DISCUSSION PAPER SERIES

IZA DP No. 3840

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November 2008

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Discussion Paper No. 3840
November 2008

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ABSTRACT

Unemployment Dynamics and the Cost of Business Cycles*

In this paper, we investigate whether business cycles can imply sizable effects on average unemployment. First, using a reduced-form model of the labor market, we show that job finding rate fluctuations generate intrinsically a non-linear effect on unemployment: positive shocks reduce unemployment less than negative shocks increase it. For the observed process of the job finding rate in the US economy, this intrinsic asymmetry is enough to generate substantial welfare implications. This result also holds when we allow the job finding rate to be endogenous, provided the structural model is able to reproduce the volatility of the job finding rate. Moreover, the matching model embeds other non-linearities which alter the average job finding rate and so the business cycle cost.

JEL Classification: E32, J64

Keywords: business cycle costs, unemployment dynamics

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* We thank Guy Laroque, Etienne Lehmann and all participants at Crest seminar (2008) and Gains seminar (2008) for helpful comments. Any errors and omissions are ours.

1 Introduction

In a very famous and controversial article, Lucas (1987) argues that the costs of business cycles are negligible: using empirically plausible values for risk aversion, he shows that individuals would only sacrifice a mere 0.008% of their consumption to get rid of all aggregate variability in consumption. This leads him to argue that further improvements in stabilization policies are not the priority. This claim is completely at odds with the neoclassical synthesis and the Keynesian legacies, but also with conventional wisdom. In subjective data studies, aggregate unemployment volatility undermines the household's perceived well-being (Wolfers (2003)). Survey data also suggest substantial benefits from improved stabilization policies (Shiller (1997)).

In this paper, we investigate whether the effects of business cycles on average consumption can imply sizable welfare costs. Business cycles may affect average consumption if aggregate shocks affect the economy asymmetrically: the consumption losses during recessions could outweigh the gains during economic booms. This implies the existence of some non-linearities in the propagation of business cycle shocks : the process of alternate expansions and contractions could lead in itself to a lower average consumption. On the other hand, shocks hitting the economy are assumed to be symmetric, even if there are no reasons to think they have to be, but it is more satisfactory to explain the asymmetry as the result of the economic structure. It is the propagation mechanism that explains the asymmetry and not the impulsion itself.

Our objective in this paper is to show that the matching unemployment theory embodies strong non-linearities which may change our view of business cycles. First, the unemployment dynamics depend non-linearly on the job finding and the job separation rates. Secondly, the canonical matching approach also predicts that the job finding rate depends itself in a non-linear way on the shocks hitting the fundamentals of the economic structure like productivity shocks. This paper aims to show that these non linearities lead to sizable welfare costs which have been surprisingly ignored despite numerous studies of the matching labor market approach.

Following Cole and Rogerson (1999), our analysis is firstly based on the reduced form of the matching model where the job finding and separation rates are considered as exogenous stochastic processes. The use of a reduced form model has the advantage of making our results robust to several changes to the standard matching model and of easily unveiling the asymmetric impact of job finding and separation rate fluctuations on unemployment. During booms, the increase in the job finding rate is partly offset by the decrease in unemployment. In recessions, the decrease in the job finding rate is amplified by the increase in unemployment. Because of this asymmetry, average unemployment is increased by fluctuations in the job finding rate. Conversely, fluctuations in the job separation rate lower the average unemployment rate[1]. We show that

[1] During booms, the decrease in the job separation rate is amplified by the increase in employment. In recessions, the increase in the job separation rate is compensated by the decrease in employment.

these asymmetries explain why the volatility and the persistence of the job finding and separation rate processes are potentially key variables in the analysis of business cycle costs. More surprisingly, we find that the welfare costs of fluctuations also depend on the structural level of unemployment. Economies in which the steady state unemployment rate is high are economies suffering from high costs of fluctuations. For the observed process of the job finding rate in the US economy, we show that the asymmetry quantitatively matters and generates sizable business cycle costs. Conversely, the observed volatility of the separation rate is too small for this non linearity to manifest: the volatility of the job separation rate has virtually no impact on average unemployment.

However, the reduced form analysis gives a correct estimate of the costs of fluctuations only if stabilization does not have any effect on the average job finding rate. Indeed, in the reduced form model, we assume that the counterfactual stabilized job finding rate is equal to the average job finding rate. But what if fluctuations also modify the average job finding rate? A structural model is then necessary to take into account how the shocks hitting the fundamentals of the economy affect the job finding rate. We consider a canonical matching model with aggregate productivity shocks. This model naturally predicts that the job finding rate is a concave function of the vacancy-unemployment ratio. This could imply that fluctuations decrease the average job finding rate. But as the vacancy-unemployment ratio is a convex function of productivity, the impact of productivity fluctuations on the average job finding rate is a priori ambiguous.

A prerequisite for quantifying these different effects is that the structural model needs to match the observed characteristics of the job finding rate. Since Shimer (2005), it is well known that the standard matching model fails to generate realistic fluctuations in the job finding rate. Productivity shocks cause strong movements in wages that offset the firm's incentive to change hiring, thus dampening the variations in the job finding rate. With such a mechanism, the welfare cost of fluctuation nearly disappears. Hall (2005) and Hall and Milgrom (2008) show that including wage rigidities can help the model to match the volatility of the job finding rate. This modification increases the sensitivity of the model to productivity shocks, but at the expense of the observed flexibility in wages Pissarides (2007). Other routes have been followed to elucidate the Shimer puzzle. Hagedorn and Manovskii (2008) show that a higher parametrization for the utility of being unemployed and a lower bargaining power for workers enable the standard matching model to yield realistic fluctuations in the unemployment rate. Whereas Hornstein and Violante (2007) introduce investment-specific technological shocks, Kennan (2006) emphasizes the role of procyclical informational rents: the gain that firms obtain by being more informed than workers increases in booms. The inclusion of turnover costs (Pissarides (2007), Mortensen and Nagypal (2003) and Silva and Toledo (2008)) and match-specific technological change (Costain and Reiter (2008)) have also been investigated.

We show, in the simple framework with wage rigidities proposed by Hall (2005), that the non-linearities embedded into the matching model may cause the business cycles to influence the average job finding rate. This framework allows us both to generate enough job finding rate volatility and to focus only on the basic non-linearity due to the matching function. It then appears that the matching elasticity to vacancy plays a crucial role in the size of the business cycle costs. The internal mechanisms of the matching model lead to quite strong additional business cycle costs when the vacancy elasticity of the matching function is sufficiently low. Considering a version of the matching model with flexible wages and fixed hiring costs (Pissarides (2007)) allows us to generalize this property and to show that the type of non-linearities is naturally dependent on the structural model. Different approaches to solving the Shimer puzzle potentially lead to different stabilized job finding rates, and imply therefore different evaluations of the cost of business cycles.

All in all, these results challenge Lucas's controversial result on business cycle costs by exploring an original mechanism. The literature following Lucas (1987) has mostly focused on the consequences of business cycles on the volatility of individual consumption. More precisely, because business cycles amplify individual income risks, they could generate higher welfare losses than Lucas's predictions when financial markets are incomplete. However, in as far as individual income fluctuations are transitory, the costs of business cycle are still low, even negligible Krusell and Smith (1999), mainly because consumption can be smoothed through capital accumulation. On the other hand, when individual income variations are permanent, and then non self-insurable, the cost of business cycles becomes substantial (Beaudry and Pages (1999), Krebs (2007)). In the literature, very few papers focus on the consequences of business cycles on average consumption. This idea has been sketched out by De Long and Summers (1988); they argue that rather than steadying economic activity at its average level, stabilization would prevent output from deviating from its potential level. Ramey and Ramey (1993) explore this mechanism in a model where firm have to pre-commit to a specific technology before starting production. In this context, stabilization enhances welfare by increasing the efficiency of production. Stabilization may also increase welfare through its effect on capital accumulation (Matheron and Maury (2000) and Epaulard and Pommeret (2003)). Barlevy (2004) shows that the business cycle costs become sizable when returns to investment are decreasing. Eliminating fluctuations reallocates investment from periods where the marginal return to investment is low to periods where this return is high, and therefore leads to a higher growth rate of consumption.

The paper is organized as follows. In the second section, we use a reduced form of the matching model to investigate the consequences of the non-linearity in the unemployment dynamics. Given the observed processes of the job separation and finding rates, we then derive its implication for the costs of business cycles. The third section takes into account the non-linearity in the job finding rate dynamics embodied into the matching model. The last section concludes.

2 Asymmetry in the unemployment dynamics: a reduced form approach

We believe that labor market frictions naturally generate asymmetries in the unemployment dynamics. Because of these asymmetries, aggregate fluctuations may have an impact on average unemployment. We first present our theoretical framework, and then analyze the unemployment dynamics. Finally, a quantitative evaluation of the business cycle costs is proposed.

2.1 Framework

Following Cole and Rogerson (1999), our theoretical framework is based on the reduced form of the matching model. We consider unemployment dynamics as the result of exogenous job separation and job finding fluctuations. By shutting down any non linearities that may affect the job finding and separation rates, the reduced form model allows us to focus on the non linearity embedded in the unemployment dynamics. Moreover, in order to isolate the effect of the labor market non-linearities on the business cycle costs, we eliminate any non-linearities in the relationship between employment and production on the one hand and between production and consumption on the other hand.

Unemployment. The unemployment dynamics arise from the entries to and exits from employment. The former are determined by the job finding rate p, the latter by the separation rate s.

$$u_{t+1} = s_t(1-u_t) + (1-p_t)u_t \tag{1}$$

Shocks. The economy is hit only by aggregate shocks which generate some fluctuations in the job finding rate p_t and in the separation rate s_t. The job finding rate and separation rate are exogenous with respect to u_t. This key assumption derives from the matching theory. The aggregate shocks affect linearly[2] both the job finding rate and the separation rate which are assumed to follow an AR(1) process:

$$p_t = (1-\rho_p)\bar{p} + \rho_p p_{t-1} + \varepsilon_t^p \tag{2}$$

$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \varepsilon_t^s \tag{3}$$

The shocks ε^p and ε^s have a zero-mean and a standard deviation equal to σ_{ε^p} and σ_{ε^s} respectively. $\bar{p}$ and $\bar{s}$ denote the average job finding rate and the average separation rate respectively.

Preferences. Workers are assumed to be risk-neutral. This assumption allows us to focus on the effects of business cycles on average consumption. We ignore the welfare cost induced

[2]We assume symmetrical shocks in order to identify the endogenous asymmetries generated by equation (1). We show in Appendix D that our results are not sensitive to this hypothesis.

by individual and aggregate consumption volatilities. Moreover, we choose to consider that the disutilities of working and of not working are both equal[3] to the same value χ. It is then straightforward to derive the representative agent intertemporal preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - \chi)$$

Production. Labor is the only factor of production. How employment variations lead to variations in aggregate production depends on the assumptions made on the returns to scale or the market structure between the final and intermediate goods. In order to isolate the impact of the labor market non-linearities, we choose an aggregate production function linear in employment $y(1-u_t)$ with y the average labor productivity, considered as constant in this reduced form analysis[4].

