Self-Fulfilling Debt Crises: Can Monetary Policy Really Help?[1]

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Abstract

This paper examines the potential for monetary policy to avoid self-fulfilling sovereign debt crises. We combine a version of the slow-moving debt crisis model proposed by Lorenzoni and Werning (2014) with a standard New Keynesian model. We consider both conventional and unconventional monetary policy. Under conventional policy the central bank can preclude a debt crisis through inflation, lowering the real interest rate and raising output. These reduce the real value of the outstanding debt and the cost of new borrowing, and increase tax revenues and seigniorage. Unconventional policies take the form of liquidity support or debt buyback policies that raise the monetary base beyond the satiation level. We find that generally the central bank cannot credibly avoid a self-fulfilling debt crisis. Conventional policies needed to avert a crisis require excessive inflation for a sustained period of time. Unconventional monetary policy can only be effective when the economy is at a structural ZLB for a sustained length of time.

1 Introduction

A popular explanation for the sovereign debt crisis that has impacted European periphery countries since 2010 is self-fulfilling sentiments. If market participants believe that sovereign default of a country is more likely, they demand higher spreads, which over time raises the debt level and therefore indeed makes eventual default more likely.[1] This view of self-fulfilling beliefs is consistent with the evidence that the surge in sovereign bond spreads in Europe during 2010-2011 was disconnected from debt ratios and other macroeconomic fundamentals (e.g., de Grauwe and Ji, 2013). However, countries with comparable debt and deficits outside the Eurozone (e.g., the US, Japan or the UK) were not impacted. This difference in experience has often been attributed to the fact that the highly indebted non-Eurozone countries have their own currency.[2] The central bank has additional tools to support the fiscal authority, either in the form of standard inflation policy or by providing liquidity, which can avoid self-fulfilling debt crises. In fact, the decline in European spreads since mid 2012 is widely attributed to a change in ECB policy towards explicit backing of periphery government debt.

The question that we address in this paper is whether central banks can credibly avert self-fulfilling debt crises. Is it the case that countries that do have their own currency (like the U.S., Japan, U.K.) will not be subject to self-fulfilling debt crises of the type seen in Europe? Similarly, would the European countries have been able to credibly avoid a sovereign debt crisis if, counterfactually, each country had their own currency? Our analysis will not apply to a specific country, European or otherwise, but rather to any country that has a central bank and would be vulnerable to a self-fulfilling debt crisis without the actions of the central bank.

We analyze a dynamic model where the central bank can use monetary policy

[1]This view was held by the ECB President Draghi himself: "... the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." (press conference, September 6, 2012). In the academic literature, versions of this argument can be found, among others, in Aguiar et al. (2013), Camous and Cooper (2014), Cohen and Villemot (2015), Conesa and Kehoe (2015), Corsetti and Dedola (2014), de Grauwe (2011), de Grauwe and Ji (2013), Gros (2012), Jeanne (2012), Jeanne and Wang (2013), Krugman (2013), Lorenzoni and Werning (2014), Navarro et al. (2014), and Miller and Zhang (2012).

[2]See for example de Grauwe (2011), de Grauwe and Jin (2013), Jeanne (2012) and Krugman (2013).

to avoid a crisis. We examine whether it is successful in avoiding a crisis and, if it is, to what extent it can reasonably be expected to be implemented. For this purpose, we combine a standard New Keynesian model with a sovereign debt crisis model. We consider the "slow moving" debt crises framework developed by Lorenzoni and Werning (2014, henceforth LW). The LW model is in the spirit of Calvo (1988), where default becomes self-fulfilling by raising the spread on sovereign debt. While Calvo presents a two-period model, the LW model is dynamic and incorporates long-term debt. The anticipation of a possible future default on long term bonds leads interest rates and debt to gradually rise over time, justifying the belief of ultimate default. This contrasts with liquidity or rollover crises, such as Cole and Kehoe (2000), where crises occur instantaneously when a sunspot arises. The slow-moving nature of the crisis appears more realistic in the context of recent crises. It also gives the central bank more time to act to support the fiscal authority.

With flexible prices, the central bank could easily inflate the debt by suprising bondholders with a large increase in inflation. This strategy is more difficult to implement with price rigidity and may only work with long-term debt. Moreover, price rigidity implies other effects of monetary policy. First, it can lower real interest rates and reduce the real cost of new borrowing. Second, it can increase output to raise government tax revenue. We examine these various effects by calibrating the maturity of debt and degree of price rigidity. We also follow the literature and consider a specification of the New Keynesian model that yields empirically consistent responses of output and inflation to monetary shocks. This allows us to determine quantitatively the relative impact of these various channels. We then focus on the extent of inflation that is needed to avoid a crisis.

Most of the paper considers the case, also analyzed in LW, where the decision to default or not takes place at a known future date T. At that time uncertainty about future fiscal surpluses is resolved. At an initial date 0 a self-fulfilling expectation shock can lead to beliefs of default at time T. Investors then demand a higher yield on new debt, which leads to a more rapid accumulation of debt between the initial period 0 and the default period T. If debt is large enough, default may occur due to insolvency. Monetary policy can be used to relax the solvency constraint both ex ante, before T, and ex post, after T. While a fixed and known T is restrictive, we find that results are little affected in an extension that considers uncertainty about T.

Our starting point is the case where the central bank is passive, keeping prices constant and the output gap at zero. There is a range of debt where countries become subject to self-fulfilling debt crises under such passive policy. The question we ask in this paper is only of interest when parameters are such that this multiplicity range for debt is not negligible (e.g., between 80 and 150% of GDP under the benchmark parameterization). No matter where we are in this range under passive policy, we find that sufficiently aggressive monetary policy can in principle preclude a self-fulfilling debt crisis. However, the policy needs to be credible and therefore not too costly, especially in terms of inflation. We find instead that the policy is very costly. For example, under the benchmark parameterization with an initial debt level in the middle of the multiplicity range (112% of GDP), optimal policy that avoids a self-fulfilling crisis implies that prices ultimately increase by a factor of 5 and the peak annual inflation rate is 24%.

We find that this result holds up quite generally. Whatever we assume about model parameters, optimal monetary policy implies very steep inflation rates as long as (i) there is a substantial multiplicity range for debt under passive policy, (ii) debt is not at the very lower end of this range. This is a result that holds independently of the specifics of the New Keynesian model or assumptions within the LW model.

It is often argued that central banks could simply buy back government debt. But this policy has implications for monetary policy and for inflation and the existing literature has already pointed out the limitations of such policies (e.g., Reis, 2013, talks about the "mystique" of central banks balance sheets). This is also true in the context of our model, even when the economy is at the zero lower bound (ZLB) and the monetary base is expanded beyond the satiation level of money demand.

This is not the first paper to analyze the impact of monetary policy in a self-fulfilling debt crisis environment. Previous work does not consider standard interest rate policies and focuses mainly on stylized two-period models with one period bonds, flexible prices, constant real interest rates and a constant output gap. The question was first analyzed by Calvo (1988), who examined the trade-off between outright default and debt deflation. Corsetti and Dedola (2014) extend the Calvo model to allow for both fundamental and self-fulfilling default. They show that with optimal monetary policy debt crises can still happen, but for larger levels of debt. They also show that a crisis can be avoided if government debt is replaced by central bank debt that is convertible into cash. Reis (2013) and Jeanne (2012) also develop stylized two-period models with multiple equilibria to illustrate ways in which the central bank can act to avoid the bad equilibrium.

Some papers consider more dynamic models, but still assume flexible prices and one-period bonds. Camous and Cooper (2014) use a dynamic overlapping-generation model with strategic default. They show that the central bank can avoid self-fulfilling default if they commit to a policy where inflation depends on the state (productivity, interest rate, sunspot). Aguiar et al. (2013) consider a dynamic model to analyze the vulnerability to self-fulfilling rollover crises, depending on the aversion of the central bank to inflation. Although a rollover crisis occurs suddenly, it is assumed that there is a grace period to repay the debt, allowing the central bank time to reduce the real value of the debt through inflation. They find that only for intermediate levels of the cost of inflation do debt crises occur under a narrower range of debt values.[3]

The rest of the paper is organized as follows. Section 2 presents the slow-moving debt crisis model based on LW. It starts with a real version of the model and then presents its extension to a monetary environment. Subsequently, it analyzes the various channels of monetary policy in this framework. Section 3 describes the New Keynesian part of the model and its calibration. Section 4 analyzes the quantitative impact of monetary policy and Section 6 concludes. Some of the technical details are left to the Appendix, while additional algebraic details and results can be found in a separate Technical Appendix.

2 A Model of Slow-Moving Self-Fulfilling Debt Crisis

In this section we present a dynamic sovereign debt crisis model based on LW. We first describe the basic structure of the model in a real environment. We then extend the model to a monetary environment and discuss the impact of monetary policy on the existence of self-fulfilling debt crises. We focus on the dynamics of

[3] There are recent models that examine the impact of monetary policy in the presence of long-term government bonds. Leeper and Zhou (2013) analyze optimal monetary (and fiscal) policy with flexible prices, while Bhattarai et al. (2013) consider a New Keynesian environment at ZLB. These papers, however, do not allow for the possibility of sovereign default. Sheedy (2014) and Gomes et al. (2014) examine monetary policy with long-term private sector bonds.

asset prices and debt for given interest rates and goods prices. The latter will be determined in a New Keynesian model that we describe in Section 3.

2.1 A Real Model

We consider a simplified version of the LW model. As in the applications considered by LW, there is a key date T at which uncertainty about future primary surpluses is resolved and the government makes a decision to default or not.[4] Default occurs at time T if the present value of future primary surpluses is insufficient to repay the debt. We assume that default does not happen prior to date T as there is always a possibility of large primarily surpluses from T onward. In one version of their model LW assume that T is known to all agents, while in another they assume that it is unknown and arrives each period with a certain probability. We mostly adopt the former assumption. In the Technical Appendix we analyze an extension where T is uncertain, which is briefly discussed in Section 4.3. While this significantly complicates the analysis, it does not alter the key findings.

The only simplification we adopt relative to LW concerns the process of the primary surplus. For now we assume that the primary surplus s_t is constant at $\bar{s}$ between periods 0 and $T-1$. LW assume a fiscal rule whereby the surplus is a function of debt. Not surprisingly, they find that the range of debt where a country is vulnerable to self-fulfilling crises narrows if the fiscal surplus is more responsive to debt. Very responsive fiscal policy could in principle eliminate the concern about self-fulfilling debt crises. In this paper, however, we take vulnerability to self-fulfilling debt crises as given in the absence of monetary policy action. We therefore abstract from such strong stabilizing fiscal policy. However, we will consider an extension where the primary surplus depends on output and is procyclical as this provides an additional avenue through which monetary policy can be effective.

A second assumption concerns the primary surplus value starting at date T. Let $\tilde{s}$ denote the maximum potential primary surplus that the government is able to achieve, which becomes known at time T and is constant from thereon. LW assume

[4] One can for example think of countries that have been hit by a shock that adversely affected their primary surpluses, which is followed by a period of uncertainty about whether and how much the government is able to restore primary surpluses through higher taxation or reduced spending.

that it is drawn from a log normal distribution. Instead we assume that it is drawn from a binary distribution, which simplifies the algebra and the presentation. It can take on only two values: s_{low} with probability ψ and s_{high} with probability $1-\psi$. When the present discounted value of $\tilde{s}$ is at least as large as what the government owes on debt, there is no default at time T and the actual surplus is just sufficient to satisfy the budget constraint (generally below $\tilde{s}$). We assume that s_{high} is big enough such that this is always the case when $\tilde{s} = s_{high}$.[5] When $\tilde{s} = s_{low}$ and its present value is insufficient to repay the debt, the government defaults.

