

The Green Paradox and  
The Misapplication of the Economics of Exhaustible Resources

by

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## 1 Introduction: The Meaning of the Green Paradox

In discussions of appropriate policy responses to climate change, the role of fossil fuels, especially oil, takes centre stage. There is a current sense of urgency to begin to reduce consumption of these fuels. A method favored by many economists is a tax on emissions of carbon dioxide, in essence on oil use. Since the marginal harm inflicted by emissions is expected to increase over several decades, a proposal consistent with much of environmental economics is that the tax should be announced as increasing through time, in step with the marginal damages.

Suppose that a global tax on fossil fuels were implemented, and that governments worldwide could commit to the future schedule of taxes deemed appropriate to balance the costs and the benefits of oil use. Would this development be salutary in the context of climate change?

One theoretic development holds that it may not. The *green paradox* states that dynamic influences may thwart the intent of the tax by giving producers an incentive to shift production toward the present. It would thereby cause an increase in damages in the short and medium terms.

Oil is an exhaustible resource. The economics of exhaustible resources is expressed through Hotelling's rule. In its simplest form the rule states that in equilibrium the price net of marginal costs, including marginal taxes, rises at the rate of interest. The argument for the green paradox is a direct application of Hotelling's rule, which prescribes the optimal timing of the extraction and use of exhaustible resources.

By changing the *relative*, net values of a unit of oil at different future dates (as compared to the original equilibrium without the tax) the tax may induce producers "to tilt" their production toward the present. Greater emissions in the present and medium term may be induced. Since

there is a fixed supply of fossil fuels in the earth, the greater emissions come at the expense of emissions in the long-term future. (In the simplest model, there is a one-to-one shift in production.) By then, other means to attack the climate problem may be available. Paradoxically, the tax, designed to help to solve the problem of climate change in the near and medium runs, may exacerbate it, and yet provide only limited relief in the future.

The green paradox merits attention from environmental economists because the theoretic issue is recast by climate change; it becomes the efficient timing of a tax instead of the equity and efficiency of having the “polluter pay” directly for the marginal damages caused. The policy issue is whether the tilt toward the present, described in theory, can be expected to play out in practice.

The fundamental question is the provenance of the short-run increases in output. The present paper expresses doubts about the analysis that gives rise to the green paradox because of doubts about the applicability of Hotelling’s rule. It begins with a brief explanation of Hotelling models. Then it reviews the application of Hotelling analysis to effects of the tax on flows. Later, it interprets a survey of the empirical analysis related to Hotelling’s rule. Finally, it considers technological and natural features of oil production. The paper concludes that the effects of a carbon tax are more likely conventional than paradoxical.

## 2 Hotelling Models

Hotelling’s (1931) model of *the economics of exhaustible resources* is a profound contribution to economic thought. It provides five major insights.

- Exhaustible resources are a form of capital.
- The price of the resource is determined in a dynamic equilibrium that regulates both the flow of the resource to market and the holding of resources as assets.
- The timing of decisions is of central significance and warrants careful analysis.
- Resources are subject to the usual market failures, *viz.* monopoly and externality.
- Exhaustibility in itself does *not* entail a special form of market failure. In particular, competitive markets are not subject to a myopic inability to allocate an exhaustible resource in way that efficiently balances the interests of the present and the future.

Units of the resource are viewed as being available to society for extraction at any time, at a known cost that depends on the quantity extracted and possibly other factors. A striking analytic result of the model is *Hotelling's rule*: in a dynamic, competitive equilibrium the price net of marginal cost of an exhaustible resource rises at the rate of interest. Under the assumptions of the model, the rule can be proved through optimal-control analysis and is mathematically incontrovertible.

The economic reasoning behind the rule is even more striking. Consider what is called herein a *type-one* Hotelling model of an exhaustible resource, in which extraction costs depend only on the quantity of the resource that is currently being extracted. In this case, the extraction cost of  $q > 0$  units of the resource is given by some function  $c(q)$ . This function is assumed to be increasing, so that extracting more units at a given time costs more, and convex, so that it becomes ever costlier to extract additional units.

