The Dynamics of Environmental Concern and the Evolution of Pollution*

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Abstract

We develop an overlapping generations model within which the evolution of pollution and the formation of environmental concern are endogenous. On the one hand, people heterogeneously concerned with environmental issues contribute to pollution which is a public bad. On the other hand, the transmission of environmental attitudes is the result of some economic choice which is affected by pollution. The model predicts that the long run proportion of environmentally concerned individuals will always be high. Though, depending on the pollution-generating technology, the transition from a low-environmentally concerned society to a high-environmentally concerned one is accompanied by two different outcomes regarding the long run level of pollution. If the technology is "clean", there is a stable steady state level of pollution. However, if it is "dirty", pollution experiences unlimited growth. This result captures some stylized facts regarding the joint evolution of environmental concern and pollution in developing nations. In the latter case, we show that intergenerational transfers from the older generation to the young working one restore the possibility to reach a stationary level of pollution.

keywords: Overlapping generations, pollution, environmental concern, cultural transmission, environmental policy.

JEL Classification: Q50, D90, J11.

1 Introduction

Many environmental issues which depend on consumer's choices could be efficiently reduced by voluntarily emission reduction actions (Human development report [2007/2008], chapter 3). Accordingly, there is a growing literature dedicated to a better understanding of consumer behavior. In particular, some studies have revealed the existence of a gap between people's perceptions and the actual level of pollution (Murch [1971]). Then, many have

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focused on the determinants of these perceptions in order to make recommendations about environmental education (Bogner and Wiseman [1997], Bickerstaff and Walker [2001]). This suggests that more environmental concern, as defined by higher perception, will necessarily generate good outcomes for environmental quality. From a static point of view, it is obvious that an increase in environmental perceptions leads to more voluntary reduction actions. However most environmental issues encompass intergenerational aspects which cannot be embedded with such a perspective. This paper adopts a dynamic approach of the impact of environmental concern on the environment.

Especially, we ask what will occur to the long-run level of pollution if preferences are endogenously shifted toward more environmental concern. We show that when consumers are able to transfer the cost of their actions to the future, the change in environmental attitudes does not always have the same implications for the long run evolution of pollution. In particular, this result crucially depends on the technology.

There is empirical evidence supporting the fact that increasing populations' concern does not necessarily generate good environmental outcomes. In particular, many kinds of pollution within the developing world have shown upward pattern while the same kinds of pollution have mostly decreased in industrialized nations. Though number of studies based on different international surveys have shown that the development of environmental values has not been confined to industrialized nations but that it has rather been a worlwide phenomenon (Dunlap, Gallup and Gallup [1992], Brechin and Kempton [1994], Dunlap and Mertig [1995], Brechin [1999], Dunlap and York [2008]). Especially air pollutants, such as nitrogen dioxide (NO_2), are experiencing a declining pattern in most developed nations : in Germany, the United States and the United Kingdom, it has respectively decreased by 48%, 12% and 26% between 1995 and 2000 (UNEP (2012)). Besides, the Health of the Planet survey (HOP) (quoted in Dunlap, Gallup and Gallup [1992]) has revealed that the share of people concerned with air pollution in the three latter nations was around 60% (60, 60, 52 respectively) in 1992. On the other hand, nitrogen dioxide levels are mostly rising within the developing world : in Brazil, India and Mexico, it has respectively increased by 59%, 34%, and 28% during the same period. However, according to the HOP survey the share of people concerned with air pollution reached 70%, 64% and 77% in 1992 in the respective nations. Hence, in those nations the increase of concern for air pollution has been accompanied by a rise in air pollution.

Many authors in the fields of environmental sociology and psychology have paid attention to this significant change in people's environmental attitudes. They identify two main forces. First, they clearly recognize a role for objective environmental conditions. Nevertheless the way it is involved remains a puzzling question. Indeed, as pointed out by Dunlap and Merting [1997] "a simple stimulus-response model of the role of objective environmental problems is much too simplistic [...] and ignores the documented complexities of environmental perception.". Second, they stress the importance of some cultural forces as transmission through social interactions. In particular for Brechin [1999] "concerns for global environmental problems [...] are likely transferred shared and learned". However, in spite of factual evidence, the question as to why increasing concern may result in higher levels of pollution has found few theoretical background[1].

We provide an explanation for both the population dynamics and the evolution of pollution by means of a formal model with microeconomic foundations. More precisely, we develop a theoretical framework whereby heterogeneity regarding environmental attitudes and the level of environmental degradations are endogenously determined. On the one hand, people heterogeneously concerned with environmental issues can voluntarily reduce pollution, a public bad, but contribute to increase it through their consumption activity. That is, the level of pollution is determined by the composition of the population. On the other hand, the transmission of environmental attitudes is the result of some economic choice which is affected by pollution.

As a first step, we consider preference heterogeneity as given. We identify different long run equilibria according to the share of environmentally concerned agents. Interestingly, we highlight a crucial role for pollution-generating technologies in the relationship between the composition of the population and the long run dynamics. That is, if the pollution-generating technology is "clean", we show that a relatively high share of agents with environmental concern is needed for pollution to reach a steady state. On the contrary, if it is "dirty", pollution stabilization requires that the share of agents with environmental concern be relatively low. In the latter case, a high share of people with environmental concern is associated with ever-increasing pollution. This can be explained as follows. Agents make a trade-off between saving for consumption and an effort to reduce pollution. The crucial feature is that consumption of the current generation affects pollution bequeathed to the next generation. Then, if the intensity of this "intergenerational consumption spill-over" is low while the efficiency of their abatement effort is high (what we call "clean" technology), as pollution rises, agents with environmental concern gradually substitute abatement effort for consumption. This is because, in this case, the cost of cleaning-up (expressed in forgone units of consumption) is low. However, if this externality is high compared to the efficiency of abatement ("dirty technology"), for agents highly affected by pollution the cost of cleaning-up is extremely high. Then as pollution rises, they gradually have an incentive to forgoe pollution abatement to compensate with consumption.

As a second step, we endogenize heterogeneity to determine which equilibria actually arise in the long run. To be consistent with the socio-psychological studies quoted before, we consider the formation of environmental attitudes as the outcome of some intergenerational cultural transmission process which interacts with environmental conditions. We build on the framework provided by Bisin and Verdier [1998, 2001] where adoption, by one child, of some cultural trait results from the interaction of a socialization effort exerted by its parents and the transmission by its social environment (as given by peers). The level of pollution is involved in this socialization process : as pollution increases the value that their offspring acquire their trait, it increases the relative socialization effort of parents endowed with environmental concern. When heterogeneity is dynamic, we find that two equilibria arise

[1] There has been some psychological explanations for the fact that environmentally friendly attitudes do not necessarily lead to reduce pollution. Those ones rely on the existence of a gap between attitudes and behaviors (see for instance Kollmuss and Agyeman [2002])

in the long run. Especially, the model predicts the spread of environmental concern within the population such that in the long run the proportion of people with environmental concern will always be high. However, depending on the pollution-generating technology, the transition from a low-environmentally concerned society to a high-environmentally concerned one may be accompanied by pollution stabilization but also by ever-increasing pollution. The intuition is as follows. In this framework, higher relative socialization efforts of parents of one type have a positive dynamic impact on the transmission of this type (and so on the future composition of the population). In our set up, initially, the functioning of the socialization mechanism as well as choices of saving both positively affect the relative socialization efforts of parents endowed with environmental concern (directly for the first effect or indirectly through the pollution level for the second effect). Therefore, the society experiences a cultural change, heading toward a relatively high share of agents with environmental concern. This one-way population change may be associated with two different long run equilibria. Indeed, the link between one given composition of the population and the long run dynamics of pollution depends upon the pollution-generating technology. Therefore, if the technology is clean, the transition toward a highly concerned society is accompanied by pollution stabilization. Nevertheless, if this is dirty, while the same cultural change occurs, pollution experiences unlimited growth.

These results have clear policy implications since sustainability (if defined as reaching a stationary state) will not be achieved for economies that do not have clean enough technology. Thereby, we focus on the implementation of environmental policies. We show that for those economies, sustainability is still achievable by applying some intergenerational transfers from the old generation of consumers to the young one.

This paper is first related to the literature which studies environmental issues in a dynamic overlapping generations framework. Our framework relies on the setting proposed by John and Pecchenino [1994] which has been taken up by Schumacher and Zou [2008]. We come closer to the latter using their idea of pollution perception. Both articles, however, consider a representative agent while we are dealing with a heterogeneous society. In this respect, we are closer to those who study the effect of some kind of heterogenity on the dynamics of pollution (Jouvet et al. [2000], Ikefuji and Horii [2007]). We differ from the latter because we consider endogenous population changes in response to changing environmental conditions. This has been dealt with in de la Croix and Gosseries [2012] (but with a representative agent) or in Raffin [2010] (with a heterogeneous society). However, none of these works introduces a microfounded mechanism of preferences transmission. In particular, our paper builds on economic models of cultural transmission as in Bisin and Verdier [1998, 2001] but within an environmental economics framework.

The remainder of this paper is organized as follows. In section 2, we set up the framework related to pollution and capital accumulation. As a first step we consider the composition of the population as given to characterize some dynamic properties. In section 3, we endogenize preference heterogeneity and study dynamic interactions between the distribution of environmental attitudes and the economic sphere (as given by pollution and capital). In section 4, we examine some public policies that allow to achieve sustainability. Section 5 concludes.

2 The Model

Consider a perfectly competitive economy with an OLG structure. Time is discrete and goes from 0 to ∞. At each date a generation is born and lives for two periods. During the first period agents supply their labor inelastically and earn a wage which can be either saved for future consumption or spent for the abatement of pollution. Furthermore, following John and Pecchenino [1994] we assume agents have preferences over consumption and the environment and can only derive utility during the second period of life (which stands for a period of retirement).

Each cohort, of constant size n, consists of two types of agents who differ in their concern for the environment. Highly concerned individuals are associated with the superscript G. Low-concerned ones are associated with the superscript T. There are n_G agents of type G, $n_T = n - n_G$ agents of type T.

2.1 Agents' Perceptions

To model environmental perception we rely on the specification introduced in Schumacher and Zou [2008]. The latter is suggested by the idea that people are probably not fully aware of the past quality of the environment. It implies that their baseline regarding pollution should be related to what they experienced so far. Accordingly, they should not be affected by the actual level of pollution but by a variation compared to their baseline[2]. Heterogeneity of agents about their environmental concern is captured by differences with regard to this baseline. In a model where agents live for two periods, the amount which affects an agent of type i, namely the perceived level of pollution is

$$H_{t+1} = P_{t+1} - h^i P_t,$$

where P_t (resp. P_{t+1}) is the pollution stock at time t (resp. $t+1$). In this specification, h^i is a parameter specific to the type of agents. On the one hand, some individuals are concerned with environmental degradations and are convinced of a harmful human impact on its natural environment. That's why they think the past level of pollution P_t was already high. Their baseline is relatively close to zero (and so is h^G) and the current level of pollution is perceived as high. On the other hand the second type of agents, the "technologists" strongly trust science and innovations. They think that unlimited growth is possible thanks to technological progress. Thereby, in their view, the past level of pollution could not be high since innovations should have solved encountered environmental issues. Their baseline is high (and so is h^T) so that their perception of P_{t+1} is low. Hence, we have $h^G < h^T$.