Consumption. Agents consume all their income[5]. Aggregate consumption is therefore equal to:

$$c_t = y(1-u_t)$$

There are no savings in our economy. This simplifying assumption is not restrictive as our results do not rely on the impossibility of individuals to smooth their income. As agents are risk-neutral, the excessive volatility of consumption implied by this assumption is not captured in the welfare calculations. The focus is here on the impact of business cycles on average consumption, which is not affected by smoothing behaviors.

The welfare cost of fluctuations. Following Lucas (1987), we compute the welfare cost of fluctuations as the cost of being in an economy hit by aggregate shocks, rather than being in an economy without aggregate shocks. In the former economy, the job finding rate and the separation rate fluctuate around their means, whereas in the latter they are set forever at their average value $\bar{p}$ and $\bar{s}$. The welfare cost of business cycles λ is defined as the percentage of the consumption flow that the agent would accept sacrificing in order to get rid of aggregate fluctuations:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\lambda)c_t\right] = \sum_{t=0}^{\infty} \beta^t \bar{c}$$

[3]The existence of a significant gap is quite controversial. There are not only job search costs in the disutility of not working, but also indirect costs such as psychological damage and skill obsolescence. Does it compensate for home production and the disutility of working? There are no clear empirical answers. We follow here the traditional approach in matching models which identifies the return to non-market activity with unemployment benefits. This view is consistent with the fact we consider that non employed people are supposed to search and are not specialized in home production as non participating individuals are.

[4]In the structural approach (Section 3), productivity will be stochastic and will drive all the aggregate series in the model.

[5]It is not necessary to present the income received by unemployed and employed workers respectively as there are no implications here. See the next section for an explicit presentation.

where $\bar{c}$ is the level of consumption in the economy without aggregate shocks. It must be emphasized that $\bar{c}$ does not necessarily coincide with the mean of consumption in the volatile economy. This was the case in Lucas's approach. When fluctuations may alter the mean of aggregates, $\bar{c}$ can no longer be measured by its mean. It needs to be derived from a counterfactual experiment based on an artificial economy without any shocks. Following here the convention in the literature, we will qualify this economy as the stabilized economy. The stabilized unemployment (or the structural unemployment) and the stabilized consumption are then equal to:

$$\bar{u} = \frac{\bar{s}}{\bar{s} + \bar{p}} \qquad \text{and} \qquad \bar{c} = y(1 - \bar{u})$$

How the stabilized economy is reached is not explicitly presented, especially nothing is said on the design and the efficiency of stabilization policies. It can be argued that the business cycle cost gives an upper bound of the benefits of stabilization policies.

Finally, our theoretical framework allows us to provide a straightforward cost of the employment fluctuations in terms of welfare. The measure of the business cycle costs is a pure measure of the employment reduction caused by fluctuations.

2.2 The analysis of the non-linearities in the unemployment dynamics

By considering equation (1), it is fairly intuitive that shocks on the job finding rate ($\Delta p_t \equiv \bar{p} - p_t$) and on the separation rate ($\Delta s_t \equiv \bar{s} - s_t$) have non linear effects on unemployment. Let us rewrite the unemployment dynamics in deviation to the steady state unemployment ($\Delta u_t \equiv u_t - \bar{u}$):

$$\Delta u_{t+1} = \underbrace{(1 - \bar{s} - \bar{p})\Delta u_t}_{\text{propagation}} - \underbrace{\Delta p_t u_t}_{\text{p-impact}} + \underbrace{\Delta s_t (1 - u_t)}_{\text{s-impact}}$$

The way shocks on the job finding and separation rates are propagated is symmetric between expansions and recessions. By contrast, the impact of these shocks depends on the level of unemployment. As unemployment is higher during a recession, the job finding rate shocks have a greater impact in recession; the decline in the job finding rate is magnified by the increase in unemployment. On the contrary, during booms, the decline in unemployment offsets the increase in the job finding rate. As a result, fluctuations in the job finding rate tend to increase average unemployment. The asymmetry stems from the difference in unemployment (or equivalently, in employment) between recessions and booms. The greater the difference, the greater the asymmetry: the standard deviation of the shocks is a key factor of the business cycle cost. Furthermore, the longer are booms and recessions, the greater are these effect: at the extreme limit, the increase in the job finding rate would be no longer operative if the expansion made unemployment disappear. During booms, the flows out of unemployment are weaker and weaker. The persistence of the shocks is another key factor of the business cycle costs.

Conversely, as the impact of the job separation rate shocks depends on the level of employment, the fluctuations in the separation rate lead unemployment to decrease more in booms than to increase in recession: fluctuations in the job separation rate tend to reduce average unemployment. Again, the volatility and the persistence of these shocks are key factors.

In this section, we assess precisely these different effects. First, as it is traditionally done in the matching approach with aggregate shocks (see for instance Hall (2005)), a steady state analysis of equation (1) is conducted and so fluctuations in the conditional steady states are considered. This analysis delivers very easily the basics of the non-linearity embedded in the unemployment fluctuations. This simple framework highlights the importance of the volatility of the job finding and separation rates for the size of business cycle costs, but also the less expected role played by the structural unemployment rate. Secondly, taking into account the inertia embedded in equation (1), we derive the full non-linear properties of the unemployment dynamics and show that the persistence of the aggregate shocks also matters.

2.2.1 Steady state analysis

Let us assume for now that the speed of convergence of unemployment is infinite: fluctuations cause unemployment to jump directly from one conditional steady state to another. A conditional steady state unemployment corresponds to the level $\widetilde{u}_i$ toward which the unemployment rate would converge if the separation and job finding rates forever keep the same value p_i and s_i, i.e. if the economy remains in the same state i. Let us define π_i the unconditional probability of being in state i. The value taken by p and s in each state i and the probability associated π_i define the Markov chains associated with p and s, consistently with equations (2) and (3). The average job finding rate $\bar{p}$ is therefore equal to $\sum_i \pi_i p_i$ and the average separation rate $\bar{s}$ to $\sum_i \pi_i s_i$. Jointly, they determine the structural unemployment rate $\bar{u}$. On the other hand, as unemployment is assumed to jump directly from one conditional steady state to another, the average unemployment in this economy is then equal to the average of the conditional steady states:

$$\tilde{u} = \sum_i \pi_i \widetilde{u}_i$$

The non-linearity embodied in equation (1) implies that average unemployment $\tilde{u}$ has no reason to coincide with structural (stabilized) unemployment $\bar{u}$.

Job finding rate shocks

To understand the specific role of the non linearity in job findings, let us assume first that the separation rate is constant and equal to its mean $\bar{s}$. This non-linearity implies that $\widetilde{u}_i$ is a convex function of the state-dependent job finding rate p_i:

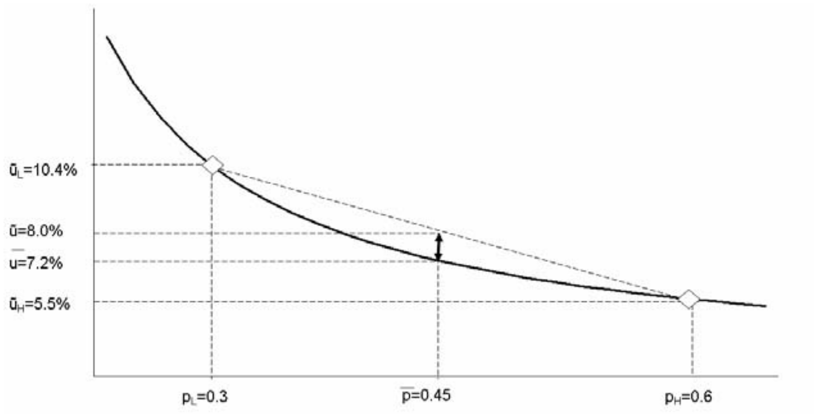
$$\widetilde{u}_i = \frac{\bar{s}}{\bar{s} + p_i} \tag{4}$$

Because unemployment is a convex function of the job finding rate, the average unemployment is higher than the structural (stabilized) unemployment $\bar{u}$, i.e. the unemployment level in the counterfactual economy where the job finding rate is forever set at its mean $\bar{p}$:

$$\bar{u} = \frac{\bar{s}}{\bar{s} + \bar{p}} < \widetilde{u} = \sum_i \pi_i \widetilde{u}_i$$

The convexity in the impact of the job finding rate on unemployment implies a higher average unemployment rate in an economy with a volatile job finding rate than in the stabilized economy. Figure 1 illustrates this result with a simple numerical example. Let us consider that the job finding rate can take only two values $p_L = 0.3$ or $p_H = 0.6$. For a job destruction rate of 3.5%, unemployment fluctuates between the two conditional steady states $\widetilde{u}_L = 10.4\%$ and $\widetilde{u}_H = 5.5\%$. If the two states are equiprobable, unemployment is on average 8% in this volatile economy; largely above the stabilized economy unemployment rate. If the job finding rate was steady at $\bar{p} = 0.45$, unemployment would be equal to 7.2%. In this stylized example, business cycles increase unemployment by 0.8 percentage point.

Figure 1: Convexity in steady state unemployment



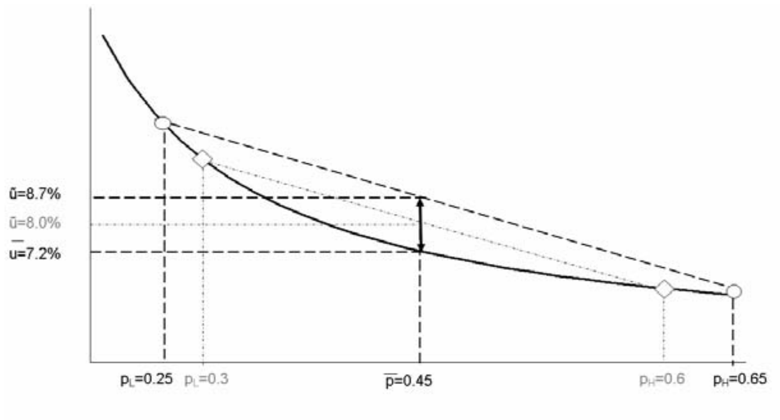
It is fairly intuitive that the gap between average unemployment $\widetilde{u}$ and stabilized unemployment $\bar{u}$ depends on the unconditional variance σ_p^2 of the job finding rate. For uniformly small deviations, using a second order Taylor expansion of equation (4), this gap can be written as [6]:

$$\tilde{u} - \bar{u} \approx \frac{\bar{s}}{(\bar{s} + \bar{p})^3} \sigma_p^2 \tag{5}$$

A mean-preserving increase in the volatility leads to a larger difference between the stabilized and fluctuating economies. The more volatile the economy, the greater the business cycle cost. Figure 2 illustrates how the variance of aggregate shocks affects the gap between $\widetilde{u}$ and $\bar{u}$ in the simple case where the job finding rate can only take two values : $p_L = 0.25$ or $p_H = 0.65$. In this configuration, the average unemployment rate in the volatile economy exceeds that of

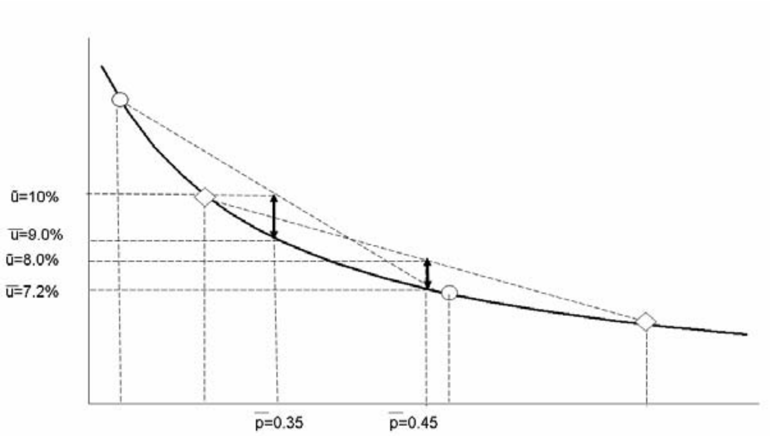
[6] See Appendix A for the derivation.