As argued by Solow (1974), the owner of a resource who wishes to maximize net present value is led to re-arrange extraction such that what is earned by the marginal unit in each time period is

equal in present-value terms. If the marginal unit at one time earns less than at another time, net present value can be increased by reallocating output from the period with the lower gain to the one with the higher gain. In symbols, let  $p(t)$  represent the price at time  $t$ ,  $r$  represent the prevailing rate of interest and  $\mu(t) = dc(q)/dq$  represent the *marginal* cost of production. Suppose that a proposed path of extraction is such that, for times  $t$  and  $s$  during production,

$$\frac{p(t) - \mu(t)}{(1+r)^t} > \frac{p(s) - \mu(s)}{(1+r)^s}.$$

Then a unit of production can be re-allocated from time  $s$  to time  $t$ , at a net gain of

$$\frac{p(t) - \mu(t)}{(1+r)^t} - \frac{p(s) - \mu(s)}{(1+r)^s} > 0.$$

The re-allocation can be done as many times as required, at a net gain each time. Ultimately, a constant,  $\lambda$  say, is determined such that for any times during which extraction takes place,

$$\frac{p(t) - \mu(t)}{(1+r)^t} = \lambda = \frac{p(s) - \mu(s)}{(1+r)^s}.$$

Another re-arrangement yields that

$$p(t) - \mu(t) = \lambda(1+r)^t : \tag{1}$$

the *net* price (price net of marginal cost) rises at the rate of interest. Equation (1) expresses Hotelling's rule. The discussion stresses that Hotelling's rule is an arbitrage condition for the use of an asset over different periods of time.

Often it is assumed that the marginal cost is constant, so that for some number  $\gamma$ ,  $c(q) = \gamma q$ . In this case, for any value of  $q$ ,  $\mu(q) = \gamma$ . The assumption allows for developing sharp mathematical

results, including the early insights of the green paradox by Sinclair (1992) and Ulph and Ulph (1994), as well as some more recent ones.

A remarkable feature of Hotelling's original paper is that he also considered what may be called a *type-two* model. In *type-two models*, cost depends on the total available reserve,  $Q$  say, as well as current output, and is written  $C(q, Q)$ . The properties of this cost function are that it is an increasing, convex function of output  $q$  and a decreasing, convex function of reserves  $Q$ . That is to say, costs are lower if reserves are higher. Also,  $C(0, Q) = 0$  for any value of  $Q$ . A type-two model delivers less sharp theoretic results than a type-one model: There is still arbitrage among marginal units of the resource but the influence of the remaining reserve on cost yields a more complicated expression of Hotelling's rule. The rule is expressed in terms of the discounted sum, over the future of production from the resource (up to the time of abandonment of the industry,  $T$  say), of the increases in cost that arise because current production affects future costs through depleting total reserves,  $Q$ . In symbols, at time  $t$  during extraction, the expression for the

Hotelling rent includes the expression,  $\sum_{s=t}^T \frac{\partial C(q(s), Q(s)) / \partial Q}{(1+r)^{s-t}}$ . This expression is harder to work

with theoretically. Type-two models are considered to be more realistic. For one thing, they can allow for extraction costs at date  $T$  to be so high that some of the resource is never extracted.

The function  $C(q, Q)$  has been a workhorse of empirical research in economics since the late 1970s. Several theoretic analyses have also utilized it. In the main, however, theorists have resorted to the simpler function  $\gamma q$ . The same observations are true of the green paradox:

Although some authors have used the function  $C(q, Q)$  in theoretic work, the simpler function is the foundation of the more striking conclusions.

Either function implies that any level of extraction  $q$  is possible if one is willing to incur the current marginal cost. Consistently with the nature of arbitrage in Hotelling models, output can be shifted at will over time. If cost is  $\gamma q$ , an unbounded level of output can be had at the constant marginal cost  $\gamma$ . There is no impediment to tilting output toward the present.

