Targeting in social networks with anonymized information

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Targeting in networks

- Seeding in networks:
  - Finding the agents most likely to disseminate information, talk about new products
  - Kempe Kleinberg Tardos (2003) : literature in CS on complexity and approximation algorithms
- Targeting in networks:
  - Price discriminate in the presence of consumption externalities: Bimpikis, Candogan, Ozdaglar (2012), Bloch Quérou (2013)
  - Subsidize effort in teams with synergies: Winter (2004), Sakovics Steiner (2012), Galeotti, Golub, Goyal (2019), Sun Zhao Zhou (2019)
  - Inject funds to banks in a financial network: Demange (2016)

Knowing the network

- **The literature typically assumes that agents (and the planner) know the entire network.**
  - May be a good assumption for small networks (i.e. banks)
  - But not for a discriminating monopolist, or for a firm seeding the market.
- Maybe it does not matter? (Akbarpour, Malladi Saberi (2018)).
- But when it does matter, how important is the fact that agents (and/or planner) do not know the network?

The Question

- A social planner wants to *target* agents in a social network
  - to subsidize effort levels
  - to price discriminate among nodes
- But the planner does not have access to all network data but only *anonymized data*. The names of the agents in the nodes are withheld (for privacy or security reasons)
- For example, the planner has access to a geographical map, organizational chart, network of aliases..
- How does this restriction affect the social planner's targeting ability?

Utilities

- Let **G** denote the social network.
- We let $g_{ij} \in \{0,1\}$ denote whether there is a link between agents i and j.
- Agents have linear-quadratic utilities:

$$U_i = a_i x_i - \frac{b_i}{2} x_i^2 + \alpha \sum_j g_{ij} x_i x_j .$$

- This corresponds to *positive externalities* generated by neighbors.
- For example, agents have *local consumption externalities* or *synergies in bilateral effort*.

The planner's objective

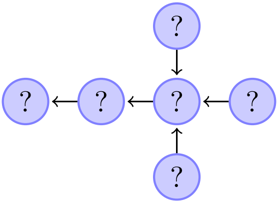
- The objective of the planner is to maximize the sum of utilities of agents.
  - If the planner is a discriminating monopolist, she chooses to maximize total surplus, and then use personalized prices to extract all the surplus
  - If the planner's objective is to maximize the production of teams, she chooses to subsidize agents' efforts in order to reach the highest output value.

The first-best contract

- In the first-best, the planner chooses to discriminate among agents:
$$\mathbf{x}^* = (\mathbf{I} - \alpha(\mathbf{G} + \mathbf{G}^T))^{-1} a\mathbf{1}.$$
- The Katx-Bonacich centrality of agent i in a network $\mathbf{G}$ is given by:
$$b_i(\mathbf{G}) = \sum_{k=0}^{\infty} \delta^k \sum_j m_{ij}^k,$$
where m_{ij}^k (the $i-j$ cell in the matrix $\mathbf{G}^k$) denotes the number of paths of length k between i and j.
- Quantities chosen in the first-best are proportional to the Katz-Bonacich centrality of agent i in the undirected weighted network $\mathbf{G} + \mathbf{G}^T$

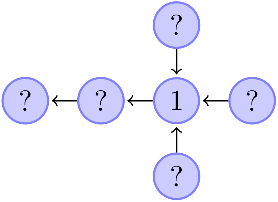
Information of the planner

- The planner only sees the network *architecture or topology*



Information of the agents

- Each agent knows at least the network topology and his location in the network



Mechanisms

- In general, the planner can select a mechanism asking agents to reveal all their information
- Here we focus on *simple mechanism*
- The planner offers a menu of contracts $\{x_l\}$, corresponding to the decisions at each and every of the locations.
- Every agent then selects a contract among the menu $\{x_l\}$.
- If all agents select different contracts, each agent obtains the contract he selected. Otherwise, if several agents select the same location, the planner punishes all agents and chooses an outcome $\mathbf{x} = \mathbf{0}$.

Incentive compatible contracts

Definition

A contract $\mathbf{x} = (x_1, ..., x_n)$ is *ex post incentive compatible* at a network architecture $\hat{g}$ if every agent i chooses the contract corresponding to his location, i.e.

$$U_i(x(\ell_g) \geq U_i(x(k(i), \ell_{-i}(g))\forall k(i) \neq \ell_i(j).$$

Proposition

Any contract $\mathbf{x}$ *where agents obtain positive payoffs is ex post incentive compatible at any network architecture* $\hat{g}$*.*

- Obvious result, due to the fact that it is always an equilibrium for all agents to announce their true location (but many other equilibria exist as well..)

Adjacent agents

Definition

A coalition S contains adjacent players if and only if, for all $i, j \in S$, $g_{ij} + g_{ji} \geq 1$.

- Players can only engineer deviations if they can communicate
- In order to communicate, agents must be connected (at least in one direction)

Group incentive compatible contracts

Definition

The mechanism is group incentive compatible then there does not exist a coalition S of adjacent players and a mapping $k(j)$ from S to N such that, for every agent i in S,

$$U_i(x(k(j)_{j \in S}, \ell_j(g)_{j \notin S}) \geq U_i(x_{\ell_g(i)})).$$

Group incentive compatible contract

Proposition

The optimal contract $\mathbf{g}^*$ *is group incentive compatible for any network architecture* $\hat{g}$ *if the size of the group is* $2, 3or4$.

