty. Even though the choices were different, the obtained utilities were similar for risk and for uncertainty.

An easy response strategy in the trade-off method is to let the outcomes of the standard sequence increase by the difference between the gauge outcomes (\mathcal{L} and ℓ in the sequence of gains \mathcal{G} and \mathcal{g} in the sequence of losses). This would bias the results in the direction of linear utility. We checked for this heuristic but found little evidence to support it, even allowing for response error. Hence, this heuristic did not affect the conclusions.

The trade-off method is chained in the sense that previous responses are used in the elicitation of subsequent choices. Chaining may lead to error propagation, where errors made in one particular choice affect later choices. We checked for the impact of error propagation using the Monte Carlo simulation methods developed by Bleichrodt and Pinto (2000) and Abdellaoui et al. (2005). In both simulations, we confirmed the conclusions from those studies that the impact of error propagation was negligible.⁶ We also repeated the parametric analysis of utility accounting for serial correlation in the error terms.⁷ The estimates were identical to the ones reported in Section 5. Hence, we conclude that the chained nature of our measurements had no impact on the results.

Let us finally discuss incentives. We used hypothetical outcomes because we wanted to detect utility curvature. For small money amounts no real utility curvature is usually be observed and the equality

⁶ Bleichrodt et al. (2010) also concluded that error propagation was negligible in their measurements using the trade-off method.

⁷ We assumed that the error terms followed an AR(1) process $\epsilon_t = \rho\epsilon_{t-1} + u_t$ with both ϵ_t and u_t normally distributed with expectation 0 and variance σ^2 and estimated using generalized least squares.

of utility for risk and for uncertainty would then automatically follow. A second reason for not using real incentives is that we wanted to include losses. Ambiguity attitudes differ between gains and losses and loss aversion is very important in explaining risk and ambiguity attitudes. Because we used substantial losses, we could not implement real incentives: it is impossible to find subjects willing to participate in an experiment in which they can lose substantial amounts of money. The literature on the importance of real incentives is mixed. Most studies found that there was little or no effect of using real instead of hypothetical choices for the kind of tasks that we asked our subjects to perform (Bardsley et al. 2010). Therefore, we believe that the limited potential advantage of using real incentives did not outweigh the advantages of being able to use larger outcomes and losses.

7. Conclusion

We performed a critical test of ambiguity models, such as multiple priors and prospect theory, that assume that utility is source-independent and the same for risk and for uncertainty. We verified this assumption and found support for it, suggesting that utility reflects attitudes towards outcomes and not ambiguity attitude. Ambiguity attitudes are better modeled by a difference in event weighting between risk and uncertainty. Moreover, we found that reference-dependence was important both in modeling attitudes towards risk and in modeling attitudes towards ambiguity. Utility was S-shaped, concave for gains and convex for losses and we observed clear evidence for loss aversion with most subjects being loss averse and the median loss aversion coefficients varying between 1.86 and 2.48. Descriptive ambiguity models should be reference-dependent.

To apply ambiguity models in practice requires methods to measure their parameters. It is often believed that this is complex. We present an easily applicable method to measure utility and loss aversion under uncertainty. Our method extends the trade-off method by being robust to sign-dependence and allows the elicitation of standard sequences that include gains, losses, and the reference point. It requires no simplifying assumptions about utility and event weighting and takes account of heterogeneity in individual preferences. We hope that our method will foster the use of ambiguity models in empirical research and practical applications.

Figure A1. Choice screen under uncertainty.

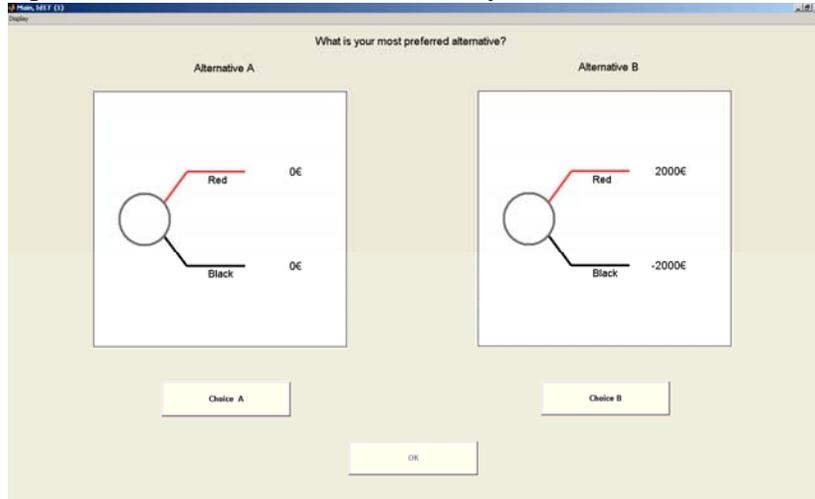


Figure A2. Scrollbar screen under uncertainty.