### 3 The Green Paradox

Even though the analysis of extraction with cost  $C(q, Q)$  is more complicated than with cost  $\gamma q$ , Sinn (2008) deftly uses arbitrage between *adjacent* periods (times  $t$  and  $t+1$ ) to make the argument for the green paradox. Let  $C(q, Q) = g(Q)q$ , where  $g'(Q) < 0$  and  $q = -dQ/dt$ . This case is special but only slightly so: it assumes constant marginal cost at any point in time, but increasing costs as the resource stock  $Q$  decreases through extraction. Again let the market price be  $p$  and the interest rate be  $r$ . The proceeds from a single unit of resource extracted at time  $t$  and invested for one period are  $p(t) - g(Q)$  directly and  $r[p(t) - g(Q)]$  in interest. If instead that unit is not extracted until  $t+1$ , and also if the change in  $Q$  over the period is neglected as being quite small, the owner gains  $p(t+1) - g(Q)$ . Let the change in price over the period be represented by  $\Delta p(t) \equiv p(t+1) - p(t)$ . Arbitrage renders the two gains equal so that

$$[p(t) - g(Q)](1 + r) = p(t+1) - g(Q).$$

Simple algebra yields a single-period generalization of Hotelling's rule:

$$\frac{\Delta p(t)}{p(t) - g(Q)} = r. \tag{2}$$

This rule necessarily holds on an equilibrium path for each time period and so links all periods.

If a tax on emissions is imposed and it rises through time, say being equal to the discounted value of the damages due to the marginal unit of emission, the result may not be what is expected by proponents of the tax. To illustrate what may happen using minimal algebra, Sinn proposes an unfamiliar type of tax. Suppose a cash-flow tax grows in such a way that the net cash flow per unit decreases through time at rate  $\delta$ :

$$1 - \tau(t) = \frac{[1 - \tau(0)]}{(1 + \delta)^t}.$$

The equilibrium condition becomes

$$\frac{[1 - \tau(0)]}{(1 + \delta)^t} [p(t) - g(Q)](1 + r) = \frac{[1 - \tau(0)]}{(1 + \delta)^{t+1}} [p(t) + \Delta p(t) - g(Q)].$$

In this situation the modified rule becomes

$$\frac{\Delta p(t)}{p(t) - g(Q)} = r + \delta + r\delta. \quad (3)$$

(The last term on the right,  $r\delta$ , is negligible if the time unit is small.) Sinn applies the following general reasoning to this condition for different values of  $\delta$ . When  $\delta = 0$ , so that the tax is constant (including if it is zero), condition (3) is the same as condition (2) throughout the period of production. Therefore, the tax is neutral (does not change the path of extraction and hence the accumulation of carbon in the atmosphere). For given values of the other variables in condition (3), the change in price  $\Delta p$  is greater when  $\delta > 0$  (price grows faster when the tax is increasing) than when  $\delta = 0$ . Sinn makes assumptions that assure that the reserves are eventually completely exhausted. With or without the tax, when the reserves are completely used up (and hence output is zero), the price must be the intercept of demand. Since the price rises faster with

the increasing tax, it must start out lower. In equilibrium at the current time  $t$ , therefore, quantity demanded and hence output must be higher. Moreover, the reserve is used up earlier.

The economics behind the result is that the rising tax changes the *relative* gains to the producer over the life of the reserve, making current extraction comparatively more attractive. With greater present extraction, the industry is in equilibrium at a lower current price.

Consequently, global warming is exacerbated. The green paradox is that a policy designed to tax the emission of carbon in a way that is proportional to damages caused leads to greater emissions. A better rule may be to have an initially high but decreasing tax (so that  $\delta$  is negative). Such a tax would encourage the producer to shift output toward the future.

Sinn observes that other forms of policy, such as a subsidy to greener forms of energy, would be subject to a green paradox as well.

Sinn's analysis is the benchmark for understanding the green paradox. Several other authors have studied the paradox, within the Hotelling framework but under somewhat different conditions than studied by Sinn, and have qualified his results.

#### 4 Evidence for Hotelling's Rule

For a mathematically incontrovertible result, Hotelling's rule has been subject to much controversy. Practitioners, whose conscious, rational decisions are supposed to implement the rule, flatly and (to this author's knowledge unanimously) deny its relevance. Strong challenges have come from academia.

What is the evidence? In an erudite review of empirical research on Hotelling's rule, Livernois (2009: 37-38) finds that the evidence does "not necessarily invalidate the conceptual message of the Hotelling Rule". This finding arises from a conviction "that mining firms think not just of the present but about the future, and that they wish to maximize the value of their assets".