Figure 2: The role of volatility



the stabilized economy by 1.5 percentage point (vs 0.8 percentage point in the case $p_L = 0.30/$ $p_H = 0.60$).

Figure 3: The role of the average job finding rate



Equation (5) indicates that the unemployment gap also depends on the mean of the job finding rate and of the separation rate. A lower value of $\bar{p}$ or a higher value of $\bar{s}$ imply a more convex economy. This result is important as it generates strong interactions between structural and cyclical unemployment. Furthermore, this suggests that labor market institutions affect the welfare costs of fluctuations as they have an impact on the average job finding and separation rates. Figure 3 displays how the structural job finding rate modifies the consequences of fluctuations on average unemployment.

Job separation rate shocks

So far, the separation rate was assumed to be constant. However, it appears clearly from equation

(1) that the asymmetry in the unemployment dynamics could also come from fluctuations in the separation rate. The resulting unemployment gap would then read:

$$\widetilde{u} - \bar{u} \approx -\frac{\bar{p}}{(\bar{s}+\bar{p})^3}\sigma_s^2 \tag{6}$$

with σ_s^2 the unconditional variance of the separation rate. Contrary to the job finding rate case, fluctuations in the separation rate tend to reduce average unemployment. Job separations decrease more in expansion than they increase in recessions. As shown by equation (6), the unemployment gap depends on the average job finding and separation rate. An increase in either of these variables weakens the impact of fluctuations on average unemployment. Further, the asymmetry embodied in equation (1) translates into average unemployment only if the separation rate is volatile enough.

Figure 4: Concavity in steady state unemployment

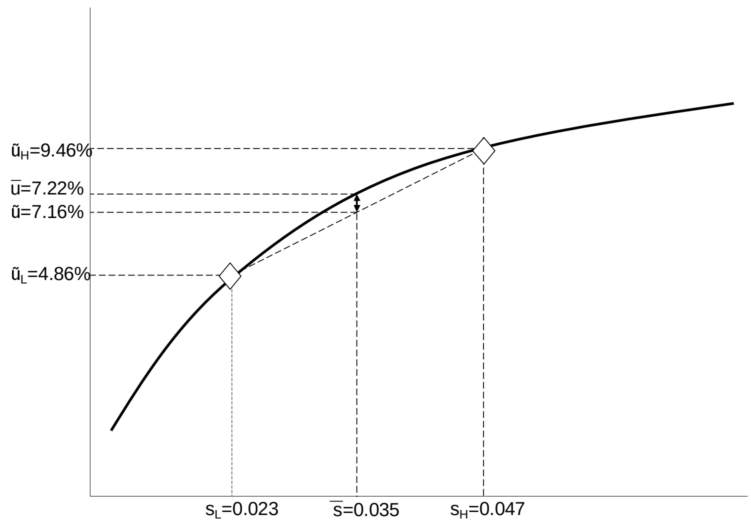


Figure 4 illustrates this result with a simple numerical example. Let us consider that the separation rate can take only two values $s_L = 0.023$ or $s_H = 0.046$, which correspond to a standard deviation of the same relative magnitude as the job finding example. For a job finding rate of 45%, unemployment fluctuates between the two conditional steady states $\widetilde{u}_H = 9.46\%$ and $\widetilde{u}_L = 4.86\%$. If the two states are equiprobable, unemployment is on average 7.16% in this volatile economy, below the stabilized economy unemployment rate. If the job separation rate was steady at $\bar{s} = 0.035$, unemployment would be equal to 7.22%. In this stylized example, business cycles decrease unemployment by 0.05 of a percentage point. It is remarkable[7] that the same standard deviation of roughly 33% for the job finding and separation rates leads to a very different magnitude for the absolute level of the unemployment gap $\widetilde{u} - \bar{u}$.

[7] We will investigate this point further in the quantitative section.

2.2.2 Considering unemployment inertia

All previous calculations have been made with the assumption that unemployment jumps directly to the conditional steady states. Under this assumption, average unemployment in the business cycle economy is equal to the average of the steady states $\tilde{u}$. This equality is no longer valid once we take into account unemployment inertia. Let us explore the implications of this inertia in the case where unemployment fluctuations are caused only by job finding rate shocks[8]. Combining equation (1) and (4), the dynamics of unemployment can be rewritten:

$$u_{t+1} = u_t + (\bar{s} + p_i)(\widetilde{u}_i - u_t) \quad (7)$$

Contrary to the previous section, unemployment does not jump to its conditional steady states ; it converges toward the level $\widetilde{u}_i$ at rate $\bar{s}+p_i$. Because of unemployment inertia, the asymmetry embodied in the conditional steady state does not necessarily manifest in average unemployment. This asymmetry affects the average unemployment rate only if the aggregate shocks are persistent enough.

An illustration. Let us illustrate this intuition by considering again the numerical example of section 2.2.1. The job finding rate can take two values $p_L = 0.3$ or $p_H = 0.6$. Figure 5 shows the adjustment path of unemployment from its stabilized level towards its respective conditional steady states $u_L = 10.4\%$ and $u_H = 5.5\%$. As shown in the right hand side of Figure 5, the dynamics of unemployment are identical in the first periods following the shock. The difference between recession and expansion arises only if either lasts long enough. The additional unemployment caused by business cycles depends not only on the mean and the volatility of the aggregate shocks, but also on their persistence. We also expect some interactions between the level of asymmetry and persistence. The higher the variance of shocks, the greater the consequence of persistence[9].

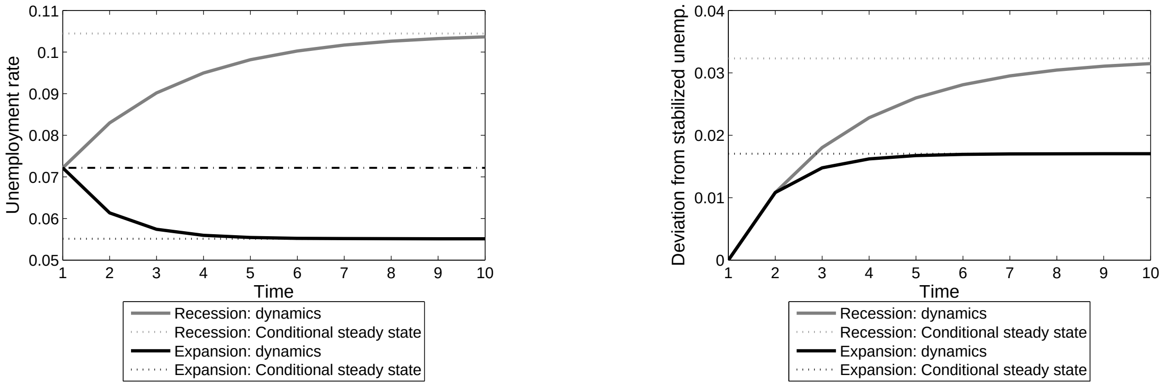
Some analytical results. To see in a synthetic formula the role of the mean, the volatility and the persistence of aggregate shocks, let us solve equation (1). We compute average unemployment in the case where only the job finding rate is fluctuating. Rewrite simply the unemployment dynamics in the following way:

$$u_{t+1} = \bar{s} + \phi_t u_t$$

[8]The analysis is similar in the case where the job separation rate is fluctuating.

[9]Note that the asymmetry in the convergence rate amplifies the role of persistence. As pointed out by Cole and Rogerson (1999), the rate at which unemployment converges toward its conditional steady states depends on the aggregate state. The rate of convergence toward its conditional steady state $\widetilde{u}_i$ is equal to $\bar{s}+p_i$. Unemployment in the volatile economy is "biased" toward favorable aggregate states as the speed of adjustment is higher in those sates.

Figure 5: The role of persistence



where $\phi_t \equiv 1 - \bar{s} - p_t$ and $\mathbb{E}[\phi] \equiv \bar{\phi} = 1 - \bar{s} - \bar{p}$. The autoregressive process defined by equation (2) implies that ϕ_t follows an autoregressive stationary process:

$$\phi_t = \rho_p \phi_{t-1} + (1 - \rho_p)\bar{\phi} - \varepsilon_t^p$$

A backward substitution gives:

$$\mathbb{E}[u] \quad = \quad \bar{s} \left(1 + \sum_{k=0}^{+\infty} \mathbb{E}[\prod_{j=0}^{k} \phi_{t-j}] \right)$$

As shown in Appendix B, the additional unemployment created by business cycles can then be approximated by [10]:

$$\mathbb{E}[u] - \bar{u} \approx \bar{s} \quad \underbrace{\frac{\sigma^2_{\varepsilon^p}}{1 - \rho_p^2}}_{\text{Exo. variance}} \quad \underbrace{\sum_{k=2}^{+\infty} (1 - \bar{s} - \bar{p})^{k-2} \sum_{i=0}^{k-1} (k - 1 - i)\rho_p^{i+1}}_{\text{Endogenous Propagation}} \tag{8}$$

The difference between average and stabilized unemployment depends on the exogenous volatility of p and on the propagation of these shocks which in turn results from the exogenous persistence ρ_p and the unemployment inertia. If p_t and hence ϕ_t were not serially correlated ($\rho_p = 0$), fluctuations in the job finding rate would not affect average unemployment. Average unemployment in the business cycles and in the stabilized economy would be identical: $\mathbb{E}[u] = \bar{u}$. When aggregate shocks are persistent, average unemployment in the business cycle economy is no longer equal to stabilized unemployment. Volatility then matters: the greater the variance of job finding rate shocks, the higher the unemployment rate. In line with our intuition, equation (8) also shows

[10] The approximation consists of neglecting moments of order above 2. This is line with our approach : as we want to understand how symmetrical shocks can yield non symmetrical effects on unemployment, we disregard in particular the consequences of non-zero skewness

some interactions between the volatility, the persistence and the mean of the job finding rate. An increase in the variance of the shocks raises average unemployment all the more so when the average job finding rate is low and the persistence of the shocks is high. The magnitude of business cycle costs will then depend on the observed characteristics of the job finding rate shocks. Business cycle costs also depend on the average separation rate : like the persistence and the mean of the job finding rate, the level of the separation rate can amplify the costs generated by job finding rate fluctuations.

Symmetrically, in the case where only the separation rate is fluctuating, we have a welfare gain of fluctuation which positively depends on the volatility and on the persistence of the separation rate[11]:

$$\mathbb{E}[u] - \bar{u} \approx -\frac{\sigma^2_{\varepsilon^s}}{1-\rho_s^2}\left[\sum_{k=1}^{+\infty}(1-\bar{s}-\bar{p})^{k-1}\sum_{i=0}^{k-1}\rho_s^{i+1} - \sum_{k=2}^{+\infty}\bar{s}(1-\bar{s}-\bar{p})^{k-2}\sum_{i=0}^{k-1}(k-1-i)\rho_s^{i+1}\right] \quad (9)$$

Again, the average job separation and finding rates interact with the volatility and the persistence of the shocks.