A key feature of the model is the presence of long-term debt. As usual in the literature, assume that bonds pay coupons (measured in goods) that depreciate at a rate of $1-\delta$ over time: κ, $(1-\delta)\kappa$, $(1-\delta)^2\kappa$, and so on.[6] A smaller δ therefore implies a longer maturity of debt. This facilitates aggregation as a bond issued at $t-s$ corresponds to $(1-\delta)^s$ bonds issued at time t. We can then define all outstanding bonds in terms of the equivalent of newly issued bonds. We define b_t as debt measured in terms of the equivalent of newly issued bonds at $t-1$ on which the first coupon is due at time t. As in LW, we take δ as given. It is associated with tradeoffs that are not explicitly modeled, and we do not allow the government to change the maturity to avoid default.

Let Q_t be the price of a government bond. At time t the value of government debt is $Q_t b_{t+1}$. In the absence of default the return on the government bond from t to $t+1$ is

$$R_t^g = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{1}$$

If there is default at time T, bond holders are able to recover a proportion $\zeta < 1$ of the present discounted value s^{pdv} of the primary surpluses s_{low}.[7] In that case the return on the government bond is

$$R_{T-1}^g = \frac{\zeta s^{pdv}}{Q_{T-1} b_T} \tag{2}$$

Government debt evolves according to

$$Q_t b_{t+1} = R_{t-1}^g Q_{t-1} b_t - s_t \tag{3}$$

[5] See Technical Appendix for details.

[6] See for example Hatchondo and Martinez (2009).

[7] One can think of ζ as the outcome of a bargaining process between the government (representing taxpayers) and bondholders. Since governments rarely default on all their debt, we assume $\zeta > 0$.

In the absence of default this may also be written as $Q_t b_{t+1} = ((1-\delta)Q_t + \kappa)b_t - s_t$. The initial stock of debt b_0 is given.

We assume that investors also have access to a short-term bond with a gross real interest rate R_t. The only shocks in the model occur at time 0 (self-fulfilling shock to expectations) and time T (value of $\tilde{s}$). In other periods the following risk-free arbitrage condition holds (for $t \geq 0$ and $t \neq T-1$):

$$R_t = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{4}$$

For now we assume, as in LW, a constant interest rate, $R_t = R$. In that case $s^{pdv} = Rs_{low}/(R-1)$ is the present discounted value of s_{low}. There is no default at time T if s^{pdv} covers current and future debt service at T, which is $((1-\delta)Q_T + \kappa)b_T$. Since there is no default after time T, Q_T is the risk-free price, equal to the present discounted value of future coupons. For convenience it is assumed that $\kappa = R - 1 + \delta$, so that (4) implies that $Q_T = 1$. This means that there is no default as long as $s^{pdv} \geq Rb_T$, or if

$$b_T \leq \frac{1}{R-1} s_{low} \equiv \tilde{b} \tag{5}$$

When $b_T > \tilde{b}$, the government partially defaults on debt, with investors seizing a fraction ζ of the present value s^{pdv}.

This framework may lead to multiple equilibria and to a slow-moving debt crisis, as described in LW. The existence of multiple equilibria can be seen graphically from the intersection of two schedules, as illustrated in Figure 1. The first schedule, labeled "pricing schedule", is a consistency relationship between price and outstanding debt at $T-1$, in view of the default decision that may be taken at T. This is given by:

$$Q_{T-1} = 1 \qquad \text{if } b_T \leq \tilde{b} \tag{6}$$

$$= \psi \frac{\zeta s^{pdv}}{Rb_T} + (1-\psi) \qquad \text{if } b_T > \tilde{b} \tag{7}$$

When $b_T \leq \tilde{b}$, the arbitrage condition (4) also applies to $t = T-1$, implying $Q_{T-1} = 1$. When b_T is just above $\tilde{b}$, there is a discrete drop of the price because only a fraction ζ of primary surpluses can be recovered by bond holders in case of default. For larger values of debt, Q_{T-1} will be even lower as the primary surpluses have to be shared among more bonds.

The second schedule is the "debt accumulation schedule," which expresses the amount of debt that accumulates through time $T-1$ as a function of prices between 0 and $T-1$. Every price Q_t between 0 and $T-1$ can be expressed as a function of Q_{T-1} by integrating (4) backwards from $T-1$ to 0:

$$Q_t - 1 = \left(\frac{1-\delta}{R}\right)^{T-1-t} (Q_{T-1} - 1) \tag{8}$$

Substituting in (3) and integrating the government budget constraint forward from 0 to $T-1$, we get (see Appendix A):

$$b_T = (1-\delta)^T b_0 + \frac{\chi^\kappa \kappa b_0 - \chi^s \overline{s}}{Q_{T-1}} \tag{9}$$

where

$$\begin{aligned} \chi^\kappa &= R^{T-1} + (1-\delta)R^{T-2} + (1-\delta)^2 R^{T-3} + ... + (1-\delta)^{T-1} \\ \chi^s &= 1 + R + R^2 + ... + R^{T-1} \end{aligned}$$

The numerator $\chi^\kappa \kappa b_0 - \chi^s \overline{s}$ in (9) corresponds to the accumulated new borrowing between 0 and T. We assume that it is positive, which happens when the primary surplus is insufficient to pay the coupons on the initial debt. A sufficient, but not necessary, condition is that the primary surplus itself is negative during this time. The debt accumulation schedule then gives a negative relationship between b_T and Q_{T-1}. When Q_{T-1} is lower, asset prices from 0 to $T-2$ are also lower. This implies a higher yield on newly issued debt, reflecting a premium for possible default at time T. These default premia lead to a more rapid accumulation of debt and therefore a higher b_T at $T-1$.

Figure 1 shows these two schedules and illustrates the multiplicity of equilibria. There are two stable equilibria, represented by points A and B. At point A, $Q_{T-1} = 1$. The bond price is then equal to 1 at all times. This is the "good" equilibrium in which there is no default. At point B, $Q_{T-1} < 1$. This is the "bad" equilibrium. Asset prices starting at time 0 are less than 1 in anticipation of possible default at time T. Intuitively, when agents believe that default is likely, they demand default premia (implying lower asset prices), leading to a more rapid accumulation of debt, which in a self-fulfilling way indeed makes default more likely.

In the bad equilibrium there is a slow-moving debt crisis. As can be seen from (8), using $Q_{T-1} < 1$, the asset price instantaneously drops at time 0 and then continues to drop all the way to $T-1$. Correspondingly, default premia gradually rise over time. Such a slow-moving crisis occurs only for intermediate levels of debt. When b_0 is sufficiently low, the debt accumulation schedule is further to the left, crossing below point C, and only the good equilibrium exists. When b_0 is sufficiently high, the debt accumulation schedule is further to the right, crossing above point D, and only a bad equilibrium exists. In that case default is unavoidable when $\tilde{s} = s_{low}$.

2.2 A Monetary Model

We now extend the model to a monetary economy. The goods price level is P_t. R_t is now the gross nominal interest rate and $r_t = R_t P_t / P_{t+1}$ the gross real interest rate. The central bank can set the interest rate R_t and affect P_t. The coupons on government debt are now nominal. The number of bonds at time $t-1$ is B_t and B_0 is given. We define $b_t = B_t / P_t$. The arbitrage equation with no default remains (4), while the government budget constraint for $t \neq T$ becomes

$$Q_t B_{t+1} = ((1-\delta)Q_t + \kappa)B_t - s_t P_t - Z_t \tag{10}$$

where s_t is now the real primary surplus, $s_t P_t$ the nominal surplus, and Z_t is a nominal transfer from the central bank.

The central bank budget constraint is :

$$Q_t B^c_{t+1} = ((1-\delta)Q_t + \kappa)B^c_t + [M_t - M_{t-1}] - Z_t \tag{11}$$

where B^c_t are government bonds held by the central bank and are its sole assets. The value of central bank assets decreases with the depreciation of government bonds and payments Z_t to the treasury. It is increased by the coupon payments and an expansion $M_t - M_{t-1}$ of monetary liabilities.

The balance sheets of the central bank and government are interconnected as most central banks pay a measure of net income (including seigniorage) to the Treasury as a dividend.[8] We will therefore consider the consolidated government budget constraint by substituting the central bank constraint into the government budget constraint:

$$Q_t B^p_{t+1} = ((1-\delta)Q_t + \kappa)B^p_t - [M_t - M_{t-1}] - s_t P_t \tag{12}$$

[8] See Hall and Reis (2013) for a discussion.

where $B_t^p = B_t - B_t^c$ is government debt held by the general public. The consolidated government can reduce debt to the private sector by issuing monetary liabilities $M_t - M_{t-1}$.

Let $\widetilde{m}$ represent accumulated seigniorage between 0 and $T-1$:

$$\widetilde{m} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}} + r_{T-2}\frac{M_{T-2} - M_{T-3}}{P_{T-2}} + ... + r_0 r_1 ... r_{T-2}\frac{M_0 - M_{-1}}{P_0} \tag{13}$$

Similarly, let m^{pdv} denote the present discounted value of seigniorage revenues starting at date T:

$$m^{pdv} = \frac{M_T - M_{T-1}}{P_T} + \frac{1}{r_T}\frac{M_{T+1} - M_T}{P_{T+1}} + \frac{1}{r_T r_{T+1}}\frac{M_{T+2} - M_{T+1}}{P_{T+2}} + ... \tag{14}$$

At time T the real obligation of the government to bond holders is $[(1-\delta)Q_T + \kappa]b_T$. The no-default condition is $b_T^p \leq \tilde{b}$, with the latter now defined as

$$\tilde{b} = \frac{s^{pdv} + m^{pdv}}{(1-\delta)Q_T + \kappa} \tag{15}$$

where

$$s^{pdv} = \left[1 + \frac{1}{r_T} + \frac{1}{r_T r_{T+1}} + ...\right] s_{low} \tag{16}$$

and Q_T is equal to the present discounted value of coupons:

$$Q_T = \frac{\kappa}{R_T} + \frac{(1-\delta)\kappa}{R_T R_{T+1}} + \frac{(1-\delta)^2\kappa}{R_T R_{T+1} R_{T+2}} + ... \tag{17}$$

In analogy to the real model, the new pricing schedule becomes

$$Q_{T-1} = \frac{(1-\delta)Q_T + \kappa}{R_{T-1}} \qquad \text{if } b_T^p \leq \tilde{b} \tag{18}$$

$$= \psi\frac{min\{0, \zeta s^{pdv} + m^{pdv}\}}{R_{T-1}b_T^p} + (1-\psi)\frac{(1-\delta)Q_T + \kappa}{R_{T-1}} \qquad \text{if } b_T^p > \tilde{b} \tag{19}$$

Since m^{pdv} can potentially be negative, in (19) the minimum return in the bad state is set at 0. The new pricing schedule implies a relationship between Q_{T-1} and b_T that has the same shape as in the real model, but is now impacted by monetary policy through real and nominal interest rates, inflation, and seigniorage.