Hotelling's framework of analysis seems to be borne out. Indeed, the practitioners that the present author has met are in substantive agreement about the framework. In the statements of the green paradox, however, as well as in the foundations of the economics of exhaustible resources, the thrust of Hotelling's message is not limited to the idea that extracting firms maximize present value. In economics, maximization of present value is an initial, working hypothesis about conduct in *any* industry.<sup>1</sup> Hotelling's rule, his analytic result, is the message that is specific to exhaustible resources and is used by resource economists as the foundation for thinking about the dynamics of resource price.

Elsewhere, Livernois (2009: 22) describes Hotelling's rule as "a condition of inter-temporal arbitrage that ensures that the last unit extracted in any time period earns the same return (in present value terms)". As Solow's (1974) definitive treatment notes, the arbitrage consists of being able to move production of units of output freely among time periods in response to changes in prices and interest rates. Of the rule, Livernois (2009: 37-38) finds that "overall one cannot conclude that the Hotelling Rule has been a significant force", and that "other factors, notably technological change, revisions to expectations regarding the resource base, and market structure, have had a more significant influence on the evolution of prices".

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<sup>1</sup> It is not, however, a good hypothesis about behavior of the national oil companies that are responsible for much of the world's output of oil. See Cairns and Calfucura, forthcoming. However, it can be accepted herein for the sake of argument.

Livernois's review suggests that the arbitrage condition has not received broad support from empirical research, in spite of econometricians' efforts to control for the other influences. Indeed, recent history does not suggest that the price of oil, even influenced heavily by the qualifications of technological change, unanticipated discoveries and market structure, is determined through inter-temporal arbitrage of individual units of production. The price rises abruptly in the mid and in the late 1970s, begins to fall from the early 1980s through most of the 1990s and from 1999 moves up steadily to a very sharp peak in 2008. Then it falls steeply but finally begins again to rise. All of these broad movements have been subject to strong fluctuations. These medium-term movements, especially those in the current century, are difficult to reconcile with the unimpeded arbitrage of units of reserves demanded by Hotelling's rule, even under uncertainty.

Instead, the empirical evidence may be indicating that the arbitrage, which underlies the predictions of Hotelling theory, may not be being realized. Is there an explanation for what appears to be a troubling departure from an incontrovertible mathematical result?

## 5 Technological Models

The key assumption, usually glossed over, is that output can be re-arranged as desired. The function  $C(q, Q)$  is not a valid representation of the technology of oil production, let alone the simpler  $cq$ . The potential to produce is obtained by a high up-front cost in drilling wells that determines an output capacity, as reasoned by Campbell (1980) in introducing what may be called a *technological* analysis of nonrenewable resources. Once capacity is installed, it is usually not changed for a significant period of time; almost always (other than for "ramping up" production, for maintenance or for production problems requiring maintenance) output in the

early years is at the level of capacity. Campbell's ideas were advanced by Crabbé (1982) and Lasserre (1985).

In the oil industry specifically, after the period of capacity production, the productivity of a well decreases through what is known as natural decline. At any time, the level of output is restricted by the more stringent of capacity and natural factors (Cairns 2009).

In essence, Hotelling models of both types one and two are short-run models. They neglect the decision to commit capital to oil production. Moreover, they neglect the facts that the technology does not allow for unlimited output and that there are other, natural limits to production at any time. The option to shift production unimpeded from one point in time to another, the free arbitrage basic to Hotelling analysis, is not available.

For analysis of the effects of policy a long-run analysis is required. Technological models are reserve-based rather than sector-based, analyzing conditions of production at the individual reserve. The behavior of the sector is an aggregate of the behavior of individual producers.

Much formal academic work has been done on extraction. Less is "known" (formally or mathematically modeled) about development than extraction. Less is known about exploration than extraction. The reason is that, as one moves vertically backwards, more complicated and intricate features are involved, and all the downstream features must be incorporated in a truly valid model. As a result, the industry cannot be fully represented in formal, mathematical models. Less formal, economic analysis has to be used to provide bridges and extensions.

While the Hotelling model enjoys widespread theoretical support, its applicability in practice has been continually challenged. Where empirical evidence of the path of price net of marginal cost

has been sought, it has been found that this variable rises more slowly than at the rate of interest, contrary to the Hotelling prediction that it rises at the rate of interest.