2.3 Quantifying the welfare cost of business cycles

To investigate whether observed fluctuations in the job finding and separation rates affect average unemployment and hence the costs of business cycles, it is necessary to estimate the AR(1) processes[12] described by equations (2) and (3). As equations (8) and (9) give only an approximation of average unemployment, we resort to simulations to obtain a more accurate estimate of the costs of business cycles. Consistently with the AR(1) estimations, we simulate job finding and separation shocks in order to obtain artificial series for the job finding and separation rates[13]. We then use them to simulate equation (1), which allows us to compute the average unemployment rate in the business cycle economy and the business cycle costs.

2.3.1 Data

The behavior of job finding and job separation rates over the business cycle is still a debated subject. It especially depends on the underlying conception of "unemployment". Contrary to Shimer (2005) who computes the job finding rate from standard unemployment, Hall (2005) uses a measure of unemployment expanded to include "discouraged workers" and "marginally attached workers". Although those workers are classified as being out of the labor force, their behavior is close to that of workers classified as unemployed. Indeed, there are significant worker

[11]We checked numerically that the difference in the brackets is positive.

[12]It must be emphasized that considering a log-normal distribution for p and s would have led to very similar results. See Appendix D for more details.

[13]We checked that all values of p and s are in the interval [0,1].

flows directly from out of the labor force to employment: Blanchard and Diamond (1990) suggest that a fraction of people classified as being out of the labor force do search for a job, though probably less intensively than unemployed workers. Cole and Rogerson (1999) argue that a model with only two labor market states (employed and unemployed) must be calibrated with care, using data including people who can be in a third labor market state (not in the labor force). Taking into account, as Hall (2005) does, the transitions from being not in the labor force to employment seems to be a reasonable way of dealing with this issue. Moreover, this approach seems particularly relevant in an analysis of the business cycle costs. Although our benchmark result is based on Hall's data, we also provide the results obtained when Shimer's approach is used.

The job finding rate

The Hall (2005) and Shimer (2005) measures of the job finding rate both exhibit pro-cyclicality. The job finding rate plunges at each recession and recovers at each expansion (Figures 7 and 8, Appendix C). Both measures show a downward trend in the 1970s and in the early 1980s. As some of these movements could be due to factors unrelated to business cycles, we primarily focus hereafter on series detrended by a low frequency filter[14]. However, note that two elements call for not detrending the data. First, the downward trend does not necessarily result from non cyclical factors ; it could also be explained by the increase in the frequency of recessions observed during the 1970s. Secondly, as the mechanism studied in this paper relies on non linearities, the method used to isolate the cyclical component of the job finding rate could affect the welfare cost of fluctuations. This is why non detrended (raw) data are also considered hereafter.

Table 1: Job finding rate statistics

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Mean $\bar{p}$	0.285		0.450	
Standard deviation σ_p	0.084	0.069	0.068	0.053
Autocorrelation	0.942	0.913	0.939	0.915

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

We then estimate the parameters characterizing the process of the job finding rate as described by equation (2). As expected, the "expanded job finding rate" has a lower mean than Shimer (2005)'s measure (Table 1). As shown in the previous section, this may exacerbate the asymmetry in the unemployment dynamics and induce larger business cycles costs. More importantly,

[14] As in Shimer (2005), we used a Hodrick-Prescott filter with smoothing parameter 10^5.

including low intensive job seekers also implies a higher variability[15]. Hall (2005)'s measure should therefore lead to a higher cost of business cycles. On the other hand, these figures are quite sensitive to the use or not of the HP filter.

The job separation rate

The discrepancy between Shimer's and Hall's measures of the separation rate is also noteworthy. Hall (2005) computes a series for the overall separation rate (which includes layoffs, quits, end of short-term contracts) from gross flows on separations. By contrast, Shimer (2005) focuses only on transitions from employment to unemployment. He infers the job separation rate from short term unemployment. As for the job finding rate, Hall's measure includes worker flows between employment and not being in the labor force. Although both series capture the NBER dated recessions quite well, their trends are completely opposed (Figures 9 and 10, Appendix C). Furthermore, Hall's job separation rate is 10 times less volatile than Shimer's for a similar persistence (Table 2).

Table 2: Job separation rate statistics

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Mean $\bar{s}$	0.031		0.034	
Standard deviation σ_s $\times 10^{-2}$	0.058	0.029	0.540	0.260
Autocorrelation	0.970	0.872	0.946	0.756

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

2.3.2 Business cycle costs with only job finding shocks

Figure 6 illustrates the asymmetry in the unemployment fluctuations relative to the stabilized economy for a particular simulation of equation (1) corresponding to a particular draw in the estimated process of the job finding rate. Positive shocks on the job finding rate reduce unemployment less than negative shocks increase it. Therefore average unemployment in the business cycle economy (dashed line) is above the unemployment rate in the stabilized economy (solid line).

Table 3 presents the welfare cost of business cycles in the US economy for the four measures of the job finding rate. Non-linearities in the unemployment dynamics are enough to generate sizable costs of business cycles. In particular, these costs are between one and two orders of magnitude greater than the costs found by Lucas (1987). The method chosen to measure the

[15] Table 1 shows the standard deviation of the job finding rate, and not of its innovation.

Figure 6: Asymmetry in the unemployment dynamics, Hall data, detrended

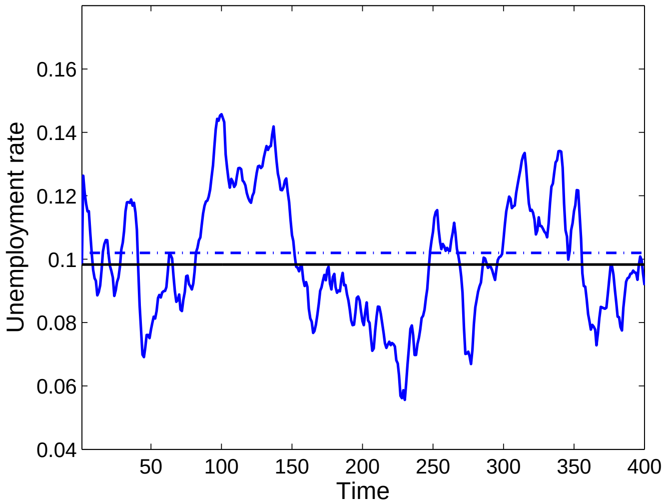


Table 3: The welfare cost of job finding rate fluctuations

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Unemployment	10.50%	10.22%	7.14%	7.08%
Stabilized unemployment	9.83%	9.83%	7.00%	7.00%
Cost of fluctuations	0.74%	0.44%	0.13%	0.08%

job finding rate has strong consequences on the welfare costs of fluctuations. When some non-employed job seekers (Hall's method) are taken into account, fluctuations in the job finding rate induce at least a 0.44% consumption loss. This cost increases to 0.74% if all job finding rate fluctuations are related to business cycles factors. As expected, Shimer's measure leads to lower business cycle costs: if the job finding rate is computed using only transitions from unemployment, the welfare cost of business cycles reduces to 0.08%. Such a result was expected as Shimer's job finding rate series display both a lower volatility and a higher mean (Table 1), two characteristics that we identified as cost-reducing.

As Table 4 shows, consistently with equation (8), volatility plays a crucial role in this result. The gap between the results inferred from the Shimer and Hall data comes mainly from differences in volatility. Considering a lower average job finding rate[16] for a relatively low volatility modifies only marginally the magnitude of the business cycle costs (0.10% versus 0.08%). The same

[16] A lower average job finding rate implicitly increases the relative volatility of the job finding rate. To understand the specific role of the mean of the job finding rate, we ensure that its relative volatility is held constant. We modify the standard deviation of the shocks to maintain the coefficient of variation constant.

Table 4: Understanding the "Hall-Shimer wedge", detrended data

From Shimer data	$\bar{p}$,σ_p,$\bar{s}$,ρ_p		$\bar{p}$,$\bar{s}$,ρ_p	σ_p,$\bar{s}$,ρ_p	$\bar{p}$,σ_p,ρ_p		$\bar{p}$,σ_p,$\bar{s}$, 0.98
From Hall data		$\bar{p}$,σ_p,$\bar{s}$,ρ_p	σ_p	$\bar{p}$	$\bar{s}$	$\bar{p}$,σ_p,$\bar{s}$, 0.98	
Unemployment	7.08 %	10.22%	7.33%	10.71%	6.53%	10.33%	7.08%
Stabilized unemployment	7.00%	9.83%	7.00%	10.62%	6.46%	9.83%	7.00%
Cost of fluctuations	0.08%	0.44%	0.35%	0.10%	0.07%	0.56%	0.09%

Notes: This table shows the welfare cost of fluctuations for job finding rate processes that share characteristics from both Hall (2005) and Shimer (2005) data. For instance, column 3 presents the same average job finding and separation rates and the same autocorrelation as Shimer (2005) but the same dispersion (measured by the coefficient of variation) as Hall (2005). In column 4, we modify the variance of the shocks to ensure that the relative volatility (again measured by the coefficient of variation) of the job finding rate is held constant. Regarding the persistence experiment (columns 7 and 8), the variance of the shocks has been to modified to maintain the unconditional variance of the job finding rate constant.

statement can be made when a different value for s is considered for a given volatility (0.07% versus 0.08%). On the other hand, if Shimer's job finding rate was characterized by a higher level of volatility (equal to Hall's data)[17], the corresponding cost of fluctuations would amount to 0.35% which is significantly different from 0.08%. However, it is also significantly different from the 0.44% cost induced from Hall's data. The remaining difference is due to the influence of the average value of the job finding rate and of the separation rate. The influence of the structural unemployment then depends on the level of the volatility: the higher the latter, the higher the influence of $\bar{p}$ and $\bar{s}$. This result reveals significant interactions between structural and cyclical unemployment[18]. A higher structural unemployment rate amplifies the welfare cost of business cycles when the volatility in unemployment is high. This result suggests that business cycles could reduce average consumption by more in continental European countries[19] which would then suffer from both higher structural unemployment and more costly unemployment fluctuations.

When we consider an arbitrarily higher persistence[20] (ρ equal to 0.98), the cost of business cycles is higher for both Hall's and Shimer's measure, but the increase is greater when volatility and structural unemployment are higher. An increase in the persistence of shocks reinforces the asymmetry in the unemployment process, especially when volatility is high.

These results bring new insights to the analysis of the welfare costs of fluctuations. Business cycle costs mainly depend on the variability of aggregate shocks, but the three characteristics of the

[17] We focus here on the relative volatility of the job finding rate, measured by the coefficient of variation.

[18] That higher unemployment is caused by a higher separation rate or a lower job finding rate influences the cost of fluctuations only marginally.

[19] To the best of our knowledge, job finding rate data for European countries do not go back enough to infer their cyclical properties. Petrongolo and Pissarides (2001) have constructed series for France and Spain, but these series start respectively in 1991 and in 1987. However, all empirical evidence points to a lower average job finding rate in Europe.