[2] This modelling has ever been used in the OLG framework to include habit in consumption (De la Croix and Michel [1999])

2.2 Pollution Accumulation

The pollution accumulation process is described by the following equation based on John and Pecchenino [1994]:

$$P_{t+1} = (1-b)P_t + \beta n_G c_t^G + \beta n_T c_t^T - \gamma n_G A_t^G - \gamma n_T A_t^T, \quad (1)$$

where $b \in [0,1]$ is a natural rate of absorption. Pollution increases with consumption of the old generation, $\beta > 0$ being the consumption externality while it can be reduced by abatement spendings of the current generation with $\gamma > 0$ being the effectivness of abatement. As in Jouvet et al. [2000], in this framework, abatement takes the form of a voluntary contribution. As highlighted by Nyborg and Howarth [2006] "The fact that people voluntarily incur private costs that serve to increase the supply of environmental quality is striking and important observation." (p.352). Indeed voluntary contributions can be shown to play an important role in the environmental framework in spite of the high number of potential subscriptors (see Kotchen [2003, 2007, 2009] for empirical evidence and theoretical proof). Prominent examples are regularly sorting wastes actions, the purchase of green products, the purchase of local products and investments in new form of home heating. In particular, according to the Eurobarometer [2008], 90 % of European Union citizens have accomplished at least one such private action for environmental preservation during the last 3 months.

2.3 Economic Choices

During the first period agents have to choose the mix of abatement-saving which maximizes utility, $U(c_{t+1}^i, H_{t+1}^i)$, which depends upon the consumption when old and the perceived level of pollution. We assume that, for any agent, $U : \mathbb{R}_{++}^2 \to \mathbb{R}$, is twice continuously differentiable and such that $U_c > 0$, $U_{cc} \le 0$, $U_H < 0$, $U_{HH} \le 0$ and $\lim_{c\to 0} U_c = \infty$, $\lim_{H\to 0} U_H = -\infty$.

An agent of type i, $i \in \{G, T\}$, faces the following budget constraints

$$w_t = s_t^i + A_t^i, \quad (2)$$

$$c_{t+1}^i = (1 + r_{t+1})\, s_t^i, \quad (3)$$

where w_t is the agent's wage (whatever the type) at time t, s_t^i is the amount of saving of a type-i individual and r_t is the interest rate at time t.

An agent of type i maximizes $U\left(c_{t+1}^i, H_{t+1}^i\right)$ subject to the budget constraints (2) and (3), as well as to

$$P_{t+1} = (1-b)P_t + \beta n_G c_t^G + \beta n_T c_t^T - \gamma A_t^i - \gamma \bar{A}_t$$

where $\bar{A}_t$ includes abatement spendings of all other agents of the cohort[3]. This leads to the following first order conditions

$$(1 + r_{t+1})U_c^i + \gamma U_H^i = 0, \qquad \forall i \in \{G, T\}. \tag{4}$$

Following Zhang [1999] and Schumacher and Zou [2008], for analytical convenience we restrict to simple class of utility functions so that we assume the following parameter to be constant

$$\sigma = -\frac{U_H^i H^i}{U_c^i c^i} > 0, \qquad \forall i \in \{G, T\}. \tag{5}$$

This assumption implies that preferences are homothetic. Note that σ does not change from one type of agent to another. As highlighted in the introduction, agents do not differ in their taste (or distaste) for pollution but only in the level they perceive.

Using (5), let us rewrite FOC as

$$\gamma\sigma s_t^i = P_{t+1} - h^i P_t, \qquad \forall i \in \{G, T\}. \tag{6}$$

Then one can express the type-i agent's best-response function as

$$A_t^i = \max\{\frac{\sigma w_t}{\sigma - 1} - \frac{(1 - b - h^i)P_t}{\gamma(\sigma - 1)} - \frac{\beta(n_G c_t^G + n_T c_t^T)}{\gamma(\sigma - 1)} + \frac{\bar{A}_t}{\sigma - 1}, 0\}, \tag{7}$$

where $\sigma > 1$, which means that there are strategic complementarities[4]. We assume that the conditions for the existence and uniqueness of a Nash equilibrium hold (see Appendix 5.1).

From equation (6), we can deduce that for a given stock of pollution at time $t + 1$, agents of type G always choose to save more than those of type T. To understand, note that the

[3]Agents are supposed to voluntarily contribute to pollution reduction because they suffer from the aggregate level of the public bad. In a similar similar but static set up, Vicary [2000] and Kotchen [2009] show that contrary to the results obtained in the pure public good model of Andreoni [1988], as the number of subscriptors grows, the level of contribution does not fall to zero. Actually, in their model, consumption activity increases the level of the public bad (or decreases it if a public good) and this cost can be compensated by a positive contribution. Compared to traditional models of pure public good, it increases the incentive to voluntarily contribute. In our framework, consumption affects the future level of the public bad. Hence, here positive contributions can be used to offset the cost of past consumption so that the same conclusion holds. However, this is not always true since not only contributions but also consumption can be used to mitigate this cost. Therefore we impose necessary but not sufficient conditions for positive contributions (in Appendix).

[4]For consistency, the magnitude of σ must be much higher than one. Otherwise abatement spendings would be negligible. To see this, note that given equation (5), the general shape of $U(c, H)$ is $B.c^a H^b$, where $B \in \mathbb{R}$ and $\frac{b}{a} = \sigma$. Hence, assuming $\sigma \sim 1$ implies $a \sim b$. Denote by $S_s = \frac{s}{w}$, the share of saving in income and $S_A = \frac{A}{w}$, the share of abatement expenditure in income. One can find $S_s = \frac{a}{a - \gamma b(A/H)}$ and $S_A = \frac{b/n}{a - \gamma b(A/H)}$. Accordingly, if the value of σ is close to one, then the share of abatement in income is n times lower than the share of saving in income. Then, for consistency the magnitude of σ must be of order n.

former have a higher demand for environmental quality. Indeed, suppose the society consists in a single representative agent of type G. The demand for environmental quality is then,

$$-P^*_{t+1} = -\frac{\gamma\sigma}{1-\sigma} + \frac{\sigma((1-b)P_t + \beta c_t)}{1-\sigma} - h^G P_t.$$

This is higher than the demand for enviromental quality if the society consisted in a single representative agent of type T, since the term $-h^G P_t$ would be replaced by $-h^T P_t$ which is lower. However, when the society consists in n agents of both types, the level of environmental quality provided by the society, which stems from the two kinds of demand is too low for type G so that the latter have to consume more. In other words, agents with environmental concern suffer more from intragenerational externalities which requires them to compensate with consumption. Our purpose is then to study how a larger number of agents with environmental concern affects the level of pollution.

2.4 The Representative Firm

The productive sector consists in a perfectly competitive representative firm which produces using the constant returns to scale production function $Y = f(K)L$. We normalize by labor supply so that output per worker is written as $y = f(k)$, where $k = K/n$. We assume that f meets the following properties : $f^{'} > 0$ and $f^{''} < 0$.

The firm maximizes its profit equalizing each marginal productivity with its market price. Hence, the marginal productivity of labor equals the wage rate and the marginal productivity of capital minus its depreciation rate equals the interest rate. Finally total savings of the young generation form the capital at time t+1,

$$w_t = f(k_t) - k_t f'(k_t), \tag{8}$$

$$r_{t+1} = f'(k_{t+1}) - \delta, \tag{9}$$

$$\frac{n_G}{n} s_t^G + (1 - \frac{n_G}{n}) s_t^T = k_{t+1}. \tag{10}$$

For analytical convenience we restrict to a Cobb Douglas production function $f(k_t) = k_t^v$ (Zhang [1999], Schumacher and Zou [2008]) where $v \in [0,1]$ is the capital share in total production. We further assume full depreciation of captital, that is $\delta = 1$.
In what follows, let us denote by $q = \frac{n_G}{n}$, the share of type-G agents. Using the previous equations we deduce the intertemporal equilibrium.

Definition 1 (Intertemporal Equilibrium)**.** *Given inital conditions (P_0, k_0), the intertemporal equilibrium is the sequence $(P_t, k_t)_{t\in\mathbb{N}}$ which satisfies the two following equations for each t*

$$P_{t+1} = \frac{(b\sigma^{'} + \bar{h}(q) - \sigma^{'})}{(1-\sigma^{'})} P_t - \frac{(\beta v + \gamma v - \gamma)\sigma^{'}}{(1-\sigma^{'})} k_t^v, \tag{11}$$

$$k_{t+1} = -\frac{(1 - b - \bar{h}(q))}{\gamma(1-\sigma^{'})} P_t - \frac{(\beta v + \gamma v - \gamma)}{\gamma(1-\sigma^{'})} k_t^v, \tag{12}$$

where $\bar{h}(q) = qh^G + (1-q)h^T$ and $\sigma^{'} = \frac{\sigma}{n} \neq 1$. Note that P_{t+1} and P_t are now measured in per capital units of pollution.

Definition 2. *The pollution-generating technology*

(1) We say that A_1 holds if the effectiveness of abatement γ is high enough compared to the consumption externality β such that the positive effect of additionnal abatement allowed by the rise of wages, overcomes the negative effect entailed by the rise in consumption, that is $\beta v + \gamma v - \gamma < 0$. The technology is "clean".

(2) We say that A_2 holds if the effectiveness of abatement is small compared to the consumption externality such that the negative effect entailed by the rise in consumption, overcomes the positive effect of additionnal abatement allowed by the rise of wages, that is $\beta v + \gamma v - \gamma > 0$. The technology is "dirty".

2.5 Existence of Steady States

A steady state exists if some vector (k, P) solves $k_{t+1} = k_t$, $P_{t+1} = P_t$ for all t. There are two steady states: (0,0) and a non-trivial one, which depends upon q, so that we denote it by $(\bar{P}(q), \bar{k}(q))$. It is given by equations (13) and (14) below.

$$\bar{P}(q) = \frac{\sigma^{'}\gamma}{1-\bar{h}(q)} \left(\frac{(1-\bar{h}(q))(\beta v + \gamma v - \gamma)}{\gamma(b\sigma^{'} + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}} \tag{13}$$

$$\bar{k}(q) = \left(\frac{(1-\bar{h}(q))(\beta v + \gamma v - \gamma)}{\gamma(b\sigma^{'} + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}}. \tag{14}$$

Proposition 1. *Existence of a non-trivial steady state*[5]

Suppose that agents are different enough so that $h^T > 1 - \sigma b > h^G$.

It exists $\tilde{q} = \frac{b\sigma^{'} + h^T - 1}{h^T - h^G} < 1$ *such that,*

(1) If the share of agents concerned about pollution is relatively high ($q > \tilde{q}$)*, a non-trivial steady state exists if, and only if,* A_1 *holds. In this case a rise of* q*, the share of type* G *agents, induces a decrease of the steady state pollution and capital stocks.*

(2) If the share of agents concerned about pollution is small enough ($q < \tilde{q}$)*, then a non-trivial steady state exists if, and only if,* A_2 *holds. In this case, a rise of* q *induces an increase of the steady state pollution and capital stocks.*

In steady state, pollution and capital meet the two following equations (obtained from rearranging equations (11) and (12)) which respectively stands for the natural evolution of pollution with respect to capital and the aggregate optimal choices line for one given distribution of environmental attitudes,

$$P = \frac{1}{b}\left((\beta v + \gamma v - \gamma)k^v - \gamma k\right), \tag{15}$$

$$P = \frac{\gamma\sigma^{'}}{(1 - \bar{h}(q))}k. \tag{16}$$

We depict in Figure 1 (resp. 2) the graphs of the functions $k \mapsto P$ where P is respectively given by equation (15) and (16), when A_1 (resp A_2) holds. Steady states are represented by intersection points between the two curves.

2.5.1 Clean Technology

Figure 1 illustrates long run equilibria of an economy endowed with a clean technology. The natural evolution of pollution in steady state is described by the C_1 curve. We draw three lines corresponding to aggregate optimal choices in steady state for different values of q. L_1 stands for the optimal choices for $q = 1$, L_2 for $\tilde{q} < q < 1$ and L_3 for $q = 0$. As one can notice from this graph, a steady state exists only when the share of agents with environmental concern is sufficiently high.

The results can be interpreted as follows. In our framework, agents suffer from pollution which is increased by consumption, but which is reduced by voluntary reduction actions. Besides, consumption generates an intergenerational spillover because it affects pollution felt by the next generation. That is, past consumption generates a cost for the current generation. In response, the latter has two options. It can either decide to voluntarily

[5] Stability is investigated in appendix.