The type-two model, too, predicts that the net price rises at less than the interest rate. But comparatively few equilibrium models have used even a type-two view of these markets. One, by Cairns and Quyen (1998: 181) for mineral exploration, argues that the modified rule for the rate of change of resource rent “involves a complicated convex sum, with endogenous weights, of the deposit-specific terms” so that the cost function  $C(q, Q)$ , involving aggregate magnitudes  $q$  and  $Q$ , does not generate the correct rule.

An alternative approach is provided by Cairns and Davis (2001). They observe that oil is produced from underground reservoirs that are under great pressure from the contained oil and gas. A well drilled into the formation, like a pin prick into a balloon, allows the pressure to be released, and with it the oil and gas. In some cases the pressure (through what is called natural drive) is sufficient to produce the oil, but at declining rates. In others, if pressure is not great enough, pumps are installed at the surface to lift the oil to the surface. For what is known as secondary production, wells can be installed at the periphery of the reservoir to inject water or gas that will drive the hydrocarbons in the reservoir toward the producing well. In all cases, the valuable natural product is oil (and gas). The scarce natural or artificial instrument of production is pressure. Technology allows for the augmentation of pressure in various ways.

## 6 Producing Properties: Appropriate Short-Run Analysis

An initial analysis is that of a *producing* property: all investments have been made and operation is in the short run. This discussion is the counterpart at the level of the reserve to a Hotelling

analysis at the level of the sector. Following Adelman (1990, 1993) and Cairns and Davis (2001) let the oil be driven to the surface by pressure,  $P(t)$ , which declines by *natural decline* at rate  $a$ , according to the equation

$$dP(t)/dt = -aq(t). \quad (4)$$

For some constant  $\pi$ , output depends on pressure according to the inequality

$$q(t) \leq \pi P(t). \quad (5)$$

As a result,  $q(t) = q(0)e^{-at}$ .

Suppose that the reserve has been prepared for exploitation at time  $t = 0$  and is abandoned when the pressure is not sufficient to raise the oil to the surface at time  $t = T$ . The reserve initially available for production is

$$R(0) = \int_0^T q(t)dt = \frac{q(0)}{a}(1 - e^{-aT}).$$

Let the net price  $v(t)$  grow at rate  $g < r$ . Then the *value* of the reserve is given by

$$V(0) = \int_0^T [v(0)e^{gt}] [q(0)e^{-at}] e^{-rt} dt = v(0)R(0) \frac{a}{a+r-g} \frac{1 - e^{-(a+r-g)T}}{1 - e^{-aT}}.$$

The value  $V(0)$  is strictly less than  $v(0)R(0)$ , which is the value given by a condition derived from Hotelling's rule called the Hotelling Valuation Principle (Miller and Upton 1983).

Production in this model is fully determined by geological features and not economic choices.

The formal optimization of its value is trivial. The economic meaning of the result, however, is not trivial. Since the reserve is extracted over time, with the net price per unit rising at less than

the interest rate, the value must be lower than given by the Hotelling valuation principle.

Adelman's perspective on the oil industry indicates that, at an individual reservoir and hence for the whole industry, the prediction of the Hotelling model does not hold.

The mathematical reason is that there is a positive shadow value of pressure. It arises from constraint (5). In an abstract sense, pressure is a *scarce resource* because an increase in pressure would allow the operator to extract more quickly and thereby to increase present value:

$\partial V(P, R) / \partial P > 0$ . A portion of the net price,  $v(t)$ , is attributed to pressure. The results hold, with minor modifications, under conditions of uncertainty as well (Davis and Cairns 1998, 1999).

The fundamental reason that the Hotelling rule does not hold is that the output from a reservoir is constrained. Since output is constrained at a producing property there is no change in output as a result of a tax (unless the tax is so high as to put the firm out of business).

The green paradox is a prediction of an *increase* in production. Are there other mechanisms that may induce an increase in short- and medium-term production?

## 7 The Development of Known Reserves

The discussion thus far, like Hotelling analysis, has abstracted from some decisions that are made by oil producers. Some reserves are known but "held on the shelf" for later development.