The debt accumulation schedule now becomes (see Appendix A):

$$b_T^p = (1-\delta)^T\frac{B_0^p}{P_T} + \frac{P_{T-1}}{P_T}\frac{\chi^\kappa \kappa B_0^p/P_0 - \chi^s \overline{s} - \widetilde{m}}{Q_{T-1}} \tag{20}$$

where

$$\chi^{\kappa} = \left[r_{T-2}...r_1r_0 + (1-\delta)r_{T-2}...r_1\frac{P_0}{P_1} + (1-\delta)^2 r_{T-2}...r_2\frac{P_0}{P_2} + ... + (1-\delta)^{T-1}\frac{P_0}{P_{T-1}}\right]$$
$$\chi^{s} = 1 + r_{T-2} + r_{T-2}r_{T-3} + ... + r_{T-2}...r_1r_0$$

The schedule again implies a negative relationship between Q_{T-1} and b_T. Monetary policy shifts the schedule through its impact on interest rates, inflation, and seigniorage.

2.3 The Impact of Monetary Policy

Monetary policy affects the paths of interest rates, prices, output and seigniorage, which in turn shifts the two schedules and therefore can affect the existence of self-fulfilling debt crises. The idea is to implement a monetary policy strategy conditional on expectations of sovereign default, which only happens in the crisis equilibrium. If this strategy is successful and credible, the crisis equilibrium is avoided altogether and the policy does not need to be implemented. It is therefore the threat of such a policy that may preclude the crisis equilibrium.

In terms of Figure 1, the crisis equilibrium is avoided when the debt accumulation schedule goes through point C or below. This is the case when

$$\frac{\chi^{\kappa}\kappa B_0^p/P_0 - \chi^s\bar{s} - \widetilde{m}}{s^{pdv} + m^{pdv} - \left((1-\delta)Q_T + \kappa\right)(1-\delta)^T B_0^p/P_T} r_{T-1} \leq \psi\frac{min\{0, \zeta s^{pdv} + m^{pdv}\}}{s^{pdv} + m^{pdv}} + 1 - \psi \tag{21}$$

Note that point C itself is not on the price schedule as its lower section starts for $b_t > \tilde{b}$. It is therefore sufficient that this condition holds as an equality, which corresponds to point C. At point C, $Q_{T-1} < 1$. All prices from 0 to $T-1$ will then be less than one, implying rising default premia that lead to an accumulation of debt. (21) gives a condition for what the central bank needs to do to counteract these rising default premia and avoid default. This condition is key and applies no matter what specific model we assume that relates interest rates, prices and output. We will refer to this as the *default avoidance condition.*

The central bank can impact condition (21) through both *ex ante* policies, taking place between 0 and $T-1$, and *ex post* policies, taking place in period T and afterwards. Ex-ante policies have the effect of shifting the debt accumulation schedule down, while ex-post policies shift the pricing schedule to the right.

Monetary policy can affect the existence of a default equilibrium through inflation, real interest rates, seigniorage and output. Inflation reduces the real value of nominal coupons on the debt outstanding at time 0. Ex-ante policy in the form of inflation prior to time T reduces the real value of coupon payments both before and after T. This is captured respectively through χ^{κ} in the numerator of (21) and the term B_0/P_T in the denominator in (21). Inflation after time T only reduces the real value of coupons after T, which is reflected in a lower value of Q_T in the denominator.

Reducing real interest rates lowers the cost of new borrowing. For ex-ante policy this is captured through both χ^{κ} and χ^{s} in the numerator of (21), which represents the accumulated new borrowing from 0 to T. For ex-post policy it shows up through a rise in s^{pdv} in the denominator of (21).[9] Expansionary monetary policy can also lead to a rise in seigniorage. Seigniorage prior to time T reduces the numerator of the left hand side of (21), while seigniorage after time T raises the denominator. Finally, we will also consider an extension where monetary policy can have a favorable effect through output. If we allow the primary surplus to be pro-cyclical, expansionary monetary policy that raises output will raise primary surpluses.

3 A Basic New Keynesian Model

The default avoidance condition (21) depends on interest rates, prices and output. We now consider a specific New Keynesian model that determines prices and output given interest rates that will be controlled by the central bank. We consider a standard NK model based on Galí (2008, ch. 3), with three extensions suggested by Woodford (2003): i) habit formation; ii) price indexation; iii) lagged response in price adjustment. These extensions are standard in the monetary DSGE literature and are introduced to generate more realistic responses to monetary shocks. The main effect of these extensions is to generate a delayed impact of a monetary policy shock on output and inflation, leading to the humped-shaped response seen in the data.

[9] There is one more subtle real interest rate rate effect, which is specific to the assumption that the central bank knows exactly when the default decision is made. By reducing the real interest rate r_{T-1} the central bank can offset the negative impact of expected default on Q_{T-1}. This is captured through the last term on the left hand side of (21).

3.1 Households

With habit formation, households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - \eta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} - z\iota_t \right) \tag{22}$$

where total consumption C_t is

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{23}$$

and N_t is labor and z is a default cost. We have $\iota_t = 0$ if there is no default at time t and $\iota_t = 1$ if there is default. The default cost does not affect households' decisions, but provides an incentive for authorities to avoid default. Habit persistence, measured by η, is a common feature in NK models to generate a delayed response of expenditure and output.

The budget constraint is

$$\begin{aligned} & P_t C_t + D_{t+1} + Q_t B_{t+1}^p + M_t = \\ & W_t N_t + \Pi_t - f(M_t, Y_t^n) + R_{t-1} D_t + R_{t-1}^g Q_{t-1} B_t^p + M_{t-1} - T_t \end{aligned} \tag{24}$$

Here D_{t+1} are holdings of one-period bonds that are in zero net supply. P_t is the standard aggregate price level and W_t is the wage level. Π_t are firms profits distributed to households and T_t are lump-sum taxes. We will abstract from government consumption, so that the primary surplus is $P_t s_t = T_t$. $f(M_t, Y_t^n)$ is a transaction cost, where $Y_t^n = P_t Y_t$ is nominal GDP and $\partial f / \partial M \leq 0$.

The first-order conditions with respect to D_{t+1} and B_{t+1}^p are

$$\widetilde{C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{25}$$

$$\widetilde{C}_t = \beta E_t R_t^g \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{26}$$

where

$$\widetilde{C}_t \equiv (C_t - \eta C_{t-1})^{-\sigma} - \eta \beta E_t (C_{t+1} - \eta C_t)^{-\sigma}$$

The combination of (25) and (26) gives the arbitrage equations (4), (18), and (19). This is because government default, which lowers the return on government bonds, does not affect consumption due to Ricardian equivalence.[10]

Let Y_t denote real output and c_t, y_t and y_t^n denote logs of consumption, output and the natural rate of output. Using $c_t = y_t$, and defining $x_t = y_t - y_t^n$ as the output gap, log-linearization of the Euler equation (25) gives the dynamic IS equation

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(i_t - E_t\pi_{t+1} - r^n) \tag{27}$$

where

$$\tilde{x}_t = x_t - \eta x_{t-1} - \beta\eta E_t(x_{t+1} - \eta x_t) \tag{28}$$

Here $i_t = ln(R_t)$ will be referred to as the nominal interest rate and $r^n = -ln(\beta)$ is the natural rate of interest. The latter uses our assumption below of constant productivity, which implies a constant natural rate of output.

3.2 Firms

There is a continuum of firms on the interval $[0, 1]$, producing differentiated goods. The production function of firm i is

$$Y_t(i) = AN_t(i)^{1-\alpha} \tag{29}$$

We follow Woodford (2003) by assuming firm-specific labor.

Calvo price setting is assumed, with a fraction $1-\theta$ of firms re-optimizing their price each period. In addition, it is assumed that re-optimization at time t is based on information from date $t-d$. This feature, adopted by Woodford (2003), is in the spirit of the model of information delays of Mankiw and Reis (2001). It has the effect of a delayed impact of a monetary policy shock on inflation, consistent with the data.[11] Analogous to Christiano et al. (2005), Smets and Wouters (2003) and many others, we also adopt an inflation indexation feature in order to generate more persistence of inflation. Firms that do not re-optimize follow the simple indexation rule

$$ln(P_t(i)) = ln(P_{t-1}(i)) + \gamma\pi_{t-1} \tag{30}$$

[10] When substituting the consolidated government budget constraint $Q_tB_{t+1}^p = R_t^gQ_{t-1}B_t^p - (M_t - M_{t-1}) - T_t$ into the household budget constraint (24), and imposing asset market equilibrium, we get $C_t = Y_t$, which is real GDP and unaffected by default. Here we assume that the transaction cost $f(M_t, Y_t^n)$ is paid to intermediaries that do not require real resources and return their profits to households. It is therefore included in Π_t.

[11] This feature can also be justified in terms of a delay by which newly chosen prices go into effect.

where $\pi_{t-1} = ln\ P_{t-1} - ln\ P_{t-2}$ is aggregate inflation one period ago.

Leaving the algebra to the Technical Appendix, these features give the following Phillips curve (after linearization):

$$\pi_t = \gamma\pi_{t-1} + \beta E_{t-d}(\pi_{t+1} - \gamma\pi_t) + E_{t-d}(\omega_1 x_t + \omega_2 \tilde{x}_t) \tag{31}$$

where

$$\begin{aligned}
\omega_1 &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{\phi+\alpha}{1-\alpha+(\alpha+\phi)\varepsilon} \\
\omega_2 &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1-\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}\frac{\sigma}{(1-\eta\beta)(1-\eta)}
\end{aligned}$$

3.3 Money Demand

Most of theresults we report are for a cashless economy. But to consider the additional role of seigniorage we use a convenient form of the transaction cost that gives rise to a standard specification for money demand when $i_t > 0$ ($m_t = ln(M_t)$)[12]:

$$m_t = \alpha_m + p_t + y_t - \alpha_i i_t \tag{32}$$

When i_t is close to zero, money demand reaches the satiation level $\alpha_m + p_t + y_t$. Under conventional monetary policy we assume that money supply does not go beyond the satiation level, so that there is a direct correspondence between the chosen interest rates and money supply.

3.4 Monetary Policy

We follow most of the literature by using a quadratic approximation of utility. Conditional on avoiding the default equilibrium, the central bank then minimizes the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \mu_x (x_t - \nu x_{t-1})^2 + \mu_\pi (\pi_t - \gamma\pi_{t-1})^2 \right\} \tag{33}$$

[12]The transaction cost $f(M_t, Y_t^n) = \alpha_0 + M_t \left(ln\left(\frac{M_t}{P_t Y_t}\right) - 1 - \alpha_m \right) / \alpha_i$ gives rise to money demand (32). This function applies for values of M_t where $\partial f / \partial M \leq 0$. Once the derivative becomes zero, we reach a satiation level and we assume that the transaction cost remains constant for larger M_t.

where ν, μ_x and μ_π a function of model parameters (see the Technical Appendix for the derivation). The central bank chooses the optimal path of nominal interest rates over $H > T$ periods. After that, we assume an interest rate rule as in Clarida et al. (1999):

$$i_t - \bar{\imath} = \rho(i_{t-1} - \bar{\imath}) + (1-\rho)(\psi_\pi E_t \pi_{t+1} + \psi_y x_t) \tag{34}$$

where $\bar{\imath} = -ln(\beta)$ is the steady state nominal interest rate. We will choose H to be large. Interest rates between time T and H involve ex-post-policy.[13]

Optimal policy is chosen conditional on two types of constraints. The first is the ZLB constraint that $i_t \geq 0$ for all periods. The second is the default avoidance condition (21) as an equality.[14]

Using the NK Phillips curve (31), the dynamic IS equation (27), and the policy rule (34) after time H, we solve for the path of inflation and output gap conditional on the set of H interest rates chosen. We then minimize the welfare cost (33) over the H interest rates subject to $i_t \geq 0$ and the no-default condition.