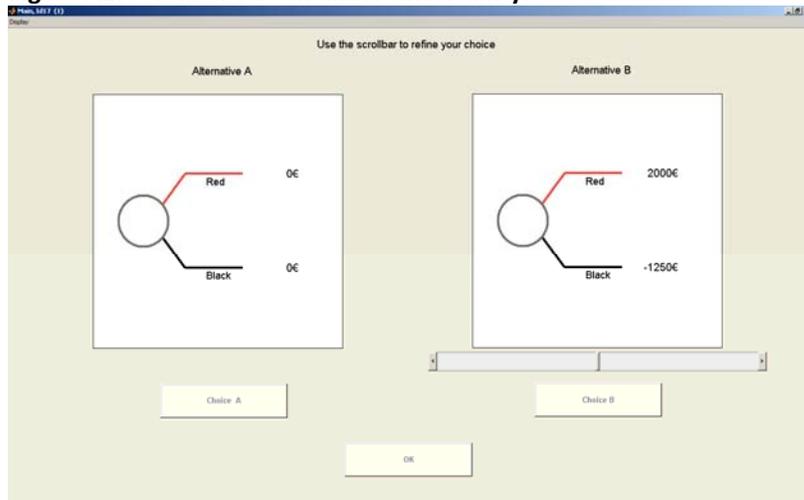


Figure A3. Confirmation screen under uncertainty.

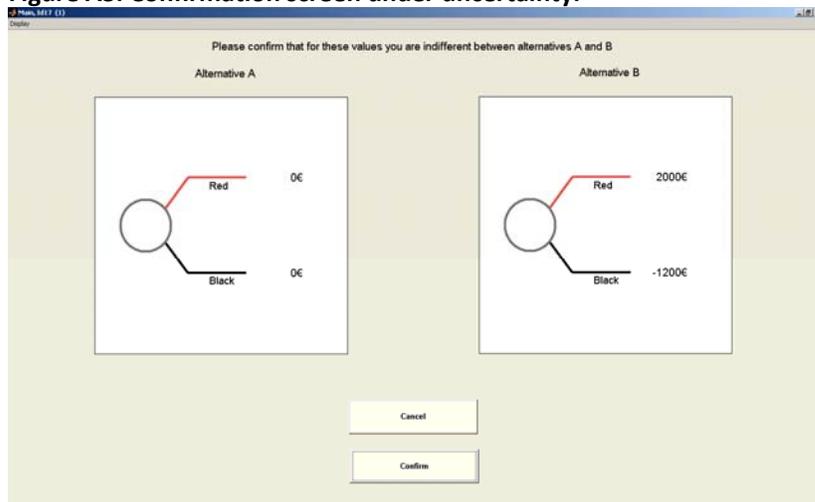


Table A1. Three illustrations of the bisection method under risk.

	Offered choices in elicitation L	Offered choices in elicitation x_1^+	Offered choices in elicitation x_2^-
1	0 vs. (2000, 0.5; -2000)	(2000,0.5;0) vs. 1000	(300,0.5;-200) vs. (800,0.5;-700)
2	0 vs. (2000, 0.5; -1000)	(2000,0.5;0) vs. 500	(300,0.5;-200) vs. (800,0.5;-450)
3	0 vs. (2000, 0.5; -1500)	(2000,0.5;0) vs. 750	(300,0.5;-200) vs. (800,0.5;-325)
Slider	Start value: -1250 Interval: [-2000,-500]	Start value: 625 Interval: [250,1000]	Start value: -388 Interval: [-576,-200]