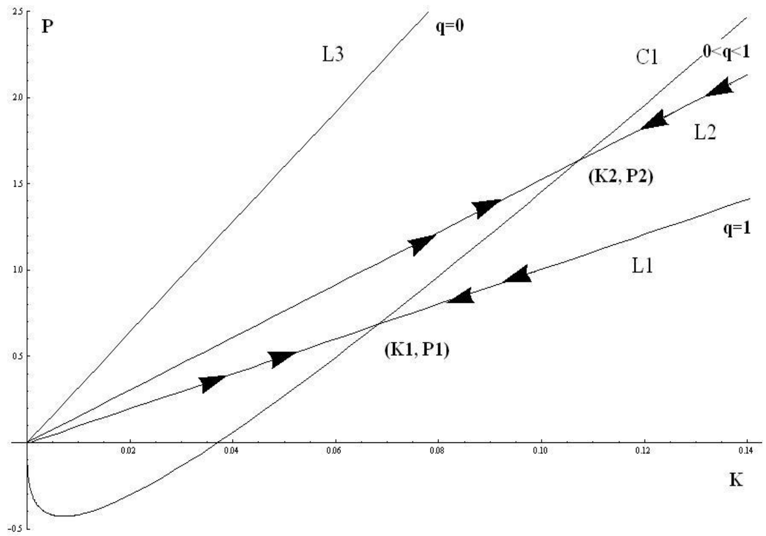

Figure 1: Long run equilibria in the (k,P) space when A_1 holds.

contribute to pollution reduction or, as consumption does not affect the current level of pollution, it can choose to increase consumption to compensate this cost.

When the technology is clean, the cost of past consumption is relatively low as compared to the efficiency of abatement. In such a case, agents of type G have an incentive to reduce the cost of past consumption by increasing their voluntary contribution. Indeed, since they have a strong perception of environmental issues, for them the cost of pollution is relatively high. But, as abatement is relatively efficient as compared to the cost of past consumption, they can reduce this cost by increasing the level of voluntary contribution without giving rise to a too large decrease in consumption[6]. That is, increasing the abatement effort does not entail a too high loss in terms of consumption so that they have an incentive to make a higher contribution (or put differently, in such a case, the cost of cleaning-up expressed in forgone consumption units is low). Hence, as the cost of past consumption increases, successive generations gradually have an incentive to substitute abatement effort for consumption such that pollution eventually stabilizes.

Nevertheless, agents of type T have an incentive to increase consumption to compensate the cost incurred by the previous generation. Actually, these agents have a weak perception of environmental issues so that for them, the cost of pollution is relatively low. More precisely, the cost of past consumption is so low compared to the efficieny of abatement, that increasing their level of voluntary contribution would generate a large decrease in the level of perceived pollution. However, as they substitute environmental quality for consumption in a constant way, this large decrease in pollution would entail a dramatic decrease in consumption. By contrast, rising consumption does not generate a too high increase in the cost of pollution. Hence, agents of type T have an incentive to consume to compensate the

[6]Note that the shape of preferences requires the ratio $\frac{C}{H}$ to be constant

cost of past consumption. As this cost increases, successive generations gradually substitute consumption for abatement such that pollution experiences an unlimited growth.

2.5.2 Dirty Technology

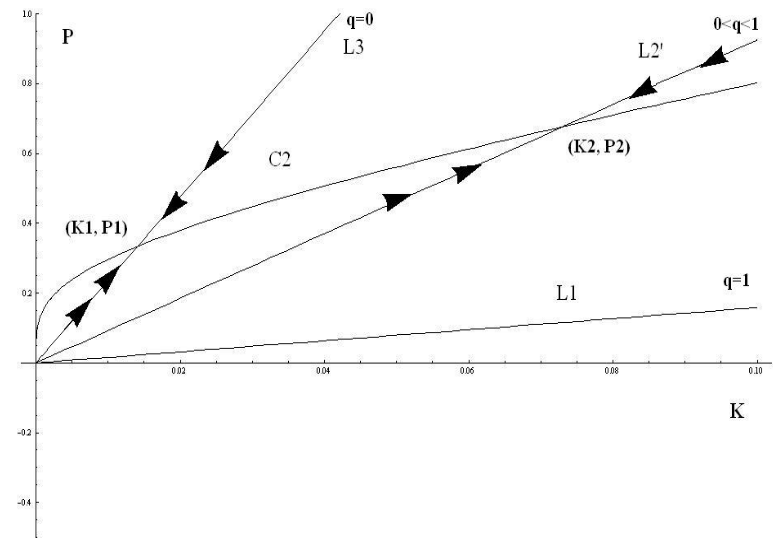

Figure 2: Long run equilibria in the (k,P) space when A_2 holds.

Figure 2 depicts long run equilibria of an economy endowed with a dirty technology. In this case, the natural evolution of pollution in steady state is described by $C2$. L_1, $L_2^{'}$, L_3 correspond respectively to the optimal choices for $q = 1$, $\tilde{q} > q > 0$ and $q = 0$. Stationary states exist only when q is sufficiently low.

When technology is dirty, the cost of past consumption is relatively high as compared to the efficiency of abatement. In this second case, agents of type G have an incentive to increase their consumption rather than their voluntary contribution to environmental quality. In fact, the technology implies that for them, the cost of pollution is dramatically high. Hence, since abatement is relatively inefficient, reducing the cost of past consumption would require a large rise in abatement effort. However, as they substitute abatement effort for consumption in a constant way, the increase in abatement would have to be accompanied by a dramatic reduction in consumption. Accordingly, they are encouraged to increase consumption which does not generate a too high loss in terms of increased pollution. Finally, as the cost of past consumption increases, successive generations substitute consumption to abatement such that pollution experiences unlimited growth.

Unlike type G, agents of type T have an incentive to make a higher voluntary contribution in order to reduce current pollution. Given the dirty technology, for these agents, the cost of pollution is not as low as with the clean technology since consumption is relatively polluting. Though, it is not as high as for agents of type G. Therefore the cost of past consumption can be compensated with a not too large increase in abatement effort. This fact means that rising the level of voluntary contribution does not entail a high decrease in consumption, so that, type-T agents have an incentive to reduce pollution. Finally, as the cost of past consumption increases, successive generations have an incentive to substitute abatement for consumption such that pollution eventually stabilizes.

To sum up, considering a heterogenous population generates new insights regarding the role played by the composition of the population. When the cost of today's actions is bequeathed future generations, the result is not straightforward and depends upon the pollution-generating technology. In particular, when the technology is not clean enough, this study reveals that a large number of people concerned with environmental issues may lead to ever increasing pollution[7].

3 Intergenerational cultural transmission of environmental attitudes

In this second part the distribution of environmental concern is no longer constant. We introduce a cultural transmission mechanism as developed in Bisin and Verdier [1998, 2001]. In this model, children are born naive and adopt a given kind of environmental attitude through imitation and social learning. Socialization is the result of two interacting types of transmission : transmission inside the family called "direct transmission" and socialization outside by peers, called "oblique transmission". More precisely, each child is first exposed to his parent's cultural trait (the perception of pollution) and adopts it with probability e^i, $i \in \{G, T\}$. If not, which occurs with probability $(1 - e^i)$, the child is socialized to the cultural trait of a role model chosen randomly in the population. Thus if direct transmission failed, the probabilities to pick up trait G and T are respectively q_t and $1 - q_t$.
Let P_t^{ij} be the probability for a child from a family with perception i to be socialized to trait j at time t. We have the following transition probabilities :

$$P_t^{GG} = e_t^G + (1 - e_t^G)q_t, \qquad P_t^{GT} = (1 - e_t^G)(1 - q_t),$$
$$P_t^{TT} = e_t^T + (1 - e_t^T)(1 - q_t), \quad \text{and} \quad P_t^{TG} = (1 - e_t^T)(q_t),$$

which enable us to characterize the dynamic law for the share of agents with environmental concern in the population,

$$q_{t+1} = q_t + q_t(1 - q_t)(e^G - e^T). \tag{17}$$

In this model, socialization is the result of an economic choice. Indeed, parents purposefuly attempt to transmit their own environmental attitude. As in Bisin and Verdier, the desire to transmit comes from the cultural intolerance hypothesis, which means that the gain to have a child with the same cultural trait is always higher than the gain to have a child with

[7] This is an existence result, always true provided that $\sigma^{'} > 1$ and $\bar{h}(q) < 1 - b$. Indeed, in this case, it is easy to show that $dP_t > 0$, $dk_t > 0$ for all t.

a different trait[8]. We assume that cultural intolerance is endogenous. As highlighted by Bisin and Verdier [2010], the preference on the part of parents for sharing their cultural trait with their children can depend on the economic and social conditions. Especially, here we assume that cultural intolerance depends upon the perceived level of pollution. We suppose the following. Let us denote by $V_t^{ij}(H_{t+1}^i)$ the gain for a parent of type i to have a child of type $j \neq i$ at time t. Thereby, $\Delta V^i(H_{t+1}^i) = V_t^{ii}(H_{t+1}^i) - V_t^{ij}(H_{t+1}^i)$ stands for the cultural intolerance function.

Assumption 1. *For all $i, j \in \{G, T\}$, $V_t^{ij} : \mathbb{R}^+ \to \mathbb{R}^+$. Besides,*

1. *We have,*
   - a. $\frac{dV^{GG}}{dH^G} > 0$,
   - b. $\frac{dV^{GT}}{dH^G} < 0$,
   - c. $V^{GG}(H^G) > V^{GT}(H^G) \quad \forall\ H^G > 0$,
   - d. $\Delta V^G(H^G)$ *positive and increasing,*
   - e. $\Delta V^G(0) = 0$.
2. *On the other hand,*
   - a. $\frac{dV^{TT}}{dH^T} < 0$,
   - b. $\frac{dV^{TG}}{dH^T} > 0$,
   - c. $V^{TT}(H^T) > V^{TG}(H^T) \quad \forall\ H^T > 0$,
   - d. $\Delta V^T(H^T)$ *positive and decreasing,*
   - e. $\lim_{H^T \to \infty} \Delta V^T(H^T) = 0$.

This assumption means that according to the level of pollution parents are more or less convinced that their child should share their opinion and values regarding the environment. For a parent of type G, the higher the perceived level of pollution, the more intolerant he is with children who do not care about environmental issues so that the higher is the relative payoff to have a child with environmental concern. Symmetrically, for a parent of type T, the lower the perceived pollution, the more intolerant he is with environmentally concerned children and the higher is the relative payoff to have a technologist child. Statement (1.e) implies that type-G parents do not demonstrate cultural intolerance toward technologists if the perceived level of pollution is nil, while statement (2.e) means, likewise, that type-T parents are not intolerant toward environmentally concerned children if the perceived level is dramatically high.

Moroever, in this model socialization is costly. The parental effort implies a welfare loss which can be understood as time spent with the child. This effort is denoted by $C(e_i)$ with $C(e_i) = \frac{e_i^2}{2C}$, where C is a positive constant.

3.1 Agents' choices

Agents live for three periods. During childhood they are only subject to socialization. The optimal choices are made during adulthood according to a two-stage maximisation procedure. In the first stage, young adults decide over the optimal combination of abatement and saving.

[8]In the baseline model of Bisin and Verdier, cultural intolerance results from imperfect empathy. Here we simply assume that parents have a gain to coexist with a child having the same cultural trait.