3.5 Calibration

We consider one period to be a quarter and normalize the constant productivity A such that the natural rate of output is equal to 1 annually (0.25 per quarter). The other parameters are listed in Table 1. The left panel shows the parameters from the LW model, while the right panel lists the parameters that pertain to the New Keynesian part of the model.

Consider first the LW parameters. We set $\beta = 0.99$, implying a 4% annualized interest rate. A key parameter, which we will see has an impact on the results, is δ. In the benchmark parameterization we set it equal to 0.05, which implies a government debt duration of 4.2 years. This is typical in the data. For example, OECD estimates of the Macauley duration in 2010 are 4.0 in the US and 4.4 for the average of the five European countries that experienced a sovereign debt crisis (Greece, Italy, Spain, Portugal and Ireland). The coupon is determined such that $\kappa = 1/\beta - 1 + \delta$.

[13] Since H will be large, the precise policy rule after H does not have much effect on the results.

[14] In the good equilibrium $i_t \geq 0$ is the only constraint and the optimal policy implies $i_t = \bar{\imath}$ each period, delivering zero inflation and a zero output gap. However, conditional on a sunspot that could trigger a default equilibrium condition (21) becomes an additional constraint.

The other parameters, T and the fiscal surplus parameters, do not have a direct empirical counterpart, but are chosen so that there is a broad range of self-fulfilling equilibria. The range of B_0 for which there are multiple equilibria under passive monetary policy ($i_t = \bar{\imath}$) is $[B_{low}, B_{high}]$, where[15]

$$B_{low} = \frac{\beta}{1-\beta}\frac{(\psi\zeta + 1 - \psi)\beta^T s_{low} + (1-\beta^T)\bar{s}}{1-(1-\zeta)(1-\delta)^T\beta^T\psi} \tag{35}$$

$$B_{high} = \frac{\beta}{1-\beta}\left(\beta^T s_{low} + (1-\beta^T)\bar{s}\right) \tag{36}$$

As already mentioned, if the range of initial debt B_0 for which multiple equilibria are feasible is very narrow, the entire problem would be a non-issue. This would be the case, for example, if fiscal surplus $\bar{s}$ or s_{low} is very positive or if the recovery rate ζ is close to one.

Under the parameters in Table 1 this range is $[0.79, 1.46]$. This means that debt is between 79% and 146% of GDP. This is not unlike debt of the European periphery hit by the 2010 crisis, where debt ranged from 62% in Spain to 148% in Greece. Note that the assumption $\bar{s} = -0.01$, corresponding to a 4% annual primary deficit, also corresponds closely to Europe, where the five periphery crisis countries had an average primary deficit of 4.4% in 2010.[16] The U.K., U.S. and Japan had even larger primary deficits at that time. We set $T = 20$ for the benchmark, corresponding to 5 years. We will see in section 4.2 that there are other parameter choices that lead to the same values of B_{low} and B_{high} without much effect on results.

The New Keynesian parameters are standard in the literature. The first 5 parameters correspond exactly to those in Gali (2008). The habit formation parameter, the indexation parameter and the parameters in the interest rate rule are all the same as in Christiano et al. (2005). We take $d = 2$ from Woodford (2003, p. 218-219), which also corresponds closely to Rotemberg and Woodford (1997). This set of parameters implies a response to a small monetary policy shock under the Taylor rule that is similar to the empirical VAR results reported by Christiano et al. (2005). The level of output and inflation at their peak correspond exactly to

[15] These values lead to equilibria at points C and D in Figure 1.

[16] It should be clear that even though we are basing some of our calibration on Eurozone crisis countries, we are not trying to match the behavior in these countries. These countries are only of interest in the context of counterfactual analysis of what would happen if an average crisis country had its own monetary policy.

that in the data. Both the output and inflation response is humped shaped like the data, although the peak response (quarter 6 and 3 respectively for inflation and output) occurs a bit earlier than in the data. We discuss the two money demand parameters in section 4.3, where we consider the role of seigniorage.

4 Can Monetary Policy Credibly Avoid a Debt Crisis?

The optimal monetary policy that we have described is credible as long as the welfare cost associated with inflation and non-zero output gaps is less than the cost of default. But rather than comparing the welfare cost to the cost of default, in reporting the results we will mainly focus on the level of inflation needed to satisfy the condition to avoid default under optimal policy. We do so for two reasons. First, the cost of default is hard to measure, including reputational costs, trade exclusion costs, costs through the financial system and political costs. In addition, even within our model the cost of inflation is very sensitive to parameters that otherwise have very little effect on the level of inflation under optimal policy. Second, we will see that the key message that an excessive amount of inflation is needed avoid a self-fulfilling default, is very robust and not affected by parameter assumptions that significantly affect the welfare cost in the model.[17]

We will first consider optimal monetary policy in a cashless economy where we abstract from seigniorage. After considering the benchmark parametrization, we show that the results are robust to significant changes in parameters. We finally consider seigniorage and extensions with a pro-cyclical fiscal surplus and uncertainty about T, none of which change the findings.

[17] At a deeper level, a problem is that there is no consensus on what the exact welfare costs of inflation and output gap are. The welfare costs of inflation depend significantly on the type of price setting (see Ambler (2007) for a discussion of Taylor pricing versus Calvo pricing). The welfare costs of inflation are also broader than the inefficiencies associated with relative price changes that inflation induces. In the model the inflation cost would be zero if all firms raised prices simultaneously. It is also well known that the representative agent nature of the model understates the welfare costs of non-zero output gaps.

4.1 Results under Benchmark Parameterization

Figure 2 shows the dynamics of inflation under optimal policy under the benchmark parameterization for $H = 40$ (which we assume throughout). The results are shown for various levels of B_0. The optimal path for inflation is hump shaped. Optimal inflation gradually rises, both due to rigidities and because the welfare cost (33) depends on the change in inflation. Eventually optimal inflation decreases as it becomes less effective over time when the original debt depreciates and is replaced by new debt that incorporates inflation expectations. When $B_0 = B_{middle} = 1.12$, which is exactly in the middle of range of debt levels giving rise to multiple equilibria, the maximum inflation rate reaches 23.8%. Inflation is over 20% for 4 years, over 10% for 8 years and the price level ultimately increases by a factor 5.3.

Such high inflation is implausible. Inflation needed to avoid default gets even much higher for higher debt levels. When B_0 reaches the upper bound B_{high} for multiple equilibria, the maximum inflation rate is close to 47% and ultimately the price level increases by a factor 25! Only when B_0 is very close to the lower bound for multiplicity, as illustrated for $B_0 = 0.8$, is little inflation needed.

In order to understand why so much inflation is needed, first consider a rather extreme experiment where all of the increase in prices happens right away in the first quarter. This cannot happen in the NK model, so assume that prices are perfectly flexible, the real interest rate is constant at $1/\beta$ and the output gap remains zero. When $B_0 = B_{middle} = 1.12$, the price level would need to rise by 42%. This is needed to lower debt so that we are no longer in the region where multiple equilibria are possible. Of course such a policy, even if possible, is not plausible either as it would involve an annualized inflation rate for that quarter of 168%.

In reality inflation will be spread out over a period of time, both because sticky prices imply a gradual change in prices and because it is optimal from a welfare perspective not to have the increase in the price level happen all at once. However, such a delay increases the ultimate increase in the price level that is needed. As the time zero debt depreciates (is repaid), inflation quickly becomes less effective as it only helps to reduce the real value of coupons on the original time zero debt. More inflation is then needed to avoid the default equilibrium.

Inflation may be limited to the extent that lower real interest rates, by lowering the costs of borrowing, help to avoid the default equilibrium. But the benefit from lower real interest rates turns out to be limited. Under the benchmark parameterization the real interest rate goes to zero for two quarters, since we reach the ZLB and inflation is initially zero, but after that it soon goes back to its steady state. In order to understand why this result is more general than the specific parameterization here, consider the consumption Euler equation, which in linearized form implies (27). It is well known that without habit formation ($\eta = 0$) this can be solved as

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0(r_t - r^n) \tag{37}$$

This precludes a large and sustained drop in the real interest rate as it would imply an enormous and unrealistic immediate change in output at time zero, especially with $\sigma = 1$ as often assumed.

The same point applies when we introduce habit formation, in which case (37) becomes

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0\left(1 - (\beta\eta)^{t+1}\right)(r_t - r^n) \tag{38}$$

For the benchmark parameterization, where $\sigma = 1$ and $\eta = 0.65$, this becomes endix. Removing the expectation operator and the r^n for convenience, we have

$$x_0 = -0.36r_0 - 0.58r_1 - 0.73r_2 - 0.83r_3 - 0.89r_4 - 0.93r_5 - 0.95r_6 - 0.97r_7 - 0.98r_8 - ... \tag{39}$$

Subsequent coefficients are very close to -1. For the path of real interest rates under optimal policy this implies $x_0 = 0.0157$. This translates into an immediate increase in output of 6.3% on an annualized basis, which is already pushing the boundaries of what is plausible.

4.2 Sensitivity Analysis

We now consider changes to both the LW and NK parameters. Several issues arise when changing these parameters. First, changes in the LW parameters change the region $[B_{low}, B_{high}]$ for B_0 under which multiple equilibria arise. For example, when ζ becomes close to 1, $B_{high} - B_{low}$ approaches zero. The same is the case when s_{low} approaches $(1-\beta^T)\bar{s}/[(1-\delta)^{-T} - \beta^T]$. As emphasized in the introduction, the entire question we address has little meaning if there is limited vulnerability even without active monetary policy. Related to that, the "effort" required from the central bank depends on the extent to which B_0/B_{low} is above 1. When the multiplicity region is narrow, B_0/B_{low} is necessarily close to 1. Second, for some NK parameters it is possible to get implausible economic outcomes, leading output to rise at a 25 to 100 APR in the first quarter. As discussed above, this matters for real interest rates. It also matters when we allow the primary surplus to depend on output in an extension below. Overall we find that when the region of multiplicity is broad enough, B_0 is well above B_{low} and the output response is even remotely plausible, optimal monetary policy always implies very steep inflation.

Table 2 shows that changes in the LW parameters significantly affect the $[B_{low}, B_{high}]$ region. One way to control for this is to consider combinations of these parameters that lead to exactly the same region as under the benchmark. The left panel of Figure 3 shows combinations of T, $\bar{s}$ and s_{low} that generate the same B_{low} and B_{high}. The panel on the right shows that this has little effect on the path of optimal inflation. Varying T from 10 to 30, while adjusting $\bar{s}$ and s_{low} to keep B_{low} and B_{high} unchanged, gives very similar paths for optimal inflation.

In Figure 4 and Table 2 we adopt a different approach. We present results when significantly varying one parameter at a time, but keeping $B_0/B_{low} = 1.42$ the same as under the benchmark parameterization. For the LW parameters this implies values of B_0 that can be relatively closer to B_{low} or B_{high}, dependent on their values for that parameter.[18]

Each panel of Figure 4 reports optimal inflation for two values of a parameter, one higher and the other lower than in the benchmark. The last two columns of Table 2 report the price level after inflation and the maximum level of inflation. Figure 4 shows that for most parameters the optimal inflation path is remarkably little affected by the level of parameters. For example, optimal inflation is only slightly higher for $T = 10$ than $T = 30$. When T is low, ex-post policies will be much more important than for higher values of T, but the overall impact on inflation is similar. Also notice that setting the probability ψ of the bad state equal to 1 has little effect on the results.