References

- Abdellaoui, M., A. Baillon, L. Placido, P. P. Wakker. 2011. The rich domain of uncertainty: Source functions and their experimental implementation. *American Economic Review* **101** 695-723.
- Abdellaoui, M., F. Vossman, M. Weber. 2005. Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Science* **51** 1384-1399.
- Abdellaoui, M., H. Bleichrodt, C. Paraschiv. 2007. Measuring loss aversion under prospect theory: A parameter-free approach. *Management Science* **53** 1659-1674.
- Abdellaoui, M., H. Bleichrodt, O. l'Haridon, C. Paraschiv. forthcoming. Is there one unifying concept of utility? an experimental comparison of utility under risk and utility over time. *Management Science*.
- Baillon, A., B. Driesen, P. P. Wakker. 2012. Relative concave utility for risk and ambiguity. *Games and Economic Behavior* **75** 481-489.
- Baltussen, G., M. Van den Assem, D. Van Dolder. 2012. Risky choice in the limelight. Available at SSRN 2057134.
- Barberis, N., M. Huang, T. Santos. 2001. Prospect theory and asset prices. *Quarterly Journal of Economics* **66** 1-53.
- Bardsley, N., R. Cubitt, G. Loomes, P. Moffatt, C. Starmer, R. Sugden. 2010. *Experimental Economics: Rethinking the Rules*. Princeton University Press, Princeton and Oxford.
- Benartzi, S., R. H. Thaler. 1995. Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics* **110** 73-92.
- Birnbaum, M. H., J. P. Bahra. 2007. Gain-loss separability and coalescing in risky decision making. *Management Science* **53** 1016-1028.
- Birnbaum, M. H. 2008. New paradoxes of risky decision making. *Psychological Review* **115** 463-501.
- Bleichrodt, H., J. L. Pinto. 2000. A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Science* **46** 1485-1496.
- Bleichrodt, H., A. Cillo, E. Diecidue. 2010. A quantitative measurement of regret theory. *Management Science* **56** 161-175.
- Booij, A. S., B. M. S. Van Praag, G. Van de Kuilen. 2010. A parametric analysis of prospect theory's functionals for the general population. *Theory and Decision* **68** 115-148.
- Booij, A. S., G. van de Kuilen. 2009. A parameter-free analysis of the utility of money for the general population under prospect theory. *Journal of Economic Psychology* **30** 651-666.
- Bostic, R., R. J. Herrnstein, R. D. Luce. 1990. The effect on the preference reversal of using choice indifference. *Journal of Economic Behavior and Organization* **13** 193-212.

- Bowman, D., D. Minehart, M. Rabin. 1999. Loss aversion in a consumption-savings model. *Journal of Economic Behavior and Organization* **38** 155-178.
- Chew, S. H., Li, K. K., Chark, R., Zhong, S. (2008). Source preference and ambiguity aversion: Models and evidence from behavioral and neuroimaging experiments. Daniel M. Houser, Kevin McGabe, eds. *Neuroeconomics. in: Advances in Health Economics and Health Services Research, Vol. 20*, JAI Press, Bingley, UK, 179-201.
- Cohen, M., J. Jaffray, T. Said. 1987. Experimental comparison of individual behavior under risk and under uncertainty for gains and for losses. *Organizational behavior and human decision processes* **39** 1-22.
- Du, N., D. V. Budescu. 2005. The effects of imprecise probabilities and outcomes in evaluating investment options. *Management Science* **51** 1791-1803.
- Ellsberg, D. 1961. Risk, ambiguity and the savage axioms. *Quarterly Journal of Economics* **75** 643-669.
- Ert, E., I. Erev. 2008. The rejection of attractive gambles, loss aversion, and the lemon avoidance heuristic. *Journal of Economic Psychology* **29** 715-723.
- Ert, E. and I. Erev. . 2012. On the descriptive value of loss aversion in decisions under risk: Five clarifications. *Working Paper Technion, Haifa*.
- Fox, C. R. & Poldrack, R. A. (2008). Prospect theory on the brain: Studies on the neuroeconomics of decision under risk. P. Glimcher, Colin F. Camerer, E. Fehr and Russell A. Poldrack, eds. *Handbook of Neuroeconomics*, Elsevier, New York, 145-173.
- Gaechter, S., E. J. Johnson, A. Herrmann. 2007. Individual-level loss aversion in risky and riskless choice. *IZA Discussion Paper No. 2961*.
- Gajdos, T., T. Hayashi, J. M. Tallon, J. C. Vergnaud. 2008. Attitude toward imprecise information. *Journal of Economic Theory* **140** 27-65.
- Ghirardato, P., F. Maccheroni, M. Marinacci. 2004. Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory* **118** 133-173.
- Ghirardato, P., M. Marinacci. 2001. Risk, ambiguity, and the separation of utility and beliefs. *Mathematics of Operations Research* **26** 864-890.
- Gilboa, I. 1987. Expected utility with purely subjective non-additive probabilities. *Journal of Mathematical Economics* **16** 65-88.
- Gilboa, I., D. Schmeidler. 1989. Maxmin expected utility with a non-unique prior. *Journal of Mathematical Economics* **18** 141-153.
- Gollier, C. 2011. Portfolio choices and asset prices: The comparative statics of ambiguity aversion. *The Review of Economic Studies* **78** 1329-1344.
- Hogarth, R. M., H. Kunreuther. 1989. Risk, ambiguity, and insurance. *Journal of Risk and Uncertainty* **2** 5-35.