[20] The variance of the shock has been to modified to maintain the overall variance of the job finding rate constant

job finding rate process deeply interact. The marginal effect of volatility crucially depends on the mean and on the persistence of the job finding rate, but also on the value of the separation rate. Furthermore, this suggests that labor market institutions have an impact on the welfare costs of business cycles, not only because they affect the amount of volatility faced by individuals (with unemployment benefits for example), but also because they affect structural unemployment.

Overall, these first results put into question the optimism of Lucas (1987) about the weakness of business cycle costs. They are obtained in a very simple framework without taking into account the individual risks associated with aggregate unemployment, which has received more attention in the literature since the seminal work of Krusell and Smith (1999). Unemployment fluctuations could then imply welfare costs through both a decrease in aggregate consumption and an increase in individual consumption volatility. There are no reason to think that these two dimensions are not cumulative[21], leading to substantial business cycle costs.

2.3.3 Business cycle costs with job separation shocks

If shocks on the job finding rate lead to business cycle costs, those on the separation rate could imply business cycle gains. Let us simulate equation (1) with job separation shocks, first without job finding rate shocks, secondly with those shocks and taking into account the co-variance between the two. We again consider both Shimer's and Hall's data. Whatever the data considered, the consumption gains brought by these fluctuations are negligible. They are even nil considering Hall's data which display so little volatility in the separation rate. We show in Table 5 the results relative to Shimer's data. Even in this case, the volatility is not enough to generate strong asymmetries in the unemployment dynamics: the business cycle gains are of the same magnitude as the business cycle costs shown by Lucas (1987).

Table 5: The welfare cost of fluctuations in the separation rate

	no shock on p		two correlated shocks	
	Raw	Detrended	Raw	Detrended
Unemployment	6.99%	7.00%	7.16%	7.10%
Stabilized unemployment	7.00%	7.00%	7.00%	7.00%
Cost of fluctuations	-0.009%	0.00%	0.18%	0.10%

Note: The two first columns show the welfare cost of fluctuations in the case where only the separation rate fluctuates. Columns 3 and 4 give the consequences of the co-variance between the job finding rate and the separation rate. These results are computed using Shimer (2005)'s data for which the covariance is $-1.6.10^{-4}$ ($-0.8.10^{-4}$) for raw (detrended) data.

Note that this does not necessarily imply that the volatility of the separation rate does not

[21] The verification of this assertion is left to further research.

significantly contribute to the volatility of the unemployment rate. Considering Shimer's data, Fujita and Ramey (2008) estimate that fluctuations in the separation rate account for at least 28% of the unemployment volatility. Even if the volatility in the job separations represents a significant proportion of the unemployment fluctuations in the business cycle, it fails to decrease average unemployment significantly[22].

However, if their specific role is limited, job separation shocks can interact with job finding ones. Taking into account its covariance with the job finding rate may enhance the consequences of job finding rate fluctuations. If the separation rate is negatively correlated to the job finding rate, movements in the separation rate cause the unemployment rate to increase further during recessions and decrease further during booms. This amplifies the asymmetry in job creation between periods of boom and recession. During booms, the increase in the job finding rate is all the more offset as the separation rate amplifies the decline in unemployment. During recessions, the increase in the separation rate enhances the magnifying effect of the rise in unemployment. A negative correlation between the job finding rate and the job separation rate in the cycle could lead to exacerbate the costs induced by the job finding volatility.

We then simulate equation (1) taking into account both shocks and the observed negative correlation (equal to -0.48 and -0.59 in the raw and detrended cases respectively) between them (only for Shimer (2005)'s data). The last two columns of Table 5, compared to those of Table 3, show how this negative correlation amplifies the welfare consequences of the job finding rate volatility. The welfare cost of fluctuations increases from 0.08% to 0.10% when considering HP filtered data, and from 0.13% to 0.18% when the data are not filtered. However, as this effect modifies only marginally the costs induced directly by the volatility in the job finding rate, we choose to disregard the fluctuations in the separation rate in the rest of the paper.

3 Endogenizing the job finding rate: a structural approach

The previous section showed that the observed volatility and persistence in the job finding rate lead to sizable costs of business cycles. The structural matching model is a candidate for generating such costs. However, Shimer (2005) shows that the standard matching model fails

[22] We indeed perform here a different exercise. In the variance decomposition, the variances are weighted by parameters which compensate for the difference in level between s and p. In our investigation of business cycle costs, the levels matter. Let us consider equations (5) and (6) as these simplified expressions are a good approximation of the business cycle costs when the processes of p and of s are highly serially correlated as in data. They show that the difference in the order of magnitude of the volatility of p and s is not compensated for by the coefficients which pre-multiply the variance in each equation. This is an intrinsic limitation to the impact of separation shocks on business cycle costs. It explains why the observed volatility of the separation rate is too small for this non linearity to manifest: the volatility of the job separation rate has no real impact on average unemployment.

to generate realistic fluctuations in the job finding rate. The standard deviation of the job finding rate is 12 times greater in the data than in the model (Shimer (2005)). An increase in labor productivity increases the expected profit from a filled job, and thus firms tend to open more vacancies. But there are internal forces in this framework which partially offset the initial increase in expected profits and then dampen the incentives for vacancy creations. Replicating the job finding rate fluctuations is then not easy.

There already exist in the literature different approaches which solve the Shimer puzzle[23]. Do we care about identifying the mechanism at the origin of the high fluctuations in the job finding rate? From the analysis conducted in the first part, it could be tempting to say that it is enough to know that at least one theory is able to replicate the job finding rate dynamics. Actually, we do care. Indeed, the results obtained in the first part are derived from a model in which the job finding rate is exogenous, and in which fluctuations are neutral regarding the average job finding rate. To assess the welfare costs of fluctuations, one must take into account the consequences of stabilization on the average job finding rate. If productivity shocks and the job finding rate are linearly related, the results found in the previous section should *a priori* be close to the endogenous job finding rate case. But if not, the average job finding rate can then be affected by business cycles. Why do we suspect the presence of a non-linear effect of business cycles on the job finding rate? The job finding rate is a non-linear function of the labor market tightness which also depends non-linearly on productivity changes. To show and quantify these different effects, a structural model is then required to generate a counterfactual stabilized economy. In this case, the costs of business cycles could differ according to the model specification.

The choice of the theoretical model is then potentially crucial. The studies aiming at elucidating the Shimer puzzle emphasize different mechanisms and none seems to close the debate. We choose to study the business cycle cost implied by the wage rigidity approach as suggested initially by Hall in a first response to the puzzle. This framework fits perfectly well with our objectives: it allows us to generate enough volatility in the job finding rate, but also to reveal, in a very transparent way, the basic non-linearity embodied in the matching model. As the wage retroaction is neutralized in the job creation condition, it allows us to focus only on the implications of the basic non-linearity introduced by the matching function, independently of other assumptions (hiring and/or separation costs, insider/outsider wages). We then present different calibrations of the matching function elasticity in order to unveil these implications. Each replicates the job finding rate process (standard deviation and mean)[24]. The implied business cycle costs are not necessarily identical and equal to that obtained in the first part.

[23]See for instance Hall (2005), Hall and Milgrom (2008), Pissarides (2007), Hagedorn and Manovskii (2008), Hornstein and Violante (2007), Kennan (2006), Mortensen and Nagypal (2003), Silva and Toledo (2008) and Costain and Reiter (2008)

[24]The persistence would be naturally matched by that of productivity shocks.

3.1 A canonical matching model

The model considered hereafter is a version of the matching model *à la* Pissarides with aggregate uncertainty and exogenous separation. The economy includes the basic assumptions of the framework presented in the previous section. Agents are risk neutral, without access to financial markets and can either be employed or unemployed. An unemployed worker gets an unemployment benefit z. Employed workers receive wage w until their job is destroyed (at rate s); we do not take into account on-the-job search and voluntary quits. Output per unit of labor is denoted by y_t and is assumed to follow a first-order Markov process according to some distribution $G(y, y') = Pr(y_{t+1} \leq y' | y_t = y)$. Jobs and workers meet pairwise at a Poisson rate $M(u, v)$, where $M(u, v)$ stands for the flows of matches and v the number of vacancies. This function is assumed to be strictly increasing and concave, exhibiting constant returns to scale, and satisfying $M(0, v) = M(u, 0) = 0$. Under these assumptions, unemployed workers find a job with a probability $p(\theta) = M(u, v)/u$ that depends on the ratio of vacancies to unemployment ($\theta = v/u$). The probability of filling a vacancy is given by $q(\theta) = M(u, v)/v$. Hereafter, we impose that the matching function is Cobb-Douglas: $M(u, v) = \varphi u^{1-\alpha} v^{\alpha}$ with $0 < \alpha < 1$.

The unemployment dynamics in the economy (equation (10)) are similar to equation (1), except that the job finding rate is now endogenous. Equations (11) and (12) define the job finding rate and the job filling rate respectively which depends[25] on the current productivity state y:

$$u' = s(1-u) + (1 - p(\theta_y))u \tag{10}$$

$$p(\theta_y) = \varphi \theta_y^{\alpha} \tag{11}$$

$$q(\theta_y) = \varphi \theta_y^{\alpha - 1} \tag{12}$$

In this economy, the only source of fluctuations is the labor productivity shocks. The welfare cost of fluctuation is therefore defined relatively to a counterfactual economy in which labor productivity remains at its average value.

3.1.1 The value functions

The worker's utility

Define U_y and W_y to be the state contingent present value of an unemployed worker and an employed worker[26]:

$$U_y = z + \beta\{(1 - p(\theta_y))\mathbb{E}_y[U_{y'}] + p(\theta_y)\mathbb{E}_y[W_{y'}]\} \tag{13}$$

[25] Throughout the paper the notation x_y indicates that a variable x is a function of the aggregate productivity level y and E_y is the expected value conditional on the current state y.

[26] For the sake of simplicity, we omit from these equations the disutility of working and not working, the lump-sum tax financing the unemployment benefits and the dividend paid by firms to workers, as these variables are assumed to be identical across individuals.

$$W_y = w_y + \beta\{(1-s)\mathbb{E}_y[W_{y'}] + s\mathbb{E}_y[U_{y'}]\} \quad (14)$$

The firm's surplus

To hire workers, firms must open vacancies at cost κ. The firm's value of an unfilled vacancy V_y is given by:

$$V_y = -\kappa + \beta\{q(\theta_y)\mathbb{E}_y[J_{y'}] + (1-q(\theta_y))\mathbb{E}_y[V_{y'}]\} \quad (15)$$

with J_y the state contingent present value of a filled job and $q(\theta_y)$ the probability of filling a vacancy conditionally on the productivity state y. When the job is filled, the firms operate with a constant return to scale technology with labor as only input. Firms face labor productivity shocks. The firm's value of a job is given by:

$$J_y = y - w_y + \beta\{(1-s)\mathbb{E}_y[J_{y'}] + s\mathbb{E}_y[V_{y'}]\} \quad (16)$$

Free entry implies $V_y = 0$ for all y. Therefore, the job creation condition is:

$$\kappa = \beta q(\theta_y)\mathbb{E}_y[J_{y'}] \quad (17)$$

3.1.2 Aggregate consumption

Aggregate consumption is equal to the aggregate production net of the vacancy costs.

$$c_y = y(1-u) - \kappa\ v \quad (18)$$

Fluctuations may alter average consumption through average unemployment, in the same way as in the reduced form analysis, but also by influencing average vacancies. More vacancies in the fluctuating economy on average could increase the business cycle costs. In other words, the business cycle costs may derive from the fluctuations of either production or hiring costs.