- For deviating pairs: one of the agents has to exchange for a quantity which is higher than the quantity at his true location.
- This is costly because first-best contract levels are already too high for the agents.
- The only possible compensation is if the sum fo externalities due to the permutation of agents in S increases
- But this cannot be the case if $|S| = 2$.

What happens with groups of size ≥ 4 remains an open problem..

Group incentive compatible contracts with transfers

Definition

The mechanism is group incentive compatible with transfers then there does not exist a coalition S of adjacent players and a mapping $k(j)$ from S to N such that,

$$\sum_{i \in S} U_i(x(k(j)_{j \in S}, \ell_j(g)_{j \notin S}) > \sum_{i \in S} U_i(x_{\ell_g(i)}).$$

Group incentive compatible contracts with transfers

With linear quadratic utilities, this definition reduces to:

$$\sum_{i \in S} \sum_{j \notin S} g_{ij} x_{\rho_S(i)} x_j \leq \sum_{i \in S} \sum_{j \notin S} g_{ij} x_i x_j \forall S, \forall \rho_S, \tag{1}$$

- Because the optimal contract $\mathbf{x}^*$ maximizes the sum of welfare, the sum of welfare of a subcoalition S only increases if the sum of welfare of players outside the coalition (in $N \setminus S$) decreases.
- Deviating players take advantage of the loss of externalities of players outside the coalition

Undirected networks

- Suppose that $g_i j = 1 \Leftrightarrow g_{ji} = 1$

Proposition

If the network g is undirected, then the optimal contract $\mathbf{x}^*$ is group incentive compatible with transfers.

Undirected networks

- Why is the optimal contract immune to group deviations in undirected networks?
- Consider pairs. A contract is immune to deviations if and only if
$$(x_i - x_j)(\sum_{k \neq j} g_{ik} x_k - \sum_{k \neq i} g_{jk} x_k) \geq 0.$$
- But because the Katz-Bonacich measure is defined recursively, $x_i^* \geq x_j^*$ if and only if $\sum_{k \neq j} g_{ik} x_k^* \geq \sum_{k \neq i} g_{jk} x_k^*$.
- Intuitively: you cannot exploit externalities on other players because externalities are always reciprocal.

Hierarchical networks

- Suppose that agents can be partitioned into a hierarchy $A_1, .., A_K$ such that $g_{ij} = 1$ only if $j \in A_k, i \in A_l$, $l > k$

Proposition

If the network g is a hierarchical network, then the optimal contract $\mathbf{x}^*$ is group incentive compatible.

Single root networks

Proposition

Suppose that the hierarchy has a single root, $|A_1| = 1$ and that all agents are connected to the root and that the hierarchy contains more than two tiers. Then agents at tiers $m = 3, .., M$ must receive the same quantity as the root agent in the optimal contract satisfying group incentive compatibility with transfers.

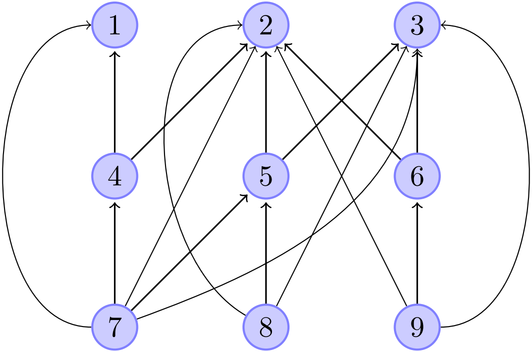
Single root networks

- Why is the optimal contract uniform (except for tier 2) in single root networks?
- The root has does not receive any externality.
- Players at tiers $m = 3, .., M$ receive externality so are willing to exchange their contracts with the contract of the root if the root has a higher level.
- Because the planner wants to maximize the level of the root, she must offer the same contract to the root and all players at tiers $m > 2$.

Nested neighborhoods

- For any three tiers m, p, q with $m < p < q$, we have for all $i, \in A_q$, for all $j \in A_p$ such that $g_{ij} = 1$,

$$\{k|g_{ik} = 1\} \cup A_m = \{k|g_{jk} = 1\} \cup A_m.$$



Nested neighborhoods

Proposition

Consider a hierarchy with nested neighborhoods such that all agents are connected to more than one root, and $|A_m| \leq |A_{m+1}|$ *for all* $m = 1, .., M-1$*. Then all agents receive the same quantity in the optimal contract satisfying group incentive compatibility with transfers.*

Nested neighborhoods

- Why is the optimal contract uniform in nested neigborhood networks?
- The root has does not receive any externality.
- Players at tiers $m = 2, .., M$ receive externality so are willing to exchange their contracts with the contract of the root if the root has a higher level.
- Because the planner wants to maximize the level of the root, she must offer the same contract to the root and all players.

Conclusions

Situations where the planner does as well with anonymized data as with full data:

- Agents can only deviate unilaterally
- Groups of deviating agents cannot make side-payments
- Rich message space and information structure: agents report their neighbors
- Influence is reciprocal
- Regular oriented trees

Conclusions

Situations where the planner does worse with anonymized than with full data:

- Single-root networks
- Nested neighborhoods networks
- Group deviations by nonadjacent agents

Conclusions

Future work:

- Alternative models of partial information of the planner (and of the agents) on the structure of the network
- When is correlation sufficient to support the first-best?
- What if the planner is restricted to using simple mechanisms?