- Jaffray, J. Y. 1989. Linear utility theory for belief functions. *Operations Research Letters* **8** 107-112.
- Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47** 263-291.
- Klibanoff, P., M. Marinacci, S. Mukerji. 2005. A smooth model of decision making under ambiguity. *Econometrica* **73** 1849-1892.
- Köbberling, V., P. P. Wakker. 2003. Preference foundations for nonexpected utility: A generalized and simplified technique. *Mathematics of Operations Research* **28** 395-423.
- Köbberling, V., P. P. Wakker. 2005. An index of loss aversion. *Journal of Economic Theory* **122** 119-131.
- Köszegi, B., M. Rabin. 2006. A model of reference-dependent preferences. *Quarterly Journal of Economics* **121** 1133-1166.
- Luce, R. D. 1991. Rank-and sign-dependent linear utility models for binary gambles. *Journal of Economic Theory* **53** 75-100.
- Maccheroni, F., M. Marinacci, A. Rustichini. 2006. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica* **74** 1447-1498.
- Miyamoto, J. M. 1988. Generic utility theory: Measurement foundations and applications in multiattribute utility theory. *Journal of Mathematical Psychology* **32** 357-404.
- Nau, R. F. 2006. Uncertainty aversion with second-order utilities and probabilities. *Management Science* **52** 136-145.
- Neilson, W. S. 2010. A simplified axiomatic approach to ambiguity aversion. *Journal of Risk and Uncertainty* **41** 113-124.
- Neilson, W. S. 2002. Comparative risk sensitivity with reference-dependent preferences. *Journal of Risk and Uncertainty* **24** 131-142.
- Noussair, C., S. Robin, B. Ruffieux. 2004. Revealing consumers' willingness-to-pay: A comparison of the BDM mechanism and the vickrey auction. *Journal of Economic Psychology* **25** 725-741.
- Novemsky, N., D. Kahneman. 2005. The boundaries of loss aversion. *Journal of Marketing Research* **42** 119-128.
- Pennings, J. M. E., A. Smidts. 2003. The shape of utility functions and organizational behavior. *Management Science* **49** 1251-1263.
- Rabin, M. 2000. Risk aversion and expected-utility theory: A calibration theorem. *Econometrica* **68** 1281-1292.
- Roca, M., R. M. Hogarth, A. J. Maule. 2006. Ambiguity seeking as a result of the status quo bias. *Journal of Risk and Uncertainty* **32** 175-194.

- Schmeidler, D. 1989. Subjective probability and expected utility without additivity. *Econometrica* **57** 571-587.
- Seo, K. 2009. Ambiguity and Second-Order belief. *Econometrica* **77** 1575-1605.
- Siniscalchi, M. 2009. Vector expected utility and attitudes toward variation. *Econometrica* **77** 801-855.
- Starmer, C., R. Sugden. 1993. Testing for juxtaposition and event-splitting effects. *Journal of Risk and Uncertainty* **6** 235-254.
- Stott, H. P. 2006. Cumulative prospect theory's functional menagerie. *Journal of Risk and Uncertainty* **32** 101-130.
- Sugden, R. 2003. Reference-dependent subjective expected utility. *Journal of Economic Theory* **111** 172-191.
- Treich, N. 2010. The value of a statistical life under ambiguity aversion. *Journal of Environmental Economics and Management* **59** 15-26.
- Tversky, A., D. Kahneman. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5** 297-323.
- Wakker, P. P. 2010. *Prospect Theory: For Risk and Ambiguity*. Cambridge University Press, Cambridge UK.
- Wakker, P. P., A. Tversky. 1993. An axiomatization of cumulative prospect theory. *Journal of Risk and Uncertainty* **7** 147-176.
- Wakker, P. P., D. Deneffe. 1996. Eliciting von neumann-morgenstern utilities when probabilities are distorted or unknown. *Management Science* **42** 1131-1150.
- Wu, G., A. B. Markle. 2008. An empirical test of gain-loss separability in prospect theory. *Management Science* **54** 1322-1335.