3.1.3 Equilibrium

The labor market equilibrium depends on the way the wage is determined in the economy. Though our benchmark is the rigid wage model, we first present the traditional equilibrium with a Nash-bargained flexible wage. It allows us to compare the non-linearities embedded in these two equilibria.

Equilibrium with flexible wages.

When a worker and an employer meet, the expected surplus from trade is shared according to the Nash bargaining solution. The joint surplus S_y is defined by $S_y = W_y + J_y - U_y$. The worker gets a fraction γ of the surplus, with γ her bargaining power. The equilibrium with flexible wages is defined by the job creation condition and the wage rule (equations (19) and (20)), plus equations (10) to (12):

$$\frac{\kappa}{q(\theta_y)} = \beta \mathbb{E}_y \left[y' - w(\theta_{y'}) + (1-s)\frac{\kappa}{q(\theta_{y'})} \right] \tag{19}$$

$$w(\theta_y) = \gamma(y + \kappa\theta_y) + (1-\gamma)z \tag{20}$$

As Shimer (2005) points out, the adjustment of wages is responsible for the insensitivity of the labor market tightness to the productivity shocks. It also makes the interplay of the non-linearities in the model more complex relative to the rigid wage equilibrium, due to the retroaction of wages in the job creation condition (equation (19)).

Equilibrium with rigid wages

Incorporating wage rigidity in the matching model is a natural way to generate enough volatility. Moreover, this allows us to focus on the basic non-linearities introduced by the matching function which exist whatever the matching model considered.

Following Hall (2005), we consider a constant wage $w_y = w$, $\forall y$. This constant wage is an equilibrium solution if $z \leq w \leq \min \pi_y$, where π_y denotes the annuity value of the expected profit[27]. The wage is set at the Nash bargaining solution relative to the average state of productivity $\bar{y}$. This wage is an equilibrium wage provided it lies in the bargaining set defined by the participation constraints of the firms and the workers.

The rigid wage equilibrium is then defined by substituting equations (19) and (20) by equations (21) and (22), again in addition to the conditions (10) to (12):

$$\frac{\kappa}{q(\theta_y)} = \beta \mathbb{E}_y \left[y' - w + (1-s)\frac{\kappa}{q(\theta_{y'})} \right] \tag{21}$$

$$w = \gamma(\bar{y} + \kappa\theta_{\bar{y}}) + (1-\gamma)z \tag{22}$$

Let us comment on these equations regarding the non-linearities they incorporate. The non-linearity arising from the unemployment dynamics (equation (10)) has been intensively investigated in the previous section. Let us concentrate here on the additional non-linearity that appears once p is endogenous.

First of all, the job finding rate depends non-linearly on the labor market tightness (equation (11)): due to congestion effects, the return of an additional vacancy to the job finding rate is decreasing. This implies an asymmetric adjustment in the job finding rate over the business cycle. The average job finding rate in the stabilized economy is potentially higher than in a

[27] This annuity value is simply computed using the value an employer attaches to a new hired worker who never receives any wage:

$$\widetilde{J}_y = y + \beta(1-s)\mathbb{E}_y[\widetilde{J}_{y'}]$$

The annuity value is then given by $\pi_y = [1 - \beta(1-s)]\widetilde{J}_y$.

economy with business cycles, which tends to create higher business cycle costs (p-concavity effect).

Secondly, equation (21), combined with the job filling rate condition (equation (12)), shows that the average labor market tightness could also be affected by productivity fluctuations. If expansions and recessions have the same marginal impact on the firm's profits, the free entry condition is satisfied for greater variations in job creation in booms than in recessions. This effect tends to increase the average job finding rate in the stabilized economy, and thus leads to lower business cycle costs (θ-convexity effect). Let us note that the flexible wage equilibrium embodies the same non-linearity when abstracting from the wage retroaction.

The total impact of productivity fluctuations on the job finding rate is then the combination of these two effects (p-concavity and θ-convexity). From equations (11) and (12), it appears clearly that the overall effect depends on the elasticity α of the matching function to vacancies.

These basic non-linearities, embedded in the matching process, are common to the whole Mortensen-Pissarides class of model. It is obvious that the flexible and rigid wage equilibria share the same fundamental non-linearities, as they stem from the intrinsic characteristics of the unemployment dynamics, of the job finding rate and of the job filling rate which are exactly the same in the two equilibria. However, in the flexible wage case, the θ-convexity effect is modified by the retroaction of the wage in the job creation condition (equations (19) and (20)). Hence, the non-linearity between y and θ_y also depends on the assumption about the wage bargaining process, and more generally on the particular assumptions considered, such as the existence or not of fixed hiring and separation costs[28]. In this sense, the flexible wage equilibrium introduces non-linearities which are not intrinsic to the matching process. As already mentioned, the use of the rigid wage framework allows us to focus on basic non-linearities which are common to a large class of model.

3.2 Quantifying the business cycles costs

We first calibrate the rigid wage economy. We then simulate the model in order to compute the business cycle costs.

3.2.1 Calibration

The model is calibrated to match US data. We normalize average labor productivity to 1 and calibrate the productivity process to match the US labor productivity standard deviation and persistence[29]. The monthly discount rate is set to 0.0042. The job separation rate is set at Hall's

[28] See the last section for more details on this point.

[29] We use the same data as Shimer (2005), the real output per worker in the non farm business sector, detrended with a HP smoothing parameter of 10^5.

estimate for the US economy, 0.031. We choose the elasticity of the matching function α to be 0.5, in Petrongolo and Pissarides (2001) range. Following Mortensen and Pissarides (1999), γ is set at 0.5. The scale of the matching function φ is chosen to pin down the US average vacancy-unemployment ratio. Unemployment benefits and vacancy costs are then calibrated to reproduce the volatility and the mean of the job finding rate over the cycle. These two targets are computed using Hall (2005)'s measure of the job finding rate (Table 1).

Table 6: Benchmark calibration of the matching model

Parameter	Calibration	Target
Labour productivity		
Average	1	Normalization
Persistence ρ_y	0.90	US data (1951-2003)
Standard deviation σ_y	0.9%	US data (1951-2003)
Discount rate r	0.0042	Corresponds to 5% annually
Job destruction rate	0.031	Hall (2005)
Elasticity of the matching function α	0.5	Petrongolo-Pissarides (2001)
Workers' bargaining power γ	0.5	Mortensen and Pissarides (1999)
Scale of the matching function φ	0.346	Matches US average v-u ratio of 0.72 (Pissarides, 2007)
Cost of vacancy κ	0.239	Matches US average job finding rate of 0.285 and job
Unemployment benefits z	0.796	finding rate volatility of 0.068

3.3 Simulation

Table 7: The welfare cost of fluctuations in a matching model with rigid wages

	Business cycles economy				Stabilized economy			Cost of
	$E(u_t)$	$E(p_t)$	σ_p	$E(\theta)$	$\bar{u}$	$\bar{p}$	$\bar{\theta}$	fluctuations
$\alpha = 0.5$	10.2%	0.285	0.069	0.72	9.82%	0.285	0.681	0.44%
$\alpha = 0.4$	10.2%	0.285	0.069	0.72	9.75%	0.288	0.686	0.55%
$\alpha = 0.6$	10.2%	0.285	0.069	0.72	9.89%	0.283	0.685	0.32%

Table 7, Line 1, presents the results for the benchmark calibration of the rigid wage model following Hall (2005). These results show that the average unemployment rate is higher in the fluctuating economy. This is also the case for the average labor market tightness, and so for the average vacancies as well. Despite higher average vacancies, the welfare costs of fluctuations induced by the structural model are identical to those obtained in the reduced form model. Actually, the effect of higher vacancies on the business cycle costs appears to be negligible[30].

[30]The increase in average vacancies has little impact on aggregate consumption because comparatively to employment, vacancies do not weigh much in aggregate net production. First, the number of vacancies is small

More striking is the absence of the influence of productivity fluctuations on the average job finding rate. This result can be considered as disappointing since the endogenization of the job finding rate should potentially bring about other asymmetries: productivity shocks may have non linear effects on the job finding rate as the result of the two antagonist effects present in the rigid wage equilibrium, namely the θ-convexity and p-concavity effects. To understand this surprising neutrality, let us approximate the response of the job finding rate using the comparative statics of the model without aggregate shocks[31]. The job finding rate is then:

$$p(y) = \left[\beta \frac{1}{\kappa} \frac{y - \bar{w}}{1 - \beta(1-s)}\right]^{\frac{\alpha}{1-\alpha}}$$

The stabilization of labor productivity either decreases or increases the average job finding rate, depending on the value of the elasticity of the matching function α. The job finding rate is a concave (convex) function of labour productivity if α is below (above) 1/2 and the stabilized job finding rate is then higher (lower) relatively to that of the volatile economy. In the benchmark calibration with $\alpha = 1/2$, the θ-convexity and the p-concavity effects exactly compensate each other.

In order to illustrate this non-linearity and its magnitude, we then simulate two other cases: $\alpha = 0.4$ and $\alpha = 0.6$ (last two lines of Table 7). The values of the parameters κ and z have been changed accordingly in order to still match the job finding rate characteristics[32]. Depending on α, the US welfare cost of fluctuations could reach 0.55% or reduce to 0.30%. Petrongolo and Pissarides (2001) estimate this elasticity to be between 0.3 and 0.5. This suggests that with $\alpha = 0.5$, our benchmark calibration gives a lower bound of the welfare costs of fluctuations. In the more realistic case ($\alpha = 0.4$), the average job finding rate in the fluctuating economy is inferior to the value which would be reached in the stabilized economy. The magnitude of the worsening in the business cycle costs is quite sizable, since it represents a third of the impact of the job finding rate's volatility and persistence. The internal mechanism of the matching model leads to quite high business cycle costs. Note that it occurs only when the labor market tightness θ (and so the job finding rate $p(\theta)$) is volatile enough to make the non-linearity operating. Replicating the volatility of both the labor market tightness and the job finding rate leads to sizable business cycle costs through different channels which are all at work in this structural model[33].

compared to the number of employed workers ($v = 8\%n$ for the benchmark calibration). Secondly, the flow cost of vacancy κ is estimated to be low relative to labor productivity ($\kappa = 0.239\bar{y}$ for the benchmark calibration)

[31] The comparative static of the model without aggregate shocks can be used to approximate the dynamic stochastic model if the shocks are persistent enough. See Mortensen and Nagypal (2003) for more details.

[32] Note that, in the $\alpha = 0.4$ case, the rigid wage defined at the median productivity is no longer in the bargaining set. The wage is then fixed at its highest value ensuring that the firm's value is still positive ($w = \text{argmin}_y \pi_y$).

[33] Note that the higher volatility implied by the unfiltered job finding rate process would lead to an even more substantial business cycle cost in the ($\alpha = 0.4$) case.