Hence, for all $i \in \{G, T\}$, equation (6) holds. In the second stage, agents take the level of pollution as given and choose their optimal effort e^i by maximizing

$$\theta \left(P_t^{ii} V_t^{ii}(H_{t+1}^i) + P_t^{ij} V_t^{ij}(H_{t+1}^i) \right) - C\left(e^i\right),$$

where θ is a discount factor[9]. This leads to the following optimal effort functions for agents of type G and T respectively,

$$e_t^G = C\theta(1-q_t)\Delta V^G(H_{t+1}^G), \qquad e_t^T = C\theta q_t \Delta V^T(H_{t+1}^T).$$

We can already see that e^G and e^T do not necessarily respect the cultural substitution property, as defined in Bisin and Verdier [2001]. When this property holds, then parents who belong to a minority have more incentive to transmit their own trait than those who belong to a majority. On the one hand, direct and oblique transmission are substitutes in the socialization process. Namely, parents of type i have less incentive to socialize their child whenever the cultural trait i is more widespread in the population because transmission by peers works better. On the other hand, parental effort is also sensitive to the variations of q through the variations of the perceived level of pollution which affects the cultural intolerance. Hence, depending on the direction and the magnitude of this second effect, the overall impact of q on e^i can be of either sign. Indeed, calculating the derivatives,

$$\frac{\partial e_t^G}{\partial q_t} = -C\theta\Delta V^G(H_{t+1}^G) + C\theta(1-q_t)\frac{d\Delta V^G}{dH^G}\frac{\partial P_{t+1}}{\partial q_t},$$

$$\frac{\partial e_t^T}{\partial q_t} = C\theta\Delta V^P(H_{t+1}^T) + C\theta q_t\frac{d\Delta V^T}{dH^T}\frac{\partial P_{t+1}}{\partial q_t},$$

[9] We think it makes sense to assume that parents decide on the level of pollution without taking into account the effect it will have on ΔV^i. Indeed, this means that they do not purposefully choose their intolerance level. Besides, this timing procedure has been used in the literature of human capital where separability provided by a sequential game is needed to obtain closed form solutions (see for instance Ehrlich and Lui [1991]). Note that here, the two-stage maximisation can be done only if education costs enter separately in the utility function (Bisin and Verdier [1998, 2000]). We could have used a different modelling strategy that is to assume that pollution interacts with parental effort to determine the probability of direct transmission. We interpret that as the impact that changing environmental conditions have on individuals through the media. That is, the higher the actual pollution level, the higher the transmission by the media. Following the mechanism untitled "Do not talk to strangers" briefly discussed in Bisin and Verdier [2001], we assume that parental effort and the media are complementary. This assumption means that both transmission by parents and socialization by the media must work for the child to be socialized to one trait (otherwise, the child picks a trait from the population as a whole). This is consistent with the environmental psychology literature arguing that environmental perceptions consist of both specific values (transmitted by parents) and a vision of objective environmental conditions (transmitted by the media). In this case, the transition probabilities would be

$$P_t^{GG} = e_t^G.f(P_t) + (1 - e_t^G.f(P_t))q_t, \qquad P_t^{GT} = (1 - e_t^G.f(P_t))(1-q_t),$$
$$P_t^{TT} = e_t^T.(1-f(P_t)) + (1 - e_t^T.(1-f(P_t)))(1-q_t), \quad \text{and} \quad P_t^{TG} = (1 - e_t^T.(1-f(P_t)))(q_t),$$

where $f(P_t)$, which is increasing, is the probability to adopt the environmental concern through the media. It can be shown that our qualitative results would not be modified.

one can see that the second term can be of either sign depending on the impact of q on pollution at $t+1$. Note, that we always have $e^G(0) > 0$, $e^T(0) = 0$ and $e^G(1) = 0$, $e^T(1) > 0$. This means that when the population is perfectly homogeneous, it is always true that parents who belong to the cultural majority make lower effort, since, in this particular case this effort is nil.

The variable H^i_{t+1} is a function of each variable at time t which can be written as $H^i(P_t, k_t, q_t)$. This function can be introduced in the intolerance functions to have an optimal parental effort depending exclusively on the variables at t,

$$e^G(k_t, P_t, q_t) = C\theta(1-q_t)\Delta V^G(H^G(P_t, k_t, q_t)),$$

$$e^T(k_t, P_t, q_t) = C\theta q_t \Delta V^T(H^T(P_t, k_t, q_t)).$$

By inserting those functions in equation (17), we obtain the dynamic law of the share of type-G agents. Thereby, in the system we are going to study, all variables are predeterminated.

Definition 3 (Intertemporal Equilibirum)**.** *Let (P_0, k_0, q_0) be the vector of intial values. The intertemporal equilibrium is the sequence $(P_t, k_t, q_t)_{t\in\mathbb{N}}$ which satisfies for each t, the following system of equations*[10]

$$P_{t+1} = \frac{(b\sigma' + \bar{h}(q_t) - \sigma')}{(1-\sigma')}P_t - \frac{(\beta v + \gamma v - \gamma)\sigma'}{(1-\sigma')}k_t^v, \quad (18)$$

$$k_{t+1} = -\frac{(1 - b - \bar{h}(q_t))}{\gamma(1-\sigma')}P_t - \frac{(\beta v + \gamma v - \gamma)}{\gamma(1-\sigma')}k_t^v, \quad (19)$$

$$q_{t+1} = q_t + q_t(1-q_t)(e^G(P_t, k_t, q_t) - e^T(P_t, k_t, q_t)). \quad (20)$$

In what follows, we investigate "plausible" states of the economy focusing on the existence of steady states when preferences heterogeneity is endogenous.

3.2 Steady states

Proposition 2. *Existence of steady sates with cultural transmission of environmental concern.*

(1) If A_1 holds, the system given by equations (18), (19) and (20) admits two steady states. One is characterized by a perfectly homogeneous population of environnmentally concerned people : $(\bar{P}(1), \bar{k}(1), 1)$ and it is locally unstable. The other one is characterized by a heterogeneous population within which, the share of agents concerned about pollution is relatively

[10]The variable q_t can be a rational number as long as we restrict the analysis to the case $\Delta V^i : \mathbb{R} \to \mathbb{Q}$, $\forall i \in \{G, T\}$. In this case, $e^i \in \mathbb{Q}$ and $q_t \in \mathbb{Q}$. The behavior of the dynamic system is decribed by a map $G : \mathbb{R}_2 \times \mathbb{Q} \to \mathbb{R}_2 \times \mathbb{Q}$. However, for the sake of simplicity, we study the same function G which maps $\mathbb{R}_3$ into $\mathbb{R}_3$.

high : $(\bar{P}(\bar{q}), \bar{k}(\bar{q}), \bar{q})$ with $(\bar{q} > \tilde{q})$, with $\tilde{q}$ defined as in Proposition (1). It can be locally stable.

(2) If A_2 holds, the system given by equations (18), (19) and (20) admits only one steady state characterized by a perfectly homogeneous population of technologist agents : $(\bar{P}(0), \bar{k}(0), 0)$ which is locally unstable. There is no steady state with a heterogeneous population unless the impact of pollution on the socialization process is negligible.

If the technology is clean, the long run equilibrium is characterized by a high share of agents with environmental concern and a steady state level of pollution. To further characterize the dynamics under A_2, we rely on a numerical application (presented in Appendix). We find that a likely observable situation is the one in which the population of environmentally concerned tends to one while pollution experiences unlimited growth. Hence, when heterogeneity is endogeneous, only two equilibria arise in the long run. Whatever the pollution-generating technology, there is a transition from a low-environmentally concerned society to a high-environmentally concerned one. However, this population change may be either accompanied by pollution stabilization or by ever-increasing pollution.

In our framework, higher relative socialization efforts of parents of one type have a positive dynamic impact on the transmission of this type. Hence, the transmission of environmental concern is triggered by two cumulative effects which act on relative parental efforts. The first one is the classical effect of cultural substitution. Indeed, let us start from an intitially low share of type-G agents. Since, agents of type G are in minority, the transmission of their trait by the outside cultural environment (as given by peers) is few effective. Accordingly, these agents are encouraged to make a relatively high effort to directly transmit their trait. The second (indirect) effect is a "saving effect". Now, let us start from an initially low stock of capital. As a first step, successive generations have an incentive to save due to high returns on capital. This generates a rise in pollution. However, worsening environmental conditions increase the value that parents-G's offspring acquire their trait such that they are encouraged to increase their relative effort. Therefore both effects positively affect the transimission of type G through their impact on relative socialization efforts. Hence, the society experiences an endogenous cultural change directed toward more individuals with environmental concern.

As pollution depends upon the composition of the population, this population change, in turn, affects environmental conditions. But, as emphasized in section 2, the relationship between one given composition of the population and the long run dynamics of pollution depends upon the pollution-generating technology. Hence, while preferences are endogenously shifted toward more environmental concerned, different outcomes regarding the long run dynamics of pollution may arise depending on the technology. When the technology is clean, we have seen that as pollution increases, agents of type G have incentive to increase voluntary contributions to pollution reduction. Therefore, in such case the transition to a highly concerned society results into pollution stabilization. However, when technology is dirty, agents of type G prefer to consume to compensate the utility loss due to rising pollution. Hence, in this second case, the spread out of environmental concern is accompanied by ever-increasing pollution.

To sum up, in this section we endogenized heterogeneity to investigate which of the various possible equilibria identified in the previous section actually set up in the long run. When pollution and cultural attitudes toward the environment are jointly determined, we expect a spread out of environmental concern which runs counter sustainability purposes for economies endowed with a dirty enough technology. This is a quite pessimistic conclusion as some economic systems should not be able to change technology easily[11]. In what follows we show that the issue can still be solved using specifc instruments.

3.3 Intergenerational transfers

Due to intergenerational externalities, the *laissez-faire* equilibrium leads to ever-increasing pollution when technology is not clean enough. This result calls for policy intervention. Here we show that some intergenerational transfers allow to achieve sustainability (in the sense of reaching a stationary level of pollution).

Suppose the government introduces a tax of rate τ on consumption of each member of the generation $t-1$ such that equation (3) turns into

$$(1+\tau)c_t^i = (1+r_t)\, s_{t-1}^i. \tag{21}$$

The FOC given by equation (6) does not change. First, if this tax is redistributed to generation t, each agent i, $i \in \{G, T\}$ of this generation faces the following new budget constraint

$$w_t + \tau \frac{\sum_i c_t^i}{n} = A_t^i + s_t^i. \tag{22}$$

The intertemporal equilibrium can be found using equations (22) and (21) instead of (2) and (3) and we deduce the following steady state values for pollution and capital

$$\bar{P}(q) = \frac{\sigma' \gamma}{1-\bar{h}(q)} \left(\frac{(1-\bar{h}(q))(\frac{\beta-\gamma\tau}{(1+\tau)} v + \gamma v - \gamma)}{\gamma(b\sigma' + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}}, \tag{23}$$

$$\bar{k}(q) = \left(\frac{(1-\bar{h}(q))(\frac{\beta-\gamma\tau}{(1+\tau)} v + \gamma v - \gamma)}{\gamma(b\sigma' + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}}. \tag{24}$$

When $b\sigma' + \bar{h}(q) - 1 < 0$ holds, which means that the necessary condition for the existence of a stable steady state is met, a sufficient condition is that $\frac{\beta-\gamma\tau}{(1+\tau)} v + \gamma v - \gamma < 0$, that is $\tau > \frac{\beta v + \gamma v - \gamma}{\gamma}$. We deduce the following proposition.

[11] Furthermore, this result also applies to some kind of pollution such as waste for which the related technology seems to correspond to what we call dirty technology worldwide.

Proposition 3. *There exists a thresold tax rate* $\tau = \frac{\beta v + \gamma v - \gamma}{\gamma}$*, above which a tax levied on consumption of members of the generation* $t-1$ *and redistributed to generation* t *as an additional income restores the possibility of convergence toward a steady state.*

Secondly, if the tax is used to finance a subsidy on abatement efforts of generation t, each agent i, $i \in \{G, T\}$ faces the following budget constraint

$$w_t = (1-s)A_t^i + s_t^i, \tag{25}$$

where s is the subsidy rate. The FOC turn into

$$\gamma \sigma s_t^i = (1-s)H_{t+1}^i. \tag{26}$$

As before, we performe the intertemporal equilibrium now using (25) and (26) instead of (2) and (6) and we deduced non-trivial-steady-state values of P and k,

$$\bar{P}(q) = \frac{\sigma' \gamma}{1-\bar{h}(q)} \left(\frac{(1-\bar{h}(q))(\frac{\beta}{(1+\tau)}v + \frac{\gamma v - \gamma}{1-s})}{\gamma(b\sigma' + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}}, \tag{27}$$

$$\bar{k}(q) = \left(\frac{(1-\bar{h}(q))(\frac{\beta}{(1+\tau)}v + \frac{\gamma v - \gamma}{1-s})}{\gamma(b\sigma' + \bar{h}(q) - 1)} \right)^{\frac{1}{1-v}}. \tag{28}$$

In this case, the sufficient condition for the existence of some stable steady state is $\frac{\beta}{(1+\tau)}v + \frac{\gamma v - \gamma}{1-s} < 0$ which can be reformulated as $\tau > 1 - s - \frac{\gamma(1-v)}{v}$.