There are three parameters, δ, γ and d, for which there are more significant differences. A lower debt depreciation δ, which implies a longer maturity of debt, implies lower inflation. The reason is that inflation is effective for a longer period of time as the time 0 debt depreciates more slowly. But even when $\delta = 0.025$, so that the duration is 7.2 years, optimal inflation is still above 10% for 6.5 years and

[18] Only for $\zeta = 0.7$ is B_0 now slightly above B_{high}. For all other parameters the B_0 is within the interval for B_0 generating multiple equilibria.

the price level ultimately triples. A lower value for the lag in price adjustment, d, also allows for a lower inflation rate. With $d = 0$ it is possible to increase inflation from the start, when debt deflation is the most powerful. But even with $d = 0$, optimal inflation still peaks close to 20% and the price level still more than quadruples as a result of years of inflation. No matter what the parameter values, an implausibly high level of inflation is needed to avert a self-fulfilling debt crisis.

Finally, we also see a clear difference when we lower the inflation indexation parameter γ. Lower indexation reduces inflation persistence. But more importantly, it directly affects optimal policy through (33). With $\gamma = 1$, only changes in inflation matter, while with $\gamma < 1$ the level of inflation is also undesirable. To avoid higher inflation levels, the central bank takes advantage of the real interest rate channel to avoid the bad equilibrium. But the sharp drop in the real interest rate leads to an unrealistic output response: with $\gamma = 0.9$, output increases at an annual rate of 24% in the first quarter. The same happens when we set $\gamma = d = \eta = 0$ as in the Gali (2008) textbook model. In that case inflation starts at 23% APR in the first quarter, but the ultimate increase in the price level is now much less, only 66%. Inflation, while still substantial, is again limited in this case because of a sharp drop in real interest rates. There is now an incredulous 25% increase in output in the first quarter, which is a 100% annualized growth rate. As emphasized above, such results have little meaning as the power of monetary policy derives entirely from output changes that are not remotely plausible.

A couple of comments are in order about welfare versus inflation. As already pointed out, the welfare cost is very sensitive to NK parameters even when inflation is little affected. For example, the benchmark case gives a welfare cost of 2.8%, measured as a one year percentage drop in consumption or output that generates the same drop in welfare. This seems quite small. But when we increase θ from 0.66 to 0.8, the welfare cost more than triples to 8.7, with very little difference in optimal inflation. If we adopt the textbook Gali model, where $\gamma = d = \eta = 0$, the welfare cost is a staggering 85% and would be even much larger if changed the model to restrict the massive increase in output in the first quarter.

Even if we substantially changed the NK model, beyond changes in parameters, the central bank still needs to meet the default avoidance condition (21). It is this condition, not the specifics of the NK model, that imposes a very large burden on the central bank. The default avoidance condition (21) depends on inflation and real interest rates. Without specifying a particular NK model, one could consider all possible paths of inflation and real interest rates that are consistent with this condition. We have tried many such paths. The general message is that either inflation is excessive or the real interest rate path implies an output path (using only the intertemporal consumption Euler equation) that is wildly implausible. For space considerations we report just two scenarios in Figure 5.

The first scenario is one where we keep the real interest rate constant at the natural rate, implying that output is also constant when using the consumption Euler equation. The entire burden then falls on inflation. Chart A shows for each level of B_0 in the multiplicity range what *constant* level of inflation (for $H = 40$ quarters) is just sufficient to meet the default avoidance condition (21). Chart B shows the corresponding price level after 40 quarters. When $B_0 = B_{middle} = 1.12$, the constant inflation rate is 13.5%. Since this is sustained for 10 years, the price level rises by a factor 3.5, something that is again more than excessive.

The second scenario reduces the annualized real interest rate in period 0 from 4% (the natural rate) to 0. After that we assume that the gap between real interest rate and the natural rate is multiplied by 0.95 each quarter and we close the remaining gap entirely after 40 quarters. The resulting real interest rate path is shown in chart C. Chart A shows that this significantly reduces the inflation that is needed, to a constant 7.9% APR for 40 quarters when $B_0 = B_{middle} = 1.12$, leading to a rise in the price level by a factor 2.1. Even this is very high though. The price level needs to more than double in ten years. But even if one considers this plausible, the real interest rate path implies a totally implausible output path when using the same dynamic IS equation (27) as before (based only on the intertemporal consumption Euler). Chart 5 shows that output will rise by 16% in one quarter (a 64% annual growth rate) and rises by 34% within one year. No plausible model would generate such output effects of monetary policy. In summary, the key message is that the default avoidance condition (21) cannot be satisfied for a remotely credible path of inflation and plausible path of output.

On may still argue that even though the default avoidance condition does not depend on the NK part of the model, it does depend on the specifics of the LW model. However, echoing the findings above, as long as we keep B_0/B_{low} the same, changing LW parameters leads to results very similar to those reported in Figure 5. The LW structure matters mainly in generating a substantial region of vulnerability. As long as that is in place, precise parameter values do not matter much. The only exception is the bond maturity that relates to δ, which is why we have attempted to stay close to the data in replicating the maturity of government bonds.

4.3 Extensions

4.3.1 Seigniorage

So far we have assumed a cashless economy. In order to consider seigniorage, we need to make an assumption about the semi-elasticity α_i of money demand. Seigniorage revenue is larger for lower values of α_i as that leads to a smaller drop in real money demand when inflation rises. Estimates of α_i vary a lot, from as low as 6 in Ireland (2009) to as high as 60 in Bilson (1978).[19] The biggest effect from seigniorage therefore comes from the lowest value $\alpha_i = 6$. But even in that case the effect is limited. When $B_0 = B_{middle}$, the maximum inflation rate is reduced from 23.8% to 19.9% and the price level ultimately increases by a factor 4.1 instead of 5.3.[20] Here we have assumed that the money supply cannot go beyond the satiation level. There is clearly some benefit from conventional seigniorage, but quantitatively it is small and does not change our conclusion that an excessive amount of inflation is needed to avoid the crisis equilibrium. This result is consistent with Reis (2013) and Hilscher et al. (2014). As Reis (2013) puts it, "In spite of the mystique behind the central bank's balance sheet, its resource constraint bounds the dividends it can distribute by the present value of seigniorage, which is a modest share of GDP".

4.3.2 Pro-cyclical fiscal policy

Nominal rigidities also give the central bank control over the accumulation of debt through the level of output that affects the primary surplus. So far we have abstracted from this channel, but we now introduce a pro-cyclical primary surplus.

[19] Lucas (2000) finds a value of 28 when translated to a quarterly frequency. Engel and West (2005) review many estimates that also fall in this range.

[20] We calibrate α_m to the U.S., such that the satiation level of money corresponds to the monetary base just prior to its sharp rise in the Fall of 2008 when interest rates approached the ZLB. At that time the velocity of the monetary base was 17. This gives $\alpha_m = -1.45$. The velocity is $4P_tY_t/M_t$ as output needs to be annualized, which is equal to $4e^{-\alpha_m}$ at the satiation level.

From 0 through $T-1$ we have

$$s_t = \bar{s} + \lambda(y_t - \bar{y}) \tag{40}$$

where $\bar{y}$ is steady-state output. We similarly assume that s_{low} is pro-cyclical: $s_{low} = \bar{s}_{low} + \lambda(y_t - \bar{y})$. We set the value of the cyclical parameter of the fiscal surplus to $\lambda = 0.1$, in line with empirical estimates.[21]

With this additional effect from an output increase, the required inflation decreases slightly. For $B_0 = B_{middle}$, the maximum inflation rate is reduced from 23.8% in the benchmark to 19.9%. The increase in the price level after inflation is reduced from 5.3 under the benchmark to 4.0, which remains excessive. Optimal policy now gives more emphasis to raising output, leading to a first quarter increase that is 10% APR, pushing the boundary of what is plausible.

4.3.3 Uncertainty about T

In the Technical Appendix we discuss one final extension, uncertainty about the date T of the default decision. This significantly complicates the model and we only consider two possible values, T_1 and T_2, which occur with probabilities p and $1-p$. The key results remain the same. As one might expect, the range for B_0 over which there are multiple equilibria is now in between that for the cases where $T = T_1$ and $T = T_2$ without uncertainty. Monetary policy after T_1 is now contingent on whether there was a default decision at T_1 or not. The key conclusion that an excessive amount of inflation is needed to avoid default (at both T_1 and T_2) remains unaltered.

4.3.4 Unconventional Policy

Leaving details to an earlier draft of this paper, Bacchetta et.al. (2015), one can also consider unconventional monetary policy, defined as an expansion of the money supply beyond the satiation level where we reach the ZLB. This can for example take the form of the central bank buying government debt or providing liquidity support to the government that obviates the need for new government borrowing. In both cases government debt to the private sector is reduced and replaced by monetary liabilities. The present discounted value of seigniorage does not change

[21] Note that since $\bar{Y} = 0.25$ for quarterly GDP, the specification implies that $\Delta s = 0.4\Delta Y$. This is consistent for example with estimates by Girouard and André (2005) for the OECD.

when the money supply increases beyond the satiation level as eventually this expansion needs to be unwound, assuming that the economy cannot be at the ZLB forever.[22] Nonetheless the advantage of this policy is that the consolidated government does not pay default premia on monetary liabilities as it does on government debt in a bad equilibrium.

We find that such policies are only viable if for a substantial period of time, lasting at least through time T, we are at a structural ZLB in that the natural real rate is zero or negative. An expansion of money beyond the satiation level that is unwound prior to time T is not helpful as the saving from not paying default premia is offset by a capital loss associated with a gradual drop in the government bond price. For example, if the consolidated government buys back its bonds at time zero and then at time $T-1$ issues new bonds in order to unwind the monetary liabilities, it buys at a higher price then it sells. This problem does not arise if the monetary liabilities are unwound, and new government bonds sold, after time T. In that case the policy is effective because no default premia are paid on monetary liabilities. But in practice such a policy is only feasible if the natural rate is close to zero. If we are not close to a structural ZLB, so that the natural real rate is well above zero, lowering the nominal interest rate to zero is deflationary. Under flexible prices it implies a negative inflation rate. Under sticky prices deflation tends to come at the cost of a substantial recession.[23]

5 Conclusion

Several recent contributions have derived analytical conditions under which the central bank can avoid a self-fulfilling sovereign debt crisis. Extreme central bank intervention, generating extraordinary inflation, would surely avoid a sovereign debt crisis. But the cost would be excessive, making such actions not credible. To address the credibility of such policies, we have adopted a dynamic model with many realistic elements that allow for an attempt of quantitative assessment. We introduced a New Keynesian model with nominal rigidities in which monetary policy has realistic effects on output and inflation. We introduced long-term bonds

[22] This will be the case when the natural real interest rate eventually becomes positive.

[23] The consumption Euler equation without habit formation implies that $E_0 \sum_{t=1}^{T} \pi_t + \sigma E_0(y_T - y_0) = -Tr^n$ when the nominal interest rate is zero, so that in general there will be a combination of falling prices and a drop in output.

in a slow-moving debt crises model and calibrated the debt maturity to what is observed in many industrialized countries. Overall our conclusion is that, in most cases, the ability of the central bank to avert self-fulfilling crises is limited. Unless debt is close to the bottom of an interval where multiple equilibria occur, monetary policy leads to very high inflation for a sustained period of time.