3.4 Non-linearities in a flexible wage economy: an illustration

Not only the non-linearity of the job finding rate due to the matching function is not specific to the rigid wage economy, but the flexible wage case adds other sources of non-linearity which depend on the specification of the matching model considered. The different mechanisms at work could modify the consequences of fluctuations on the average job finding rate. It is not possible to check this assertion in all cases of the flourishing literature on the Shimer puzzle. Let us illustrate this point by considering the Pissarides (2007) approach which is very close to the canonical matching model[34]. It consists of introducing a fixed cost K of recruiting. It modifies equation (15) as follows:

$$V_y = -\kappa + \beta\{q(\theta_y)\mathbb{E}_y(J_{y'} - K) + (1 - q(\theta_y))\mathbb{E}_y V_{y'}\}$$

The key point is that it makes the vacancy cost less dependent on the labor market tightness. In the traditional framework, following a positive productivity shock, tightness increases, and so it increases the vacancy cost, which puts a brake on the expansion of the job finding rate. In that case, when considering insider wage contracts, the equilibrium is defined by[35]:

$$\begin{aligned} \frac{\kappa}{q(\theta_y)} + K &= \beta\mathbb{E}_y\left[y' - w(\theta_{y'}) + (1-s)(\frac{\kappa}{q(\theta_{y'})} + K)\right] \\ w(\theta_y) &= \gamma(y + \kappa\theta_y + \beta K p(\theta_y)) + (1-\gamma)z \end{aligned}$$

The additional non-linearity between y and θ_y, coming from the wage equation, results from the consideration of a fixed hiring cost and an insider wage contract. This clearly illustrates that additional non-linearities can be introduced in the matching model through alternative assumptions, leading to higher or lower business cycle costs.

Appendix F shows that the condition which ensures that the job finding rate is lowered by business cycles is less stringent in this flexible wage environment. Wage adjustments modify the response of vacancies to productivity shocks, which in turn alter the non-linearity in the job finding rate. For the same value of α, the flexible wage framework leads to a higher stabilized job finding rate, and therefore to a higher cost of fluctuations. All in all, this result gives new implications to the Shimer puzzle literature: the way this puzzle is solved matters for the magnitude of business cycles costs. The next question is then to determine the most empirically relevant model which will then deliver the "realistic" magnitude of the costs generated by unemployment fluctuations.

[34] Even closer, Hagedorn and Manovskii (2008) have responded to the Shimer puzzle by suggesting that the problem is more in the way the model is calibrated than in the model itself. They show that the volatility of the job finding rate is high only if the non-market activity z is calibrated sufficiently close to the average productivity. The key point is the size of the percentage changes of profits in response to (productivity) shocks. Note that it implies calibrating z at a much higher value than a strict interpretation as an unemployment benefit would imply. This is not consistent with our assumption to not differentiate the disutility of working and of not working.

[35] Note that aggregate consumption (equation 18) is now: $c_y = y(1-u) + \kappa v - K * M(u, v)$.

4 Conclusion

This paper brings new insight to the business cycle cost analysis. It shows that non-linearities in the unemployment dynamics caused by frictions on the labor market can generate sizable costs of fluctuation. Using a reduced-form model of the labor market, these costs are estimated to be almost two orders of magnitude greater than those of Lucas (1987). Our results emphasize that the welfare cost of fluctuations does not only depend on the variability of aggregate shocks. The persistence of these shocks, but also the level of structural unemployment have great implications. Furthermore, a high structural unemployment rate magnifies the welfare consequences of the volatility and the persistence of macroeconomic shocks. These findings extend to the case where the job finding rate is endogenized, provided the model is able to generate realistic fluctuations in the job finding rate. It could suggest that business cycles may reduce average consumption by more in continental European countries which would then suffer from both higher structural unemployment and more costly unemployment fluctuations. Business cycle costs would not be alike across countries.

We also show in the rigid wage version *à la* Hall (2005) that the internal mechanisms of the matching model matter for the magnitude of business cycle costs as they impact the average job finding rate through different non-linearities. It then remains to go further in the identification of the factors at the origin of unemployment fluctuations. The answer to this question is also important for stabilization policies. In this paper, nothing is said about the design and the efficiency of stabilization policies. It can be argued that the business cycle cost gives an upper bound of the benefits of stabilization policies and we agree that the cost of fluctuations and the benefits of stabilization policies must be distinguished, even if they are closely connected. An estimation of stabilization policy presupposes a view of the type of shocks that affect the economy, of the way they affect it, but also of the amount of variability that stabilization policies can (or should) remove. As is standard in the matching literature, we assumed that labor productivity shocks are the only source of fluctuations. Considering other shocks could bring about different non-linearities, and induce different interactions with the job finding rate volatility. This question is left for further research.

We neglect in this paper several dimensions strongly related to unemployment, which could have changed our quantitative measure of the business cycle costs. First, employment is not considered as providing extra welfare disutility relative to unemployment. This question is highly debated among the profession. Without any doubt, our choice increases the magnitude of business cycle costs. On the other hand, unemployment benefits do not lead to distortive taxation in our theoretical framework. Business cycles, by increasing average unemployment, could imply higher taxes, which would, in turn, weigh employment down. This could have been counted as a cost of the business cycles. Another dimension which could magnify these costs is the loss of human capital generated by unemployment spells reflected in the permanent decrease in wages observed in data. Is it compensated for by more intense human capital accumulation in expansion? All these points would deserve to be addressed to obtain a more general assessment of the welfare cost of unemployment fluctuations.

From a methodological standpoint, this paper pleads for not linearizing models of business cycles. We show that the non-linearities embodied in the matching model lead to significant business cycle costs. One may suspect that the intensive use of log-linearization in the DGSE approach is misleading. This is the case for the labor market but it could be also the case for other markets. A reappraisal of business cycle models on this basis would be a fruitful area for research.

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A The unemployment asymmetry gap

The conditional steady state can be written as a function of the job finding rate:

$$\widetilde{u}_i = \widetilde{u}(p_i)$$

Let $\nu_i = p_i - \bar{p}$, the conditional unemployment rate is therefore:

$$\widetilde{u}_i = \widetilde{u}(\bar{p} + \nu_i)$$

Because the unemployment rate is a convex function of the job finding rate, volatility in the job finding rate affects average unemployment. The unemployment gap ψ_p between an economy characterized by a stable job finding rate and an economy with a volatile job finding rate can be computed as follows:

$$\sum_i \pi_i \widetilde{u}(\bar{p} + \nu_i) = \bar{u} + \psi_p$$

A second order approximation of the left hand side yields:

$$\sum_i \pi_i \left[\widetilde{u}(\bar{p}) + \nu_i \widetilde{u}'(\bar{p}) + \frac{\nu_i^2}{2} \widetilde{u}''(\bar{p}) \right] \approx \bar{u} + \psi_p$$

Which gives:

$$\begin{aligned} \psi_p &\approx \frac{\sigma_p^2}{2} \widetilde{u}''(\bar{p}) \\ \psi_p &\approx \sigma_p^2 \frac{\bar{s}}{(\bar{s} + \bar{p})^3} \end{aligned}$$

A similar calculation gives for the job separation rate:

$$\begin{aligned} \psi_s &\approx \frac{\sigma_s^2}{2} \widetilde{u}''(\bar{s}) \\ \psi_s &\approx -\sigma_s^2 \frac{\bar{p}}{(\bar{s} + \bar{p})^3} \end{aligned}$$

B Average unemployment: analytical results

The unemployment dynamics read:

$$u_{t+1} = s + (1 - s - p_t)u_t \tag{23}$$

Define $\phi_t \equiv 1 - \bar{s} - p_t$ and $\mathbb{E}[\phi] \equiv \bar{\phi} = 1 - \bar{s} - \bar{p}$.

$$\phi_t = \rho\phi_{t-1} + (1-\rho)\bar{\phi} - \varepsilon_t^p$$

Where ε_t^p is the innovation of the job finding rate process. It is iid, has mean zero and variance $\sigma^2_{\varepsilon_p}$.

The unemployment dynamics can be written:

$$u_{t+1} = \bar{s} + \phi_t u_t \tag{24}$$

A backward substitution gives:

$$\begin{aligned} u_{t+1} &= \bar{s} + \sum_{k=0}^{+\infty}\prod_{j=0}^{k}\phi_{t-j}\bar{s} \\ \mathbb{E}[u_{t+1}] &= \bar{s}\left(1 + \sum_{k=0}^{+\infty}\mathbb{E}[\prod_{j=0}^{k}\phi_{t-j}]\right) \end{aligned}$$

We can write ϕ infinite moving average representation:

$$\phi_t = \bar{\phi} - \Sigma_{j=0}^{+\infty}\rho_p^j\varepsilon_{t-j}^p$$

And the mean of unemployment can be written as:

$$\mathbb{E}[u] = \bar{s}\left(1 + \mathbb{E}\Sigma_{k=0}^{+\infty}[(\bar{\phi} - \Sigma_{j=0}^{+\infty}\rho_p^j\varepsilon_{t-j}^p)...(\bar{\phi} - \Sigma_{j=0}^{+\infty}\rho_p^j\varepsilon_{t-j-k}^p)]\right)$$

Which can be approximated by:

$$\mathbb{E}[u] \approx \bar{s}\left(\frac{1}{1-\bar{\phi}} + \frac{\sigma^2_{\varepsilon^p}}{1-\rho_p^2}\sum_{k=2}^{+\infty}(\bar{\phi}^{k-2}\sum_{i=0}^{k-1}(k-1-i)\rho_p^{i+1})\right)$$

A similar calculation gives the consequences of job separation rate fluctuations on the average unemployment rate:

$$\mathbb{E}[u] - \bar{u} \approx -\frac{\sigma^2_{\varepsilon^s}}{1-\rho_s^2}\left[\sum_{k=1}^{+\infty}(1-\bar{s}-\bar{p})^{k-1}\sum_{i=0}^{k-1}\rho_s^{i+1} - \sum_{k=2}^{+\infty}\bar{s}(1-\bar{s}-\bar{p})^{k-2}\sum_{i=0}^{k-1}(k-1-i)\rho_s^{i+1}\right]$$