Proposition 4. *For any subsidy rate* $s > 0$*, there exists a thresold tax rate* $\tau = 1 - s - \frac{\gamma(1-v)}{v}$*, above which a tax levied on consumption of members of the generation* $t-1$ *used to subsidy abatement efforts of the generation* t *restores the possibility of convergence toward a steady state.*

Transfers partially internalize the intergenerational spillover and thus reduce the cost of past consumption. Furthermore, these policies affect relative prices : the transfer to younger generation lowers the price of abatement while the tax levied on the old generation increases the price of consumption. Accordingly, agents have an incentive to substitute larger amounts of abatement for consumption. That is, from a given tax level (relative or not to the subsidy level), increasing the level of voluntary contribution does not entail a relatively high loss in terms of consumption anymore. Therefore a more and more concerned society will have a higher incentive to voluntarily contribute to pollution reduction. Hence, economies endowed with a dirty technology can still achieve sustainability by establishing a transfer system between generations.

4 Conclusion

We study dynamic interactions between the level of environmental degradations and the distribution of attitudes toward the environment. To this aim, we develop an overlapping generations framework which allows us to specify a stylized cultural transmission mechanism and also to include intergenerational aspects of pollution. This generates new results. Even if environmental concern widely spreads within the population, the transition from a low-environmentally concerned to a high-environmentally concerned society is not necessarily accompanied by positive outcomes regarding environmental quality. More particularly, we show that if there are strong intergenerational externalities, which cannot be efficiently reduced through abatement efforts, as environmental concern spreads, pollution experiences an unlimited growth. This result may actually account for what has occurred within the developing world for various kinds of pollution. Furthermore, it also stands for the situation of industrialized nations regarding waste accumulation (during the last decades, waste stocks have continously grown despite rising concern from northern populations). Those conclusions call for policy intervention.

Hence we deduce that contrary to what is suggested by studies in environmental psychology, in this case recommendations in terms of environmental education are clearly irrelevant. Nevertheless we show that the issue can still be solved. More particularly, an intergenerational transfer from the older generation to the working one can be proved to prevent the unlimited growth of environmental degradation (allowing the system to reach a stationary state).

Our analytical framework relies on simplifying assumptions. First, agents only differ in their environmental perceptions. However, differences in income among the two types of agents could have important implications for the dynamics of pollution. Introducing wealth inequalities may be a valuable extension. A second limitation is the fixity of the pollution-generating technology which is shown to have a crucial role in this framework. Hence, including technological change (either endogenously or as an exogenous variable) would certainly provide interesting results.

5 Appendix

5.1 Existence and uniqueness of the Nash equilibrium

Definition 4. *A Nash equilibrium at t is a vector of abatement efforts* $\left(A^G_{t,1}, \cdots A^G_{t,n_G}, A^T_{t,n_G+1}, \cdots A^T_{t,n}\right)$ *such that for each agent k of type i*

$$A^i_{t,k} = \max\{\frac{\sigma w_t}{\sigma - 1} - \frac{(1-b-h^i)P_t}{\gamma(\sigma-1)} - \frac{\beta(n_G c^G_t + n_T c^T_t)}{\gamma(\sigma-1)} + \frac{\bar{A}_t}{\sigma-1}, 0\}.$$

Let $\Theta \equiv \{s_t \in \mathbb{R}, 0 \geq s_{t,k} \geq w_t, k = 1 \cdots n\}$ be a compact and convex set. The function $A^i_{t,k} = \max\{\frac{\sigma w_t}{\sigma-1} - \frac{(1-b-h^i)P_t}{\gamma(\sigma-1)} - \frac{\beta(n_G c^G_t + n_T c^T_t)}{\gamma(\sigma-1)} + \frac{\bar{A}_t}{\sigma-1}, 0\}$ is continous from Θ into Θ. Then Brouwer's fixed point theorem implies that a fixed point exists which is a Nash equilibrium.

For unicity, set $b^i = \frac{\sigma\gamma w_t - (1-b-h^i)P_t - \beta(n_G c^G_t + n_T c^T_t)}{\gamma(\sigma-1)}$. The Nash equilibrium is unique if the system $AX = b$, $X \geq 0$ admits a unique solution. In this system A is the symmetric matrix with non diagonal elements equal to $\frac{-1}{\sigma-1}$ and diagonal elements equal to 1, $X \equiv \left(A^G_{t,1}, \cdots A^G_{t,n_G}, A^T_{t,n_G+1}, \cdots A^T_{t,n}\right)$ and $b \equiv (b^G_1, \cdots b^G_{n_G}, b^G_{n_G+1}, \cdots b^G_n)$. Unicity of the solution is equivalent to A being invertible which is true when the determinant of A is strictly different from zero. With the variable change $\sigma - 1 = \lambda$, one can find

$$\det A = \left(-\frac{1}{\lambda}\right)^n \det(M_{-}\lambda I),$$

where M is the symmetric matrix with non-diagonal elements equal to one and diagonal elements equal to zero. The latter term of this product is the caracteristic polynomial of this matrix which vanisches on the spectrum of M which is $\{-1, n-1\}$. Accordingly, the system $AX = b$ has a unique solution for $\lambda \neq -1$ and $\lambda \neq n-1$, that is for $\sigma \neq 0$ and $\sigma \neq n$. Hence, there exists a unique Nash equilibrium provided that $\sigma \neq 0$ and $\sigma \neq n$.

This equilibrium may correspond to a corner solution. To ensure that interior equilibria are not unlikely, we impose an additional condition. Since all agents of type i are identical, from (7) we can deduce

$$A^i_t = k^v \frac{(\gamma\sigma'(1-v) - v\beta)}{\sigma'\gamma(\sigma'-1)} + P_t \frac{(b\sigma' + \bar{h} - h^i - \sigma' + \sigma' h^i)}{\sigma'\gamma(\sigma'-1)},$$

with $\sigma' = \frac{\sigma}{n}$ and $\bar{h} = \frac{n_G}{n} h^G + \frac{n-n_G}{n} h^T$. An interior equilibrium exists if this sum is positive. This condition cannot hold for all values of the state variables and all values of q. We can nonetheless ensure the likelihood of interior equilibria by imposing,

$$(1)\ \text{If}\quad \sigma' < 1\ (\sigma < n),\quad \text{then}\quad \sigma' < \frac{\beta v}{\gamma(1-v)},$$

$$(2)\ \text{If}\quad \sigma' > 1\ (\sigma > n),\quad \text{then}\quad \sigma' > \frac{\beta v}{\gamma(1-v)}.$$

5.2 Proof of Proposition 1

The first part of Proposition 1 regarding the existence of steady states is trivial. For the second, differentiating the functions $\bar{P}(q)$ and $\bar{k}(q)$ with respect to q we obtain,

$$\frac{\partial \bar{P}(q)}{\partial q} = \frac{\bar{P}(h^G - h^T)}{(1-v)(1-\bar{h}(q))}\left((1-v) + \frac{b\sigma'}{(1-b\sigma' - \bar{h}(q))}\right) \tag{29}$$

$$\frac{\partial \bar{k}(q)}{\partial q} = -\frac{\bar{k}b\sigma'(h^G - h^T)}{(1-v)(1-\bar{h}(q))(b\sigma' + \bar{h}(q) - 1)}. \tag{30}$$

The sign of the above derivatives depends upon the sign of $b\sigma' + \bar{h}(q) - 1$. If $b\sigma' + \bar{h}(q) - 1 < 0$, which involves A_1, then both dervatives are negative. If $b\sigma' + \bar{h}(q) - 1 > 0$, which involves A_2, then both derivatives are positive.

5.3 Stability of the non-trivial steady state for the q exogenous case

Proposition 5. *A necessary condition for local stability.*

(1) When A_1 holds, stability of the steady state requires that the cost of an additional unit of saving is relatively high ($\sigma' < 1$) along the transition path.

(2) When A_2 holds, stability of the steady state requires that the cost of an additional unit of abatement is relatively high ($\sigma' > 1$) along the transition path.

Proof. To study the local stability of the non-trivial steady state, we linearize the two-dimensional map consisting of equations (13) and (14) around $(\bar{P}(q), \bar{k}(q))$. Denote by Df_2 the Jacobian matrix,

$$Df_2(\bar{k}, \bar{P}) = \begin{pmatrix} -\frac{v(b\sigma' + \bar{h}(q) - 1)}{(1-\bar{h}(q))(1-\sigma')} & -\frac{(1-b-\bar{h}(q))}{\gamma(1-\sigma')} \\ -\frac{\sigma' v\gamma(b\sigma' + \bar{h}(q) - 1)}{(1-\bar{h}(q))(1-\sigma')} & \frac{(b\sigma' + \bar{h}(q) - \sigma')}{(1-\sigma')} \end{pmatrix}.$$

The characteristic polynomial of Df_2 can be written as

$$P(\lambda) = \alpha_0\lambda^2 + \alpha_1\lambda + \alpha_2,$$

where $\alpha_0 = (1-\sigma')(1-\bar{h}(q))$, $\alpha_1 = \sigma'(b-1)(\bar{h}(q) - 1) + vb\sigma' - (1-\bar{h}(q))(v + \bar{h}(q))$, and $\alpha_2 = -\bar{h}(q)v(b\sigma' + \bar{h}(q) - 1)$.

The steady state is locally stable if the magnitude of each eigenvalue of the above matrix is smaller than one. We denote the roots of P by $\lambda_1 = \frac{-\alpha_1 + \sqrt{\Delta}}{2\alpha_0}$ and $\lambda_2 = \frac{-\alpha_1 - \sqrt{\Delta}}{2\alpha_0}$. We want to show that, at least one of the two roots of the characteristic polynomial has magnitude larger than 1 if, on the one hand, $\sigma' > 1$ and $\beta v + \gamma v - \gamma < 0$, or if, on the other hand,

$\sigma^{'} < 1$ and $\beta v + \gamma v - \gamma > 0$.

(1) Let us consider the case of $\beta v + \gamma v - \gamma < 0$. The existence of a steady state implies $\alpha_2 > 0$ since $b\sigma^{'} + \bar{h}(q) - 1$ is negative. Suppose that $\sigma^{'} > 1$, which implies $\alpha_0 < 0$. Then ${\alpha_1}^2 - 4\alpha_0\alpha_2 > {\alpha_1}^2$ (since $4\alpha_0\alpha_2 < 0$) so that $\sqrt{\Delta} > \alpha_1$. Hence $\lambda_1 = \frac{-\alpha_1+\sqrt{\Delta}}{2\alpha_0} < 0$. Especially, we show that we always have $\lambda_1 < -1$. $\lambda_1 < -1 \Rightarrow -\alpha_1 + 2\alpha_0 > -\sqrt{\Delta}$ which is equivalent to $\Delta > (-\alpha_1 + 2\alpha_0)^2$ (since $-\alpha_1 + 2\alpha_0 > \sqrt{\Delta} \Leftrightarrow \lambda_2 < -1$ and we are done), or also to $\alpha_1 - \alpha_2 - \alpha_0 < 0$. However, after simplification, we find $\alpha_1 - \alpha_2 - \alpha_0 = (v-1)(\bar{h}(q) - 1)(b\sigma^{'} + \bar{h}(q) - 1)$ which is always negative since $b\sigma^{'} + \bar{h}(q) - 1$ is negative. Finally, when $\beta v + \gamma v - \gamma < 0$ and $\sigma^{'} > 1$, one of the eigenvalues, is always lower than -1 and the steady state is unstable.