In the context of eurozone periphery countries, the exercise we conduct in this paper should be thought of as a counterfactual analysis: what could happen if these countries had their own independent central bank? We have not examined the situation where the central bank of a currency union aims to avoid sovereign default in periphery countries of the union. The analysis would be quite different. Specifically, the ECB could buy government bonds of the periphery countries that experience high default premia and sell government bonds of countries that are not subject to a sovereign debt crisis. No monetary liabilities need to be issued in the process, generating no inflation.

The ECB could keep interest rates on new debt of the periphery governments equal to their no-default levels and buy all new bonds that would otherwise be sold to the private sector at that low interest rate. The threat alone of doing so is sufficient, which is, in our view, what happened under the OMT policy in the summer of 2012 and the famous Draghi statement "to do whatever it takes". Such a threat was credible as such an intervention would not overwhelm the ECB.[24] This explains why sovereign spreads quickly fell due to the change in policy. But such a policy applies to a periphery and is of no help if a central bank aims to avoid a self-fulfilling sovereign debt crisis associated with its central government. Analogously, it would not work if the ECB aimed to avoid a self-fulfilling sovereign debt crisis across the entire Eurozone.

Several extensions are worthwhile considering for future work. We have focused on a closed economy. In an open economy monetary policy also affects the exchange rate, which affects relative prices and output. Moreover, we have only considered one type of self-fulfilling debt crises, associated with the interaction between sovereign spreads and debt. It would be of interest to also consider rollover crises or even a combination of both types of crises. This also provides an oppor-

[24]For example, in 2010 the sum of all the periphery country government deficits together (Greece, Ireland, Portugal, Spain, Italy) amounted to 13% of the ECB balance sheet. And a self-fulfilling default can be avoided even if only a portion of these financing needs are covered by the ECB.

tunity to consider the optimal maturity of sovereign debt, which we have taken as given.

Appendix

A. Derivation of the Debt Accumulation Schedule.

We derive the debt accumulation schedule in the general case of Section 2.4. The debt accumulation schedule in the cashless economy (Section 2.2) is a special case of this where $M_t = 0$ at all times. We first derive a relationship between Q_0 and Q_{T-1}. Integrating forward the one-period arbitrage equation (4) from $t = 1$ to $t = T - 1$, we have:

$$Q_0 = A^{\kappa}\kappa + A^{Q}Q_{T-1} \tag{41}$$

where

$$A^{\kappa} = \frac{1}{R_0} + \frac{1-\delta}{R_0R_1} + \frac{(1-\delta)^2}{R_0R_1R_2} + ... + \frac{(1-\delta)^{T-2}}{R_0R_1R_2...R_{T-2}} \tag{42}$$

$$A^{Q} = \frac{(1-\delta)^{T-1}}{R_0R_1R_2...R_{T-2}} \tag{43}$$

Next consider the consolidated budget constraint (12):

$$Q_tB_{t+1}^p = ((1-\delta)Q_t + \kappa)B_t^p - \upsilon_t - s_tP_t \tag{44}$$

where $\upsilon_t = [M_t - M_{t-1}]$. The government budget constraint at $t = 0$ is:

$$\frac{Q_0B_1^p}{P_0} = ((1-\delta)Q_0 + \kappa)\, b_0^p - \overline{s} - \frac{\upsilon_0}{P_0} \tag{45}$$

For $1 < t < T$

$$\frac{Q_tB_{t+1}^p}{P_t} = r_{t-1}\frac{Q_{t-1}B_t^p}{P_{t-1}} - \overline{s} - \frac{\upsilon_t}{P_t} \tag{46}$$

Using equations (46) and (45) and integrating forward, we obtain

$$\begin{aligned} \frac{Q_{T-1}B_T^p}{P_{T-1}} &= r_{T-2}...r_1r_0\frac{Q_0B_1^p}{P_0} - \overline{s}(1 + r_{T-2} + r_{T-2}r_{T-1} + ... + r_{T-2}r_{T-1}...r_1) \\ &- \left[r_1...r_{T-2}\frac{\upsilon_1}{P_1} + r_2...r_{T-2}\frac{\upsilon_2}{P_2} + ... + \frac{\upsilon_{T-1}}{P_{T-1}}\right] \end{aligned} \tag{47}$$

Combining equation (47) with (45) and (41), we obtain:

$$\begin{aligned} \frac{Q_{T-1}B_T^p}{P_{T-1}} = \; & r_{T-2}...r_1r_0(1-\delta)b_0^pQ_0 + r_{T-2}...r_1r_0\kappa b_0^p \\ & -\overline{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + ... + r_{T-2}...r_1r_0) \\ & - \left[r_0...r_{T-2}\frac{\upsilon_0}{P_0} + r_1...r_{T-2}\frac{\upsilon_1}{P_1} + ... + \frac{\upsilon_{T-1}}{P_{T-1}}\right] \end{aligned} \tag{48}$$

Using equations (41)-(43), we can rewrite equation (48) as

$$\begin{aligned}\frac{Q_{T-1}B_T^p}{P_{T-1}} &= \frac{P_0}{P_{T-1}}(1-\delta)^T b_0^p Q_{T-1} \qquad (49)\\ &+r_{T-2}...r_1r_0\left[1+\frac{1-\delta}{R_0}+\frac{(1-\delta)^2}{R_0R_1}+...+\frac{(1-\delta)^{T-1}}{R_0R_1R_2...R_{T-2}}\right]\kappa b_0^p\\ &-\overline{s}(1+r_{T-2}+r_{T-2}r_{T-3}+...+r_{T-2}...r_1r_0)\\ &-\left[r_0...r_{T-2}\frac{v_0}{P_0}+r_1...r_{T-2}\frac{v_1}{P_1}+...+\frac{v_{T-1}}{P_{T-1}}\right]\end{aligned}$$

Using the expression for v_t, the last term in brackets is equal to $\tilde{m}$ as defined in (13). This yields (20).

B. Avoiding Self-fulfilling Default at a Structural ZLB

Assume that the discount rate β is 1 for $t = 0,..,\bar{T}$ with $\bar{T} \geq T-1$ and it is a constant $\beta < 1$ for $t > \bar{T}$. Under this assumption the central bank can keep the interest rate zero through time $\bar{T}$, and raise it to $1/\beta$ after time $\bar{T}$, while keeping inflation and the output gap at zero all along. So we have $R_t = 1$ for $t = 0,..,\bar{T}$ and $R_t = 1/\beta$ for $t > \bar{T}$. The price level is always 1. We then have $Q_T = \kappa\sum_{i=0}^{\bar{T}-T}(1-\delta)^i + (1-\delta)^{\bar{T}-T+1}$.

Using (**??**) we have $\tilde{m} = M_{T-1} - M_{-1}$. Let $\bar{M}$ be the level of money demand starting at $\bar{T}+1$, when we are no longer at the ZLB. Then $m^{pdv} = \bar{M} - M_{T-1}$. Define $\Delta M = M_{T-1} - M_{-1}$ and $dm = \bar{M} - M_{-1}$. Then $\tilde{m} = \Delta M$ and $m^{pdv} = dm - \Delta M$. Assuming that we are already at the ZLB at time -1, dm is negative.

Using the results from section 2.4, the pricing schedule is then

$$\begin{aligned} Q_{T-1} &= (1-\delta)Q_T + \kappa & \text{if } B_T^p \leq \tilde{b} \qquad (50)\\ &= \psi h(\Delta M) + (1-\psi)((1-\delta)Q_T + \kappa) & \text{if } B_T^p > \tilde{b} \qquad (51)\end{aligned}$$

where $h(\Delta M) = 0$ if $\zeta s^{pdv} + dm - \Delta M \leq 0$ and otherwise $h(\Delta M) = \frac{\zeta s^{pdv}+dm-\Delta M}{B_T^p}$, and

$$\tilde{b} = \frac{s^{pdv} + dm - \Delta M}{(1-\delta)Q_T + \kappa} \qquad (52)$$

The debt accumulation schedule is

$$B_T^p = (1-\delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\overline{s} - \Delta M}{Q_{T-1}} \qquad (53)$$

Passive monetary policy takes the form $\Delta M = 0$. The central bank then does not expand the money supply between -1 and $T-1$. Under passive monetary policy there are multiple equilibria when B_0 is within a range that we have called $[B_{low}, B_{high}]$. The condition $B_0 \leq B_{high}$ implies that the debt accumulation schedule crosses at or below $(1-\delta)Q_T + \kappa$ when $B_T^p = \tilde{b}$ (point D in Figure 1) and $\Delta M = 0$. This can be written as

$$\frac{\chi^\kappa \kappa B_0^p - T\bar{s}}{\tilde{b} - (1-\delta)^T B_0^p} \leq (1-\delta)Q_T + \kappa \tag{54}$$

Substituting the expression for $\tilde{b}$ with $\Delta M = 0$, this becomes

$$s^{pdv} + dm - (1-\delta)^T B_0^p((1-\delta)Q_T + \kappa) \geq \chi^\kappa \kappa B_0^p - T\bar{s} \tag{55}$$

Now assume that $\Delta M \geq \chi^\kappa \kappa B_0^p - T\bar{s}$ conditional on a sunspot shock. We will show that this is a sufficient condition to avoid the bad equilibrium. If there were a bad equilibrium with default, we know from the pricing schedule that $Q_{T-1} < (1-\delta)Q_T + \kappa$. In that case the debt accumulation schedule, together with the assumption $\Delta M \geq \chi^\kappa \kappa B_0^p - T\bar{s}$, implies

$$B_T^p \leq (1-\delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\bar{s} - \Delta M}{(1-\delta)Q_T + \kappa} \tag{56}$$

It can be shown that the right hand side is less than or equal to $\tilde{b}$, so that $B_T^p \leq \tilde{b}$ and there cannot be a default equilibrium. The condition that the right hand side is less than $\tilde{b}$ is

$$(1-\delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\bar{s} - \Delta M}{(1-\delta)Q_T + \kappa} \leq \frac{s^{pdv} + dm - \Delta M}{(1-\delta)Q_T + \kappa} \tag{57}$$

Multiplying both sides by $(1-\delta)Q_T + \kappa$, we can rewrite this as (55), which holds if there are multiple equilibria under passive monetary policy.

Notice that $\Delta M = \chi^\kappa \kappa B_0^p - T\bar{s}$ is exactly the amount that implements the liquidity support policy described in Section 5. Alternatively, ΔM could be used to buy outstanding debt in the secondary market at any time before time T. The latter would be the buyback policy, also described in Section 5.