A more sophisticated analysis admits the possibility that the pressure constraint can be alleviated, to an extent, by investment in the reservoir. Can the possibility of hastening the development of such reserves give support to the green paradox, albeit after a development period of several years?

The choice of when to develop reserves is a question of timing that is resolved by a generalization of Hotelling's insights. What is known of the response of the holders of reserves is that *optimal* development takes place when the rate of increase of the present value of developing the reserve falls to the rate of interest (Cairns and Davis 2007, Davis and Cairns 2012). Instead of there being an economically meaningful choice of the timing of the production of individual units of a stock, the choice of timing is of the original investment. That choice is coincident with a choice of the level of investment. In equilibrium, what rises at the rate of interest is the net present value of the entire project, evaluated at the point of investment. Once made, the investment "locks in" the maximal rate of production and, in practice, the actual rate of production in the short run.

If a tax on emissions is introduced, what is the effect on the development decision? The tax affects all units of output throughout the period of production from a given reserve. Its effect is on, not the "tilt" of production at particular points of time, but the timing and level of initial investments in a reserve (including any anticipated future investments in secondary or tertiary recovery). Development may well be earlier (or later), depending on the revised prediction of the unfolding of the market through time by the investing firm. The rate of change of the net present value of a given project is not an easy variable to predict, even for a potential investor who has a strong incentive to predict it.

For a given choice of *timing* of investment in a reserve  $R(t)$ , Adelman (1990: 6) considers the choice of the *level* of investment and hence the choice of pressure. Investment is given as the product of an "investment factor"  $k$ , the decline factor  $a$  (now endogenous to the choice of the level and placement of investment) and the initial level of output  $q(0)$ , so that the value to be maximized (with respect to  $a$ , because of the determination of investment, as well as  $q$ ) is

$$V_3[P(0), R(0), a] = \int_0^T v(t)q(t)e^{-rt} dt - kaq(0).$$

Adelman's derives a cubic equation for  $a$ . It has one real root that determines the level of investment uniquely. The level of investment is dependent on the *path* of the net price of the product. If anticipated net prices are lowered by the tax, the level of capacity is lower.

In Adelman's formulation, the decline starts in immediately. For production using secondary and tertiary methods, decline may not be immediate but fairly constant for a time at the level of capacity set by the investment. A model of mining by Cairns (1998, 2001) confirms Adelman's result that investment determines the level of production. It also explains the fact that, at most properties, output is observed to be constant over a period early in the productive life of the reserve. As in Adelman's model, in Cairns's model the net present value is given by the discounted value of output net of the cost of investment. A finding is that the marginal cost of invested capital is equated to the discounted value of its shadow price over its lifetime. Therefore, the shadow value of investment must be positive on some non-degenerate interval. The positive shadow value indicates that the level of output is constrained by capacity.

In the case of oil it follows from the fact that pressure declines with output according to equation (5) that the interval of time on which production is constrained by capacity must be early in the life of the reserve  $[0, T]$ , and that after the interval of producing at capacity the pressure constraint becomes effective. Cairns and Davis (2001) consider another type of constraint, a regulatory one for determining the allocation of production in a reservoir that is common property to more than one producer, and find a similar result.

In sum, at any time at any producing property in the oil industry, output is constrained by the most stringent of a number of constraints. The short-term increase in output assumed in the green paradox cannot be obtained from currently active reserves without some increase in investment.

This putative investment is not profitable in the original equilibrium; if it were, it would have been made. The tax on emissions reduces the profitability of such investment; it is, in effect, imposed on each unit of output, reducing the net price. Moreover, if the green paradox is to be believed, the price of product and hence the net price is lower at each instant. There is no reason to suppose that the investment will take place in response to the tax, or that an increase in present production will occur. On the other hand, some marginal resources that are in production may become unprofitable as a result of the tax and be shut in.

The net effect of the tax is that there is no appreciable increase in output from reserves that are currently in production. There is no reason to believe in a green paradox if one bases the analysis on current abilities to increase supplies in response to a tilt in the time profile of net prices resulting from a tax. On the contrary, the tax can be expected to result in a reduction in production, and a reduction in emissions of greenhouse gases.