(2) Let us consider the case of $\beta v + \gamma v - \gamma > 0$, which implies $\alpha_2 < 0$ since $b\sigma^{'} + \bar{h}(q) - 1$ is positive. Consider $\sigma^{'} < 1$, which implies $\alpha_0 > 0$. Then $\Delta = {\alpha_1}^2 - 4\alpha_0\alpha_2 > {\alpha_1}^2$, so that $\sqrt{\Delta} > \alpha_1$. Hence $\lambda_1 = \frac{-\alpha_1+\sqrt{\Delta}}{2\alpha_0}$ is always positive, but this is not the case for λ_2, which, as we will we show, is always lower than -1 as long as $\sigma^{'} < 1$. $\lambda_2 < -1$ is equivalent to $2\alpha_0 - \alpha_1 < \sqrt{\Delta}$, or $\Delta > (-\alpha_1 + 2\alpha_0)^2$ (since $\lambda_1 > 0 \Leftrightarrow 2\alpha_0 - \alpha_1 > -\sqrt{\Delta}$). When $\alpha_0 > 0$, this is equivalent to $\alpha_1 - \alpha_2 - \alpha_0 = (v-1)(\bar{h}(q) - 1)(b\sigma^{'} + \bar{h}(q) - 1) > 0$, which is always positive since $b\sigma^{'} + \bar{h}(q) - 1 > 0$. Hence, when $\beta v + \gamma v - \gamma > 0$ and $\sigma^{'} < 1$, one of the eigenvalues is always lower than -1 and the steady state is unstable. □

When the condition of Proposition (5) holds, simulations show that the steady state can be locally stable and this is true for various combinations of the parameters.

5.4 Proof of proposition 2

The question of the existence of steady states can be reduced to the existence of intersection points between the PP, kk and qq loci in the (P_t, k_t, q_t) space. First, the PP and kk loci intersects at (0,0), so that a vector (0,0,q) is a steady state of the three-dimensional system, provided that q belongs to the qq locus.

Then, let $S_1 \equiv \{\{k(q), P(q)\}, q \in [0, \tilde{q}[\}$, and $S_2 \equiv \{\{k(q), P(q)\}, q \in]\tilde{q}, 1]\}$ be the sets of non-zero intersection points between the PP and kk loci in the (P_t, k_t, q_t) space when A_2, respectively A_1 hold. Hence, necessary and sufficient conditions for the three-dimensional system to be at steady state are (P_t, k_t) belongs to either one of the two sets and q_t belongs to the qq locus. That is $q_t = 0$, $q_t = 1$ and q_t solving $e_t^G - e_t^P = 0$.
Hence $(\bar{P}(0), \bar{k}(0), 0)$ is a steady state of the system when $(\bar{P}(0), \bar{k}(0)) \in S_1$ and $(\bar{P}(1), \bar{k}(1), 1)$ is a steady state if $(\bar{P}(1), \bar{k}(1)) \in S_2$. Furthermore q_t is on the qq locus if q_t solves $e_t^G - e_t^T = 0$ which is equivalent to

$$\frac{1}{1-q_t} = 1 + \frac{\Delta V^G(H_{t+1}^G)}{\Delta V^T(H_{t+1}^P)}. \tag{31}$$

As we know that steady states of the system must be such that (P_t, k_t) are in either S_1 or S_2, we can find solutions in the restricted sets $S_1 \times [0, \tilde{q}[$ or $S_2 \times]\tilde{q}, 1]$.

The system admits a fixed point $(\bar{P}, \bar{k}, \bar{q})$ where $0 < \bar{q} < 1$ if the equation

$$\frac{1}{1-q} = 1 + \frac{\Delta V^G(P(q)(1-h_G))}{\Delta V^T(P(q)(1-h_P))}, \tag{32}$$

i. has at least one solution in $[0, \tilde{q}[$, $(P(q), k(q))$ are $\in S_1$ (which implies A_2 holds);

ii. has at least one solution in $]\tilde{q}, 1]$, $(P(q), k(q))$ are $\in S_2$ (which implies A_1 holds).

To study the existence of solution to this equation let us define

$$\Psi(q) \equiv \frac{1}{1-q}, \quad q \in [0, 1]$$

and

$$\Phi_1(q) \equiv 1 + \frac{\Delta V^G(P(q)(1-h_G))}{\Delta V^T(P(q)(1-h_P))} = 1 + \frac{\Delta V^G(q)}{\Delta V^T(q)}, for \quad q \in [0, \tilde{q}[\quad and \quad (P(q), k(q)) \text{ in } S_1,$$

$$\Phi_2(q) \equiv 1 + \frac{\Delta V^G(P(q)(1-h_G))}{\Delta V^T(P(q)(1-h_P))} = 1 + \frac{\Delta V^G(q)}{\Delta V^T(q)}, for \quad q \in]\tilde{q}, 1] \quad and \quad (P(q), k(q)) \text{ in } S_2.$$

We need to determine whether $\Phi_1(q) - \Psi(q) = 0$ admits a solution on $[0, \tilde{q}[$ on the one hand, and whether $\Phi_2(q) - \Psi(q) = 0$ has a solution on $]\tilde{q}, 1]$ on the other hand.

(1) Let us study the existence of fixed point (where q is different than zero) on $S_1 \times [0, \tilde{q}[$.

First, Ψ is strictly increasing on $[0, \tilde{q}[$, $\Psi(0) = 1$ and $\Psi(\tilde{q}) = \frac{1}{1-\tilde{q}} > 1$.
On the other hand, $\forall q \in [0, \tilde{q}[$,

$$\frac{d\Phi_1(q)}{dq} = \frac{dP(q)}{dq} \times \frac{\frac{d\Delta V^G(H^G(q))}{dH^G}(1-h_G)\Delta V^T(H^T(q)) - \frac{d\Delta V^T(H^T(q))}{dH^T}(1-h_T)\Delta V^G(H^G(q))}{(\Delta V^T(H^T(q)))^2}.$$

By assumption, $\frac{d\Delta V^G}{dH^G} > 0$ and $\frac{d\Delta V^P}{dH^P} < 0$. The second term of the product is thus positive and $\frac{\partial \Phi_1}{\partial q}$ has the same sign as $\frac{\partial P}{\partial q}$ on $[0, \tilde{q}[$. We know, from Proposition 1 that $\frac{\partial P(q)}{\partial q} > 0$ $\forall q \in [0, \tilde{q}[$. Accordingly Φ_1 is strictly increasing on $[0, \tilde{q}[$.

Moroever, $\Phi_1(0) = 1 + \dfrac{\Delta V^G(0)}{\Delta V^P(0)}$ is strictly larger than 1 since, by assumption, the functions ΔV^G and ΔV^P are stricly positive as long as for $i \in \{G, T\}$ $H^i > 0$, which is true, since here $H^i = P(0)(1-h_i) > 0$.

Furthermore we have,

$$\lim_{q \to \tilde{q}^-} \quad P(q) = +\infty,$$
$$\lim_{H^G \to \infty} \quad \Delta V^G(H^G) = \delta < \infty,$$
$$\lim_{H^P \to \infty} \quad \Delta V^P(H^P) = 0,$$

so that $\Phi(\tilde{q}) = \lim\limits_{q \to \tilde{q}^-} \Phi_1(q) = +\infty$[12].

Let us define the continous function $g^1(q) = \Phi_1(q) - \Psi(q)$. Then $g^1(0) > 0$, $\lim_{q \to \tilde{q}^-} g^1(q) = \infty$. For $g^1(q) = 0$ to have a solution on $[0, \tilde{q}[$, a necessary condition is that g^1 is decreasing on the interval $[q_1, q_2]$, with $q_1 \geq 0$ and $q_2 < \tilde{q}$, or $\frac{dg^1}{dq} = \frac{d\Phi_1}{dq} - \frac{d\Psi}{dq} < 0$ on $[q_1, q_2]$. That is, we need to find $\frac{d\Phi_1}{dq} < \frac{d\Psi}{dq}$ on the relevant interval. However, $\frac{d\Psi}{dq} = \frac{1}{(1-q)^2}$ is precisely low for low values of q, so that, the necessary condition does not hold if the derivative of Φ_1 is not small enough. More particularly, g^1 must decrease enough such that it reaches the x-axis. Then for that condition to be true, the derivative of g^1, $\frac{dg^1}{dq}$, must be sufficiently negative, which means that the derivative of Φ_1, $\frac{d\Phi_1}{dq}$, needs to be very close to zero.

Therefore, a steady state is unlikely on $S_1 \times [0, \tilde{q}[$. It requires that the impact of a change in pollution on the transmission process, as given by the term $\frac{d\Phi_1}{dq}$, be negligible so that the effect of cultural substitution (a change in the cultural composition of the population), $\frac{d\Psi}{dq}$, which is weak, can overcome it[13].

(2) Let study the existence of solutions in $]\tilde{q}, 1]$.

Ψ is strictly increasing on $]\tilde{q}, 1]$. $\Psi(1) = \lim\limits_{q \to 1} \Psi(q) = +\infty$ and as we just noted $\Psi(\tilde{q})$ is a finite number which is superior to one.

On the other hand, the derivative of Φ_2 on the interval $]\tilde{q}, 1]$ has the same sign as $\frac{\partial P}{\partial q}$ on the same interval. From Section 2, we know that the latter is negative on $]\tilde{q}, 1]$. So that Φ_2 is strictly decreasing on $]\tilde{q}, 1]$.

At q=1, $\Phi_2(1) = 1 + \dfrac{\Delta V^G(P(1)(1-h_G))}{\Delta V^P(P(1)(1-h_P))}$ is a finite number larger than one.

Furthermore, $\lim\limits_{q \to \tilde{q}^+} P(q) = +\infty$, hence $\Phi(\tilde{q}) = \lim\limits_{q \to \tilde{q}^+} \Phi_1(q) = +\infty$.

[12] $\Delta V^G(H^G)$ cannot be infinite since otherwise the optimal effort, which stands for the probability of direct transmission, could be higher than one as long as C, or θ, are not nil. Likewise, $\Delta V^T(H^T)$ must be finite in zero.

[13] To make sure of the unlikelihood of a steady state and to confirm the conditions on its existence, we relied on a numerical example which is presented in Appendix 5.6

Finally, we have $\Phi_2(\tilde{q}) > \Psi(\tilde{q})$ and $\Phi_2(1) < \Psi(1)$. Φ_2 and Ψ are both continuous on $]\tilde{q}, 1]$. Moreover Ψ is strictly increasing on $]\tilde{q}, 1]$, while Φ_2 is decreasing on the same interval. Accordingly there is a unique $q \in]\tilde{q}, 1]$ such that

$$\Psi(q) = \Phi_2(q).$$

5.5 Stability of steady states

5.5.1 Stability of the steady states with homogeneous population

To study the local stability of each steady state linearize the system consisting of equations (16), (17) and (18) around $(\bar{P}(0), \bar{k}(0), 0)$ on the one hand, and around $(\bar{P}(1), \bar{k}(1), 1)$, on the other hand.