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Table 1 Calibration			
Lorenzoni-Werning parameters		New Keynesian parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	$\sigma = 1$	elasticity of intertemporal subsitution
$\delta = 0.05$	coupon depreciation rate	$\phi = 1$	Frisch elasiticity
$\kappa = 0.06$	coupon	$\varepsilon = 6$	demand elasticity
$T = 20$	quarters before default decision	$\alpha = 0.33$	capital share
$\zeta = 0.5$	recovery rate	$\theta = 0.66$	Calvo pricing parameter
$\psi = 0.95$	probability low surplus state	$\eta = 0.65$	habit parameter
$s_{low} = 0.02$	low state surplus	$\gamma = 1$	indexation parameter
$\overline{s} = -0.01$	surplus before T	$d = 2$	lag in price adjustment
		$\rho = 0.8$	persistence in interest rate rule
		$\psi_\pi = 1.5$	inflation parameter in interest rule
		$\psi_y = 0.1$	output parameter in interest rule

Table 2 Sensitivity Analysis				
Parameters	B_{low}	B_{high}	maximum inflation	price level after inflation
Benchmark	0.79	1.46	23.8	5.3
Lorenzoni-Werning Parameters				
$T = 10$	1.15	1.71	25.8	6.3
$T = 30$	0.56	1.22	21.4	4.5
$\delta = 1/40$	0.89	1.46	15.1	3.0
$\delta = 1/10$	0.71	1.46	37.5	12.0
$\bar{s} = -0.02$	0.58	1.28	23.2	5.1
$\bar{s} = 0$	1.00	1.64	23.9	5.3
$s_{low} = 0.01$	0.28	0.63	23.2	5.1
$s_{low} = 0.03$	1.27	2.25	23.7	5.3
$\zeta = 0.3$	0.46	1.46	26.4	6.1
$\zeta = 0.7$	1.08	1.46	21.3	4.5
$\psi = 0.7$	0.98	1.46	22.0	4.7
$\psi = 1$	0.75	1.46	24.0	5.4
$\beta = 0.98$	0.61	1.17	26.7	6.7
$\beta = 0.995$	0.90	1.62	21.8	4.6
New Keynesian Parameters				
$\theta = 0.5$	0.79	1.46	23.7	5.0
$\theta = 0.8$	0.79	1.46	22.9	5.5
$\eta = 0$	0.79	1.46	22.3	4.9
$\eta = 0.8$	0.79	1.46	23.8	5.3
$\epsilon = 4$	0.79	1.46	24.1	5.3
$\epsilon = 8$	0.79	1.46	23.2	5.2
$d = 0$	0.79	1.46	19.6	4.2
$d = 4$	0.79	1.46	29.6	7.2
$\sigma = 2$	0.79	1.46	22.7	5.0
$\sigma = 0.5$	0.79	1.46	24.4	5.4
$\phi = 2$	0.79	1.46	23.6	5.3
$\phi = 0.5$	0.79	1.46	23.7	5.2
$\gamma = 0.9$	0.79	1.46	18.0	3.2
$\gamma = \eta = d = 0$	0.79	1.46	23.0	1.7

Figure 1 Multiple Equilibria Lorenzoni-Werning Model

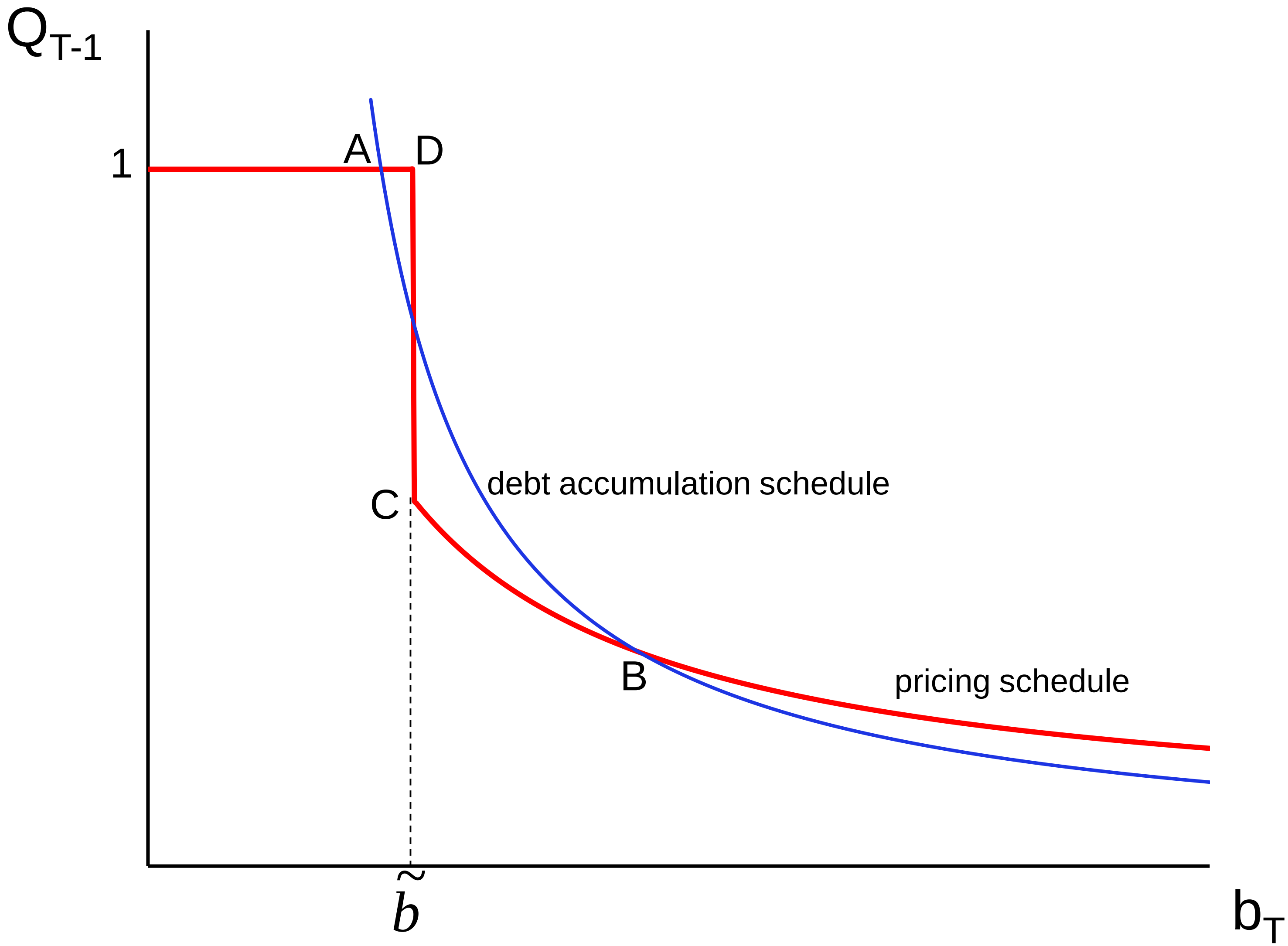


Figure 2 Benchmark NK Model: Inflation Needed to Avoid Default

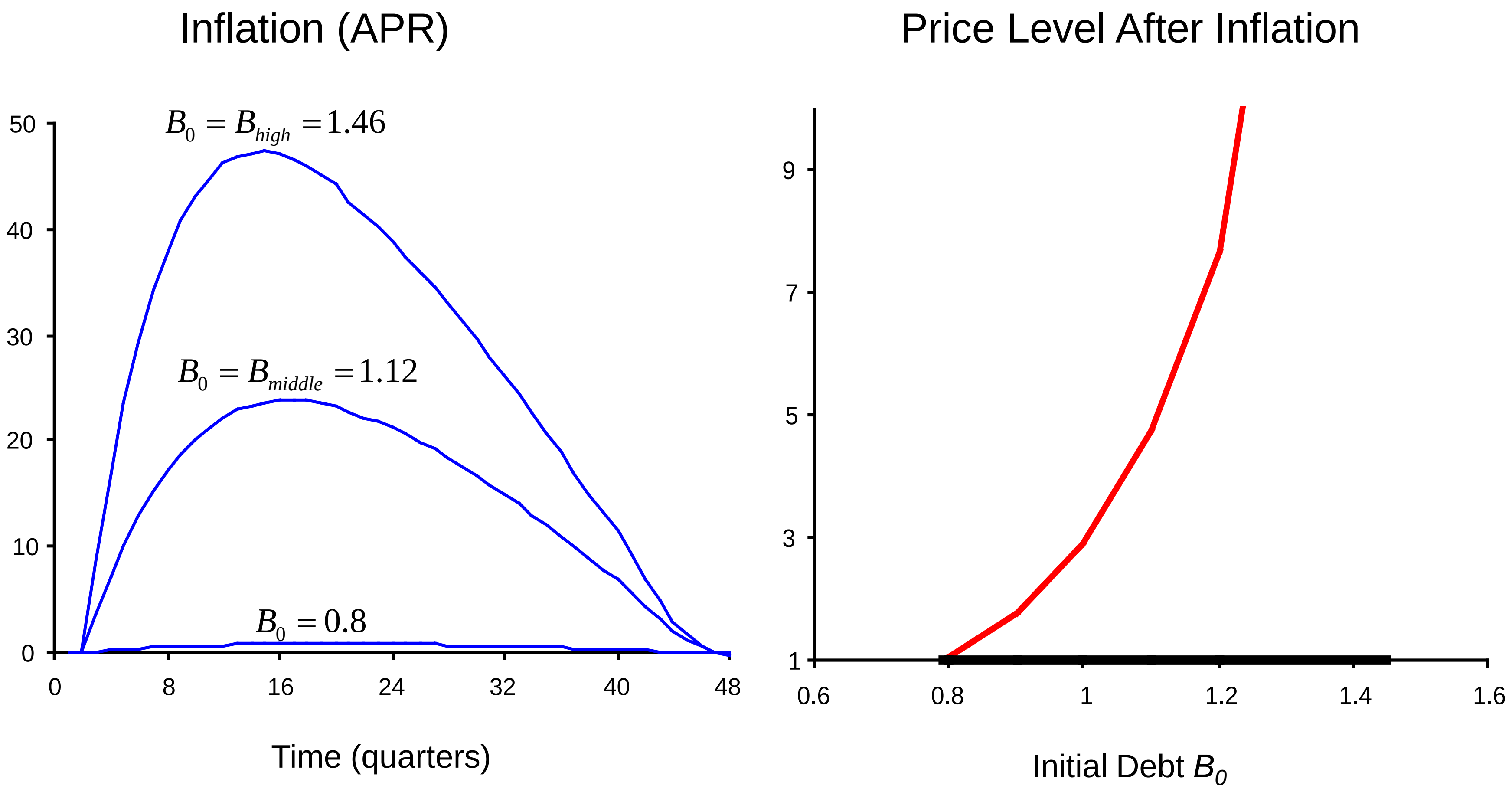
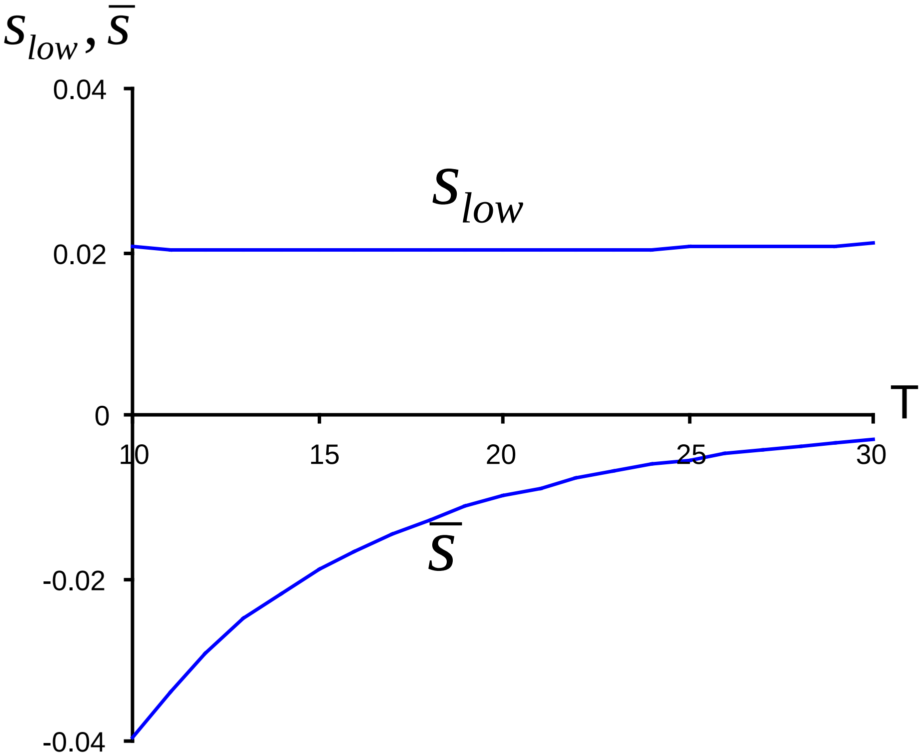