C Job finding rate and separation rate data

Figure 7: Job finding rate - Shimer data

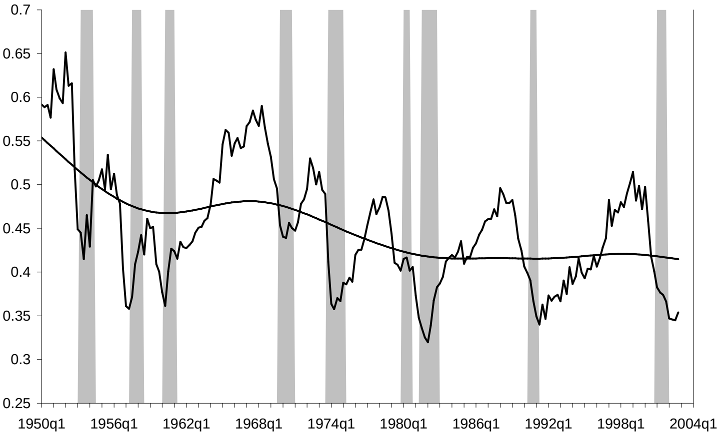


Figure 8: Job finding rate - Hall data

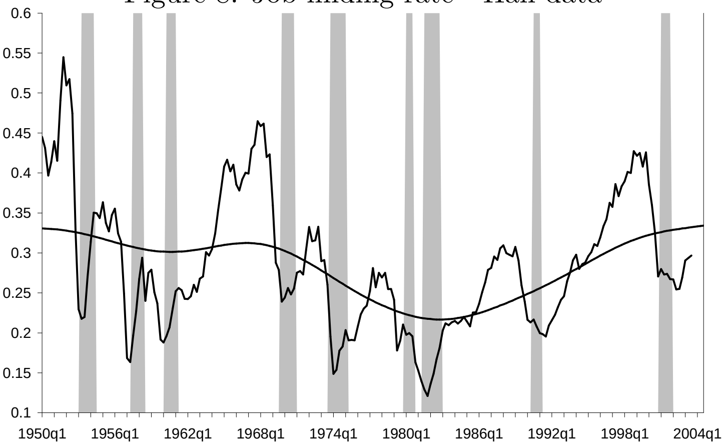


Figure 9: Separation rate - Shimer data

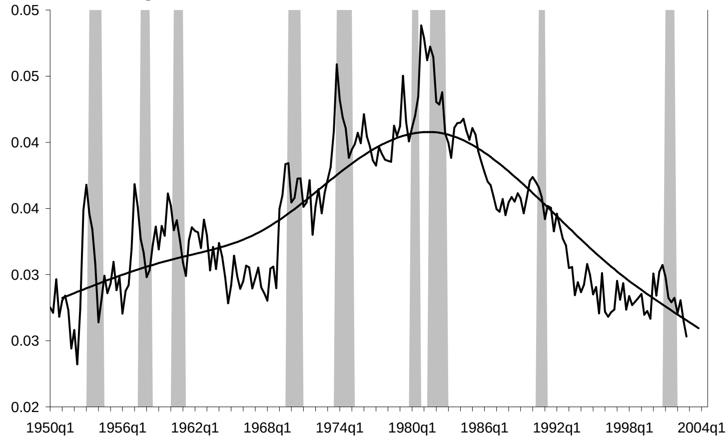
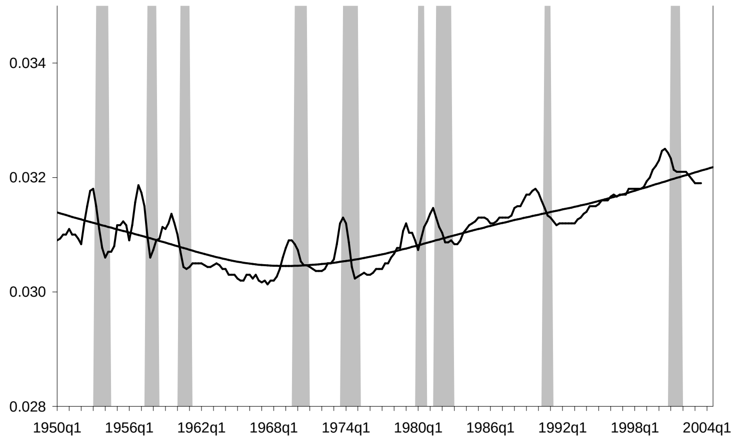


Figure 10: Separation rate - Hall data



D Alternative specification: log-normal shocks

We estimate the following AR(1) processes:

$$ln(p_t) = a_{lp} + \rho_{lp} ln(p_{t-1}) + \varepsilon_t^{lp} \quad (25)$$

$$ln(s_t) = a_s + \rho_{ls} ln(s_{t-1}) + \varepsilon_t^{ls} \quad (26)$$

Table 8 and 9 shows that the implied characteristics for the level of p and s are relatively close to those displayed in Tables 1 and 2. This is particularly the case when considering H-P filtered data. Using the HP filter makes the choice of estimating in log or in level pointless. On the other hand, without filtering, it appears that the log-linearity decreases the cost of business cycles. As the distribution of p is shifted to the right, the non-linear effect of the fluctuations in p on u is weaker: p fluctuates over a less convex region (Figure 11).

Figure 11: The implications of log-normality

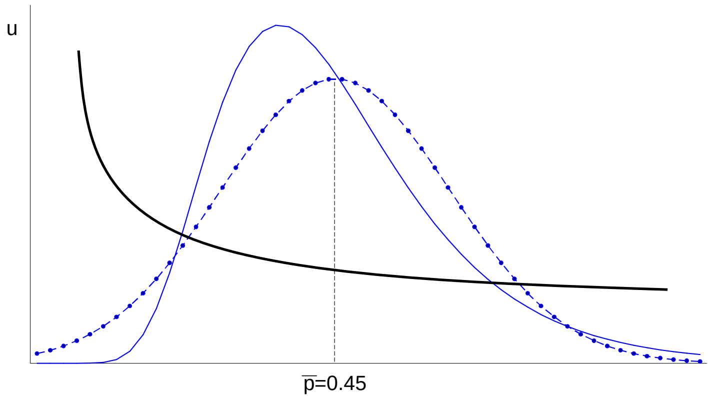


Table 8: Job finding rate fluctuations under log-normality

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Estimated process of $ln(p)$				
Standard deviation of ε^{lp}	0.098	0.097	0.049	0.048
Autocorrelation ρ_{lp}	0.941	0.913	0.941	0.914
Implied dynamics of p				
Mean $\bar{p}$	0.285		0.450	
Standard deviation σ_p	0.084	0.068	0.068	0.053
Autocorrelation	0.942	0.913	0.939	0.915
Business cycle cost	0.63%	0.43%	0.13%	0.08%

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

E Non linearities in the endogenous job finding rate: the rigid wage case

This Appendix gives the condition, for a rigid wage economy, under which the average job finding rate is higher in the volatile economy than in the stabilized economy. Following Shimer (2005) and Mortensen and Nagypal (2003), we approximate the response of the job finding rate in the dynamic stochastic model using the comparative statics of the model without aggregate shocks. The expected surplus from a filled job can then be written simply as:

$$J_y = y - \bar{w} + \beta(1-s)J_y$$

And the free entry condition:

$$-\kappa + \beta q(\theta_y)J_y = 0$$

Table 9: Job separation rate fluctuations under log-normality

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Estimated process of $ln(s)$				
Standard deviation of ε^{ls}	0.005	0.004	0.055	0.051
Autocorrelation ρ_{ls}	0.970	0.873	0.941	0.734
Implied dynamics of s				
Mean $\bar{s}$	0.031		0.034	
Standard deviationσ_s $\times 10^{-2}$	0.058	0.029	0.540	0.260
Autocorrelation	0.970	0.873	0.946	0.734
Business cycle cost	0%	0%	-0.006%	0%

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both data are detrended with a HP smoothing parameter of 10^5.

Which yields:

$$\frac{\kappa}{q(\theta_y)} = \beta \frac{y - \bar{w}}{1 - \beta(1 - s)}$$

As $q(\theta_y) = \theta_y^{-1+\alpha}$, the equilibrium value of the vacancy-unemployment ratio is:

$$\theta_y = \left[\beta \frac{1}{\kappa} \frac{y - \bar{w}}{1 - \beta(1 - s)} \right]^{1/(1-\alpha)}$$

Which gives

$$p(y) = \left[\beta \frac{1}{\kappa} \frac{y - \bar{w}}{1 - \beta(1 - s)} \right]^{\frac{\alpha}{1-\alpha}}$$

The stabilisation of the productivity process affects the job finding rate in a non linear fashion. The consequences on the average job finding rate depend on α.

$$\frac{\partial^2 p}{\partial y^2} = \frac{(2\alpha - 1)\alpha}{(1 - \alpha)^2} \left[\beta \frac{1}{\kappa} \frac{y - \bar{w}}{1 - \beta(1 - s)} \right]^{\frac{3\alpha - 2}{1-\alpha}} \left(\frac{\beta}{\kappa(1 - \beta(1 - s))} \right)^2$$

The job finding rate is a concave function of labor productivity if $\alpha < 1/2$. In this case, stabilizing the process of labor productivity increases the average job finding rate.

F Non linearities in the endogenous job finding rate: the flexible wage case

In this Appendix, we show that the condition ensuring the concavity of the job finding rate is less stringent when wages are flexible than when wages are rigid. As in Appendix E, we use the comparative statics of the model without aggregate shocks to approximate the response of the job finding rate.

For a level y of productivity, the equilibrium vacancy-unemployment ratio θ is characterized by the following equation.

$$H(\theta, y) \equiv \frac{\kappa}{q(\theta)} + \beta K + \frac{\gamma\beta(\kappa\theta + p(\theta)\beta K)}{1-\beta(1-s)} - \frac{\beta(1-\gamma)(y-z)}{1-\beta(1-s)} = 0$$

From which we derive:

$$\begin{aligned}\frac{\partial\theta}{\partial y} &= -\frac{\partial H/\partial y}{\partial H/\partial\theta} \\ &= \frac{\frac{\beta(1-\gamma)}{1-\beta(1-s)}}{-\kappa\frac{q'(\theta)}{q(\theta)^2} + \frac{\gamma\beta}{1-\beta(1-s)}[\kappa + \beta K p'(\theta)]}\end{aligned}$$

The second-order derivative of $p(y)$ can then be written:

$$\frac{\partial^2 p}{\partial y^2} = \alpha(1-\alpha)\frac{[\bar{\Psi}(1-\gamma)]^2}{(\Gamma(\theta) + \gamma\bar{\Psi}\beta K p'(\theta))^2}\left[-\theta^{\alpha-2} + \frac{\alpha\theta^{-2}}{\Gamma(\theta) + \gamma\bar{\Psi}\beta K p'(\theta)}[\kappa + \gamma\bar{\Psi}\beta K\theta^{2\alpha-1}]\right]$$

where $\bar{\Psi} = \frac{\beta}{1-\beta(1-s)}$ and $\Gamma(\theta) = -\kappa\frac{q'(\theta)}{q(\theta)^2} + \frac{\gamma\beta\kappa}{1-\beta(1-s)}$

The job finding rate is a concave function of productivity if:

$$\frac{\partial^2 p}{\partial y^2}(\alpha) < 0$$

$$\begin{aligned}-\theta^{\alpha-2}[\kappa(1-\alpha)\theta^{-\alpha} + \bar{\Psi}\gamma\kappa + \bar{\Psi}\gamma\beta K\alpha\theta^{\alpha-1}] + \alpha\theta^{-2}[\kappa + \bar{\Psi}\gamma\beta K\theta^{2\alpha-1}] &< 0 \\ -\kappa(1-\alpha)\theta^{-2} - \bar{\Psi}\gamma\kappa\theta^{\alpha-2} + \alpha\theta^{-2}\kappa &< 0\end{aligned}$$

This condition can be rewritten:

$$G(\alpha) \equiv 2\alpha - 1 - \frac{\beta}{1-\beta(1-s)}\gamma\theta^{\alpha} < 0$$

If $\alpha = 1/2$, then:

$$G(1/2) \equiv -\frac{\beta}{1-\beta(1-s)}\gamma\theta^{1/2} < 0$$

The job finding rate is a strictly concave function of productivity when $\alpha = 1/2$ (i.e at the rigid wage threshold). Because $2\alpha - 1 < 0$ for $\alpha < 1/2$, this restriction is also satisfied for any $\alpha \in [0, 1/2]$. Then, we deduce that the concavity of the job finding rate is more probable in the case of flexible wages.