Conceivably, investing earlier at some properties held on the shelf may, to an extent, offset a lower capacity level. The timing of investment responds to the rate of change of present value, and is subtle. Uncertainty about the timing of introduction and properties of the tax, however, may cause investors to delay investment until the uncertainty is resolved.

## 8 Undiscovered Reserves

Exploration, too, requires sunk investments (Cairns and Quyen 1998). Exploration is a “set-up” cost (Hartwick, Kemp and Long 1985) as opposed to an investment in productive capacity (Campbell 1980). It involves a sunk cost but no constraint on arbitrage of the output from a reserve that has been set up. For an interpretive survey of the economics of exploration, see Cairns (1990).

An emissions tax would shift the distribution of returns from investments in exploration (which depend on returns from eventual development) in any mineral province “to the left”, and a tax increasing continually through time would entail a continual shift to the left.

An increase or a decrease in exploration may result. As predicted by the green paradox, there may be a “black-gold rush” to realize the value of exploration provinces earlier than in the original equilibrium, with a reduction in price and a possible increase in the variance of returns. Davis and Cairns (2012) argue that the strike time under uncertainty is still when the expected return falls to the rate of interest. Or there may be a holding back of exploration in the face of lower returns to the sunk investment and an increase in the value of waiting if the variance does increase.

In either case, any current *increase* in exploration depends on whether there is spare capacity in drilling rigs and in exploration professionals. In equilibrium, spare capacity is low. After all, investments are made to reap quasi rents, not to sit idle in anticipation of a possible tax and the consequent reduction of both scarcity and quasi rents. Because there is a long lead time from the start of exploration through development to production, the relevant changes due to a tax are between one and two decades forward.

## 9 Other Forms of Capital

Investments in knowledge – in basic and applied research in exploration, development and extraction (set-up costs) and in the training of professionals (largely capacity costs) – are also sunk and have long lead times before coming into service. Because such investments are likely to be embodied in new vintages of capital, there is a very limited effect of investments in knowledge on wells currently in service or under development. The returns to these investments are likely to be reduced as compared to those in the original equilibrium before the tax.

Professionals, especially more promising minds, may shy away from training in an industry that is expected to be subject to increasing taxation, reduced rents, and societally mandated attempts to develop substitutes for its product.

The tax would not have an immediate effect on sunk capacity in refining and transportation. It is difficult to perceive an incentive to increase new investment in these activities in a way that would help to facilitate the realization of the predictions of the green paradox.

The development of substitutes takes long lead times and uncertain innovation. A credible commitment to a rising tax on emissions may hasten research into substitutes, with some success in the medium term, and also a reduction in the attractiveness of the oil industry *per se*. It is true that the credibility of announced measures is a severe problem for climate policy. But the green paradox, too, relies on it.

## 10 Conclusion: *Hotelling reigns but does not rule.*

Hotelling's rule is a result from a simple model that illustrates that non-renewable resources are a form of capital and should be analyzed as such. His important insight about far-sighted decision making is borne out. Hotelling models, however, assume the preponderance of an exhaustibility

constraint and free allocation of resources over time. In reality, exhaustibility is peripheral to the analysis of the oil industry. The dominating constraint is that allocation is capped in one of a number of ways.

Technological models make a qualitative break from type-one and -two Hotelling models concerning the form of decisions in the oil industry. In Hotelling models, decisions are taken at each instant about the level of flow of units of the resource. In technological models, decisions about flows are atrophied. Extraction requires a combination of a discovered reserve with fixed capital. The basic decisions are about the timing and level of investment in capacity.

Models of several important aspects of oil production must be stitched together if one wishes to begin to analyze the dynamics of the industry in a way that is relevant to policy. Some of these features have been pointed to herein. Some are becoming more fully understood. Others have not yet been subject to rigorous research. Each, to have credibility for policy analysis, requires long and deep research.

In the particular case of the green paradox, the professional economist can at best return the Scottish verdict, *not proven*. It is not possible to prove a green paradox given the current limitations of mathematical analysis of a complicated, multi-faceted industry.

The present paper suggests that many influences may be inimical to the predictions of the paradox. A reasonable environment or finance minister should not hesitate to implement incentive-based policies such as carbon taxes (even taxes that rise through time), cap and trade, subsidies for substitutes or others that are under current discussion.

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