Figure 3 Sensitivity Analysis LW Parameters

$\mathrm{T}, \bar{\mathrm{s}}$ and $\mathrm{s}_{\mathrm{low}}$ for same $[\mathrm{B}_{\mathrm{low}}, \mathrm{B}_{\mathrm{high}}]$



Inflation when $B_0 = B_{middle} = 1.12$

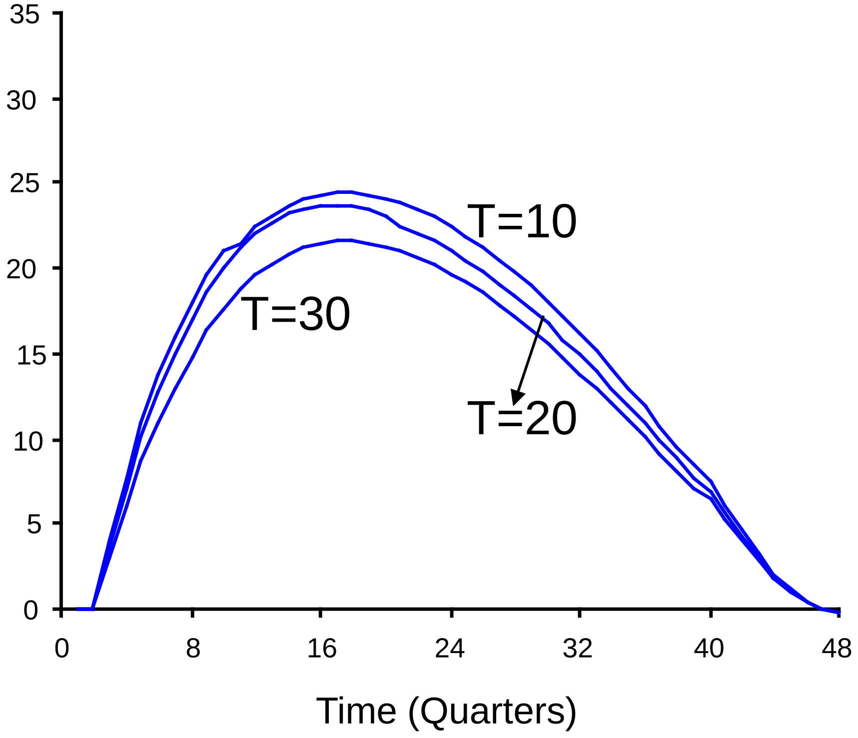


Figure 4 Sensitivity Analysis Optimal Inflation (B_0/B_{low}=1.42)

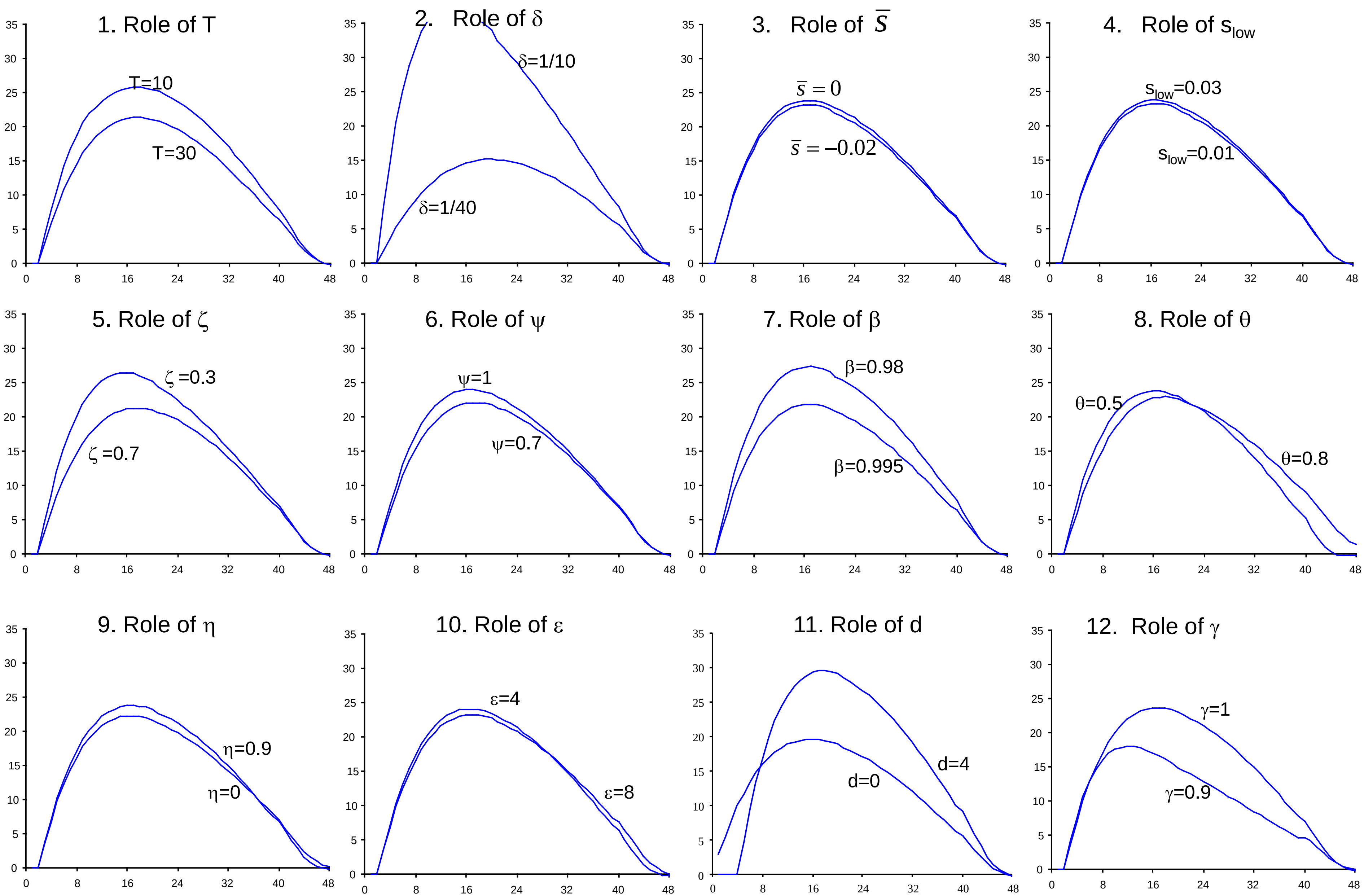


Figure 5 Constant Inflation Needed to Avoid Default*

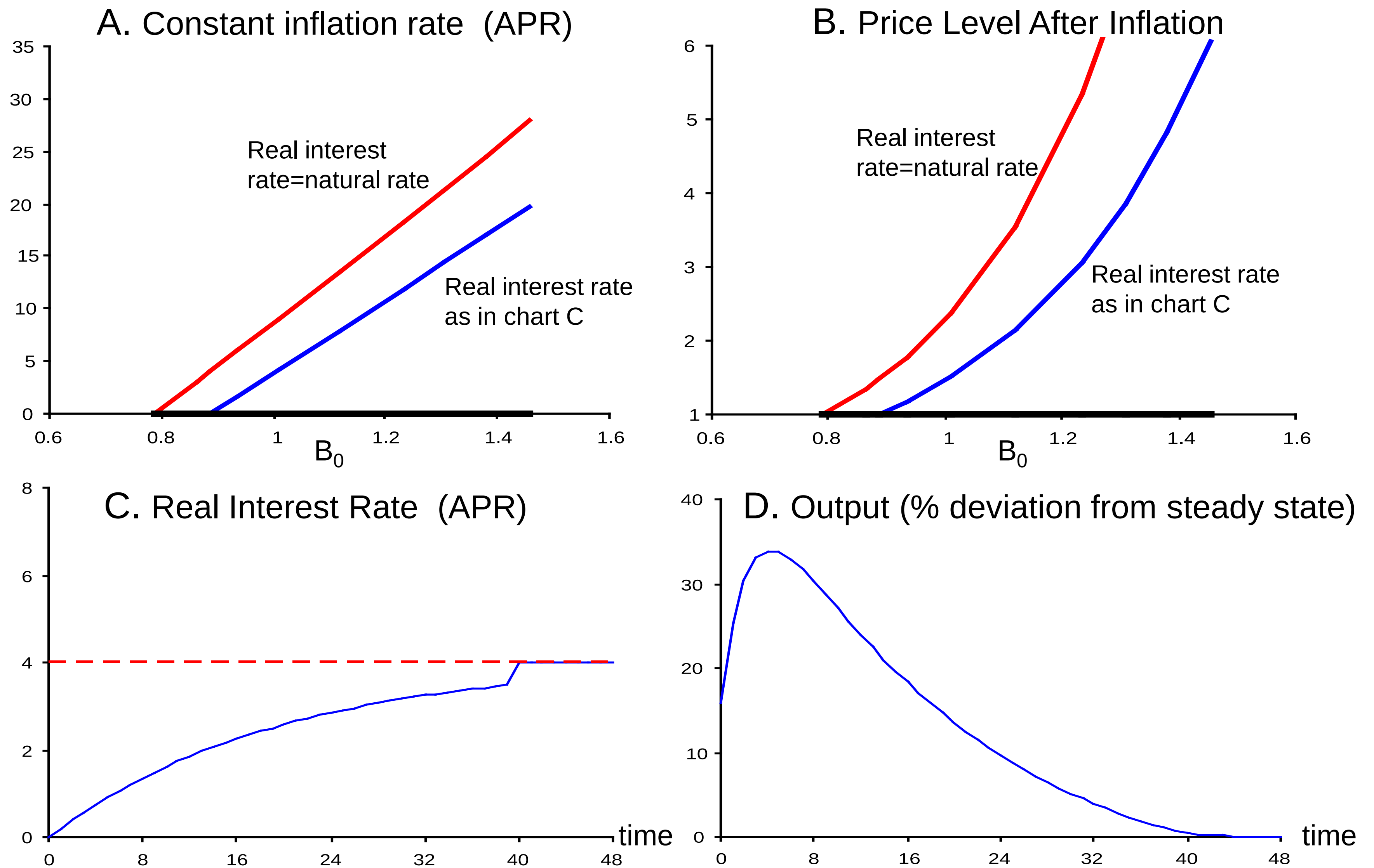


*Chart A shows the constant inflation rate (40 quarters) that avoids default for different B_0, assuming either that the real rate remains equal to the natural rate or that the real rate is significantly reduced as Chart C. Chart B shows the corresponding increase in the price level after 40 quarters. Chart D shows the output path that corresponds to the real interest rate path from chart C, using the dynamic IS equation.