(1) The Jacobian matrix evaluated at $(\bar{P}(0), \bar{k}(0), 0)$ is

$$\begin{pmatrix} \frac{-v(b\sigma' + \bar{h}(0) - 1)}{(1-\bar{h}(0))(1-\sigma')} & -\frac{(1-b-\bar{h}(0))}{\gamma(1-\sigma')} & \frac{\gamma(h^G - h^P)}{(1-\sigma')}\bar{P} \\ \frac{-\sigma'\gamma v(b\sigma' + \bar{h}(0) - 1)}{(1-\bar{h}(0))(1-\sigma')} & \frac{(b\sigma' + \bar{h}(0) - \sigma')}{(1-\sigma')} & \frac{(h^G - h^P)}{(1-\sigma')}\bar{P} \\ 0 & 0 & 1 + e^G(0) - e^T(0) \end{pmatrix}.$$

We can express its characteristic polynomial as

$$Q_1(\lambda) = ([1 + e^G(0) - e^T(0)] - \lambda)(\alpha_0\lambda^2 + \alpha_1\lambda + \alpha_2),$$

where α_0, α_1 and α_2 are defined as in Section 5.3. The second term of this product is precisely $P(\lambda)$, the characteristic polynomial of Df_2 as defined in Section 5.3, evaluated at $q = 0$. Hence, two of the eigenvalues can be of magnitude less than one as long as the two-dimensional system of section 2 is stable. That is Df_2 has two eigenvalues of magnitude less than one (which at least requires that $\sigma' > 1$). However, the third eigenvalue is equal to $1 + e^T(0) - e^G(0)$. This is always higher than one since the socialization mechanism implies $e^G(0) > 0$ and $e^T(0) = 0$. Hence, the system consisting of (16), (17) and (18) is unstable around $(\bar{P}(0), \bar{k}(0), 0)$.

(2) The Jacobian matrix evaluated at $(\bar{P}(1), \bar{k}(1), 1)$ is

$$\begin{pmatrix} \frac{-v(b\sigma' + \bar{h}(1) - 1)}{(1-\bar{h}(1))(1-\sigma')} & -\frac{(1-b-\bar{h}(1))}{\gamma(1-\sigma')} & \frac{\gamma(h^G - h^P)}{(1-\sigma')}\bar{P} \\ \frac{-\sigma'\gamma v(b\sigma' + \bar{h}(1) - 1)}{(1-\bar{h}(1))(1-\sigma')} & \frac{(b\sigma' + \bar{h}(1) - \sigma')}{(1-\sigma')} & \frac{(h^G - h^P)}{(1-\sigma')}\bar{P} \\ 0 & 0 & 1 + e^T(1) - e^G(1) \end{pmatrix}.$$

Its characteristic polynomial can be express as

$$Q_2(\lambda) = ([1 + e^T(1) - e^G(1)] - \lambda)(\alpha_0\lambda^2 + \alpha_1\lambda + \alpha_2).$$

Again if stability holds for the two-dimensional system studied in section 5.3 (that is if at least $\sigma' < 1$), then two of the eigeinvalues have magitude smaller than one. Then, the three-dimensional system is stable if and only if $\lambda_3 = 1 + e^T(1) - e^G(1)$, has magnitude smaller than one. However, within our set up, $e^T(1) > 0$ while $e^G(1) = 0$, so that we always have $\lambda_3 > 1$ and the system consisting of equations (16), (17) and (18) is never stable around $(\bar{P}(1), \bar{k}(1), 1)$.

5.5.2 Stability of the steady state with endogenous population changes

When linearizing the system at $\left(\bar{P}(\bar{q}), \bar{k}(\bar{q}), \bar{q}\right)$, we obtain the following Jacobian matrix

$$\begin{pmatrix} \frac{-v(b\sigma' + \bar{h}(\bar{q}) - 1)}{(1-\bar{h}(\bar{q}))(1-\sigma')} & -\frac{(1-b-\bar{h}(\bar{q}))}{\gamma(1-\sigma')} & \frac{\gamma(h^G - h^P)}{(1-\sigma')}\bar{P} \\ \frac{-\sigma'\gamma v(b\sigma' + \bar{h}(\bar{q}) - 1)}{(1-\bar{h}(\bar{q}))(1-\sigma')} & \frac{(b\sigma' + \bar{h}(\bar{q}) - \sigma')}{(1-\sigma')} & \frac{(h^G - h^P)}{(1-\sigma')}\bar{P} \\ \frac{-G(\bar{q},\bar{P})\sigma'\gamma v(b\sigma' + \bar{h}(\bar{q}) - 1)}{(1-\bar{h}(\bar{q}))(1-\sigma')} & \frac{G(\bar{q},\bar{P})(b\sigma' + \bar{h}(\bar{q}) - \sigma')}{(1-\sigma')} & 1 + \frac{G(\bar{q},\bar{P})(h^G - h^T)\bar{P}}{(1-\sigma')} + \bar{q}(1-\bar{q})\beta C(-\Delta V^T - \Delta V^G) \end{pmatrix},$$

where

$$G(\bar{q}, \bar{P}) = \bar{q}(1-\bar{q})\beta C\left((1-\bar{q})\frac{d\Delta V^G}{dH^T} - \bar{q}\frac{d\Delta V^T}{dH^T}\right).$$

The expressions of the eigenvalues are far more complicated. Our qualitative study is not sufficient to determine whether this steady state may be stable. Hence we rely on a parametrized example. We take into account the various parameter restrictions (including the necessary condition for stability for the two-dimensional system), then use what we view as plausible values and trace the equilibrium trajectories of each state variables for different vectors of initial conditions. Parameter values are $h^G = 0,3$, $h^T = 0,8$, $\beta = 8,5$, $\gamma = 4,5$, $v = 0,3$, $b = 0,35$, $\sigma' = 0,65$, $C = 2$, $\theta = 0.4$ and we set $\Delta V^G(H^G) = 1 - e^{-H^G}$ and $\Delta V^T(H^T) = e^{-H^T}$.

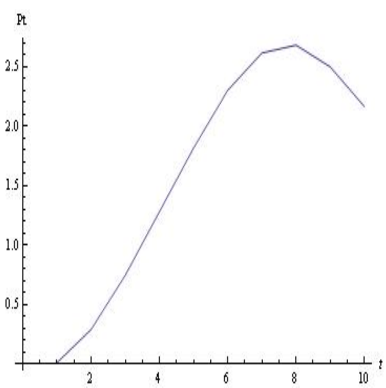

Figure 3: P_t for $P_0 = 0.01$

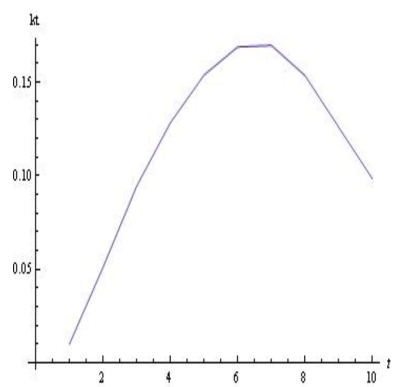

Figure 4: k_t for $k_0 = 0.01$

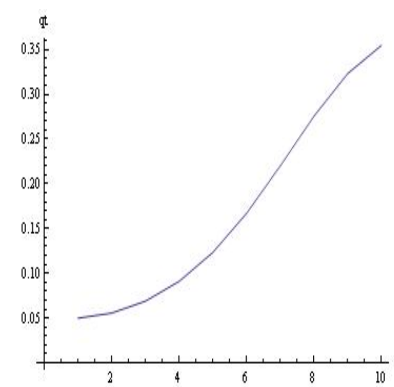

Figure 5: q_t for $P_0 = 0.05$

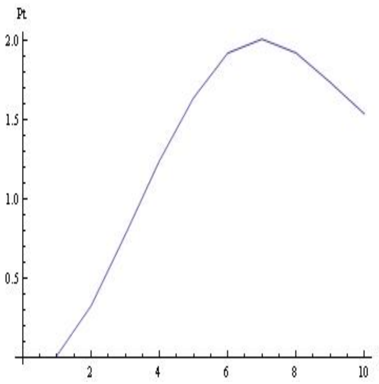

Figure 6: P_t for $P_0 = 0.015$

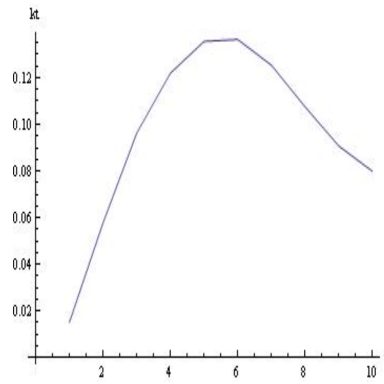

Figure 7: k_t for $k_0 = 0.015$

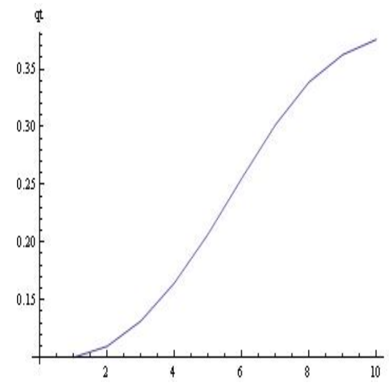

Figure 8: q_t for $q_0 = 0.1$

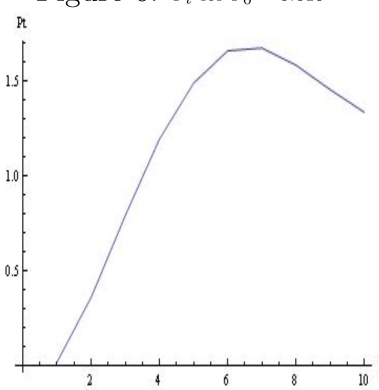

Figure 9: P_t for $P_0 = 0.02$

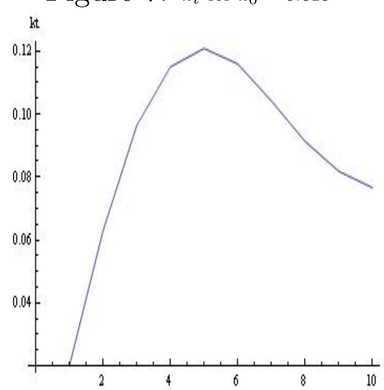

Figure 10: k_t for $k_0 = 0.02$

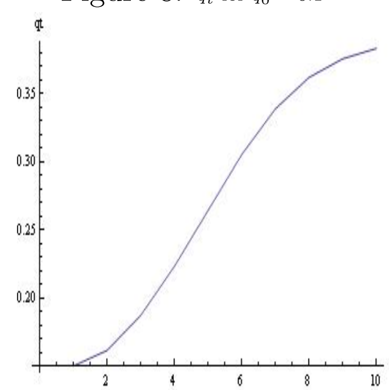

Figure 11: q_t for $q_0 = 0.15$

Those simulations show that pollution and capital first increase and then decrease to reach a stationary state. The share of people concerned with environmental issues monotically increases until its steady state value. This numerical example confirms that the steady state characterized by a heterogenous population can be stable.

5.6 Unlikelihood of a steady state : a numerical example

To check our conclusions regarding the existence of a steady state when A_2 holds, we draw the functions Φ_1 and Ψ on $[0, \tilde{q}[$.
We include the various parameter restrictions and choose what we view as plausible values: $h^G = 0,3$, $h^T = 0,8$, $\beta = 8$, $\gamma = 2$, $v = 0,3$, $b = 0,35$, $\sigma' = 1,5$, $C = 2$, $\theta = 0,4$.
In order to show that a steady state is unlikely we use different functional forms for the functions ΔV^G and ΔV^T (for which the few hypothesis of section 3 hold). Actually, we made no assumption on the marginal behavior of the two functions of interest. However, given that the likelihood of a steady state is linked to the growth rate of Φ_1 to have stronger conclusions it is interesting to test the existence for concave as well as for convex forms. Nevertheless addi-

tional restrictions are needed. As highlighted in footnote, $\Delta V^G(H^G)$ must have a finite limit when H^G tends to infinity, because e^G, as a probability, must be bounded by one. Therefore, since increasing and continuous, $\Delta V^G(H^G)$ cannot be convex on $\mathbb{R}_+$. It can be on some interval, but it necessarily exists some $\tilde{H^G}$ such that for all $H^G \geq \tilde{H^G}$, $\Delta V^G(H^G)$ is concave. Moroever, since ΔV^T is continuous, decreasing and such that $\lim_{H^T \to \infty} \Delta V^T(H^T) = 0$, if concave, it cannot be concave on its entire defintion set. Namely, it necessarily exists some $\tilde{H^T}$ such that for all $H^T \geq \tilde{H^T}$, $\Delta V^T(H^T)$ is convex.
To be general, we include cases for which the curvature of the ΔV^i changes. Hence, on the one hand, we consider functional forms such that $\Delta V^G(H^G)$ be concave on $\mathbb{R}_+$ and $\Delta V^T(H^T)$ be convex on $\mathbb{R}_+$. On the other hand, we consider the case for which $\Delta V^G(H^G)$ is convex on $[0, \tilde{H^G}]$ and $\Delta V^T(H^T)$ is concave on $[0, \tilde{H^T}]$, where, for $i \in \{G, T\}$, $\tilde{H^i}$ is defined such that ΔV^i has an inflexion point in $\tilde{H^i}$.
Moreover, we must take into account the fact that the two functions have to be bounded. We have assumed, $\Delta V^G(H^G) \in [0, \delta_G]$. In addition, we must have $\Delta V^G(H^G) \in [0, \delta_T]$. Namely, $\Delta V^T(0)$ must be finite since e^T has to be always lower than one.

For the case with a change in the curvature, we choose the following functional forms,

$$\Delta V^G(H^G) = a - ae^{\frac{-H^G}{b}} \qquad \text{and} \qquad \Delta V^T(H^T) = ae^{\frac{-H^T}{b}}, \tag{33}$$

and, for the case with an inflexion point we select,

$$\Delta V^G(H^G) = a - ae^{\frac{-{H^G}^2}{b}} \qquad \text{and} \qquad \Delta V^T(H^T) = ae^{\frac{-{H^T}^2}{b}}. \tag{34}$$

The value set is $[0, a[$ for ΔV^G, $]0, a]$ for ΔV^T, so that C can always be chosen such that e^G and e^T are lower than one. We will not try several values of a because it will not impact on the function Φ_1 (since it simplifies in the ratio $\frac{\Delta V^G}{\Delta V^T}$). However, the parameter b affects Φ_1. More precisely, the lower b, the lower $|\frac{d\Delta V^i}{dH^i}|$. Hence, by varying b, we are able to assess how important is the magnitude of $\frac{d\Delta V^i}{dH^i}$ (that is of Φ_1') for the existence of a steady state. In other words, we can check the importance of the impact of pollution on the evolution of environmental concern.

Figures 18, 19, 20 are the graphs for the funtion $\Phi_1^b(q) = 1 + \frac{1 - e^{(-P(q)(1-h^G)\frac{1}{b})}}{e^{(-P(q)(1-h^T)\frac{1}{b})}}$ where b takes three different values. In figures 24, 25, 26 we draw the graphs which represent the function $\Phi_1^b(q) = 1 + \frac{1 - e^{((-P(q)(1-h^T))^2\frac{1}{b})}}{e^{((-P(q)(1-h^P))^2\frac{1}{b})}}$ for the same three different values of b.

A steady state exists only in Figure 14. An intersection point exits only when b is very small ($b = \frac{1}{50}$ for the first kind of functions and less than $\frac{1}{50}$ for the other kind). Hence the derivative of Φ_1 has to be very small for a steady state to exist.

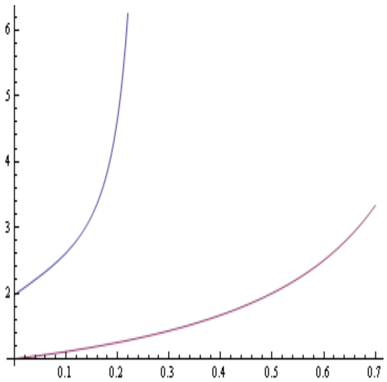

Figure 12: $b = 1$

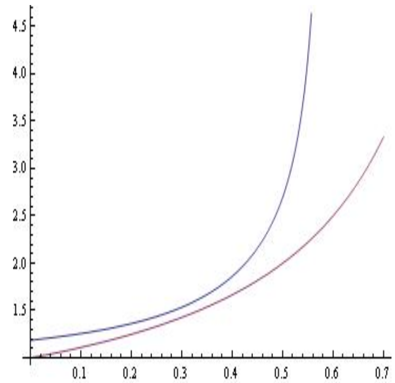

Figure 13: $b = 10$

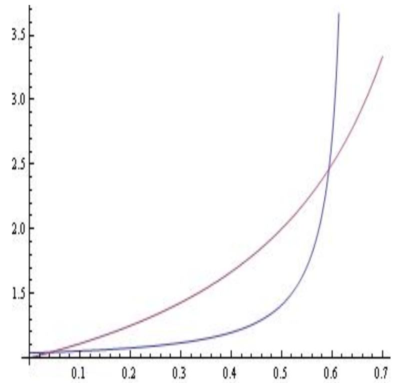

Figure 14: $b = 50$

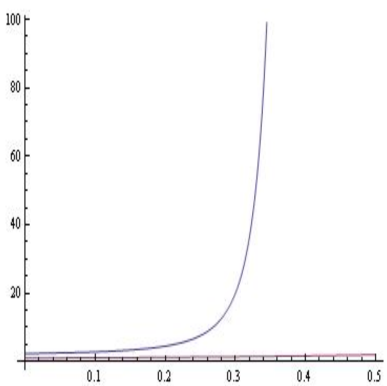

Figure 15: $b = 1$

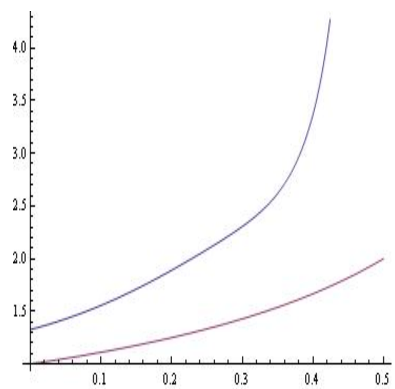

Figure 16: $b = 10$

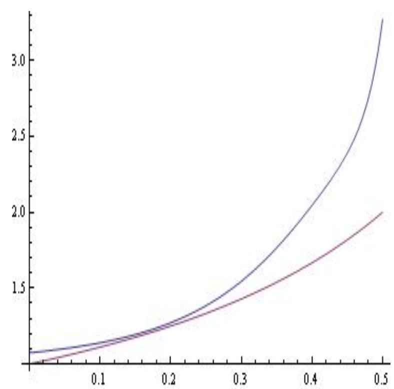

Figure 17: $b = 50$

5.7 Equilibrium trajectories under A_2

When the technology is dirty, the system does not converge toward a stationary state. In this case, we would like to know the path followed by state variables at equilibrium. Given our results in Section 2 and the direction of the change in the composition of the population (the spread of environmental concern), we can have the intuition that states variable will increase so that pollution will rise unboudedly. This numerical example confirms the intuititon. We use the same parameter values as in Section 5.6. Furthermore, we set $\Delta V^G(H^G) = 1 - e^{-H^G}$ and $\Delta V^T(H^T) = e^{-H^T}$ (which meet assumption (1)). The graphs below represent the equilibrium trajectories of each state variable for ten periods and to be more general we show paths for three vectors of initial conditions.

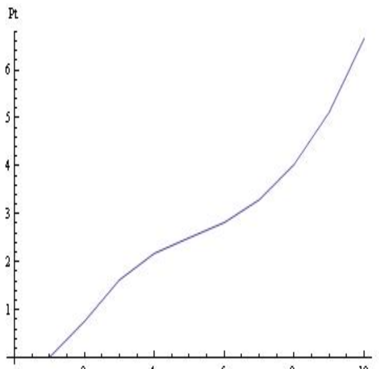

Figure 18: P_t for $P_0 = 0.01$

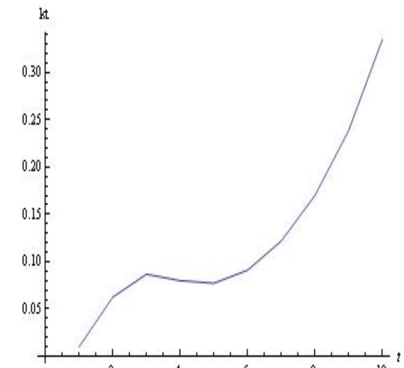

Figure 19: k_t for $k_0 = 0.01$

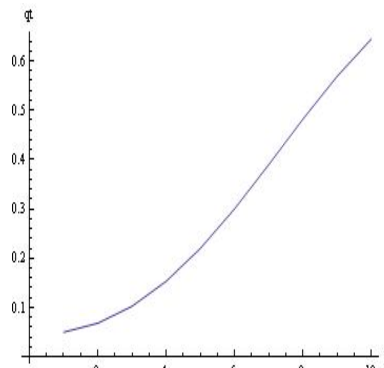

Figure 20: q_t for $P_0 = 0.05$

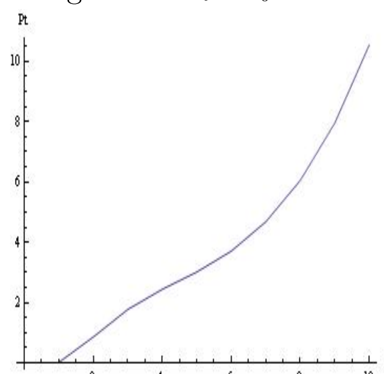

Figure 21: P_t for $P_0 = 0.015$

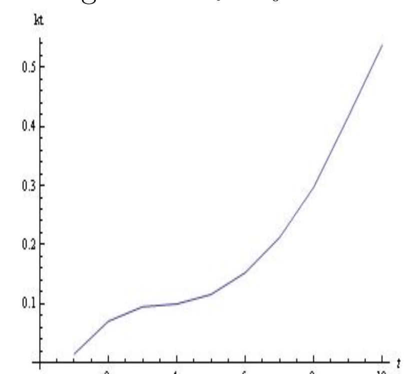

Figure 22: k_t for $k_0 = 0.015$

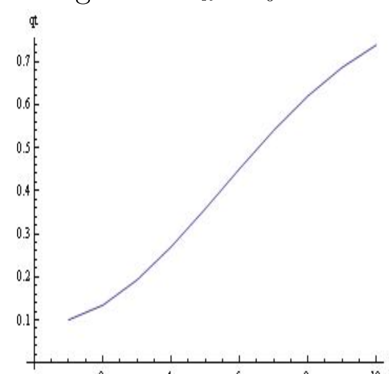

Figure 23: q_t for $q_0 = 0.1$

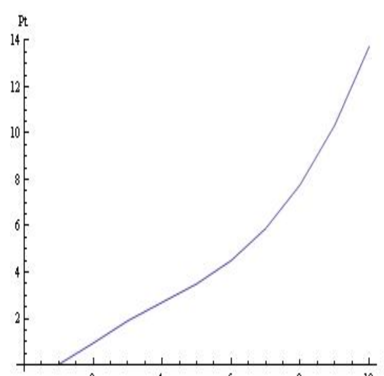

Figure 24: P_t for $P_0 = 0.02$

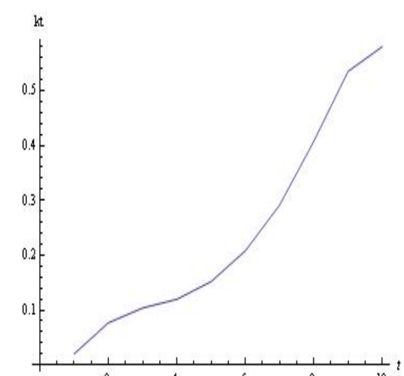

Figure 25: k_t for $k_0 = 0.02$

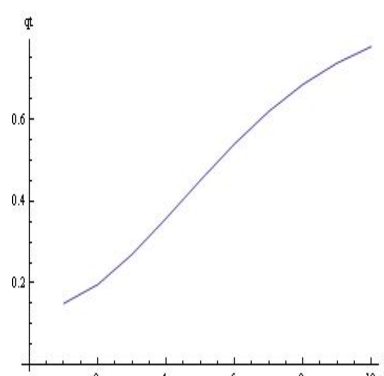

Figure 26: q_t for $q_0 = 0.15$

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